International Conference
on Industrial Engineering and Systems Management
IESM 2009
May 13-15
MONTRÉAL - CANADA

Quantity Oriented Resource Allocation Strategy on Multiple Resources Projects under Stochastic Conditions ⋆

Anabela Tereso [a], Madalena Araújo [a], Rui Moutinho [a], Salah Elmaghraby [b]

[a] *Universidade do Minho, 4800-058 Guimarães, PORTUGAL*

[b] *North Carolina State University, Raleigh, NC 27695-7906, USA*

Abstract

Previous developments from the first author and other researchers were made on devising models for the total cost optimization of projects described by activity networks under stochastic conditions. Those models only covered the single resource case.

The present paper will discuss the case of multiple resources. More precisely, we introduce a strategy of allocation of those resources in order to minimize the waste arising from their latent idleness on their consumption within the same activity. On this strategy we elect one resource as "pivot" and write equations that describe all the quantities for the other resources, on the same activity.

Key words: Project Management and Scheduling, Stochastic Activity Networks, Resource Allocation, Multiple Resources

List of Acronyms

`RCPSP`: *Resource Constraint Project Scheduling Problem*
`AoA`: *Activity-on-Arc*
`DP`: *Dynamic Programming*
`EMA`: *Electromagnetic Algorithm*
`EVA`: *Evolutionary Algorithm*
`SRpco`: *Single Resource Project Cost Optimization*
`MRpco`: *Multiple Resources Project Cost Optimization*
`QOras`: *Quantity Oriented Resource Allocation Strategy*
`WBras`: *Waste Balance Resource Allocation Strategy*

1 Introduction

This paper follows the researches made by several contributors starting from the first research made by the first author (see [1]). Those works address a `RCPSP` (*Resource Constraint Project Scheduling Problem*) where we want to minimize the total project cost involving multimodal activities under stochastic conditions. More precisely, given an project on its planning phase, where all its activities and related requirements are established, we want to determine which is the optimal allocation of resources. This

⋆ Corresponding author A. Tereso.
Email addresses: `anabelat@dps.uminho.pt` (Anabela Tereso), `mmaraujo@dps.uminho.pt` (Madalena Araújo), `rumout@gmail.com` (Rui Moutinho), `elmaghra@eos.ncsu.edu` (Salah Elmaghraby).

allocation yields the minimum total project cost, which is composed by the sum of the allocation cost and the delay cost.

All the researches so far, work with AoA (*Activity-on-Arc*) network representation of the project activities and their precedences. By treating multimodal activities, we deal with activities which their duration time is measured as a function of the resource allocation. As an example, if a certain activity requires 6 days with 1 man allocated, it should require only 2 days when the allocation is 3 men. However, in the stochastic context, that relation between allocation and duration is not deterministic. This requires that we take the *Work Content* of an activity as a probabilistic distribution. In particular, we have been using the exponential distribution for that matter.

Since the first approach to this problem, on MATLAB using DP (*Dynamic Programming*) [2], the subsequent researches improved the computational approach by migrating the initial work to JAVA [6]. Another MATLAB implementation, but using a global optimization algorithm [3], the EMA (*Electromagnetic Algorithm*), was also migrated to JAVA [7]. Then, the use of the EVA (*Evolutionary Algorithm*) was applied to the same problem [5]. All of these implementations, only cover the SRPCO (*Single Resource Project Cost Optimization*) models where the total project cost C is obtained by

$$C = \mathcal{E}\left[\sum_{a \in \mathrm{A}} (c \times x_a \times W_a) + c_L \times \max\Big(0, \Upsilon_n - T\Big)\right] \tag{1}$$

where the following notation applies:

A: Set of project activities;
c: Quantity (of resource) cost per unit
c_L: Project delay cost per time unit
$x_a \in \mathbb{R}^+$: Allocated quantity of resource on activity a such $l_a \leqslant x_a \leqslant u_a$
$W_a \sim \mathrm{Exp}(\lambda_a)$: Work content of activity a
Υ_n: Evaluated realization time of last node
T: Schedule project realization time

This paper addresses the more general case – the MRPCO (*Multiple Resources Project Cost Optimization*) problem. In this case, the project has a set of resources and each activity may require several of them instead of just one. On the SRPCO models, the behavior of an activity was fully described by its single resource. However, on the new MRPCO models we need to describe that behavior for several resources. Each resource will have its own work content and allocation constraints according to each activity's needs.

1.1 The Impact of Resource Multiplicity

The extension of the project cost evaluation from the SRPCO model is quite straightforward. The multiplicity of resources simply induces a sum of the allocation cost of each resource of each activity. Thus, the total project cost C is

$$C = \mathcal{E}\left[\sum_{a \in \mathrm{A}} \sum_{r \in \mathrm{R}_a} (c_r \times x_r^a \times W_r^a) + c_L \times \max\Big(0, \Upsilon_n - T\Big)\right] \tag{2}$$

where the extended notation applies (refer to Eq. (1)):

R_a: project resources subset needed by activity a
c_r: quantity cost per unit of resource r
$x_r^a \in \mathbb{R}^+$: allocated quantity of resource r on activity a such $l_r^a \leqslant x_r^a \leqslant u_r^a$
$W_r^a \sim \mathrm{Exp}(\lambda_r^a)$: Work content of resource r on activity a

To each resource allocation to an activity is associated an individual duration Y_r^a evaluated similar to the SRPCO model.

$$Y_r^a = \frac{W_r^a}{x_r^a} \tag{3}$$

The actual activity duration – Y_a – is, therefore, the maximum of those individual ones.

$$Y_a = \max_{r \in \mathrm{R}_a} \Big(Y_r^a\Big) \tag{4}$$

Clearly it makes little sense to expend more of a resource (and incur a higher cost) to have the activity duration under this resource less than its duration under any other resource. Thus, it is desired to have (in expectation)

$$\mathcal{E}\left[Y_i^a\right] = \mathcal{E}\left[Y_j^a\right], \forall i, j \in \mathrm{R}_a \tag{5}$$

To ensure allocation vectors leading to the desired equality we devise the following strategy.

2 Quantity Oriented Strategy

The QORAS (*Quantity Oriented Resource Allocation Strategy*) tries to secure equality on individual durations, yielded by the resources within an activity, by electing one of those resources as the pivot. Then, using the equality on Eq. (5), the quantities for all resources are, immediately, known.

For ease of explanation, we shall assume the existence of only two resources (indexed by 1 and 2) in the context of an activity $a \in \mathrm{A}$, i.e. $\#\mathrm{R}_a = 2$

Given the two resources, Eq. (4) becomes,

$$Y_a = \max\left(Y_1^a, Y_2^a\right) \tag{6}$$

where, by Eq. (3)

$$Y_1^a = \frac{W_1^a}{x_1^a} \qquad Y_2^a = \frac{W_2^a}{x_2^a} \tag{7}$$

From the equality condition, it follows

$$\mathcal{E}\left[Y_1^a\right] = \mathcal{E}\left[\frac{W_1^a}{x_1^a}\right] = \frac{\mathcal{E}\left[W_1^a\right]}{x_1^a} \tag{8}$$

$$\mathcal{E}\left[Y_2^a\right] = \frac{\mathcal{E}\left[W_2^a\right]}{x_2^a} \tag{9}$$

$$\tag{10}$$

Thus, the equality of the individual durations (in expectation) is

$$\frac{\mathcal{E}\left[W_1^a\right]}{x_1^a} = \frac{\mathcal{E}\left[W_2^a\right]}{x_2^a} \tag{11}$$

and putting all the allocated quantities, say, in terms of x_1^a, we obtain

$$x_2^a = \frac{\mathcal{E}\left[W_2^a\right]}{\mathcal{E}\left[W_1^a\right]} x_1^a \tag{12}$$

From the last equation we can put the allocation discussion. The process is not yet complete since we need to enforce that all the resulting allocated quantities lie within their feasible intervals.

In order to guide us throughout the allocation method, we start the following running example.

Example – Part 1 of 2

Assume that we have only two resources (indexed as 1 and 2) in an arbitrary fixed activity a. Suppose, also, that

$$x_1^a \in [0.5, 2] \quad \mathcal{E}\left[W_1^a\right] = 1 \qquad x_2^a \in [1.5, 3] \quad \mathcal{E}\left[W_2^a\right] = 2 \tag{13}$$

By using resource 1 as the pivot, we have from Eq. (12)

$$x_2^a = 2x_1^a \tag{14}$$

In Table 1 there are four allocations resulting from the application of Eq. (14) to a set of four feasible allocations of resource 1. As the table shows, some evaluated x_2^a are out of bounds.

Table 1
QOras – Problems while allocating a resource by decision over another

x_1^a	$x_2^a = 2x_1^a$	x_2^a in bounds?
0.5	1.0	**no (smaller)**
1.0	2.0	yes
1.5	3.0	yes
2.0	4.0	**no (greater)**

In order to correct the invalid allocations we may simply truncate the values into the feasible region. So, whenever the invalid allocation is smaller (greater) than the lower (upper) bound, we enforce the value to be equal to the lower (upper) bound. Thus, let $\hat{x}_2^a$ be the corrected value of the allocation of the resource 2:

$$\hat{x}_2^a = \min\left(u_2^a, \max\left(l_2^a, \frac{\mathcal{E}\left[W_2^a\right]}{\mathcal{E}\left[W_1^a\right]} x_1^a\right)\right) \tag{15}$$

But, while the values for resource 2 are corrected, we must observe that it is required to adjust x_1^a as well.

Example – Part 2 of 2

Continuing our example, we must increase the x_2^a from 1.0 to $1.5 = l_2^a$ and decrease from 4.0 to $3.0 = u_2^a$, on the notable cases, respectively. Table 2 synthesizes these corrections and their implications over the proportionality relation. When x_2^a increases (decreases) the correct value for x_1^a must decrease

Table 2
QOras – $\hat{x}_2^a$ correction

x_2^a	$\hat{x}_2^a$	Correction	Implication on x_1^a
1.0	1.5	increase	must decrease
2.0	2.0	not needed	no
3.0	3.0	not needed	no
4.0	3.0	decrease	must increase

(increase) in proportion. By using a simple equation, derived from Eq. (14), we can evaluate the value of x_1^a corresponding to the newly adjusted $\hat{x}_2^a$. So, for $\hat{x}_2^a = 1.5$ it follows

$$1.5 = 2\hat{x}_1^a \Leftrightarrow \hat{x}_1^a = 0.75 \tag{16}$$

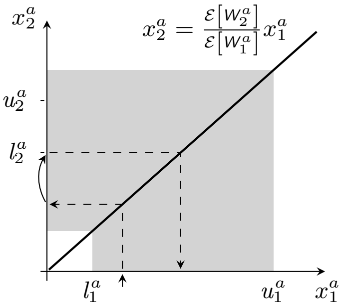
By similar reasoning, when $\hat{x}_2^a = 3.0$, $\hat{x}_1^a = 1.5$.

Both values of $\hat{x}_1^a$ are within the feasible region. Therefore, the proportion between the two resources is successfully restored. On Table 3 are presented the final corrected allocations.

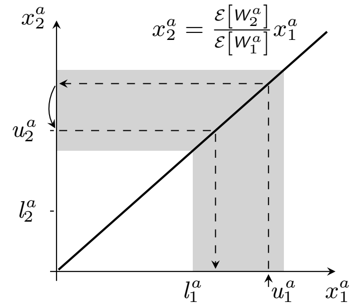
Table 3
QOras – Allocations before and after correction

x_1^a	x_2^a	$\hat{x}_1^a$	$\hat{x}_2^a$
0.50	1.00	0.75	1.50
1.00	2.00	1.00	2.00
1.50	3.00	1.50	3.00
2.00	4.00	1.50	3.00

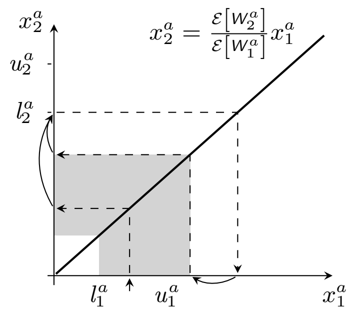
As just seen on the example, the use of a simple equation was sufficient to restore the proportionality relation. We will now formalize that correction and study its implications on the other allocations. We will do that by observing the three possible scenarios:



(a) Allocation interval of x_2^a is a subset of the range of the function of Eq. (12)



(b) Range of function of Eq. (12) intersects the allocation interval of x_2^a but does not includes it



(c) Range of function of Eq. (12) does not intersect the allocation interval of x_2^a

Fig. 1. `QOras` – Typical scenarios on the corrective mechanism on the context of two resources

Case $\boldsymbol{x_2^a \in \left[l_2^a, u_2^a\right]}$ All allocated quantities are within their feasible space. Nothing is further required.

Case $\boldsymbol{x_2^a < l_2^a}$ Requires correction by putting $x_2^a = l_2^a$. The pivot also needs correction in order to restore the relation between the quantities. Thus,

$$\hat{x}_1^a = \frac{\mathcal{E}\left[W_1^a\right]}{\mathcal{E}\left[W_2^a\right]} l_2^a \tag{17}$$

Case $\boldsymbol{x_2^a > u_2^a}$ Analogously to the $x_2^a < l_2^a$ case, it goes $\hat{x}_2^a = u_2^a$ and

$$\hat{x}_1^a = \frac{\mathcal{E}\left[W_1^a\right]}{\mathcal{E}\left[W_2^a\right]} u_2^a \tag{18}$$

After restoring the proportionality relation, there remains the possibility that the resulting $\hat{x}_1^a$ laying out of its feasible region in which case we will need to truncate it, thus,

$$\hat{x}_1^a = \min\left(u_1^a, \max\left(l_1^a, \frac{\mathcal{E}\left[W_1^a\right]}{\mathcal{E}\left[W_2^a\right]}\alpha\right)\right) \tag{19}$$

where $\alpha = l_2^a$ case $\hat{x}_2^a < l_2^a$ or $\alpha = u_2^a$ case $\hat{x}_2^a > u_2^a$.

This truncation, however, may raise a second need for restoration of the proportionality relation. But, this in turn will start a vicious cycle of corrections. Suffice it to say that this particular scenario only occurs when the range of function on Eq. (12) does not intersect the feasible interval of x_2^a (see Fig. 1). Which means that it is impossible to secure the equality between Y_1^a and Y_2^a.

3 Discussion and Conclusion

In the two resources only case scenario, it is still quite manageable to assess the impossibility of equal individual durations. But, the generalization to an arbitrary number can be tricky as the corrective mechanism itself brings unexpected complexity.

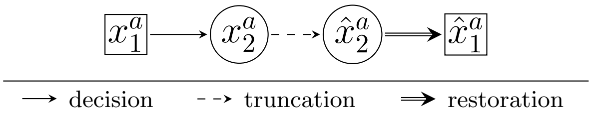


Fig. 2. `QOras` – Corrective mechanism scheme (two resources) with resource 1 as pivot

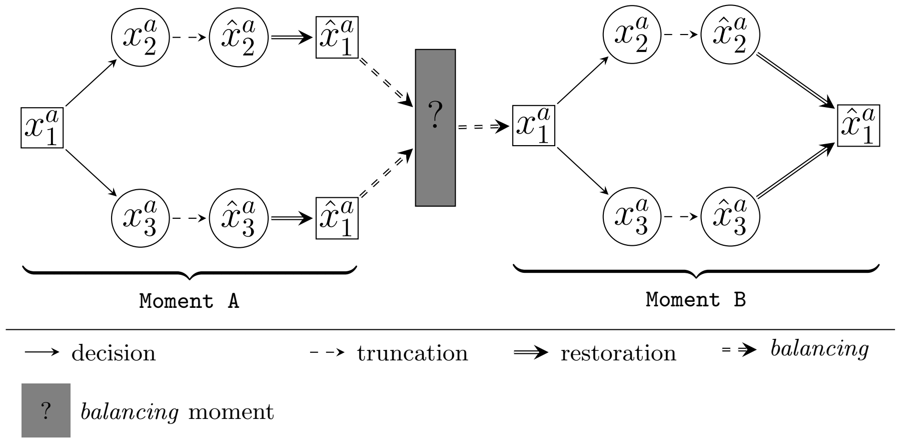


Fig. 3. `QOras` – Corrective mechanism scheme (three resources) with resource 1 as pivot

Fig. 2 represents the corrective mechanism with only two resources, which may be satisfactory. However, with three or more resources a new problem has to be addressed, namely that of incompatibility between the results secured from the two non-pivotal resources.

The expansion of the decision equation is straightforward:

$$\frac{\mathcal{E}\left[W_1^a\right]}{x_1^a} = \frac{\mathcal{E}\left[W_2^a\right]}{x_2^a} = \frac{\mathcal{E}\left[W_3^a\right]}{x_3^a} \tag{20}$$

from which we secure:

$$x_2^a = \frac{\mathcal{E}\left[W_2^a\right]}{\mathcal{E}\left[W_1^a\right]} x_1^a \quad \text{and} \quad x_3^a = \frac{\mathcal{E}\left[W_3^a\right]}{\mathcal{E}\left[W_1^a\right]} x_1^a \tag{21}$$

When the corrective procedure is applied a new problem arises. Fig. 3 gives a schematic of what can happen and the corrective action that may be taken. At first the two $\hat{x}_1^a$'s that result from applying the correction to x_2^a and x_3^a independently are different, leading to inconsistency in the decision, which needs to be treated. A new mechanism must evaluate a suitable allocation of the pivot resource in order to achieve the desired result. Naturally, the issue becomes increasingly more complex as more resources are involved.

Although the `QOras` is intuitive, it is incapable of enforcing equal durations and is difficult to generalize to more than two resource.

Instead of pursuing ways to improve `QOras`, we decided to diverge our approach and look for other strategies of multiple resources allocation, namely the `WBras` (*Waste Balance Resource Allocation Strategy*) presented last year in `EngOpt2008` [4].

References

[1] Anabela P. Tereso. *Gestão de Projectos – Alocação adaptativa de recursos em redes de actividades multimodais*. PhD thesis, Universidade do Minho, 2002.

[2] Anabela P. Tereso, M. Madalena T. Araújo, and Salah E. Elmaghraby. Adaptive Resource Allocation in Multimodal Activity Networks. *International Journal of Production Economics*, 92(1):1–10, November 2004.

[3] Anabela P. Tereso, M. Madalena T. Araújo, and Salah E. Elmaghraby. The Optimal Resource Allocation in Stochastic Activity Networks via the Electromagnetism Approach. *Ninth International Workshop on Project Management and Scheduling (PMS'04)*, April 2004.

[4] Anabela P. Tereso, M. Madalena T. Araújo, Rui Moutinho, and Salah E. Elmaghraby. Project management: multiple resources allocation. Rio de Janeiro - Brazil, June 1-5 2008. International Conference on Engineering Optimization (EngOpt2008). Full text published in the proceedings (ISBN 978857650156-5).

[5] Anabela P. Tereso, Lino A. Costa, Rui A. Novais, and Madalena T. Araújo. The Optimal Resource Allocation in Stochastic Activity Networks via the Evolutionary Approach: A Platform Implementation in Java. Beijing, China, May 30 – June 2 2007. International Conference on Industrial Engineering and Systems Management (IESM' 2007). Full paper published in the proceedings (ISBN 978-7-89486-439-0).

[6] Anabela P. Tereso, João Ricardo M. Mota, and Rui Jorge T. Lameiro. Adaptive Resource Allocation to Stochastic Multimodal Projects: A Distributed Platform Implementation in Java. *Control and Cybernetics Journal*, 35(3):661–686, 2006.

[7] Anabela P. Tereso, Rui A. Novais, and M. Madalena T. Araújo. The Optimal Resource Allocation in Stochastic Activity Networks via the Electromagnetism Approach: A Platform Implementation in Java. Reykjavc, Iceland, July 2006. 21st European Conference on Operational Research (EURO XXI). Submitted to the "Control and Cybernetics Journal" (under revision).