Finite Element Analysis of Sandwich Structures

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ABSTRACT

Sandwich construction may be defined as a three-layer type of construction where a relatively weak, low-density core material supports and stabilizes thin layers of high strength face material.Its typical features, namely high strenght-thin and low strength-thick materials, interfaces, bonding and load transfer suggest that each of the layers will perform accordingly to its material characteristics and laminate position.

Most of the theories used for the analysis of such structures are based either in the Kirchhoff or Mindlin assumptions. The first model does not accounts for transverse shear deformations while the second assumes a first order shear-deformation behaviour. However, both models consider for all the layers a common and unique rotation of the middle-surface normal. In the model which will be described in this paper, it is assumed that each layer (skin or core) can rotate independently, due to their different material characteristics. With this assumption, each layer can deform locally, this being a more accurate model for high-stress gradient areas.

In each layer, transverse shear deformation is considered by the imposition of Mindlin-type kinematic relations. In the displacement-based finite-element model, each node possesses 9 degrees of freedom, three displacements of the plate middle-surface and two rotations of the normal of each layer middle-surface. Displacement continuity at the interfaces is imposed. The transverse shear stresses at the middle-surface of the layers are accuratelly computed, although constant in each layer. However, in this model shear correction factors are not used, which simplifies the usual Mindlin-type models, in which these factors are calculated through cylindrical bending assumptions.

In this paper, the three-layer sandwich element formulation for linear static analysis will be described. The four, eight and nine-noded isoparametric plate elements are considered. Numerical examples are discussed in order to access the model accuracy.

INTRODUCTION

Sandwich structures are one of the most sucessful areas of research and development in composite materials field. The literature in this field is relatively large. Allen[1] has published theories for sandwich plate analysis. Pagano[2,3] presents exact solutions for simple cases, considering the transverse shear deformation effects. Numerical methods, such as the Finite Element Method (FEM), are necessary in practical applications as they are able to modelize general geometries, boundary conditions, loadings and materials. Among the significant contributions in this field, Khatua et al[4] and Kolar et al[5] developed elements based on displacement fields, and Kraus[6] presents an assumed stress hybrid formulation for an orthotropic sandwich plate element. Manwenya et al[7] published the formulation of a multilayer quadratic isoparametric plate element with layer-wyse shear deformation theory involved, and Ferreira[8] studied a three-layer sandwich plate, by considering the shear deformation effects of both skins and core. In the present paper, this latter work is briefly discussed.

THEORY

Modelling a layered structure like a sandwich plate, one frequently proceeds to a kind of homogeneization to account for all the kinematic effects (membrane, bending, shear and interactions). However, this procedure does not capture completely the local effects of heterogeneous, highly non-symmetric sandwich structures. We propose a more accurate formulation which considers a three-layer plate (fig.1), each one of these can locally deform. The present approach considers the shear deformation effects in each layer.

The present shear deformation theory for general sandwich plates has been developed by assuming the displacement field in the following form:

In each layer, i, the middle-layer displacement field is found by

$$\{u_{i0}\}=\{ u_{i0} , v_{i0} , w_i \}^T, \qquad i=1,2,3 \tag{1}$$

in which u_{i0} , v_{i0} and w_i represents the i^{th} layer displacements about the x,y and z axes. For the first layer (top) we assume (by displacement continuity) that

$$\begin{aligned} u_{10} &= u_0 + \frac{h_2}{2}\theta_{y2} + \frac{h_1}{2}\theta_{y1} \\ v_{10} &= v_0 - \frac{h_2}{2}\theta_{x2} - \frac{h_1}{2}\theta_{x1} \\ w_1 &= w_0 \end{aligned} \tag{2}$$

in which h_i represents the i^{th} layer thickness and θ_{xi}, θ_{yi} the i^{th} layer rotations of the "normal" about the x and y axes. For layer 2 and 3 (core and bottom layer, respectively), expressions similar to (2) can be found. It is supposed that the laminate middle-surface is (conveniently) placed at core middle-surface. By this supposition we can write

$$\{u_{20}, v_{20}, w_{20}\}^T = \{u_0, v_0, w_0\}^T \tag{3}$$

The middle-surface displacement field of the 3^{rd} layer (bottom) is expressed as

$$\begin{aligned} u_{30} &= u_0 - \frac{h_2}{2}\theta_{y2} - \frac{h_3}{2}\theta_{y3} \\ v_{30} &= v_0 + \frac{h_2}{2}\theta_{x2} + \frac{h_3}{2}\theta_{x3} \\ w_3 &= w_0 \end{aligned} \tag{4}$$

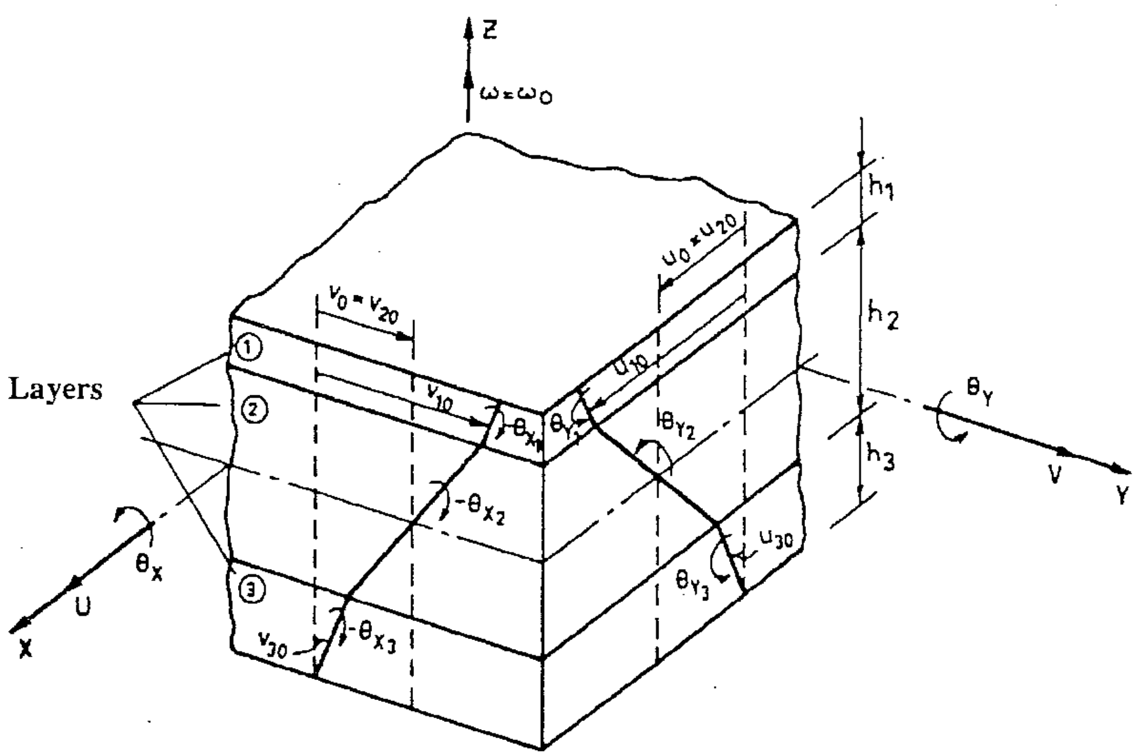

Fig.1 - Three-layer sandwich element - Geometry and displacement components

By setting z_i as a local i^{th} layer z coordinate as

$$-\frac{h_i}{2} \leq z_i \leq \frac{h_i}{2} \tag{5}$$

the i^{th} layer displacement field is found as

$$\{u_i\} = \{u_{i0}\} + z_i\ \{\theta_i\} \tag{6}$$

For the first layer (layer 1), for example, expression (6) can be developed as

$$\{u_1\} = \{u_{10}\} + z_1\ \{\theta_1\} \tag{7}$$

or

$$\begin{Bmatrix} u_1 \\ v_1 \\ w_1 \end{Bmatrix} = \begin{Bmatrix} u_0 + \frac{h_2}{2}\theta_{y2} + \frac{h_1}{2}\theta_{y1} \\ v_0 - \frac{h_2}{2}\theta_{x2} - \frac{h_1}{2}\theta_{x1} \\ w_o \end{Bmatrix} + z_1 \begin{Bmatrix} \theta_{y1} \\ -\theta_{x1} \\ 0 \end{Bmatrix} \tag{8}$$

For the other layers similar expressions can be found.

The strain - displacement relations for the i^{th} layer are as follows:

$$\left\{\varepsilon_{xi}, \varepsilon_{yi}, \gamma_{xyi}, \gamma_{xzi}, \gamma_{yzi}\right\}^T = \left\{\frac{\partial u_i}{\partial x}, \frac{\partial v_i}{\partial y}, \frac{\partial u_i}{\partial y} + \frac{\partial v_i}{\partial x}, \frac{\partial u_i}{\partial z} + \frac{\partial w_i}{\partial x}, \frac{\partial v_i}{\partial z} + \frac{\partial w_i}{\partial y}\right\}^T \tag{9}$$

In each layer, the strain tensor is divided into membrane $\{\varepsilon^m\}_i$, bending $\{\varepsilon^b\}_i$, shear $\{\varepsilon^s\}_i$, and membrane-bending, $\{\varepsilon^{mb}\}_i$:

$$\{\varepsilon\}_i = \begin{Bmatrix} \varepsilon^m \\ \varepsilon^s \end{Bmatrix}_i + \begin{Bmatrix} \varepsilon^{mb} \\ 0 \end{Bmatrix}_i + z_i \begin{Bmatrix} \varepsilon^b \\ 0 \end{Bmatrix}_i \tag{10}$$

The membrane strains are constant through the sandwich thickness and are given for the i^{th} layer as:

$$\left\{\varepsilon^m\right\}_i = \left\{\frac{\partial u_0}{\partial x}, \frac{\partial v_0}{\partial y}, \frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x}\right\}^T \tag{11}$$

The bending strains are expressed by

$$\left\{\varepsilon^b\right\}_i = \left\{\frac{\partial \theta_{yi}}{\partial x}, -\frac{\partial \theta_{xi}}{\partial y}, -\frac{\partial \theta_{xi}}{\partial x} + \frac{\partial \theta_{yi}}{\partial y}\right\}^T \tag{12}$$

The shear strains are given by

$$\left\{\varepsilon^s\right\}_i = \left\{\frac{\partial w_0}{\partial x} + \theta_{yi}, \frac{\partial w_0}{\partial y} - \theta_{xi}\right\}^T \tag{13}$$

Finally, the membrane-bending strains for the top layer (layer 1) are expressed as

$$\left\{\varepsilon^{mb}\right\}_1 = \left\{\begin{array}{c} \frac{h_1}{2}\frac{\partial\theta_{y1}}{\partial x} + \frac{h_2}{2}\frac{\partial\theta_{y2}}{\partial x} \\ -\frac{h_1}{2}\frac{\partial\theta_{x1}}{\partial y} - \frac{h_2}{2}\frac{\partial\theta_{x2}}{\partial y} \\ \frac{h_1}{2}\left(\frac{\partial\theta_{y1}}{\partial y} - \frac{\partial\theta_{x1}}{\partial x}\right) + \frac{h_2}{2}\left(\frac{\partial\theta_{y2}}{\partial y} - \frac{\partial\theta_{x2}}{\partial x}\right) \end{array}\right\} \quad (14)$$

The membrane-bending strains vanish for the core, as result of the sandwich middle-surface position. The expression for the bottom layer (layer 3) is given by

$$\left\{\varepsilon^{mb}\right\}_3 = \left\{\begin{array}{c} -\frac{h_2}{2}\frac{\partial\theta_{y2}}{\partial x} - \frac{h_3}{2}\frac{\partial\theta_{y3}}{\partial x} \\ \frac{h_2}{2}\frac{\partial\theta_{x2}}{\partial y} + \frac{h_3}{2}\frac{\partial\theta_{x3}}{\partial y} \\ \frac{h_2}{2}\left(-\frac{\partial\theta_{y2}}{\partial y} + \frac{\partial\theta_{x2}}{\partial x}\right) + \frac{h_3}{2}\left(-\frac{\partial\theta_{y3}}{\partial y} + \frac{\partial\theta_{x3}}{\partial x}\right) \end{array}\right\} \quad (15)$$

Notice that through this approach the membrane-bending effects are automatically achieved.

The material constitutive relations for the ith layer can be written as $\{\sigma_i\}=[D_i]\ \{\varepsilon_i\}$ or

$$\left\{\begin{array}{c} \sigma_1 \\ \sigma_2 \\ \tau_{12} \\ \tau_{13} \\ \tau_{23} \end{array}\right\}_i = \left[\begin{array}{cc} [D_1] & [0] \\ [0] & [D_2] \end{array}\right]_i \left\{\begin{array}{c} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \\ \gamma_{13} \\ \gamma_{23} \end{array}\right\}_i \quad (16)$$

where $\{\sigma_i\}$ represents the stress tensor for the ith layer referred to the layer coordinate axes (1,2,3) as shown in fig.2, and $[D_i]$ is the reduced material stiffnesses matrix of the ith layer and the following relations hold between these matrix coefficients and the engineering elastic constants:

$$[D_1]_i = \begin{bmatrix} \dfrac{E_1}{1-\nu_{12}\nu_{21}} & \dfrac{\nu_{21}E_1}{1-\nu_{12}\nu_{21}} & 0 \\ \dfrac{\nu_{21}E_1}{1-\nu_{12}\nu_{21}} & \dfrac{E_2}{1-\nu_{12}\nu_{21}} & 0 \\ 0 & 0 & G_{12} \end{bmatrix}_i \tag{17}$$

$$[D_2]_i = \begin{bmatrix} G_{13} & 0 \\ 0 & G_{23} \end{bmatrix}_i \tag{18}$$

The stress-strain relation for the ith layer in the laminate coordinate axes (x,y,z) are written as

$$\{\sigma'\}_i = [D']_i \{\varepsilon'\}_i \tag{19}$$

where

$$\{\sigma'\}_i^T = \{\sigma_x\ ,\ \sigma_y\ ,\ \tau_{xy}\ ,\ \tau_{xz}\ ,\ \tau_{yz}\ \}_i \tag{20}$$

$$\{\varepsilon'\}_i^T = \{\varepsilon_x\ ,\ \varepsilon_y\ ,\ \gamma_{xy}\ ,\ \gamma_{xz}\ ,\ \gamma_{yz}\ \}_i \tag{21}$$

$$[D']_i = [\theta]^T_i\ [D]_i\ [\theta]_i \tag{22}$$

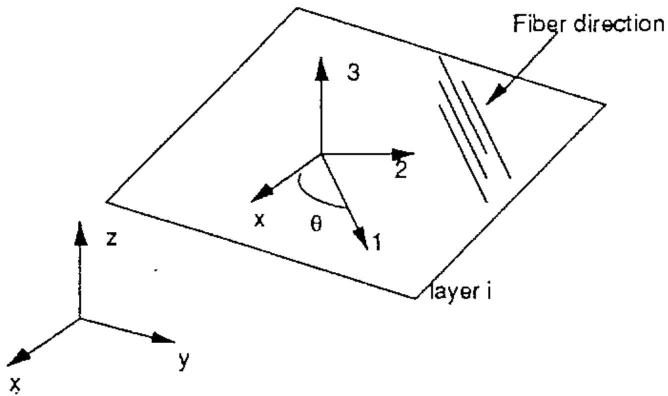

Fig.2 - Lamina coordinate systems (x,y,z) - Global and (1,2,3) - Local
(note: z and 3 are the same and normal to the plane (x,y) or (1,2))

where $[\theta]_i$ represents the transformation matrix of the stress/strain vectors between the lamina and the laminate coordinate systems according to the usual transformation rule given in Jones[9] . Notice that in the present model no shear correction factors are applied to the shear elastic coefficients.

By integrating the stresses through the plate thickness, the generalized stress-resultant for the three-layer sandwich (fig.3) are obtained as:

$$\{N\} = \{N_x, N_y, N_{xy}\}^T = \sum_{i=1}^{3} \{N_x, N_y, N_{xy}\}_i^T = \sum_{i=1}^{3} \int_{z_i^{bot}}^{z_i^{top}} \left\{\sigma_x, \sigma_y, \tau_{xy}\right\}_i^T dz =$$

$$= \sum_{i=1}^{3} \left\{\sigma_x, \sigma_y, \tau_{xy}\right\}_i^T h_i = \sum_{i=1}^{3} [D_1]_i \left[\{\varepsilon^m\}_i + \{\varepsilon^{mb}\}_i\right] h_i \qquad (23)$$

The i th -layer moments are calculated as

$$\{\bar{M}\} = \{M_x, M_y, M_{xy}\}^T = \sum_{i=1}^{3} \{M_x, M_y, M_{xy}\}_i^T = \sum_{i=1}^{3} \int_{z_i^{bot}}^{z_i^{top}} \left\{\bar{\sigma}_x, \sigma_y, \tau_{xy}\right\}_i^T z_i \, dz =$$

$$= \int_{z_i^{bot}}^{z_i^{top}} [D_1]_i \left[\{\varepsilon^m\}_i + \{\varepsilon^{mb}\}_i + z_i \{\varepsilon^b\}_i\right] z_i \, d z_i = [D_1]_i \{\varepsilon^b\}_i \frac{h_i^3}{12} \qquad (24)$$

(note that in this case $z_i^{top} = h_i/2$ and $z_i^{bot} = - h_i/2$)

The laminate moments are calculated by summing the latter (i th -layer) moments produced by the i th -layer position in z-direction. These latter moments are expressed as

$$\{\overline{M}\} = \{\overline{M}_x, \overline{M}_y, \overline{M}_{xy}\}^T = \sum_{i=1}^{3} \{\overline{M}_x, \overline{M}_y, \overline{M}_{xy}\}_i^T = \sum_{i=1}^{3} \int_{z_i^{bot}}^{z_i^{top}} [D_1]_i \left[\{\varepsilon^m\}_i + \{\varepsilon^{mb}\}_i\right] z \, d z_i =$$

$$= \sum_{i=1}^{3} [D_1]_i \left[\{\varepsilon^m\}_i + \{\varepsilon^{mb}\}_i\right] \frac{\left(z_i^{top}\right)^2 - \left(z_i^{bot}\right)^2}{2} \qquad (25)$$

The laminate moments are then obtained as follows

$$\{M\} = \{\overline{M}\} + \sum_{i=1}^{3} \{M\}_i = \sum_{i=1}^{3} [D_1]_i \left[\{\varepsilon^b\}_i \frac{h_i^3}{12} + [D_1]_i \left[\{\varepsilon^m\}_i + \{\varepsilon^{mb}\}_i\right] \frac{\left(z_i^{top}\right)^2 - \left(z_i^{bot}\right)^2}{2}\right. \qquad (26)$$

The shear stress-resultants are expressed as

$$\{Q\} = \{Q_{xz}, Q_{yz}\}^T = \sum_{i=1}^{3} \{Q_{xz}, Q_{yz}\}_i^T = \sum_{i=1}^{3} \int_{z_i^{bot}}^{z_i^{top}} \{\tau_{xz}, \tau_{yz}\}_i^T dz_i = \sum_{i=1}^{3} [D_2]_i \{\gamma_{xz}, \gamma_{yz}\}_i^T h_i \qquad (27)$$

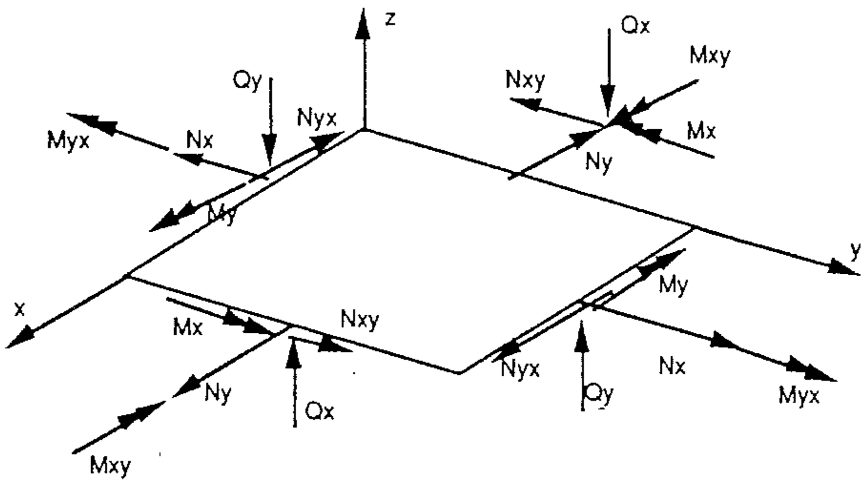

Fig.3 - Generalized stress-resultants for the three-layer sandwich plate

FINITE ELEMENT FORMULATION

In the standard finite element technique, the solution domain is discretized into n subdomains (elements) such that

$$\Pi(\delta) = \sum_{e=1}^{n} \Pi^{e}(\delta) \tag{28}$$

where Π and Π^e are the total potential energy of the system and the element, respectively. The element potential can be expressed in terms of the internal strain energy U^e and the external work done W^e for an element *e* as

$$\Pi^e(\delta) = U^e - W^e \tag{29}$$

in which $\{\delta\}$ is the vector of unknown displacement variables in the problem and is defined by

$$\{\delta\} = \{u_0, v_0, w_0, \theta_{x1}, \theta_{y1}, \theta_{x2}, \theta_{y2}, \theta_{x3}, \theta_{y3}\}^T \tag{30}$$

By assuming the same interpolation function to define all the components of the generalized displacement vector $\{\delta\}$, we can write

$$\{\delta\} = \sum_{i=1}^{NN} [N_i]\{\delta_i\} \tag{31}$$

in which NN is the number of nodes of the element, $[N_i]$ is the interpolating (shape) function matrix associated with node i and $\{\delta_i\}$ is the part of $\{\delta\}$ corresponding to node i.

The strain vector is related to the displacements vector as

$$\{\varepsilon\}=[B]\{\delta\} \tag{32}$$

in which [B] is the so-called strain-displacement matrix, containing the shape functions and their derivatives. The [B] matrix is divided into membrane, bending, shear and membrane-bending matrices, according to the relations (30) and (9) to (15).

The elastic static stiffness matrix is achieved by the minimization of the internal strain energy[8] and is evaluated by summing up the contribuition of the three layers as

$$K_{jk}=\sum_{i=1}^{3}[K_{jk}]_i \tag{33}$$

or

$$\begin{aligned}K_{jk}=&\sum_{i=1}^{3}\int_A\left[\left[B_j^m\right]_i^T\left[D_1\right]_i\left[B_k^m\right]_i h_i\right]dA+\sum_{i=1}^{3}\int_A\left[\left[B_j^m\right]_i^T\left[D_1\right]_i\left[B_k^{mb}\right]_i h_i\right]dA+\\&+\sum_{i=1}^{3}\int_A\left[\left[B_j^{mb}\right]_i^T\left[D_1\right]_i\left[B_k^m\right]_i h_i\right]dA+\sum_{i=1}^{3}\int_A\left[\left[B_j^{mb}\right]_i^T\left[D_1\right]_i\left[B_k^{mb}\right]_i h_i\right]dA+\\&+\sum_{i=1}^{3}\int_A\frac{h_i^3}{12}\left[\left[B_j^b\right]_i^T\left[D_1\right]_i\left[B_k^b\right]_i\right]dA+\sum_{i=1}^{3}\int_A\left[\left[B_i^s\right]_i^T\left[D_2\right]_i\left[B_k^s\right]_i h_i\right]dA\end{aligned} \tag{34}$$

NUMERICAL EXAMPLE AND DISCUSSION

Validity of the finite element formulations of the present model is established by comparing results for sandwich plate problems with those available in the form of exact, closed form and other finite element solutions. This example is selected from Srinivas[10], in which a square, simply supported along all four edges plate is analysed for different material and layer thickness configurations. The plate is loaded by a transversely uniform pressure, the 8-noded Serendipity element was used throughout, and the stresses are evaluated at the nearest gauss point. The plate geometry, finite element mesh and boundary conditions are illustrated in fig.4. The adimensional core material characteristics are

$$[D_1]_{core}=\begin{bmatrix}3.802 & 0.879 & 0\\ 0.879 & 1.996 & 0\\ 0 & 0 & 1.0\end{bmatrix}$$

$$[D_2]_{core}=\begin{bmatrix}0.608 & 0\\ 0 & 1.015\end{bmatrix} \tag{35}$$

The skins (layers 1 and 3) material characteristics are proportional to the core characteristics and are calculated through the use of factors

$$c_k = \frac{[D_{ij}]_k}{[D_{ij}]_{core}} \;, \; k=1,3 \tag{36}$$

Five diferent cases are studied. These seek to simulate several sandwich configurations, such as an isotropic homogeneous laminate (case 1), a symmetric sandwich (cases 2 and 3) and non-symmetric sandwich (cases 4 and 5).

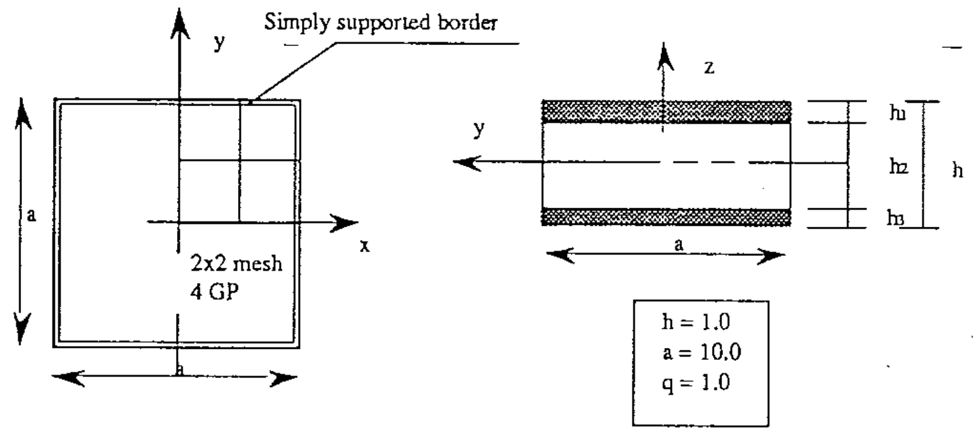

Fig.4 - Geometry, mesh, material and boundary conditions of a square, simply supported sandwich plate.

The following general observations can be made from the results presented in tables 1-5 and figs. 5-7: (i) Deflection and inplane stresses are accurately predicted without refining the mesh, as the 2x2 mesh in a quarter plate gives sufficiently accurate results, (ii) For non-symmetric sandwich, this model presents a better bending behaviour, with very accurate deflections. Notice that for cases 4 and 5, the most difficult to model, the predicted inplane stresses are more accurate than those predicted by Barros[11] and Figueiras[12], (iii) With the present model (without the use of shear correction factors) a very accurate prediction of transverse shear stresses can be directly achieved, in each middle-surface.

Table 1- Maximum in-plane stresses σ_x/q (x,y=0) and displacements in a simply supported square sandwich plate (case 1) $h_1/h=0.1$, $h_2/h=0.8$, $h_3/h=0.1$, $c_1=1$, $c_3=1$

	In-plane stresses ....>--Layer 1--		--Core	–	--Layer 3--		
Source	w*	Top	Bottom	Top	Bottom	Top	Bottom
Srinivas[exact]	181.05	36.012	28.538	28.538	-28.545	-28.545	-35.937
Figueiras	183.99	36.223	28.978	28.978	-28.978	-28.978	-36.223
Barros	181.22	35.693	28.560	28.560	-28.560	-28.560	-35.693
Kirchhoff	168.38	36.098	28.878	28.878	-28.878	-28.878	-36.098
Present analysis	180.5	38.819	30.712	30.712	-30.712	-30.712	-38.819

Table 2- Maximum in-plane stresses σ_x/q (x,y=0) and displacements in a simply supported square sandwich plate (case 2) $h_1/h=0.1$, $h_2/h=0.8$, $h_3/h=0.1$, $c_1=10$, $c_3=10$

Source	w*	In-plane stresses ....>--Layer 1-- Top	Bottom	--Core -- Top	Bottom	--Layer 3-- Top	Bottom
Srinivas[exact]	41.906	65.332	48.857	4.903	-4.860	-48.609	-65.083
Figueiras	41.922	65.226	48.735	4.8735	-4.8735	-48.735	-65.226
Barros	41.980	63.247	54.020	5.4020	-5.4020	-54.020	-63.247
Kirchhoff	31.241	66.953	53.563	5.3563	-5.3563	-53.563	-66.953
Present analysis	41.930	64.150	47.720	4.772	-4.772	-47.720	-64.150

$$w^* = \frac{w_{min} G_{12}(core)}{h\,q}$$

Table 3- Maximum in-plane stresses σ_x/q (x,y=0) and displacements in a simply supported square sandwich plate (case 3) $h_1/h=0.1$, $h_2/h=0.8$, $h_3/h=0.1$, $c_1=50$, $c_3=50$

Source	w*	In-plane stresses ....>--Layer 1-- Top	Bottom	--Core -- Top	Bottom	--Layer 3-- Top	Bottom
Srinivas[exact]	16.753	67.213	37.473	0.76826	-0.74196	-37.157	-66.900
Figueiras	16.850	58.307	46.646	0.9330	-0.9330	-46.646	-58.307
Barros	16.812	58.069	46.463	0.9290	-0.9290	-46.463	-58.069
Kirchhoff	6.762	72.457	57.966	1.1593	-1.1593	-57.966	-72.457
Present analysis	16.814	66.490	35.970	0.7194	-0.7194	-35.970	-66.490

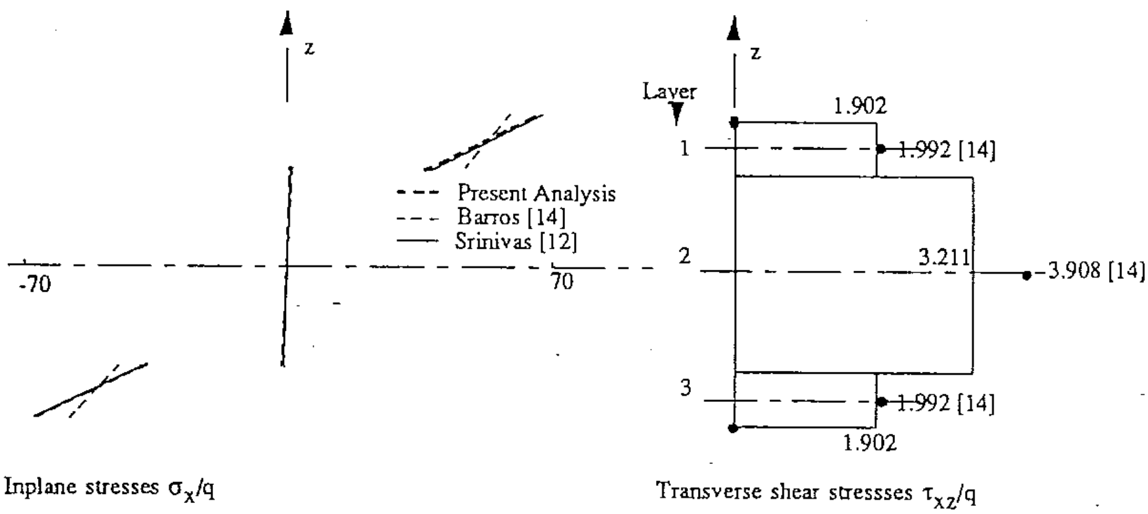

Fig. 5 - In-plane and transverse shear stresses- case 3

Table 4- Maximum in-plane stresses σ_x/q (x,y=0) and displacements in a simply supported square sandwich plate (case 4) $h_1/h=0.1$, $h_2/h=0.8$, $h_3/h=0.1$, $c_1=50$, $c_3=10$

In-plane stresses ....>		--Layer 1--		--Core	--	--Layer 3--	
Source	w*	Top	Bottom	Top	Bottom	Top	Bottom
Srinivas[exact]	28.297	91.610	37.307	0.76477	-4.4503	-44.512	-55.207
Figueiras	28.432	82.238	47.287	0.9457	-4.6464	-46.464	-53.455
Barros	28.300	81.774	47.013	0.9404	-4.6193	-46.193	-53.148
Kirchhoff	17.855	90.037	51.771	1.0354	-5.0871	-50.871	-58.524
Present analysis	28.260	90.440	35.950	0.719	-4.335	-43.350	-54.110

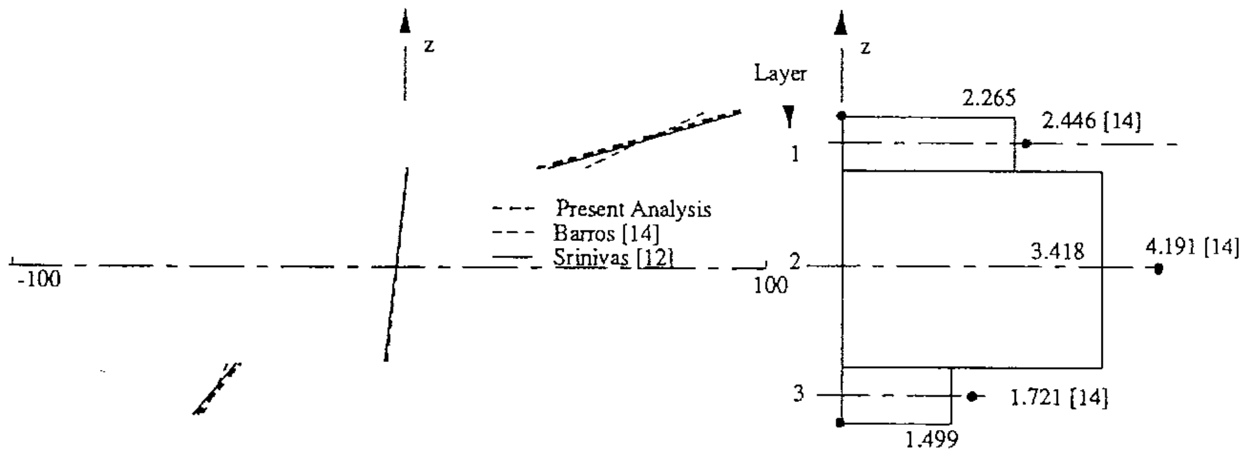

Fig. 6 - In-plane and transverse shear stresses- case 4

Table 5- Maximum in-plane stresses σ_x/q (x,y=0) and displacements in a simply supported square sandwich plate (case 5) $h_1/h=0.1$, $h_2/h=0.6$, $h_3/h=0.3$, $c_1=10$, $c_3=10$

In-plane stresses ....>		--Layer 1--		--Core	--	--Layer 3--	
Source	w*	Top	Bottom	Top	Bottom	Top	Bottom
Srinivas[exact]	34.549	63.756	50.236	5.0409	-0.33609	-3.4028	-43.771
Figueiras	34.921	63.504	53.218	5.3218	-0.8407	-8.497	-39.354
Barros	34.706	63.061	52.856	5.2856	-0.8436	-8.436	-39.089
Kirchhoff	25.342	67.060	56.199	5.6199	-0.89729	-8.9729	-41.559
Present analysis	34.533	62.540	49.908	4.908	-0.3180	-3.180	-43.210

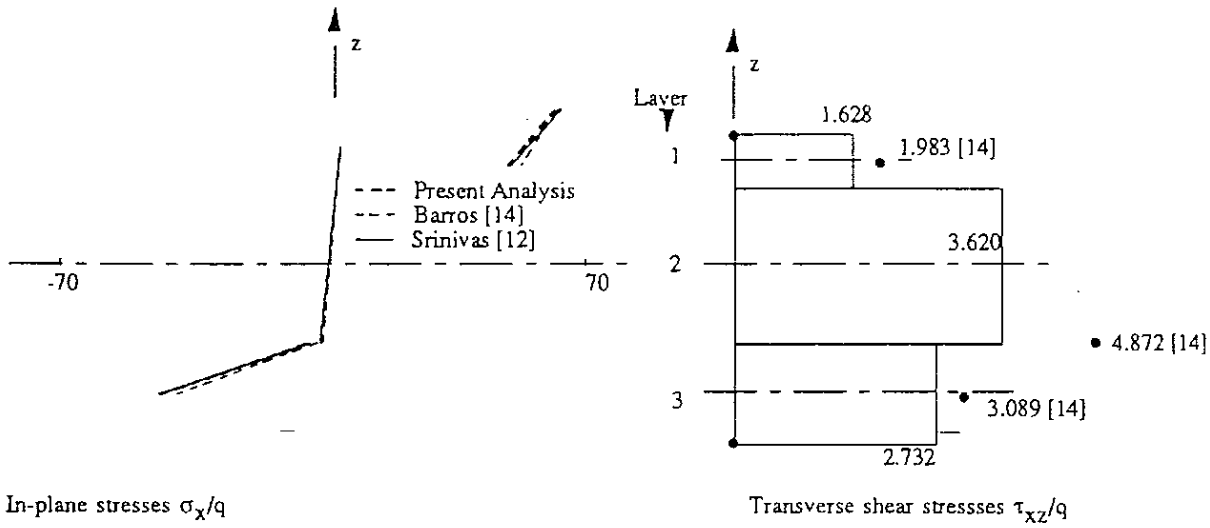

Fig. 7 - In-plane and transverse shear stresses- case 5

CONCLUSIONS

A very accurate model for sandwich plate finite element analysis is presented. This model is more accurate than the Mindlin first-order shear-deformation model[12,14] and much superior than Kirchhoff models. Both deflections and inplane stresses are precisely predicted, while transverse shear stresses are also predicted very precisely in each layer middle-surface. Shear correction factors, usually employed in first order shear deformation theories, like the Mindlin theory, are not used in the present approach.

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