RELIABILITY ANALYSIS OF SHEAR STRENGTHENING EBR FRP MODELS

J. L.T. Lima[1] and J. A. O. Barros[2],

[1] MSc Student, Dep. of Civil Eng., School of Eng., Univ. of Minho, Azurém, 4810 058 Guimarães, Portugal

[2] Associate Prof., Dep. of Civil Eng., School of Eng., Univ. of Minho, Azurém, 4810 058 Guimarães, Portugal

João Lima is a MSc Student of the Structural Division, ISISE, Department of Civil Engineering, University of Minho, Portugal. He received his Civil Engineering degree from the Faculty of Civil Engineering of Porto University, Portugal. His research interests include structural strengthening and design of RC structures.

Joaquim Barros is Associate Professor with Aggregation and Director of the Laboratory of the Structural Group of the Department of Civil Engineering, ISISE, University of Minho. He is a member of ACI Committees 440 and 544, a member of *fib* TG 9.3 and TG 8.3, and a council member of the IIFC. His research interests include structural strengthening, composite materials, fiber reinforced concrete and finite element analysis.

ABSTRACT

This work presents a statistically oriented study aiming to assess the reliability of some of the most well known design models available for the prediction of the contribution of fiber

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[2] Author to whom the correspondence should be sent (barros@civil.uminho.pt).

reinforced polymer (FRP) systems applied according to the externally bonded reinforcing (EBR) technique for the shear strengthening of reinforced concrete (RC) beams. Relevant data was collected from experimental programs carried out in recent years in the context of the shear strengthening with FRP, and an extended database was obtained. Using this data, the performance of *fib*, ACI, Italian and Australian design guidelines was appraised by means of comparing the contribution of the FRP shear systems predicted by the analytical formulations with those registered experimentally. In general, the obtained results were not very promising, since a large scatter of the design safety factor was observed and, for some cases, the contribution of the FRP systems predicted by the design models was highly unconservative, which may be a serious concern as these formulations may be currently being used in design practice.

KEYWORDS: Analytical, EBR, FRP, Design, Models, Shear, Strengthening, Database

INTRODUCTION

FRP shear reinforcement of RC beams has been widely studied in the last decade with a large number of scientific publications exalting the effectiveness of this strengthening technique. Experimental studies conducted worldwide on RC beams strengthened in shear with externally bonded FRP over the last years clearly demonstrate the reliability and effectiveness of such technique for structural retrofitting. For elements with shear resistance deficiencies, a higher load carrying capacity may be achieved by bonding FRP reinforcement systems with the fibers as orthogonal as practically possible to the critical shear crack plane for an optimal configuration, or with the fibers normal to the beam axis for a more practical setting. Common shear strengthening configurations (Fig.1) include the full wrapping of the cross section (O), U jacketing along 3 sides (U) and side bonding on the beam web (S). Additional mechanical anchorage systems can be provided to enhance the effectiveness of U or S configurations when the available bond length is short (U+ and S+). Each of the aforementioned strengthening configurations may be set in several possible arrangements (Fig. 2), including variations in the fiber orientation, the use of discrete strips or continuous sheets, and the overlay of sheets with different fiber orientations (O^X, U^X, S^X), among others.

ANALYTICAL FORMULATIONS FOR FRP SHEAR REINFORCEMENT DESIGN

As an outcome of the increasing demand of FRP strengthening systems, stimulated by a continuous growth in field applications, several proposed analytical formulations (Triantafillou, 1998, Khalifa *et al.*, 1998, Monti and Liotta, 2005) have been implemented into reference design guidelines, providing the guidance for design, detailing, and installation of FRP based strengthening systems.

The present study addresses the shear provisions included in *fib* (2001), ACI 440 (2002), CNR (2004) and the Australian Standard (2006) design guidelines. The later follows an analytical model previously introduced by Chen and Teng (C&T) (2003a, 2003b). All of the aforementioned design models rely on the approach where shear strength of a strengthened member is attained by the sum of the contributions from concrete, V_c, steel reinforcement, V_s, and FRP, V_f, as follows:

$$V_r = V_c + V_s + V_f \tag{1}$$

where V_c and V_s may be calculated according to provisions existing in current design codes, independently of the adopted FRP strengthening system. The methodology to estimate the design value of the FRP contribution in shear, V_{fd}, according to each of the aforementioned design proposals is briefly described in Tables 1 and 2. In case of the ACI-440 design model V_{fd} can be determined as ϕV_f , being ϕ a shear strength reduction factor. Figure 3 shows the notation adopted to define the geometric properties of a generic beam reinforced in shear with externally bonded FRP.

PREDICTIVE PERFORMANCE OF THE ANALYTICAL FORMULATIONS

Database assembly

The use of databases associated with the modern statistical analysis and data-mining software packages (R, 2008) settles the basis for knowledge discovery by means of registering, sharing and manipulating results from a large number of experimental tests conducted worldwide by several different researchers. This kind of approach, used in the present work, is particularly suitable for the study of complex phenomena, such as shear behavior of RC beams strengthened with FRP, where the number of variables involved is large and their relative importance is not yet determined. To assess the accuracy of the theoretical predictions obtained with the aforementioned analytical formulations, a data base (DB) containing more than 250 experimental results of RC beams strengthened with EBR FRP was collected from published literature, and previous compiled databases (Bousselham and Chaallal, 2004, Aprile and Benedetti 2004) were upgraded. The criteria adopted in this task was to collect the largest amount of data with a wide spectrum of test results regarding the beams' geometry, concrete properties, longitudinal steel reinforcement ratios, shear steel reinforcement ratios, FRP properties and strengthening configurations. From the collected data it was found that the vast majority of the tests (≈83 %) were conducted with rectangular cross sections (R), with an average height of around 350 mm, where 54 % of the tested beams had a concrete compressive strength between 20 and 30 MPa, and the most used strengthening system was type U (≈50 %). It is also noticeable that approximately 51 % of the tested beams did not have any shear reinforcement at all, and all of them had a large longitudinal reinforcement ratio, ρ_{sl}, with a mean value of about 3 %. From the above it is possible to establish that the beam characteristics available from laboratory specimens is far from general, with an asymmetric predominance of certain physical and geometric characteristics of the tested beams. A significant number of test results, which were used in the calibration of several analytical formulations, were obtained with unrealistic geometric conditions and reinforcement settings. These observations were manually flagged as "suspicious" and thus not considered in the present analysis. Aiming to reduce the influence of erroneous and inconsistent data present in the DB, even after the pre-trial operation, the analysis was performed not only in the integral database (IDB), but also in partial subsets of the data - reduced databases (RDB).

General statistical analysis procedures

The performance of *fib*, ACI, CNR and C&T design models is appraised using the collected data registered in the DB. For each described design model, the obtained values of V_{fd} are compared with $V_{f,exp}$ and a χ factor corresponding to the $V_{f,exp}/V_{fd}$ ratio is evaluated. On the performed analysis V_{fd} is the design value of the FRP contribution for the global shear resistance predicted by the design codes and $V_{f,exp}$ is the FRP contribution obtained based on experimental results as follows:

$$V_{R,\exp}^{ref} = V_c + V_s \ ; \quad V_{R,\exp}^{str} = V_c + V_s + V_{f,\exp} \ \therefore \ V_{f,\exp} = V_{R,\exp}^{str} - V_{R,\exp}^{ref} \tag{2}$$

where $V_{R,\exp}^{ref}$ is the shear resistance of the unstrengthened reference control tested beam, $V_{R,\exp}^{str}$ is the shear resistance of the strengthened tested beam and V_c, V_s and $V_{f,\exp}$ are, respectively, the concrete, stirrup and FRP contribution to the global shear resistance. This approach, based on the assumption that the superimposition principle can be applied to this phenomenon, may not be absolutely realistic, as there are multiple interactions between the intervenient parameters. However, this strategy allows for cost-effective analysis procedures that makes this kind of study possible.

In order to allow a direct comparison between the referred design models, it was considered in all the calculations that the critical shear crack inclination, θ_{cr}, is 45º, even though some of the analytical formulations admit the use of different values for θ_{cr}, and previous studies (Barros *et al.*, 2007) reveal a better adjustment of the predicted V_f values when $\theta_{cr} \neq 45º$.

It should be emphasized that in the following sections $\chi = V_{f,exp}/V_{fd}$, where $V_{f,exp}$ is calculated according to equation (2) and V_{fd} is determined from the formulations described in Tables 1 and 2. Therefore, only the contribution of the FRP shear strengthening configurations is compared, assuming for the contribution of concrete and stirrups for the beam's shear resistance ($V_c + V_s$) the result obtained in the corresponding unstrengthened beam.

Results obtained using the integral database (IDB)

Figure 4 plots the predicted against experimental values, where a 45º solid line, χ=1.0, establishes the division between the safe predictions from the unconservative ones. A complementarily line traces an "ideal safety trend" corresponding to χ=1.5 and the data scatter is adjusted with a linear regression dashed line that reveals the global trend. The quality of the adjustment is determined by the R^2 parameter presented in the figures. A large scatter is observed in the experimental *vs* predicted design values for all of the considered analytical formulations, mainly in the range between $0 < V_{f,exp} < 100$ kN. Based on the plotted results it can be conjectured that the *fib* and C&T design models provide the results that are most compatible with the theoretical behavior assumed as ideal. Table 3 summarizes the main descriptive statistical measures regarding the χ factor, namely minimum (MIN) and maximum (MAX) values, the average (AVG) that represents a global safety factor associated with the design procedure, the standard deviation (STD) and the coefficient of variation (COV) that are indicators of accuracy. The first quartile (Q1) that cuts off the lowest 25% of data, the median (MED) corresponding to the 50th percentile, and the third quartile (Q3) that cuts off the highest

25% of data are also included. The obtained results show that the *fib* design model presents, on average, the lowest safety factor while the most conservative predictions are attained with CNR. The largest scatter is obtained by the CNR model (COV=0.73), while the least scattered model is *fib* (COV=0.55). The C&T model globally presents a good performance with an average value of χ =1.43 and COV = 0.58.

Taking into consideration that the behavior of a strengthened beam is too dependent of the adopted FRP configuration system, and the design models studied distinguish the O, U and S strengthening type, the predictive performance of these models should be evaluated attending each kind of configuration. Figure 5 presents a "box and whiskers" plot of the χ ratio variation related with the strengthening configuration. The box plot diagram (BP) graphically depicts the statistical five-number summary, which consists of the smallest non-outlier observation, lower quartile (Q1), median, upper quartile (Q3), and largest non-outlier observation, where the outliers are determined according to the condition:

$$\chi \notin \left[Q_1 - 1.5 \cdot \left(Q_3 - Q_1 \right) ; 1.5 \cdot \left(Q_3 - Q_1 \right) \right] \Rightarrow \text{outlier} \tag{3}$$

Based on the obtained results it is possible to conclude that all kinds of strengthening configurations other than O, U or S systems, generally lead to a poor performance of the analytical formulations, proving that the provisions currently available in the design codes cannot predict with enough accuracy the contribution of more sophisticated FRP arrangements such as the overlay of sheets with different fiber orientations or the use of special anchorage devices.

In the case of *fib* model it is noticeable that the O, U and S configuration systems present close values for the mean of χ parameters, which are also close to the global mean of χ represented in the figure as a horizontal dashed line. The highest mean value of χ corresponds to O type of configuration while the smallest corresponds to S type, a pattern followed by the ACI and C&T design models.

For CNR it is noticeable that S type configuration presents a very poor performance resulting that the global mean value of χ may be largely influenced by those inconsistent results.

Results obtained using the RDB

The high scatter found in the previous analysis performed over a DB with more than 250 beams with highly differentiated characteristics, proves that none of the studied design models simulates with enough accuracy the generic behavior of RC beams strengthened in shear with externally bonded FRP. It was also found that all the aforementioned design proposals provided a large amount of unsafe values for V_{fd}, especially in the range $0 < V_{f,exp} < 100$ kN. Such can be related with a significant number of experimental results where, without a clear understanding, the load carrying increase due to the FRP reinforcement is either null or extremely small, with a possible disturbing effect in the global performance of the considered analytical models. From the above considerations, the consistency of results obtained with the IDB was appraised by means of removing from the analysis those observations, which in the judgment of the authors, lead into incoherent results. A reduced database (RDB) containing 130 beams extracted from the IDB was assembled. A beam was removed from DB when fulfils one of the following conditions: *i)* statistical outliers; *ii)* beams reinforced with bidirectional fibers; *iii)* reinforcement systems with special anchorage mechanisms; *iv)* beams that show poor performance in all of the aforementioned design models ($\chi < 0.25$).

Figure 6 presents the obtained results with the RDB, providing for each design model a scatter plot of the V_{fd} *vs* $V_{f,exp}$ relationship, an histogram of the χ ratio distribution and a box plot of the χ ratio variation related with the reinforcement configuration.

The values in Table 4 show that, despite the global improvement in the design models performance with the RDB, the results follow the same trend as for the IDB analysis, thus ratifying the consistency of the collected data and the conclusions extracted from the IDB.

Reliability analysis

The previous analysis based on statistical measures showed that the C&T design model may be assumed as the one with the best performance with a χ ratio closer to 1.5 and a COV closer to the minimum observed. Nevertheless, from a structural safety point of view, a classification system based only on the main descriptive statistics measures regarding the behavior of the χ factor may not provide enough information to assess the reliability of a design proposal, considering that for structural purposes having χ=0.5 is worst than χ=2.0, which is not taken into account on the statistical analysis.

To overcome this limitation a weighed penalty classification system was applied to the DB, based on the "Demerit Points Classification" (DPC) model proposed by Collins (2001), where a penalty (PEN) is assigned to each range of χ ratios according to Table 5, and the total of penalties determines the performance of each design model.

From Table 6 it can be noticed that the *fib* design model presents the weakest performance, with the highest number of penalty points corresponding to 40% of Predictions Against Safety (PAS, χ<1), while the best results are attained by the CNR design proposal with the lowest of number of PAS (20%). The CNR model also provides the highest number of extremely conservative (χ>3) values (32%), followed by the ACI design model (20%).

Figure 7 plots the safe (PSS) *vs* unsafe (PAS) predictions diagrams for both the IDB and the RDB. From their analysis, it is mandatory to emphasize that all the studied design models show a poor performance taking into account the large amount of unsafe predictions for the design value of the FRP contribution in shear.

The influence of the strengthening system in the reliability analysis is represented in Figure 8, where it is possible to observe that the CNR model is the only one where the type S configuration presents the most favorable results, while all other proposals present a general behavior where the O configuration system has the lowest number of PAS and the S configuration system has the highest number of PAS.

Considering the analysis performed with the RDB data, the ACI design model presents results close to the CNR model without having so many extremely conservative values of χ, as the CNR proposal. It is also noticeable that the ACI model is the only one without unsafe results attained in at least one configuration.

CONDITIONING FACTORS OF PREDICTIVE PERFORMANCE – PARAMETRIC STUDY

Based on the results obtained in the previous analysis it is possible to establish that the studied design models cannot predict with enough accuracy the contribution of externally bonded FRP reinforcement for shear strengthening of RC beams. In general, the adopted analytical formulations present a lack of robustness, as shown by the high number of predictions against safety attained in the present study. This is a serious concern in terms of the use of these models as design guidelines. Such poor performance indicates that the relative influence of the considered parameters is deficiently simulated, and the effect of others parameters, not explicitly taken into account, should not be neglected.

The FRP reinforcement configuration and application technique plays a major role in the effectiveness of the EBR strengthening system. To attend this fact, the quantification of V_{fd} by each of the studied models is dependent on the shear strengthening configuration and other specificities of the application technique.

Figure 9 shows the variation of the χ ratio for different shear strengthening configurations, according to the studied design models. For the *fib* design model, the beams strengthened with a type S configuration reveal a much worst performance when compared with O or U configurations. This indicates that the determination of the FRP effective strain, ε_{fe}, should explicitly consider the case of S strengthening configuration, where the available effective bond length L_e is necessarily different from the U type of strengthening configuration.

In the case of the ACI design model Figure 9 shows that the beams strengthened with a type O configuration reveal an average value for the χ ratio significantly higher than the ones observed for types S and U. This indicates that the effective strain limitation imposed by the ACI design model, $\varepsilon_{fe} = 0.004 \leq 0.75\varepsilon_{fu}$, may be excessively severe, conducting to highly conservative results. In case of the CNR design model the same figure shows that the predicted values for type S strengthening configuration are not well adjusted, with some χ ratios clearly above the mean values obtained with the O and U configuration types. The C&T design model presents the worst performance for type S strengthening configuration suggesting a possibility of enhancement with a better calibration of the $L_{\max}$ parameter in the analytical formulation.

Figure 10 presents the variation of the χ parameter regarding the concrete strength classes C1 to C4 as defined in this figure. Generally speaking, the design models show the tendency of predicting higher χ values with the increase of f_{ck}. The ACI design proposal is the one with a better correlation between the strength class increase and the enhancement of the strengthening system performance, while the other design proposals show similar behavior trends with a very bad performance for concrete compressive strengths bellow 20 MPa. This indicates that the aforementioned design models seem to be not suited for application to low strength concrete beams.

The influence of the FRP reinforcement ratio, ρ_f, is evaluated in Figure 11. Despite that all studied formulations consider the influence of ρ_f in the quantification of V_{fd}, it is shown that the design models performance is still dependent on this parameter. Except for the case of the CNR design model, the studied formulations present a pronounced trend of χ reduction with the increase of ρ_f.

Figure 12 presents the influence of the longitudinal reinforcement ratio in the predictive performance of the studied analytical models. Despite the fact that none of the design models explicitly consider the influence of ρ_{sl} in the prediction of V_{fd}, the attained results reveal that such parameter should not be neglected, with a general trend for higher χ ratios with the increase of ρ_{sl}.

The influence of shear reinforcement ratio is represented in Figure 13, where a clear pattern of performance reduction is observed with the increase of stirrups percentage. Such poor performance shown by beams with a medium/large amount of stirrups, in particular when $\rho_{sw} > \rho_{sw,\min}$, may be due the fact that the most of the tests supporting the calibration of the analytical formulations adopted by the studied design codes was conducted on beams with none or a very small stirrup percentage. On the other hand, the collected experimental data demonstrate that the orientation of the critical shear crack, θ_{cr}, may be quite different from the suggested value of 45º recommended by the design codes, and θ_{cr} depends on the existing conventional shear reinforcement in the strengthened beam. This fact has direct implications in the FRP contribution to the shear resistance, since it collaborates for the deficient predictions obtained in many cases with the studied formulations. The interaction between conventional steel reinforcement and FRP strengthening systems, as well as its effect in the FRP strengthening effectiveness, have been studied by several authors, either regarding flexural (Barros *et al.*, 2007) or shear retrofitting (Ali *et al.*, 2006, Bousselham and Chaallal, 2006, Pellegrino and Modena, 2006). From these studies one can establish that this interaction significantly decreases the performance of externally bonded FRP strengthening technique, which, in the opinion of the authors, indicates that this influence should be explicitly considered in the analytical formulations. Furthermore, all of the studied design models define the global shear resistance based on the assumption that the superimposition principle, $V_r = V_c + V_s + V_f$, is applicable, admitting, therefore, that the FRP strengthening system does not interfere with the contribution of the V_c and V_s items, determined independently and summed. This semi-empirical approach, despite being adopted by some of the most relevant reinforced concrete design codes (ACI 318, 2002, Eurocode 2, 2004) for current shear design, may lead to unrealistic results in many cases (ACI 445, 1999, Hawkins *et al.*, 2005) and its applicability should be questioned for FRP shear strengthening design.

Modified formulations for the evaluation of stirrups contribution to shear strength, as those proposed by Pellegrino and Modena (2008) or Ali *et al.* (2006) should be adopted for a more realistic design approach. The latter was implemented in the Australian Design Guideline as:

$$V_{tr-pl} = V_{uc} + k_{us}V_{us} + k_{tp}V_{tp} \tag{4}$$

where the reduction factors k_{us} and k_{tp} have the purpose of simulating the decrease of FRP shear strengthening effectiveness with the increase of the percentage of existing steel stirrups. In Eq. (4) $V_{tr\text{-}pl}$ is the transverse shear capacity of a beam with stirrups strengthened with externally bonded transversal plates, V_{uc} is the concrete component of shear capacity of unstrengthened member, V_{us} is the stirrup component of shear capacity and V_{tp} is the maximum transverse plate component of shear capacity. However, no indication is given in the Australian Design Guideline for the evaluation of the k_{us} and k_{tp} factors.

CONCLUSIONS

Based on an extensive literature review regarding the shear strengthening of reinforced concrete (RC) beams with fiber reinforced polymer (FRP) systems applied according to the externally bonded reinforcing (EBR) technique, a comprehensive database was assembled containing experimental results of more than 250 beams. The results obtained from a statistical analysis carried out on such database demonstrate that none of the analytical formulations predicts with enough accuracy the contribution of the EBR FRP systems for the shear strengthening of RC beams. A large scatter of the $\chi = V_{f,exp}/V_{fd}$ was found in all the studied design models, even when a reduced database (RDB) was used in the analysis. Using the RDB the average of the χ factor varies between 1.4 (*fib*) and 2.9 (CNR) and the coefficient of variation is comprehended between 43% (*fib*) and 57% (CNR). From a statistical point of view the C&T model can be pointed out as the one with the best performance, since it always combines an appropriate global safety factor (AVG χ = 1.67) with the one of most least scattered behaviors (COV χ = 47%). The large amounts of calculated V_{fd} values that are against safety suggest that all of the aforementioned models are still not robust enough for generalized practical design purposes.

A reliability analysis and classification based on structural safety was also implemented. Among the studied formulations, the *fib* design model presented the most unsafe results of all studied codes, while the safest results were attained with the CNR design code provisions. CNR also provided the largest amount of extremely conservative predictions, especially for the side bonding type strengthening configuration, and the second best performance was attained, by close, with the ACI model.

The influence of some parameters not explicitly considered on the analytical models was assessed, proving that the performance of the aforementioned design models is subordinate to the global attained shear force gain. Furthermore, the influence of conventional steel reinforcement (longitudinal and transversal) proved to be significant, and none of the studied analytical models explicitly considers these parameters to determine the FRP contribution to shear.

The collected database provided an important source for data mining techniques in order to decouple the interactions between all the phenomena involved. The conducted parametric study allowed the identification of some limitations regarding the applicability of the *fib*, ACI, CNR and C&T design models to current design practice, and verify that some parameters considered by the analytical formulations are still not properly calibrated, resulting in a poor performance of all the aforementioned models for prediction of FRP contribution to the global shear resistance of a strengthened beam.

As mentioned previously, the results obtained in the present study are not very promising, and a question related to the development of FRP shear strengthening design guidelines remains: Where should we go from here?

The authors' opinion is that the use of a widest range of available data, such as the collected database, may be useful for re-calibration previously developed analytical formulations that will lead to simple, cost-effective, design guidelines suitable for design practice. Thus, additional efforts towards the enrichment of developed databases worldwide should be made, and the setting of a web-based international database is suggested for this purpose.

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REFERENCES

ACI 445R–99. (1999). Recent approaches to shear design of structural concrete. *Reported by joint ACI-ASCE Committee 445*.

ACI 318-02/318R-02. (2002). Building code requirements for structural concrete and commentary, *Reported by ACI Committee 318*.

ACI 440.2R-02. (2002). Guide for the design and construction of externally bonded FRP systems for strengthening of concrete structures. *Reported by ACI Committee 440*.

Ali, M., Oehlers, D. and Seracino, R. (2006). Vertical shear interaction model between external FRP transverse plates and internal steel stirrups. *Engineering Structures*, **28**, 381-389.

Aprile, A. and Benedetti, A. (2004) Coupled flexural-shear design of R/C beams strengthened with FRP. *Composites Part B: Engineering* **35**, No.1, 1-25.

Barros, J., Dias S. and Lima, J. (2007). Efficacy of CFRP-based techniques for the flexural and shear strengthening of concrete beams. *Cement and Concrete Composites*, **29**, No. 3, 203-217.

Bousselham, A. and Chaallal, O. (2006). Effect of transverse steel and shear span on the performance of RC beams strengthened in shear with CFRP. *Composites Part B: Engineering*, **37**, No.1, 37-46.

Bousselham, A., and Chaallal, O. (2004). Shear strengthening reinforced concrete beams with fiber reinforced polymer: assessment of influencing parameters and required research. *ACI Structural Journal*, **110**, No.2, 219-227.

Chen, J.F. and Teng, J.G. (2003a). Shear Capacity of FRP Strengthened RC Beams: FRP Rupture. *Journal of Structural Engineering*, **129**, No. 5, 615–625.

Chen, J.F. and Teng, J.G. (2003b). Shear Capacity of FRP Strengthened RC Beams: FRP Debonding. *Construction and Building Materials*, **17**, No. 1, 27– 41.

CIDAR, (2006). Design guideline for RC structures retrofitted with FRP and metal plates: beams and slabs. *Draft 3 - submitted to Standards Australia*, The University of Adelaide.

CNR-DT200, 2004. Guidelines for design, execution and control of strengthening interventions by means of fibre reinforced composites. *National Research Council - Italy*.

Collins, M.P. (2001). Evaluation of shear design procedures for concrete structures. *A Report prepared for the CSA technical committee on reinforced concrete design*.

European Standard EN 1992. (2004). Eurocode 2: Design of concrete structures - Part 1-1: General rules and rules for buildings. English version.

fib Bulletin 14. (2001). Externally Bonded FRP reinforcement for RC structures. *Technical report, Task Group 9.3 FRP (fibre reinforced polymer) reinforcement for concrete structures*.

Hawkins, N., Kuchma, D., Mast, R., Marsh, M. and Reineck, K. (2005). NCHRP Report 549 – Simplified shear design of structural concrete members. *Reported by Transportation Research Board.*

Khalifa et al. (1998). Contribution of externally bonded FRP to shear capacity of RC flexural member. *ASCE Journal of Composites for Construction*, **2**, No. 4, 195-202.

Monti, G. and Liotta, M. (2005). FRP-strengthening in shear: tests and design equations. *7th International Symposium on Fiber Reinforced Polymer (FRP) Reinforcement for Concrete Structures (FRP7RCS)*, ACI Symposium Publication 230.

Pellegrino, C. and Modena, C. (2008). An experimentally based analytical model for the shear capacity of FRP-strengthened reinforced concrete beams. *Mechanics of Composite Materials*, **44**, No. 3, 231-244.

Pellegrino, C. and Modena, C. (2006). Fiber reinforced polymer shear strengthening of reinforced concrete beams: Experimental study and analytical modeling. *ACI Structural Journal*, **103**, No. 2, 720-728.

Pellegrino, C. and Modena, C. (2002). Fiber reinforced polymer shear strengthening of reinforced concrete beams with transverse steel reinforcement. *ASCE Journal of Composites for Construction*, **6**, No. 2, 104-111.

R Development Core Team. (2008). R: A Language and Environment for Statistical Computing. *R Foundation for Statistical Computing*.

Triantafillou, T. (1998). Shear strengthening of reinforced concrete beams using epoxy bonded FRP composites. ACI Structural Journal **11**, No. 9, 107-115.

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Fig. 12 – Influence of the longitudinal reinforcement ratio on the predictive performance of the models.

Fig. 13– Influence of the transversal reinforcement on the predictive performance of the models.

Table 1 - V_{fd} calculation methodology

***fib* design proposal:**	**ACI design proposal:**
$V_{fd} = 0.9 \cdot \varepsilon_{fed} \cdot E_f \cdot \rho_f \cdot b_w \cdot d \cdot \left(\cot\theta + \cot\beta\right) \cdot \sin\beta$ $\rho_f = \frac{2 \cdot t_f \cdot w_f}{b_w \cdot s_f}$ (strips) ; $\rho_f = \frac{2 \cdot t_f \cdot \sin\beta}{b_w}$ (cont.) $\varepsilon_{fed} = \frac{0.8 \cdot \varepsilon_{fe}}{\gamma_f}$; $\gamma_f = 1.2 \ / \ 1.3 \ / \ 1.35$ *i)* Full wrapping configuration (O): $\varepsilon_{fe} = 0.17 \cdot \left(\frac{f_{cm}^{2/3}}{E_f \cdot \rho_f / 1000} \right)^{0.30} \cdot \varepsilon_{fu}$ *ii)* Side bonding or U jacketing configuration (U, S): $\varepsilon_{fe} = \min \begin{cases} 0.65 \cdot \left(\frac{f_{cm}^{2/3}}{E_f \cdot \rho_f / 1000} \right)^{0.56} \cdot 10^{-3} \\ 0.17 \cdot \left(\frac{f_{cm}^{2/3}}{E_f \cdot \rho_f / 1000} \right)^{0.30} \cdot \varepsilon_{fu} \end{cases}$	$V_{fd} = \phi \cdot \psi_f \cdot \left(2 \cdot t_f \cdot \frac{w_f}{s_f} \cdot f_{fe} \cdot (\sin\beta + \cos\beta) \cdot d_f \right)$ $\phi = 0.85$; $\psi_f = 0.95$ (O) ; $\psi_f = 0.85$ (U,S) $f_{fe} = E_f \cdot \varepsilon_{fe}$ *i)* Full wrapping configuration (O): $\varepsilon_{fe} = 0.004 \le 0.75 \cdot \varepsilon_{fu}$ *ii)* Side bonding or U jacketing configuration (U, S): $\varepsilon_{fe} = k_v \cdot \varepsilon_{fu} \le 0.004$ $k_v = \frac{k_1 \cdot k_2 \cdot L_e}{11900 \cdot \varepsilon_{fu}} \le 0.75$ $k_1 = \left(\frac{f_{ck}}{27} \right)^{2/3}$; $L_e = \frac{23300}{\left(t_f \cdot E_f\right)^{0.58}}$ $k_2 = \frac{d_f - L_e}{d_f}$ (U) ; $k_2 = \frac{d_f - 2 \cdot L_e}{d_f}$ (S)
Notation: ε_{fed} - design value of effective FRP strain; ε_{fe} - mean value of effective FRP strain; ε_{fu} - FRP ultimate tensile strain; γ_f - partial factor for FRP reinforcement; ρ_f - FRP reinforcement ratio; E_f - elasticity modulus of FRP reinforcement; f_{cm} - concrete average compressive strength;	 ϕ - shear strength reduction factor; ψ_f - additional reduction factor for FRP; k_v - bond reduction coefficient; k_1 - modif. factor regarding the concrete strength; k_2 - modif. factor regarding the FRP configuration; L_e - effective bond length of FRP reinforcement; f_{ck} - concrete characteristic compressive strength;

Table 2 - V_{fd} calculation methodology (Cont.)

CNR design proposal:

i) Full Wrapping configuration (O)

$$V_{fd} = \frac{1}{\gamma_{Rd}} \cdot 0.9 \cdot d \cdot f_{fed} \cdot 2 \cdot t_f \cdot (\cot\theta + \cot\beta) \cdot \frac{w_f}{s_f'}$$

$$f_{fed} = f_{fdd} \cdot \left[1 - \frac{1}{6} \cdot \frac{L_e \cdot \sin\beta}{\min\{0.9 \cdot d, h_w\}}\right] +$$

$$+ \underbrace{\frac{1}{2}(\phi_R \cdot f_{fd} - f_{fdd}) \cdot \left[1 - \frac{L_e \cdot \sin\beta}{\min\{0.9 \cdot d, h_w\}}\right]}_{\geq 0}$$

$$\phi_R = 0.2 + 1.6 \cdot \frac{r_c}{b_w} \quad ; \quad 0 \leq \frac{r_c}{b_w} \leq 0.5$$

$$L_e = \sqrt{\frac{E_f \cdot t_f}{2 \cdot f_{ctm}}} \quad ; \quad f_{fdd} = \frac{0.80}{\gamma_{fd}} \cdot \sqrt{\frac{2 \cdot E_f \cdot G_{fk}}{t_f}}$$

$$G_{fk} = 0.03 \cdot k_b \cdot \sqrt{f_{ck} \cdot f_{ctm}} \quad ;$$

$$k_b = \sqrt{\frac{2 - w_f / s_f'}{1 + w_f / 400}} \geq 1$$

ii) U jacket configuration (U)

$$f_{fed} = f_{fdd} \cdot \left[1 - \frac{1}{3} \cdot \frac{L_e \cdot \sin\beta}{\min\{0.9 \cdot d, h_w\}}\right]$$

iii) Side bonding configuration (S)

$$V_{fd} = \frac{1}{\gamma_{Rd}} \cdot \min\{0.9 \cdot d, h_w\} \cdot f_{fed} \cdot 2 \cdot t_f \cdot \frac{\sin\beta}{\sin\theta} \cdot \frac{w_f}{s_f'}$$

$$f_{fed} = f_{fdd} \cdot \frac{z_{red,eq}}{\min\{0.9 \cdot d, h_w\}} \cdot \left(1 - 0.6 \cdot \sqrt{\frac{L_{eq}}{z_{red,eq}}}\right)^2$$

$$z_{red,eq} = z_{red} + L_{eq}$$

$$z_{red} = \min\{0.9 \cdot d, h_w\} - L_e \cdot \sin\beta$$

$$L_{eq} = \frac{s_{uf}}{f_{fdd} / E_f} \cdot \sin\beta$$

CIDAR (C&T) design proposal:

$$V_{fd} = 2 \cdot f_{fed} \cdot t_f \cdot \frac{w_f}{s_f'} \cdot h_{fe} \cdot (\cot\theta + \cot\beta) \cdot \sin\beta$$

$$h_{fe} = z_b - z_t \quad ; \quad z_b = 0.9 \cdot d - d_{fb} \quad ; \quad z_t = d_{ft}$$

$$f_{fed} = D_f \cdot f_{fd,\max}$$

i) Failure by FRP rupture (O)

$$D_f = 0.5 \cdot \left(1 + \frac{z_t}{z_b}\right)$$

$$f_{fd,\max} = \begin{cases} \frac{1}{\gamma_f} \cdot \phi_R \cdot f_{fu}, & \varepsilon_f \leq 1.5\% \\ \frac{1}{\gamma_f} \cdot \phi_R \cdot E_f \cdot \varepsilon_f, & \varepsilon_f > 1.5\% \end{cases}$$

$$\phi_R = 0.80 \;;\; \gamma_f = 1.25$$

ii) Failure by FRP debonding (U , S)

$$D_f = \begin{cases} \frac{2}{\pi \cdot \lambda} \cdot \frac{1 - \cos\left(\frac{\pi}{2} \cdot \lambda\right)}{\sin\left(\frac{\pi}{2} \cdot \lambda\right)}, & \lambda \leq 1 \\ 1 - \frac{\pi - 2}{\pi \cdot \lambda}, & \lambda > 1 \end{cases} \quad ;$$

$$\lambda = L_{\max} / L_e$$

$$L_{\max} = \begin{cases} \frac{h_{fe}}{\sin\beta}, & (U) \\ \frac{h_{fe}}{2 \cdot \sin\beta}, & (S) \end{cases} \quad ; \quad L_e = \sqrt{\frac{E_f \cdot t_f}{\sqrt{f_{ck}}}}$$

$$f_{fd,\max} = \min \begin{cases} \frac{1}{\gamma_f} \cdot \phi_R \cdot f_{fu} \\ \frac{1}{\gamma_f} \cdot 0.35 \cdot \beta_L \cdot \beta_w \cdot \sqrt{\frac{E_f \cdot \sqrt{f_{ck}}}{t_f}} \end{cases}$$

$$\beta_L = \begin{cases} \lambda, & \lambda \leq 1 \\ 1, & \lambda > 1 \end{cases} \quad ; \quad \beta_w = \sqrt{\frac{2 - w_f / (s_f' \cdot \sin\beta)}{1 + w_f / s_f' \cdot \sin\beta}}$$

Notation:

γ_{Rd} - partial factor for the resistance model (1.2);	ϕ_R - reduction factor due to local stress in corners;
f_{ctm} - average concrete tensile strength;	λ - normalized maximum bond length;
f_{fed} - design value for the FRP effective stress;	D_f - stress distribution factor;
f_{fd} - design value for the ultimate FRP stress;	$f_{fd,\max}$ - maximum design stress in FRP;
f_{fdd} - design value for the FRP debonding stress;	f_{fu} - ultimate FRP tensile stress;
G_{fk} - bonded joint specific fracture energy;	h_{fe} - effective height of the bonded reinforcement;
k_b - covering / scale coefficient;	β_L - bond length coefficient;
s_{uf} - FRP slip at debonding (0.20mm);	β_w - strip width coefficient;

Table 3 – Statistical values of the χ factor computed from the IDB

χ	Min	Q1	MED	AVG	Q3	MAX	STD	COV
fib	0.00	0.730	1.198	1.22	1.718	3.278	0.666	0.546
ACI	0.00	0.980	1.903	2.017	2.831	5.961	1.255	0.622
CNR	0.00	1.126	2.108	2.528	3.541	9.261	1.846	0.730
C&T	0.00	0.875	1.370	1.431	1.962	5.454	0.826	0.577

Table 4 – Statistical values of the χ factor computed from the RDB

χ	Min	Q1	MED	AVG	Q3	MAX	STD	COV
fib	0.071	0.868	1.347	1.352	1.745	3.278	0.616	0.456
ACI	0.337	1.197	2.081	2.128	2.831	5.463	1.092	0.513
CNR	0.380	1.658	2.397	2.794	3.654	8.931	1.705	0.610
C&T	0.177	1.010	1.443	1.609	2.011	5.454	0.808	0.502

Table 5 – Demerit points classification criteria

$\chi = V_{f,exp}/V_{fd}$	Classification	Penalty
< 0.75	Extremely Dangerous	10
[0.75-1.00[	Dangerous	5
[1.00-1.25[	Reduced Safety	2
[1.25-1.75[	Appropriate Safety	0
[1.75-3.00[	Conservative	1
≥ 3.00	Extremely Conservative	2

Table 6 – Reliability analysis based on structural safety

χ	*fib*		ACI		CNR		C&T	
	Nº samples	Total	Nº samples	Total	Nº samples	Total	Nº samples	Total
< 0.75	55	550	32	320	28	280	45	450
0.75 - 1.00	30	150	22	110	15	75	23	115
1.00 - 1.25	26	52	18	36	16	32	30	60
1.25 - 1.75	53	0	26	0	27	0	43	0
1.75 - 3.00	47	47	65	65	58	58	65	65
> 3.00	1	2	42	84	68	136	6	12
∑ PEN	212	801	205	615	212	581	212	702