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Tests for the Null Hypothesis of Cointegration: a Monte Carlo Comparison

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Abstract

The aim of this paper is to compare the relative performance of several tests for the null hypothesis of cointegration, in terms of size and power in finite samples. This is carried out resorting to Monte Carlo simulations, considering a range of plausible data-generating processes. As of this writing, there is no study providing guidance on the use of this type of procedures in empirical situations, with the exception of the limited studies of McCabe *et al.* (1997) and Haug (1996). We also analyse the impact on size and power of choosing different procedures to estimate the long run variance of the errors. We found that the parametrically adjusted test of McCabe *et al.* (1997) is the most well-balanced test in terms of power and size distortions.

Key Words: Cointegration; Tests; Monte Carlo

JEL Classification: C12; C22

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1 Introduction

The problem of testing for cointegration among economic variables has been a central issue in the literature of cointegrated time series. Usually, testing is carried out by means of residual based procedures that consist of extensions of unit root tests, that is, one tests whether the residuals from the cointegrating regression contain a unit root or, by contrast, are $I(0)$. The null hypothesis is, thus, of no cointegration, against the alternative of cointegration. However, this approach seems unnatural, especially if one takes the existence of a long run equilibrium relationship between the variables as the hypothesis of interest, stemming from economic theory (for example, the link between consumption and disposable income). With cointegration as the null hypothesis, one would reject it only if the data would provide strong evidence against the maintained hypothesis, unlike the situation where the hypotheses are reversed.

There have been some attempts to test directly for cointegration[1]. One route has been to extend existing univariate test procedures to test for an $I(0)$ null against an $I(1)$ alternative, such as the tests advocated by Kwiatkowski, Phillips, Schmidt and Shin (1992, henceforth KPSS), Leybourne and McCabe (1994) and Saikkonen and Lukkonen (1993), among others (see Stock, 1994 for a survey). These tests emerged from the apparently unrelated literature on testing for unit moving average roots (see Tanaka, 1990, for example) and testing time-varying parameters (see Nabeya and Tanaka, 1988, *inter alia*), and are one-sided LM tests with asymptotic optimal local power properties (see Stock, 1994). In this way, Leybourne and McCabe (1993), Shin (1994), Harris and Inder (1994) and McCabe, Leybourne and Shin (1997) devised tests that generalize the so-called KPSS statistic to the context of cointegration. The main difference between these versions lies on the proposed estimation method to obtain the residuals and the variance, subsequently used to construct the test statistic.

A related test was suggested by Hansen (1992), although it was primarily conceived to test for parameter instability in cointegration models. Under the alternative hypothesis, each coefficient in the model is allowed to follow a random walk, so by testing the stability of the estimated parameters, one is also testing for cointegration. On the other hand, Park (1990) developed a test for the null of cointegration based on the addition of superfluous regressors to the cointegrating regression. More recently, Xiao (1999) proposed a residual based test that examines the fluctuation of the residuals from a regression.

The aim of this paper is to compare the relative performance of these testing approaches, in terms of size and power in finite samples. This is carried out resorting to Monte Carlo simulation, considering a range of plausible data-generating processes. As of this writing, there

[1] Despite the "conceptual pitfalls" discussed by Phillips and Ouliaris (1990).

is no study providing guidance on the use of this type of procedures in empirical situations, with the exception of the limited studies of McCabe *et al.* (1997) and Haug (1996). Moreover, it would be useful to know which tests are best suited for conducting confirmatory analysis, that is, applying tests for the null of cointegration in conjunction with the standard tests for the null of no cointegration. If the two approaches give consistent results (i.e., there is an acceptance *and* a rejection of the nulls), one may conclude whether the series are cointegrated or not, whereas if both tests either reject or accept their respective null hypotheses, the results are inconclusive. See, for example, Shin (1994) and Maddala and Kim (1998) for a discussion, as well as Charemza and Syczewska (1998) and Carrion-i-Silvestre, Sansó-i-Rosselló and Ortuño (2001) for an application to the univariate case.

Besides the distinctions in the way each test is constructed, another important issue investigated in this paper is the impact on size and power of choosing different procedures to estimate the long run variance of the errors. Most of the tests analyzed here depend on the estimation of this nuisance parameter, and it is well known that the use of semi-parametric estimators may lead to substantially oversized tests in samples of small size. Some results are known for stationarity tests (see Lee, 1996 and Hobijn, Franses and Ooms, 1998), but there is little evidence concerning tests for cointegration, although one may expect similar conclusions.

The paper is organized as follows. Section 2 presents a general model of cointegration and reviews the methods for estimating the long run variance. In section 3, the tests for the null of cointegration are presented. The DGPs for the Monte Carlo experiments, as well as the simulation results, are analysed in section 4. Section 5 summarizes and concludes.

2 The Model

Since each test was derived under a specific model, it is difficult to present a common formulation for all tests. Nevertheless, we may write a general model as

$$y_t = \alpha + x_t^{'}\beta + u_t, \tag{1}$$

where y_t is a scalar $I(1)$ process and x_t is a vector $I(1)$ process of dimension k, such that $\Delta x_t = v_t$, v_t being a k-vector stationary process. For simplicity, we concentrate on the single equation specification with an intercept, although more general specifications could be considered (e.g., containing time trends). The variables y_t and x_t are said to be cointegrated if u_t is $I(0)$, whereas if u_t is $I(1)$ there is no long run equilibrium relationship between y_t and x_t.

Some tests differ on how the disturbance term is specified under the alternative hypothesis of no cointegration, as will be seen later. Under the null hypothesis of cointegration, $\zeta_t = (u_t, v_t^{'})'$

follows a general stationary process obeying some mild regularity conditions (as stated in Phillips and Durlauf, 1986, for example) and satisfies a multivariate invariance principle, such that

$$T^{-1/2}\sum_{t=1}^{[Tr]}\zeta_t \Rightarrow B(r), \qquad \text{as } T\to\infty.$$

Here, "$\Rightarrow$" denotes weak convergence, [.] is the integer part operator and $B(r)$ is a $k+1$ dimensional Brownian motion defined on $r\in[0,1]$, with long run covariance matrix $\Omega = \lim_{T\to\infty} Var(T^{-1/2}\sum\zeta_t)$. These conditions allow for any stationary and invertible ARMA process, possibly with heterogeneous innovations.

We partition $B(r)$ conformably with $\zeta_t=(u_t,v_t^{'})'$ as $B(r)=[B_1(r),B_2(r)']'$ and

$$\Omega=\begin{bmatrix}\omega_{11} & \Omega_{12}\\ \Omega_{21} & \Omega_{22}\end{bmatrix},$$

where ω_{11} is the long run variance of u_t. We will restrict our attention to the case where cointegration among the regressors is excluded, so that Ω_{12} is positive definite. If we allow for correlation between u_t and v_t, then an asymptotically efficient estimation method should be used to account for the endogeneity of the regressors, such as the fully-modified OLS method of Phillips and Hansen (1990) or the leads and lags OLS estimator of Saikkonen (1991), for example. In this case, we are also interested in the long run variance of u_t conditional on v_t, defined by $\omega_{1.2}=\omega_{11}-\Omega_{12}\Omega_{22}^{'}\Omega_{21}$, which plays an important role in the construction of the test statistics studied in this paper.

The long run variance is usually estimated from

$$s^2(l)=\hat{\gamma}_0+2\sum_{j=1}^{l}w(j,l)\hat{\gamma}_j, \tag{2}$$

where $\hat{\gamma}_j = T^{-1}\sum_{t=j+1}^{T}\hat{u}_t\hat{u}_{t-j}$ $(j=0,1,...T)$ is the estimate of the j-th autocovariance and $w(j,l)$ is a kernel function depending on a bandwidth parameter (or truncation lag) l. Thus, the estimation depends on the chosen kernel and on the procedure to truncate the number of autocovariances.

Here, we compare three widely used kernels, namely the Parzen, the Bartlett and the Quadratic Spectral (QS) kernels (see Andrews, 1991). Concerning the truncation lag selection, we compare an exogenous procedure suggested by KPSS (1992), among others, and the data-dependent bandwidth selection procedure of Andrews (1991). The first method selects l based on a deterministic function of the sample size T, namely $l(x)=\text{integer}[x(T/100)^{1/4}]$ (see Schwert, 1989, KPSS, 1992 and Lee, 1996). The problem here is the choice of x: if x is to large, the long run variance will be overestimated and the tests will have low power, whereas if

x is small, tests will tend to be oversized. We consider several values for x ($x = 4, 10, 12, 16$)[2]. The second method is data-based and non-parametric, consisting of obtaining, for a given kernel, the optimal bandwidth parameter sequence that minimizes asymptotic mean square errors. Andrews (1991) suggested the use of an automatic plug-in bandwidth estimator, which has the form

$$\begin{aligned} \hat{l} &= 1.1147[\hat{\alpha}(1)T]^{1/3}, && \text{(Bartlett kernel)} \\ \hat{l} &= 2.6614[\hat{\alpha}(2)T]^{1/5}, && \text{(Parzen kernel)} \\ \hat{l} &= 1.3221[\hat{\alpha}(2)T]^{1/5}, && \text{(QS kernel)}, \end{aligned}$$

where $\hat{\alpha}(1)$ and $\hat{\alpha}(2)$ are obtained from fitting AR(1) regressions to the residuals. This procedure has the advantage of removing the arbitrariness associated with the choice of l as in the first method.

Further refinements have been recently proposed, namely a "pre-whitening" procedure (see Andrews and Monahan, 1992) that filters the residuals with an AR regression in order to make them closer to white noise and then calculate the spectral density at the origin. However, this method renders null of cointegration tests inconsistent, since under the alternative of no cointegration (i.e., a unit root in the residuals) the AR estimate would be close to unity, the long run variance would tend to infinity and, thus, the tests statistics close to zero[3] (see discussion in Lee, 1996 and Shin, 1994).

Another possibility is the procedure proposed by Newey and West (1994), which is similar to the method of Andrews (1991), although they use non-parametric estimation to construct the automatic plug-in bandwidth estimator, instead of AR(1) regressions. The application of this method to stationarity tests is documented in Hobijn *et al.* (1998). Despite the obvious interest in comparing the Newey-West and Andrews's procedures, we will not pursue that here.

3 Tests for the Null Hypothesis of Cointegration

In this section, we will briefly describe the cointegration tests examined in the subsequent Monte Carlo study. As said earlier, we may group the tests into four different categories, according to the way the test is constructed.

[2] $l = 10$ is the value recommended by Shin (1994) for the computation of his test.

[3] This fact helps to explain the poor performance of null of cointegration tests in Haug's (1996) study, since he uses prewhitening to construct the test statistics.

3.1 Variable addition test

Park (1990) suggested an approach for testing the null hypothesis of cointegration, which consists of including a set of superfluous regressors z_t in the cointegration regression, so that

$$y_t = \alpha + x_t^{'}\beta + z_t^{'}\delta + e_t. \tag{3}$$

If the variables in 1 are truly cointegrated, then the added regressors in 3 will not be significant, while the opposite holds if the regression is spurious. Standard significance tests (such as Wald) will be able to discriminate between the two situations. The test statistic may be written as

$$J_1 = \frac{RSS_1 - RSS_2}{\hat{\omega}_{1.2}}, \tag{4}$$

where RSS_1 and RSS_2 are, respectively, the residual sum of squares from 1 and 3. The denominator is an estimate obtained with a consistent estimator of the (conditional) long run variance of u_t. A particular advantage of this test is that under the null hypothesis of cointegration (i.e., $\delta = 0$), J_1 has a limiting $\chi^2(p)$ distribution, where p is the dimension of the set of superfluous regressors, therefore avoiding extensive tabulations. If the alternative is true, J_1 diverges at a rate dependent on the chosen bandwidth for $\hat{\omega}_{1.2}$.

Park (1990) recommends the inclusion of the most irrelevant polynomial time trends or artificially generated random walks as superfluous regressors. For the Monte Carlo experiments, we use $z_t = (t, t^2, t^3)$. Regarding the estimation method, the J_1 test may be implemented with any asymptotically efficient procedure[4], and we use the Canonical Cointegration Regression (CCR) method developed by Park (1992), which is similar to FM-OLS[5].

3.2 Fluctuation Test

A different approach is followed by Xiao (1999), by deriving a residual based test for the null of cointegration based on the fluctuation of the residuals $\hat{u}_t$ from the cointegrating regression. The idea is quite simple: if cointegration holds, the residuals will replicate the stationary behaviour of the errors and will display a limited amount of fluctuation, whilst if the residuals will fluctuate too much the converse should be true.

The fluctuation principle was originally proposed to study the stability of the estimated coefficients of a model (see Ploberger, Krämer and Kontrus, 1989, for example). Using the

[4]Some preliminary simulations using CCR and FM-OLS revealed that the estimation method has no impact on the performance of the test.

[5]With FM-OLS, the dependent variable is modified and then regressed on the regressors to obtain estimates free of nuisance parameters, whereas with CCR, both the dependent variable and the regressors are modified before applying OLS (see Park, 1992 for details).

FM-OLS method, Xiao (1999) constructs a statistic that is asymptotically free of nuisance parameters, based on the recursive estimates statistic

$$R_T = \max_{i=1...T} \frac{i}{\sqrt{\hat{\omega}_{1.2}T}} \left| \frac{1}{i}\sum_{t=1}^{i} \hat{u}_t^+ - \frac{1}{T}\sum_{t=1}^{T} \hat{u}_t^+ \right|, \tag{5}$$

where $\hat{u}_t^+$ are the residuals resulting from FM-OLS estimation. The limiting distribution of R_T is non-standard and depends only on the dimension of the set of regressors. Xiao (1999) tabulated R_T for models with no constant, with constant or with trend.

3.3 L_c Test

Hansen (1992) proposed some LM-type structural change tests in cointegrated models, making use of the FM-OLS method. A versatile feature of those tests is the possibility of using them as cointegration tests. In fact, if the alternative hypothesis is the intercept following a random walk, then structural change testing becomes cointegration testing, albeit with the null hypothesis of cointegration. Decomposing u_t in 1 such that $u_t = w_t + v_t$, being w_t a random walk and v_t a stationary term, the model then becomes

$$y_t = \alpha_t + x_t^{'}\beta + v_t, \tag{6}$$

with $\alpha_t = \alpha + w_t$, that is, the intercept "absorbs" the random walk w_t when there is no cointegration.

Having this fact in consideration, Hansen (1992) suggested the use of the statistic

$$L_c = T^{-1}\sum_{t=1}^{T} \hat{s}_t^{'}\hat{V}_t^{-1}\hat{s}_t, \tag{7}$$

to test the null of cointegration, where $\hat{s}_t$represents the scores of the FM-OLS estimates and $\hat{V}_t^{-1}$ is a weighting matrix based upon an estimate of the covariance matrix of the second-order errors. However, this statistic was designed to test the stability of the whole cointegration vector, so there are advantages in regarding a version that tests only (partial) structural change in the intercept. Hao (1996) has shown that such a test may be carried out by employing the KPSS statistic to test for the null of cointegration. This is considered next.

3.4 Tests based on the KPSS statistic

The tests previously presented are tests for a general null hypothesis of stationary errors (i.e. u_t in 1 is $I(0)$), against a general alternative of $I(1)$ errors. However, it is possible to formulate with more detail the behaviour of u_t, both under the null and the alternative. Assume for the moment that y_t and x_t do not cointegrate and that u_t in 1 may be decomposed into the sum of a random walk and stationary component,

$$u_t = \gamma_t + \varepsilon_t, \tag{8}$$

where the random walk is $\gamma_t = \gamma_{t-1} + \eta_t$, with $\gamma_0 = 0$[6] and η_t distributed as $i.i.d.(0, \sigma_\eta^2)$, while the stationary part ε_t is distributed as $i.i.d.(0, \sigma_\varepsilon^2)$[7] and is assumed independent of η_t (note the similarity with the model in 6).

Cointegration results from this formulation when $\sigma_\eta^2 = 0$, so that $\gamma_t = 0$ and no longer is a random walk. Therefore a test for cointegration has the null hypothesis $H_0 : \sigma_\eta^2 = 0$ against the alternative $H_1 : \sigma_\eta^2 > 0$. This is the well known unobserved components representation, although one could also write it as a moving average model (see Stock, 1994). Following KPSS (1992), an asymptotically equivalent test to the locally best invariant (LBI) test of H_0 against H_1 uses the LM statistic

$$L = T^{-2} \frac{\sum_{t=1}^{T} S_t^2}{s^2(l)}, \tag{9}$$

where S_t is the partial sum process $S_t = \sum_{t=1}^{i} \hat{u}_t$ of the residuals from 1 and $s^2(l)$ is a consistent estimate of the ω_{11}. Allowing for correlation between ε_t and v_t calls for the use of an efficient estimation method and the denominator should be replaced by $\hat{\omega}_{1.2}$, as discussed in section 2. Different versions of this approach, using distinct estimation methods, have been proposed, as can be seen next.

3.4.1 Leybourne-McCabe test

Generalizing KPSS (1992), Leybourne and McCabe (1993) suggested a simple version of 9 by considering an OLS regression of 1 and using the corresponding residuals $\hat{u}_t$ to construct the test statistic that we will denote as

$$LM = T^{-2} \frac{\sum_{t=1}^{T} (\sum_{j=1}^{t} \hat{u}_j)^2}{\hat{\omega}_{11}}. \tag{10}$$

Leybourne and McCabe (1993) suggested estimating $\hat{\omega}_{11}$ with a simple truncated autocovariances estimator. In our simulations, we employ the kernel estimators mentioned in section 2.

3.4.2 Harris-Inder Test

The previous test, by using OLS, does not take into account the potential problems that arise from second-order biases in the estimation of the cointegrating regression. Harris and Inder

[6] This implies no loss of generality, since (1) contains an intercept.

[7] The $i.i.d.$ assumption of ε_t may be relaxed so that ε_t may follow a stationary process as discussed for u_t and v_t in section 2.

(1994), using the FM-OLS method, suggested an extension of the KPSS test that considers this issue[8]. The test statistic may be written as

$$HI = T^{-2}\frac{\sum_{t=1}^{T}(\sum_{j=1}^{t}\hat{u}_j^{+})^2}{\hat{\omega}_{1.2}}, \tag{11}$$

where $\hat{u}_t^{+}$ represent the FM-OLS residuals as in 8. Note that an estimate of $\hat{\omega}_{1.2}$ is used rather than $\hat{\omega}_{11}$ as in 10, reflecting the fact that one is accounting for the possible endogeneity of x_t.

3.4.3 Shin Test

Another way of circumventing the problem of endogenous regressors is to use the estimation method advocated by Saikkonen (1991). The procedure consists of introducing past and future values of Δx_t, so that the regression becomes

$$y_t = \alpha + x_t^{'}\beta + \sum_{j=-n}^{n}\Delta x_{t-j}^{'}\pi_j + u_t^{*}. \tag{12}$$

The truncation parameter n should increase with T at an appropriate rate and may be chosen with any model selection criterion such as AIC or BIC. To simplify, we use the rule $n = [T^{1/3}]$, which is also common, and the same value is used for both leads and lags of Δx_t. Applying OLS to the modified regression in 12 will yield efficient estimates, see Saikkonen (1991) for a more detailed discussion.

A version of 9 may be constructed with the residuals $\hat{u}_t^{*}$ from 12, as

$$S = T^{-2}\frac{\sum_{t=1}^{T}(\sum_{j=1}^{t}\hat{u}_j^{*})^2}{\hat{\omega}_{1.2}}, \tag{13}$$

thus resulting the Shin test statistic. Shin (1994) recommends fixing $l = 10$ when estimating $\hat{\omega}_{1.2}$ as a compromise between size distortion and low power.

3.4.4 McCabe-Leybourne-Shin Test

Unlike all the tests discussed previously, which used a non-parametric procedure to correct for excess correlation in the disturbances, McCabe *et al.* (1997) devised a parametric approach to test for the null of cointegration. They extend the parametric adjustment procedure of Leybourne and McCabe (1994) to the cointegration case, by considering a different formulation for the error component in 8. In fact, they assume that u_t follows

$$\Phi(L)u_t = \gamma_t + \varepsilon_t, \tag{8'}$$

[8]Harris and Inder (1994) consider a slightly different model that permits correlation between the random walk and stationary components of the error.

where $\Phi(L) = 1 - \phi_1 L - ... - \phi_p L^p$ is a stable autoregressive polynomial of order p, with γ_t and ε_t as defined previously. Under the null hypothesis of cointegration u_t is a stationary AR(p) process, whereas if $\sigma_\eta^2 > 0$, u_t becomes non-stationary, with an ARIMA$(p, 1, 1)$ representation (see Leybourne and McCabe, 1994).

In order to implement the test, McCabe *et al.* (1997) advocate the use of Saikkonen's dynamic least squares method to estimate 1, but the autoregressive coefficients ϕ_j in 14 should be obtained by maximum likelihood by fitting an ARIMA$(p, 1, 1)$ model to $\hat{u}_t^*$, the residuals from 12. In the simulation study, p is fixed ($p = 2$), since the test does not appear to be sensitive to the order of the autoregression[9]. The test statistic is then constructed with the "second stage" residuals $\hat{\varepsilon}_t = \hat{u}_t^* - \sum_{i=1}^p \phi_i \hat{u}_{t-i}^*$ from 14 as

$$MLS = T^{-2} \frac{\sum_{t=1}^T (\sum_{j=1}^t \hat{\varepsilon}_j)^2}{\hat{\sigma}^2}, \tag{14}$$

with $\hat{\sigma}^2 = T^{-1} \sum_{i=t}^T \hat{\varepsilon}_t$ being a consistent estimator of σ_ε^2.

This test has, at least theoretically, some advantages comparatively to other KPSS versions. Indeed, the test is consistent at a faster rate, i.e. $O(T)$, and does not depend on any lag truncation parameter. This should be apparent even in terms of its finite sample performance. It also allows for cointegration among the regressors, unlike othe tests. However, according to Hobijn *et al.* (1998), this test is not consistent for the alternative of a pure random walk, although this has recently been disputed by Lanne and Saikkonen (2000) for its univariate version.

4 Monte Carlo Simulations

To evaluate the finite sample performance of the null of cointegration tests discussed above, we develop a series of Monte Carlo experiments. Of course, direct comparisons of the relative performances of the tests may be difficult, since they are affected by the choice of the DGPs. Nevertheless, we try to provide some tentative remarks supported by the results of the simulation study. The general DGP is similar to McCabe *et al.* (1997) and is based on the models previously presented:

$$\begin{aligned} y_t &= \alpha_t + x_t + \varepsilon_t, \\ \alpha_t &= \alpha_{t-1} + \eta_t, \qquad \alpha_0 = 1, \qquad \eta_t \sim i.i.d.(0, \sigma_\eta^2), \\ x_t &= x_{t-1} + \zeta_t, \qquad \zeta_t \sim i.i.d.(0, 1), \end{aligned} \tag{15}$$

[9] This was confirmed with some preliminary simulations and is also evident from the results in McCabe *et al.* (1997), although they recommend a data-dependent selection criterion, with general-to-specific testing.

$$\varepsilon_t \quad = \quad \rho\varepsilon_{t-1} + \omega_t - \theta\omega_{t-1}, \qquad \omega_t \sim i.i.d.(0,1),$$

with η_t independent of ζ_t and ε_t. The parameters are allowed to take values $\rho = (0, 0.5, 0.9, 1)$ and $\theta = 0.4$ (with $\rho = 0.5$). It is also considered the effects of correlation between the errors (endogeneity of regressors), with $corr(\zeta_t, \omega_t) = \gamma = 0.7$ (with $\rho = 0$ and $\theta = 0$, for simplicity) as in McCabe *et al.* (1997). The selected sample sizes are $T = 100$ and 200 and the number of replications is 10000. All simulations were programmed in GAUSS and we used the module COINT 2.0 to implement long run variance estimation. All results that are mentioned but not reported are available upon request.

To analyse size and power of the tests under this DGP, we selected the values $(0, 0.01, 0.1)$ for σ_η^2 (as in McCabe *et al.,* 1997, for instance). Table 1 presents the estimates of rejection frequencies of the different tests at the 5% level of significance under the null hypothesis ($\sigma_\eta^2 = 0$) for the *MLS* test and the kernel-based tests, computed with the long run variance estimated by the data-dependent procedure, while Table 2 displays the corresponding results when the fixed bandwidth method with the QS kernel was used to estimate the long run variance[10]. Concerning the first test, it is relatively well behaved, except for the case of very persistent errors[11]. Note that the correction for endogeneity works quite well, although the test slightly overrejects (for $T = 100$) and underrejects (for $T = 200$) with ARMA errors.

As for the remaining tests, a general conclusion one can draw is that the choice of kernel does not seem to play an important role, considering the negligible differences between the results produced by each of them, which confirms the results of other studies (Lee, 1996 and Hobijn *et al.,* 1998, for instance). On the other hand, the method for the determination of the bandwidth induces distinct performances, since in general we obtain worse results when a fixed bandwidth is used. In this case, an expected trade-off occurs: small values of x will produce better size results for less autocorrelated errors, while larger x's are needed for larger values of ρ. Apparently, the best balanced choice would be to fix x equal to 10 or 12, as recommended by Shin (1994) for his test.

Considering each test, we observe that the Shin test tends to be conservative, even for $\rho = 0.9$, when the data-dependent procedure is used, especially with the QS and Parzen kernels. By contrast, the Harris-Inder test is normally very far from attaining the correct size and performs quite badly when correlation between ζ_t and ω_t is introduced, which is somehow intriguing. Equally, Park's J_1 test is badly oversized in both Tables, even for $\rho = 0$, dramatically so for $\rho = 0.9$, and for the endogeneity case in Table 1 the problems do not diminish asymptotically.

[10] Results for the other kernels were not significantly different, so we will omit them to save space.

[11] Additional simulations have shown that the size distortions of the *MLS* test are insignificant for a wide range of values of ρ, becoming more considerable only for values larger than 0.8.

This also constitutes a problem for the R_T fluctuation test, although it performs substantially better with the other DGPs. The LM test does reasonably well (especially if one considers that there is no correction for endogeneity), while Hansen's L_c test tends to be slightly oversized, even when there is no autocorrelation in the errors.

In terms of power, the rejection frequencies when the null is false at the 5% level of significance are shown in Tables 3 and 4 for data-dependent long run variance estimation, and Table 5, for the case of $\sigma^2_\eta = 0.1$ with fixed bandwidth using the QS kernel. To avoid misrepresenting power due to size distortions, size-adjusted power is also calculated (in parentheses in the tables) based on size-corrected critical values obtained from the corresponding results in Tables 1 and 2. Again, the differences in the results from using different kernels are not substantial, even though the Bartlett kernel tends to produce lower power for $\sigma^2_\eta = 0.01$ while delivering higher power for $\sigma^2_\eta = 0.1$ (Tables 3 and 4). On the other hand, if we compare Table 5 with the first part of Table 4, we observe that power is lower for tests using fixed bandwidth estimation, except for some cases with $x = 4$. In fact, as expected, power deteriorates as x grows and, thus, higher powers are attained when $x = 4$.

It is clear that power is very low for all DGPs when $\sigma^2_\eta = 0.01$. The Shin test has the highest size-corrected power of all tests, which increases with ρ, while it remains approximately the same for other tests, around 5%-6%, despite higher nominal powers. Examining now Table 4, it is possible to see that power declines with increasing ρ and that the Shin test is again the most well balanced procedure in terms of size-corrected power for this set of DGPs. Park's J_1 test does well when autocorrelation is low and for the ARMA structure under study, but has its (size-corrected) power greatly affected by excessive disturbances correlation. It is interesting to notice that in all Tables the L_c test is the best for the case of endogenous regressors, while HI is the worst, which is odd, since both tests use the same method for correcting second-order biases (FM-OLS). We should also mention that we were unable to closely replicate the results of McCabe *et al.* (1997), which report higher powers for the MLS and Shin test, although this could be explained by some differences in the computation of the test procedures and in the DGPs.

In order to refine the previous analysis, we compute the power of the tests as a function of the ratio of the disturbances variances $\lambda = \sigma^2_\eta/\sigma^2_\varepsilon$. As discussed in KPSS (1992), if λ is close to zero, the process is stationary (which means cointegration) and it will tend to a random walk (no cointegration) as λ approaches infinity. This allow us to examine relative power of the tests in the continuum of the alternative hypothesis, rather than concentrating on two possible points ($\sigma^2_\eta = 0.01, 0.1$), as in the previous experiment. We considered several values of λ ranging from

0.0001 to 10^6, and for simplicity we set ρ and θ equal to zero.

Figures 1 and 2 show the results for the tests computed with data-dependent long run variance estimation and Figures 3 and 4 reports the case of tests with the fixed bandwidth method with $x = 10$, using the QS kernel. The results for the MLS test are graphed in each figure so that the comparison is clear[12]. The most striking feature in this set of experiments is the non-monotonic power of the Shin and LM test when the data-based estimation is employed. In fact, when λ is larger than 0.4-0.5, power is reduced to a level of around 25-30%.

On the other hand, the MLS test does very well, especially for $T = 200$ and when λ gets larger, while the J_1 test performs better for $T = 100$ and smaller λ. It is also clear that power stabilizes after λ attains a given value (that varies with the sample size), except for the Shin and LM tests, as noted before. Comparing the results of the different long run variance estimation methods, we observe that power is generally lower when the band is fixed at this particular value, although we could search for a value that would produce similar powers.

Finally, we also consider the case of a pure random walk as the alternative hypothesis, when $\rho = 1$ and $\sigma_\eta^2 = 0$, which is the standard setting for the study of null of no cointegration tests. This will allow us to check the claim of Hobijn *et al.* (1998) regarding the inconsistency of the McCabe *et al.* (1997) test. From Table 6 it is clear that the MLS does not appear to be inconsistent, with very reasonable power, growing with the sample size. On the other hand, the Shin test and the LM display the lowest powers, at the levels predicted by Figures 1 to 4. Moreover, tests based on data-dependent estimation exhibit lower power, with the exception of the J_1 test. It is interesting to notice that the Parzen kernel seems to work better, especially with fixed bandwidth.

5 Conclusion

Although less often used, tests for the null hypothesis of cointegration may be a useful instrument in the analysis of economic time series. Unlike tests for the null of no cointegration, there is no evidence on the properties of the different tests and their relative merits. This study tries to fill this gap by analysing the performance of several tests that have been recently proposed. Conducting a series of Monte Carlo experiments, we found that no test dominates the others in every situation under study.

However, we would recommend the use of the MLS test, given its reasonable and well-balanced overall performance. First, this test has, at least theoretically, some advantages, namely a faster rate of convergence. Secondly, its computation is free of the problems associated with

[12]The graphs are truncated for larger values of λ since the results do not change significantly after unity.

all the other tests, that is, how the scaling long run variance is obtained. As could be seen from our experiments, the performance of the tests depends heavily on the method chosen for the estimation of this nuisance parameter. Although it seems preferable to use a data-dependent procedure, mainly to control for size distortions, this may lead to substantial reductions in power. Nevertheless, there is still room for improvements on the performance of this parametric-based test, as the attempts of Lanne and Saikkonen (2000) point out.

References

[1] Andrews, D. W. K. (1991), Heteroskedasticity and Autocorrelation Consistent Covariance Matrix Estimator, *Econometrica,* 59, 817-858.

[2] Andrews, D. W. K. and Monahan, J. C. (1992), An Improved Heteroskedasticity and Autocorrelation Consistent Covariance Matrix Estimator, *Econometrica,* 60, 953-966.

[3] Charemza, W. W. and Syczewska, E. M. (1998), Joint Application of the Dickey-Fuller and KPSS Tests, *Economics Letters*, 61, 17-31.

[4] Carrion-i-Silvestre, J. L., Sansó-i-Rosselló, A. and Ortuño, M. A. (2001), Unit Root and Stationairty Tests' Wedding, *EEconomics Letters*, 70, 1-8.

[5] Hansen, B. E. (1992), Tests for Parameter Instability in Regressions with $I(1)$ Processes, *Journal of Business and Economic Statistics*, 10, 321-335.

[6] Hao, K. (1996), Testing for Structural Change in Cointegrated Regression Models: Some Comparisons and Generalizations, *Econometric Reviews*, 15, 401-429.

[7] Harris and Inder (1994), A Test of the Null of Cointegration, in C. Hargreaves (ed.), *Nonstationary Time Series Analysis and Cointegration,* Oxford University Press, Oxford, 133-152.

[8] Haug, A. A. (1996), Tests for Cointegration: a Monte Carlo Comparison, *Journal of Econometrics,* 71, 89-115.

[9] Hobijn, B., Franses, P. H. and Ooms, M. (1998), Generalizations of the KPSS-test for Stationarity, Econometric Institute Report 9802/A

[10] Kwiatkowski, D., Phillips, P. C. B., Schmidt, P. and Shin, Y. (1992), Testing the Null Hypothesis of Stationarity against the Alternative of a Unit Root, *Journal of Econometrics*, 54, 159-178.

[11] Lanne, M. and Saikkonen, P. (2000), Reducing Size Distortions of Parametric Stationarity Tests, manuscript.

[12] Lee, J. (1996), On the Power of Stationarity Tests Using Optimal Bandwidth Estimates, *Economics Letters*, 51, 131-137.

[13] Leybourne, S. J. and McCabe, B. P. M. (1994), A Consistent Test for a Unit Root, *Journal of Business and Economic Statistics,* 14, 435-446.

[14] Maddala, G. S. and Kim, I. M. (1998), *Unit Roots, Cointegration and Structural Change*, Cambridge University Press, Cambridge.

[15] McCabe, B. P. M. and Leybourne, S. J. and Shin, Y. (1997), A Parametric Approach to Testing for the Null of Cointegration, *Journal of Time Series Analysis,* 18 (4), 395-413.

[16] Nabeya, S. and Tanaka, , K. (1988), Asymptotic Theory of a Test for the Constancy of Regression Coefficients Against the Random Walk Alternative, *Annals of Statistics,* 16, 218-235.

[17] Park, J. Y. (1990), Testing for Unit Roots and Cointegration by Variable Addition, in T. Fomby and G. F. Rhodes (eds.), *Advances in Econometrics*, Vol. 8, JAI Press, 107-133.

[18] Park, J. Y. (1992), Canonical Cointegrating Regressions, *Econometrica*, 60, 119-143.

[19] Phillips, P. C. B. and Durlauf, S. (1986), Multiple Time Series Regression with Integrated Processes, *Review of Economic Studies*, 53, 473-496.

[20] Phillips, P. C. B. and Hansen, B. (1990). Statistical Inference in Instrumental Variables Regression with I(1) Processes, *Review of Economic Studies*, 57, 99-125.

[21] Phillips, P. C. B. and Ouliaris, S. (1990), Asymptotic Properties of Residual-based Tests for Cointegration, *Econometrica,* 58, 165-193.

[22] Ploberger, W., Krämer, W. and Kontrus, K. (1989), A New Test for Structural Stability in the Linear Regression Model, *Journal of Econometrics*, 40. pp. 307-318.

[23] Saikkonen, P. (1991), Asymptotically Efficient Estimation of Cointegrating Regressions, *Econometric Theory,* 7, 1-21.

[24] Saikkonen, P. and Lukkonen, R. (1993), Testing for a Moving Average Unit Root in ARIMA Models, *Journal of the American Statistical Association,* 88, 596-601.

[25] Schwert, G. W. (1989), Tests for Unit Roots: a Monte Carlo Investigation, *Journal of Business and Economics Statistics*, 7, 147-159.

[26] Shin, Y. (1994), A residual-based Test of the Null of Cointegration against the Alternative of No Cointegration, *Econometric Theory*, 10, 91-115.

[27] Stock, J. H. (1994), Unit Roots, Structural Breaks and Trends, in R. F. Engle and D. L. McFadden (eds.), *Handbook of Econometrics*, Vol. IV, Elsevier, Amsterdam, 2740-2841.

[28] Tanaka, K. (1990), Testing for a Moving Average Unit Root, *Econometric Theory*, 6, 433-444.

[29] Xiao, Z. (1999), A Residual Based Test for the Null Hypothesis of Cointegration, *Economics Letters*, 64, 133-141.

6 Appendix

Table 1 - Empirical Size ($\sigma_\eta^2 = 0$)

	$T = 100$					$T = 200$				
	ρ					ρ				
Tests	0	0.5	0.9	$\theta = 0.4$ $(\rho{=}0.5)$	$\gamma = 0.7$	0	0.5	0.9	$\theta = 0.4$ $(\rho{=}0.5)$	$\gamma = 0.7$
MLS	0.04	0.07	0.51	0.078	0.06	0.05	0.05	0.48	0.025	0.052
QS										
S	0.02	0.03	0.03	0.051	0.038	0.03	0.04	0.05	0.051	0.045
HI	0.06	0.12	0.41	0.168	0.581	0.06	0.09	0.27	0.102	0.583
LM	0.05	0.07	0.11	0.061	0.101	0.05	0.07	0.09	0.059	0.103
L_c	0.09	0.11	0.21	0.097	0.074	0.09	0.11	0.12	0.094	0.079
R_T	0.04	0.05	0.17	0.06	0.306	0.04	0.05	0.10	0.047	0.342
J_1	0.09	0.26	0.74	0.348	0.221	0.07	0.18	0.58	0.223	0.211
$Bartlett$										
S	0.022	0.036	0.061	0.061	0.039	0.031	0.05	0.077	0.061	0.045
HI	0.057	0.115	0.274	0.124	0.577	0.052	0.098	0.198	0.089	0.584
LM	0.051	0.082	0.142	0.075	0.101	0.051	0.08	0.119	0.069	0.104
L_c	0.093	0.13	0.141	0.107	0.079	0.089	0.13	0.131	0.11	0.081
R_T	0.041	0.065	0.108	0.056	0.305	0.044	0.069	0.082	0.054	0.337
J_1	0.08	0.28	0.707	0.333	0.219	0.067	0.203	0.569	0.234	0.212
$Parzen$										
S	0.02	0.03	0.03	0.049	0.037	0.03	0.04	0.05	0.051	0.044
HI	0.06	0.12	0.41	0.179	0.579	0.05	0.09	0.28	0.105	0.582
LM	0.05	0.07	0.11	0.061	0.101	0.05	0.06	0.08	0.057	0.103
L_c	0.09	0.10	0.22	0.096	0.07	0.09	0.10	0.12	0.091	0.077
R_T	0.04	0.05	0.19	0.061	0.307	0.04	0.06	0.10	0.044	0.34
J_1	0.10	0.28	0.76	0.382	0.223	0.07	0.19	0.61	0.248	0.212

NOTE: Rejection frequencies at the 5% significance level.

Table 2 - Empirical Size with Fixed Bandwidth (σ^2_η= 0)

		$T = 100$					$T = 200$				
		ρ					ρ				
Tests	x	0	0.5	0.9	$\theta = 0.4$ $(\rho{=}0.5)$	$\gamma = 0.7$	0	0.5	0.9	$\theta = 0.4$ $(\rho{=}0.5)$	$\gamma = 0.7$
S	4	0.042	0.06	0.24	0.042	0.044	0.045	0.065	0.342	0.051	0.046
	10	0.064	0.056	0.117	0.057	0.057	0.051	0.052	0.134	0.052	0.051
	12	0.067	0.058	0.101	0.067	0.07	0.058	0.051	0.115	0.055	0.053
	16	0.115	0.086	0.096	0.107	0.117	0.06	0.057	0.093	0.059	0.063
HI	4	0.099	0.124	0.393	0.101	0.571	0.068	0.09	0.424	0.072	0.574
	10	0.199	0.201	0.311	0.195	0.572	0.115	0.115	0.233	0.108	0.568
	12	0.245	0.24	0.316	0.25	0.591	0.138	0.139	0.227	0.14	0.583
	16	0.351	0.326	0.373	0.339	0.61	0.181	0.184	0.237	0.181	0.582
LM	4	0.057	0.074	0.315	0.058	0.113	0.053	0.074	0.383	0.057	0.114
	10	0.075	0.067	0.147	0.068	0.123	0.058	0.06	0.156	0.058	0.112
	12	0.081	0.067	0.13	0.076	0.137	0.065	0.058	0.131	0.06	0.117
	16	0.101	0.084	0.109	0.098	0.154	0.07	0.066	0.10	0.067	0.126
L_c	4	0.092	0.11	0.393	0.089	0.06	0.085	0.12	0.524	0.094	0.068
	10	0.151	0.106	0.147	0.0128	0.072	0.102	0.089	0.198	0.096	0.063
	12	0.185	0.136	0.128	0.161	0.091	0.116	0.099	0.156	0.106	0.06
	16	0.303	0.23	0.142	0.275	0.147	0.13	0.114	0.124	0.119	0.071
R_T	4	0.05	0.056	0.266	0.044	0.328	0.048	0.063	0.366	0.049	0.351
	10	0.116	0.08	0.117	0.092	0.376	0.067	0.053	0.121	0.055	0.363
	12	0.157	0.101	0.109	0.128	0.413	0.08	0.059	0.096	0.067	0.382
	16	0.272	0.198	0.142	0.232	0.466	0.116	0.084	0.079	0.101	0.403
J_1	4	0.209	0.273	0.708	0.213	0.301	0.119	0.173	0.641	0.127	0.241
	10	0.446	0.406	0.704	0.445	0.536	0.248	0.258	0.511	0.249	0.35
	12	0.522	0.527	0.719	0.525	0.61	0.314	0.311	0.518	0.316	0.406
	16	0.662	0.673	0.788	0.668	0.738	0.409	0.409	0.551	0.416	0.517

NOTE: Rejection frequencies at the 5% significance level using the QS kernel.

Table 3 - Empirical Power ($\boldsymbol{\sigma_\eta^2}$= 0.01)

	$T = 100$					$T = 200$				
	ρ					ρ				
Tests	0	0.5	0.9	$\theta = 0.4$	$\gamma = 0.7$	0	0.5	0.9	$\theta = 0.4$	$\gamma = 0.7$
MLS	0.059 (0.069)	0.085 (0.061)	0.54 (0.064)	0.079 (0.05)	0.067 (0.057)	0.073 (0.079)	0.064 (0.061)	0.482 (0.06)	0.025 (0.046)	0.092 (0.09)
QS										
S	0.062 (0.13)	0.096 (0.141)	0.244 (0.278)	0.052 (0.05)	0.047 (0.059)	0.084 (0.117)	0.087 (0.10)	0.165 (0.165)	0.057 (0.054)	0.086 (0.093)
HI	0.065 (0.055)	0.122 (0.05)	0.401 (0.053)	0.171 (0.051)	0.582 (0.051)	0.085 (0.079)	0.097 (0.057)	0.264 (0.05)	0.085 (0.06)	0.601 (0.056)
LM	0.058 (0.055)	0.07 (0.047)	0.103 (0.05)	0.063 (0.05)	0.091 (0.058)	0.083 (0.082)	0.076 (0.055)	0.085 (0.054)	0.063 (0.053)	0.139 (0.081)
L_c	0.097 (0.057)	0.105 (0.048)	0.209 (0.047)	0.098 (0.051)	0.111 (0.057)	0.122 (0.075)	0.117 (0.054)	0.113 (0.047)	0.112 (0.059)	0.133 (0.097)
R_T	0.045 (0.056)	0.056 (0.053)	0.174 (0.053)	0.061 (0.052)	0.317 (0.054)	0.068 (0.079)	0.061 (0.055)	0.093 (0.048)	0.053 (0.056)	0.372 (0.077)
J_1	0.097 (0.051)	0.268 (0.053)	0.728 (0.051)	0.349 (0.05)	0.234 (0.056)	0.098 (0.068)	0.185 (0.051)	0.56 (0.048)	0.234 (0.05)	0.252 (0.073)
Bartlett										
S	0.062 (0.127)	0.097 (0.132)	0.179 (0.162)	0.061 (0.051)	0.047 (0.058)	0.083 (0.116)	0.093 (0.094)	0.141 (0.105)	0.065 (0.055)	0.086 (0.093)
HI	0.064 (0.058)	0.117 (0.052)	0.273 (0.05)	0.124 (0.051)	0.583 (0.051)	0.084 (0.08)	0.105 (0.058)	0.196 (0.052)	0.99 (0.058)	0.605 (0.055)
LM	0.058 (0.058)	0.085 (0.051)	0.143 (0.051)	0.075 (0.051)	0.113 (0.057)	0.08 (0.078)	0.089 (0.059)	0.116 (0.051)	0.075 (0.052)	0.139 (0.081)
L_c	0.10 (0.057)	0.129 (0.051)	0.141 (0.05)	0.107 (0.05)	0.096 (0.064)	0.121 (0.075)	0.135 (0.055)	0.129 (0.046)	0.11 (0.051)	0.139 (0.097)
R_T	0.046 (0.056)	0.067 (0.051)	0.109 (0.051)	0.056 (0.05)	0.314 (0.053)	0.07 (0.078)	0.077 (0.057)	0.086 (0.053)	0.059 (0.055)	0.373 (0.077)
J_1	0.088 (0.056)	0.281 (0.053)	0.705 (0.05)	0.335 (0.049)	0.232 (0.058)	0.092 (0.072)	0.211 (0.051)	0.559 (0.052)	0.241 (0.05)	0.256 (0.073)
Parzen										
S	0.063 (0.131)	0.097 (0.149)	0.258 (0.297)	0.05 (0.051)	0.047 (0.059)	0.082 (0.116)	0.088 (0.104)	0.179 (0.185)	0.054 (0.054)	0.085 (0.092)
HI	0.067 (0.055)	0.126 (0.049)	0.418 (0.05)	0.18 (0.049)	0.58 (0.05)	0.083 (0.08)	0.101 (0.057)	0.278 (0.056)	0.113 (0.057)	0.601 (0.056)
LM	0.058 (0.055)	0.066 (0.048)	0.108 (0.049)	0.062 (0.049)	0.111 (0.057)	0.08 (0.078)	0.074 (0.06)	0.081 (0.05)	0.06 (0.053)	0.139 (0.081)
L_c	0.096 (0.057)	0.098 (0.048)	0.22 (0.048)	0.095 (0.05)	0.086 (0.06)	0.12 (0.076)	0.111 (0.055)	0.111 (0.042)	0.092 (0.05)	0.133 (0.097)
R_T	0.045 (0.056)	0.052 (0.051)	0.188 (0.05)	0.062 (0.051)	0.315 (0.054)	0.069 (0.078)	0.06 (0.053)	0.102 (0.049)	0.052 (0.057)	0.372 (0.077)
J_1	0.103 (0.051)	0.285 (0.054)	0.756 (0.05)	0.384 (0.05)	0.234 (0.056)	0.103 (0.072)	0.20 (0.052)	0.559 (0.05)	0.258 (0.05)	0.252 (0.073)

NOTE: Rejection frequencies at the 5% significance level. In parentheses are the size-adjusted rejection frequencies based on finite-sample critical values.

Table 4 - Empirical Power ($\boldsymbol{\sigma}_{\eta}^{2}$= 0.1)

	$T = 100$					$T = 200$				
	ρ					ρ				
Tests	0	0.5	0.9	$\theta = 0.4$ (ρ=0.5)	$\gamma = 0.7$	0	0.5	0.9	$\theta = 0.4$ (ρ=0.5)	$\gamma = 0.7$
MLS	0.309 (0.328)	0.177 (0.143)	0.541 (0.068)	0.109 (0.076)	0.418 (0.392)	0.662 (0.671)	0.361 (0.355)	0.512 (0.077)	0.078 (0.132)	0.789 (0.785)
QS										
S	0.424 (0.547)	0.23 (0.291)	0.258 (0.29)	0.095 (0.093)	0.367 (0.408)	0.698 (0.753)	0.402 (0.432)	0.203 (0.203)	0.215 (0.212)	0.721 (0.738)
HI	0.433 (0.403)	0.263 (0.141)	0.411 (0.052)	0.24 (0.07)	0.759 (0.094)	0.705 (0.692)	0.421 (0.335)	0.303 (0.061)	0.322 (0.22)	0.903 (0.242)
LM	0.419 (0.411)	0.212 (0.17)	0.111 (0.055)	0.138 (0.117)	0.446 (0.341)	0.701 (0.70)	0.394 (0.345)	0.112 (0.072)	0.276 (0.256)	0.728 (0.636)
L_c	0.469 (0.387)	0.209 (0.117)	0.204 (0.048)	0.14 (0.079)	0.581 (0.516)	0.771 (0.704)	0.423 (0.29)	0.128 (0.054)	0.264 (0.175)	0.877 (0.838)
R_T	0.378 (0.40)	0.154 (0.149)	0.177 (0.051)	0.103 (0.086)	0.616 (0.246)	0.694 (0.713)	0.339 (0.328)	0.108 (0.058)	0.211 (0.217)	0.863 (0.531)
J_1	0.549 (0.455)	0.462 (0.175)	0.75 (0.056)	0.407 (0.107)	0.674 (0.449)	0.82 (0.793)	0.622 (0.421)	0.611 (0.069)	0.541 (0.265)	0.901 (0.787)
Bartlett										
S	0.431 (0.552)	0.244 (0.289)	0.187 (0.168)	0.111 (0.096)	0.379 (0.42)	0.714 (0.767)	0.423 (0.424)	0.172 (0.128)	0.237 (0.218)	0.73 (0.745)
HI	0.442 (0.425)	0.268 (0.166)	0.286 (0.048)	0.208 (0.102)	0.781 (0.133)	0.727 (0.722)	0.442 (0.34)	0.231 (0.062)	0.323 (0.243)	0.92 (0.294)
LM	0.427 (0.424)	0.24 (0.18)	0.154 (0.055)	0.159 (0.121)	0.457 (0.349)	0.714 (0.711)	0.425 (0.352)	0.149 (0.075)	0.299 (0.261)	0.734 (0.641)
L_c	0.491 (0.404)	0.251 (0.133)	0.14 (0.048)	0.165 (0.085)	0.615 (0.557)	0.791 (0.721)	0.464 (0.295)	0.154 (0.056)	0.303 (0.193)	0.895 (0.858)
R_T	0.396 (0.422)	0.183 (0.15)	0.114 (0.05)	0.113 (0.10)	0.652 (0.299)	0.715 (0.73)	0.37 (0.326)	0.101 (0.061)	0.241 (0.231)	0.886 (0.578)
J_1	0.544 (0.48)	0.482 (0.175)	0.721 (0.054)	0.456 (0.111)	0.684 (0.464)	0.829 (0.812)	0.645 (0.42)	0.60 (0.067)	0.553 (0.277)	0.909 (0.80)
Parzen										
S	0.413 (0.542)	0.224 (0.294)	0.272 (0.307)	0.093 (0.093)	0.352 (0.395)	0.693 (0.751)	0.386 (0.423)	0.20 (0.209)	0.207 (0.206)	0.699 (0.714)
HI	0.424 (0.396)	0.26 (0.136)	0.403 (0.049)	0.245 (0.07)	0.746 (0.076)	0.702 (0.696)	0.412 (0.318)	0.313 (0.058)	0.315 (0.212)	0.891 (0.192)
LM	0.41 (0.401)	0.204 (0.167)	0.115 (0.055)	0.134 (0.112)	0.436 (0.33)	0.697 (0.696)	0.381 (0.343)	0.111 (0.072)	0.264 (0.249)	0.713 (0.619)
L_c	0.456 (0.376)	0.188 (0.11)	0.222 (0.048)	0.13 (0.07)	0.56 (0.503)	0.762 (0.692)	0.393 (0.272)	0.129 (0.05)	0.236 (0.157)	0.864 (0.822)
R_T	0.366 (0.393)	0.142 (0.14)	0.193 (0.051)	0.099 (0.082)	0.60 (0.215)	0.677 (0.695)	0.314 (0.297)	0.113 (0.059)	0.194 (0.206)	0.844 (0.481)
J_1	0.556 (0.453)	0.482 (0.177)	0.774 (0.055)	0.501 (0.109)	0.67 (0.442)	0.827 (0.796)	0.63 (0.41)	0.641 (0.065)	0.561 (0.266)	0.896 (0.781)

See Notes to Table 3.

Table 5 - Empirical Power with Fixed Bandwidth ($\boldsymbol{\sigma}_{\eta}^{2}$= 0.1)

		$T = 100$					$T = 200$				
		ρ					ρ				
Tests	x	0	0.5	0.9	$\theta = 0.4$ (ρ=0.5)	$\gamma = 0.7$	0	0.5	0.9	$\theta = 0.4$ (ρ=0.5)	$\gamma = 0.7$
S	4	0.242 (0.266)	0.15 (0.133)	0.245 (0.052)	0.114 (0.094)	0.30 (0.314)	0.552 (0.57)	0.373 (0.334)	0.385 (0.071)	0.277 (0.239)	0.644 (0.653)
	10	0.19 (0.166)	0.121 (0.111)	0.125 (0.056)	0.099 (0.092)	0.218 (0.21)	0.392 (0.386)	0.279 (0.274)	0.164 (0.07)	0.222 (0.218)	0.45 (0.446)
	12	0.183 (0.149)	0.116 (0.105)	0.113 (0.056)	0.097 (0.085)	0.207 (0.169)	0.357 (0.336)	0.261 (0.259)	0.146 (0.071)	0.201 (0.185)	0.394 (0.388)
	16	0.189 (0.089)	0.133 (0.083)	0.106 (0.058)	0.129 (0.068)	0.197 (0.101)	0.313 (0.294)	0.23 (0.212)	0.112 (0.063)	0.192 (0.181)	0.335 (0.307)
HI	4	0.394 (0.297)	0.273 (0.153)	0.397 (0.053)	0.216 (0.108)	0.652 (0.038)	0.647 (0.603)	0.47 (0.381)	0.467 (0.074)	0.371 (0.281)	0.596 (0.051)
	10	0.405 (0.133)	0.302 (0.081)	0.318 (0.051)	0.271 (0.065)	0.553 (0.022)	0.521 (0.361)	0.395 (0.263)	0.271 (0.067)	0.327 (0.201)	0.649 (0.011)
	12	0.419 (0.089)	0.336 (0.072)	0.332 (0.05)	0.308 (0.069)	0.539 (0.017)	0.487 (0.30)	0.394 (0.228)	0.26 (0.061)	0.343 (0.173)	0.598 (0.012)
	16	0.468 (0.07)	0.402 (0.061)	0.38 (0.05)	0.385 (0.058)	0.54 (0.023)	0.469 (0.194)	0.391 (0.138)	0.262 (0.062)	0.343 (0.131)	0.552 (0.01)
LM	4	0.36 (0.345)	0.228 (0.184)	0.331 (0.053)	0.168 (0.139)	0.382 (0.277)	0.644 (0.638)	0.47 (0.381)	0.467 (0.074)	0.349 (0.288)	0.654 (0.53)
	10	0.29 (0.235)	0.178 (0.151)	0.165 (0.152)	0.142 (0.111)	0.307 (0.196)	0.48 (0.458)	0.395 (0.263)	0.271 (0.067)	0.279 (0.254)	0.502 (0.388)
	12	0.276 (0.215)	0.173 (0.142)	0.144 (0.058)	0.129 (0.111)	0.291 (0.169)	0.487 (0.30)	0.394 (0.228)	0.26 (0.061)	0.269 (0.232)	0.454 (0.343)
	16	0.253 (0.16)	0.175 (0.119)	0.125 (0.103)	0.155 (0.099)	0.267 (0.124)	0.381 (0.341)	0.287 (0.256)	0.129 (0.074)	0.239 (0.202)	0.395 (0.273)
L_c	4	0.375 (0.28)	0.239 (0.139)	0.401 (0.047)	0.192 (0.101)	0.42 (0.392)	0.692 (0.624)	0.501 (0.364)	0.56 (0.069)	0.394 (0.258)	0.773 (0.739)
	10	0.244 (0.09)	0.168 (0.085)	0.155 (0.054)	0.151 (0.07)	0.218 (0.167)	0.479 (0.352)	0.344 (0.248)	0.236 (0.067)	0.271 (0.195)	0.527 (0.489)
	12	0.232 (0.051)	0.171 (0.057)	0.137 (0.053)	0.161 (0.061)	0.191 (0.102)	0.412 (0.262)	0.307 (0.207)	0.193 (0.07)	0.242 (0.152)	0.435 (0.40)
	16	0.252 (0.019)	0.221 (0.039)	0.149 (0.049)	0.23 (0.042)	0.173 (0.04)	0.319 (0.167)	0.245 (0.132)	0.132 (0.157)	0.193 (0.10)	0.308 (0.255)
R_T	4	0.301 (0.301)	0.174 (0.165)	0.272 (0.051)	0.127 (0.121)	0.478 (0.097)	0.604 (0.614)	0.414 (0.383)	0.41 (0.076)	0.315 (0.284)	0.744 (0.33)
	10	0.24 (0.119)	0.144 (0.094)	0.122 (0.056)	0.12 (0.071)	0.341 (0.027)	0.405 (0.367)	0.285 (0.279)	0.152 (0.07)	0.218 (0.225)	0.494 (0.049)
	12	0.232 (0.075)	0.158 (0.077)	0.114 (0.05)	0.133 (0.071)	0.312 (0.021)	0.353 (0.278)	0.263 (0.239)	0.121 (0.065)	0.204 (0.183)	0.429 (0.029)
	16	0.265 (0.052)	0.209 (0.059)	0.145 (0.052)	0.206 (0.065)	0.321 (0.021)	0.29 (0.149)	0.225 (0.149)	0.098 (0.064)	0.19 (0.129)	0.334 (0.015)
J_1	4	0.62 (0.384)	0.469 (0.162)	0.728 (0.053)	0.389 (0.104)	0.672 (0.363)	0.817 (0.745)	0.631 (0.448)	0.674 (0.07)	0.507 (0.209)	0.865 (0.717)
	10	0.743 (0.274)	0.616 (0.132)	0.705 (0.058)	0.559 (0.099)	0.753 (0.244)	0.821 (0.62)	0.649 (0.36)	0.563 (0.071)	0.573 (0.254)	0.84 (0.578)
	12	0.776 (0.242)	0.671 (0.122)	0.721 (0.056)	0.611 (0.086)	0.79 (0.21)	0.825 (0.549)	0.693 (0.342)	0.564 (0.072)	0.60 (0.234)	0.84 (0.531)
	16	0.845 (0.205)	0.781 (0.105)	0.79 (0.052)	0.741 (0.074)	0.845 (0.175)	0.846 (0.48)	0.733 (0.29)	0.584 (0.06)	0.667 (0.18)	0.858 (0.444)

Table 6 - Power with the Random Walk Alternative

Tests	$T = 100$		$T = 200$	
MLS	0.773		0.893	
QS	Data-based	$x = 10$	Data-based	$x = 10$
S	0.261	0.501	0.292	0.814
HI	0.541	0.684	0.655	0.876
LM	0.343	0.663	0.351	0.871
L_c	0.359	0.686	0.535	0.915
R_T	0.278	0.534	0.44	0.817
J_1	0.936	0.928	0.941	0.96
Bartlett				
S	0.261	0.606	0.299	0.875
HI	0.481	0.757	0.478	0.921
LM	0.35	0.753	0.35	0.919
L_c	0.258	0.792	0.261	0.953
R_T	0.229	0.637	0.233	0.889
J_1	0.924	0.943	0.93	0.969
Parzen				
S	0.267	0.703	0.303	0.93
HI	0.597	0.836	0.757	0.958
LM	0.337	0.831	0.343	0.957
L_c	0.419	0.882	0.635	0.981
R_T	0.335	0.753	0.535	0.943
J_1	0.944	0.955	0.951	0.977

NOTE: Rejection frequencies at the 5% significance level. The QS kernel is used with the fixed bandwith.

Figure 1: Power as a function of λ ($T = 100$, automatic bandwidth)

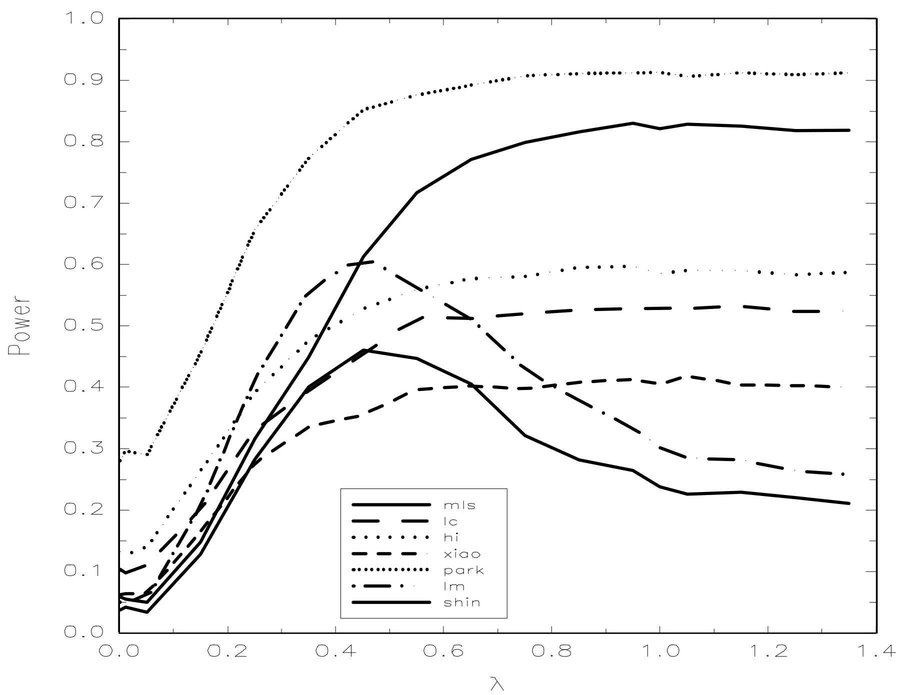

Figure 2: Power as a function of λ ($T = 200$, automatic bandwidth)

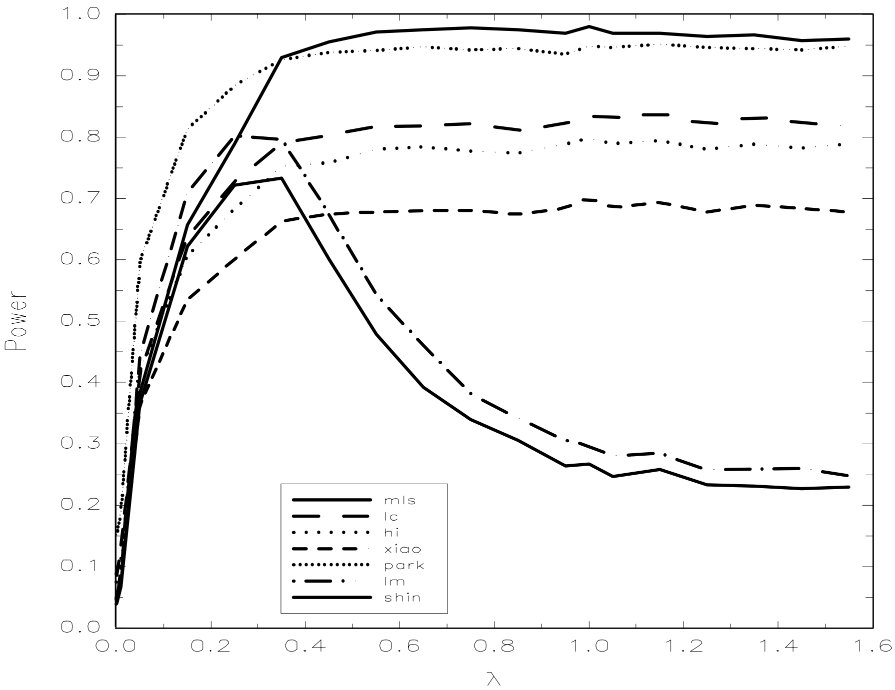

Figure 3: Power as a function of λ ($T = 100$, fixed bandwidth)

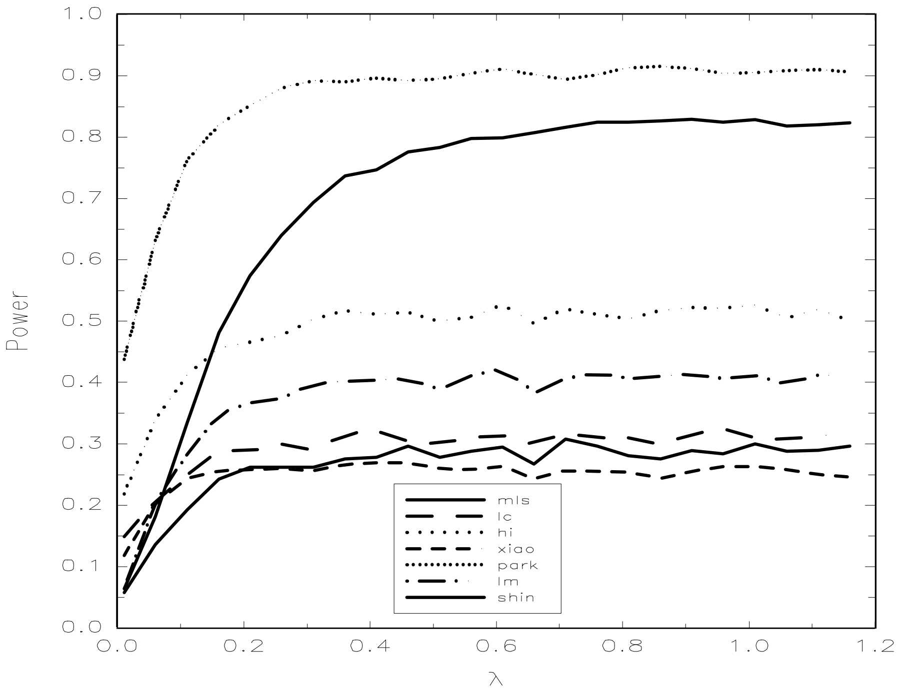

Figure 4: Power as a function of λ ($T = 200$, fixed bandwidth)