Applying the Theta Model to Short-Term Forecasts in Monthly Time Series

Gilson Adamczuk Oliveira[*], Marcelo Gonçalves Trentin[+], Anselmo Chaves Neto[**]

[*] Department of Mechanical Engineering, Federal Technological University of Paraná, Via do Conhecimento, km 1, Pato Branco, Brazil
[+]Department of Electrical Engineering, Federal Technological University of Paraná, Via do Conhecimento, km 1, Pato Branco, Brazil
[**] Department of Statistic, Federal University of Paraná, Centro Politécnico - Jardim das Américas, Curitiba, Brazil

Email: gilson@utfpr.edu.br; marcelo@utfpr.edu.br; anselmo@ufpr.br

Abstract

This paper addresses demand forecasting for monthly data using the univariate Theta method. The model is relatively recent and was developed by Nikopoulos and Assimakopoulos (2000). It is based on the concept of modifying the local curvatures of the time series, obtained by a theta (Θ) coefficient. The central idea is to decompose the time series into at least two theta lines L(Θ) representing a long term period and the other, a short-term one. The Forecast is a combination of the forecasts obtained by adjusting the theta lines achieved in the decomposition. The model was applied to the historical record of four Products from the mechanical metal sector using L(Θ = 0) and L(Θ = 2). The choice of these two values for the Θ (0 and 2) coefficients is the simplest case for the model, which was used in the M3-competition of Makridakis and Hibon (2000). The forecasts were compared with traditional methods and the model performed well, as it presented average MAPE's of 5.99%, 14.44%, 16.53% and 7.78% respectively, for the four Products studied over the last three months which were separated for validation.

Keywords: demand forecasting; time series; decomposition; combining forecasts; theta model.

1 Introduction

One of the major difficulties of the sector that manages the area of materials in a company lies in the need to take decisions about future activities. Normally there is a historical record of data, and attempts are made to forecast data for the near future, and the short and medium term. Generally, in industrial environments, demand forecasting and sales of products is necessary and fundamental for materials planning and strategic and operational decisions.

The Demand Forecast is the basis for the strategic planning of production, sales and finances of any business. From these data, companies can draw up plans for capacity, cash flow, sales, production and stocks, manpower, purchases and so forth.

In the modern world companies face fierce competition. With that they need to seek more information about the scenarios they may face. The use of forecasts assists administrators in decision making, especially in uncertain market. Statistical tools are of great importance for this type of activity, not being allowed simply informal predictions. The forecasting models based on time series are then recommended. Better forecasts allow the creation of planning more consistent and reliable.

Medium- and small-sized businesses do not always have the human resources and software available that would enable them to use time series models involving great depth statistics. For this reason, companies find it difficult to implement the more complex forecasting models of time series. In this context, simpler models which are known for their efficiency are desired and supply this need. In addition to smoothing models, a variation known as the Theta Model (Assimakopoulos; Nikolopoulos, 2000) is presented as a good alternative.

The simplest case of the theta model involves the decomposition of the time series into two periods. A long-term period, which seeks to investigates the trend from the entire historical record. A second short

period investigates the latest changes. Based on this central idea, the forecast is matched by the tendency of the long-term with the local changes which have been occurring at the present time.

This study sets out to discuss the theta model, by seeking to prove its performance, and by making forecasts in sales series, which were obtained from a company in the mechanical metal sector.

To prove this, we used historical data on monthly sales of products from a company company which handles stainless steel products. Employing an electronic spreadsheet, there was a predictions performance comparison between model theta and traditional models of time series forecasting, said automatic. The forecast horizon is three months. As a performance criterion, we used the absolute average percentage error (MAPE).

Section 1 shows why the issue is important; Section 2 gives a brief theoretical framework on the subject. The theta model is described in Section 3. Section 4 presents the methodology and the application in the company is described in Section 5. The findings are reported in Section 6.

2 Theoretical Framework

Time series are observations ordered in time, a sequence of values that do not follow a non-random order (Morettin; Tolói, 2004). The analysis of time series is based on the hypothesis that the future is a continuation of the past, at least of the recent past, in which the trends of growth or decline observed should remain in the future, as well as the seasonality or cyclicality observed in the past. This can be considered a temporal model (Corrêa; Gianesi; Caon, 2001).

The representation of time series is normally done by considering the time series (a realization of a stochastic process, namely, processes controlled by probabilistic laws), such as X_t, in which observations are recorded at discrete instants and equally distributed, with a notation of $X_1, X_2 .... X_n$. The classical form of writing a series is:

$$X_t = f(t) + a_t \tag{1}$$

where $f(t)$ is a completely determined function (a systematic, deterministic part) and a_t is a random sequence, independent of $f(t)$, it being the case that the random variables a_t are not correlated and have a zero mean and constant variance. The random variable a_t is also called white noise when the distribution is normal, i.e., $a_t \sim N(0, \sigma_a^2)$. Many of the properties observed in a time series X_t can be captured and take on the following form of classical decomposition follows:

$$X_t = T_t + S_t + C_t + a_t \tag{2}$$

in which T_t is the component of trend; S_t the seasonal component; C_t is a cyclical component and a_t is a random or noise component, which is the non-explained part which is expected to be purely random. The seasonal and cyclical components are repeated at each fixed interval, in which periodic variations can be captured by these components. Given a set of time series observations collected up to the instant t and a model that represents these phenomena, the forecast of the value of the series at the time $t + h$ series can be obtained.

Several procedures have been drawn up for time series decomposition. Each method seeks to remove the non-observable components as precisely as possible. These procedures, after each of the components has been identified, enable the behavior of the series to be better understood and thus enable more accurate values to be forecast (Wheelwright; Makridrakis, 1985).

There are two main objectives in time series analysis. The first is to identify the nature of the phenomenon generating the sequence of observations. The second is to forecast future values of the time series. To achieve these goals, there is a need to identify the pattern that generates the data of the sequence

observed. Having identified the pattern of behavior, it is possible to extrapolate the pattern identified and predict future events (Statsoft, 2007). Generally what is done is to identify the structure of the process generating the series, within a class of predefined models, and under some conditions the parameters are estimated and forecasts made. Or moreover, the sequence is modeled according to the premises of softening the variation and adjusting the updated trend in time and then finally the forecasts are made.

The methods of time series forecasting can be divided into two groups: automatic ones, which can be directly applied with the aid of a computer; non-automatic ones, which require the intervention of a specialist so that they can be applied (Morettin; Tolói, 2004). The main ones are: moving average, simple exponential smoothing, linear exponential smoothing and seasonal exponential smoothing and Winter's linear. Among the automated methods, the exponential smoothing models stand out because they are simple to implement and the good results achieved. Among the non-automatic ones, which require the user to have a greater statistical base and knowledge, mention should be made of the autoregressive moving average models (AR, MA and ARMA), autoregressive integrated moving averages (ARIMA), Kalman and AEP filters, ARARMA models, and multivariate ARMA models (MARMA). Among the non-automatic ones, ARIMA models are the main representatives (Morettin; Tolói, 2004).

An opportune review of the literature on exponential smoothing, from the original studies by Brown and Holt, can be found in Gardner (1985) and later updated in Gardner (2006). Box and Jenkins (1976) present the ARIMA methodology, which is widely used in time series analysis, and it has great prominence given that has been widely announced and is flexible. Gooijer and Hyndman (2006) reviewed the past twenty-five years of time series forecasts, and this study is a primary source of references for studies on the subject, besides which they found that there had been tremendous progress in several areas, but many topics still require further studies.

The accuracy of forecast is not based only on the forecast horizon desired, but is also strongly influenced by the characteristics of the observations of the series under study. The combination of forecasts of more than one model increases reliability and reduces large deviations in the projected values, it not necessarily being the most sophisticated models that obtain the best results (Wheelwright; Makridrakis, 1985).

In various studies there is controversy between the results obtained with simple models and those obtained using more complex ones which require users to have greater knowledge. Depending on the application and who will use the model, the simplest method is the one that has most to recommend it (FILDES *et a.l*, 1998) (Makridakis; Hibon, 2000).

On concluding the M3-Competition, Makridakis and Hibon (2000) confirmed that methods they considered were simpler obtained good results in the competition and in some cases were even better in relation to others that are regarded as being statistically sophisticated such as ARIMA and ARARMA. In this context the theta model is presented as a quick and simple alternative. This can be considered an automatic method, since the choice of the Θ coefficients is fixed.

The theta model involves the decomposition of the time series into a set of new series. The resulting series are called theta lines $L(\Theta)$, and maintain the average and the slope of the original data, but not their curvatures. This is based on modifying the local curvature of a time series seasonally adjusted by the theta coefficient (Θ).The coefficient Θ is applied directly to the second difference of the time series. Each of the theta lines is extrapolated separately and the forecasts are combined with equal weights (Assimakopoulos, Nikolopoulos, 2000).

Different combinations of theta lines can be used for each forecast horizon. Hyndman and Koehler (2006) claim that the theta model in particular presented very good forecasts in the M3-Competition (Makridakis; Hibon, 2000), as it obtained one of the best performances among all other models (Gooijer; Hyndman, 2006).

Hyndman and Billah (2003) consider that the theta model is equivalent to simple exponential smoothing with drift (SES-d). The drift considered corresponds to half the value of the slope of a linear regression adjusted to the data. Thus, it provides a form of control that limits the possibility of the model producing inaccurate forecasts (Koehler, 2006). Nikolopoulos and Assimakopoulos (2005) disagree with this

consideration, and state that the theta model is more generic than exponential smoothing. The model is described in the following section.

3 The theta model

The challenge of the method proposed was to increase the degree of exploiting the useful information embedded in the data before applying a forecasting method. This information from an intuitively point of view has short and long term components. These components are identified in the theta model and are then extrapolated separately (Assimakopoulos; Nikopoulos, 2000).

The model is based on the concept of modifying the local curvatures of the time series. This alteration is obtained by using the *theta* co-efficient, or simply Θ, which is applied directly to the second difference of the time series. The X_i data of the series can be written in as:

$$X_i = X_1 + (i-1).(X_2 - X_1) + \left(\sum_{t=2}^{i-1} (i-t).X''_{t+1} \right) \tag{3}$$

where in the time *t*:

$$X''_t = X_t - 2X_{t-1} + X_{t-2} \tag{4}$$

The Y_i points of a theta line by definition are:

$$Y_i = Y_1 + (i-1).(Y_2 - Y_1) + \Theta.\left(\sum_{t=2}^{i-1} (i-t).X''_{t+1} \right) \tag{5}$$

The problem falls back on minimizing quadratic errors, namely:

$$\min(\sum_i e_i^2) = \min(\sum_i (Y_i - X_i)^2)\,. \tag{6}$$

As shown in Assimakopoulos and Nikopoulos (2000) the result of this minimization implies that both the mean and the slope of the theta lines are equal to those of the original series. Different combinations of theta lines can be used for each forecast horizon. One of the simpler cases is the combination of two theta lines with Θ=0 and Θ=2, i.e., the data can be decomposed as follows:

$$X_t = \frac{1}{2}(L(\Theta = 0) + L(\Theta = 2)) \tag{7}$$

The L(Θ = 0) is the linear regression of the data. This is shown in Assimakopoulos and Nikopoulos (2000). The L(Θ = 2) can be obtained in a simplified form using the previous equation. Thus:

$$L(\Theta = 2) = 2.X_t - L(\Theta = 0) \tag{8}$$

The L(Θ = 0) describes the series as a linear trend. Since the L(Θ = 2) doubles the local curvatures by extending the short-term action. In the forecasts L(Θ = 0) is extrapolated by a linear trend and L(Θ = 2) is extrapolated by simple exponential smoothing (SES). The simple combination of two forecasts gives the final $\hat{X}_t(h)$ forecast for the theta model, namely:

$$\hat{X}_t(h) = \frac{1}{2}(\hat{L}(\Theta = 0)_h + \hat{L}(\Theta = 2)_h) \tag{9}$$

where $\hat{L}(\Theta = 0)_h$ and $\hat{L}(\Theta = 2)_h$ are the forecasts by linear regression and SES of the respective *theta* lines.

Figure 1 shows the lines theta and the respective forecasts and real data of series 30 of the M3-competition. Use of the coefficients Θ (0 and 2) was made to produce the forecasts estimates in the M3-Competition of Makridakis and Hibon (2000). The steps followed were:

- Step 0: test of seasonality. The criterion used was the t-test for auto-correlating data with a one-year lag (12 observations for monthly series). The T value was calculated compared with *t*= 1.645 to a 10% significance level
- Step 1: seasonal decomposition of the data by the classical multiplicative method
- Step 2: decomposition of the series into L(Θ = 0) and L(Θ = 2)
- Step 3: extrapolation. The L(Θ = 0) is extrapolated by linear regression and L(Θ = 2) by SES
- Step 4: combination. The forecasts produced in the extrapolation of the theta lines are combined with equal weights
- Step 5: The seasonal indices obtained in the decomposition of step 1 are incorporated into the forecasts.

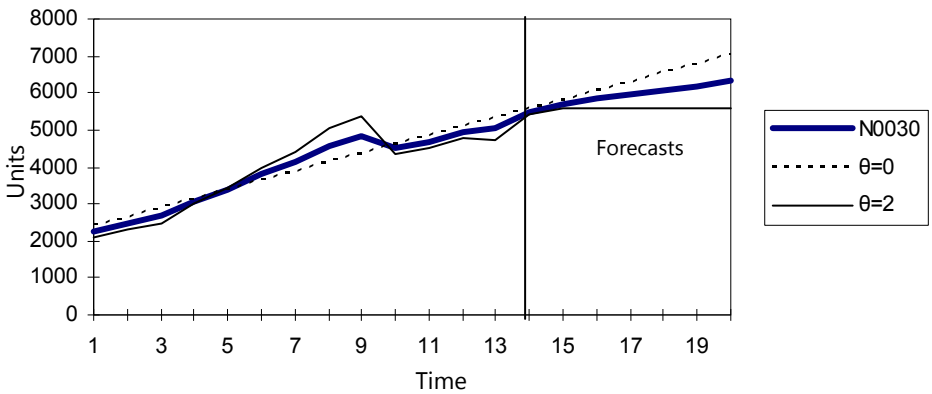

Figure 1: Series 30 of the M3-competition and theta forecasts. Source: Assimakopoulos and Nikopoulos (2005)

Assimakopoulos and Nikopoulos (2005) suggest how the model with L(Θ = 0) and L(Θ = 2) can be applied using an electronic spreadsheet:

- Step 0: seasonal decomposition of the data by the classical multiplicative method, if necessary
- Step 1: Apply the linear regression of the data L(Θ = 0), prepare the regression line and forecasts
- Step 2: Prepare the values of L(Θ = 2) using $L(\Theta = 2) = 2.X_t - L(\Theta = 0)$
- Step 3: extrapolate the L(Θ = 2) with SES, optimized by the Microsoft Excel Solver) or by another simpler method such as moving averages
- Step 4: combine forecasts with equal weights obtained by linear regression and SES.

4 Methodology

In order to apply the model, the choice of Θ coefficients, due to the good performance obtained in the M3-Competition, was restricted to two theta lines L(Θ = 0) and L(Θ = 2). The other methodological steps and decisions adopted in this study were:

- Seasonality: no seasonality tests were carried out. Data were analyzed graphically where a check was made that there was no significant seasonality in the historical data used
- Sequence of application of the method: steps 1-4 were followed as described in item 3.1 for the electronic spreadsheet
- Measurement of accuracy: to evaluate the forecasts obtained the criterion of MAPE (mean absolute percentage error) was adopted. Although some authors criticize this criterion in the M3-Competition, this was the main criterion, with the advantage of being able to stipulate an average MAPE between the series analyzed, which show order of magnitude differences

$$MAPE = \frac{\sum_{i=1}^{n} \left| \frac{(x_i - \hat{x}_i)}{x_i} \right| .100}{n} \tag{10}$$

- Comparison with other methods: the forecasts obtained were compared to automatic methods which were adjusted by using statistical software

5 Application

The metal mechanic sector company provided the historical record monthly sales of four products A, B, C and D.

The company is regarded as being of medium to large, located in Parana state, southern Brazil. The company operates in the market for over 20 years. It has approximately 300 employees. Manufactures and sells finished products to the lines of residential kitchen furniture, serving the Brazilian market and export. Its products are geared for both the higher social classes and for the most popular. Was not obtained permission from the company to disclose more information about its structure and also of its products.

Products A, B and C are traditional and have long been on the market. They are shown in Figure 2. Product D has a more recent historical record and is shown in Figure 3.

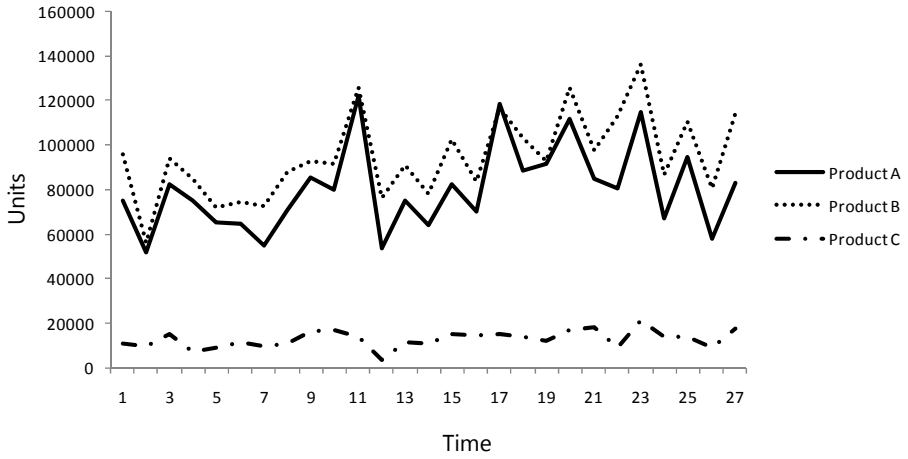

Figure 2: Historical record of Products A, B and C from January/2005 to March/2007. Source: company studied

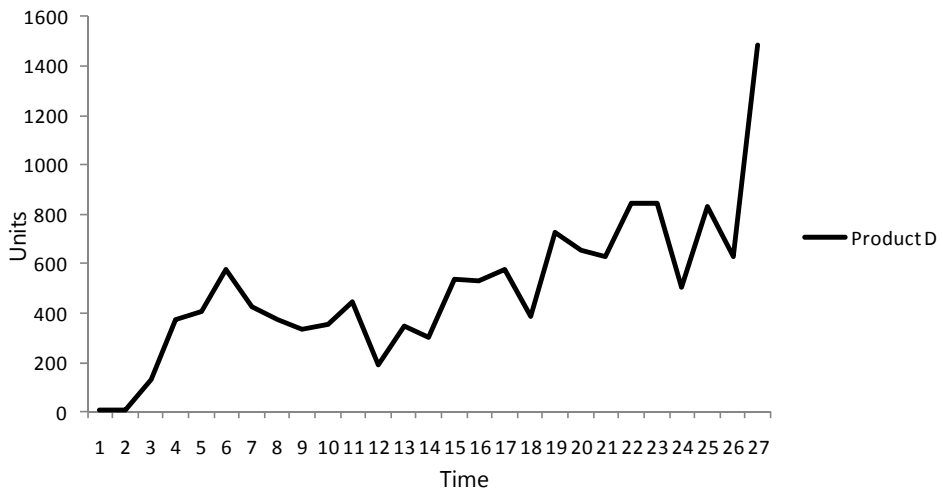

Figure 3: Historical record of Product D from January/2005 to March/2007. Source: company studied

The historical record supplied by the company covers 30 months starting in Jan/2005. The months from April to June 2007 were separated for validation and comparison with automatic methods adjusted by the smallest root mean square error (RMSE). Tables 1, 2 and 3 show, respectively, the real values, the forecasts using the theta model and the forecasts using other automatic methods adjusted by the historical data.

Table 1: Real demand values for the Products from April to June 2007

Period	Product A	Product B	Product C	Product D
04/2007	83010	109159	20617	1009
05/2007	84250	134084	15512	1176
06/2007	73019	130291	17355	960

Source: Authors

Tables 2 and 3 enable it to be affirmed that the *theta* model, in its simplest application, with L(Θ = 0) and L(Θ = 2), obtained results that are at least equivalent to those of other automated methods. In a comparison with the MAPE's of the four Products, an average of 11.19% was obtained using the theta model against 12.07% using automatic methods automatic adjusted by the smallest RMSE.

Table 2: Forecasts and MAPE's obtained using the theta model

Period	Product A		Product B		Product C		Product D	
	Forecast	MAPE	Forecast	MAPE	Forecast	MAPE	Forecast	MAPE
04/2007	84112	1.33 %	105072	3.74 %	14576	29.30 %	981	2.81 %
05/2007	84525	0.33 %	105697	21.17 %	14673	5.40 %	996	15.31 %
06/2007	84937	16.32 %	106323	18.40 %	14771	14.89 %	1010	5.22 %
Average MAPE		5.99 %		14.44 %		16.53 %		7.78 %

Source: Authors

Table 3: Forecasts and MAPE's obtained with automatic methods adjusted by the smallest RMSE

Period	Product A (linear tendency) RMSE = 18449.1		Product B (linear tendency) RMSE = 16385.8		Product C (linear tendency) RMSE = 3666.13		Product D (quadratic tendency) RMSE = 188.56	
	Forecast	MAPE	Forecast	MAPE	Forecast	MAPE	Forecast	MAPE
04/2007	91803	10.59 %	112164	2.75 %	15600	24.33 %	1018	0.88 %
05/2007	92628	9.94 %	113415	15.41 %	15795	1.83 %	1071	8.90 %
06/2007	93453	27.98 %	114666	11.99 %	15089	13.05 %	1125	17.21 %
Average MAPE		16.17 %		10.05 %		13.07 %		9.00 %

Source: Authors

The historical data of the product A and the forecasts made for times 28 to 30, respectively $\hat{X}_{27}(1)$, $\hat{X}_{27}(2)$, $\hat{X}_{27}(3)$, are shown in the last three columns of table 4. L(Θ = 2) was extrapolated by simple exponential smoothing (SES) with α = 3,448042E-03.

Table 4: Data used in the analysis of demand for product A

t	(X_t)	L(Θ = 0) (y=825,23t+68696,2)	L(Θ = 2) $L(\Theta=2)=2.X_t-L(\Theta=0)$
1	74863	69521,43	80205,33
2	51867	70346,66	33386,46
3	82692	71171,89	94211,63
4	74920	71997,12	77843,00
5	65568	72822,35	58313,37
6	64955	73647,58	56262,98
7	54956	74472,82	35438,42
8	70970	75298,05	66641,75
9	85469	76123,28	94814,88
10	80183	76948,51	83416,65
11	121836	77773,74	165897,94
12	53624	78598,97	28648,31
13	74831	79424,20	70237,16
14	64319	80249,43	48388,01
15	82299	81074,66	83524,06
16	70102	81899,90	58304,62
17	118274	82725,13	153822,31
18	88665	83550,36	93779,56
19	91316	84375,59	98256,09
20	111738	85200,82	138275,34
21	84745	86026,05	83464,59
22	80762	86851,28	74673,64
23	114657	87676,51	141637,69
24	67092	88501,74	45681,62
25	94758	89326,98	100189,14
26	57968	90152,21	25784,55
27	83307	90977,44	75635,60
$\hat{X}_{27}(1)$	84112	91802,67	76421,59
$\hat{X}_{27}(2)$	84525	92627,90	76421,59
$\hat{X}_{27}(3)$	84937	93453,13	76421,59

Source: Authors

For instance, the forecasting for the moment 28, being in time 27, can be obtained by equation (9) and computed by (11):

$$\hat{X}_{27}(1) = \frac{1}{2}(91802{,}67 + 76421{,}59) = 84112 \tag{11}$$

To the effect of the method see Figure 4 which shows the real series of product A and the theta lines L(Θ = 0) and L(Θ = 2) and forecasting (data in Table 4).

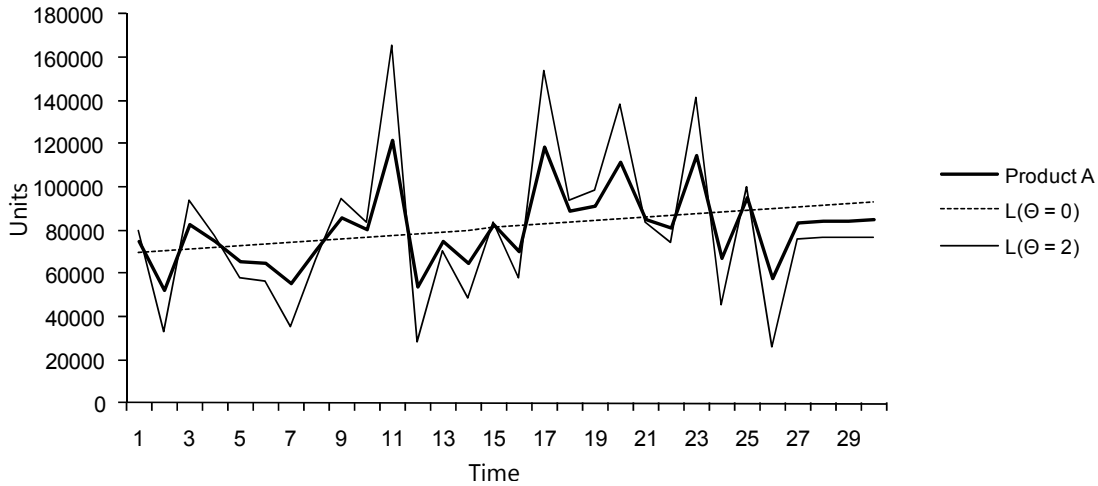

Figure 4: Product A series and theta forecasts. Source: company studied

6 Conclusions

The aim of this study which was to apply the theta model was achieved. The theta method with two lines (Θ= 0 and Θ= 2) is simple and does not require extensive training or knowledge before it can be applied. It is flexible because it can be applied, with good results, in stationary monthly series or with a trend whether or not combined with seasonality. It also brings the advantage of one being able to apply it by using a spreadsheet.

The forecasts obtained were compared with traditional methods and the model performed well, as it presented average MAPE's of 5.99%, 14.44%, 16.53% and 7.78% respectively for the four products studied in the last three months separated for validation. The MAPE's obtained in the forecasts confirm the results of the M3-Competition of Makridakis and Hibon (2000), which suggests the theta method is a fast and efficient method for obtaining good forecasts.

A disadvantage that may be cited is the lack of confidence intervals for the forecasts in the study by Nikopoulos and Assimakopoulos (2000). Hyndman and Billah (2003) suggest how confidence intervals can be built in.

One suggestion for further studies is to use more than two theta lines and (or) with different values of Θ = 0 and Θ= 2. This should be carefully analyzed in practical applications to ensure that the advantage of the simplicity of the method is not diminished.

Acknowledgements

We are grateful to CAPES and the Araucaria Foundation, for their financial support which enabled this study to be conducted.

References

ASSIMAKOPOULOS, V.; NIKOLOPOULOS, K. The theta model: a decomposition approach to forecasting. International Journal of Forecasting v.16 p. 521 –530, 2000.

ASSIMAKOPOULOS, V.; NIKOLOPOULOS, K. Fathoming the Theta Model. 25th International Symposium on Forecasting, ISF, San Antonio, Texas, USA, 2005.

BOX, G. E. P.; JENKINS, G. M. Time Series Analysis: forecasting and control, Ed. Holden Day, 1976.

CORRÊA, H. L.; GIANESI, I. G. N.; CAON, M. Planejamento, Programação e Controle da Produção. 4th ed., São Paulo: Atlas, 2001.

FILDES, R.; HIBON, M.; MAKRIDAKIS, S.; MEADE N. Generalising about univariate forecasting methods: further empirical evidence. International Journal of Forecasting, v.14, p. 339 –358b, 1998.

GARDNER, E. S., Jr. Exponential smoothing: The state of the art. Journal of Forecasting, vol. 4, p. 1–28,1985.

GARDNER, E. S., Jr. Exponential smoothing: The state of the art—Part II. International Journal of Forecasting, vol. 22, p. 637–666, 2006.

GOOIJER, J.G.; HYNDMAN R.J. 25 years of time series forecasting. International Journal of Forecasting, Vol 22, p. 443–473, 2006.

HYNDMAN, R.J.; BILLAH, B. Unmasking the Theta method. International Journal of Forecasting, v.19 (2), p. 287-290, 2003.

HYNDMAN, R.J.; KOEHLER, A.B. Another look at measures of forecast accuracy. International Journal of Forecasting, v.22, p. 679–688, 2006.

KOEHLER, A. Discussion. International Journal of Forecasting, v. 22, p. 667–670, 2006.

MAKRIDAKIS, S.; HIBON, M. The M3-Competition: results, conclusions and implications. International Journal of Forecasting, v. 16, p. 451 –476, 2000.

MORETTIN, P.A.; TOLOI, C.M.C. Análise de Séries Temporais. São Paulo: Edgard Blücher, 2004

NIKOLOPOULOS, K.; ASSIMAKOPOULOS, V. Fathoming the Theta model. 25th International Symposium on Forecasting, ISF, San Antonio, Texas, USA, 2005.

STATSOFT, INC. Electronic Statistic Textbook. Tulsa, OK: Statsoft,. Available at: http://www.statsoft.com./textbook/stathome/html. Consulted on 10/08/2007.

WHEELWRIGHT, S. C.; MAKRIDAKIS, S. Forecasting Methods for Management. 4th ed. , New York: John Wiley & Sons Inc., 1985.