Non linear probabilistic analysis of reinforced concrete structures

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Summary

The behaviour of reinforced concrete bridges is doted of uncertainty as main parameters, like the ones related to material properties, are variable and not deterministic. In order to take this into consideration, a non linear probabilistic analysis should be developed. This paper presents an application of it on the evaluation of the structural behaviour of two batches of reinforced concrete beams, which were loaded, in laboratory, up to failure. However, when performing it, a previous attention must be paid to the intervenient parameters. In fact, on the one hand it is important to consider the highest number of parameters as possible but, on the other hand, this implies higher computational costs. In order to avoid this, it is essential to identify, by developing a sensitivity analysis, all critical parameters. A comparison of numerical results with obtained experimental data is executed, being, the advantages of such kind of analysis, pointed out.

Keywords: reinforced concrete structures; structural behaviour; sensitivity analysis; uncertainty; non linear probabilistic analysis.

1. Introduction

When evaluating the behaviour of a reinforced concrete structures it is, in many situations, desirable to develop a non linear numerical analysis, due to the non linearity of the behaviour of existent materials. On other way, and in order to consider the uncertainty of main parameters, probabilistic techniques should be also introduced on such kind of analysis, leading so to a non linear probabilistic analysis. Examples of this kind of analysis can be seen in [1, 2, 3]. In this paper the executed probabilistic non linear analyses, are realized using SARA platform, which is a combination of the software ATENA (non linear structural analysis software) [4] and FREET (reliability analysis software) [5, 6, 7].

Such analyzes falls upon the evaluation of the behaviour of two batches of reinforced concrete beams, tested, in laboratory, up to failure. A deterministic numerical model is firstly developed, calibrated and simplified. A sensitivity analysis, in order to identify critical parameters, is then executed. Afterwards, and once identified the probabilistic density function and the correlation coefficients of those parameters, a full probabilistic analysis is developed. From such analysis, the probabilistic density function of each output parameter is obtained. In other way, experimental data was grouped by beam typologies and characterized by a random density function. An index, which characterizes, in a more rigorous way, the approximation between both curves, is then defined.

2. Experimental Analysis

2.1 Simply supported beams

The first batch of laboratory tested beams, a group of 36 elements, were four point loaded, till failure. Such beams, with a rectangular section of 75 x 150 mm and a 1.50 m span length were concreted at same time. The test scheme was also the same for all beams. The loads were applied by an electro mechanic actuator with 150 kN capacity, and were positioned at 1/3 and 2/3 of the span.

Each beam was supported in two elements, placed symmetrically in relation to their symmetric axle. While one of those elements restricts only the vertical displacement, the other restricts both the vertical and the horizontal one. Figure 1 presents the pattern scheme of the isostatic beams laboratory tests [7].

The beams present the same concrete (C25/30) and longitudinal steel reinforcement (S500B), accordingly to [8]. For longitudinal reinforcement it was used diameters of reinforcing bars of $\phi 6$, $\phi 8$ and $\phi 10$. For transversal reinforcement a diameter of $\phi 4$ was considered. The concrete cover was considered to be 1.0 cm in all sides of the beam.

The laboratory test was developed with displacement control, using a displacement transducer which is positioned inside the electro mechanic actuator, at a velocity of 0.0005 mm/s. It was measured the applied load, by a load cell positioned inside the actuator, and the mid span displacement, by a displacement transducer (Linear Variable Differential Transformer - LVDT). Figure 1 presents an image of such tests.

The repeatability was then achieved, by using the same materials and providing the same laboratory conditions during all the developed tests. However, each beam presents a specific longitudinal reinforcement and space between transversal reinforcement. Taking this into consideration, tested beams were divided by typologies. In order to study different failure modes, 4 typologies were selected, representing 19 tested beams (Table 1).

Typology 1 presents a low percentage of longitudinal reinforcement and, consequently, a bending failure mode with a high steel extension, may appear. Typology 3 and 4 present a higher quantity of steel reinforcement and, as a result, they are more susceptible to shear failure modes. However, for typology 4, there are no shear failure modes due to the small space between stirrups. Typology 2, present a quantity of longitudinal reinforcement and of stirrups within the normal one, being the obtained failure mode a bending one, with concrete crushing (Figure 1).

a b c

Fig. 1: a) Scheme of developed laboratory tests; b) Bending failure mode; c) Shear failure mode

Table 1: Analyzed typologies

Typologies	Longitudinal reinforcement	Transversal reinforcement	Failure modes	Number of tested beams
1	$3\phi 6$	@0.10	Bending – High strain in reinforcing steel	2
2	$2\phi 8$	@0.10	Bending – Concrete crushing	3
3	$2\phi 10$	@0.10	Shear / Bending – Concrete crushing	8
4	$2\phi 10$	@0.075 (Supports) + @0.158 (Mid span)	Bending – Concrete crushing	3

2.2 Mixed supported beams

The second battery of laboratory tested beams is constituted by a group of 32 elements that were loaded, in laboratory, till failure. Such beams, with a rectangular section of 75 x 150 mm and a span of 1.50 m were concreted at same time. The test scheme was also the same for all beams. The loads were applied by an electro mechanic actuator with 150 kN capacity, and were positioned at 1/3 and 2/3 of the span.

Those beams are supported in two elements, a simply and a partial clamp support. While one of those elements restricts only the vertical displacement, the other restricts both the vertical and the horizontal one and, also, partially the bending moment. In this situation there is no symmetry and the structure is one degree hyperstatic. Figure 2 presents the pattern scheme of the respective laboratory tests and a detail of the partial clamp support, materialized by two steel plates which were tightened by two bolts into the structures.

The beams present the same concrete (C25/30) and longitudinal steel reinforcement (S500B), accordingly to [8]. For longitudinal reinforcement it was used diameters of reinforcing bars of ϕ6 and ϕ8. For transversal reinforcement a diameter of ϕ4 was considered. The concrete cover was variable in the longitudinal way and fixed as 1.0 cm in the transversal direction.

The laboratory test was developed with displacement control, using a displacement transducer which is positioned inside the electro mechanic actuator, at a velocity of 0.0005 mm/s. It was measured the applied load, by a load cell positioned inside the actuator, the mid span displacement, by a displacement transducer (Linear Variable Differential Transformer - LVDT) and the reaction at simply support, by a load cell with 200 kN of capacity. Figure 2 presents an image of such tests.

As it was measured the applied load by the actuator and the load cell, positioned in the simply support, it was possible to determine the bending moment at the clamp support, by using the moment static equilibrium. By calculating the clamp bending moment, it will be possible to control the structural behavior, respectively, the clamp restraint degree, within the laboratory test. The results obtained from this analysis will confirm the partial clamp effect of the support.

The repeatability was achieved, by using the same materials and providing the same laboratory conditions during all developed tests. However, each beam presents a specific longitudinal reinforcement, space between transversal reinforcement and longitudinal concrete cover. Taking this into consideration the tested beams were divided by typologies.

The obtained failure modes were the same, for all beam typologies, respectively, a bending failure with concrete crushing. This is essentially due to the fact that it was used a small space between stirrups, which reduced the probability of obtaining a shear failure mode. The structural behaviour consists in a first elastic phase. Then, when the first hinge appears, on the partial clamp support, the beam stiffness reduces and the structural response becomes non linear. Later on, the second hinge appears, above the application point of the load, which is located near the simply support. A mechanism of plastic hinges appears, and the global failure of the structure is achieved (Figure 2).

In this situation, only one typology, defined by two beams with longitudinal reinforcement of 2ϕ8 (superior) and 3ϕ6 (inferior), stirrups spaced of 0.08 m (mid span) and 0.03 m (supports), and with a cover of 2 cm, was analyzed.

a b c

Fig. 2: a) Scheme of developed laboratory tests; b) Failure mechanism; c) Partial clamp support

3. Numerical Analysis

3.1 Numerical model

3.1.1 Simply supported beams

The simply supported beams, tested at laboratory, and which scheme is presented at Figure 1, were studied by using a plane stress field non linear numerical model. Materials, concrete and reinforcing steel, were modelled taking into consideration values from [8]. It was adopted the same steel for longitudinal and transversal reinforcement. A uniform mesh of quadrilateral elements was adopted.

The 4 typologies, which were experimentally studied in laboratory (Table 1), were numerically studied. In order to simulate the laboratory test, which was executed till failure, some considerations were taken. It was adopted a displacement control numerical test and two load cases, respectively, one representing the real supports (LC1) and one other representing the applied displacement (LC2), accordingly to Figure 3.

In order to develop the non linear analysis, a non linear search algorithm was adopted. The used algorithm, to work with the displacement control test, was the Newton-Raphson one. An arc-length search algorithm was also considered near failure load. During the numerical test, it was monitored the applied load and the midspan displacement.

LC1 LC2

Fig. 3: Adopted load cases (LC1 and LC2).

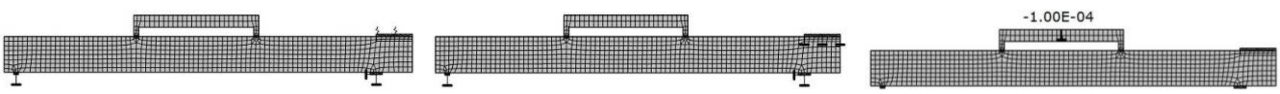
3.1.2 Mixed supported beams

The mixed supported beams, simply supported in one side and partially clamped on the other, tested at laboratory, and which scheme is presented at Figure 2, were studied by using a plane stress field non linear numerical model. The used materials were characterized by means of laboratory characterization tests (Table 4). A uniform mesh of quadrilateral elements was adopted.

The clamp support is simulated, in a plane stress field environment, by restricting both the vertical and the horizontal displacements. As the clamp effect is not total since beginning, due to concrete accommodation, vertical spring elements, which work only in compression, are adopted. Consequently, the vertical restrain of the steel plate is only partial in the beginning of the test. The total vertical restrain is only warranted in one more advanced phase of the test.

In order to simulate the laboratory test, which was executed till failure, some aspects were broached. It was adopted a displacement control numerical test and it were considered three load cases (Figure 4), respectively, one representing the real supports with spring elements placed at clamp support (LC1), the other representing the real supports and totaling restraining the vertical displacement at clamp support (LC2) and one other representing the applied displacement (LC3).

To develop the non linear analysis of reinforced concrete beams, a non linear search algorithm was used. The used algorithm, to work with the displacement control test, was the Newton-Raphson one. An arc-length search algorithm was also considered near failure load. During the analysis it was directly monitored the displacement at beam mid span, the applied load and the reaction at left simply support. In an indirect way, the bending moment at clamp support was also determined.



LC1 LC2 LC3

Fig. 4: Adopted load cases (LC1, LC2 and LC3).

3.2 Sensitivity analysis

3.2.1 Simply supported beams

The previously developed numerical model was then applied on the analysis of typologies which are described in Table 1. From such analysis, three types of failure modes, respectively, bending mode with high strain in reinforcing steel or with concrete crushing, and shear mode, were obtained (Figure 5).

Bending failure Shear failure

Fig. 5: Failure modes.

In order to study the influence of some parameters, namely the ones related to material properties, on the structural behavior, a sensitivity analysis is performed. The following results were obtained:

Typology 1: 1) In general the obtained failure mode is bending with concrete crushing. However, it is verified that, for low values of steel yield strength (σ_y), a bending failure mode of bending with high extension in the steel appears; 2) Initial tangent stiffness grows in same order of elasticity modulus (E_c) and in reverse order of tensile strength (f_t); 3) Cracking load grows in same order of elasticity modulus (E_c) and of tensile strength (f_t); 4) Post-cracking stiffness and failure load grows in same order of compressive strength (f_c), of fracture energy (G_f), of steel elasticity modulus (E_s) and of steel yield strength (σ_y); 6) Failure load grows in same order of steel yield strength (σ_y).

Typology 2: 1) In general the obtained failure mode is bending with concrete crushing. However, it is verified that, for high values of tensile strength (f_t) and of fracture energy (G_f) a failure mode of bending with high extension in the steel appears; 2) Initial tangent stiffness grows, slightly, in same order of elasticity modulus (E_c) and in reverse order of tensile strength (f_t); 3) Cracking load grows, slightly, in same order of elasticity modulus (E_c) and in same order of tensile strength (f_t); 4) Post-cracking stiffness grows in same order of compressive strength (f_c), of fracture energy (G_f), of steel elasticity modulus (E_s) and of steel yield strength (σ_y); 5) Failure load grows in same order of compressive strength (f_c), of fracture energy (G_f) and of steel yield strength (σ_y);

Typology 3: 1) In general the obtained failure mode is shear one. It is verified, however, that for high values of elasticity modulus (E_c), of tensile strength (f_t), of compressive strength (f_c) and of steel elasticity modulus (E_s), as well as for low values of fracture energy (G_f) and of steel yield strength (σ_y), the failure mode is by bending with concrete crushing; 2) Initial tangent stiffness grows, slightly, in same order of elasticity modulus (E_c) and in reverse order of tensile strength (f_t); 3) Cracking load grows, slightly, in same order of elasticity modulus (E_c) and of tensile strength (f_t); 4) Post-cracking stiffness grows, slightly, in same order of elasticity modulus (E_c), of compressive strength (f_c), of fracture energy (G_f), of steel elasticity modulus (E_s) and of steel yield strength (σ_y), and, slightly, in a reverse order of tensile strength (f_t); 4) Failure load grows in same order of compressive strength (f_c), of fracture energy (G_f) and of steel yield strength (σ_y);

Typology 4: 1) In general the obtained failure mode is shear one. It is verified, however, that for high values of elasticity modulus (E_c), of tensile strength (f_t), and of compressive strength (f_c), as well as for low values of fracture energy (G_f) and of yield strength (σ_y), the failure mode is by bending with concrete crushing; 2) Initial tangent stiffness grows, slightly, in same order of elasticity modulus (E_c) and in reverse order of tensile strength (f_t); 3) Cracking load grows, slightly, in same order of elasticity modulus (E_c) and of tensile strength (f_t); 4) Post-cracking stiffness grows, slightly, in same order of elasticity modulus (E_c), of compressive strength (f_c), of fracture energy (G_f), of steel elasticity modulus (E_s) and of steel yield strength (σ_y); 4) Failure load grows in same order of compressive strength (f_c), of fracture energy (G_f) and of yield strength (σ_y).

As a general conclusion, it is verified that while the influence of parameters like elasticity modulus (E_c) and tensile strength (f_t) is observed for lower loads, parameters as the compressive strength (f_c), the fracture energy (G_f), the steel elasticity modulus (E_s) and the steel yield strength (σ_y) present an influence for loads placed between 25% - 100% of failure load. Transversal reinforcement parameters do not affect the structural behavior.

3.2.2 Mixed supported beams

The numerical model, which was previously developed, was, further on, applied on the study of the analyzed typology. From respective analysis it was obtained a failure mechanism, characterized by the formation of two plastic hinges, the first one beside the applied load, which is placed nearer the simply support, and the second one at clamp support (Figure 6). The obtained failure mode is bending one, with concrete crushing.

First hinge　　Second hinge

Fig. 6: Failure mechanism.

A sensitivity analysis was then executed with the aim of studying the influence of some parameters, namely the ones related to material properties, on the structural behavior. From this analysis, it is verified that the superior longitudinal and transversal reinforcement parameters almost not affect the structural behavior.

The following results were obtained: 1) In general the obtained failure mode is bending with concrete crushing. The respective failure appears earlier during the analysis as the compressive strength (f_c) diminishes, and with the slightly increase of the elasticity modulus (E_c), of the tensile strength (f_t), of the steel elasticity modulus (E_s) and of the steel yield strength (σ_y). It is, also, verified that for high values of fracture energy (G_f) the obtained failure mode is the shear one; 2) Initial tangent stiffness grows in same order of tensile strength (f_t); 3) Cracking load grows in same order of tensile strength (f_t); 4) Post-cracking stiffness grows, slightly, in same order of compressive strength (f_c), of fracture energy (G_f), of steel elasticity modulus (E_s) and of steel yield strength (σ_y) and, in a reverse order of tensile strength (f_t); 5) Failure load grows, slightly, in same order of compressive strength (f_c), of fracture energy (G_f) and of steel yield strength (σ_y).

As a general conclusion, it is verified that the most important parameters are the tensile strength (f_t), the fracture energy (G_f), the steel elasticity modulus (E_s) and the steel yield strength (σ_y), presenting, both an influence for loads which are placed between 25% - 100% of failure load.

3.3 Probabilistic analysis

3.3.1 Simply supported beams

In order to evaluate the structural behaviour of tested beams, considering the uncertainty of some input variables, a non linear probabilistic analysis is developed. Such analysis is based in previously described numerical model and, takes into consideration, the variability of critical parameters, identified by the sensitivity analysis. In this situation, there was no statistical characterization of such parameters and so, it was needed to define their probabilistic density functions (Table 2) and correlation coefficients, by using the existent bibliography [3, 9]. Afterwards, a full probabilistic analysis, using simulation techniques, was developed.

Table 2: Statistical values for critical parameters probability density function.

Parameters		Distribution	Average value	COV
Elasticity modulus [GPa]	E_c	Normal	30.000	0.100
Compression strength [MPa]	f_c	Normal	28.000	0.100
Tensile strength [MPa]	f_t	Normal	1.500	0.200
Fracture energy [N/m]	G_f	Normal	51.400	0.100
Steel yield strength [MPa]	σ_y	Normal	540.000	0.050
Steel elasticity Modulus [GPa]	E_s	Normal	200.000	0.050

From developed full probabilistic non linear analysis, a probability density function for numerical failure load is obtained. Also, for each analyzed typology it is possible to determine the same function but for experimental failure load. In order to study the approximation of both functions, an approximation index (θ) was used. Such index is defined by the following expressions:

$$\theta = \mu_Z / \sigma_Z \quad (1)$$

being Z the approximation function (Experimental Value – Numerical Value), and μ_Z and σ_Z the respective average and standard deviation value. Table 3 presents the main results of this analysis.

Table 3: Results from probabilistic non linear analysis.

Typology	Experimental failure load [kN]			Numerical failure load [kN]			Approximation index (θ)
	Distribution law	Average value [kN]	Standard deviation [kN]	Distribution law	Average value [kN]	Standard deviation [kN]	
1	Normal	24.805	0.247	Normal	24.572	1.028	0.220
2	Normal	29.983	1.333	Normal	28.606	1.191	0.772
3	Normal	41.278	4.522	Normal	36.642	3.568	0.807
4	Normal	44.007	2.104	Normal	37.863	3.613	1.473

From analysis of Table 3, it is possible to conclude that typology 1 and 3, related to bending failure modes, present a lower approximation index, which means that both probability density functions, for experimental and numerical data, are near each other. In other way, typology 3 and 4 associated to shear failure modes, present a higher liability index. In fact, the shear failure mode is directly correlated to a large possibility of structural responses.

3.3.2 Mixed supported beams

With the objective of evaluating the behaviour of tested beams a non linear probabilistic analysis is executed. This analysis considers the developed numerical model and, the randomness of critical parameters. In this case, values of probabilistic density functions (Table 4) and of correlation coefficients were defined accordingly to characterization tests. Afterwards, a full probabilistic analysis was developed. Table 5 presents the main results of this analysis.

Table 4: Statistical values for critical parameters probability density function.

Parameters		Distribution	Average value	Standard deviation
Elasticity modulus [GPa]	E_c	Normal	28.011	0.075
Compression strength [MPa]	f_c	Normal	30.769	0.050
Tensile strength [MPa]	f_t	Normal	2.669	0.100
Fracture energy [N/m]	G_f	Normal	103.912	0.100
Steel yield strength [MPa]	σ_y	Normal	461.028	0.050
Steel elasticity modulus [GPa]	E_s	Normal	190.542	0.075

Table 5: Results from probabilistic non linear analysis.

Parameter	Experimental data			Numerical results			Approximation index (θ)
	Distribution law	Average value	Standard deviation	Distribution law	Average value	Standard deviation	
Failure load [kN]	Normal	29.390	1.598	Normal	30.013	1.918	0.250 (-)
Bending moment [kN.m]	Normal	6.904	0.674	Normal	4.346	1.856	1.299

From analysis of Table 5 it is possible to conclude that, in a general way, and for both the failure load and the bending moment at clamp support, obtained experimental data is near the numerical results, as the approximation index is very low.

4. Conclusions

This paper presents an application of a full probabilistic non linear analysis of two batches of reinforced concrete beams which were loaded in laboratory till failure. The first batch is characterized by simply supported beams, while the second one is defined by mixed supported beams, simply supported in one side and partially clamped on the other. The behaviour of those structures, taking into consideration the uncertainty of main parameters, is then studied.

The respective analysis is divided on following steps: 1) Develop a numerical model; 2) Execute a sensitivity analysis to identify critical structural parameters; 3) Determine a random distribution function and correlation coefficient between each parameter; 3) Develop a full probabilistic non linear analysis; 4) Determine the probabilistic density function for both numerical and experimental results; 5) Calculate approximation index (θ), which relates the proximity of both results.

Main conclusions of this analysis are: 1) The applicability of the developed method through the analysis of the behaviour of reinforced concrete structures, as bridges; 2) The comparison between numerical and experimental data, using this method, is made in a consistent way, as the variability of most important parameters is considered; 3) For these two batches of tested beams, the obtained numerical results are near the experimental data; 4) In some situations, like the ones related to shear failure modes, even in the presence of a detailed numerical model, the variability of responses is so high that it is very difficult to evaluate the structural behaviour.

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