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Topological Relationships between a Circular Spatially Extended Point and a Line: Spatial Relations and their Conceptual Neighborhoods

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***Abstract*—This paper presents the topological spatial relations that can exist in the geographical space between a Circular Spatially Extended Point and a Line and describes the use of those spatial relations in the identification of the conceptual neighbourhood graphs that state the transitions occurring among relations. The conceptual neighbourhood graphs were identified using the snapshot model and the smooth-transition model. In the snapshot model, the identification of neighbourhood relations is achieved looking at the topological distance existing between pairs of spatial relations. In the smooth-transition model, conceptual neighbours are identified analysing the topological deformations that may change a topological spatial relation. The graphs obtained were analysed as an alternative to map matching techniques in the prediction of the future positions of a mobile user in a road network.**

***Index Terms*—Topological spatial relations, Conceptual neighbourhood graph, Snapshot model, Smooth-transition model, Spatially Extended Point.**

I. Introduction

Spatial relations between geometric objects have been classified into several types [1, 2], including direction relations [3], distance relations [4] and topological relations [5]. Topological relations are those spatial relations preserved under continuous transformations of the space, such as rotation or scaling.

Research on topological spatial relations between different types of objects (points, lines and regions) has been undertaken for many years, identifying the topological spatial relations between them, and demonstrating their geometric realization, proving the existence of such relations. Some of the works undertaken so far include the identification of the topological spatial relations between regions [6], between lines [6], between regions and lines [6, 7], between regions with broad boundaries [8], between a spatially extended point and a region [9], between broad lines [10], and between lines with broad boundaries [10], only to mention a few.

The relevance of identification of such topological spatial relations is associated with the need to conceptualize the spatial relations that can exist among several objects in the geographical space. The obtained models can be used as a computational framework for spatial reasoning. Their implementation in a system, like a Geographical Information System, allows the representation and manipulation of complex objects associated with complex realities.

The work described in this paper is associated with the topological spatial relations existing between a Circular Spatially Extended Point and a Line. A Circular Spatially Extended Point is a region-like object characterized by the inclusion of a point and a region that defines the area of influence of that point. In the scope of this work, the Circular Spatially Extended Point represents a complex object in the sense that the point and its region of influence are not dissociable (Fig. 1).

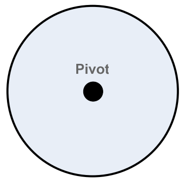

Fig. 1 – A circular spatially extended point

The identification of the topological spatial relations between a Circular Spatially Extended Point and a Line was first addressed by the authors of this paper in [11] to use them in the prediction of mobile users' future positions in a context-aware mobile environment. With the topological spatial relations it is possible to identify the conceptual neighborhood graphs that state the possible transitions between spatial relations and, therefore, the possible movements that a mobile user can do in a road network. The use of a Circular Spatially Extended Point is associated with the need to associate a certain degree of uncertainty to the position of a mobile user. A similar approach was followed by Wuersch and Caduff [12, 13] for pedestrian navigation using the topological spatial relations existing between two Circular Spatially Extended Points, one representing the user's location and the other representing a waypoint that is used to define paths for pedestrians in a pedestrian guiding system.

The abstractions usually used to represent spatial objects, such as single points, single lines and single regions, and also their complex data types, complex points, complex lines and complex regions, and for whom the topological spatial relations existing between them were already identified [14], cannot be used for the representation of the particular

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integration of a point and a region in the same object. This object is here identified as a Circular Spatially Extended Point. For regions with broad boundaries, for example, the two regions that integrate the object "region with broad boundary" are 2-dimensional components, not representing the 0-dimensional part of a Circular Spatially Extended Point (its pivot) [15]. In emerging applications areas, like context-aware mobile environments, location-based services, ubiquitous computing, among others, the position of a mobile user constitutes the key for providing specific context-aware services. However, this position usually integrates a certain degree of uncertainty associated to the sensing technology. Although technologies like the Global Positioning System provide quite accurate estimates, the position provided by other means like cellular networks positioning systems is typically much less precise. Having this limitation and the need to properly deal with it, the use of a Circular Spatially Extended Point allows the representation of such uncertainty and also the definition, in a specific application, of the maximum uncertainty value through the specification of the radius of the Circular Spatially Extended Point.

The need for the identification of the topological spatial relations was motivated by a specific application domain - context-aware mobile environments - presenting this paper an example of how the topological spatial relations existing between a mobile user and a road network can be used to assign the user to a specific road segment. However, this research is of general use since the adopted principles were not adapted or strictly designed to a specific application.

The motivation for this work is the identification of the topological spatial relations between a Circular Spatially Extended Point and a Line, and the need for the corresponding conceptual neighborhood graphs is here expressed. The following sections are dedicated to the synthesis of the conceptual framework adopted for the identification of such spatial relations and to the identification of the corresponding conceptual neighborhood graphs. Section 2 characterizes the objects in analysis in this work and presents the topological spatial relations that can exist between these objects. This is followed by the identification of the conceptual neighborhood graphs using two distinct approaches [7]: the snapshot model (section 3) and the smooth-transition model (section 4). In section 5 the two graphs are compared and the main differences between them are identified and discussed. Section 6 presents an example of the use of the identified spatial relations and conceptual neighborhood graphs, and section 7 concludes summarizing the work undertaken.

II. Topological Spatial Relations between a Circular Spatially Extended Point and a Line

A Circular Spatially Extended Point (CSEP) can be considered as a region-like concept. A CSEP (Fig. 1) has its own interior, boundary and exterior. While it shares the same concepts of interior, boundary and exterior of a region, the CSEP is distinguished from a general region by the identification of a point within the interior called the pivot. The pivot is conceptually similar to a 0-dimension object. A major difference between a usual point and a pivot is that a pivot has an area of influence that defines the boundary of the CSEP [9].

From a geometrical point of view, a simple line, representing a linear curve, has a boundary with two simple points, each of which has no extension [6, 14] (Fig. 2). The definition of a simple line usually refers to a 1-dimensional object of $\Re^2$ with no self-intersections [16]. Closed lines are lines without end-points [16], so they lay out of the definition of simple line and consequently are not considered in the scope of the work presented in this paper.

Fig. 2 – A simple line

The formalism used for the identification of the topological spatial relations between a CSEP and a line is based on the algebraic approach proposed by Egenhofer (the 4- and 9-intersections models) [6]. The topological spatial relations were identified [11], using a 4x3 matrix as proposed in [9]. The conditions that allowed the identification of the spatial relations were revised and their formal proofs were also undertaken, work that is presented in this paper. The 4x3 matrix represents the intersections ($\cap$) between the pivot ($P^{\bullet}$), interior (P°), boundary (∂P) and exterior (P^-) of a CSEP (P) and the interior (L°), boundary (∂L) and exterior (L^-) of a line (L) (Fig. 3).

Fig. 3 – Parts of a CSEP and a line

Each relation (R) between a CSEP (P) and a line (L) is characterized by 12 (4x3) intersections with empty (∅) or non-empty (¬∅) values depending on how the geographical objects are related (Equation 1).

$$R(P,L)=\begin{bmatrix} P^{\bullet}\cap L^{\circ} & P^{\bullet}\cap \partial L & P^{\bullet}\cap L^{-} \\ P^{\circ}\cap L^{\circ} & P^{\circ}\cap \partial L & P^{\circ}\cap L^{-} \\ \partial P\cap L^{\circ} & \partial P\cap \partial L & \partial P\cap L^{-} \\ P^{-}\cap L^{\circ} & P^{-}\cap \partial L & P^{-}\cap L^{-} \end{bmatrix} \quad (1)$$

The several conditions proposed by Egenhofer and Herring [6] for the identification of the topological relations between regions, lines and points in a Geographic Database were analyzed. Following these authors' suggestions, 9 conditions were adopted and adapted to the specific context of this work. These conditions are associated with the definition of the topological spatial relations that can exist between regions, between a region and a line, and between a non-point object (a region or a line) and a point, and are here described as conditions 1 to 9. Additional conditions were defined attending to the particular case of the definition of the topological relations between a CSEP and a line. These conditions are referred as condition 10 to condition 14. The complete set of conditions and their formal proofs are now described.

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Condition 1. The exteriors of the two objects (P and L) intersect with each other (Equation 2).

$$R(P,L) \neq \begin{bmatrix} - & - & - \\ - & - & - \\ - & - & - \\ - & - & \phi \end{bmatrix} \tag{2}$$

Proof: Knowing that $P^{\bullet} \cup P^{\circ} \cup \partial P \cup P^{-} = \Re^2$ and that $L^{\circ} \cup \partial L \cup L^{-} = \Re^2$, the statement $P^{-} \cap L^{-} = \varnothing$ can only be possible either if: i) $P^{\bullet} \cup P^{\circ} \cup \partial P = \Re^2$; ii) $L^{\circ} \cup \partial L = \Re^2$; or iii) $P^{\bullet} \cup P^{\circ} \cup \partial P \cup L^{\circ} \cup \partial L = \Re^2$. However, all these conditions are impossible since the objects P, L and $P \cup L$ are bounded and $\Re^2$ is unbounded.

Condition 2. If P's boundary intersects L's exterior then P's interior must intersect L's exterior as well, and *vice-versa* (Equation 3, where $\vee$ means *or*).

$$R(P,L) \neq \begin{bmatrix} - & - & - \\ - & - & \phi \\ - & - & \neg\phi \\ - & - & - \end{bmatrix} \vee \begin{bmatrix} - & - & - \\ - & - & - \\ - & - & - \\ \phi & \neg\phi & - \end{bmatrix} \tag{3}$$

Proof: Assuming that the constraint rules are false, then $\partial P \cap L^{-} = \neg\varnothing \Rightarrow P^{\circ} \cap L^{-} = \varnothing$ and $\partial L \cap P^{-} = \neg\varnothing \Rightarrow L^{\circ} \cap P^{-} = \varnothing$. Knowing that $L^{\circ} \cup \partial L \cup L^{-} = \Re^2$, this leads to $P^{\circ} \cap (L^{\circ} \cup \partial L \cup L^{-}) = P^{\circ} \cap \Re^2 = \varnothing$, which is a contradiction to the assumed non-emptiness of the interior of a CSEP, here represented by a region, so $P^{\circ} \cap \Re^2 = \neg\varnothing$. For the other rule, and knowing that $P^{\bullet} \cup P^{\circ} \cup \partial P \cup P^{-} = \Re^2$, this leads to $L^{\circ} \cap (P^{\bullet} \cup P^{\circ} \cup \partial P \cup P^{-}) = L^{\circ} \cap \Re^2 = \varnothing$, which is a contradiction to the assumed non-emptiness of the interior of a line, so $L^{\circ} \cap \Re^2 = \neg\varnothing$.

Condition 3. P's boundary intersects with at least one part of L and *vice-versa* (Equation 4).

$$R(P,L) \neq \begin{bmatrix} - & - & - \\ - & - & - \\ \phi & \phi & \phi \\ - & - & - \end{bmatrix} \vee \begin{bmatrix} - & - & - \\ - & \phi & - \\ - & \phi & - \\ - & \phi & - \end{bmatrix} \tag{4}$$

Proof: Knowing that $P^{\bullet} \cup P^{\circ} \cup \partial P \cup P^{-} = \Re^2$, $L^{\circ} \cup \partial L \cup L^{-} = \Re^2$, and that only non-empty parts of both objects are considered, it is obtained that $\partial P \cap \Re^2 = \neg\varnothing$ and that $\partial L \cap \Re^2 = \neg\varnothing$. These statements are equivalent to $\partial P \cap (L^{\circ} \cup \partial L \cup L^{-}) = \neg\varnothing$ and $\partial L \cap (P^{\bullet} \cup P^{\circ} \cup \partial P \cup P^{-}) = \neg\varnothing$, which verify the constraint rules expressed in equation 4.

Condition 4. If both interiors are disjoint then P's interior cannot intersect with L's boundary (Equation 5).

$$R(P,L) \neq \begin{bmatrix} - & - & - \\ \phi & \neg\phi & - \\ - & - & - \\ - & - & - \end{bmatrix} \tag{5}$$

Proof: Assuming that both interiors are disjoint, $P^{o} \cap L^{o} = \varnothing$, and that P's interior intersects L's boundary, $P^{o} \cap \partial L = \neg\varnothing$, the concept of simple line is not accomplished since the two end points that represent the boundary of the line are contiguous to the points that integrate the interior of the line and cannot be disaggregated from them. So, it is impossible for a line to be disjoint from the interior of a region and at the same time its boundary be intersected by the region's interior.

Condition 5. If L's interior intersects with P's interior and exterior, then it must also intersect with P's boundary (Equation 6).

$$R(P,L) \neq \begin{bmatrix} - & - & - \\ \neg\phi & - & - \\ \phi & - & - \\ \neg\phi & - & - \end{bmatrix} \tag{6}$$

Proof: As a simple line integrates two end points that represent the boundary of the line and that are contiguous to the connected set of points that integrate the interior of the line, it is impossible the intersection of L's interior with the interior and the exterior of P without also intersecting P's boundary (between P's interior and exterior we have P's boundary).

Condition 6. P's interior always intersects with L's exterior (Equation 7).

$$R(P,L) \neq \begin{bmatrix} - & - & - \\ - & - & \phi \\ - & - & - \\ - & - & - \end{bmatrix} \tag{7}$$

Proof: Let's assume that the condition is wrong, then $P^{o} \cap L^{-} = \varnothing$. To confirm this condition, the statement $P^{o} = L^{o} \cup \partial L$, or the statement $P^{o} = L^{o}$, must be verified. Since P is a region-like object (2-dimensional) and L represents a simple line object (1-dimensional) this leads to an impossible situation since they cannot be equal.

Condition 7. P's boundary always intersects with L's exterior (Equation 8).

$$R(P,L) \neq \begin{bmatrix} - & - & - \\ - & - & - \\ - & - & \phi \\ - & - & - \end{bmatrix} \tag{8}$$

Proof: Let's assume that the condition is wrong, then $\partial P \cap L^{-} = \varnothing$. To confirm this condition, the statement $\partial P = \partial L \cup L^{o}$ must be verified. Since a simple line has two end-points, a non-empty boundary, and the boundary of a region is a closed line with no end-points, the statement is not verified since the boundary of a region is not equal (in conceptual terms) to a simple line.

Condition 8. L's interior must intersect with at least one of the four parts of P (Equation 9).

$$R(P,L) \neq \begin{bmatrix} \phi & - & - \\ \phi & - & - \\ \phi & - & - \\ \phi & - & - \end{bmatrix} \qquad (9)$$

Proof: Knowing that $P^{\bullet} \cup P^{\circ} \cup \partial P \cup P^{-} = \Re^2$, $L^{\circ} \cup \partial L \cup L^{-} = \Re^2$, and that only non-empty parts of objects are considered, it is possible to say that $L^{o} \cap \Re^2 = \neg\varnothing$. These statements are equivalent to $L^{o} \cap (P^{\bullet} \cup P^{\circ} \cup \partial P \cup P^{-}) = \neg\varnothing$, which verifies the constraint rules expressed in equation 9.

Condition 9. P's pivot can only intersect with a single part of L (Equation 10).

$$R(P,L) \neq \begin{bmatrix} \neg\phi & \neg\phi & - \\ - & - & - \\ - & - & - \\ - & - & - \end{bmatrix} \vee \begin{bmatrix} - & \neg\phi & \neg\phi \\ - & - & - \\ - & - & - \\ - & - & - \end{bmatrix} \vee \begin{bmatrix} \neg\phi & - & \neg\phi \\ - & - & - \\ - & - & - \\ - & - & - \end{bmatrix} \qquad (10)$$

Proof: Since $P^{\bullet}$ is 0-dimensional geometric primitive, representing a position, and by definition it has no boundary (a simple point can be specified as having the following characteristics: $\partial P = \varnothing$ and $P^{\circ} = P$ ([14]), it can only be intersected by one of the three parts considered for a line, L°, ∂L or L^{-}. This leads to the conditions $P^{\bullet} \cap L^{\circ} = \neg\varnothing \vee P^{\bullet} \cap \partial L = \neg\varnothing \vee P^{\bullet} \cap L^{-} = \neg\varnothing$.

Condition 10. P's pivot must intersect with at least one part of L (Equation 11).

$$R(P,L) \neq \begin{bmatrix} \phi & \phi & \phi \\ - & - & - \\ - & - & - \\ - & - & - \end{bmatrix} \qquad (11)$$

Proof: Knowing that $P^{\bullet} \cup P^{\circ} \cup \partial P \cup P^{-} = \Re^2$, $L^{\circ} \cup \partial L \cup L^{-} = \Re^2$, and that only non-empty parts of objects are considered, it is possible to say that $P^{\bullet} \cap \Re^2 = \neg\varnothing$. These statements are equivalent to $P^{\bullet} \cap (L^{\circ} \cup \partial L \cup L^{-}) = \neg\varnothing$, which verifies the constraint rules expressed in equation 11.

Condition 11. If P's interior intersects with L's interior, and P's exterior intersects with L's boundary, then the P's boundary must intersect with L's interior (Equation 12).

$$R(P,L) \neq \begin{bmatrix} - & - & - \\ \neg\phi & - & - \\ \phi & - & - \\ - & \neg\phi & - \end{bmatrix} \qquad (12)$$

Proof: As a simple line integrates two end points that represent the boundary of the line and that are contiguous to the connected set of points that integrate the interior of the line, it is impossible the intersection of L's interior with P's interior and the intersection of L's boundary with P's exterior, without L's interior also intersecting P's boundary (between P's interior and exterior we have P's boundary).

Condition 12. The boundary of a simple line L (simple lines are one-dimensional, continuous features embedded in the plane [14]) can only intersect with at most two parts of P (Equation 13).

$$R(P,L) \neq \begin{bmatrix} - & \neg\phi & - \\ - & \neg\phi & - \\ - & \neg\phi & - \\ - & - & - \end{bmatrix} \vee \begin{bmatrix} - & - & - \\ - & \neg\phi & - \\ - & \neg\phi & - \\ - & \neg\phi & - \end{bmatrix} \vee \begin{bmatrix} - & \neg\phi & - \\ - & - & - \\ - & \neg\phi & - \\ - & \neg\phi & - \end{bmatrix} \vee \begin{bmatrix} - & \neg\phi & - \\ - & \neg\phi & - \\ - & - & - \\ - & \neg\phi & - \end{bmatrix} \qquad (13)$$

Proof: A simple line has a boundary that integrates two points, each one of them being a 0-dimensional geometric primitive. These two points can only intersect, each of them, one part of the CSEP P. This leads to the intersection of the boundary of L (∂L) with at most two parts of P since the two points of ∂L can intersect the same part of P, with exception to the pivot of P ($P^{\bullet}$) that can only be intersected by one of the two points of ∂L.

Condition 13. If L's boundary intersects P's pivot, then P's interior must intersect L's interior (Equation 14).

$$R(P,L) \neq \begin{bmatrix} - & \neg\phi & - \\ \phi & - & - \\ - & - & - \\ - & - & - \end{bmatrix} \qquad (14)$$

Proof: Assuming that the constraint rule is false, then $\partial L \cap P^{\bullet} = \neg\varnothing \Rightarrow P^{\circ} \cap L^{\circ} = \varnothing$. Knowing that $L^{\circ} \cup \partial L \cup L^{-} = \Re^2$, this leads to $P^{\circ} \cap (L^{\circ} \cup \partial L \cup L^{-}) = P^{\circ} \cap \Re^2 = \varnothing$, which is a contradiction to the assumed non-emptiness of the interior of a CSEP, here represented by a region, so $P^{\circ} \cap \Re^2 = \neg\varnothing$.

Condition 14. If L's interior intersects P's pivot, then P's interior must intersect L's interior (Equation 15).

$$R(P,L) \neq \begin{bmatrix} \neg\phi & - & - \\ \phi & - & - \\ - & - & - \\ - & - & - \end{bmatrix} \qquad (15)$$

Proof: Assuming that the constraint rule is false, then $L^{\circ} \cap P^{\bullet} = \neg\varnothing \Rightarrow P^{\circ} \cap L^{\circ} = \varnothing$. Knowing that $L^{\circ} \cup \partial L \cup L^{-} = \Re^2$, this leads to $P^{\circ} \cap (L^{\circ} \cup \partial L \cup L^{-}) = P^{\circ} \cap \Re^2 = \varnothing$, which is a contradiction to the assumed non-emptiness of the interior of a CSEP, here represented by a region, so $P^{\circ} \cap \Re^2 = \neg\varnothing$.

The adoption of a 4x3 matrix for the definition of the intersections between the pivot ($P^{\bullet}$), interior (P°), boundary (∂P) and exterior (P^{-}) of P, and the interior (L°), boundary (∂L) and exterior (L^{-}) of L, results in the identification of 4096 (2^{12}) different matrices. In this set, with a very large number of possible combinations, only a reduced number of matrices represent valid topological relations for the objects in analysis.

In order to support the process of generation of the 4096 different matrices and the elimination of the invalid ones, a computational approach was followed using Mathematica® [17]. This implementation allowed the identification of the 4096 matrices, the definition of the several conditions (Conditions 1 to 14) and the automatic elimination of the invalid patterns associated with those conditions (Equations 2 to 15).

After the application of the 14 conditions, 38 matrices were left as possible ones. Each one of these matrices was manually analyzed to certify its validity. As all the matrices were considered valid, no more conditions were defined. This analysis was undertaken through the geometric realization of the 38 different topological spatial relations, validating the relations in terms of their existence.

Table III presents the identified topological relations (with their geometric realization) and their corresponding matrices. In those matrices, the absence of intersection is represented by 0 (∅) and its presence by 1 (¬∅).

III. Conceptual Neighborhood Graph with the Snapshot Model

Geographic objects and phenomena may gradually change their location, orientation, shape, and size over time. A qualitative change occurs when an object deformation affects its topological relation with respect to other object. Models for changes of topological relations are relevant to spatio-temporal reasoning in geographic space as they derive the most likely configurations and allow for predictions (based on inference) about significant changes [18].

In a conceptual neighborhood graph, nodes represent spatial relations and edges are created to link similar relations. Different definitions of similarity lead to different graphs involving the same set of relations. Usually, conceptual neighborhood graphs are built considering situations of continuous change, representing the possible transitions from one relation to other relations. Those graphs are useful for reducing the search space when looking for the next possible situations that might occur [19].

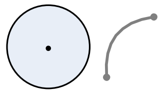
One of the approaches to identify a conceptual neighborhood graph is to use the snapshot model. This model compares two different topological relations with no knowledge of the potential transformations that may have caused the change [7]. The comparison is made by considering the topological distance between two topological relations [18]. This distance determines the number of corresponding elements, empty (∅) and non-empty (¬∅), with different values in the corresponding intersection matrices.

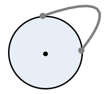
The definition of topological distance (τ) between two spatial relations (R_A and R_B) given by Egenhofer and Al-Taha [18] is the sum of the absolute values of the differences between corresponding entries of the intersections verified in the corresponding matrices (M_A and M_B). The adoption of this definition and its adaptation to the context of this work, 12-intersection matrices [11, 20], lead to the topological distance calculation as described by Equation 16.

$$\tau_{R_A,R_B} = \sum_{i=1}^{3}\sum_{j=1}^{4} \left| M_A[i,j] - M_B[i,j] \right| \qquad (16)$$

As an example, consider the topological spatial relations illustrated in Table I. Using relations R_1 and R_2, and their corresponding matrices M_1 and M_2, the calculated topological distance between these two topological spatial relations is 2.

Table I – Topological distance: an example





$$M_1 - M_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$$M_1 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix} \qquad M_2 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix} \qquad \tau_{R_1,R_2} = 2$$

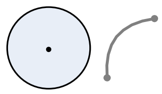
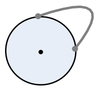
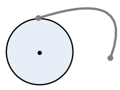
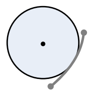
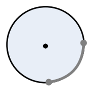
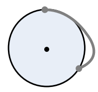
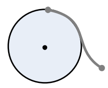
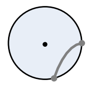
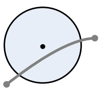
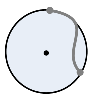
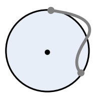
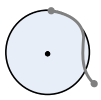
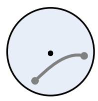
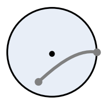
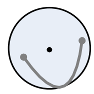
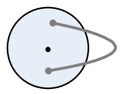
The calculation of the topological distances, using Equation 16, showed that for the majority of the topological relations the minimum distance to their neighborhoods is 1. The minimum topological distance (Table II) between one relation and its neighborhoods is 2 only in the case of relation R_{21}.

Table II – Topological distance (snapshot model)

	R_1	R_2	R_3	R_4	R_5	R_6	R_7	R_8	R_9	R_{10}	R_{11}	R_{12}	R_{13}	R_{14}	R_{15}	R_{16}	R_{17}	R_{18}	R_{19}	R_{20}	R_{21}	R_{22}	R_{23}	R_{24}	R_{25}	R_{26}	R_{27}	R_{28}	R_{29}	R_{30}	R_{31}	R_{32}	R_{33}	R_{34}	R_{35}	R_{36}	R_{37}	R_{38}
R_1	0	2	1	1	4	3	2	4	2	5	4	3	4	5	5	4	3	6	5	6	4	7	6	6	7	6	6	4	7	6	5	6	7	7	6	5	8	7
R_2	2	0	1	3	2	1	2	2	4	3	2	3	4	3	5	4	5	4	3	4	6	5	4	6	7	6	4	6	5	4	5	6	5	7	6	7	6	5
R_3	1	1	0	2	3	2	1	3	3	4	3	2	5	4	6	5	4	5	4	5	5	6	5	7	8	7	5	5	6	5	4	7	6	8	7	6	7	6
R_4	1	3	2	0	3	2	1	5	1	4	3	2	5	6	4	3	2	5	4	7	3	6	5	7	6	5	7	3	6	5	4	7	8	6	5	4	7	6
R_5	4	2	3	3	0	1	2	2	4	1	2	3	4	3	3	4	5	2	3	4	6	3	4	6	5	6	4	6	3	4	5	6	5	5	6	7	4	5
R_6	3	1	2	2	1	0	1	3	3	2	1	2	5	4	4	3	4	3	2	5	5	4	3	7	6	5	5	5	4	3	4	7	6	6	5	6	5	4
R_7	2	2	1	1	2	1	0	4	2	3	2	1	6	5	5	4	3	4	3	6	4	5	4	8	7	6	6	4	5	4	3	8	7	7	6	5	6	5
R_8	4	2	3	5	2	3	4	0	4	1	2	3	2	1	3	4	5	2	3	2	6	3	4	4	5	6	2	6	3	4	5	4	3	5	6	7	4	5
R_9	2	4	3	1	4	3	2	4	0	3	2	1	4	5	3	2	1	4	3	6	2	5	4	6	5	4	6	2	5	4	3	6	7	5	4	3	6	5
R_{10}	5	3	4	4	1	2	3	1	3	0	1	2	3	2	2	3	4	1	2	3	5	2	3	5	4	5	3	5	2	3	4	5	4	4	5	6	3	4
R_{11}	4	2	3	3	2	1	2	2	2	1	0	1	4	3	3	2	3	2	1	4	4	3	2	6	5	4	4	4	3	2	3	6	5	5	4	5	4	3
R_{12}	3	3	2	2	3	2	1	3	1	2	1	0	5	4	4	3	2	3	2	5	3	4	3	7	6	5	5	3	4	3	2	7	6	6	5	4	5	4
R_{13}	4	4	5	5	4	5	6	2	4	3	4	5	0	1	1	2	3	2	3	4	6	5	6	2	3	4	4	6	5	6	7	2	3	3	4	5	4	5
R_{14}	5	3	4	6	3	4	5	1	5	2	3	4	1	0	2	3	4	1	2	3	7	4	5	3	4	5	3	7	4	5	6	3	2	4	5	6	3	4
R_{15}	5	5	6	4	3	4	5	3	3	2	3	4	1	2	0	1	2	1	2	5	5	4	5	3	2	3	5	5	4	5	6	3	4	2	3	4	3	4
R_{16}	4	4	5	3	4	3	4	4	2	3	2	3	2	3	1	0	1	2	1	6	4	5	4	4	3	2	6	4	5	4	5	4	5	3	2	3	4	3
R_{17}	3	5	4	2	5	4	3	5	1	4	3	2	3	4	2	1	0	3	2	7	3	6	5	5	4	3	7	3	6	5	4	5	6	4	3	2	5	4
R_{18}	6	4	5	5	2	3	4	2	4	1	2	3	2	1	1	2	3	0	1	4	6	3	4	4	3	4	4	6	3	4	5	4	3	3	4	5	2	3
R_{19}	5	3	4	4	3	2	3	3	3	2	1	2	3	2	2	1	2	1	0	5	5	4	3	5	4	3	5	5	4	3	4	5	4	4	3	4	3	2
R_{20}	6	4	5	7	4	5	6	2	6	3	4	5	4	3	5	6	7	4	5	0	4	1	2	2	3	4	2	6	3	4	5	4	3	5	6	7	4	5
R_{21}	4	6	5	3	6	5	4	6	2	5	4	3	6	7	5	4	3	6	5	4	0	3	2	4	3	2	6	2	5	4	3	6	7	5	4	3	6	5
R_{22}	7	5	6	6	3	4	5	3	5	2	3	4	5	4	4	5	6	3	4	1	3	0	1	3	2	3	3	5	2	3	4	5	4	4	5	6	3	4
R_{23}	6	4	5	5	4	3	4	4	4	3	2	3	6	5	5	4	5	4	3	2	2	1	0	4	3	2	4	4	3	2	3	6	5	5	4	5	4	3
R_{24}	6	6	7	7	6	7	8	4	6	5	6	7	2	3	3	4	5	4	5	2	4	3	4	0	1	2	4	6	5	6	7	2	3	3	4	5	4	5
R_{25}	7	7	8	6	5	6	7	5	5	4	5	6	3	4	2	3	4	3	4	3	3	2	3	1	0	1	5	5	4	5	6	3	4	2	3	4	3	4
R_{26}	6	6	7	5	6	5	6	6	4	5	4	5	4	5	3	2	3	4	3	4	2	3	2	2	1	0	6	4	5	4	5	4	5	3	2	3	4	3
R_{27}	6	4	5	7	4	5	6	2	6	3	4	5	4	3	5	6	7	4	5	2	6	3	4	4	5	6	0	4	1	2	3	2	1	3	4	5	2	3
R_{28}	4	6	5	3	6	5	4	6	2	5	4	3	6	7	5	4	3	6	5	6	2	5	4	6	5	4	4	0	3	2	1	4	5	3	2	1	4	3
R_{29}	7	5	6	6	3	4	5	3	5	2	3	4	5	4	4	5	6	3	4	3	5	2	3	5	4	5	1	3	0	1	2	3	2	2	3	4	1	2
R_{30}	6	4	5	5	4	3	4	4	4	3	2	3	6	5	5	4	5	4	3	4	4	3	2	6	5	4	2	2	1	0	1	4	3	3	2	3	2	1
R_{31}	5	5	4	4	5	4	3	5	3	4	3	2	7	6	6	5	4	5	4	5	3	4	3	7	6	5	3	1	2	1	0	5	4	4	3	2	3	2
R_{32}	6	6	7	7	6	7	8	4	6	5	6	7	2	3	3	4	5	4	5	4	6	5	6	2	3	4	2	4	3	4	5	0	1	1	2	3	2	3
R_{33}	7	5	6	8	5	6	7	3	7	4	5	6	3	2	4	5	6	3	4	3	7	4	5	3	4	5	1	5	2	3	4	1	0	2	3	4	1	2
R_{34}	7	7	8	6	5	6	7	5	5	4	5	6	3	4	2	3	4	3	4	5	5	4	5	3	2	3	3	3	2	3	4	1	2	0	1	2	1	2
R_{35}	6	6	7	5	6	5	6	6	4	5	4	5	4	5	3	2	3	4	3	6	4	5	4	4	3	2	4	2	3	2	3	2	3	1	0	1	2	1
R_{36}	5	7	6	4	7	6	5	7	3	6	5	4	5	6	4	3	2	5	4	7	3	6	5	5	4	3	5	1	4	3	2	3	4	2	1	0	3	2
R_{37}	8	6	7	7	4	5	6	4	6	3	4	5	4	3	3	4	5	2	3	4	6	3	4	4	3	4	2	4	1	2	3	2	1	1	2	3	0	1
R_{38}	7	5	6	6	5	4	5	5	5	4	3	4	5	4	4	3	4	3	2	5	5	4	3	5	4	3	3	3	2	1	2	3	2	2	1	2	1	0

Based on the calculated topological distances, a conceptual neighborhood graph was identified. Fig. 4 presents the obtained graph, where the closest relations of each topological relation are connected.

Table III – Topological spatial relations between a circular spatially extended point and a line

R_1	R_2	R_3	R_4	R_5	R_6	R_7	R_8
							
$\begin{bmatrix}0&0&1\\0&0&1\\0&0&1\\1&1&1\end{bmatrix}$	$\begin{bmatrix}0&0&1\\0&0&1\\0&1&1\\1&0&1\end{bmatrix}$	$\begin{bmatrix}0&0&1\\0&0&1\\0&1&1\\1&1&1\end{bmatrix}$	$\begin{bmatrix}0&0&1\\0&0&1\\1&0&1\\1&1&1\end{bmatrix}$	$\begin{bmatrix}0&0&1\\0&0&1\\1&1&1\\0&0&1\end{bmatrix}$	$\begin{bmatrix}0&0&1\\0&0&1\\1&1&1\\1&0&1\end{bmatrix}$	$\begin{bmatrix}0&0&1\\0&0&1\\1&1&1\\1&1&1\end{bmatrix}$	$\begin{bmatrix}0&0&1\\1&0&1\\0&1&1\\0&0&1\end{bmatrix}$
R_9	R_{10}	R_{11}	R_{12}	R_{13}	R_{14}	R_{15}	R_{16}
							
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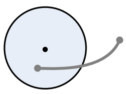
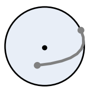
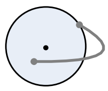
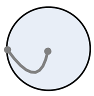
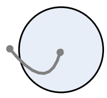
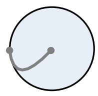
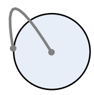
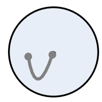
R_{17}	R_{18}	R_{19}	R_{20}	R_{21}	R_{22}	R_{23}	R_{24}
							
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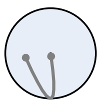
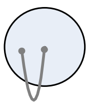
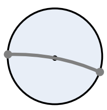
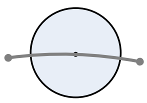
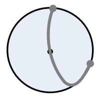
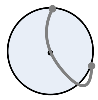
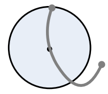
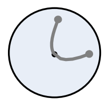
R_{25}	R_{26}	R_{27}	R_{28}	R_{29}	R_{30}	R_{31}	R_{32}
							
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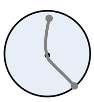
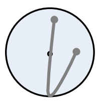
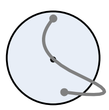
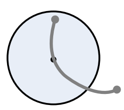
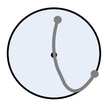
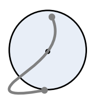
R_{33}	R_{34}	R_{35}	R_{36}	R_{37}	R_{38}		
							
$\begin{bmatrix}1&0&0\\1&1&1\\0&1&1\\0&0&1\end{bmatrix}$	$\begin{bmatrix}1&0&0\\1&1&1\\1&0&1\\0&0&1\end{bmatrix}$	$\begin{bmatrix}1&0&0\\1&1&1\\1&0&1\\1&0&1\end{bmatrix}$	$\begin{bmatrix}1&0&0\\1&1&1\\1&0&1\\1&1&1\end{bmatrix}$	$\begin{bmatrix}1&0&0\\1&1&1\\1&1&1\\0&0&1\end{bmatrix}$	$\begin{bmatrix}1&0&0\\1&1&1\\1&1&1\\1&0&1\end{bmatrix}$		

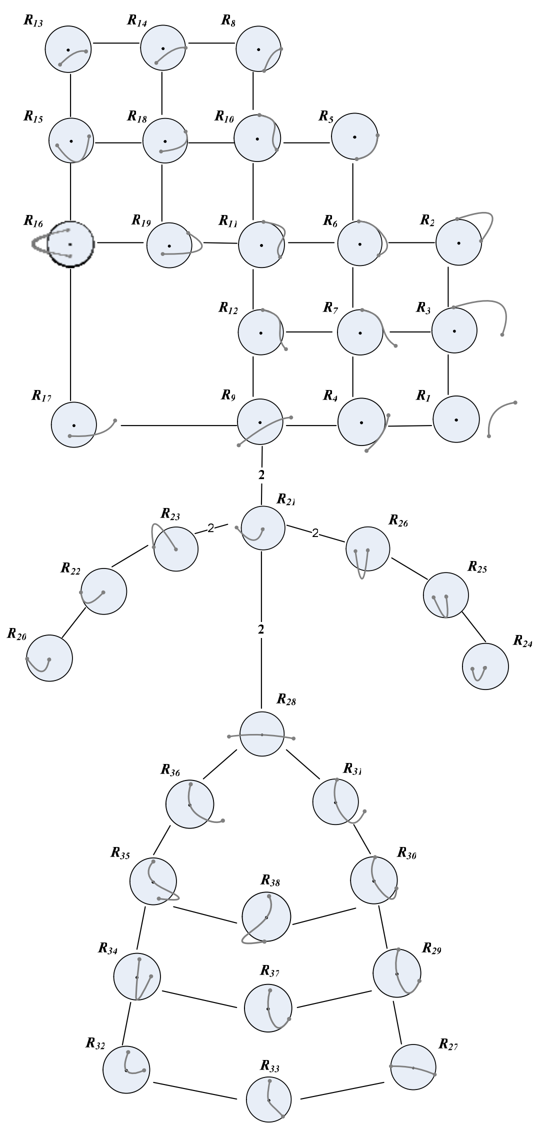


Fig. 4 – Conceptual neighborhood graph: snapshot model

The graph (Fig. 4) is virtually divided in three parts. In the upper part, the 19 topological relations do not include any intersection between the pivot of the CSEP and the line. If the pivot of the spatially extended point is ignored, making a CSEP equal to a region, these 19 topological spatial relations correspond to the 19 topological spatial relations identified in [7] for line-region relations. The middle of the graph contains the relations in which one of the boundaries of the line intersects the pivot of the CSEP. The lower part of the graph contains the topological relations in which the pivot of the CSEP is intersected by the interior of the line. These three parts are linked by relation R_{21} that presents edges to relations R_9, R_{23}, R_{26} and R_{28} with the minimum topological distance of 2. All the other edges, and as previously mentioned, link spatial relations with topological distance equal to 1.

IV. Conceptual Neighborhood Graph with the Smooth-transition Model

The smooth-transition model states that two relations are conceptual neighbors if there is a smooth-transition from one relation to the other. Egenhofer and Mark [7] define a smooth-transition as an infinitesimally small deformation that changes the topological relation. Attending to the adopted 12-intersection matrix, the existence of a smooth-transition means that an intersection or its *adjacent* intersection changes from empty to non-empty or reverse. The concept of adjacency between the several parts (interior, boundary and exterior) of a region (R) is formalized as [7]:

Adjacent (R°) = ∂R
Adjacent (∂R) = R° and R^-
Adjacent (R^-) = ∂R

In the context of this work, the notion of adjacency needs to be adapted to the several parts of a CSEP [21]. For a CSEP (P) we have:

Adjacent ($P^\bullet$) = P°
Adjacent (P°) = $P^\bullet$ and ∂P
Adjacent (∂P) = P° and P^-
Adjacent (P^-) = ∂P

Following the work of Egenhofer and Mark [7], the changes that can occur in the smooth-transition model between a line and a region are associated with moving the boundary of the line to an adjacent part of the region or pushing the interior of the line to an adjacent part of the region. In this work this principles are adopted and adapted in order to change the parts of a region to the parts of a CSEP.

For the definition of the conditions that allow the identification of the conceptual neighbors, the notion of *extent* was introduced [7]. It represents the number of non-empty intersections existing between the line and the four parts of the CSEP. If the interior of the line is completely located in the exterior of the CSEP then the extent of this relation is 1 (*Extent(P, L°)=1*). This is the case of relation R_1. If the interior of the line intersects the four parts of the CSEP then the extent of the relation is 4 (*Extent(P, L°)=4*) and this is verified in relations like R_{28} or R_{30}.

Using the *Adjacent* and *Extent* concepts, the smooth--transitions that can occur between a CSEP (P) and a line (L) can be formalized as follows:

Condition I. If the two boundaries of L intersect the same part of P then the intersection must be extended to the adjacent parts of P (Equation 17).

$$Extent(P,\partial L)=1 \Rightarrow \forall_{i\in\{P^\bullet,P^\circ,\partial P,P^-\}}\left(M[i,\partial L]=\neg\phi\right): \left(M_{Neighbor}\left[Adjacent(i),\partial L\right]:=\neg\phi\right) \quad (17)$$

Condition II. If the two boundaries of L intersect different parts of P then the intersection must be extended to the adjacent parts of P (Equation 18).

$$Extent(P,\partial L)=2 \Rightarrow \forall_{i\in\{P^\bullet,P^\circ,\partial P,P^-\}}\left(M[i,\partial L]=\neg\phi\right): \left(M_{Neighbor}[i,\partial L]:=\phi\right)\wedge\left(M_{Neighbor}\left[Adjacent(i),\partial L\right]:=\neg\phi\right) \quad (18)$$

Condition III. The intersection of L's interior must be moved to an adjacent part of P (Equation 19).

$$\forall_{i\in\{P^\bullet,P^\circ,\partial P,P^-\}}\left(M\left[i,L^\circ\right]=\neg\phi\right): \left(M_{Neighbor}\left[Adjacent(i),L^\circ\right]:=\neg\phi\right) \tag{19}$$

Condition IV. The intersection of L's interior with the parts of P must be reduced (Equations 20, 21 and 22).

$$Extent(P,L^\circ)=2\Rightarrow\forall_{i\in\{P^\bullet,P^\circ,\partial P,P^-\}}\left(M\left[i,L^\circ\right]=\neg\phi\right): \left(M_{Neighbor}\left[i,L^\circ\right]:=\phi\right) \tag{20}$$

$$Extent(P,L^\circ)=3\Rightarrow\forall_{i\in\{P^\bullet,P^\circ,\partial P,P^-\}}\left(M\left[i,L^\circ\right]=\neg\phi\right): \left(M_{Neighbor}\left[i,L^\circ\right]:=\phi\right) \tag{21}$$

$$Extent(P,L^\circ)=4\Rightarrow\forall_{i\in\{P^\bullet,P^\circ,P^-\}}\left(M\left[i,L^\circ\right]=\neg\phi\right): \left(M_{Neighbor}\left[i,L^\circ\right]:=\phi\right) \tag{22}$$

The established conditions to the smooth-transitions may generate impossible patterns (in terms of the topological spatial relations that can really exist). This impossible patterns need to be identified and eliminated from the set of valid ones (possible conceptual neighbors). One simple validation can be done by checking if the identified conceptual neighbor does match with one of the intersections matrices that are the possible topological spatial relations. If not, certainly that represents an impossible pattern. Although this simple validation, Egenhofer and Mark [7] defined two consistency constraints that are here adopted and extended in order to consider the specific case of the topological spatial relations that can exist between a CSEP (P) and a line (L). These constraints limit the possible transitions that can occur following conditions I to IV in order to guarantee that the identified patterns are valid. In that sense, these constraints are equivalent to some of the conditions used in the identification of the topological spatial relations that can exist between a CSEP and line.

Constraint I. If L's interior intersects with P's interior and exterior, then it must also intersect P's boundary (Equation 23).

$$M\left[P^\circ,L^\circ\right]=\neg\phi\wedge M\left[P^-,L^\circ\right]=\neg\phi\Rightarrow M\left[\partial P,L^\circ\right]:=\neg\phi \tag{23}$$

Constraint II. If L's boundary intersects with P's interior (exterior), then L's interior must intersect P's interior (exterior) (Equations 24 and 25).

$$M\left[P^\circ,\partial L\right]=\neg\phi\Rightarrow M\left[P^\circ,L^\circ\right]:=\neg\phi \tag{24}$$

$$M\left[P^-,\partial L\right]=\neg\phi\Rightarrow M\left[P^-,L^\circ\right]:=\neg\phi \tag{25}$$

Constraint III. P's pivot can only intersect with a single part of L (Equations 26, 27 and 28).

$$M\left[P^\bullet,L^\circ\right]=\neg\phi\Rightarrow M\left[P^\bullet,\partial L\right]:=\phi\wedge M\left[P^\bullet,L^-\right]:=\phi \tag{26}$$

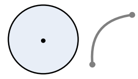
$$M\left[P^\bullet,\partial L\right]=\neg\phi\Rightarrow M\left[P^\bullet,L^\circ\right]:=\phi\wedge M\left[P^\bullet,L^-\right]:=\phi \tag{27}$$

$$M\left[P^\bullet,L^-\right]=\neg\phi\Rightarrow M\left[P^\bullet,L^\circ\right]:=\phi\wedge M\left[P^\bullet,\partial L\right]:=\phi \tag{28}$$

In order to exemplify the use of these conditions to identify the conceptual neighborhood graph using the smooth-transition model, let us consider Condition I and the corresponding Equation 17. Taking R_1 and its corresponding M_1, Table IV shows the neighbors identification process. For the initial relation R_1, and after the application of Equation 17, a matrix is identified with a valid pattern that corresponds to R_3 meaning that an edge linking these two relations in the conceptual neighborhood graph is needed. Another example, using the same Equation 17, is also presented in Table IV. For the initial relation R_{13}, and as P's interior has two the adjacent parts, P's pivot and P's boundary, two matrices are identified, each one of them corresponding to a valid pattern: R_{14} and R_{24}. In this case, two of the neighbors of R_{13} are R_{14} and R_{24}.

Table IV – Smooth-transition model: an example

R_1

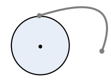


M_1

$$\begin{bmatrix}0&0&1\\0&0&1\\0&0&1\\1&1&1\end{bmatrix}$$

$$Extent(P,\partial L)=1$$
$$\Rightarrow M_{Neighbor}\left[Adjacent(P^-),\partial L\right]:=1$$
$$\Leftrightarrow M_{Neighbor}\left[\partial P,\partial L\right]:=1$$
$$\Leftrightarrow\begin{bmatrix}0&0&1\\0&0&1\\0&1&1\\1&1&1\end{bmatrix}$$

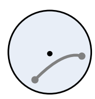
R_3



M_3

$$\begin{bmatrix}0&0&1\\0&0&1\\0&1&1\\1&1&1\end{bmatrix}$$

R_{13}

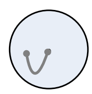


M_{13}

$$\begin{bmatrix}0&0&1\\1&1&1\\0&0&1\\0&0&1\end{bmatrix}$$

$$Extent(P,\partial L)=1$$
$$\Rightarrow M_{Neighbor}\left[Adjacent(P^\circ),\partial L\right]:=1$$
$$\Leftrightarrow M_{Neighbor}\left[P^\bullet,\partial L\right]:=1$$
$$\Leftrightarrow\begin{bmatrix}0&1&0\\1&1&1\\0&0&1\\0&0&1\end{bmatrix}$$

R_{24}

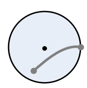


M_{24}

$$\begin{bmatrix}0&1&0\\1&1&1\\0&0&1\\0&0&1\end{bmatrix}$$

$$Extent(P,\partial L)=1$$
$$\Rightarrow M_{Neighbor}\left[Adjacent(P^\circ),\partial L\right]:=1$$
$$\Leftrightarrow M_{Neighbor}\left[\partial P,\partial L\right]:=1$$
$$\Leftrightarrow\begin{bmatrix}0&0&1\\1&1&1\\0&1&1\\0&0&1\end{bmatrix}$$

R_{14}



M_{14}

$$\begin{bmatrix}0&0&1\\1&1&1\\0&1&1\\0&0&1\end{bmatrix}$$

Applying Condition I to Condition IV, Constraint I to Constraint III, and their respective equations (17 to 28), the several links between relations in the conceptual neighborhood graph were identified. The corresponding graph has 83 edges linking 38 topological spatial relations, and is shown in Fig. 5.

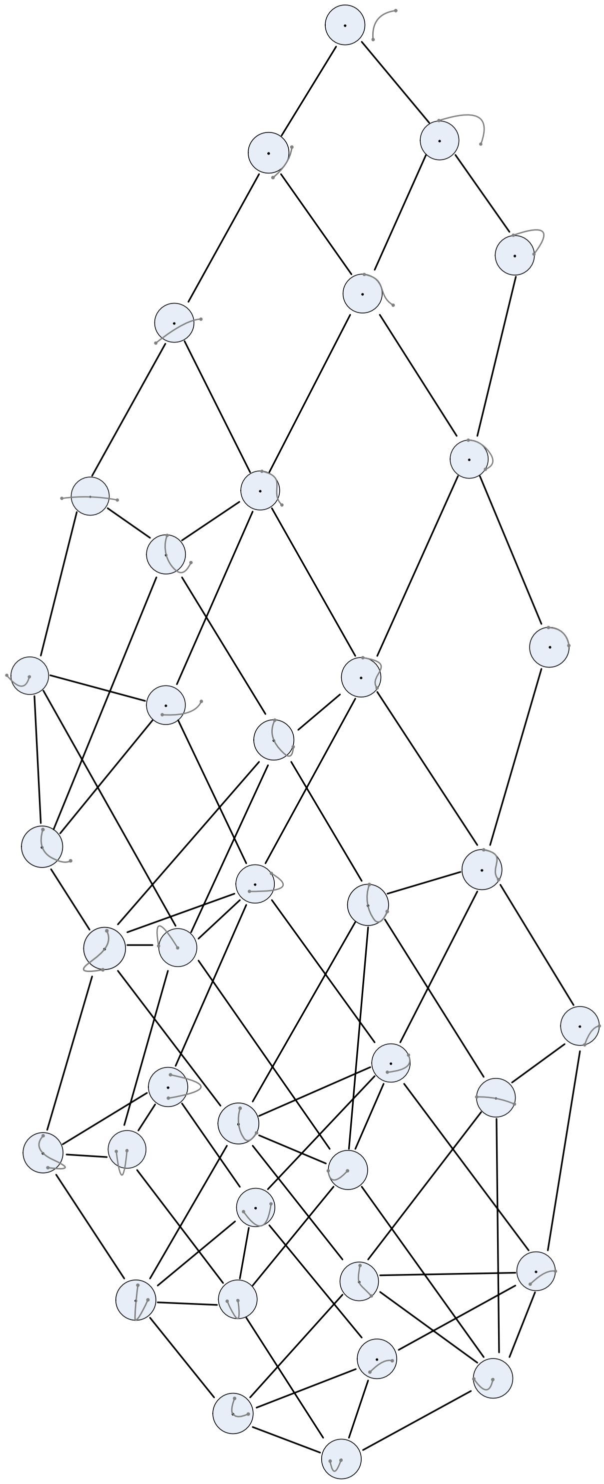


Fig. 5 – Conceptual neighborhood graph: smooth-transition model

The complexity of the graph results from the fact that 11 relations have 5 conceptual neighbors, and 6 relations have 6 conceptual neighbors. By comparison, in the graph obtained through the snapshot model each relation has a maximum of 4 neighbors, resulting in a total of 51 edges.

V. Comparison of the Two Conceptual Neighborhood Graphs

The analysis of the two conceptual neighborhood graphs highlighted the main differences between them. It also allowed the validation of the two graphs, as the transitions between spatial relations were analyzed to check whether they are possible or not. These verifications ensure that the graphs accomplish the principles that guided their identification. One of the main differences between the two graphs, as shown in Table V, is the list of the possible transitions between the 38 topological spatial relations.

Table V – Possible transitions among relations

Snapshot Model	*Smooth-transition Model*
1 → 3, 4	**1** → 3, 4
2 → 3, 6	**2** → 3, 6
3 → 1, 2, 7	**3** → 1, 2, 7
4 → 1, 7, 9	**4** → 1, 7, 9
5 → 6, 10	**5** → 6, 10
6 → 2, 5, 7, 11	**6** → 2, 5, 7, 11
7 → 3, 4, 6, 12	**7** → 3, 4, 6, 12
8 → 10, 14	**8** → 10, 14, $27_{=}$
9 → 4, 12, $17_{\_}$, $21_{\_}$	**9** → 4, 12, $28_{=}$
10 → 5, 8, 11, 18	**10** → 5, 8, 11, 18, $29_{=}$
11 → 6, 10, 12, 19	**11** → 6, 10, 12, 19, $30_{=}$
12 → 7, 9, 11	**12** → 7, 9, 11, $17_{=}$, $31_{=}$
13 → 14, 15	**13** → 14, 15, $24_{=}$, $32_{=}$
14 → 8, 13, 18	**14** → 8, 13, 18, $20_{\equiv}$, $33_{=}$
15 → 13, 16, 18	**15** → 13, 16, 18, $25_{=}$, $34_{=}$
16 → 15, $17_{\_}$, 19	**16** → 15, 19, $26_{=}$, $35_{=}$
17 → $9_{\_}$, $16_{\_}$	**17** → $12_{=}$, $19_{=}$, $21_{\equiv}$, $36_{=}$
18 → 10, 14, 15, 19	**18** → 10, 14, 15, 19, $22_{\equiv}$, $37_{=}$
19 → 11, 16, 18	**19** → 11, 16, $17_{=}$, 18, $23_{\equiv}$, $38_{=}$
20 → 22	**20** → $14_{\equiv}$, 22, $24_{=}$, $27_{=}$, $33_{\equiv}$
21 → $9_{\_}$, 23, $26_{\_}$, 28	**21** → $17_{\equiv}$, 23, 28, $36_{\equiv}$
22 → 20, 23	**22** → $18_{\equiv}$, 20, 23, $25_{=}$, $29_{=}$, $37_{\equiv}$
23 → 21, 22	**23** → $19_{\equiv}$, 21, 22, $26_{=}$, $30_{=}$, $38_{\equiv}$
24 → 25	**24** → $13_{=}$, $20_{=}$, 25, $32_{=}$
25 → 24, 26	**25** → $15_{=}$, $22_{=}$, 24, 26, $34_{=}$
26 → $21_{\_}$, 25	**26** → $16_{=}$, $23_{=}$, 25, $35_{=}$
27 → 29, 33	**27** → $8_{=}$, $20_{=}$, 29, 33
28 → 21, 31, $36_{\_}$	**28** → $9_{=}$, 21, 31
29 → 27, 30, 37	**29** → $10_{=}$, $22_{=}$, 27, 30, 37
30 → 29, 31, 38	**30** → $11_{=}$, $23_{=}$, 29, 31, 38
31 → 28, 30	**31** → $12_{=}$, 28, 30, $36_{=}$
32 → 33, 34	**32** → $13_{=}$, $24_{=}$, 33, 34
33 → 27, 32, 37	**33** → $14_{=}$, $20_{\equiv}$, 27, 32, 37
34 → 32, 35, 37	**34** → $15_{=}$, $25_{=}$, 32, 35, 37
35 → 34, $36_{\_}$, 38	**35** → $16_{=}$, $26_{=}$, 34, 38
36 → $28_{\_}$, $35_{\_}$	**36** → $17_{=}$, $21_{\equiv}$, $31_{=}$, $38_{=}$
37 → 29, 33, 34, 38	**37** → $18_{=}$, $22_{\equiv}$, 29, 33, 34, 38
38 → 30, 35, 37	**38** → $19_{=}$, $23_{\equiv}$, 30, 35, $36_{=}$, 37

The notation used in Table V is as follows:

- n, for common transitions among relations in the two graphs;
- $n_{\_}$, for transitions allowed in the graph obtained by the snapshot model and not possible in the graph obtained by the smooth-transition model;

- $n_=$ and $n_\equiv$, for transitions allowed in the graph obtained by the smooth-transition model and not possible in the graph obtained by the snapshot model.

From the analysis of Table V one can see that the graph obtained following the smooth-transition model integrates almost all the edges (transitions) identified by the snapshot model. Two exceptions are verified: one is associated with relation R_{17} and the other with relation R_{36}. In all other cases, the graph obtained by the smooth-transition model allows more transitions since it looks for small deformations that change the topological relations. In what concerns R_{17} and R_{36}, the snapshot model includes transitions from those relations to other relations with topological distance 1. Although this is the minimum value for the topological distance it does not correspond to the smallest amount of changes that can affect the objects. Looking at R_{17}, this relation has transitions to relation R_9 and relation R_{16}. In the smooth-transition model these transitions are not possible since R_{17} has one of the line's boundaries intersecting the interior of the CSEP and the other boundary intersecting the exterior of the CSEP. Any small deformation in R_{17} includes the movement of one of the line's boundaries to an *Adjacent* part of the intersected component of the CSEP. Following this, the intersection between one line's boundary and the CSEP's interior is moved to the *Adjacent* parts of CSEP's interior (its pivot and its boundary), allowing the transitions to relation R_{12} and relation R_{21}, or the intersection between the other line's boundary and the CSEP's exterior is moved to the *Adjacent* part of CSEP's exterior (its boundary), allowing the transition to relation R_{19}. The other possible transition for R_{17} allowed in the smooth-transition model is obtained moving the line's interior to an *Adjacent* part of CSEP's interior (its pivot in this specific case since the boundary already has an intersection in this relation) leading to relation R_{36}.

Looking at the possible transitions for R_{36} in the snapshot model, which are different from those allowed in the smooth-transition model, one can see that the differences are due to the movement of the line's boundaries, as explained above for R_{17}.

This analysis shows that topological distance equal to 1 is not synonym of a small change. Table VI presents the possible transitions identified for the smooth-transition model and the respective topological distances. In this table one can see that many of the identified transitions are associated with topological distances of 2 (represented with the symbol $_=$ and in Table V) and some topological distances of 3 (represented with the symbol $_\equiv$ in Table V). The topological distance of 3 is associated with relations that present the pivot of the CSEP intersected either by the line's interior or by the line's boundary, being these relations a start or an end relation in the conceptual neighborhood graph. As a CSEP's pivot can only intersect with a single part of the line, moving the line's boundary or the line's interior to intersect the pivot implies that any other intersection with the pivot must be removed. This increases the topological distance between the relations. This is also the situation verified with many of the transactions with topological distance 2 existing in the graph obtained by the smooth-transition model.

To conclude this analysis we refer that in what concerns relations R_1 to R_7, the transitions allowed in the two graphs are exactly the same (Table V) since these relations are associated with lines that do not intersect the CSEP's interior avoiding the *Adjacent* parts of this interior, which would be the pivot and the boundary of the CSEP and would increase the number of possible transitions.

Table VI – Topological distance (smooth-transition model)

	R_1	R_2	R_3	R_4	R_5	R_6	R_7	R_8	R_9	R_{10}	R_{11}	R_{12}	R_{13}	R_{14}	R_{15}	R_{16}	R_{17}	R_{18}	R_{19}	R_{20}	R_{21}	R_{22}	R_{23}	R_{24}	R_{25}	R_{26}	R_{27}	R_{28}	R_{29}	R_{30}	R_{31}	R_{32}	R_{33}	R_{34}	R_{35}	R_{36}	R_{37}	R_{38}
R_1	0	2	1	1	4	3	2	4	2	5	4	3	4	5	5	4	3	6	5	6	4	7	6	6	7	6	6	4	7	6	5	6	7	7	6	5	8	7
R_2	2	0	1	3	2	1	2	2	4	3	2	3	4	3	5	4	5	4	3	4	6	5	4	6	7	6	4	6	5	4	5	6	5	7	6	7	6	5
R_3	1	1	0	2	3	2	1	3	3	4	3	2	5	4	6	5	4	5	4	5	5	6	5	7	8	7	5	5	6	5	4	7	6	8	7	6	7	6
R_4	1	3	2	0	3	2	1	5	1	4	3	2	5	6	4	3	2	5	4	7	3	6	5	7	6	5	7	3	6	5	4	7	8	6	5	4	7	6
R_5	4	2	3	3	0	1	2	2	4	1	2	3	4	3	3	4	5	2	3	4	6	3	4	6	5	6	4	6	3	4	5	6	5	5	6	7	4	5
R_6	3	1	2	2	1	0	1	3	3	2	1	2	5	4	4	3	4	3	2	5	5	4	3	7	6	5	5	5	4	3	4	7	6	6	5	6	5	4
R_7	2	2	1	1	2	1	0	4	2	3	2	1	6	5	5	4	3	4	3	6	4	5	4	8	7	6	6	4	5	4	3	8	7	7	6	5	6	5
R_8	4	2	3	5	2	3	4	0	4	1	2	3	2	1	3	4	5	2	3	2	6	3	4	4	5	6	2	6	3	4	5	4	3	5	6	7	4	5
R_9	2	4	3	1	4	3	2	4	0	3	2	1	4	5	3	2	1	4	3	6	2	5	4	6	5	4	6	2	5	4	3	6	7	5	4	3	6	5
R_{10}	5	3	4	4	1	2	3	1	3	0	1	2	3	2	2	3	4	1	2	3	5	2	3	5	4	5	3	5	2	3	4	5	4	4	5	6	3	4
R_{11}	4	2	3	3	2	1	2	2	2	1	0	1	4	3	3	2	3	2	1	4	4	3	2	6	5	4	4	4	3	2	3	6	5	5	4	5	4	3
R_{12}	3	3	2	2	3	2	1	3	1	2	1	0	5	4	4	3	2	3	2	5	3	4	3	7	6	5	5	3	4	3	2	7	6	6	5	4	5	4
R_{13}	4	4	5	5	4	5	6	2	4	3	4	5	0	1	1	2	3	2	3	4	6	5	6	2	3	4	4	6	5	6	7	2	3	3	4	5	4	5
R_{14}	5	3	4	6	3	4	5	1	5	2	3	4	1	0	2	3	4	1	2	3	7	4	5	3	4	5	3	7	4	5	6	3	2	4	5	6	3	4
R_{15}	5	5	6	4	3	4	5	3	3	2	3	4	1	2	0	1	2	1	2	5	5	4	5	3	2	3	5	5	4	5	6	3	4	2	3	4	3	4
R_{16}	4	4	5	3	4	3	4	4	2	3	2	3	2	3	1	0	1	2	1	6	4	5	4	4	3	2	6	4	5	4	5	4	5	3	2	3	4	3
R_{17}	3	5	4	2	5	4	3	5	1	4	3	2	3	4	2	1	0	3	2	7	3	6	5	5	4	3	7	3	6	5	4	5	6	4	3	2	5	4
R_{18}	6	4	5	5	2	3	4	2	4	1	2	3	2	1	1	2	3	0	1	4	6	3	4	4	3	4	4	6	3	4	5	4	3	3	4	5	2	3
R_{19}	5	3	4	4	3	2	3	3	3	2	1	2	3	2	2	1	2	1	0	5	5	4	3	5	4	3	5	5	4	3	4	5	4	4	3	4	3	2
R_{20}	6	4	5	7	4	5	6	2	6	3	4	5	4	3	5	6	7	4	5	0	4	1	2	2	3	4	2	6	3	4	5	4	3	5	6	7	4	5
R_{21}	4	6	5	3	6	5	4	6	2	5	4	3	6	7	5	4	3	6	5	4	0	3	2	4	3	2	6	2	5	4	3	6	7	5	4	3	6	5
R_{22}	7	5	6	6	3	4	5	3	5	2	3	4	5	4	4	5	6	3	4	1	3	0	1	3	2	3	3	5	2	3	4	5	4	4	5	6	3	4
R_{23}	6	4	5	5	4	3	4	4	4	3	2	3	6	5	5	4	5	4	3	2	2	1	0	4	3	2	4	4	3	2	3	6	5	5	4	5	4	3
R_{24}	6	6	7	7	6	7	8	4	6	5	6	7	2	3	3	4	5	4	5	2	4	3	4	0	1	2	4	6	5	6	7	2	3	3	4	5	4	5
R_{25}	7	7	8	6	5	6	7	5	5	4	5	6	3	4	2	3	4	3	4	3	3	2	3	1	0	1	5	5	4	5	6	3	4	2	3	4	3	4
R_{26}	6	6	7	5	6	5	6	6	4	5	4	5	4	5	3	2	3	4	3	4	2	3	2	2	1	0	6	4	5	4	5	4	5	3	2	3	4	3
R_{27}	6	4	5	7	4	5	6	2	6	3	4	5	4	3	5	6	7	4	5	2	6	3	4	4	5	6	0	4	1	2	3	2	1	3	4	5	2	3
R_{28}	4	6	5	3	6	5	4	6	2	5	4	3	6	7	5	4	3	6	5	6	2	5	4	6	5	4	4	0	3	2	1	4	5	3	2	1	4	3
R_{29}	7	5	6	6	3	4	5	3	5	2	3	4	5	4	4	5	6	3	4	3	5	2	3	5	4	5	1	3	0	1	2	3	2	2	3	4	1	2
R_{30}	6	4	5	5	4	3	4	4	4	3	2	3	6	5	5	4	5	4	3	4	4	3	2	6	5	4	2	2	1	0	1	4	3	3	2	3	2	1
R_{31}	5	5	4	4	5	4	3	5	3	4	3	2	7	6	6	5	4	5	4	5	3	4	3	7	6	5	3	1	2	1	0	5	4	4	3	2	3	2
R_{32}	6	6	7	7	6	7	8	4	6	5	6	7	2	3	3	4	5	4	5	4	6	5	6	2	3	4	2	4	3	4	5	0	1	1	2	3	2	3
R_{33}	7	5	6	8	5	6	7	3	7	4	5	6	3	2	4	5	6	3	4	3	7	4	5	3	4	5	1	5	2	3	4	1	0	2	3	4	1	2
R_{34}	7	7	8	6	5	6	7	5	5	4	5	6	3	4	2	3	4	3	4	5	5	4	5	3	2	3	3	3	2	3	4	1	2	0	1	2	1	2
R_{35}	6	6	7	5	6	5	6	6	4	5	4	5	4	5	3	2	3	4	3	6	4	5	4	4	3	2	4	2	3	2	3	2	3	1	0	1	2	1
R_{36}	5	7	6	4	7	6	5	7	3	6	5	4	5	6	4	3	2	5	4	7	3	6	5	5	4	3	5	1	4	3	2	3	4	2	1	0	3	2
R_{37}	8	6	7	7	4	5	6	4	6	3	4	5	4	3	3	4	5	2	3	4	6	3	4	4	3	4	2	4	1	2	3	2	1	1	2	3	0	1
R_{38}	7	5	6	6	5	4	5	5	5	4	3	4	5	4	4	3	4	3	2	5	5	4	3	5	4	3	3	3	2	1	2	3	2	2	1	2	1	0

VI. Topological Distance in Predicting Movement in Space

As already stated in this paper, several models for topological relations have been developed. These models provide a computational basis for spatial reasoning relating the formal ground needed by an information system and the human perception of the geographic space [8].

Spatial relations between geographic objects are time-dependent and can change due to various phenomena. Models of changes are relevant to spatio-temporal reasoning as they allow for predictions related with the objects in analysis [18]. The objects involved are expected not to make discontinuous changes such as jumps, nor may the deformations destroy the topology of a single object, for example, tearing it into pieces.

For Galton [22], the phenomenon of movement arises whenever the same object occupies different positions in space at different moments. The given definition suggest that a theory of movement must include a *theory of time*, a *theory of space*, a *theory of objects* and a *theory of position*. Time can be treated as instants or as intervals, and an ordering relation need to be established, representing temporal sequences. Space integrates elements as points, lines or regions as fundamental entities. Objects present specific characteristics being capable of motion (or different types of motion), or not. Position integrates the theory of objects with the theory of space, with each object occupying a certain part of space at a specific time.

Looking at position, this work takes into account the topological constraints present in the space in which the user is moving, whilst trying to predict his/her future positions. The first step of this prediction process is concerned with the assignment of the user to a specific road segment in the road network. For this task, the topological distance can be used instead of the geometrical distance. The second step is associated with the movement of the user along the road network, after his/her assignment to a road segment.

A user that moves from one point to another in a road network generates a trajectory. A user's trajectory can be defined as a sequence of connected road segments or as a sequence of connected vertices between two locations [23]. Knowing a start point, the initial position, and an end point, the target destination, trajectories can be generated for the user.

If the destination of the user is unknown, the anticipation of his/her next position can be achieved following the transitions allowed in one of the conceptual neighborhood graphs identified in the previous sections. As an example, let us consider the road network represented in Fig. 6. In this example, the position of the user (in gray), obtained through a GPS receiver, does not overlap a specific road segment.

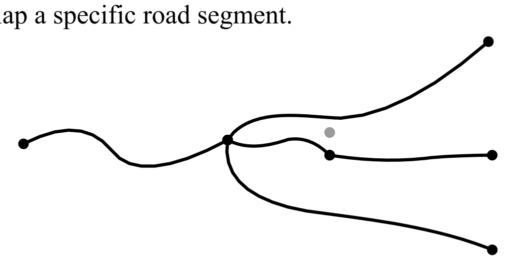


Fig. 6 – A user and a part of a road network

Analyzing the example, and being the user in an imprecise point in terms of the road network, how can the user be assigned to a specific road segment? The answer can be using the minimum topological distance that allows the intersection of the CSEP's pivot and the road segments. Using the area of uncertainty associated with the user's position, Fig. 7 presents the actual scenario, now with each road segment identified by a specific label.

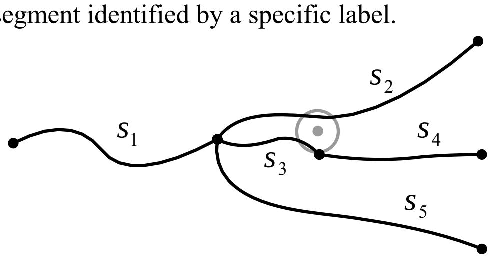


Fig. 7 – A user represented by a CSEP

Analyzing the topological spatial relations that exist between the CSEP and the neighbors' road segments – s_2, s_3 and s_4 – it is possible to verify that the spatial relations are R_9, R_{12} and R_3, respectively. Following the conceptual neighborhood graphs obtained through the snapshot model and the smooth-transition model, the closest spatial relations in which an intersection with the CSEP's pivot is possible is R_{21} (for s_2, s_3 and s_4), in the case of the snapshot model, and R_{28} (for s_2) and R_{31} (for s_3 and s_4), in the case of the smooth-transition model. Tables II and VI give us the topological distances among those relations:

- Snapshot model: i) topological distance of 2 between R_9 and R_{21}; ii) topological distance of 3 between R_{12} and R_{21}; iii) topological distance of 5 between R_3 and R_{21}.
- Smooth-transition model: i) topological distance of 2 between R_9 and R_{28}; ii) topological distance of 2 between R_{12} and R_{31}; iii) topological distance of 4 between R_3 and R_{31}.

Following the snapshot model, the user would be assigned to s_2 since this road segment presents the minimum topological distance. Looking at the smooth-transition model, as it allows small deformations that change the topological relation, the user could be assigned to s_2 or to s_3 since both alternatives present the same topological distance.

The question that can now be posted is: ignoring the topological spatial relations that can exist between the objects in analysis and the conceptual neighborhood graphs with the possible transitions, is it possible to predict the user's position?

Map matching methods are used to locate a mobile user on a road network map. A simple way of performing map matching is to assign the position of the mobile user to the nearest road segment [24]. Although this method is simple to implement it can ignore alternative paths as only the nearest distance is considered and it can be difficult to implement in dense urban road networks. In order to improve the location capabilities, other methods have been proposed and developed. They usually consider historical information about the user's motion (his/her past locations).

The prediction system that is envisaged in this work does not consider any previous knowledge about the user's motion, for privacy reasons, and opens new possibilities in the exploration of the paths that can be followed by a mobile user, as several road segments can be associated to the user through the use of a CSEP.

If the geometrical representation of the user is done recurring to a single point that locates the user in a particular location, the prediction of the user's next position depends upon the map matching location strategy used. Following the example presented in Fig. 6, Fig. 8 shows the assignment of the user to the nearest road segment present in the road network in analysis. As the user is not geometrically represented by a CSEP that topologically relates he/she to the other line segments, the user is located on segment s_2, without considering the s_3 and s_4 ways.

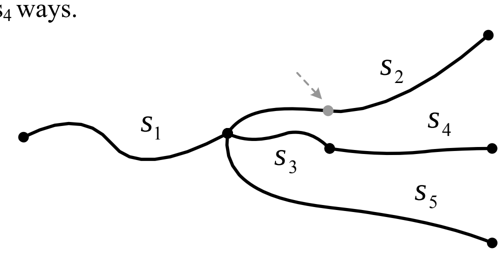


Fig. 8 – Assignment of the user to the nearest road segment

The first step of the prediction process can consider the topological distance as an alternative or as a complement of the geometric distance (since a combination of both metrics can be considered). In the second step, the transitions allowed in the conceptual neighborhood graphs can be used to predict user's future movements. In this case, graph paths can be generated considering the several alternatives present in the road network and the probability of following such alternatives (considering for instance the traffic load associated to each road segment).

VII. CONCLUSION

This paper presented the topological spatial relations that can exist between a CSEP and a line and identified the conceptual neighborhood graphs that state the transitions that can occur between relations. Two graphs were obtained. One, using the principles associated with the snapshot model, looks for the topological distances between relations, and the other using the principles associated with the smooth-transition model verifies any small deformation that changes the topological relation.

The two graphs were analyzed in order to verify if the identified transitions were possible or not, and also compared in order to identify the main differences between them. The graph obtained through the smooth-transition model presents a more complex structure integrating more edges. This means that more transitions are allowed.

This work constitutes a basis for dealing with spatial objects that can be represented geometrically by a CSEP and a line, and is suitable for reasoning about gradual changes in topology. These changes can be associated with objects' motion and/or deformations over a period of time [8].

After ongoing implementation of a prototype that follows the prediction approach introduced in this paper, it will be possible to analyze the importance of the topological distance in the prediction process and how this metric can be combined with the geometrical distance in map matching techniques.

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