Effect of Measurement Uncertainty in Control Charts

Pedro S. Hack*, Carla S. ten Caten*

* Departamento de Engenharia de Produção e Transportes, Escola de Engenharia, Universidade Federal do Rio Grande do Sul, Campus Centro, 90035-190 Porto Alegre, Rio Grande do Sul, Brazil

Email: pedrohack@producao.ufrgs.br, tencaten@producao.ufrgs.br

Abstract

This article presents the effects of considering the probability distributions that come from measurement uncertainty in control charts. It was developed a model of control charts in which is inserted the variability of measurement uncertainty in the form of uncertainty regions that affect the points in the charts and also the control limits. Thus, the control chart has its probabilities of type I and II errors altered, and the operator uses the total probabilities that the process has gone out of control, based on data from both central tendency and variability charts.

Keywords: quality control; statistical process control; measurement uncertainty.

1 Introduction

Online control methods are primary tools for enhancing performance and reducing the variability of processes (MONTGOMERY, 2004). Thus, control charts are largely used for monitoring processes in the manufacturing and services area due to its ease of understanding and operation (ZHANG; WU, 2005). Even though control charts present a satisfactory performance as to the effects of random errors, their sensitivities are shortfall for detecting systematic errors (EPPE; DE PAUW, 2009) and, consequently, other process control charts were introduced in order to complement the existing ones, such as the exponentially weighted moving average (EWMA), the cumulative sum (CuSum) and the multivariate control charts (VARGAS, 2004).

Classic rules of statistical control consider a process as out of control when one of its samples is out of the control limits of the used chart, however, the values represented by the samples as well as the control limits are considered deterministic when they are essentially stochastic (GÜLBAY; KAHRAMAN, 2007). Studies made by Zarandi *et al.* (2008) and Gülbay and Kahraman (2007) show the effects of considering control limits as value ranges with probability distributions associated, achieving better results in comparison to traditional charts.

Although other works discuss supplemental rules to classic control limits (MARGAVIO *et al.*, 1994; ZHANG; WU, 2005; GAURI; CHAKRABORTY, 2009), variation of models (VARGAS *et al.*, 2004; EPPE; DE PAUW, 2009) and the different applications (HANSLIK *et al.*, 2001; DULL; TEGARDEN, 2004), there are very few studies on the effect of the measurement system (specifically the uncertainty of this system) on the process statistical control and, with the fast development of the automatic data acquisition systems, there is a concern with using effectively and efficiently the great amount of data available on the process (CHEN, 2010).

All the value obtained from the measurement (or series of measurements) are subject to the intrinsic properties of the measurement system used, which are responsible for making it not a deterministic value, but a range of values distinguished by a probability distribution. This distribution characterizing the value of a measurand is called measurement uncertainty (MU) and can alter significantly the decision making based directly or indirectly on the measures (CARBONE *et al.*, 2003).

The uncertainty is a measurement system that expresses the range in which the true value of the measurand is and also represents the degree of confidence on the result (DÉSENFANT; PRIEL, 2006). Since

the publication of the Guide to the Expression of Uncertainty in Measurement (1993), there has been a growing recognition that the MU is not less critical than the value of the measurement itself (REZNIK; DABKE, 2004).

This paper focuses on studying the effect of the MU on the creation of control charts regarding the detection of special causes, considering probability distributions associated with control limits as well as to samples and their impact on the sensitivity and the rate of false alarms. The effect on type I and II errors due to the MU is also showed. Therefore, the aim of this work is developing a methodology to calculate the individual probability of each sample in a control chart to indicate that the process is actually in or out of the statistical control, considering not only its position on the chart, but also the probability distribution of this position resulting from the MU. This probability is associated with an important indicator that increases even more the strength of the control chart.

This paper is organized as follows. On Section 2, reviews on the subject of control charts and uncertainty of measurement are presented. The methodological procedures used on the research follow on Section 3. On Section 4 the mathematical development for obtaining the expressions which indicate the probabilities of samples being out of control are presented. Section 5 has an example of application and discussion of results, followed by the conclusion, on Section 6.

2 Theoretical Framework

The concept of uncertainty as a quantifiable attribute is relatively new in the history of metrology, although the error analysis is a practice of this science for years. Even when all error components are studied and their corrections are applied, an uncertainty still remains on the value of the result, generating doubt on how well this result represents the value of the quantity being measured (ISO GUM, 2008).

The International Vocabulary of Metrology (1993) defines measurement uncertainty as the parameter associated with the measurement result, which characterizes the dispersion of values that could be reasonably attributed to the measurand. For Désenfant and Priel (2006), every time decisions are made based on measurements, the MU is the simpler and more accepted way to express confidence on the result, besides being required on international quality standards.

In 1993, the International Organization for Standardization (ISO) published the first version of Guide to the Expression of Uncertainty in Measurement (GUM), unifying a method for calculating uncertainty, based on the first-order Taylor series expansion and on the law of propagation of uncertainty, described in equation (1).

$$u_c^2(y) = \sum_{i=1}^{N} \left(\frac{\partial f}{\partial x_i}\right)^2 u^2(x_i) + 2\sum_{i=1}^{N-1}\sum_{j=i+1}^{N} \frac{\partial f}{\partial x_i}\frac{\partial f}{\partial x_j} u(x_i, x_j) \quad (1)$$

Where i and j represent the random variables on which the measurand depends, $u(x_i)$ and $u(x_j)$ the individual uncertainty of these variables, $u_c(y)$ represents the standard uncertainty of the measurand, $\partial f/\partial x_i$ and $\partial f/\partial x_j$ are the coefficients of sensitivity of each variable and $u(x_i, x_j)$ the uncertainty of the correlated variables x_i and x_j.

With equation (1), all measurement uncertainties are propagated to the measurand, using as base the mathematical model relative to the measurement system used. Based on the central limit theorem, it can be assumed that the final result of the measurement takes an approximately normal distribution (or t-student) centered on the measurements average and with a standard deviation equal to the standard uncertainty.

In order to obtain a high coverage probability (usually 95%) on the uncertainty estimate, it is necessary to multiply the standard uncertainty by a coverage factor k, which depends on the degrees of freedom of the sources that contribute to the uncertainty, resulting in expanded uncertainty, hence, the equation (2).

$$U_{95\%} = ku_c(y) \quad (2)$$

The experimental standard deviation is an important contribution to the MU; however, other elements may influence the result and should be studied, e.g. repeatability, reproducibility, inherited uncertainty and equipment technical specifications. If all uncertainty sources are not studied and considered, there is a risk of underestimating or even overestimating the MU of the measurand (JORNADA; CATEN, 2007).

According to Pendrill (2006) when there are uncertainties in measurement, there are also the risks of making a wrong decision, and to these risks there are always associated costs. To Rossi and Crenna (2006), when guarantying compliance of a process with technical requirements, the MU strongly affects the decision making process. When the uncertainties of measurement are not considered in procedures of quality control, the potential costs of the process could increase due to the use of an inefficient control.

When acquiring data for control chart by variables, all data gathered are subject to uncertainties of their measurement systems and, according to Gülbay and Kahraman (2007), even control charts by attributes have an associated stochastic property. When building both stages of the control chart application (implementation stage and monitoring stage), the values are traditionally considered deterministic and the presence of a value out of the control region is considered a special cause, without there being concern with the confidence that this value is actually in the specific region of the chart and what is the probability of the process being in or out of control, focusing only on the classic indicators of type I and type II errors.

When building any control chart it is necessary to calculate at least three control limits: UCL, CCL, LCL (upper, central and lower control limit, respectively), traditionally calculated with three standard deviations of distance from the average for both sides. To each one of these limits, as well as the points that represent the samples, there are intrinsically associated sources of uncertainty of measurement. If these uncertainties are not correctly studied and considered, the conclusions made concerning the control chart could be mistaken, generating potential unnecessary stops in production, rework or discarding (MONTGOMERY, 2004).

Control charts have two kinds of associated errors (CARBONE *et al.*, 2003). Type I or α errors are associated with the risk of the chart indicating that the process is out of control when that is not true (false alarm). Type II or β errors correspond to the probability of the process becoming out of control without the chart indicating so. Unlike type I, type II errors cannot be determined without knowledge of the real average of the process, however, the sampling strategy should be determined in such a way to minimize the probability of the chart not detecting changes of considerable magnitude in the process. Thus, larger sample sizes can detect deviations in the average more rapidly and easily.

The distance in the control limits related to the average is directly related to the error probabilities α and β. According to Montgomery (2004), when moving the limits to the central line, the risks of type I errors are increased and type II decreased, while when moving them away, the opposite is true. Different control charts have different sensitivities to false alarms for points outside the limits (ZHANG and WU, 2005); however, these analyses do not consider the uncertainty associated with calculations of their different parameters, focusing only on supplemental rules of control.

When considering uncertainty of the measurement system, probability intervals are considered both for control limits and samples, which results in an alteration in the risks of type I and II errors. When evaluating uncertainty in the control chart it is possible to estimate the probability of the process being in or out of control limits for each point of the sampling average, regardless of their location on the chart, or even, the probability that a false alarm occurs due to type I and II errors.

According to Chen (2010), the presence of false alarms is one of the main factors that lead to operators rejecting the employment of certain types of control charts. Therefore, when providing a probability estimate of making a wrong decision when investigating the occurrence of a special cause, the operator is offered an important tool to help them in this decision.

Consequently, it is important to correctly evaluate the MU associated with control charts and their potential impact on the statistical process control, quantifying its benefits and the efforts required to achieve this goal.

3 Methodological Procedures

The work was developed in three structured stages: (*i*) research on literature for publications related to the subject matter; (*ii*) study of which effects of uncertainty would be addressed from the control chart standpoint; (*iii*) preparation of the systematics to evaluating uncertainty in control charts and the evaluation of its impacts on the statistical process control.

On the initial stage searches were done the Science Direct, Emerald and SciELO online databases for scientific papers containing relevant contributions on the MU and control charts in a correlated way. In addition to these publications, books considered relevant for the area related to the research were included.

Subsequently, on the first stage, a study based on the gathered bibliography was done on which approaches of the topic required more attention and where were the methodological gaps that should be developed. Based on this research the stage of developing the methodology to consider the effect of the MU on the statistical process control was started. Once the methodology was defined, an example of the application that would facilitate the visualization of the method and provide a discussion of the results was elaborated.

4 Effect of Uncertainty of Measurement on Control Charts

When incorporating the uncertainty variation sources and probability distribution, both on individual samples of the control chart and on their control limits, one starts working with value ranges for both of them, which represents the doubt, or uncertainty, of their real values, i.e., uncertainty regions are used to define the samples and the limits on the control chart, and not the deterministic values. This effect of the MU on the control charts can be observed on Figure 1.

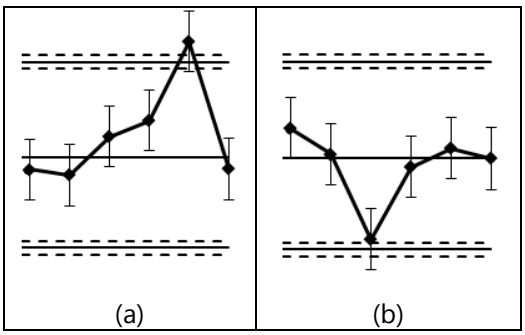

Figure 1 – Control chart considering uncertainties of measurement

It is possible to observe on Figure 1 that both the control limits and samples of the control chart acquire an uncertainty region around them, what defines the doubt of their real positioning on the chart.

If part of the uncertainty region of a sample outside the limits is comprised inside the critical control limits, this sample starts having the probability of belonging to that region and, therefore, of the process being under statistical control (type I error). Likewise, a sample within the control limits that has part of its uncertainty region outside the limits has a probability of being out of the region under control (type II error). In Figure 1(a) there is a sample out of the control limits (considered a special cause), however, when the uncertainties involved in the measurement system are considered, it starts having a probability of being under the upper control limit. Similarly, on Figure 1(b) there is a sample located within the limits

that starts having the probability of being considered out of the control region when the uncertainty is evaluated.

In order to evaluate whether the process is under statistical control, it is necessary to obtain the real probability that each sample is in or out of the control region, based on their uncertainty regions.

The probability of a specific sample being comprised inside the control limit uncertainty region (region where the probability of representing the special cause starts) is determined by the integral of the of the probability distribution functions of the sample and the limit.

$$P = \int f_{sample}(x) * f_{limit}(x) \tag{3}$$

Where $f(x)$ is the function of the probability distribution of the random variable x.

With the aim of obtaining this probability in real time and maintaining the online process control, the user responsible for the control chart would have to calculate the integral of the equation (3) or solve it numerically. Since this calculation would make the method operationalization in real time harder, it is possible to use critical control limits alternatively, which corresponds to extremity values of the control limits uncertainty region, defined as the following:

$$\begin{cases} UCL_{crit} = UCL \pm \dfrac{U_{95\%}(CL_{crit})}{2} \\ LCL_{crit} = LCL \pm \dfrac{U_{95\%}(CL_{crit})}{2} \end{cases} \tag{4}$$

The user might work using external (adding the uncertainty portion to the upper limit and subtracting from the lower limit) or internal (doing the opposite procedure) critical limits. The criterion of using external or internal critical limits for the control limits should be achieved weighting on the client's and producer's risk. When using internal critical limits the control of the chart becomes stricter, as it comes to be the worst uncertainty case associated with the limits and, therefore, the chance of type I error is increased and the chart indicates the process as out of control with a higher frequency (producer's risk). Similarly, when using external critical limits, more type II errors occur and the process may be falsely considered under control more frequently (client's risk).

When using critical limits, the integral of the equation (3) is reduced to the comparison of the sample uncertainty region (characterized by a normal distribution) to the critical limit value used, exempting the convolution integral calculation and enabling the use of the tabulated values or electronic spreadsheets.

Whatever the user-defined critical control limit is, the probability of the sample being out of the control limits due to the MU distribution comes to be:

$$P_i = P(X_i \geq UCL_{crit}) + P(X_i \leq LCL_{crit}) \tag{5}$$

Where P_i is the probability of the ith sample being out of the control limits according to its uncertainty region, UCL_{crit} is the upper critical control limit and LCL_{crit} the lower critical control limit.

And the equation (5) may still be rewritten using the standard normal distribution:

$$P_i = [1 - \Phi(LCS_{crit})] + \Phi(LCI_{crit}) \tag{6}$$

Where $\Phi(x)$ is the probability of a normally distributed variable being under x.

Since the variability due to the MU is much lower than the process variability, in a real application only one of the uncertainty region tails will be able to have the region out of the control limits at the same time, thus, the equation (6) may be rewritten as follows:

$$P_i = \begin{cases} 1 - \Phi(LCS_{crit}), & x_i \geq \bar{\bar{x}} \\ \Phi(LCI_{crit}), & x_i < \bar{\bar{x}} \end{cases} \tag{7}$$

Since the graphical position of the sample represents its average, to samples graphically out of the control limits, the probability P_i will be in the interval between 0.5 and 1, otherwise, in the interval between 0 and 0.5.

4.1. Type I and II Errors Considering Measurement Uncertainty

A sample out of the control limits is subject to the risk of being a false alarm, defined by α (usually 0.27% on traditional control charts), however, when the MU is taken into consideration, this risk comes to depend on the critical control limits used. The type I error is defined as a probability of a sample out of the control limits deriving from a common cause, i.e., the probability of a false alarm occurring. However, when using critical control limits, the probability α is altered, resulting in a $\alpha_{critical}$, which represents the probability of a sample normally distributed being out of the critical control limits, without the process being out of the statistical control, calculated according to (8).

$$\alpha_{crit} = 1 - P(LCL_{crit} \leq \bar{X} \leq UCL_{crit}) \tag{8}$$

The opposite of a false alarm could also occur, i.e., the process might be out of the statistical control, even when all samples are within the control limits (type II error). The probability of occurrence of such an error is defined by β. Since the probability of β also depends on the critical control limits calculated based on αcritical. When using critical control limits, the probability of occurrence of type II errors is also redefined:

$$\beta_{crit} = \Phi\left(Z_{\alpha_{crit}/2} - \frac{\delta\sqrt{n}}{\sigma}\right) - \Phi\left(-Z_{\alpha_{crit}/2} - \frac{\delta\sqrt{n}}{\sigma}\right) \tag{9}$$

Where $Z_{\alpha_{crit}/2}$ is the point of the standard normal distribution corresponding to the upper percentage $\alpha_{crit}/2$, δ is the distance relative to the average to which a special cause is to be detected and σ is the process standard deviation.

Once the type I and II errors are calculated taking into account the critical limits, it is necessary to calculate when the control chart will correctly indicate the special cause when considering the MU.

According to the equation (7), when considering the uncertainties of measurement, the samples have a probability of being out of the limits (represented by their uncertainty regions); however, according to the equation (8), a sample out of the limits may still be a false alarm. The probability of a sample being effectively located out of the control limits (P_i) and this representing the fact that the process is out of control, $P_{OC,inside}$, is given by the combination of equations (7) and (8):

$$P_{OC,inside} = (P_i)(1 - \alpha_{crit}) \tag{10}$$

The process may still be out of control even when the sample is graphically inside the control limits, $P_{OC,outside}$, in this case this probability of type β error is combined with the probability of the sample being inside the limits $(1 - P_i)$:

$$P_{OC,outside} = (1 - P_i)\beta_{crit} \tag{11}$$

Finally, the probability of any sample – regardless of their graphical location on the control chart – represent a special cause due to process out of the statistical control, P_{OC}, is given by the sum of the probabilities of equations (10) and (11).

$$P_{OC} = (P_i)(1 - \alpha_{crit}) + (1 - P_i)\beta_{crit} \tag{12}$$

Table 1 presents the probabilities the samples graphically inside and out of the control limits actually representing common causes resulting from under control processes and special causes resulting from processes out of control, both for traditional control charts and control charts that consider the MU.

Table 1 – Comparison of probabilities between traditional control charts and those that consider the measurement uncertainty

	Traditional Control Chart	Control Chart Considering the MU

Sample Graphically Out of the Control Limits		
Process Out of Control	$(1-\alpha)$	$(P_i)(1-\alpha_{crit})+(1-P_i)\beta_{crit}$
Process Under Control	α	$1-[(P_i)(1-\alpha_{crit})+(1-P_i)\beta_{crit}]$
Sample Graphically Inside the Control Limits		
Process Out of Control	β	$(P_i)(1-\alpha_{crit})+(1-P_i)\beta_{crit}$
Process Under Control	$(1-\beta)$	$1-[(P_i)(1-\alpha_{crit})+(1-P_i)\beta_{crit}]$

The final probability of the process being out of control considering the charts of central tendency and the variability of a specific sample being out of control is given by the combination of independent probabilities of the two charts:

$$P_{OC,total} = 1-(1-P_{OC,CT})(1-P_{OC,V}) \tag{13}$$

Where $P_{OC,CT}$ is the probability referring to the central tendency chart and $P_{OC,V}$ the probability referring to the variability chart.

It can be observed that in the traditional control chart the probabilities referring to the rejection of the hypotheses of the process control are generalists (i.e., applied to the whole chart and not the samples) and are also different for the samples inside and out of the limits. On the control chart considering the MU the opposite is true, each control chart sample has its own probability of representing the process out of control, and the expression for calculating this probability is the same, regardless of the sample's graphical position.

5 Application and Discussion

Figure 2 exemplifies a $\overline{X}$ and R control chart with sample size 5, considering the uncertainty of the measurement system. For demonstrative effect, the example considered only the effects of the standard deviation from the mean, the instrument inherited uncertainty and the instrument resolution. The contributions for the calculation of uncertainty are presented on Table 2. For the example presented, the external critical control limits were adopted. In order to simplify the method operationalization, the standard deviation from the average component may be considered the same for all pieces, making standard uncertainties of every piece equivalent, i.e. $u^2(x_i) \cong u^2(x)$.

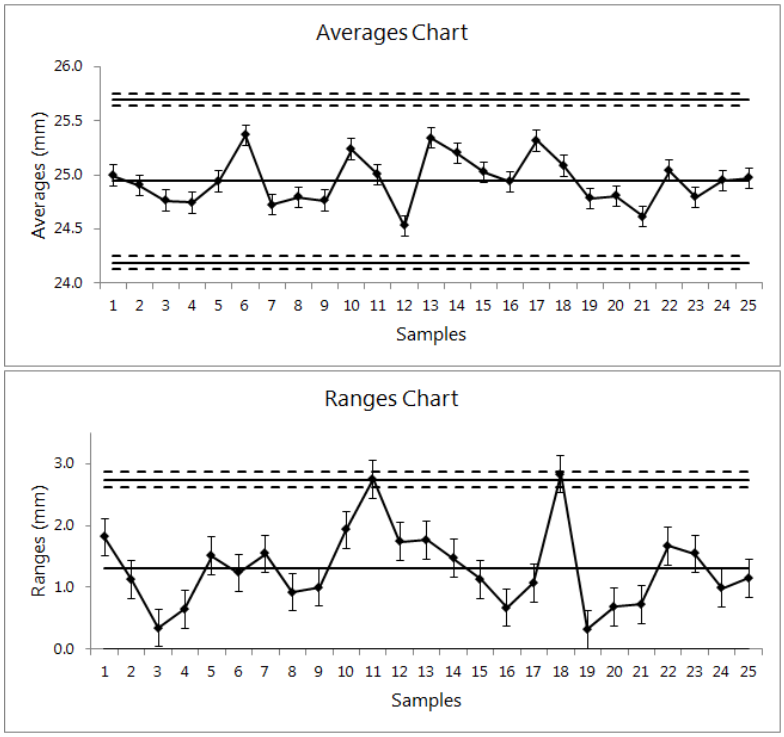

Figure 2 – Control Charts $\bar{X}$ and R considering measurement uncertainty

The uncertainty of the points on the control charts and of the critical control limits for the $\bar{X}$ and R charts of the Figure 2 were calculated with the use of equations (14) to (17) (see Appendix A).

$$u^2(\bar{X}) = \frac{u^2(x)}{n} \tag{14}$$

$$u^2(R) = 2u^2(x) \tag{15}$$

$$u_c^2(CL_{crit,\bar{X}}) = \frac{u^2(x)(1+2C^2)}{n} \tag{16}$$

$$u_c^2(CL_{crit,R}) = \frac{C^2 2u^2(x)}{n} \tag{17}$$

Equations (16) and (17) were used to compose the critical limits (external) according to equation (4).

Table 2 – Contributions to the uncertainty calculation of each sample

Effect	Variation	Distribution	Divisor	Contribution to Unc. (u_i)	Degrees of Freedom
Std. Dev. from the Avg.	0.01	t	$\sqrt{2}$	0.007	2
Inherited Uncertainty	0.01	t	2	0.05	∞
Resolution	0.01	Uniform	$\sqrt{3}$	0.00287	∞

On Figure 2 it is possible to notice that the uncertainty regions of the samples for the range chart are greater than those for the average charts, due to the fact that uncertainty for the averages has in its denominator the sample size. Consequently, greater sample sizes tend to reduce the average chart samples' uncertainty region, in addition to reducing the probability of type II error. These two reasons increase the recommendation for greater sample sizes when planning control charts.

Using equations (1) and (2) with the data from Table 2 and later equations (7) to (13), the probabilities for each sample of the process being out of control were obtained. The estimate of β was obtained

considering a deviation from the average of two standard deviations. The results are in Table 3. The first column identifies the sample number, columns 2 and 3 their average and range and the fourth column presents the probability of the sample being a special cause, considering its average and range. The columns 7 to 12 are the second part of data. The last row has the magnitudes of uncertainty regions of average and range.

Table 3 – Probabilities of the process in Figure 2 being out of control

#	$\bar{X}$	R	$P_{OC,CT}$	$P_{OC,V}$	$P_{OC,total}$	#	$\bar{X}$	R	$P_{OC,CT}$	$P_{OC,V}$	$P_{OC,total}$
1	24.994	1.811	0.024%	0.024%	0.049%	14	25.199	1.467	0.024%	0.024%	0.049%
2	24.904	1.119	0.024%	0.024%	0.049%	15	25.023	1.125	0.024%	0.024%	0.049%
3	24.763	0.341	0.024%	1.259%	1.283%	16	24.935	0.665	0.024%	0.025%	0.049%
4	24.742	0.643	0.024%	0.026%	0.050%	17	25.315	1.066	0.024%	0.024%	0.049%
5	24.941	1.508	0.024%	0.024%	0.049%	18	25.085	2.828	0.024%	38.866%	38.881%
6	25.367	1.231	0.024%	0.024%	0.049%	19	24.781	0.314	0.024%	1.952%	1.976%
7	24.724	1.542	0.024%	0.024%	0.049%	20	24.803	0.680	0.024%	0.025%	0.049%
8	24.792	0.919	0.024%	0.024%	0.049%	21	24.613	0.722	0.024%	0.025%	0.049%
9	24.762	0998	0.024%	0.024%	0.049%	22	25.038	1.666	0.024%	0.024%	0.049%
10	25.236	1.926	0.024%	0.024%	0.049%	23	24.791	1.540	0.024%	0.024%	0.049%
11	25.001	2.747	0.024%	20.756%	20.775%	24	24.947	0.986	0.024%	0.024%	0.049%
12	24.527	1.741	0.024%	0.024%	0.049%	25	24.968	1.144	0.024%	0.024%	0.049%
13	25.339	1.763	0.024%	0.024%	0.049%						
	$U_{95\%}(\bar{X})$ =0.096 $UCL_{crit,\bar{X}} = 25.753$ $LCL_{crit,\bar{X}} = 24.134$						$U_{95\%}(R) = 0.304$ $UCL_{crit,R} = 2.871$ $LCL_{crit,R} = 0.000$				

Through Table 3 it is possible to observe that the samples with greater probabilities of being out of statistical control are numbers 11 and 18 (20.77% and 38.88%, respectively). This is due mainly to the high variability inside the sample (range), and also to the fact that the expanded uncertainty of range has a great magnitude when compared to the uncertainty of the average.

The traditional control chart does not consider type II error in the identification of special causes, since according to its approach this probability is the same for all chart points. On the other hand, when the MU is considered, the probability of occurrence of a type II error is different for each sample, since it is associated with the probability of the sample actually being inside the control limits and, thus, it is modified by the sample uncertainty region and the position of its central tendency. Alternatively it is possible to use the control chart with uncertainties disregarding the β error, in order to maintaining the same methodology used by the traditional control chart. In this case, the probability of the samples being out of control is reduced in all of the chart's points.

A facilitator in the operationalization of control chart considering uncertainties is that uncertainties of type B depend of factors intrinsic to the measurement process and for that reason are not altered. Similarly, uncertainty of type A may be considered equivalent in all measurands (according to the example), enabling uncertainties to be kept on a database and only altered when the production is altered or the measurement system undergoes some calibration or alteration.

6 Conclusions

The effect of the measurement system and its uncertainty is generally disregarded in the use of control charts. Consequently, this work presented a study of the effects of measurement uncertainty on traditional control charts, aiming at developing a methodology to calculate the probability of a control chart sample representing a special cause, considering not only its position on the chart, but also the uncertainty of probability associated with it due to the MU.

Through the law of propagation of uncertainty and the control chart properties, an equation that determines the probability of each sample in a control chart representing a special cause was presented, considering type I and II errors in both control charts. With this equation, the control chart user needs only the measurement system data used to calculate the uncertainty.

As an example, the $\bar{X}$ and R control charts considering the effects of the MU were presented, demonstrating the probabilities of a sample inside the control limits representing a special cause. In this example, it was observed that the uncertainty of measurement has a greater effect on the range chart, due to the equating. A way of reducing the uncertainty region for both charts is increasing the sample size, what would also contribute to the reduction of type II error.

In the future, we suggest a comparison between the traditional control chart and the control chart with uncertainties, in order to determine which are best suited for specific situations and also comparing their average run length (number of samples needed until there is a false alarm) for the different parameters used. We also suggest incorporating the MU in the supplemental rules of control charts (tendency, points in sequence over or under the central limit, alarm limits etc.). It could also be investigated the effect of considering or not the type II error in the control chart with uncertainties. The sensitivity study of the MU in the capacity analysis is also very promising, since it directly impacts the costs to the manufacturer.

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Appendix A. Development of Equations 14 to 17

Following the hypothesis that the standard deviation from the average uncertainty is approximately the same for every part produced, $u^2(x_i) \cong u^2(x)$, the sample uncertainties for the average and range charts can be obtained applying equation (1) on the equations for average and range::

$$u^2(\overline{X}) = \frac{\sum_{i=1}^{n} u^2(x_i)}{n^2} \cong \frac{\sum_{i=1}^{n} u^2(x)}{n^2} \cong \frac{n\left(u^2(x)\right)}{n^2} \cong \frac{u^2(x)}{n} \tag{A.1}$$

$$u^2(R) = u^2(x_{max}) + u^2(x_{min}) \cong 2u^2(x) \tag{A.2}$$

The limits of the average chart are calculated as follows:

$$CL_{crit,\overline{X}} = \overline{X} \pm C \cdot \overline{R} \tag{A.3}$$

Where C is the constant that depends on the sample size.

Applying (A.3) in (1):

$$u_c^2\left(CL_{crit,\overline{X}}\right) = 1u^2(\overline{X}) + C^2 u^2(\overline{R}) \tag{A.4}$$

The uncertainty of $\overline{X}$ is calculated as presented in (A.1) and the uncertainty of $\overline{R}$ is calculated as follows:

$$u^2(\overline{R}) = \frac{\sum_{i=1}^{n} 2u^2(x_i)}{n^2} \cong \frac{2u^2(x)}{n} \tag{A.5}$$

Thus, applying (A.1) and (A.5) in (A.4), one obtains:

$$u_c^2\left(CL_{crit,\overline{X}}\right) = \frac{u^2(x)}{n} + C^2\frac{2u^2(x)}{n} = \frac{u^2(x)(1+2C^2)}{n} \tag{A.6}$$

The control limits of the range chart are calculated as follows:

$$CL_{crit,R} = C \cdot \overline{R} \tag{A.7}$$

Applying (A.7) in (1):

$$u_c^2\left(CL_{crit,R}\right) = C^2 u^2(\overline{R}) \tag{A.8}$$

Applying (A.5) in (A.8):

$$u_c^2\left(CL_{crit,R}\right) = \frac{C^2 2u^2(x)}{n} \tag{A.9}$$