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A Growth Strategy Model

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Abstract

A firm´s growth model is described, which simulates a no-access to credit context and maximizes the Gross Value Added (GVA), finding an optimal growth path. It builds on both the fundamental accounting identity and an A.C type production function. The former is written as the GVA equal to labour costs plus capital multiplied by a management factor. The growth model computes GVA depending on estimates of financial and depreciation factors and on a decision concerning a change on labour cost, decision that equally affects investment. A number of constrains are set to assure minimum values of available cash, appropriate values of the constant A and of unit labour costs, depending on the firm's history and best practices in the sector. Validation was made using 15 Portuguese industry divisions' data, for the period 1995-2005: From 1995-2000, data was used to calibrate the model; then, estimations were simulated for the period 2001-2005 and compared with actual data. Results indicate that the model predicts the GVA with average yearly deviations below 8% on the first three years. For capital, the errors were below 25%. This quasi-experience is not perfect because real data reflects a context where credit was used. This effect was analysed and the error was found to be positively correlated to the amount of credit used in reality.

Keywords: economic growth model; gross value added; optimization

1 Introduction

In the pursuit of sustainable growth, firms may experiment different strategies such as organization management, lean production, marketing strategies, R&D efforts and others. These strategies may align the firm with multiple drivers of growth such as overall increase in demand for the segments in the firm's portfolio, gains in market share or the expansion into new markets.

Underlying all specific choices of strategy the essential decision in a firm regards the allocation of the value it has created. Managers are faced with two fundamental options: The distribution of value to the owners and/or to the workers, for one, and, alternatively, investment in capital and technology. From a macroeconomic view, distribution originates the consumption that is necessary to maintain and increase the level of knowledge in individuals, and investment generally triggers value creation (Fernandes, 2005). From the sectorial and firm level views, the same reasoning applies but distribution to workers and owners has more direct impact on work force productivity and capital attractiveness. Finding the right balance between these two options, distribution and investment, is equivalent to finding an optimal growth path, which maximizes added value in a given period of time.

Many macroeconomic models have addressed the issue of growth, concerning society as a whole. Those macro models can be transferred to the firm's level. As such, modelling gross value added (GVA) for a firm, considering the dichotomy between investment and distribution, has the potential to be used as a tool for an objective assessment of alternative growth strategies and as a guide to managers for making successful decisions.

The main issue this communication addresses is the decision between investment and distribution, taking into account probable evolutions of other economic and social variables, at firm and sectorial levels.

The objectives of the study are to find the answers to the following questions: i) How to build an economic growth model for a firm or a sector that accurately describes the process of adding value, centred on the dichotomy of distribution/investment? ii) How to maximize value added in a given time-

period, defining an optimal growth path? And iii) how successful is the model expected to be, measuring by its application to past real economic data?

In section 2 the main macroeconomic growth models are reviewed. Section 3 describes the methodology, the model and algorithms used in this study. In section 4 the results of a theoretical evaluation of the model's behaviour are discussed. Section 5 describes a linear program, in which the optimal growth path is computed; and in section 6 the model is validated using data from divisions of the Portuguese economy. Finally conclusions are taken in section 7.

2 Review of Literature

Macroeconomic models provide a simplified version of the real economy, studying the parameters and conditions for growth and progress in society. These models are applied to countries' economies and aim at understanding why some have witnessed prolonged periods of growth and others have not. In a macro system such as a country's economy, growth is measured by the increase in gross domestic product, which results necessarily from the increase in value added within each one of the firms present in the economy. For this reason it can be said that firms and sectors are a fractal representation of a whole economy and, consequently, the conclusions taken from macroeconomic models can be used for the study of growth at the firm level.

Macro models have as framework a treble identity stating that aggregate output equals aggregate income and aggregate expense. After the works of Keynes, it became established that saving ($S=s.Y$) equals investment (I) and that this parameter sets up the value of GVA (Y) through $\Delta I=s.\Delta Y$, where s stands for the saving coefficient.

Marking the beginning of modern growth theory, Harrod (1952) and Domar (1946) proposed a model that used a production function in which output grows in proportion to capital, and defined the rate of growth ($\hat{y}$) as being proportional to the saving coefficient and to the accelerator $\sigma=\Delta Y/\Delta C$, such that $\hat{y}=\sigma.s$.

In the fifties, Solow (1956) and Swan (1956) developed a model that uses two equations: a neoclassical production function with inputs capital C and labour N (nº of workers); and a capital accumulation equation. The model described assumes diminishing returns to capital resulting that an economy would only grow if its number of workers increased, while output per capita remained constant. Solow introduced one exogenous parameter in the production function: a technical progress parameter that serves as the engine of growth, allowing for a positive growth rate of output per capita. The model concluded that an economy converges toward a balanced growth path where the model's parameters grow at constant rates.

Later came along models of endogenous growth. These used production functions such as $Y=A.C$, in which output Y is proportional to capital C, and A represents a parameter with no dimensions proportional to technical progress, whatever form this progress may take. Romer (1990) made the technical progress parameter endogenous and many other models more complex and more complete surged, giving centre-stage either to R&D expenditure, household consumption, human capital and so on.

The models mentioned above established fundamental rules which are still core to current macroeconomic models. These are that (1) aggregate demand equals aggregate supply; (2) output equals income and expense; (3) saving equals investment; (4) investment plus final consumption equals expense; (5) capital depreciates with time; and (6) output is somehow proportional to increases in capital.

3 Methodology

A new model for a firm's growth will be here proposed, what can be linearly extrapolated to economic sectors. It presents a production function based on production factors that are simultaneously accounting

variables, and considers GVA as the output. The dynamics of growth for this model are described next, along with initial conditions and inputs, intermediate variables, calibration variables, and the algorithm that computes the yearly values of GVA.

The building and validation of the model consists of the following three steps: 1- A theoretical study, depicting different cases in which the effects of different input values are observed over a 10 year period. 2- An optimal growth path is determined, computed by an optimization program developed with linear programming. 3-The model is validated comparing its results with actual data from chosen Portuguese economic divisions; Data from 1995 to 2000 is used to calibrate corrective variables, and data from 2001 to 2005 is compared with the model's results. For each division yearly average relative errors are calculated for GVA and correlated with the use of debt in the division.

3.1 A Model for a Firm

Basically, a firm uses inputs, which are resources obtained in the market, it has a production function that represents how those inputs interrelate with the management structure and how they integrate the production process, and delivers an output product, which is returned to the market.

A firm's GVA (Y) can be computed by adding labour costs (L) with capital costs (Cm), the latter calculated as the sum of capital depreciation, financial costs and profits from ordinary activity. On the other hand, the GVA equals income R, which is the sum of labour income (L) with the return on capital (m.C), where m will be described as the management factor. From the expense view, Y also equals expense D, given by the sum of investment (I) with final consumption (Cf). Identity (1) represents the model's production function written from the three view points of the economic process.

$$Y=L+Cm=R=L+m.C=D=I+Cf \tag{1}$$

The level of investment controls how much new capital will be available for production. This condition, coupled with the principle of GVA being somehow proportional to increases in capital, results on how investment affects GVA. However, the inputs of labour and capital do not give a full explanation of the output value. The yield and efficiency of the productive process vary according to the management structure of a firm, insofar as two firms with the same labour force and the same capital value may have different outputs.

The value of workers' knowledge and skills, which is the labour value, fixes the value of their contribution to production. Management decides the average wage (w), which, multiplied by the number of workers in the firm (N), returns the value of labour L=N.w. Additionally, managerial decisions govern what forms of capital to acquire and use and how it should be organized. They also control how workers use capital, consequently impacting the value of the use of capital. This overall impact of the management structure is represented in the production function by the factor m, which is referred to as the management factor (Fernandes, 2009) and represented as Y=L+m.C.

Considering the multiple ways management impacts the value of capital, we can disaggregate the management factor (m), making explicit each component contribution to value added: 1- A depreciation factor (δ) is the average asset's depreciation rate, such that (δ.C) represents the amount of capital that is added to production every year; 2- A financial factor (f) such that (f.C) represents the cost of financing the firm's assets; and 3- An organization factor (o) representing the remaining part of GVA (o.C) that is proportional to the contribution of the firm's organizational structure to value added (Serzedelo & Fernandes, 2011). Identity (2) shows the production function, with the management factor divided into its different constituent factors. This production function is based on the accounting identity and every one of its terms can be identified rigorously in the firms' financial maps.

$$Y=L+m.C=L+(\delta+f+o).C \tag{2}$$

This production function enables not only an analysis of the quantity of investment but also of the quality of investment, the latter in terms of how investing in different forms of capital, with a higher or lower degree of technological components, impacts value creation.

3.2 A Growth Model for a Firm

Y_0 represents GVA in year zero; Y_1 represents GVA in year one and so on. When given the initial conditions for an economy to perform, which establish the static identities that characterize year zero $Y_0=R_0=D_0$, the model is able to compute each set of static identities that characterize the following years, $Y_n=R_n=D_n$. Figure 1 shows a schematic view of how the model operates. Table 1 shows the other inputs, intermediate variables and specific equations reflecting the dynamics of growth.

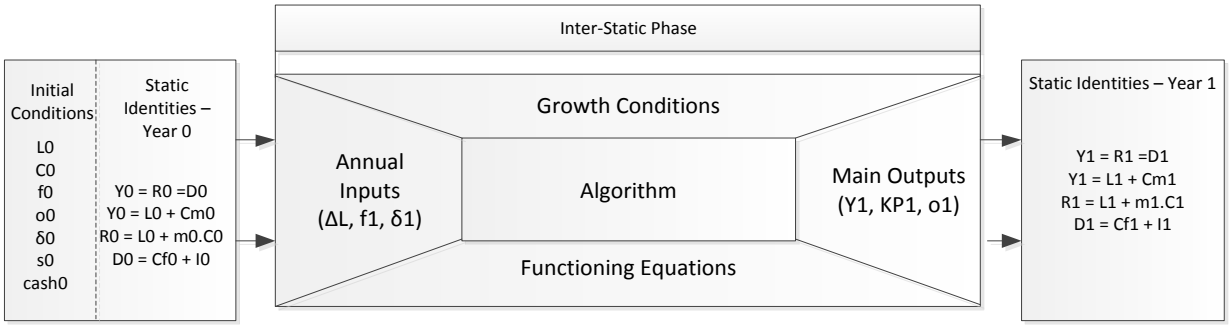

Figure 1: The static identities and the inter-static phase

Table 1: The model's variables

Input Variables	Intermediate Variables - Year 0	Growth Conditions	Output Variables - Year 1
• KPref – Reference Knowledge Productivity • cc1 – calibration coef. 1 • cc2 – calibration coef. 2	• m0=f0+δ0+o0 – Management factor • Cm0=m0.C0 – Capital Costs • f0.C0 – Financial costs • δ0.C0 – Depreciation Costs • o0.C0 – Profits from ordinary activity • Cf0=(1-s0)D0 – Final consumption • CP0=Y0/C0 – Capital Productivity • KP0=Y0/L0 – Knowledge Productivity	• ΔL/L=ΔCf/Cf	• Cf1'=Cf0.L1/L0
		• Δca= (L1+f1.C1)-cash0	• Cf1=Cf1'+Δca – Corrected final consumption • Cf1=Cf1'+[L1+(f1).(C0-δ1.C0+D0-Cf1')]-cash0
		• I1=D0-Cf1	• In1=I1-δ1.C0 – New investment • C1=C0-δ1.C0+I1
		• ΔCP1: IF(KP'≥Kpref, cc1(KPref-KP), 0) • ΔCP2=Δ (L/C)/cc2	• CP1=CP0 +ΔCP1+ΔCP2
		• Y=CP.C	• Cm1=Y1-L1 • o1.C1=Cm1-(f1+δ1).C1 • m1=f1+δ1+o1 • cash1=cash0-L1-f1.C1+Cf1 – Available cash

The first assumption for growth, which relates income to expense, is that the rate of change of labour income ΔL/L equals the rate of change of final consumption ΔCf/Cf. The second assumption, which relates to the production side of the economic process, is that GVA is proportional to capital Y=CP.C. The third and fourth assumptions are that the firm does not distribute profit to owners and does not have access to credit.

For investment there are two equations: The first one splits investment (I) into two components: One that equals capital depreciation (δ.C); and the other representing new investment (In). The second equation is used in the inter-static phase and represents the assumption that investment in year one is equal to the difference between the expense of year zero (D_o) and the final consumption of year one (Cf_1).

The following steps summarize how the model works: 1st – given initial conditions (L_0; C_0; s_0; f_0; δ_0; o_0 and $cash_0$); 2nd – When (L) changes, (Cf) changes, (I) changes and the model computes the value for the new investment (In); 3rd – Accordingly, $C_1=C_0-\delta_n.C_0+I$; 4th – C_1 establishes the new value added (Y_1), considering

the new capital productivity (CP_1). 5th – The model also computes the new value of knowledge productivity (KP_1), and the new value of the organization factor (o_1).

From the initial conditions we have the amount of cash the firm has at its disposal. To prevent it from being idle, the model uses it either to increase or decrease final consumption and consequently to decrease or increase investment. The model works this out calculating the difference between cash and the sum of labour cost with financial costs. If cash exceeds these costs, then that difference is subtracted from final consumption and therefore investment increases. The exact opposite happens if cash is not sufficient to cover these costs.

The model has two other corrections, both affecting CP. One reflects the effect on CP of an excessive reduction of labour income (L) relative to Y. For each economic sector or division, the highest value of knowledge productivity found amongst its firms is set as the reference knowledge productivity (KP_{ref}). For a firm to increase its value of KP over the reference value could be equivalent to excessively decreasing labour costs and creating an unsustainable social situation for its workers or a situation where capital could not be used adequately. This correction (ΔCP1) is calculated as shown in Table 1, and in it we find the model´s variable cc1 which is calibrated for each division. The other correction (ΔCP2) reflects the effect that changes in the ratio of labour to capital have on CP. How much a variation in this ratio affects CP will depend on a coefficient cc2, which should be calibrated for each division.

4 Theoretical Results

Putting the model to work, we depict different situations along periods of 10 years. Initial conditions are the following: L_0=50; C_0=500; f=0.018; δ=0.094; o=0; s=0.41; cash=60. Consequently the model computes m_0=0.112; Cm=56; $f_0.C_0$=9; $\delta_0.C_0$=47; $o_0.C_0$=0; Y_0=106; Cf_0=63; CP_0=0.212 and KP_0=2.12. The calibration coefficients are constant with cc1=0.2, cc2=2.5 and KP_{ref}=3. This is a balanced path. All figures are in units of value except CP, KP, f, δ, o and the calibration coefficients which are dimensionless. Now we change the inputs and watch how outputs change:

A. From the third year on, labour costs increase at a constant rate and the financial and depreciation coefficients stay unchanged.

The firm follows a balanced growth path along the first two years, showing a constant GVA. On the third and subsequent years we introduce a constant annual increase rate of 2% for labour costs (ΔL/L=0.02). Figure 2 shows the results. The GVA declines every year, starting a downturn.

B. From the third year on, labour costs decrease at a constant rate and the financial and depreciation coefficient stay unchanged.

The firm follows a balanced growth path along the first two years, showing a constant GVA. On the third and subsequent years we introduce a constant decrease rate of 2% for labour costs (ΔL/L=-0.02). Fig. 3 shows the results. The GVA increases every year, starting an expansion period.

C. Depreciation coefficient with a one-year increase and back to the initial value. Labour and the financial coefficient stay unchanged.

The firm follows a balanced growth path along the first three years, showing a constant GVA. On the third year we increase the value of the depreciation coefficient approximately 30% from 0,094 to 0.124. In the subsequent years the value of the depreciation coefficient returns to its original value of 0.094. Fig. 4 shows the results. The GVA by the firm starts to decline and keeps decreasing every year.

D. Financial coefficient with a one-year increase and back to the initial value. Labour and the depreciation coefficient stay unchanged.

The firm follows a balanced growth path along the first three years, showing a constant GVA. On the third year we increase the value of the financial coefficient approximately 30% from 0,018 to 0.024. In the subsequent years the value of the financial coefficient returns to its original value of 0.018. Fig. 5 shows

the results. The GVA by the firm declines on the third year and stays at that new value in the subsequent years.

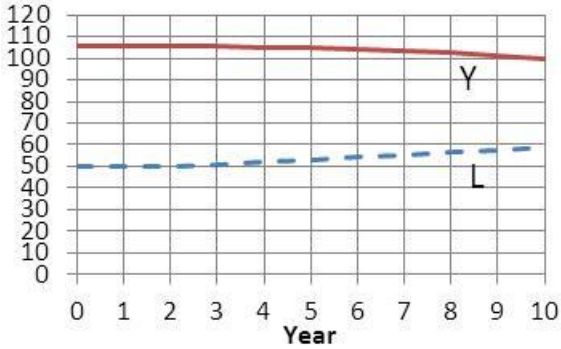

Figure 2: Case A

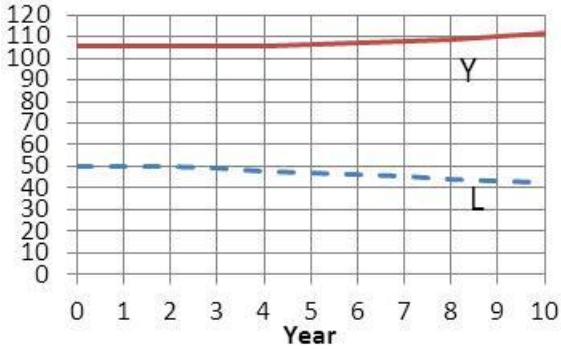

Figure 3: Case B

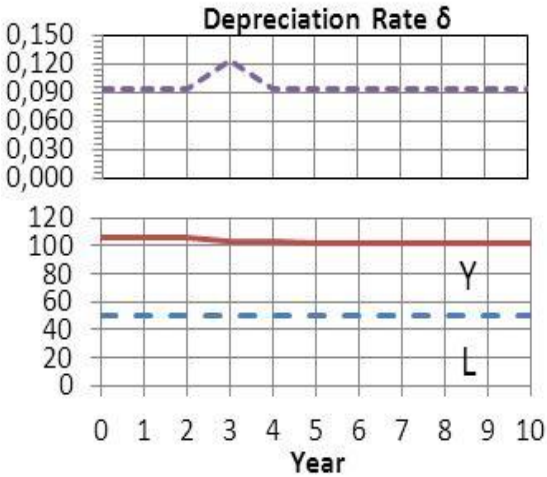

Figure 4: Case C

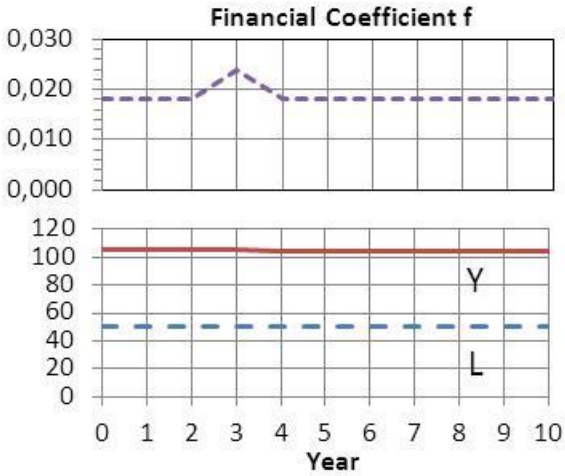

Figure 5: Case D

5 The Optimal Growth Path

A linear program was developed with the software GAMS, maximizing the sum of annual values of GVA for a given time-period. Sets were created to represent each year, the initial conditions were defined as scalars and the model's growth conditions and functioning equations described as parameters. At each year there were several hypotheses *h* concerning the annual labour growth rate, each one representing a different path that computed new values of the model's outputs. In the program, if the number of hypothesis is *h* and the number of years is *n*, the number of solutions is h^n. All these solutions are calculated for all years and the optimal solution is considered to be the path that maximizes the accumulated GVA. Growth was simulated using the same initial conditions used in the theoretical results chapter. The simulation was for a period of four years, given three hypotheses of annual labour growth rates ΔL/L (h1=+0.05, h2=0, h3=-0.05) and keeping the values of the other annual inputs of the model, f and δ, constant. The optimal path was found to maximize the sum of annual GVA at 531 value units. It consists of the sequence of three consecutive years of labour negative growth rates h3=-0.05 followed by a labour positive growth rate h1=+0.05 in the fourth year. Fig. 6 shows the optimal path of L and Y, represented by the dark dashed line and dark solid line, respectively.

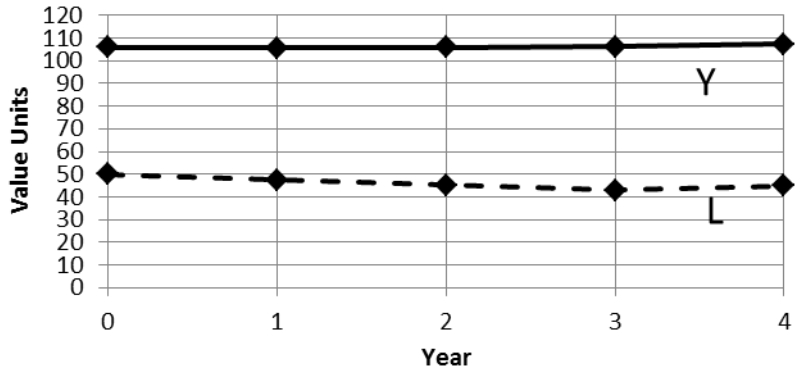

Figure 6: The Optimal growth path

6 Model Validation

The source of data for the study was the European Communities BACH database (BACH, 2012), which contains aggregated and harmonized information on the annual accounts of non-financial economic activity sectors of European countries. The sectorial breakdown is at the two digit level (NACE Rev. 2). The data used is from thirteen divisions of section A - Manufacturing (10, 12, 13, 14, 15, 16, 18, 20, 21, 22, 25, 27, 31) and two divisions of section F – Construction (42, 43), of the Portuguese economy, considering firms of all sizes.

6.1 Initial Conditions and Calibration

Data from 1995-2000 was used to calibrate the model, finding cc1 and cc2, while actual data from 2001-2005 was used to compare with the model's predictions for the same period.

Considering the initial conditions shown in fig.1, we find explicit in the database the values of GVA, capital and labour costs. Investment is computed adding capitalized production to the net acquisitions of intangible and tangible fixed assets. The depreciation factor is computed dividing depreciation on intangible and tangible fixed assets by the value of fixed assets. The financial factor is computed dividing the total financial charges by fixed assets, and the organizational factor is computed dividing the profits from ordinary activity by the value of fixed assets.

According to how the model computes capital productivity, the calibration coefficient cc2 was determined. Using the data of all 15 divisions, CP was correlated with the labour/capital ratio. It was found a high linear correlation (R^2=0.917). The correlation between CP and L/C was also found for each division individually, and, from there, cc2 was defined as the inverse of the slope of the linear regression line.

As for cc1, it was found that the calibration period did not provide a reliable measure of how capital productivity should be affected in cases where knowledge productivity exceeded the reference value KP_{ref}. For this reason, in the model's estimations, any potential correction to the capital productivity due to KP exceeding KP_{ref} was not taken into account.

The year 2000 was set as year zero of the estimation period. Additionally, for each of the following years, the actual values of ΔL, δ and f were given as inputs.

6.2 Validation

As an example of the model's performance, the results of the model's estimations for four divisions are shown in figures 7 to 10. The dark grey and light grey columns refer to actual GVA and estimated GVA, respectively. The solid lines and the dashed lines refer to actual capital and estimated capital, respectively, while the solid lines with triangle marks refer to labour costs. Results indicate that the model estimates GVA more accurately than it estimates capital. Table 2 shows the average relative errors found for GVA and capital estimations at each year. For GVA, errors are less than 9% in the first 4 years, and for capital

they are less than 25%. According to the model's condition of GVA being proportional to capital, the deviations between actual and estimated capital productivity, which were on average 20%, explain how the average relative errors of the GVA estimations can be lower than the average relative errors of the capital estimations.

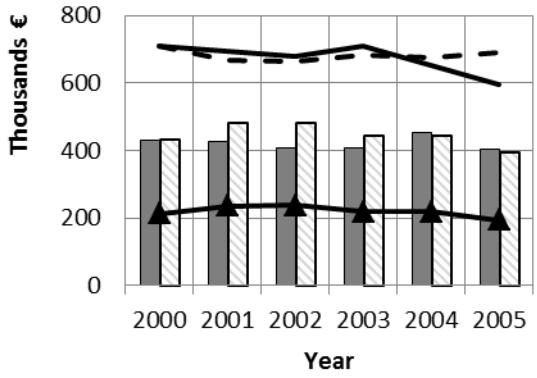

Figure 7: Estimations vs. Real Data: NACE Divison C20 – Chemical Industry

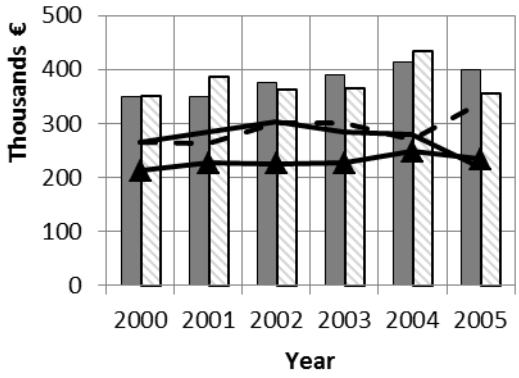

Figure 8: Estimations vs. Real Data: NACE Division C27 – Electric Equipment Manufacturing

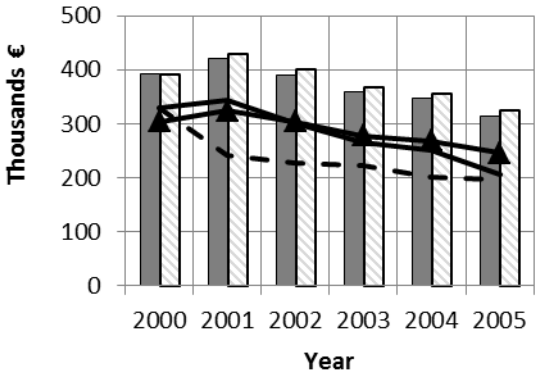

Figure 9: Estimations vs. Real Data: NACE Division C14 – Clothing Industry

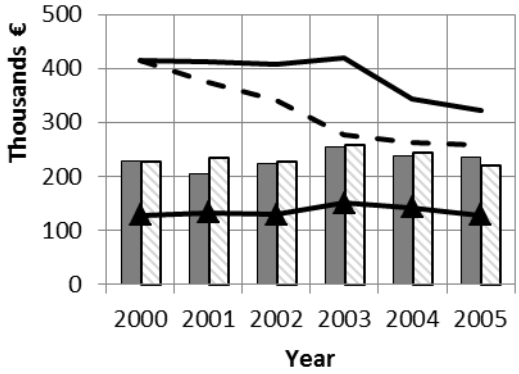

Figure 10: Estimations Vs Real Data: Nace Division C16 – Wood and Cork Industry

Table 2: Yearly Average Relative Error of the GVA and Capital Estimations

	Year 1	Year 2	Year 3	Year 4	Year 5
GVA	0.064	0.062	0.082	0.066	0.105
Capital	0.136	0.175	0.230	0.236	0.330

This validation method implied comparing results from a model where credit is not allowed with a reality where that condition does not exist. As such, the differences between actual and estimated capital may reflect how the model's context differs from reality in terms of access to credit. To better understand how capital expenditures relate to credit, we used real data to correlate variations in capital relative to total assets (ΔC/Total Assets) with variations in credit relative to total assets (ΔCredit/Total Assets). To analyse how this correlation relates to the errors of our capital estimations, the divisions were divided into two groups according to the error of capital estimation in year one. What we found was that, for divisions where the error of capital estimation is higher than 15%, there is a high linear correlation (R^2=0.76) of the variation in credit with the variation in capital (Fig. 11). In divisions where the error of capital estimation is lower than 5%, there is a low correlation (R^2=0.29) of the variation in credit with the variation in capital (Fig. 12).

The results seem to explain why in divisions where credit assumes a stronger role as a driver of capital expenditure the errors of the model's estimations were higher.

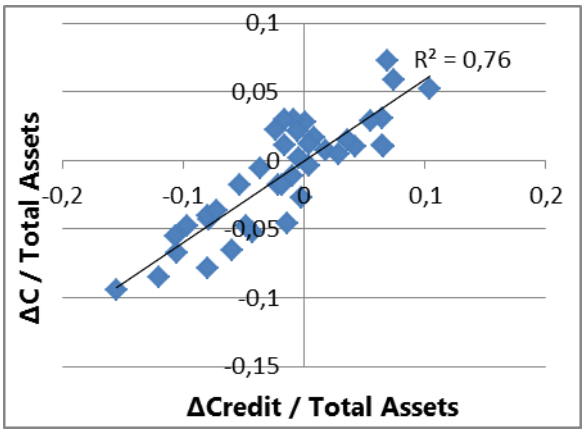

Figure 11: Correlation between variation in capital with variation in credit for divisions with error of capital estimation higher than 15%.

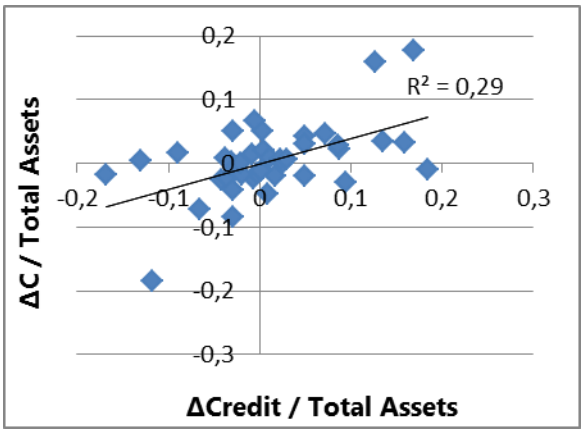

Figure 12: Correlation between variation in capital with variation in credit for divisions with error of capital estimation lower than 5%.

7 Conclusions

A growth model for a firm was proposed, assessing the implications of management decisions between investment and distribution of the value created. Abstract situations were modelled and an optimal growth path found by a linear program. The model was applied to different divisions of the Portuguese economy, its estimations compared with the division's actual economic performance. Results showed that GVA was predicted within a 9% inaccuracy and capital within a 25% inaccuracy. The estimation error for capital correlated well with the use of debt, which existed in reality but was not allowed in the model.

The success of this model seems to underline the effective importance of the management decision on what to do with the value created – either distribute it by workers and/or owners; or invest it in the production line. The model shows clearly the benefits and dangers of either, and allows for finding an optimal growth path.

The realities of the market, such as sales and disposal of assets, and many other fiscal, legal, financial and organizational issues are far more complex in reality than what was simulated by the model. Still, we conclude that the model contributes to the clarification of the fundamental conditions that are necessary but by no means sufficient for growth.

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