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Validation of an elastoplastic model to predict secant shear modulus of natural soils by experimental results

J.A. Santos & A. Gomes Correia
Technical University of Lisbon, Portugal

A. Modaressi & F. Lopez-Caballero
Ecole Centrale Paris, France

R. Carrilho Gomes
Barreiro School of Technology, Politechnical Institute of Setúbal, Portugal

ABSTRACT: The main purpose of this paper is to investigate the parameters of an elastoplastic constitutive law in modelling the non-linear soil stiffness from very small strains to pre-failure conditions. A simple approach is presented to derive model parameters related with shear hardening.

1 INTRODUCTION

The purpose of this paper is to validate an elastoplastic constitutive law in modelling the non-linear soil stiffness. Some of the important factors that affect soil behaviour such as the strain level and the stress conditions are taken into account in the formulation of the model (Hujeux 1985). The influence of other factors which control the stiffness degradation such as the plasticity index and the initial state (OCR, void ratio, stress state, etc.) are considered via the model parameters. The ability of the model to simulate the cyclic behaviour of sandy and remoulded clayey samples has already been explored and verified, and a methodology to identify some of the model parameters developed (Modaressi & Lopez-Caballero 2001). The aim of this paper is to improve the existing procedures and to investigate parameters that control the secant shear modulus giving special attention to natural soils.

Cyclic behaviour of different kind of soils is compared based on available laboratory data including some experimental results obtained by the authors (resonant column and cyclic torsional shear tests) and others found in the literature. The study is based on a key parameter defined by the authors and called reference "threshold" shear strain γ_r^t or $\gamma_{0.7}$ (Santos & Gomes Correia 2000, Gomes Correia et al. 2001, Santos & Gomes Correia 2001). This parameter is defined as the shear strain for a stiffness degradation factor of $G/G_0 = 0.7$ in which G_0 is the initial shear modulus for very small strain ($\gamma \approx 10^{-6}$) and G is the secant modulus of soil.

Based on this key parameter almost a unique strain-dependent shear modulus degradation curve can be defined using a normalised shear strain given by: $\gamma^* = \gamma/\gamma_{0.7}$ (Santos 1999).

The comparison of the model's response with experimental measurements made in this work contributes to the validation of the model and the generalization of its parameter identification methodology to natural soils. Only the non-linear soil stiffness is analysed in this work. The energy dissipation or damping generated during cyclic loading is not issued in this paper.

2 UNIQUE STRAIN-DEPENDENT SHEAR MODULUS CURVE FOR SOIL

It is well known that the strain-dependent curve G/G_0 depends mainly on soil plasticity in cohesive soils (Vucetic & Dobry 1991) and is affected by the mean effective stress in cohesionless soils (Ishibashi & Zhang 1993). But it was found that all the influences of these factors can be considered in a simple form when using the normalised shear strain γ^* defined previously.

The meaning of the key parameter $\gamma_{0.7}$ which is a reference threshold shear strain is closely related to the concept of volumetric threshold shear strain γ_v^t (Vucetic 1994) which represents the limit beyond which the soil structure starts to change irreversibly: in drained conditions permanent volume change will take place, whereas in undrained conditions pore water pressure will build up.

In a practical point of view, the reference threshold shear strain defines the beginning of significant stiffness degradation.

These evidences suggest the idea to perform the normalisation using the reference threshold shear strain and it was shown that it is possible to define almost a unique strain-dependent stiffness degradation curve for sands and clays. Figure 1 shows how the results of Vucetic & Dobry (1991), and Ishibashi & Zhang (1993), can be fitted inside two simple boundary curves, for soils with different plasticity index ($PI = NP$ to 50%) and subjected to confining pressure varying between 1 to 400kPa.

Santos (1999) proposed two equations to define the lower and upper bound values of G/G_0 as a function of γ^* (for $10^{-6} \leq \gamma \leq 10^{-2}$):

$$\text{lower bound}\begin{cases} 1 & \gamma^* \leq 10^{-2} \\ \dfrac{1 - tanh\left[0.48\, ln(\gamma^* / 1.9)\right]}{2} & \gamma^* > 10^{-2} \end{cases} \qquad (1)$$

$$\text{upper bound}\begin{cases} 1 & \gamma^* \leq 10^{-1} \\ \dfrac{1 - tanh\left[0.46\, ln(\dfrac{\gamma^* - 0.1}{3.4})\right]}{2} & \gamma^* > 10^{-1} \end{cases} \qquad (2)$$

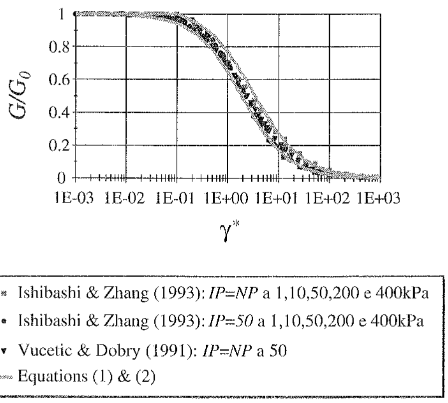

Figure 1. Stiffness degradation curves in γ^* scale.

A hyperbolic function can also be used to fit test results in an easier way (Teachavorasinskun et al. 1991, Gomes Correia et al. 2001). Simple regression analysis shows that the previous boundary curves can be fitted by a mean curve defined by the following relationship:

$$G/G_0 = 1 / [\, 1 + a \times (\gamma / \gamma_{0.7})] \qquad (3)$$

Based on the least squares method the best fitting was obtained with a = 0.385.

Figure 2 presented by Gomes Correia et al. (2001) shows an attempt to generalize the proposed methodology to Brazilian tropical soils: lateritic and saprolitic soils. Despite the different isotropic consolidated stresses, degree of saturation and overconsolidated ratios of more than 60 resonant column tests, all results are in good agreement after normalisation.

Based on the proposed normalisation only two parameters are needed to characterize the non-linear secant stiffness of soil:

1. the initial modulus G_0 which defines the rigidity of soil at very small strains;
2. the reference "threshold" strain $\gamma_{0.7}$ which characterizes the degree of non-linearity for medium strain level.

The shape of the secant stiffness curve can be well described by a hyperbolic function like equation (3).

The expertise gained from these experimental studies is applied in this work to improve the existing procedures and to show the potentiality of the elastoplastic constitutive law.

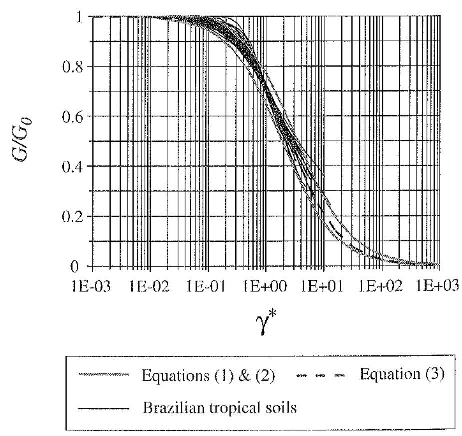

Figure 2. Stiffness degradation curves in γ^* scale. Brazilian tropical soils.

3 CONSTITUTIVE MODEL

To model soil behaviour, the elastoplastic multi-mechanism model developed by Aubry et al. (1982) and Hujeux (1985) is used in this work. The model is defined in terms of effective stresses and is based on the representation of four coupled elementary plastic mechanisms: three deviatoric plastic deformation mechanisms in three orthogonal planes ($k = 1,2 \ldots 3$) and one isotropic mechanism ($k = 4$). The model bases on a Coulomb type failure criterion and follows the critical state concept. The evolution of hardening is related with the plastic strain associated with each of the mechanisms k ($k = 1,2 \ldots 4$).

Adopting the soil mechanics sign convention (positive for compression) the deviatoric primary yield surface of the k mechanism is given by:

$$f_k(p'_k, \varepsilon_v^p, r_k) = q_k - sin\varphi'_{pp} \cdot p'_k \cdot F_k \cdot r_k \tag{4}$$

with

$$F_k = 1 - b\, ln\left(\frac{p'_k}{p_c}\right) \tag{5}$$

$$q_k = \left[\frac{1}{4}\left(\sigma'_{ii} - \sigma'_{jj}\right)^2 + \sigma'^2_{ij}\right]^{1/2} \tag{6}$$

$$p'_k = \frac{1}{2}\left(\sigma'_{ii} + \sigma'_{jj}\right) \tag{7}$$

$$p_c = p_{co}\, exp(\beta \varepsilon_v^p) \tag{8}$$

φ'_{pp} is the critical state friction angle; β is the plasticity compression modulus and p_{co} represents the critical state stress for the initial void ratio.

The b parameter controls the shape of the yield surface and varies from $b = 0$ to 1 passing from Coulomb type surface to a Cam-Clay type one.

The F_k function defines the influence of the void ratio and/or consolidation ratio on the hardening through the plastic volumetric strain ε_v^p.

The internal variable r_k defines the degree of the mobilised friction associated with each k deviatoric mechanisms. The degree of the mobilised friction is related with the plastic shear strain by the following incremental relationship:

$$dr_k = \frac{d\lambda(1 - r_k)^2}{a_k(r_k)} \tag{9}$$

in which $d\lambda$ is the plastic multiplier obtained from the consistency condition ($df_k = 0$). When no plastic volumetric strain takes place $d\lambda = d\gamma^p$.

The soil behaviour is classified into four domains: elastic, pseudo-elastic, hysteretic and mobilized domains according to r_k values:

- *elastic domain* when $r_k = r^{ela}$
- *pseudo-elastic domain* when $r^{ela} \leq r_k \leq r^{hys}$
- *hysteretic domain* when $r^{hys} \leq r_k \leq \leq r^{mob}$
- *mobilised domain* when $r^{mob} \leq r_k \leq 1$

In the elastic domain $r_k = r^{ela}$ the soil behaviour is described by the elastic properties which are defined as a function of the mean effective stress.

The dilatancy behaviour of soil is described by Roscoe's rule that requires two more parameters: ψ – dilatancy angle of the characteristic state line and α_ψ – scalar representing the amplitude of dilatancy.

4 MODEL PARAMETERS FOR SANDS

The model parameters can be classified in two categories: (i) parameters that can be measured by laboratory or field tests; (ii) parameters that cannot be directly measured (Modaressi & Lopez-Caballero 2001).

This paper will focus on the second set of parameters related with the shear hardening. In numerical simulations the simple shear loading is assumed.

Elastic domain: in this domain the soil behaviour is elastic; the stiffness which depends on the void ratio and the mean effective stress is constant and equal to G_0. This behaviour remains until the shear stress $\tau \leq p' sin\varphi' F$. For sands b is small ($b = 0.2$) and F depends on the initial state. The r^{ela} can thus be determined using the following relationship:

$$r^{ela} = \frac{\tau}{\tau_{pp}} = \frac{G_0 \gamma_t^e}{p' sin\varphi' F} \tag{10}$$

γ_t^e is the elastic threshold shear strain.

Hysteretic and Mobilised domains: outside the elastic domain the degree of mobilised friction is related with the plastic strain by equation (9). The a_k parameter which characterizes the evolution of the hardening is numerical and can be determined in order to obtain the best fitting of the experimental G-γ and ξ-γ curves (Modaressi & Lopez-Caballero 2001).

To simplify the matching and to avoid extensive calculations a new approach is proposed for sands under drained conditions. The basic idea is to define a "standard" shape for the $a_k = f(r_k)$ curve. Afterwards, the curve is affected by a matching factor according to the experimental data.

For this work the following a_k curve was adopted:

- $a = 0$ when $r_k = r^{ela}$
- a varying linearly with r when $r^{ela} \leq r_k \leq r^{mob}$
- a remains constant when $r^{mob} \leq r_k \leq 1$

The matching factor can be determined by means of a single point in the G-γ curve. The authors suggest to use the $\gamma_{0.7}$ to perform the matching. For this strain level:

$$\gamma^p_{0.7} = \gamma_{0.7} - \gamma^e_{0.7} \approx \tau/G - \tau/G_0 = 0.3\gamma_{0.7} \tag{11}$$

The degree of the mobilised friction is equal to:

$$r_{0.7} = \frac{\tau_{0.7}}{\tau_{pp}} = \frac{0.7\,G_0\gamma_{0.7}}{p'sin\varphi'F} \tag{12}$$

The integration of equation (9) with $d\lambda = d\gamma^p$ provides the relationship to determined a_k values:

$$a(r^*) = \frac{\gamma^p}{\frac{1}{1-r^*} + \frac{1}{r^*}ln(1-r^*)} \quad \text{with } r^* = r - r^{ela} \tag{13}$$

The values of $\gamma^p_{0.7}$ and obtained from equations (11) and (12) are used to computed $a_{0.7}$ from equation (13). The a_k curve is completely defined by the shape and the $a_{0.7}$ value.

5 CASE STUDY

To illustrate the parameters identification strategy the experimental data obtained from resonant column (RC) and cyclic torsional shear (CTS) tests on Toyoura sand (Santos 1999) is used to calibrate the model. The tests conditions are summarised in Table 1.

Other parameters were estimated from available experimental data (Ishihara 1993): $\varphi'_{pp} = 31°$; $\psi = 31°$; $\beta = 43$ and $\alpha_\psi = 1$.

According to the proposed approach the following parameters were computed:

$r^{ela} = 1.44 \times 10^{-3}$ by equation (10) with $\gamma^e_t = 10^{-6}$

$r_{0.7} = 0.253$ by equation (12) with $\gamma_{0.7} = 2.5 \times 10^{-4}$

$a_{0.7} = 4.08 = 10^{-4}$ by equation (13)

Figure 3 shows the a_k curve obtained from the "standard" shape multiplied by a factor to match the value at point ($r_{0.7}$;$a_{0.7}$). For this particular case the point reached the asymptotic value at $r^{mob} = 0.8$.

Once the a_k curve is defined the numerical simulation was performed considering cyclic simple shear loading. Figure 4 shows the very good agreement between the numerical and the experimental results. This figure shows also how the results can be fitted inside the two boundary curves described by equations (1) and (2).

Table 1. Parameters for Toyoura sand.

Drained tests	e_o	G_0 (MPa)	p' (kPa)	p_{co} (MPa)
RC + CTS	0.74	100.4	80	2.5

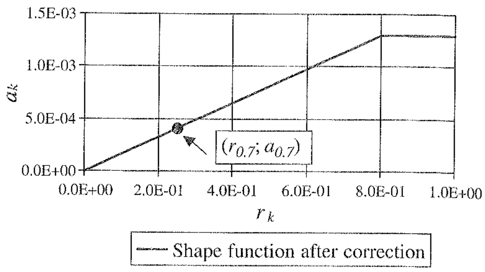

Figure 3. a_k curve after matching.

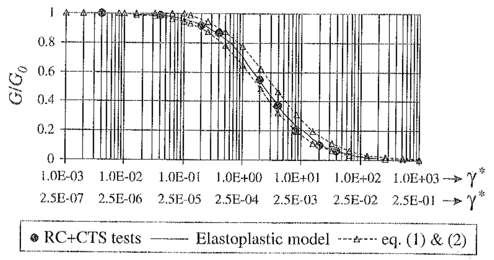

Figure 4. Experimental versus numerical results.

6 CONCLUSIONS

These preliminary results point out the potentiality of the model to fit the non-linear stiffness of soil.

For sands under drained conditions simple relationships were presented to derive the a_k curve which controls the soil stiffness degradation. The proposed approach seems to be reliable and easy-to-use to derive model parameters although it must be tested for more cases.

It is important to emphasize that only the non-linear soil-stiffness was studied in this work.

For cyclic loading the soil damping must be analysed. It is known that the hyperbolic law – equations (9) and (13) – overestimates the damping for large deformations.

This problem can be overcome by introducing an additional parameter that changes the scale of the a_k curve during shearing. This parameter can be interpreted as a damage factor induced by the loading. The ongoing research studies show that it is possible to fit the ξ-γ curve as well as the stiffness degradation curve in sands.

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