Application of control charts for monitoring the machining process of the inside diameter of a steel cylinder

Murilo T. Maia*, Elisa Henning**, Olga M. F. C. Walter***, Andrea C. Konrath[+], Custodio C. Alves[++]
* Production and Systems Department, Santa Catarina State University, Campus Avelino Marcante, 89219-710, Joinville-Santa Catarina, Brazil
**Department of Mathematics, Santa Catarina State University, Campus Avelino Marcante, 89219-710, Joinville-Santa Catarina, Brazil
***Production and Systems Department, Federal University of Santa Catarina, Campus Trindade, PO Box 476 - 88040-900, Florianópolis-Santa Catarina, Brazil
[+]Department of Computer Science and Statistics, Federal University of Santa Catarina, Campus Trindade, PO Box 476 - 88040-900, Florianópolis-Santa Catarina, Brazil
[++] Production and Systems Department, University of the Region of Joinville, Tenente Antônio João St., Bom Retiro, PO Box 246-89201-972, Joinville-Santa Catarina, Brazil

Email: murilot.maia@gmail.com, dma2eh@joinville.udesc.br, olgaformigoni@gmail.com, andreack@gmail.com, custodio.alves@univille.net.

Abstract

The quality of products and processes has proven to be crucial for companies to compete in the market. The Statistical process control enables them to foresee problems in the process, thereby ensuring the quality of manufactured items. This paper aims to present a proposal for monitoring a machining process, with the help of this tool using control charts. This research arose from the need of monitoring the rectification process of the inside diameter of a steel cylinder, a component of a product manufactured in a company in southern Brazil. The parameters must follow the design specifications, so the final product will not have performance problems. Thus, for the monitoring of this process are applied Shewhart X -S control charts for three measurements performed in sections of a cylinder, analyzing them separately. To complement, a multivariate chart Hotelling's T^2 is applied to monitor simultaneously these three measurements. The two approaches are compared in terms of performance and usage aspects. The results obtained in terms of performance were similar. However, the Shewhart control chart is easier to use and simpler to interpret by staff. The results enabled the company to know the stability of the process, facilitating the decision-making on actions taken for improvement.

Keywords: Statistical Process Control, Shewhart chart, Hotelling's T^2 chart, Machining process.

1 Introduction

The quality of products and processes has proven to be crucial for companies to compete in the market. Statistical process control (SPC) is an efficient and powerful formal technique used for monitoring processes. It allows us to predict problems in the process, ensuring the quality of the items that are produced. SPC consists of a collection of tools for resolving problems, useful in obtaining a stable process. The control chart is one such tool, and probably the most technically sophisticated (MONTGOMERY, 2004).

A control chart is a visual statistical tool that highlights the presence of special causes. It basically consists of plotting lines representing the Upper Control Limit (UCL) and Lower Control Limit (LCL), the mean or target of the process (Central Line - CL) and the observed points, which represent, in general, a statistic related to the variable of interest. If one or more points are beyond the control limits, it indicates that the process can be out of statistical control, i.e. there may be a problem in the process (SAMOHYL, 2009). The most popular univariate control chart for variables is the Shewhart control chart, which achieved success due to its simplicity.

In the industrial process there are many situations that require simultaneous monitoring and control of more than one quality characteristic. Monitoring these quality characteristics independently is possible,

but can be very misleading. This monitoring where many quality characteristics of interest are related is known as multivariate statistical process control (BERSIMIS; PSARAKIS; PANARETOS, 2007). Monitoring a process that has multiple quality characteristics is always a challenge to the quality controllers or practitioners. The Hotelling T^2 control chart is widely used to monitor multivariate observations (CHENOURIA; VARIYATH, 2011).

The motivation for this paper came from the need to apply statistical process control in a machining cell of a pilot project at a company. Because the item is produced in batches and the feature being studied is crucial for it, statistical monitoring is required, which allows the quick detection of problems. The part being studied is measured in three sections, so there are three (3) variables to be studied. Each section of the part is analyzed individually, because if there is one that does not meet the specification, the part must be rejected or reprocessed. This makes it possible to use the Shewhart $\overline{X}-S$ chart.

However, as there are three (3) variables being studied, each of which are correlated, it is possible to apply the Hotelling T^2 chart. By using the concepts of SPC, this study proposes a way to do the monitoring of these variables, thus ensuring the stability of the process.

The main goal of this paper is to present a proposal for the monitoring of the sections of the internal diameter of the part being studied, using control charts. We also want to compare the monitoring between the Shewhart $\overline{X}-S$ chart and the Hotelling T^2 chart for this process.

This article is structured as follows: in section 2, we present the theoretical framework of control charts; in section 3, there is the methodology used to develop this work; section 4 presents the results and discussion; and in section 5, there are the final remarks.

2 Statistical Control Charts

SPC, and in particular quality control techniques such as control charts, have become more and more important, due to the key role they play in industry. The main goal of statistical quality control is to reach a level of quality that becomes more and more the primary factor in the consumer's decision regarding products and services (ALVES; SAMOHYL, 2004).

The processes must be continuously monitored to detect the presence of identifiable causes. Monitoring the characteristics of a process, it is possible to know and act on the causes of variability. By knowing the causes of process variation, it is possible to propose an action plan to eliminate special causes. The control chart is the primary tool used for monitoring processes and indicating the presence of special causes, which allows detection and the taking of corrective actions (COSTA; EPPRECHT; CARPINETTI, 2008).

A control chart consists of an axis, which refers to the average of the sample values, and two parallel lines, called the upper control limit (UCL) and the lower control limit (LCL). A process is in statistical control when the variation of the samples in the chart is small, oscillating around the threshold of central control, without going outside the limits of upper and lower control. This variation is due to random causes intrinsic to the process (COSTA; EPPRECHT; CARPINETTI, 2008). If there are subgroups that go outside the LCL and UCL, the process is under the action of special causes, and actions must be taken to identify and remove them.

The implementation of control charts in a process occurs in two phases. In the first phase, we check if the process is under control by looking at the initial observations presented in the chart (CHENOURIA; VARIYATH, 2011). Also at this stage, a detailed study is done to check if the process is in statistical control by analyzing their characteristics in order to verify that the data are independent and identically distributed (VINING, 2009). After verification of the control charts, we seek to identify special causes. If

special causes are identified in the production process, we seek to identify these causes in order to eliminate them.

According to Samohyl (2009), subgroups that were under the proven influence of identifiable causes should never be used. Without being part of the control chart, these subgroups must remain recorded with the actions taken to remove the causes that took the process out of statistical control. After eliminating the subgroups that take the process out of control, the limits are recalculated. A new analysis is then done. If there are no special causes in the process, then passes it to phase II, which is the monitoring process with the limits calculated in phase I. In phase II, the information obtained in phase I is used in the construction of the control charts that are used to test if the process remains under control when future observations are monitored (HENNING *et al.*, 2011).

The performance of a control chart is usually measured by parameters related to the distribution of the time required for the chart to send a signal. The average run length (ARL) is an additional parameter, representing the average number of samples necessary to detect a change that has occurred in the process (ALVES; SAMOHYL, 2004). As said by Samohyl (2009), if it is a false positive, then it is called ARL_0; if it is true, it is called ARL_1.

2.1 Control Charts $\overline{X} - S$

The $\overline{X}$ chart, developed by Shewhart in the 1920s, is the most widely used control chart in SPC today. Its use has been shown to be effective for monitoring the process and improve the result continuously and permanently (SAMOHYL, 2009). The control limits are usually set considering three (3) standard errors above and below the average $\overline{X}$. The ARL_0, in this case is 370. In other words, a false positive is accepted every 370 samples. To define the control limits in the $\overline{X}$ chart, we use Equations 1, 2, and 3.

$$UCL = \overline{\overline{X}} + A_3\overline{S}; \quad (1)$$

$$CL = \overline{\overline{X}}; \quad (2)$$

$$LCL = \overline{\overline{X}} - A_3\overline{S}, \quad (3)$$

where $\overline{\overline{X}}$ corresponds to the average of the average of the samples, $\overline{S}$ corresponds to the average standard deviation, and A_3 corresponds to the factor for the construction of the control charts, tabulated for various sample sizes *n*. Tables can be found in traditional SQC references such as Montgomery (2004), Costa, Carpinetti and Epprecht (2008) and Samohyl (2009).There is a chart for process variability monitoring, but in practice it is seldom used. The S variability chart is based directly on the standard deviation of the subgroups. However, the accuracy of the S chart is better than the R chart, showing fewer false positives, and its use should be considered in factories where the culture of quantitative methods is more evolved and appreciated (SAMOHYL, 2009). For the limits of the control chart of standard deviation S, we use Equations 4, 5, and 6.

$$UCL = B_4\overline{S}; \quad (4)$$

$$CL = B_4\overline{S}; \quad (5)$$

$$LCL = B_3\overline{S}; \quad (6)$$

where $\overline{S}$ is the average standard deviation, and constants B_3 and B_4 are tabulated for various values of *n*.

According to Montgomery (2004) and Costa, Carpinetti and Epprecht (2008), for control charts like the Shewhart control chart to be effective, two assumptions must be previously analyzed and validated. The first is that the observations of the quality characteristic of interest are independent, with no significant

autocorrelation in the data. The second is that the observations are normally distributed. The control limits are constructed based on the assumptions that the underlying distribution of the quality characteristic is normal and there is no major contamination due to outliers (MOUSTAFA, 2009).

2.2 Multivariate Control Charts

There are many situations where simultaneous monitoring is necessary to control two or more related quality characteristics, and monitor whether these characteristics can be misleading (MONTGOMERY, 2004). For these situations, specific tools should be used to detect, identify, and analyze the meaningful causes of variability in a process. Multivariate control charts represent one of these techniques being used to simultaneously control several characteristics that indicate the quality of a single production process (ALVES, HENNING; SAMOHYL, 2008).

The most familiar monitoring and control procedure of a multivariate process is the Hotelling T^2 control chart, for monitoring the mean vector of the process. It is directly analogous to the univariate Shewhart $\overline{X}$ chart (MONTGOMERY, 2004). Hotelling (1947) was the first researcher to observe the weakness of the univariate statistical control charts in his pioneering paper. In the following decades many contributors have established studies in the same field and an extensive literature can be found, like for e.g.: Aparisi (1997), Jackson (1956, 1959), Jackson and Morris (1957), Ryan (1989), Tracy, Young and Mason (1992), Pignatiello and Pignatiello Jr. (1999), Khoo *et al*. (2004) and Champ and Jones-Farmer (2007).

Among the existing multivariate charts, the Hotelling T^2 control chart is the best known in the literature, and its applicability is most recommended for processes that have several quality characteristics. These characteristics are correlated and need to be monitored together. The T^2 test statistic is based on Equation 7,

$$T^2 = n\left(\overline{X} - \overline{\overline{X}}\right)' S^{-1}\left(\overline{X} - \overline{\overline{X}}\right), \tag{7}$$

where $\overline{X}$ corresponds to the vector of means, and S represents the covariance matrix of the process.

The application of the Hotelling T^2 multivariate control chart is done in two steps. In phase I, the limits are calculated using Equation 8 (TRACY; YOUNG; MASON, 1992; BERSIMIS; PSARAKIS; PANARETOS, 2007),

$$\text{UCL} = \frac{p(m-1)(n-1)}{mn-m-p+1} F_{\alpha,p,mn-m-p+1} \tag{8}$$

where *p* is the number of variables, *m* is the number of samples, *n* is the sample size, and *F* equals Snedecor's *F* distribution with a degree of freedom for the numerator equal to α (equivalent to the rate of false positives), and for the denominator equal to *p*, *mn-m-p*+1.

For phase II of the application of the multivariate chart, the equation of the upper control limit is given by the Equation 9 (TRACY; YOUNG; MASON, 1992; BERSIMIS; PSARAKIS; PANARETOS, 2007)

$$\text{UCL} = \frac{p(m+1)(n-1)}{mn-m-p+1} F_{\alpha,p,mn-m-p+1}. \tag{9}$$

The lower control limit (LCL) for the two phases is equal to zero. For the use of multivariate control charts, it is also necessary to verify the assumptions of normality and independence. If the multivariate normal is not an appropriate model, there is very little literature available on alternative multivariate charting techniques (Bersimis; Psarakis; Panaretos, 2007) such multivariate non-parametric statistical process techniques (CHAKRABORTI; VAN DER LAAN; BAKIR, 2001). Autocorrelated multivariate process is also an area that must be further investigated.

3 Methodology

This paper, considered from the point of view of its nature, can be considered as applied research (BARROS; LEHFELD, 2000; GIL, 2010), contributing to practical ends. It aims to solve problems or concrete needs found in reality. From the point of view of the form of the approach to the problem, the research is quantitative, since to reach its proposed goals, the obtained results are analyzed using information from the control charts.

As a computational resource for the monitoring of data, the normality and autocorrelation analysis was done using the R software program (R DEVELOPMENT CORE TEAM, 2012).

In Figure 1, the steps in the application of SPC in this paper are shown.

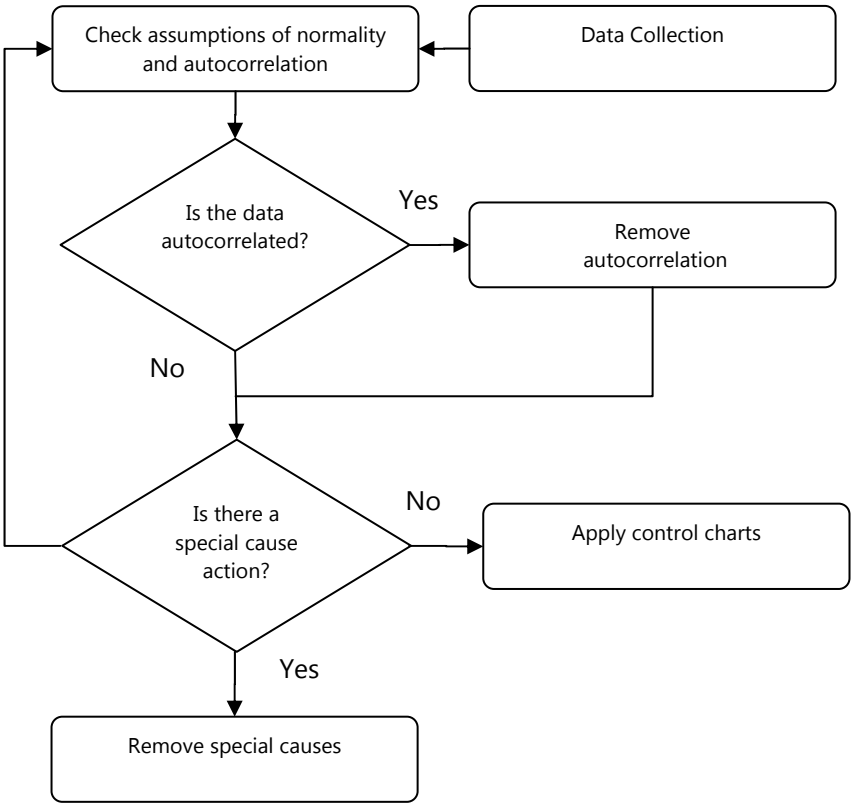

Figure 1: Flowchart of the application of SQC to the data of the sections of the cylinder

Data collection occurred in the process of grinding the inner diameter of a 1020 steel cylinder. The piece is separated into three (3) sections for measurement: top, middle, and bottom. Data from these measured sections are available in the annex. The grinding of the inner diameter is performed to increase the diameter and decrease the roughness of the surface. These are the two main characteristics to ensure product quality in this process.

4 Results

For reasons of confidentiality, the name of the company where this work was done will be referred to as Company. Working for over 40 years in the market, the company is headquartered in southern Brazil, and produces one of the main components for the white line. Their product is responsible for circulating the refrigerant in a refrigeration system.

The machining cell in question went through a transition from design to manufacturing. Thus, no method had been used yet to check the stability of the process. It was just checked if the parts were within specification using measurements. The need to implement SPC in the machining cell was then seen. Initially, we tested the normality hypothesis by means of the Shapiro-Wilk hypothesis. The outcome of the normality test for first 30 subgroups of the top, middle, and bottom sections of the cylinder is presented in Table 1. With the exception of the top section, all the other variables show normal distribution. Thus, as a small departure from normality may not influence the results very much (MONTGOMERY, 2004), we chose to apply $\overline{X}-S$ control charts to the original data (Figure 2). Although it is not shown in this study, the data showed no autocorrelation.

Table 1: First normality analysis of the data and definition of the control limits

Section	Top	Middle	Bottom
p-value	0.0009	0.7305	0.2466

Source: Self-authorship

In Figure 2, two samples (9 and 17) which are beyond the control limits can be seen - sample 17 in the $\overline{X}$ chart (Figure 2a) and sample 9 in the S chart (Figure 2b). The control limits had to be recalculated since correction errors of the parameters of the machine were identified by operators. Thus, subgroups 9 and 17 were removed from the analysis. With the removal of these subgroups, the assumptions were checked again, and the data were normally distributed and not autocorrelated.

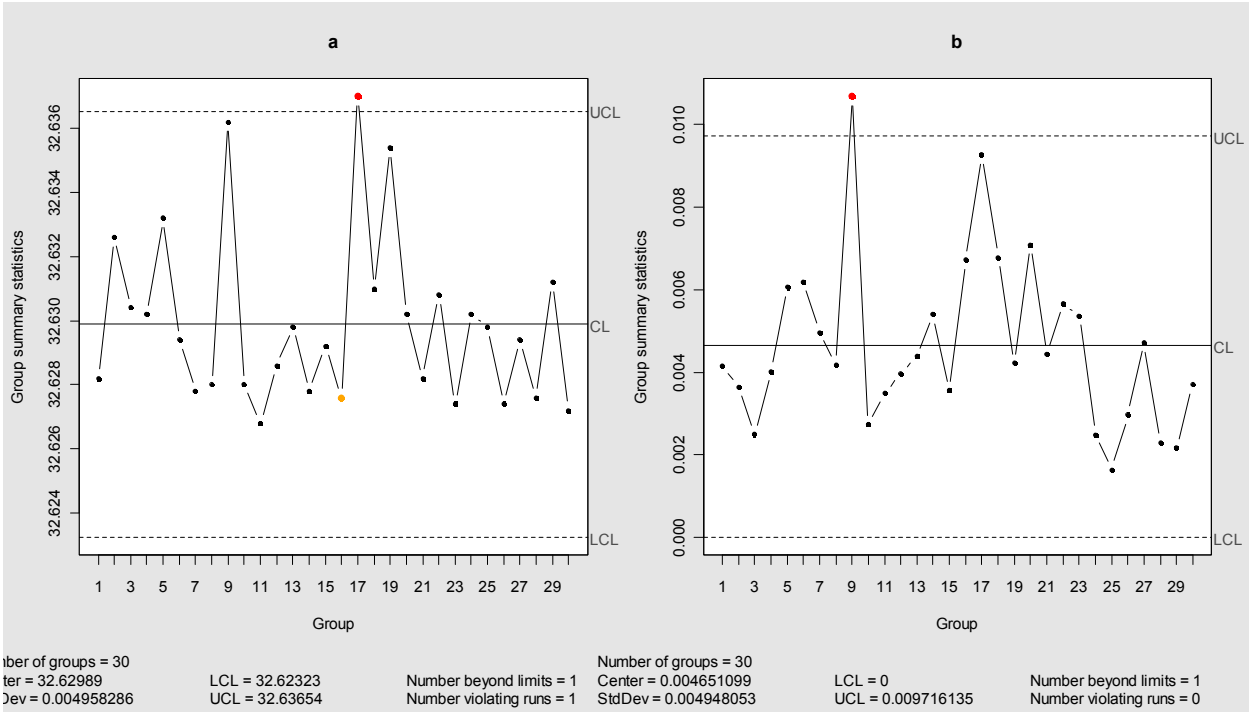

Figure 2: $\overline{X}$ (a) and S (b) control charts for the data.

The new control limits for the top (LCL=32.62323; CL=32.62989; UCL=32.63654), middle (LCL = 32.62491; CL = 32.62335; UCL = 32.6354 and bottom (LCL = 32.62348, CL = 32.62984, UCL = 32.63621) sections can be verified in Figures 3, 4, and 6, respectively. They represent phase II, i.e. the monitoring phase discussed below.

Figures 3, 4, and 5 demonstrate that with all the sections measured and also considering the average, the process is kept under statistical control. This analysis did not consider the additional rules, as it would

imply a change in the ARL_0. The process, however, is stable only under the action of the natural variability of the process, with measurements within specification.

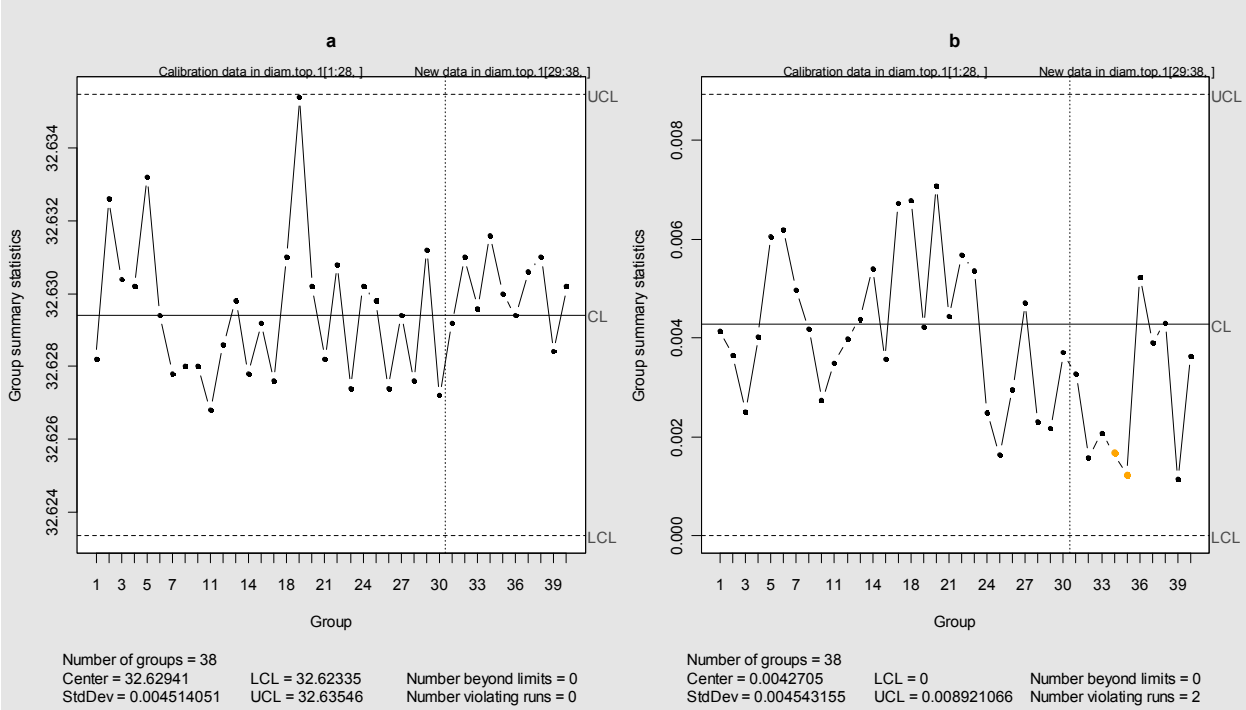

Figure 3: Xbar (a) and S (b) chart of the top section during the monitoring phase of the process.

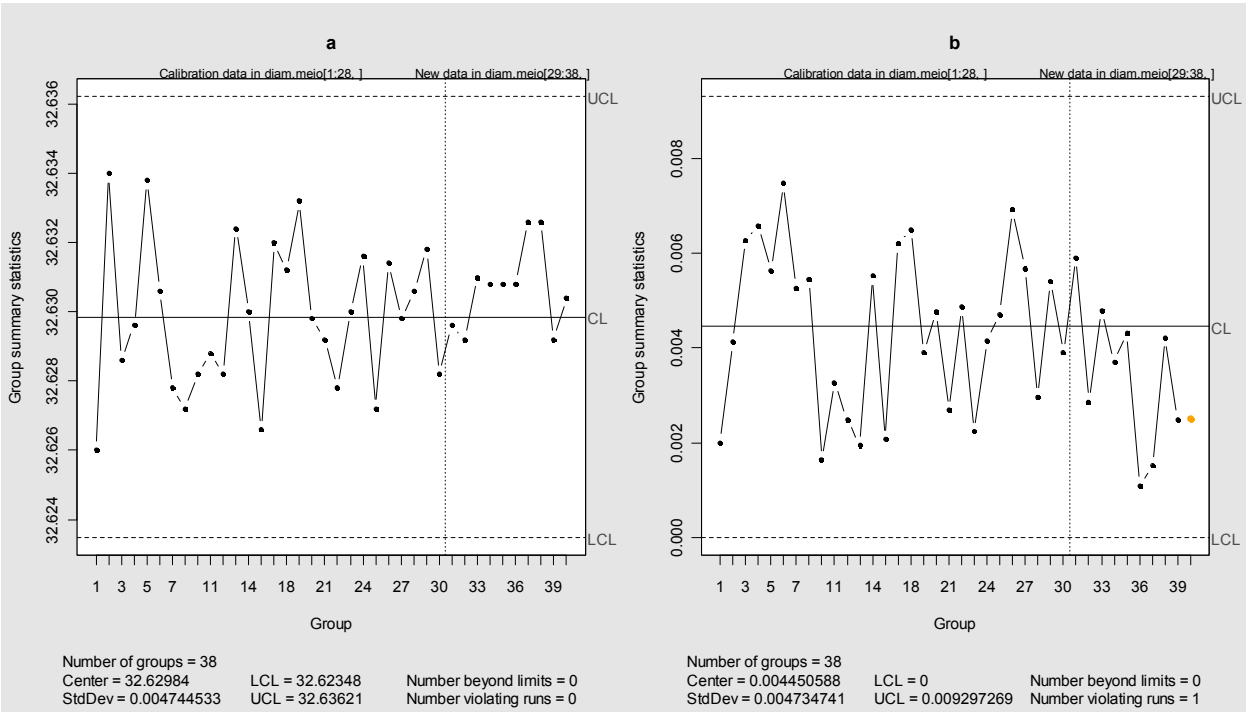

Figure 4: Xbar (a) and S (b) chart of the middle section during the monitoring phase of the process.

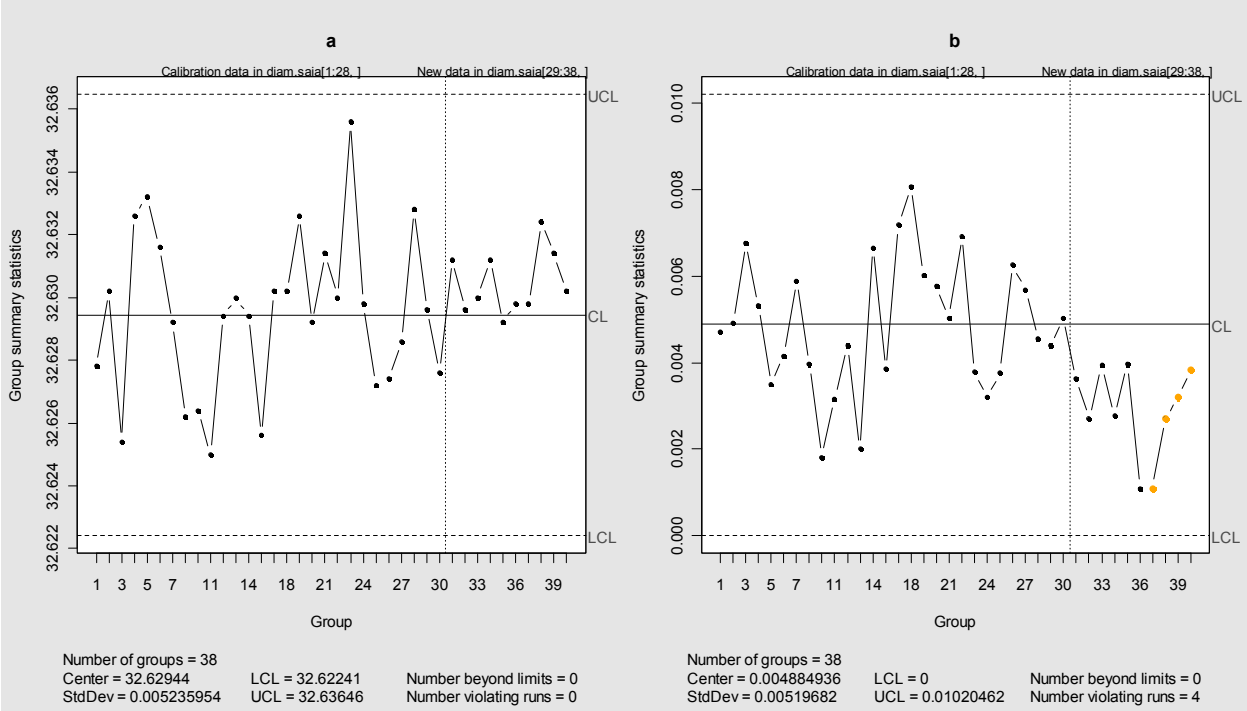

Figure 5: Xbar (a) and S (b) chart of the bottom section during the monitoring phase of the process.

Starting from the development of this work, we can begin the process of SPC in this cell. It is recommended, after a certain amount of time, to recalculate the control limits. Furthermore, feedback from the client about the quality of the product is always desirable.

In applying the Hotelling T^2 chart, the three variables studied (top, middle, and bottom) were considered, with an initial number of 30 subgroups. As with the Shewhart control chart, the subgroup size was equal to 5 (five) and a value of 370 was used for the ARL_0. As was reported in the individual charts, subgroups 9 and 17 had special causes, and these were excluded after the initial analysis. However, in this analysis, only sample 17, which also reported a special cause, was above the upper control limit. Nonetheless, it was decided to remove these subgroups, and once the limits were recalculated, the process proved to be under statistical control.

To check the normality of the data, a test was performed for multivariate normality - Mardia's test - using the QRMlib package in R (MCNEIL; ULMAN, 2011). The data can be considered normally distributed, with an asymmetry *p*-value = 0.1044016 and a kurtosis *p*-value = 0.4504732. To estimate the covariance matrix, the successive differences estimator was applied.

After checking the stability of the process in phase I, we then moved to the monitoring phase (phase II), using the data of the samples during the monitoring of the process. In addition to the initial 28 subgroups, ten more subgroups collected after setting the control limits were also used. As in phase I, we considered five samples per subgroup. The control limits of phase II were LCL = 0 and UCL = 15.85587. Figure 6 shows the Hotelling T^2 chart during the phase II monitoring.

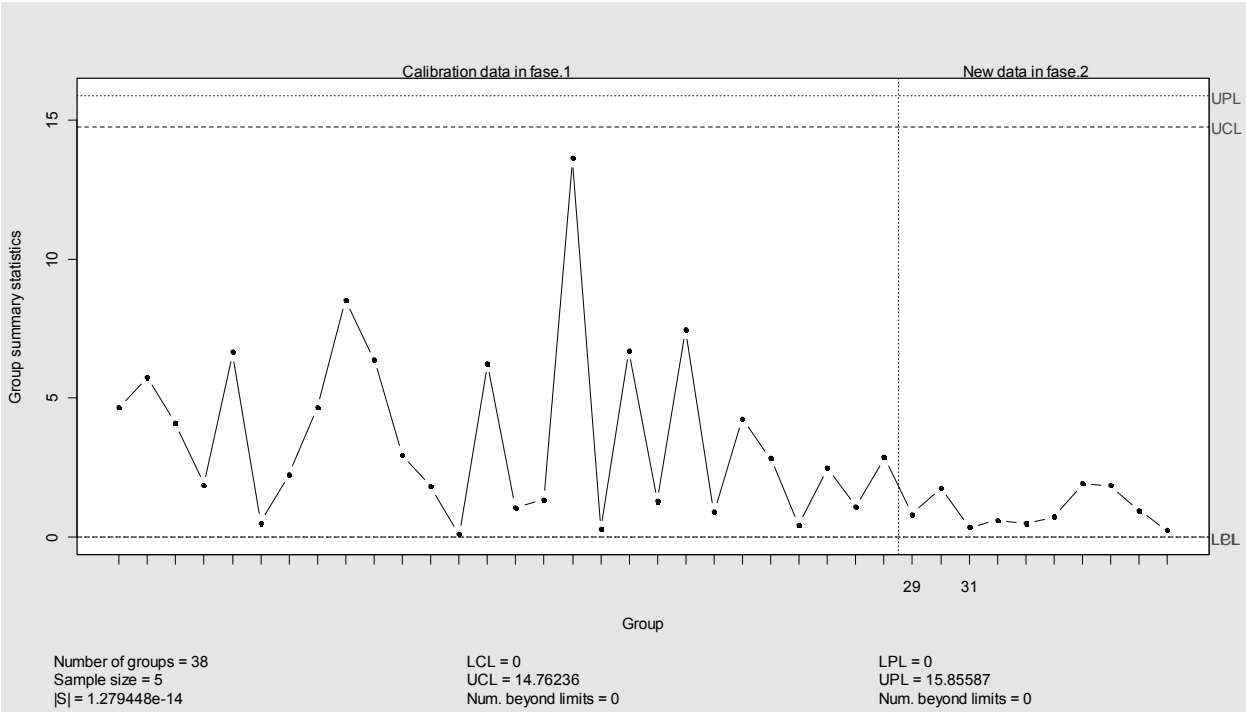

Figure 6: Hoteling T^2 control chart (phase I and phase II). Source: R software application

Thus, it can be seen that the two approaches - univariate from the Shewhart-style charts, and multivariate from the Hotelling T^2 charts (Figure 6) - had similar results in terms of performance. But as the company was in the process of implementing SPC in that cell, the application of the Shewhart-style charts is easier to interpret by the operator of the shop floor. But the Hotelling T^2 chart can be an option to be considered in the future.

5 Conclusion and Final Considerations

The main objective of this study was to propose the application of control charts for the statistical process monitoring of the grinding of the inner diameter of a steel cylinder. Shewhart-style univariate and Hotelling T^2 multivariate control charts were applied. Both the Shewhart control chart and the Hotelling T^2 control chart had almost the same performance in the analysis of the stability of the process, proving to be efficient in the analysis of the process studied. Thus, it was possible to verify the stability of this process around the design specifications.

We believe that the use of the Shewhart chart is a good choice at the beginning of the SPC implementation work in the cell. For the company, at that time, it was operationally easier to implement, considering both the operators, as the computational resources available for online monitoring.

Therefore, after a while, when this practice is more established, it is suggested to proceed to using the Hotelling T^2 chart itself, or even multivariate charts that are more sensitive to small shifts in the process mean, such as MCUSUM (Multivariate Cumulative Sum) chart or MEWMA (Multivariate Exponential Moving Average) chart.

Faster signaling of out-of-control processes leads to reduced losses, which can be significant in cases where small deviations from the established mean of the quality characteristic are turned into losses and less competitiveness. Thus, using MCUSUM and MEWMA charts, it is possible to check the quality of the products with higher accuracy, and also identify the adaptation of the operators to these new tools on the production line.

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