On the Optimal Control of a Cascade of Hydro-Electric Power Stations

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In this paper we present a model for a cascade of hydro-electric power stations where some of the stations have reversible turbines. The objective of our work is to optimize the profit of power prodution. The problem is considered in the framework of discrete-time optimal control and is solved using numerical methods. The simulation uses real data.

Keywords: Energy Policy and Planning, Natural Resources, Optimal Control

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1. Introduction

Water is becoming a scarce resource and its use has attained, in more advanced countries, a certain degree of sophistication. This has impact on how the water is used to produce electric energy. The management of multireservoir systems has attracted the attention of many researchers (Labadie, 2004; Ladurantaye et al, 2009, e.g.). It is especially important if there is also a possibility of reusing the downstream water in a situation of drought. This may be implemented in modern reversible hydroelectric power stations, associated with reservoirs along a river basin with a cascade structure, where it is possible both to discharge water from upstream to produce electric power and to pump from downstream to refill an upstream reservoir. Here we present a model for a cascade of hydro-electric power stations where some of the stations have reversible turbines. There are restrictions on the water level in the reservoirs and the objective is to optimize the profit of power production. The problem is considered in the framework of a discrete-time optimal control and is solved using numerical methods. The simulation uses real data.
The paper is organized as follows: the model is presented and the problem is stated

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in section 2; in section 3 the computational experiments are described and obtained results are presented; section 4 contains some conclusions.

2. Problem statement

Consider a cascade of hydro-electric power stations like the one shown in Figure 1. The dynamics of water volumes, $V_i(t)$, in the reservoirs $i = 1, 2, \ldots, I$, at time t, is

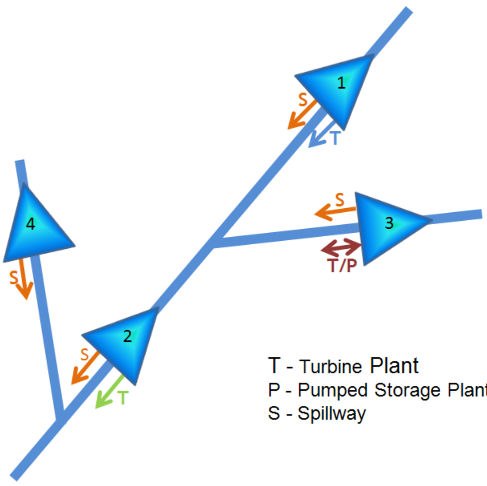

Figure 1. Example of a cascade of hydro-electric power stations.

described by the following discrete-time control system

$$
\begin{aligned}
& V_i(t) = V_i(t-1) + a_i - q_i(t) - s_i(t) + \sum_{m \in M_i} q_m(t) + \sum_{n \in N_i} s_n(t), \qquad (1) \\
& t = 1, 2, \ldots, T, \quad i = 1, 2, \ldots, I, \\
& V_i(0) = V_i^{in} \qquad i = 1, 2, \ldots, I,
\end{aligned}
$$

where $V^{in} = (V_1^{in}, \ldots, V_I^{in})$ is the vector of initial stored water volumes in the reservoirs $i = 1, \ldots, I$, M_i represents the set of reservoir indices from which the water flow comes to reservoir i, from pumping or turbining, N_i is the set of reservoir indices contributing to the spillway to reservoir i, $q(t) = (q_1(t), \ldots, q_I(t))$, and $s(t) = (s_1(t), \ldots, s_I(t))$, $t = 1, 2, \ldots, T$, are the controls, representing the turbined/pumped volumes of water and spillways for each reservoir at time t. The controls and the water volumes satisfy the following constraints:

$$
\zeta_i \left(h_i(t) - h_i^0 \right) - q_i^{0P} \leq q_i(t) \leq q_i^{0T} \left(h_i(t)/h_i^0 \right)^{\frac{1}{2}} \ , \qquad (2)
$$

$$
Z_i^{min} \leq Z_i(t) \leq Z_i^{max} \ , \qquad (3)
$$

$$
V_i^{in} - a_i \leq V_i(T), \qquad (4)
$$

where

$$Z_i(t) = Z_i^0 + \alpha_i \left(\frac{V_i(t)}{V_i^0} - 1 \right)^{\beta_i}$$

and

$$h_i(t) = Z_i(t) - \max \{Z_j(t), \xi_i\}.$$

Here j stands for the number of the downstream reservoir receiving water from reservoir i, $h_i(t)$ are the differences between water levels (see Figure 2), V_i^0, $i = 1, 2, \ldots, I$, are the minimal water volumes; $Z_i(t)$, $i = 1, 2, \ldots, I$, are the water levels in the reservoirs; Z_i^0, Z_i^{min}, and Z_i^{max} stand for the imposed nominal, minimal and maximal water levels (meters above sea level) respectively; h_i^0, $i = 1, 2, \ldots, I$, are nominal heads, and ξ_i, $i = 1, 2, \ldots, I$, are tailwater levels; q_i^{0T}, $i = 1, 2, \ldots, I$, and q_i^{0P}, $i = 1, 2, \ldots, I$, are the nominal turbined and pumped water volumes; a_i, $i = 1, 2, \ldots, I$, are the incomming flows; finally $\alpha_i, \beta_i, \zeta_i$, $i = 1, 2, \ldots, I$, are positive constants.

Consider the following discrete-time optimal control problem with mixed constraints. The functional, representing the profit, has the form

$$P(q, s, V^{in}) = \int_0^T price(t) \left(\sum_{i=1}^{I} r_i(t) \right) dt.$$

The head losses in reservoir i at instant t, $\Delta h_i(t)$, are given by

$$\Delta h_i(t) = \Delta h_i^{0T} \left(\frac{q_i(t)}{q_i^{0T}} \right)^2.$$

The functions $r_i(t), i = 1, 2, \ldots, I$, are given by

$$r_i(t) = \begin{cases} 9.8 * q_i(t) * \left(h_i(t) - \Delta h_i^T(t)\right) * \mu_i^T * (1 - \phi_i), & \text{if } q_i(t) \geq 0, \\ 9.8 * q_i(t) * \left(h_i(t) + \Delta h_i^P(t)\right) * 1/\mu_i^P * (1 - \phi_i), & \text{if } q_i(t) < 0. \end{cases}$$

The functions $r_i(t)$ connect the amounts of turbined water and the values of the gross head. The dynamics is described by discrete-time optimal control system (1) with constraits (2)-(4).

The optimal values V_i^{in}, $i = 1, 2 \ldots, I$, give the mean volumes of water that are necessary to keep in the reservoirs when the incomming flows are a_i, $i = 1, 2, \ldots, I$.

For example, in the case of the two reservoir system shown in Figure 3, the respective optimization problem has the form:

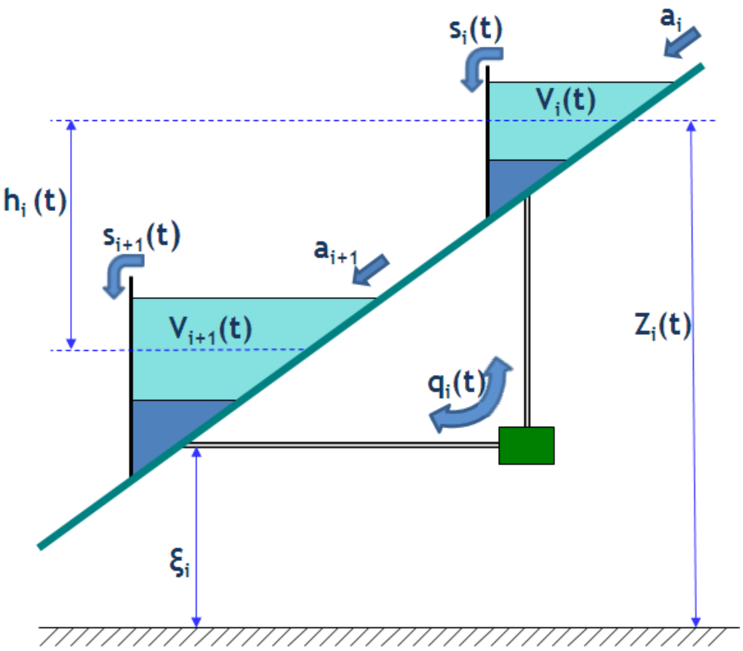

Figure 2. Two cascade reservoirs.

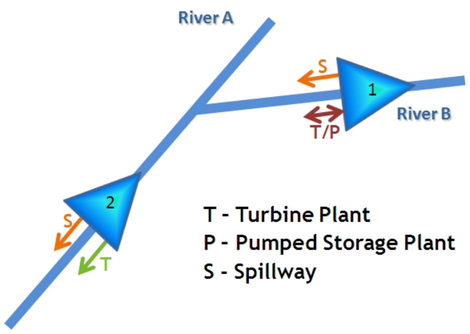

Figure 3. Two cascade reservoirs.

$$P(q, s, V_0) = \int_0^T price(t) \left(\sum_{i=1}^{2} r_i(t) \right) dt \to \max,$$

$$V_1(t) = V_1(t-1) + a_1 - q_1(t) - s_1(t),$$

$$V_2(t) = V_2(t-1) + a_2 - q_2(t) - s_2(t) + q_1(t) + s_1(t),$$

$$V_i(0) = V_i^{in} \quad i = 1, 2,$$

$$Z_i(t) = Z_i^0 + \alpha_i \left(\frac{V_i(t)}{V_i^0} - 1 \right)^{\beta_i} \ , \ i = 1, 2$$

$$h_1(t) = Z_1(t) - \max\left\{ Z_2(t), \xi_1 \right\},$$

$$h_2(t) = Z_2(t) - \xi_2,$$

$$\zeta_1 \left(h_1(t) - h_1^0 \right) - q_1^{0P} \le q_1(t) \le q_1^{0T} \left(h_1(t)/h_1^0 \right)^{\frac{1}{2}},$$

$$0 \le q_2(t) \le q_2^{0T} \left(h_2(t)/h_2^0 \right)^{\frac{1}{2}},$$

$$Z_i^{min} \le Z_i(t) \le Z_i^{max} \ , \ i = 1, 2$$

$$V_i^{in} - a_i \le V_i(T) \ , \ i = 1, 2.$$

where $t = 1, 2, \ldots, T$.

A typical one day price function, $price(t)$, is shown in Figure 3.

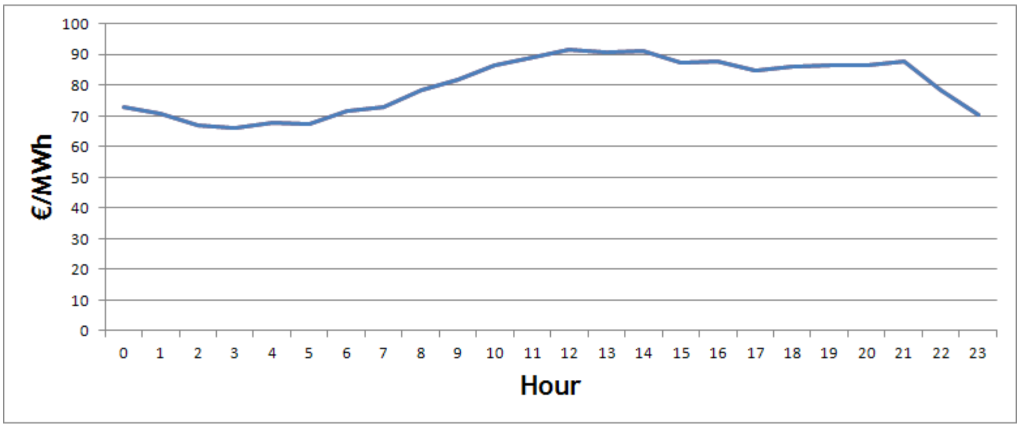

Figure 4. One day real market prices of electricity.

It should be noted that the high variability of prices certainly has a great influence on the economically efficient use of water in the reservoirs to produce energy. The restrictions are determined not only by economical reasons of producing electricity, but also by ecological reasons and other uses of the reservoir water by the nearby population. It is known that there is a higher use of electricity at 13h and 21h which is related with domestic consumption and daily cycles, and we can see that the price always increase at those times. One can then expect that this fact has influence in the water management.

In the next section we study the above two reservoir system as well as the more involved four reservoir system shown in Figure 5.

The problem of the profit optimization includes two main issues: one is how to control the turbined/pumped water flows and the other one is how to project a cascade of hydro-electric power stations. In particular we study the effectiveness of introducing a reversible link L between reservoirs 2 and 4.

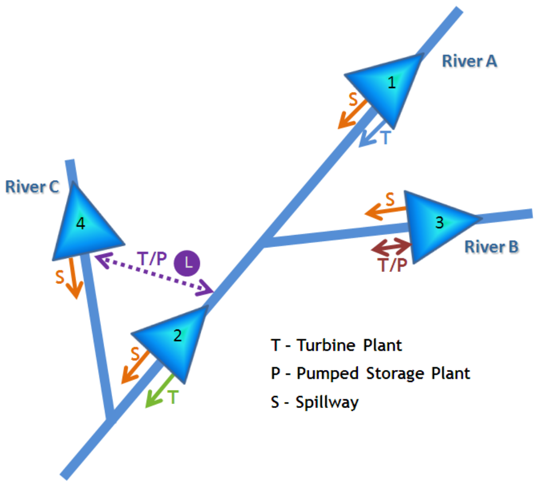

Figure 5. Four cascade reservoirs.

3. Computacional experiments and results

Computational experiments with both models were fulfilled with real data of the water levels and flows, as well as the market prices of electricity. The time period considered was one day, 24 hours, because of the great variability of intra-day electricity prices. Several type of days were tried, such as dry, mildly wet and wet days, as well as different days of the week. Only a sample of these results is presented. The optimization problems were solved using a penalty function method. The problems had to be solved numerically because their complexity does not allow for an analytical solution to be found.

In the case of two reservoirs for a very dry day the results are shown in Figure 6.

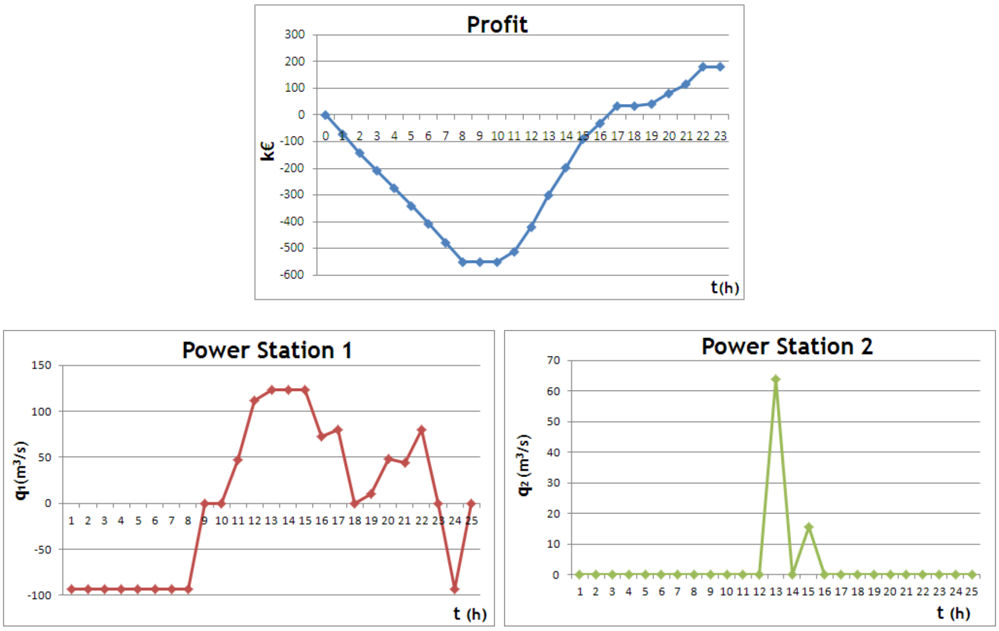

Figure 6. Example for two reservoirs: profit and power station controls.

The calculations were done with the market prices of electricity shown in Figure 4. It should be noticed that the hydroelectric power stations associated with the two reservoirs only produce electricity when the price is high enough to justify that production. The system chooses to produce energy mainly at meals time. As it was a very dry day, the system had a small amount of water to manage. Because of this, power station 1 being reversible pumped when the price was lower, allowing the reuse of the water from reservoir 2. Pumping required a certain cost but this increased the amount of available water in reservoir 1 allowing to discharge more, even out of peak hours, augmenting the profit. The variation of the water flows associated with power stations 1 and 2 can be seen in Figure 6. Before 8 o'clock the system only pumps and costs money, but from 10 o'clock onwards, the system produces energy and recovers giving a profit. It should be noted that the *Profit* in Figure 6, is net profit. The optimal trajectory of the volume of water in the reservoirs can be seen in Figure 7.

For the more complex cascade of four reservoirs (Figure 5), again with the same day market prices for the electricity, the obtained results are presented in Figure 8 (Link L is included). It can be noticed a similar behaviour as in the previous case: electricity is produced when high prices justify the production. Now, reservoirs 3

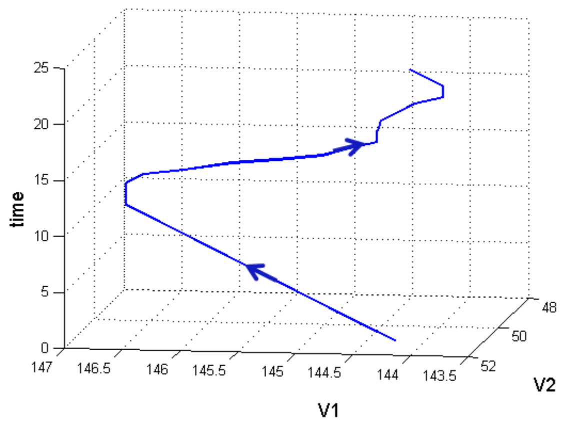

Figure 7. The optimal trajectory.

and 4 are reversible and because of that water is pumped at dawn as in the previous case.

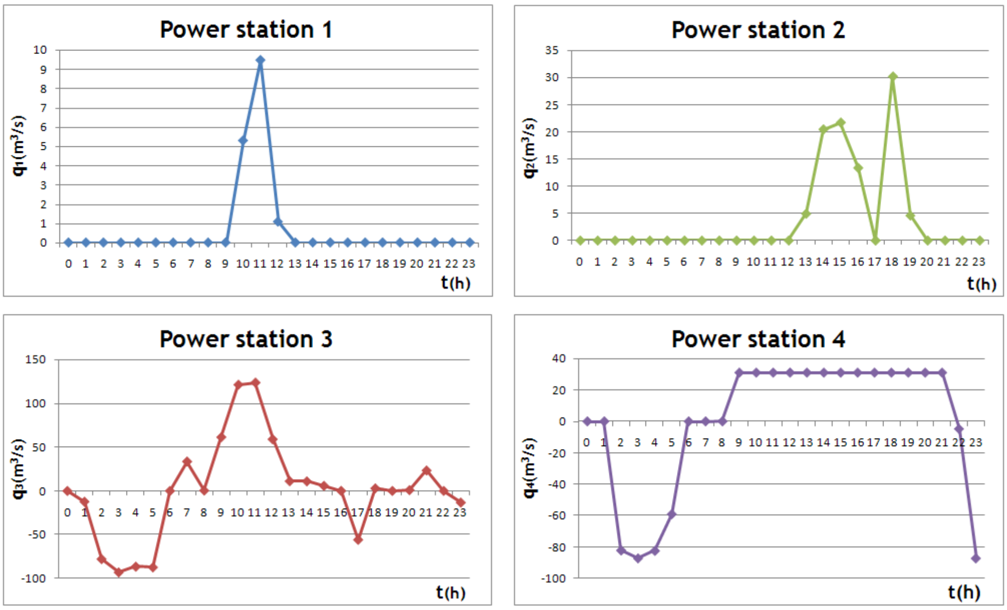

Figure 8. Four cascade reservoirs: power station controls.

We also consider an intuitive water management scheme, that is, all the water is used to produce electricity when its price reaches the highest value and pumping is the option when the price is low enough, allowing later to use a bigger volume of water for energy production. The results with this naif policy are presented in Figure 9. From Figure 10 we can see that the control algorithm used provides an intelligent water management with a final optimal profit much better than the simple one.

For a 24 hour period the profit obtained using the "optimal"policy, was 255348.32€ and the profit with the naif policy was 136033.05 €. The above considerations show that the use of optimal control methods can be important to manage water in the best way.

Now let us illustrate how this model can be used to plan a cascade of hydro-electric power stations. For example, we study the utility of link L in the cascade

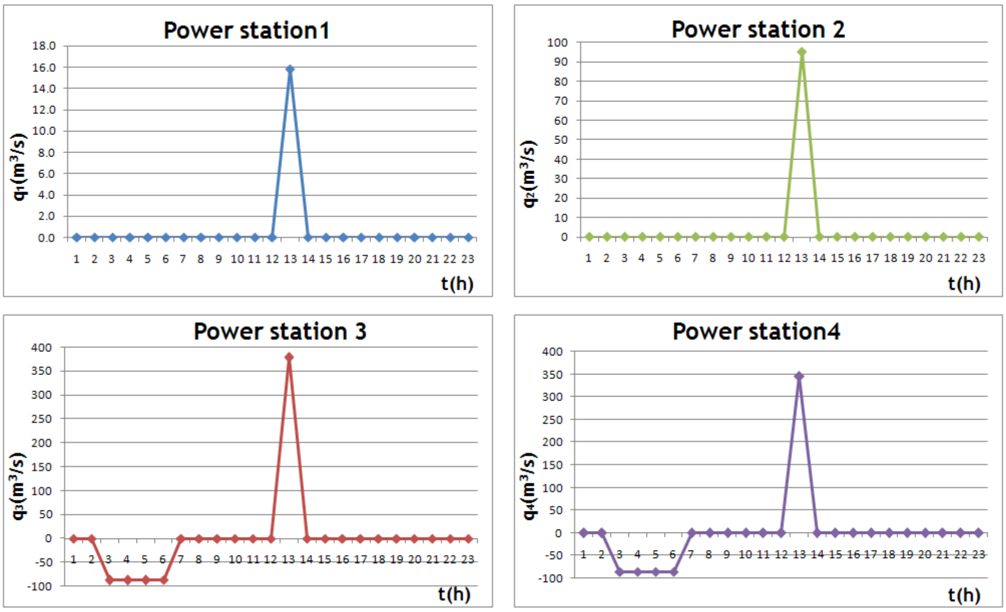

Figure 9. A naif control policy with four cascade reservoirs.

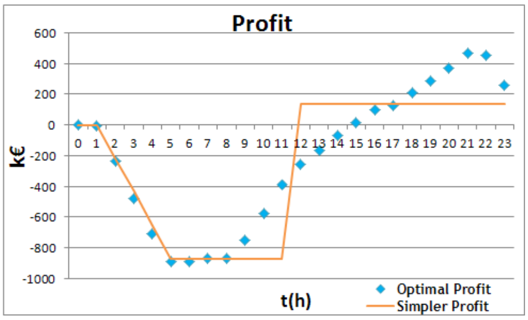

Figure 10. Comparison with a simple policy.

of four reservois as shown in Figure 5.

The same optimal control problems were solved with and without link L (see Figure 5). The results are summarized in the following table:

		Wet	Average	Dry
	Cascade Inflow (m^3)	555.6	277.8	95.2
With link L	Profit (k€)	387.6	359.1	261.9
	Turbined Flow (m^3)	1095.9	1277.2	1140.8
	Pumped Flow (m^3)	892.6	832.3	1001.6
Without link L	Profit (k€)	329.7	320.7	112.5
	Turbined Flow (m^3)	1102.2	924.5	785.8
	Pumped Flow (m^3)	713.0	535.3	646.6

The profit in the case with the link L, with two reversible power stations, has better values than in the case where the link L is out, even if there is no lack of water. For a dry day, the profit obtained with link L has approximately doubled the one without link L. Since the water to be managed by the system is very little, the inclusion of a reversible reservoir is essential to its reuse. For a wet day, the disposable water is enough. Since the level of water in each reservoir is nearer the maximum admissible level, it is more difficult to manage the water and the situation becomes less flexible. Anyway, the link is advantageous because the system continues to reuse the water of the reservoir 2 having always a bigger profit. We can conclude that the inclusion of a reversible reservoir is advantageous, and it shall be as more advantageous as less water the system has, that is, as far away is the volume of water from its upper limit.

4. Conclusions

A cascade of hydroelectric power stations was considered with a possibility of turbining and pumping in some of the power stations. This was translated into a discrete-time optimal control problem which was solved numerically. The data used in our numerical experiments were real. The developed approach can be used to plan and to manage cascade power stations.

5. Acknowledgements

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