An Empirical Investigation of Some Hedging Strategies for Crude Oil Market

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Abstract

The hedge strategies allow negotiators that have short and long positions in the market protection against price fluctuations. This paper examines the performance of bivariate volatility models for the crude oil spot and future returns of the Western Texas Intermediate – WTI type barrel prices. Besides the volatility of spot and future crude oil barrel returns time series, the hedge ratio strategy is examined through the hedge effectiveness. Thus this study shows hedge strategies built using methodologies applied in the variance modelling of returns of crude oil prices in the spot and future markets, and covariance between these two market returns, which correspond to the inputs of the hedge strategy shown in this work. From the studied models the bivariate GARCH in a Diagonal VECH and BEKK representations was chosen, using three different models for the mean: a bivariate autoregressive, a vector autoregressive and a vector error correction. The methodologies used here take into consideration the denial of assumptions of homoscedasticity and normality for the return distributions making them more realistic than other methodologies used in the financial literature. The data used is logarithm returns of daily prices quoted in dollars per barrel from November 2008 to May 2010 for spot and future contracts, in particular the June contract.

Keywords: Bivariate Volatility Models; Minimum Variance Hedge Ratio; Hedge Effectiveness; Crude Oil Market.

1 Introduction

All countries consume crude oil or oil products. Both producers and consumers are highly concerned about crude oil prices. The crude oil prices are being directly affected by several economic, political, geopolitical, technological factors, and also oil reserves, available stocks and weather conditions, among others. On other hand the crude oil price fluctuations influence directly the world economy. Compared to financial assets crude oil prices have had an elevated volatility in recent years. Studies of crude oil price movements and co-movements are highly complex. Therefore the academics and practitioners are developing many studies about themes related with crude oil prices. Economic agents indirectly involved in crude oil negotiations, such as firm or government planners, are looking for related petroleum price forecasting models, while the agents directly involved are looking for the hedge strategies studies as well. The hedge strategies allow negotiators that have short and long positions in the market, protection against price fluctuations.

Another motivation for this work is the relevance of crude oil international market growth, the biggest market among the commodity markets. This led to a development of derivative markets of this commodity, in particular, future contract markets, or simply future markets. This development brought sophisticated strategies. Among them strategies there are many for risk reduction of physical positions, investments in crude oil or others related to this commodity movements.

In an informational efficient market, future and spot prices must be associated. Consequently these prices are determinant for hedge strategies studies. The hedge strategies allow negotiators that have short and long positions in the market protection against prices fluctuations. In this study the minimum variance model, the most widespread hedge strategy, was selected among several models for hedge strategies. The risk part that could be eliminated with minimum variance hedge ratio, or MV hedge ratio, can be

determinated using a measure from the hedging effectiveness introduced in the finance literature by Ederington (1979).

The aim of this paper is to examine the performance of two bivariate volatility models for the crude oil spot and future returns of the Western Texas Intermediate – WTI type barrel prices, and for the mean a bivariate autoregressive, a vector autoregressive and a vector error correction models. Besides that it assesses the volatility of spot and future crude oil barrel returns time series and the hedge ratio strategy that is evaluated through the hedge effectiveness.

The remaining of this work is organized in the following form: the hedge and volatility models are covered in section 2; the sample used and the methodological approach are presented in section 3 and 4, respectively; the empirical results are presented in section 5; and to conclude the final remarks are given in the section 6.

2 Hedge and Volatility Models: a Brief Overview

A hedge strategy can be accomplished with future contracts. A large number of researches has been contributing for the hedge theory development and application in several markets. Among the works done some of the most important studies about hedge theory and applications in financial assets and commodities markets, must be highlighted. Working (1953) developed and formalized concepts on this subject. Johnson (1960) and Stein (1961) introduced in the literature the calculation for the number of contracts for an investment position hedge, or the optimal hedge ratio. Ederington (1979) presented the hedge effectiveness as a reduction of risk. Figlewski (1984) studied the hedging performance and basis risk. These studies have a strong assumption of homoscedasticity. In other studies elaborated after these, this assumption stopped being considered. Studies presented by Baillie and Myers (1991), Myers (1991), Ghosh (1993) and Park and Switzer (1995) took into consideration the time-varying feature of hedge ratio. Additionally other works continue improving hedge theory and application, as an example Castelino (1992) and Chance (1998). Among recently accomplished hedge works it can be mentioned that: Lien and Wilson (2001) used volatility models, such as Stochastic volatility (SV) and Generalized Autoregressive Conditional Heteroskedasticity (GARCH), for hedge strategies in crude oil markets; Javali-Naini and Manesh (2006) showed application of multivariate volatility models in the formulation of hedge; Lautier (2010) studied dynamic hedging strategies in crude oil market; Lee (2010) investigated petroleum futures through speculative and hedging activities, or relation of futures trading and volatility; and Chang et al. (2011) who examined the performance of multivariate volatility models in crude oil markets calculating optimal hedge ratio. While these models appear adequate for efficient markets, other models were presented in the finance literature for inefficient markets. However the Engle and Granger (1987) study emphasized that two price time series of efficient markets are not cointegrated. Several suggested works used models that take into consideration the stochastic trend between spot and future prices, or the cointegration of these prices. Lien and Luo (1994), Ghosh (1993) and Lien (1996) highlighted this. Regarding the long-run cointegrating relationship between spot and future markets Lien (1996) proposed the vector error correction model, a vector autoregressive model with the cointegration term or the error correction term.

For the determination of hedge ratio estimate, volatility is fundamental. Several methods allow the volatility, or variance, estimates of crude oil return distributions. These estimates can be accomplished with univariate or multivariate volatility models. These models must take into consideration the heteroskedasticity of returns time series, or return distributions of crude oil prices. That is, taking into consideration the time-varying characteristic of hedge ratio. In a study about the United Kingdom´s inflation behaviour, Engle (1982) presented a more realistic volatility model than previously presented in financial literature: the Autoregressive Conditional Heteroskedasticity Model – ARCH model. This seminal work elaborated by Engle (1982) shows the way to estimate conditional variance observing the heteroskedasticity characteristic of financial time series. There is a family of models constructed from ARCH model. Bollerslev (1986) introduced a generalization of ARCH model designated by Generalized

Autoregressive Conditional Heteroskedasticity Model – GARCH model. Engle and Bollerslev (1986) proposed another model, similar to an Exponentially Weighed Moving Average (EWMA) model, the Integrated GARCH – IGARCH. And Engle et al. (1987) suggested the ARCH in mean or ARCH-M, in which the conditional variance influences the mean. Besides estimating the variance it is necessary to estimate the covariance between spot and future returns. Bollerslev et al. (1986) generalized the ARCH-M model proposing VECH, the multivariate model. An important constraint of the VECH model refers to a covariance matrix which must be definite positive. For that reason Engle et al. (1995) proposed another parameterization for multivariate GARCH model named BEKK. The BEKK model has fewer restrictions and is easier to implement than the VECH model. Another multivariate model constructed was the Dynamic Conditional Correlation (DCC), which is a different model built from the one Bollerslev (1986) proposed. In this model the covariation is dynamic and the correlation coefficient is not. According to Engle (2002), this model consists of estimating the arguments in two steps: univariate GARCH series and after that the correlations. Baillie et al. (1991) applied the univariate and multivariate models, specifically the ARCH model in the VECH version, with several parameterizations, to estimate hedge ratio of selected commodities. Bollerslev (2009) presented a glossary of ARCH acronyms that were present in the financial literature.

3 The Data - Sample Used

To reach the objective of this work the collected data consisted of daily crude oil prices of WTI type in the spot and future markets, specifically the June contract, quoted in US$ per barrel from November 2008 to May 2010, while the spot price series were obtained from Energy information Administration – EIA, the official Energy Statistics from United States of America. The future prices of June contract were obtained from Bloomberg web site. The plot of these time series is shown in Figure 1.

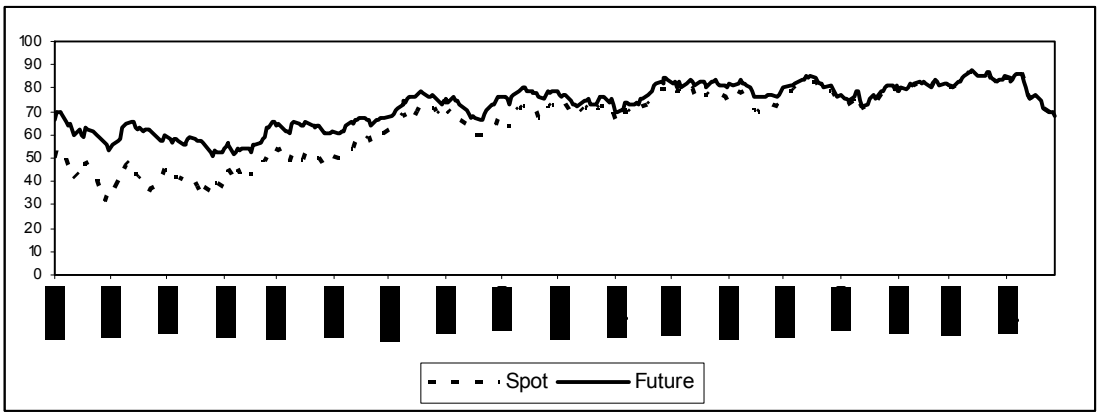

Figure 1: Spot and Future WTI Prices

The plot presented indicates a strong association between crude oil spot and future prices. It can also be observed the basis variation and the convergence of prices at the contract expire date. From these daily prices time series the return time series are calculated as follows:

$$R_t = \ln\left(\frac{price_t}{price_{t-1}}\right) \quad , \qquad [1]$$

where R_t refers to return of the price at time t, $price_t$ = quote the price at time t, $price_t$ = quote price at time $t-1$.

The spot and future return time series are the data used in this work to estimate the volatility models implemented for hedge strategies that were calculated. Table 1 presents the statistics of returns summary. The descriptive statistics for the returns series of crude oil prices, presented in Table 1, shows a very low average for spot and future returns, near zero. But the standard deviation for the two time series is higher,

once these markets volatility are very high. Another characteristic here and in the financial assets time series in general is the high kurtosis, which indicates fat tails distributions. The skewness coefficients are positive which demonstrate that these series have a longer right than left tail therefore have greater gains than losses. This occurs for spot prices that are slightly higher. It is must be highlighted that the normality can not be accepted as expected, as generally occurs with return time series of financial assets or commodities. Moreover it can be observed that Jarque-Bera statistics of crude oil returns in spot and future markets are statistically significant, therefore the distribution of these series is not normal.

Table 1: Statistics of Returns Summary

Statistics	Spot	Future
Mean	0.00084	0.00006
Median	-0.00014	0.00044
Maximum	0.13546	0.08055
Minimum	-0.12743	-0.07244
Std. Deviation	0.03458	0.02187
Skewness	0.17081	0.04586
Kurtosis	5.79999	3.99820
Jarque-Bera	123.32780	15.57447
(p-value)	(0.000000)	(0.00000)
N	372	372

4 Methodological Approach

The hedging procedure consists in mixing or associating short or long positions in a constructed portfolio trying to reduce. That is, to minimizing the returns variations of an asset, or barrel of crude oil as dealt in this work. The return of portfolio with spot and future position, can be formulate in the following form:

$$R_{Pt} = R_{St} - h_t R_{Ft} \quad , \qquad [2]$$

where R_{Pt} is the portfolio return at time *t*, R_{St} is the spot return at time at time *t*, R_{Ft} is the future return at time *t*. The variance of the hedged portfolio conditioned on the information available at time *t – 1* can be represented by the expression:

$$Var(R_{Pt}|I_{t-1}) = Var(R_{St}|I_{t-1}) - 2h_t \operatorname{cov}(R_{St}, R_{Ft}|I_{t-1}) + h_t^2 Var(R_{Ft}|I_{t-1}) \ , \qquad [3]$$

where $Var(R_{St} \mid I_{t-1})$ and $Var(R_{Ft} \mid I_{t-1})$ are variance conditional and $\operatorname{cov}(R_{St,}\ R_{Ft} \mid I_{t-1})$ is the covariance conditional of the spot and futures returns, respectively. The optimal hedge ratio is the h_t which minimizes the conditional variance, or the risk, of the hedged portfolio. As showed Baillie and Myers (1991), the partial derivative of the conditional variance with respect to h_t is the optimal hedge ratio at time t conditioned on the information available at time *t -1*, given by:

$$h_t|I_{t-1} = \frac{\operatorname{cov}(R_{St}, R_{Ft}|I_{t-1})}{Var(R_{Ft}|I_{t-1})} \ . \qquad [4]$$

For compare the performance of optimal hedge ratio between the models, or methodologies, used in this work as suggested in Ku et al. (2007) it can be used the hedging effective index HE given by:

$$HE = \frac{Var_{unhedged} - Var_{hedged}}{Var_{unhedged}} \ , \qquad [5]$$

where Var_{hedged} represent the variance of hedged portfolio and $Var_{unhedged}$ is the variance of spot returns, or unhedged portfolio. As observes Tansuchat et al. (2010): "a higher HE indicates a higher hedging effectiveness and larger risk reduction, such that a hedging method with a higher HE is regarded as a

superior hedging strategy". Another definition for the hedging effective index – $HE*$ is the proportion of the variance eliminated thought a hedge strategy and can be denoted as (see Hull (2005)):

$$HE* = h^2 \frac{Var(R_{Ft} | I_{t-1})}{Var(R_{St} | I_{t-1})} \quad . \qquad [6]$$

In this way, the better hedge effectiveness is closed to one. And the number of future contracts necessary to accomplish the hedge at the time *t* is given by:

$$N_t^* = h_t \frac{SpotQuote}{FutureQuote} \quad . \qquad [7]$$

To estimate the parameters presented here volatility models must be used. The volatility models used in this study were estimated taking into consideration the Student´s t distribution as described below. To estimate the variance or volatility and covariance this work implemented two bivariate GARCH models. The first bivariate GARCH model applied here is the VECH diagonal presented by Bollerslev et al. (1988) that consists of estimating the follow equation proposed in Ding and Engle (2001):

$$H_t = C + D \bullet e_{t-1} e_{t-1}^T + G \bullet H_{t-1} \quad , \qquad [8]$$

where • is the Hadamard product and H_t represents the variance-covariance matrix at time t. The bivariate case to the ARCH(1) and GARCH(1,1), respectively, can be denoted as follows:

$$\begin{bmatrix} h_{11,t} \\ h_{21,t} \\ h_{22,t} \end{bmatrix} = \begin{bmatrix} c_{11} \\ c_{21} \\ c_{22} \end{bmatrix} + \begin{bmatrix} d_{11} & 0 & 0 \\ 0 & d_{22} & 0 \\ 0 & 0 & d_{33} \end{bmatrix} \bullet \begin{bmatrix} e_{1,t-1}^2 \\ e_{2,t-1} e_{1,t-1} \\ e_{2,t-1}^2 \end{bmatrix} \qquad [9]$$

$$\begin{bmatrix} h_{11,t} \\ h_{21,t} \\ h_{22,t} \end{bmatrix} = \begin{bmatrix} c_{11} \\ c_{21} \\ c_{22} \end{bmatrix} + \begin{bmatrix} d_{11} & 0 & 0 \\ 0 & d_{22} & 0 \\ 0 & 0 & d_{33} \end{bmatrix} \bullet \begin{bmatrix} e_{1,t-1}^2 \\ e_{2,t-1} e_{1,t-1} \\ e_{2,t-1}^2 \end{bmatrix} + \begin{bmatrix} g_{11} & 0 & 0 \\ 0 & g_{22} & 0 \\ 0 & 0 & g_{33} \end{bmatrix} \bullet \begin{bmatrix} h_{11,t} \\ h_{21,t} \\ h_{22,t} \end{bmatrix} . \qquad [10]$$

The other bivariate GARCH model employed in this work is the BEKK model which can be expressed in its general ARCH variation as follows:

$$H_t = C'C + \sum_{i=1}^{q} D_i' e_{t-i} e_{t-i}' D_i + \sum_{j=1}^{p} G_j' H_{t-j} G_j \qquad [11]$$

$$\begin{bmatrix} \sigma_{11,t} & \sigma_{12,t} \\ \sigma_{21,t} & \sigma_{22,t} \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} \\ 0 & c_{22} \end{bmatrix}' \begin{bmatrix} c_{11} & c_{12} \\ 0 & c_{22} \end{bmatrix} + \begin{bmatrix} d_{11} & 0 \\ 0 & d_{22} \end{bmatrix}' \begin{bmatrix} e_{1,t-1}^2 & e_{1,t-1} e_{2,t-1} \\ e_{2,t-1} e_{1,t-1} & e_{2,t-1}^2 \end{bmatrix} \begin{bmatrix} d_{11} & 0 \\ 0 & d_{22} \end{bmatrix} . \qquad [12]$$

For the mean three different models were applied. The first one was the autoregressive model AR(1) which can be expressed as follows for the bivariate case:

$$\begin{bmatrix} r_{1,t} \\ r_{2,t} \end{bmatrix} = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} + \begin{bmatrix} b_{1,1} \\ b_{1,2} \end{bmatrix} \bullet \begin{bmatrix} r_{1,t-1} \\ r_{2,t-1} \end{bmatrix} + \begin{bmatrix} e_{1,t} \\ e_{2,t} \end{bmatrix} . \qquad [13]$$

The second model applied was the vector autoregressive (VAR) model, presented in the econometric literature by Sims (1980). The VAR with lag 1 can be represented by the expression:

$$\begin{cases} R_{St} = \alpha_{11} + \alpha_{12} R_{St-1} + \alpha_{13} R_{Ft-1} + e_{1t} \\ R_{Ft} = \alpha_{21} + \alpha_{22} R_{St-1} + \alpha_{23} R_{Ft-1} + e_{2t} \end{cases} \quad . \qquad [14]$$

The third model applied was the vector error correction (VEC) model presented by Engle and Granger (1987). In its the simplest form, the VEC model with lag 1 can be described as:

$$\begin{cases} R_{St} = \alpha\theta_{t-1} + \alpha_1 R_{St-1} + \alpha_2 R_{Ft-1} + e_{1t} \\ R_{Ft} = -\beta\theta_{t-1} + \beta_1 R_{St-1} + \beta_2 R_{Ft-1} + e_{2t} \end{cases} . \qquad [15]$$

The θ in the previous expression is the difference between future and spot prices and represents the cointegrating variable (see Lien (1996)). As proposed by Kroner and Sultan (1993), the VEC model can also be expressed in the following form:

$$\begin{cases} R_{St} = \alpha_{1S} + \alpha_{2S}(S_{t-1} - \delta F_{t-1}) + e_{St} \\ R_{Ft} = \alpha_{1F} + \alpha_{2F}(S_{t-1} - \delta F_{t-1}) + e_{Ft} \end{cases} , \qquad [16]$$

where S and F refers to future and spot prices. These two forms of VEC models were estimated for this work. This way the hedge ratio can be calculated in the following manner:

$$h_t | I_{t-1} = \frac{\mathrm{cov}(e_{1t}, e_{2t})}{Var(e_{2t})} . \qquad [17]$$

The autoregressive model (AR), and vector autoregressive model (VAR), appear to be adequate in efficient markets. When the efficient market hypothesis cannot be accepted the cointegration variable must be included the VAR model. Consequently the vector error correction (VEC) is the adequate model to determine the hedge ratio. The models suggested here were constructed assuming a bivariate Student's t-distribution to spot and future returns, and the freedom degree was jointly estimated with the models. This way for each AR, VAR and VEC model types different models were estimated and among these a model of each one was selected. The Akaike criterion was chosen in order to select a model from each type, that is, among the models with parameter estimates considered statistically significant. The determinations of hedge ratio were accomplished from each type of model selected and to compare the different methods, hedge effectiveness were calculated. That is, this was done to elaborate the comparison among the model types used in this work.

5 Empirical Results

The selected volatility model was the VECH diagonal, used here for the three different models estimated. The first was the autoregressive model for the average, without intercept or the parameter a, as mentioned before. The variance equation was a GARCH model and the matrix C, matrix D and matrix G are rank one, indefinite and indefinite, respectively. The t distribution was used with 7 degrees of freedom, estimated in the model. The results of the volatility obtained from the AR model are shown in Figure 2 through the variance of the spot and future returns. The volatility plot shows that the crude oil market was greatly affected by the 2008 global crises. This becomes clear in the volatility peak periods near the end of 2008 and the period that followed it shows a slump in the volatility returning to a high volatility by the end of January 2009 falling steadily after April 2009. A slight rise can be noted after July 2009 and from April on the average volatility shows level well bellow the one observed on the semester following the global crises as shown in Figure 2 both for the spot market and future market, where the volatility is always higher than the spot.

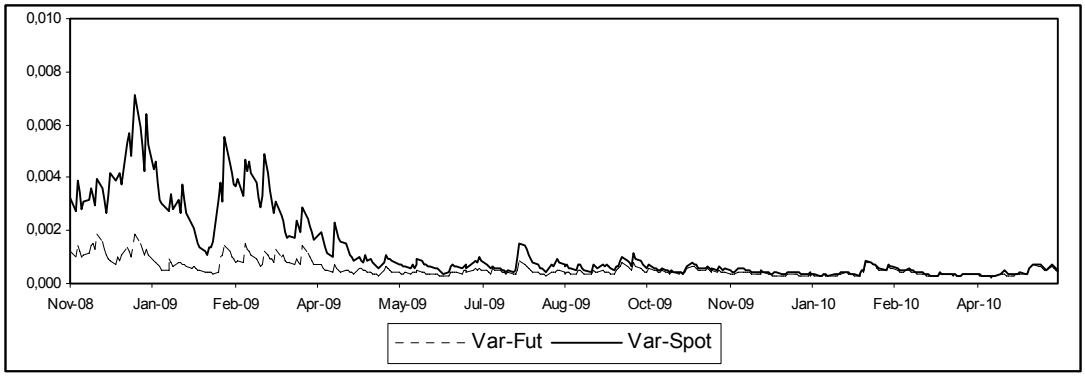

Figure 2: Spot and Future Returns Volatility of the WTI Prices

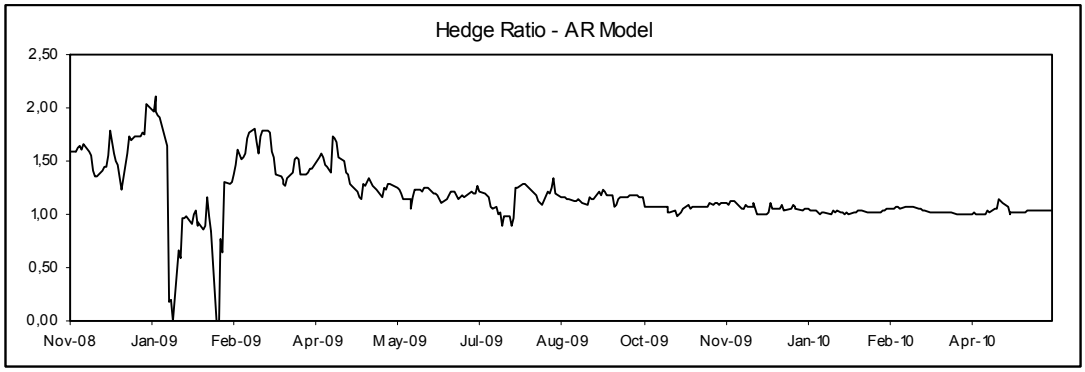

Figure 3: Minimum Variance Hedge Ratio – AR Model

Figures 3 and 4 present the hedge ratio and the hedge effectiveness obtained respectively. As these figures show, both the hedge ratio and the hedge effectiveness picture what happens to volatility of the markets once these volatilities are important parameters in determining these two measures. Concerning the hedge effectiveness, the focus of this work, it must be observed that in the first semester post-crises this measures showed a decrease that became more accentuated in January and February 2009. The hedge effectiveness nears one in the period between June 2009 and the end of the studied period, except in the month of July 2009, when a sharp fall was observed in November 2009 and April 2010 following the volatility of the spot and future markets.

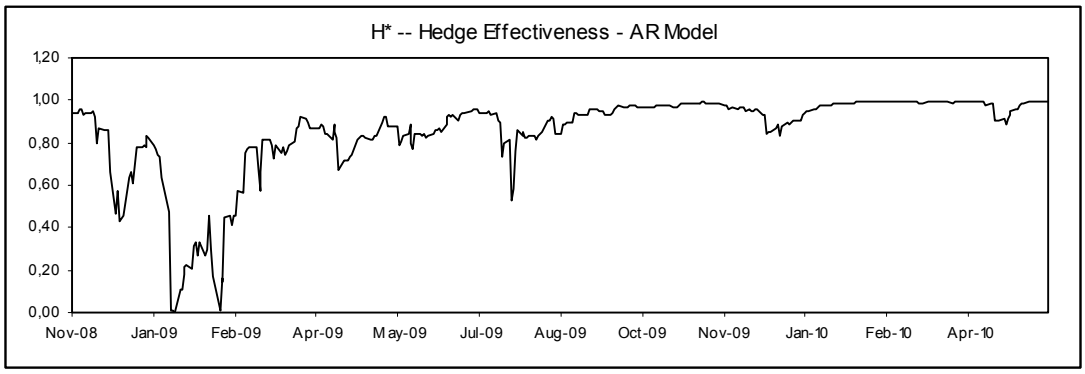

Figure 4: Hedge Effectiveness (H*) – AR Model

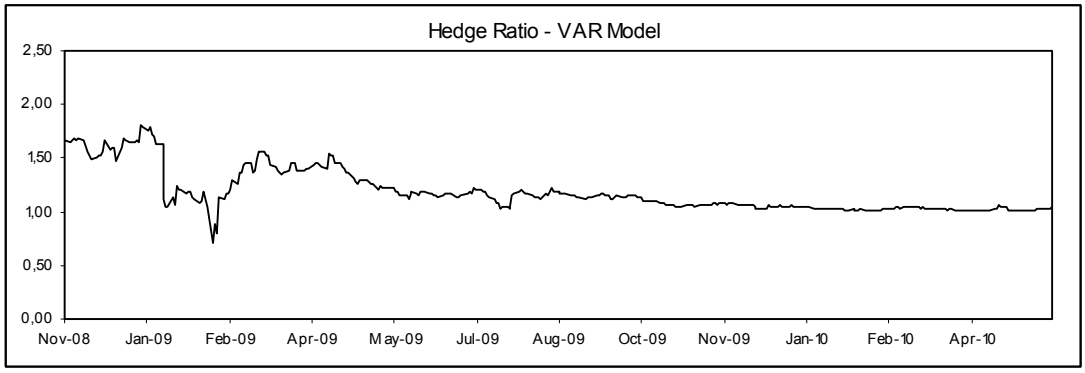

Figure 5: Minimum Variance Hedge Ratio – VAR Model

Figures 5 and 6 show the hedge ratio and hedge effectiveness of the second or the VAR model results. The variance equation of the GARCH model, or VECH diagonal model, of the VAR model presented the same characteristics of the autoregressive model while the degree of freedom of the Student's t-distribution was close to 7. In the third model the VEC model proposed by Kroner and Sultan (1993) showed in equation 17 was selected. And the results were similar to the VAR models and the degree of freedom of the Student's t-distribution was close to 8. Figures 7 and 8 present the hedge ratio and the hedge effectiveness of the VEC model respectively. Figure 8 indicate that the best results were obtained with the VAR model. It is possible to infer, from these plots, that the hedge ratio and the hedge effectiveness prompt reply to the market volatility, specially, the spot market volatility. The crude oil market was much affected by the 2008 global crisis until April 2009, as the plots presented here show. After this period the hedge effectiveness is close to one except around July 2009. It is important to observe that the VEC model presented relevant results as from April 2009 when the worst period of the global crisis for the financial markets occurred. Akin the estimated hedge effectiveness with the AR model, when the VAR and VEC models are used the hedge effectiveness shows a variability directly influenced by the spot and future markets volatility.

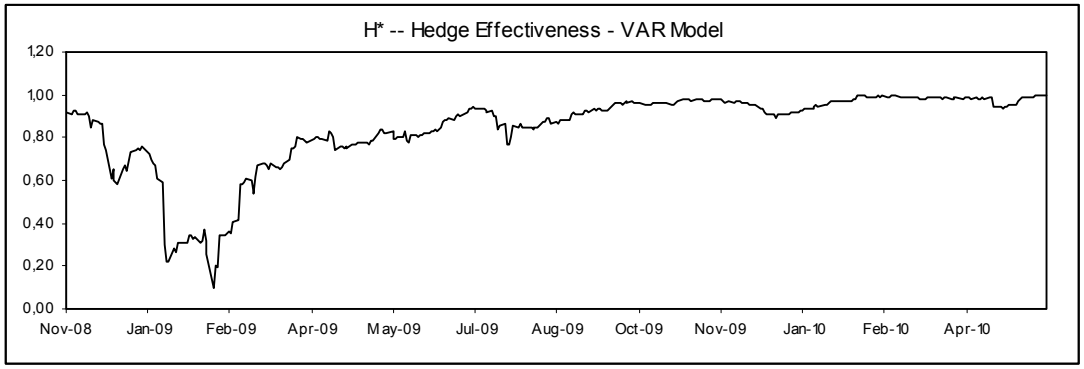

Figure 6: Hedge Effectiveness (H*) – VAR Model

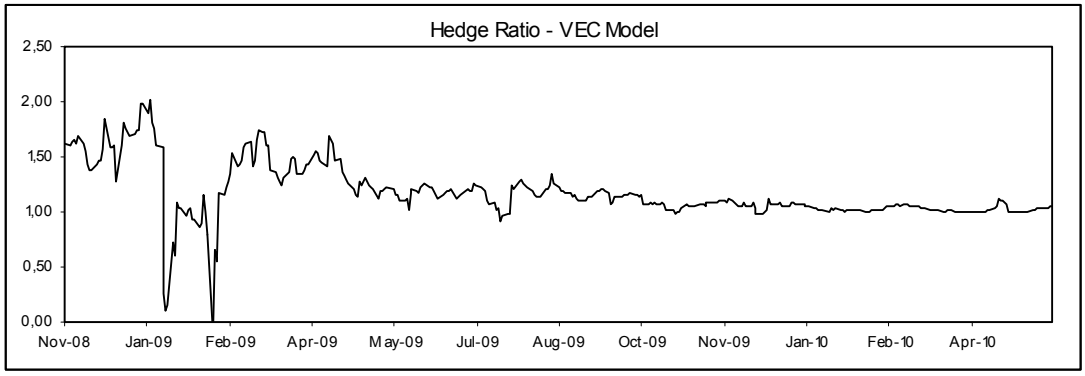

Figure 7: Minimum Variance Hedge Ratio – VEC Model

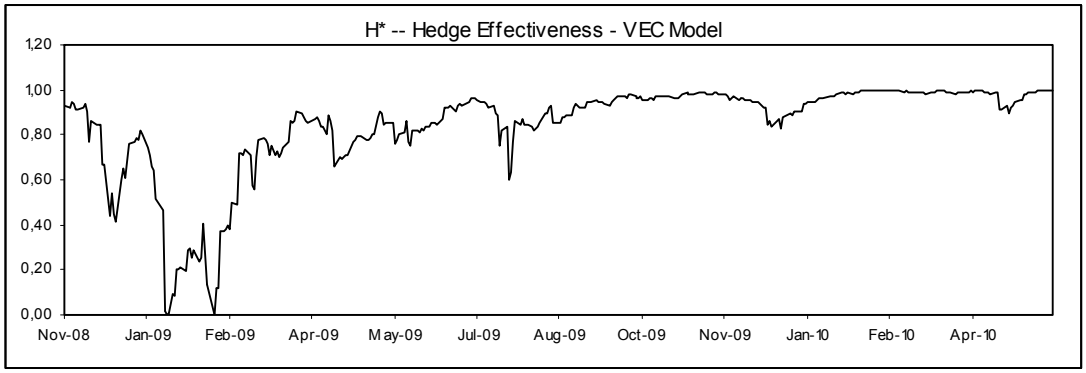

Figure 8: Hedge Effectiveness (H*) – VEC Model

The effectiveness of the hedge average estimated with the AR, VAR and VEC models were around 0.8 in the studied period. It must be highlighted that the VAR model presented the smallest variability, and degree of freedom was in the 7 to 8 interval for the three models selected.

6 Final Remarks

The aim of this paper is to show hedge strategies for the crude oil market, the volatility estimates and how to perform it. This way, classical models were implemented to carry out and compare the minimum variance hedge obtained.

Among several methodologies this work implemented some alternatives using the bivariate autoregressive model, the vector autoregressive and the vector error correction models for the average of the future and spot returns. While a bivariate GARCH model was used for the volatility, or the variance. The GARCH model was used in the VECH diagonal and the BEKK parameterization. Therefore the results presented here were obtained from the selection of these implemented models. The results of the effectiveness hedge indicate that the VECH diagonal and the vector autoregressive model presented the best results. Despite its relevance the results obtained in this study can be generalized considering the sample used here only. Therefore other studies should be conducted using different approaches.

Given the relevance of the theme dealt here, hedge of crude oil prices, it is import point out that the inferences can be enlarged with the utilization of other models, other methodologies or other samples. Bayesian and other classical models may improve hedge ratio estimates.

References

Baillie, R., Myers, R. (1991). Bivariate GARCH Estimation of the Optimal Commodity Futures Hedge. Journal of Applied Econometrics, v. 6, n. 2, pp. 109-124.

Bollerslev, T. (1986). Generalized Autoregressive Conditional Heteroskedasticity. Journal of Econometrics, v.31, n.3, pp. 307-327.

Bollerslev, T. (1990). Modeling the Coherence in Short-Run Nominal Exchange Rates: A Multivariate Generalized Arch Model, The Review of Economics and Statistics, v.72, n.3, pp. 498-505.

Bollerslev, T. (2009). Glossary to ARCH (GARCH), In Bollerslev, T., Russel, J., Watson, M. (Org.). *Volatility and Time Series Econometrics: Essays in Honour of Robert F. Engle*, Oxford University Press, Oxford.

Bollerslev, T., Engle E, R. F., Wooldridge, J. M. (1988). A Capital Asset Pricing Model with Time-Varying Covariances, The Journal of Political Economy, v.96, n. 1, pp. 116-131.

Castelino, M. (1992). Hedge Effectiveness: Basis Risk and Minimum-Variance Hedging, The Journal of Futures Markets. 12, n.2, pp.187-201.

Chance, D.(1998). *An Introduction to Derivatives*. Orlando, FL: Dryden Press.

Chang, C., McAleer, M., Tansuchat, R. (2011). Crude Oil Hedging Strategies Using Dynamic Multivariate GARCH. Kier Discussion Paper Series, Kyoto Institute of Economic Research, Discussion Paper No. 743.

Ding, Z., Engle, R. (2001). Large Scale Conditional Covariance Matrix Modeling, Estimation and Testing. in http://ssrn.com/abstract=1296437. (Jul 2010).

Ederington, L. (1979). The Hedging Performance of the New Futures Market. The Journal of Finance, v. 34, n. 1, pp. 157-170.

Engle, R. (1982). Autoregressive Conditional Heteroskedasticity with Estimates of the Variance of the United Kingdom Inflation. Econometrica, v.50, n.4, pp. 987-1007.

Engle, R. (2002). Dynamic conditional correlation: a simple class of multivariate GARCH models. Journal of Business and Economic Statistics, v.20, n.3, pp. 339-350.

Engle, R., Bollerslev, T. (1986). Modeling the Persistence of Conditional Variances. Econometric Review, v.5, n.1, pp.1-50.

Engle, R., Granger, C. (1987). Cointegration and Error Correction: Representation, Estimation, and Testing. Econometrica, v. 55, n. 2, pp. 251-276.

Engle, R., Kroner, K. (1995). Multivariate Simultaneous Generalized Arch. Econometric Theory. v.11, n.1, pp.122-150.

Engle, R., Lilien, D., Robins, R. (1987). Estimating Time Varying Risk Premia in the Term Structure: The Arch-M Model. Econometrica. v.55, n.2, pp.391-407.

Figlewski, S. (1984). Hedging with Stock Index Futures: Theory and Application in a New Market. Journal of Futures Markets, v. 5, n. 7, pp. 183-199.

Ghosh, A. (1993). Hedging with Stock Index futures: estimation and Forecasting with Error Correction Model. Journal of Futures Markets, v. 13, n. 7, pp. 743-752.

Hannan, E., Quinn, B. (1979). The Determination of the Order of an Autoregression. Journal of the Royal Statistical Society. Series B, v.41, n.2, pp. 190-195.

Hull, J. (2002). *Fundamentals of Futures and Options Markets* (4th ed.). New Jersey: Prentice Hall.

Javali-Naini, A., Manesh, M. (2006). Price Volatility, Hedging, and Variable Risk Premium in the Crude Oil Market. OPEC Review, v. 30, n. 2, pp. 55-70.

Johnson, L. L. (1960). The Theory of Hedging and Speculation in Commodity Futures. The Review of Economic Studies, v.27, n.3, pp. 139-151.

Kroner, F., Sultan, J. (1993). Time Varying Distribution and Dynamic Hedging with Foreign Currency Futures. Journal of Financial and Quantitative Analysis, v. 28, n. 4, pp. 535-551.

Ku, Y., Cheng, H., Cheng, K. (2007). On the Application of the Dynamic Conditional Correlation Model in the Estimating Optimal Time-varying Hedge Ratios. Applied Economics Letter, v. 14, pp. 503-509.

Lautier, D. (2010). Dynamic Hedging Strategies: An Application to the Crude Oil Market. in http://ssrn.com/abstract=1579635. (Mar 2011).

Lee, N. (2010). Quantile Speculative and Hedging Behaviors in Petroleum Futures Markets. International Research Journal of Finance and Economics, Issue 53, pp. 84-99.

Lien, D. (1996). The Effect of the Cointegrating Relationship on Futures Hedging: A Note. Journal of Futures Markets, v. 16, n. 7, pp. 773-780.

Lien, D., Lou, X. (1994). Multiperiod Hedging in the Presence of Conditional Heteroscedasticity. Journal of Futures Markets, v. 14, pp. 927-955.

Lien, D., Wilson, B. (2001). Multiperiod Hedging in Presence of Stochastic Volatility. International Review of Financial Analysis, v. 10, n. 4, pp. 395-406.

Myers, R. (1991). Estimating Time Varying Hedge Ratio on Futures Markets. Journal of Futures Markets, v. 11, pp. 39-53.

Park, T., Switzer, L. (1995). Time-varying Distributions and the Optimal Hedge Ratios for Stock Index Futures. Applied Financial Economics, v. 5, n. 3, pp. 131-137.

Sims, C. (1980). Macroeconomics and Reality. Econometrica, v. 48, n. 1, pp. 1-48.

Stein, J. L. (1961). The Simultaneous Determination of Spot and Futures Prices. American Economic Review, v.51, n.5, pp. 1012-1025.

Tansuchat, R., Chang, C., McAleer, M. (2010). Crude Oil Hedging Strategies Using Dynamic Multivariate GARCH. in http://ssrn.com/abstract=1531187. (Mar 2011).

Working, H. (1953). Futures Trading and Hedging. American Economic Review, v. 43, n. 3, pp. 314-343.