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Competitive Targeted Advertising with Price Discrimination*

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Abstract

This paper investigates the effects of price discrimination by means of targeted advertising in a duopolistic market in which advertising plays two major roles. It transmits relevant information to otherwise uninformed consumers and it acts as a price discrimination device. We look at the firms' optimal advertising and pricing decisions in two settings, namely mass advertising/non-discrimination strategies and targeted advertising/price discrimination strategies. In the case of targeted advertising, we show that firms advertise more in its weak market than in its strong market. The analysis highlights that targeted advertising might constitute a tool to dampen price competition. We show that average prices with mass advertising/non-discrimination can be below those with targeted advertising/price discrimination (regardless of the market segment). We also find that, when advertising costs are not too high, price discrimination by means of targeted advertising can boost industry profits at the expense of consumer and overall welfare. Finally, we show that overall welfare and consumer surplus falls when firms use targeted advertising instead of mass advertising.

1 Introduction

In many markets firms invest in advertising to create awareness for products, prices and special offers. The informative view of advertising claims that the primary role of advertising is to transmit information about (new) products' existence and/or price to otherwise uninformed consumers.[1] Until very recently, the scope of targeted advertising was relatively limited and firms' advertising strategies were mostly tailored to traditional media and mass audiences. However,

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[1] The persuasive view of advertising holds that the main role of advertising is to increase a consumer's willingness to pay for the advertised product and/or change the consumers' tastes. Advertising therefore increases product differentiation and consumers' brand loyalty. For a review of models in the persuasive view of advertising see Bagwell's (2003) comprehensive survey on the *"The Economic Analysis of Advertising."*

the increasing use of the Internet, smartphones and tablets coupled with the development of sophisticated methods for tracking and analysing detailed information about consumers are challenging conventional wisdom regarding retailers' advertising and pricing strategies.[2] This allows advertising content to vary according to the consumer viewing it and increases the scope for targeted advertising and pricing.[3]

Although economists have long been concerned in understanding the profit and welfare effects of price discrimination and advertising separately, little is known to date about the competitive and welfare effects of price discrimination enabled by targeted advertising. In fact, there are interactions between price discrimination and targeted advertising that need to be taken into account.

The main theme of this paper is, therefore, to investigate how the availability of targeted technologies may affect the firms' advertising and pricing choices to different segments of the market and the corresponding level of profits, consumers' surplus and social welfare. Put differently, the paper aims to find an answer to the following question: Who are the winners and the losers when retailers move from mass advertising/uniform pricing to targeted advertising/price discrimination?

With this goal in mind, the paper proposes a static game of duopoly competition with two firms, A and B, offering their goods directly to consumers and investing in advertising to create awareness.[4] We suppose that the set of potential buyers is composed of two distinct segments of equal mass, namely segment a and segment b. Consumers in segment i prefer product i over product j by a degree equal to $\gamma > 0$. This parameter measures the degree of a consumer's preference towards one of the firms (e.g., due to brand preference, location, switching costs). As in Stahl (1994) a potential consumer cannot be an actual buyer unless firms invest in advertising. Thus, by investing in advertising firms endogenously segment the market into captive (partially informed), selective (fully informed) and uninformed customers.

Two advertising strategies are analyzed in this paper. We first consider the case in which retailers use a mass advertising campaign. In this case, they choose an advertising reach to the entire market and all the ads have the same content. Specifically, we assume that all ads quote the same price, thereby retailers follow a uniform pricing policy. We then consider the case where retailers can use a targeted advertising campaign instead. In this case firm i chooses an advertising intensity to the strong group of consumers (segment i) and to the weak group of consumers (segment j). Ads tailored to different segments may quote different prices, implying that firms can engage in price discrimination through the use of targeted advertising.

[2] See, Anderson (2012) for an analysis of the economics of advertising and the internet.

[3] A Wall Street Journal investigation found that the Staples website displays different prices to people after tracking their locations. More than that, Staples appeared to consider the person's distance from a competitor's physical store. If a competitor had stores within 20 miles or so, Staples.com usually showed a discounted price. See this story on http://classroom.wsj.com/cre/2013/03/01/a-complex-web/

[4] It is worth noting that the scope of our model is not restricted to the study of competition between new products. It can also be useful to understand firms' strategic interaction (concerning pricing and advertising choices) when they are competing for *new consumers*, who are not aware of the products/services and their prices.

The model addressed in this paper fits well advertising and pricing policies that are nowadays possible through the use of mobile devices. With the smartphone population growing each day,[5] firms can now connect with consumers at the right place and the right time and with the right message, in a manner that was not previously possible.[6] As an illustrative example consider the case of a consumer who is standing in front of a certain store (e.g. a coffee shop), using a mobile application that incorporates location based advertising.[7] If the coffee shop has access to location based advertising tools, it is able to know that this potential customer is actually standing in front of its door and send him/her a relevant advertising message. Evidently, the consumer may also be tracked by other coffee shops in the neighbourhood. For example, the coffee shop down the street, knowing that the potential customer is closer to its competitor, may send him/her a compelling advertising message with a special shopping offer (discounts or other rewards). If the last ad is compelling enough, it can entice consumers to travel to the more distant store.[8] This kind of advertising/pricing strategies have been recently employed by stores and brands like Starbucks, Taco Bell, Tasti-D-Lite, Macy's and Pepsi.[9]

Similarly, the CEO of the New York City-based startup PlaceIQ said recently that PlaceIQ can be used to lure potential customers away from a competitor's location. Through the use of this technology, Lexus could potentially identify mobile phone users at an Audi dealership and serve them a mobile ad directing them to the nearest Lexus lot.[10,11]

This paper offers new insights to the literature on price discrimination based on customer recognition. First, in contrast to the usual finding that price discrimination reduces all segment prices, we find that moving from mass advertising/non-discrimination can raise average prices. Second, we find that price discrimination by means of targeted advertising does not necessarily

[5] In November 2012 Comscore announced that the US smartphone market had passed the 50% penetration threshold. It also announced that EU5 markets (France, Germany, Italy, Spain and UK) were reporting a 55% smartphone penetration. See http://www.comscore.com. Moreover, according to eMarketer's forecast, more than 70 percent of all cell phones in these countries will be smartphones by the end of 2016.

[6] According to a recent survey, eMarketer estimates that mobile ad spending will increase from $8.4 billion in 2012 to almost $37 billion in 2016. See http://www.forbes.com/sites/chuckjones/2013/01/04/mobile-ad-spending-forecast-to-increase-4x-over-the-next-4-years/

[7] Location-based advertising (also know as hyper-local advertising) is a type of advertising which takes advantage of a consumer's real world position. Using this real world position, location-based advertising is able to deliver relevant ads for products and services that are in close proximity to consumers' current location.

[8] See this story on http://www.acquisio.com/marketing-101/understanding-location-based-advertising/

[9] Taco Bell fast food chain sends special offers to people using a mobile app when they are in the general vicinity of a Taco Bell. The goal is to to drive traffic to its locations, as well as to promote its new product. See http://www.mobilecommercedaily.com/taco-bell-taps-mobile-to-drive-in-store-traffic-for-new-doritos-cool-ranch-tacos

and http://news.cnet.com/8301-1035_3-57579746-94/location-information-to-make-mobile-ads-more-valuable/

[10] See http://adage.com/article/digital/smg-track-mobile-ads-lead-store-visits/240661/

[11] For instance, *CBS News* reports that nowadays it is possible to analyse the recent movements of a mobile device user among stores in a shopping mall, and to predict whether a particular store will be the next destination for the mobile device user. If, say, restaurant A is more likely to be visited than restaurant B, different offers can be sent by the two restaurants to that mobile device user. It also states that mobile device users attending a large venue may be tracked and provided coupons for the vendors that they are more likely to pass based on their recent travel patterns in and around the venue. See http://www.cbsnews.com/8301-505124_162-57342567/amazon-big-brother-patent-knows-where-youll-go/

lead to the classic prisoner dilemma result that usually arises in markets exhibiting best-response asymmetry[12] and full informed consumers. Specifically, we show that, at least when advertising is not too expensive, profits with targeted advertising/price discrimination are above their mass advertising counterparts. Third, when advertising costs are not too high, we show that price discrimination by means of targeted advertising can boost industry profits at the expense of consumer surplus and welfare. Thus, the paper sheds light on the importance of taking into account different forms of market competition when public policy tries to evaluate the profit and welfare effects of price discrimination.

Finally, the paper also contributes to the literature on target advertising by shedding light on the firms' advertising strategies to each segment of the market. An interesting finding of the paper is that retailers may advertise less intensively to their strong market than to their weak market. The reason is that under price discrimination firms may have an incentive to strategically reduce the advertising intensity to be targeted to its strong market as a way to induce the rival to play less aggressively in this segment.

Related literature The paper is mainly related to the literature on informative advertising[13] and to the literature on price discrimination based on customer recognition. The literature on informative targeted advertising is rather recent and it has been evolving along two major lines. The first line studies the effects of targeted advertising technologies on prices and competition when firms can directly target different consumers.[14] The second line of the literature assumes that firms are not able to directly target their messages to different groups of consumers, taking into consideration the intermediary role played by media platforms.[15] As we assume that firms have the ability to directly target their ads to specific segments of consumers (which is frequently the case in online and mobile markets), our paper is more closely related to the first line of the literature, particularly to Iyer, et al. (2005), Galeotti and Moraga-Gonzalez (2008) and Brahim, et al. (2011).

Iyer, et al. (2005) develop a model of targeted advertising in which price discrimination can be employed. Each firm has an exogenous captive segment of consumers, who cannot be induced to switch, and they compete for the remaining selective consumers, who buy from the firm offering the best deal. In their model, when firms decide to advertise their product to a specific segment, all consumers in that segment become fully informed. In contrast, in the present paper, consumers are endogenously segmented into captive or selective due to firms' advertising choices. Even though firms may have some advantage over their competitors due to γ, conditional on being fully informed, in our model, any consumer may be induced to switch,

[12] Corts (1998) refers that the market exhibits best response asymmetry when one firm's "strong" market is the other's "weak" market. A market is designated as "strong" if in comparison to uniform pricing a firm wishes to increase its price there. The market is said to be "weak" if the reverse happens.

[13] A good survey on the economics of advertising can be found on Bagwell (2005).

[14] See for example, Roy (2000), Iyer, et al (2005); Esteban, et al. (2006); Esteban and Hernández (2007); Galeotti and Moraga-González (2008) and Brahim, et al. (2011).

[15] See for example, Bergemann and Bonatti (2011), Chandra (2009); Athey and Gans (2009); Gal-Or and Gal-Or (2005); Gal-Or and Gal-Or (2006); Gal-Or, Gal-Or, May and Spangler (2006), among others.

which is not the case in Iyer, et al. (2005). They conclude that targeted advertising leads to higher profits in relation to mass advertising independent of price discrimination being used.

Galeotti and Moraga-Gonzalez (2008) look at the firms' advertising and pricing strategies in a homogeneous product market where segmentation is based on consumer attributes that are completely unrelated to tastes. The paper compares market outcomes under mass advertising with uniform pricing and targeted advertising with price discrimination. Assuming that one market segment is exogenously more profitable then the other, the paper shows that the possibility of market segmentation may lead to positive profits within an otherwise Bertrand-like setting. In this paper, an increase in advertising costs increases the profitability of market segmentation with firms having unequal sizes. Regarding pricing strategies they find that, with targeted advertising, the price distribution of the less attractive market dominates (in the sense of first order stochastic dominance) the price distribution of the other market. The price distribution under mass advertising is in-between these two distributions (again, in the sense of first order stochastic dominance).

Brahim, et al. (2011) investigate the profit effects of targeted advertising in a Hotelling competition model with no price discrimination. They show that firms advertise more intensively in their strong markets than in their weak markets. They show that targeted advertising can reduce firms' profits. Although in their model price discrimination is not permitted they argue that if price discrimination were possible the negative effects of targeted advertising on profits would still be present.

The paper is also related to the literature on competitive price discrimination with customer recognition.[16] In these models it is generally the case that the market exhibits best-response asymmetry, and profit typically decreases with price discrimination. Thisse and Vives (1988) provide a useful model for the understanding of the profit effects of price discrimination in these markets. They consider two firms located at the extremes of the segment $[0, 1]$, with consumers uniformly distributed in this line segment. Firms can observe the location (or brand preference) of each individual consumer and price accordingly. The strong (close) market for one firm is the weak (distant) market for the other firm. In this setting they show that each consumer's location is a market to be contested, price discrimination intensifies competition, and prices and profits fall down. Consequently, firms face a prisoner's dilemma.[17] Price discrimination based on customer recognition has also been examined by Bester and Petrakis (1996), Chen (1997), Villas-Boas (1999), Fudenberg and Tirole (2000), Taylor (2003), Esteves (2010) and Gehrig, et al. (2011), (2012).[18] In all of these approaches consumers are perfectly informed about products and

[16] Chen (2005), Fudenberg and Villas-Boas (2007) and Esteves (2009b) present literature surveys on price discrimination with customer recognition.

[17] Esteves (2009b) extends the Thisse and Vives model to a two-dimensional differentiation model and shows that price discrimination might not necessarily lead to the prisoner's dilemma result. This happens when firms observe the location of consumers in the less differentiation dimension and price discriminate accordingly while they remain ignorant about their location in the more differentiated dimension.

[18] For other recent papers on price discrimination and customer recognition see also Chen and Pearcy (2010), Esteves and Vasconcelos (2012), Esteves and Regiani (2012), Ghose and Huang (2006), Ouksel and Eruysal (2011), Shy and Stenbacka (2012).

prices, there is no role for advertising and profits fall with price discrimination. Esteves (2009a) departs from this hypothesis by assuming that advertising is needed to create awareness for a firm product. She investigates the effects of price discrimination in an informative advertising model. There are important differences between Esteves (2009a) and the present paper with regard to the timing of the game and the model assumptions. Esteves (2009a) proposes a two-period model with two firms launching an homogeneous product. In period one firms choose a mass advertising intensity and a uniform price to the entire market. Since advertising endogenously segments the market into captive and price sensitive consumers, by observing the consumers' previous shopping decisions, in the second period, firms can distinguish a selective from a captive consumer and price accordingly. She shows that only one of the firms will have information to employ price discrimination. As a result of that, she shows that all firms might become better off with price discrimination.

The rest of this paper is organized as follows. Section 2 describes the main ingredients of the model. Section 3 analyses the benchmark case in which firms employ a mass advertising technology that forces them to adopt a uniform pricing policy. Section 4 analyses the equilibrium advertising and pricing strategies when firms can price discriminate through the use of perfect targeted advertising. Section 5 stresses the competitive effects of targeted advertising with price discrimination. Section 6 focus on the impact of targeted advertising on social welfare. Finally, Section 7 concludes and an appendix collects the proofs that were omitted from the text.

2 Model Assumptions

Consider a market with two firms, $i = A, B$. Each firm is launching a new good, produced at a constant marginal cost, which is assumed to be zero without loss of generality.[19] On the demand side, there is a unit mass of consumers, each of whom wishes to buy a single unit of either good A or B. Consumers have a common reservation price v and they are initially uninformed about the existence and the price of the good. As in Stahl (1994) a potential consumer cannot be an actual buyer unless firms invest in advertising.[20,21] We suppose that the set of potential buyers is composed of two distinct segments of equal mass, segment a and segment b. Consumers in segment i prefer product i over product j by a degree equal to $\gamma > 0$. As in Shilony (1977), Raju, et al. (1990) and Esteves (2010), γ can be defined as a measure of the degree of a consumer's preference towards his favorite product.[22] It can also be conceived as the minimum difference between the prices of the two competing products necessary to induce consumers to buy the

[19] The assumption of zero marginal costs can be relaxed without altering the basic nature of the results derived throughout the model.

[20] Implicitly it is assumed that search costs are prohibitively high for new products.

[21] Other papers addressing informative advertising (with mass advertising technologies) are, for example, Butters (1977), Grossman and Shapiro (1984), Stegeman (1991).

[22] Even though the paper considers that the market is segmented according to brand preference, the model also accommodates other interpretations for the parameter γ, such as search costs, transportation costs or switching costs. For example, in a location model like Shilony (1977) consumers can purchase costlessly from the neigbourhood firm but they incur a transport cost if they go to the more distant firm.

least preferred product. In other words, consumers in segment i buy product i as long as its price is not undercut by more than γ by firm j. As a result, each firm has a strong and a weak segment of consumers. For firm A, for instance, segment a is its strong segment, while segment b is its weak segment.

Although consumers are endowed with preferences over brands, without advertising they have no information about which products exist, their characteristics and price. By conveying information to otherwise uninformed consumers, advertising is a key element in generating demand for a product.

The game is static and proceeds as follows. Firms choose advertising intensities and prices *simultaneously* and non-cooperatively. The advertising messages of each firm contain truthful[23] and complete information about the existence and the price of its product. After firms have sent their ads independently, the potential demand of each firm is made of a group of captive customers and a group of selective customers.[24] Selective consumers receive ads from both firms; they buy from the firm offering them the highest surplus. Captive consumers receive ads from only one of the firms. Therefore, they purchase from the only known firm as long as the full price[25] is below v.

Advertising is a costly activity for firms. The advertising technology is exogenously given and it is the same for both firms. Two advertising technologies will be studied throughout this paper, a mass advertising technology (Section 3) and a targeted advertising technology (Section 4).

When firms use a mass advertising technology, they send the same message to all consumers in the market and so they are forced to follow a uniform pricing policy. All the ads quote the same price. In this case, the problem of firm i consists in choosing an optimal advertising reach, ϕ_i, and the corresponding uniform pricing strategy p_i, $i = A, B$. The cost of reaching a fraction ϕ of consumers is given by the function $A(\phi) = \lambda\eta(\phi)$. Following the literature on informative advertising (e.g. Butters (1977) and Tirole (1988)), it is assumed that the cost of reaching consumers increases at an increasing rate. This can be formally written as $A_\phi > 0$ and $A_{\phi\phi} \geq 0$.[26] The latter condition means that it is increasingly more expensive to inform an additional customer or, likewise, to reach a higher proportion of customers. It is also assumed that there are no fixed costs in advertising, i.e. $A(0) = 0$. The quadratic technology proposed in Tirole (1988) is not based upon an underlying technology of message production. However, it has the advantage of being extremely simple to manipulate algebraically. It is given by $A(\phi) = \lambda\eta(\phi) = \lambda\phi^2$. Since in the present model there is a large number of buyers, normalized to one, λ can be identified with the cost per ad. In what follows, whenever a functional form is needed, we will use the quadratic technology.

[23] In the USA, for instance, the FTC prohibits advertisers from making false and deceptive statements about their products (see www.ftc.gov/bcp/conline/pubs/buspubs/ad-faqs.htm).

[24] The remaining consumers receive no ad from either firm. They are uninformed and excluded from the market.

[25] Consider, for instance, the case of a consumer in segment a. The full price of good A coincides with the price quoted by firm A, whereas the full price of good B is given by the sum of γ and the price quoted by firm B. The same analysis applies, *mutatis mutandis* for consumers in segment b.

[26] Subscripts denote partial derivatives.

When firms are able to use targeted advertising, they can target ads to specific segments of the market. This amounts to say that each firm may send two types of ads: ads targeted to the strong segment of consumers (those who prefer its own product) and ads targeted to the weak group of consumers (those who prefer the rival's product). Within each segment, messages are randomly distributed among consumers. Since ads targeted to different segments of the market can quote different prices, targeted advertising can also be used as an effective tool for price discrimination. In this set-up, the problem of firms consists in choosing an optimal advertising reach and an optimal pricing strategy to each segment of the market. The advertising intensities ϕ_i^o and ϕ_i^r respectively denote the advertising reach of firm i in its own (strong) market and firm i's advertising reach targeted to the rival's market (firm i's weak market). Ads targeted to each segment will have different prices, respectively given by p_i^o and p_i^r, $i = A, B$. In line with Iyer, et al. (2005), Galeotti and Moraga-Gonzalez (2008) and Brahim, et al. (2011), we assume that (i) the advertising cost function is additive separable in ϕ_i^o and ϕ_i^r and, (ii) the cost of reaching a fraction ϕ_i^k of consumers, $k = \{o, r\}$, is given by the quadratic function $A(\phi_i^k) = \lambda \left(\phi_i^k\right)^2$.

3 Mass advertising

This section investigates optimal pricing and advertising strategies when firms use a mass advertising technology and follow a uniform pricing policy. There are two components to firm i's strategy: its advertising level (denoted by ϕ_i) and its price (denoted by p_i).

After firms have sent their ads independently, a proportion ϕ_i and ϕ_j of customers is reached by firm i and j's advertising, respectively. In each segment of the market firm i has a fraction $\phi_i \left(1 - \phi_j\right)$ of captive customers, who are only aware of its product and a fraction $\phi_i \phi_j$ of selective consumers, who are fully informed about the existence and the price of both products.

Consider first the case of selective consumers. In segment a they compare the net utility of purchasing good A at price p_A, $v - p_A$, with the net utility of purchasing good B at price p_B, $v - \gamma - p_B$. Considering $p_i \leq v - \gamma$, $i = A, B$, we have that a selective consumer in segment a buys good A if and only if $p_A - \gamma \leq p_B$. Analogously, a selective consumer in segment b buys good A if and only if $p_A + \gamma < p_B$. Similar reasoning is applied to obtain the conditions under which different types of selective consumers buy good B.

As far as concerns the behavior of captive consumers, a captive consumer to firm A in segment a, buys its good if $p_A \leq v$. If instead the captive consumer belongs to segment b, he only buys good A when $p_A \leq v - \gamma$.

Firm i's demand when firms use a mass advertising technology, D_i, is then equal to:

$$D_i\left(p_i, p_j\right) = \begin{cases} 0 & if \quad p_i > v \\ \frac{1}{2}\phi_i\left(1 - \phi_j\right) & if \quad p_j + \gamma < v - \gamma < p_i \leq v \\ \frac{1}{2}\phi_i\left(1 - \phi_j\right) + \frac{1}{2}\phi_i\phi_j & if \quad v - \gamma < p_j + \gamma < p_i \leq v \\ \phi_i\left(1 - \phi_j\right) & if \quad p_j + \gamma < p_i \leq v - \gamma \\ \phi_i\left(1 - \phi_j\right) + \frac{1}{2}\phi_i\phi_j & if \quad p_j - \gamma \leq p_i < p_j + \gamma \leq v - \gamma \\ \phi_i\left(1 - \phi_j\right) + \phi_i\phi_j & if \quad 0 \leq p_i < p_j - \gamma \end{cases}$$

Firm i's profits is equal to

$$\pi_i = p_i D_i \left(p_i, p_j\right) - A\left(\phi_i\right). \tag{1}$$

3.1 Equilibrium Analysis

In this section, we study the equilibrium price and advertising strategies. Our analysis is focused on the symmetric Nash equilibrium. Given the rival's strategies, ϕ_j and p_j, the problem of firm i consists of choosing the advertising intensity, ϕ_i, and the pricing policy, p_i that maximize its profit.

Proposition 1 *(i) The prices (v, v) and the advertising intensities $\left(\frac{v}{4\lambda}, \frac{v}{4\lambda}\right)$ constitute a symmetric Nash Equilibrium in pure strategies when $v \leq 2\gamma$ and $\lambda > \frac{v}{4}$.*

(ii) The prices $(v-\gamma, v-\gamma)$ and the advertising intensities $\left(\frac{2(v-\gamma)}{v-\gamma+4\lambda}, \frac{2(v-\gamma)}{v-\gamma+4\lambda}\right)$ constitute a symmetric Nash Equilibrium in pure strategies when $v > 2\gamma$ and $\lambda > \max\left\{\frac{v(v-\gamma)}{4(v-2\gamma)}, \frac{(v-2\gamma)(v-\gamma)}{4\gamma}\right\}$.

(iii) When $v > 2\gamma$ and $\lambda < \max\left\{\frac{v(v-\gamma)}{4(v-2\gamma)}, \frac{(v-2\gamma)(v-\gamma)}{4\gamma}\right\}$ there is no symmetric pure strategy equilibrium in prices.

Proof. See the Appendix.

From Proposition 1 it follows that when v is sufficiently large, and λ is small enough, there is no symmetric price equilibrium in pure strategies. The intuition behind this result is the following. If advertising is cheap enough (small λ), selective consumers have a non-negligible impact on firms' profits. As long as v is sufficiently high, the existence of a positive fraction of selective consumers with a preference for the rival firm creates a tension between the firm's incentives to price low in order to attract consumers belonging to its weak market and the firm's incentives to price high in order to extract rents from consumers belonging to its strong market. In equilibrium each firm follows a mixed pricing strategy as an attempt to prevent the rival from systematically predicting its price, which in turn makes undercutting less likely.

Lemma 1. *When $v > 2\gamma$ and $\lambda < \max\left\{\frac{v(v-\gamma)}{4(v-2\gamma)}, \frac{(v-2\gamma)(v-\gamma)}{4\gamma}\right\}$ firms will have incentives to compete for all the selective consumers in the market.*

Lemma 1 states that firms compete for all the selective consumers when $v > 2\gamma$ and $\lambda < \max\left\{\frac{v(v-\gamma)}{4(v-2\gamma)}, \frac{(v-2\gamma)(v-\gamma)}{4\gamma}\right\}$. When case *(i)* in Proposition 1 occurs, both firms quote price v, choosing to be a monopolist in its strong market. When case *(ii)* in Proposition 1 holds, both firms quote price $v-\gamma$. Firms serve not only the consumers in its strong market but also the captive consumers with a preference for the rival's brand. However, in this case, firms choose not to compete for the weak selective consumers who always buy their most preferred brand. In contrast, if the conditions in Lemma 1 hold each firm will have incentives to serve not only the selective consumers in its strong market but also the selective consumers in its weak market.

As we are interested in studying the competitive effects of targeted advertising and price discrimination in the remainder of the paper we will concentrate on the range of prices for which firms compete for all consumers in the market, i.e. $p \leq v - \gamma$.[27]

Proposition 2 below characterizes the symmetric mixed strategy equilibrium when Lemma 1 holds, pointing out the conditions under which such equilibrium exists. Suppose that firm j randomly selects a price from the c.d.f (cumulative distribution function), $F_j(p)$. In a symmetric mixed strategy equilibrium, both firms follow the same pricing strategy, thus, for the sake of simplicity write $F_i(p) = F_j(p) = F(p)$.

Regarding the group of selective consumers, when firm i charges price p, three events are relevant. First, if $p_j > p + \gamma$ firm i captures the whole group of selective consumers in the market. This occurs with probability $[1 - F(p + \gamma)]$ and yields a total revenue equal to $p\left[\phi_i\left(1 - \phi_j\right) + \phi_i\phi_j\right]$. Second, firm i captures no selective consumer if $p_j < p - \gamma$. This happens with probability $F(p - \gamma)$. In this case firm i revenue comes only from its captive consumers, being equal to $p\phi_i\left(1 - \phi_j\right)$. Finally, each firm serves its group of strong selective consumers if $p - \gamma < p_j < p + \gamma$. This event occurs with probability $[F(p + \gamma) - F(p - \gamma)]$ and yields a total revenue equal to $p\left[\phi_i\left(1 - \phi_j\right) + \frac{1}{2}\phi_i\phi_j\right]$. In that case, for a given ϕ_i and ϕ_j, firm i's expected profit, can be written as follows:

$$E\pi_i = p\phi_i\left(1 - \phi_j\right) + p\phi_i\phi_j\left[1 - \frac{1}{2}F(p + \gamma) - \frac{1}{2}F(p - \gamma)\right] - A(\phi_i).$$

In a mixed strategy Nash equilibrium, for a given ϕ_i and ϕ_j, any price chosen from a firm's price support should generate the same expected profit. Therefore:

$$p\phi_i\left(1 - \phi_j\right) + p\phi_i\phi_j\left[1 - \frac{1}{2}F_j(p + \gamma) - \frac{1}{2}F_j(p - \gamma)\right] - A(\phi_i) = k^m - A(\phi_i).$$

Or, equivalently:

$$p\phi_i\left(1 - \phi_j\right) + p\phi_i\phi_j\left[1 - \frac{1}{2}F_j(p + \gamma) - \frac{1}{2}F_j(p - \gamma)\right] = k^m.$$

The next proposition provides a complete characterization of the Nash equilibrium in the benchmark case in which firms use mass advertising technologies and cannot employ price discrimination strategies.

Proposition 2. *Given the conditions in Lemma 1, in the benchmark case where firms follow a mass advertising campaign with no price discrimination:*

(i) each firm's price is randomly chosen from the c.d.f

$$F^m(p) = \left\{ \begin{array}{ccc} 0 & if & p < p_{\min} \\ 1 - \frac{2}{(\phi^m)^2}\left[\frac{k^m}{p+\gamma} - \phi^m\left(1 - \phi^m\right)\right] & if & p_{\min} \leqq p \leqq p_{\max} - \gamma \\ 2 - \frac{2}{(\phi^m)^2}\left[\frac{k^m}{p-\gamma} - \phi^m\left(1 - \phi^m\right)\right] & if & p_{\max} - \gamma \leqq p < p_{\max} \\ 1 & if & p \geqq p_{\max} \end{array} \right\}$$

[27] The equilibrium results arising when $p > v - \gamma$ are available from the authors upon request.

with

$$p_{\max} = \frac{2k^m}{\phi^m (2-\phi^m)} + \gamma,$$
$$p_{\min} = p_{\max} - 2\gamma,$$

and

$$k^m = \frac{\gamma}{2} (2-\phi^m)^2 \left(1 + \sqrt{1 + (\phi^m)^2 (2-\phi^m)^{-2}}\right).$$

(ii) Each firm chooses an advertising reach $\phi^m \in [0,1]$, implicitly given by:

$$\frac{1}{2} (p_{\max} - \gamma) (2-\phi^m) = A_\phi (\phi^m).$$

(iii) Each firm earns an overall expected profit of

$$E\pi^m = \phi^m A_\phi (\phi^m) - A(\phi^m). \tag{2}$$

Proof.

See the Appendix.

Remark 1. *For the quadratic technology, firms compete for all consumers in the market when:*

$$(\text{A1}) \quad : \quad v > \left(\sqrt{2} + 3\right) \gamma;$$

$$(\text{A2}) \quad : \quad \frac{\gamma\left(\sqrt{2}+1\right)}{4} \leq \lambda \leq \frac{(v-3\gamma)(v-\gamma)}{8\gamma}.$$

Proof. See the Proof of Corollary 1.

Since we are interested in studying situations in which firms compete for all consumers in the market, in the remainder of the paper we shall assume that, *for the quadratic technology, where* $A(\phi) = \lambda\phi^2$ the follow assumptiom holds:

Assumption 1. *For the quadratic technology, we assume conditions (A1) and (A2) hold.*

Under Assumption 1, when the advertising cost function is the quadratic one there is no Nash Equilibrium in pure strategies and the mixed strategy equilibrium in prices is described in Corollary 1.

Corollary 1. *When assumptions (A1) and (A2) hold and $A(\phi) = \lambda\phi^2$ each firm chooses an advertising reach $\phi^m \in [0,1]$, equal to*

$$\phi^m = \frac{2\left(\gamma^2 + 8\lambda\gamma\right)^{1/2}}{\left(\gamma^2 + 8\lambda\gamma\right)^{1/2} + 4\lambda}, \tag{3}$$

from which we obtain:

$$p_{\max} = \left(\gamma^2 + 8\lambda\gamma\right)^{1/2} + \gamma$$
$$p_{\min} = p_{\max} - 2\gamma$$

and an overall expected profit equal to:

$$E\pi^m = \frac{4\lambda\left(\gamma^2 + 8\lambda\gamma\right)}{\left[\left(\gamma^2 + 8\lambda\gamma\right)^{1/2} + 4\lambda\right]^2}.$$

Proof. See the Appendix.

Proposition 3. *Each firm serves its group of strong selective customers with probability* $q^m \in [0,1]$ *which is equal to*

$$q^m = 1 - \frac{8\left(k^m\right)^2}{\left(\phi^m\right)^4}\left[\ln\left(\frac{\left(p_{\max} - \gamma\right)^2}{p_{\max}\left(p_{\max} - 2\gamma\right)}\right) - \frac{1}{\left(p_{\max} - \gamma\right)^2}\right].$$

For the quadratic advertising technology:

$$q^m = 1 - 32\lambda^2\left[\ln\left(\frac{\gamma}{8\lambda} + 1\right) - \frac{1}{\gamma\left(8\lambda + \gamma\right)}\right].$$

Proof. See the Appendix.

4 Targeted advertising and price discrimination

This section investigates the firms' advertising and pricing decisions when they have the possibility to target ads with different price content to specific segments of the market. Now firm i's strategy is to choose the levels of advertising to be targeted to its own strong market and to the rival's (firm i's weak) market, respectively given by ϕ_i^o and ϕ_i^r. The prices quoted in each type of ads are respectively given by p_i^o and p_i^r. Recall that o stands for firm i's own strong market, while r stands for the rival's strong market (i.e. firm i's weak market).

The firms' targeting ability is assumed to be perfect, i.e $\Pr(\text{fall in } i \mid \text{targeted to } i) = 1$ while $\Pr(\text{fall in } i \mid \text{targeted to } j) = 0$. This means that there is no leakage of ads between the groups.[28] Type$-a$ consumers are only aware of p_A^o and p_B^r, since the remaining prices namely p_A^r and p_B^o are quoted in the ads targeted to type$-b$ consumers.

Again, in each segment of the market, potential consumers can be divided into captive, selective and uninformed consumers. In segment i, after firms have sent their ads independently, a proportion ϕ_i^o and ϕ_j^r of customers is reached, respectively, by firm i and j advertising. Thus, firm i's demand in this market segment is made of a group of captive (locked-in) customers, $\phi_i^o\left(1 - \phi_j^r\right)$, and a group of selective customers, $\phi_i^o\phi_j^r$, $i, j = A, B$ with $i \neq j$.

Look first at firm A overall demand. Firm $A's$ sales in segment a (at price $p_A^o \leq v$) are equal to:

$$D_A^o = \left[\frac{1}{2}\phi_A^o\left(1 - \phi_B^r\right) + \frac{1}{2}\phi_A^o\phi_B^r \Pr(p_A^o < p_B^r + \gamma)\right]$$

[28] Galeotti and Moraga-Gonzalez (2008) and Brahim, et al. (2011) also assume that there is no leakage of ads between segments.

and firm $A's$ sales in segment b (at price $p_A^r \leq v - \gamma$) are equal to:

$$D_A^r = \left[\frac{1}{2}\phi_A^r (1 - \phi_B^o) + \frac{1}{2}\phi_A^r \phi_B^o \Pr(p_A^r + \gamma < p_B^o)\right].$$

The same analysis applies to obtain firm B's demand D_B^r in segment a (at price $p_B^r \leq v - \gamma$) and firm B's sales D_B^o in segment b (at price $p_B^o \leq v$).

As there is no leakage, the two segments are totally independent. For a given strategy of the rival firm, firm i's expected profit conditional on ads and prices targeted to segment $k = o, r$, is equal to

$$E\pi_i^k = p_i^k D_i^k - A\left(\phi_i^k\right).$$

Look first at segment a. Captive consumers to firm A are only aware of p_A^o, while captive consumers to firm B are only aware of p_B^r. The selective consumers in segment a know both p_A^o and p_B^r. Firm i's expected profit in its own strong market, denoted $E\pi_i^o$, is given by:

$$E\pi_i^o = p_i^o \frac{\phi_i^o}{2}\left[(1 - \phi_j^r) + \phi_j^r \Pr(p_i^o < p_j^r + \gamma)\right] - A(\phi_i^o). \tag{4}$$

Similarly, firm i's expected profit in the rival's market (i.e. firm i's weak market), denoted $E\pi_i^r$, is given by:

$$E\pi_i^r = p_i^r \frac{\phi_i^r}{2}\left[(1 - \phi_j^o) + \phi_j^o \Pr\left(p_i^r + \gamma < p_j^o\right)\right] - A(\phi_i^r). \tag{5}$$

In each segment k, given firm j's advertising and pricing strategy, firm i chooses the advertising level (ϕ_i^k) and the price (p_i^k) in order to maximize its expected profit defined by (4), in the case of its strong market, and by (5) in the case of its weak market.

Proposition 4. *There is no symmetric price equilibrium in pure strategies.*

Proof. See the Appendix.

Proposition 5. *When target is perfect and price discrimination is permitted there is a symmetric Nash equilibrium in which:*

(i) Regarding the strong market, each firm i, $i = A, B$ chooses a price randomly from the distribution $F_i^o(p)$ given by

$$F_i^o(p) = \begin{cases} 0 & if \quad p \leq p_{j\min}^r + \gamma \\ \frac{1}{\phi_i^{o*}}\left[1 - \frac{(v-\gamma)(1-\phi_i^{o*})}{p-\gamma}\right] & if \quad p_{j\min}^r + \gamma \leq p \leq v \\ 1 & if \quad p \geq v \end{cases}$$

where $p_{j\min}^{r*} = (v - \gamma)(1 - \phi_i^{o*})$. The advertising reach targeted to the strong market ϕ_i^{o*} is implicitly given by

$$\frac{1}{2}v - \phi_i^{o*}(v - \gamma) = A_{\phi_i^o}\left(\phi_i^{o*}\right) \tag{6}$$

or equivalently by,

$$p_{i\min}^{o*} - \frac{1}{2}v = A_{\phi_i^o}\left(\phi_i^{o*}\right)$$

with $A_{\phi_i^o}(0) < \frac{1}{2}v$. *The equilibrium profit in this market is:*

$$E\pi_i^{*o} = \phi_i^{o*} A_{\phi_i^{o*}}(\phi_i^{o*}) + \frac{1}{2}(\phi_i^{o*})^2 (v-\gamma) - A(\phi_A^{o*}). \tag{7}$$

(ii) Regarding the weak market, each firm chooses a price randomly from the distribution $F_i^r(p)$ *given by*

$$F_i^r(p) = \begin{cases} 0 & if \quad p \le p_{j\min}^r \\ \frac{1}{\phi_i^{r*}}\left[1 - \frac{v(1-\phi_j^{o*})+\gamma\phi_j^{o*}}{p+\gamma}\right] & if \quad p_{j\min}^r \le p \le v-\gamma \\ 1 & if \quad p \ge v-\gamma \end{cases}$$

The advertising reach targeted to the weak market ϕ_i^{r*} *is implicitly given by*

$$\frac{1}{2}(v-\gamma) - \frac{1}{2}\phi_j^{o*}(v-\gamma) = A_{\phi_i^{r*}}(\phi_i^{r*}), \tag{8}$$

or, equivalently by,

$$\frac{1}{2}p_{j\min}^r = A_{\phi_i^{r*}}\left(\phi_i^{r^*}\right), \tag{9}$$

where ϕ_j^{o*} *solves condition (6) and* $A_{\phi_i^r}(0) < \frac{1}{2}\left(1-\phi_j^{o*}\right)(v-\gamma)$. *The corresponding equilibrium profit in this market is:*

$$E\pi_i^{*r} = \phi_i^{*r} A_{\phi_i^{*r}}(\phi_i^{*r}) - A(\phi_i^{*r}). \tag{10}$$

Proof.
See the Appendix.

Corollary 2. *When target is perfect and firms use the quadratic technology with* $A(\phi_i^k) = \lambda\left(\phi_i^k\right)^2$, *under (A1) and (A2), for* $\lambda \ge \frac{(5v-9\gamma)^{1/2}(v-\gamma)^{1/2}-(v-\gamma)}{8}$, *there is a symmetric Nash equilibrium in which the firms' equilibrium advertising intensity targeted to their own strong market is given by:*

$$\phi^{o*} = \frac{v}{4\lambda + 2(v-\gamma)},$$

yielding equilibrium profits equal to

$$E\pi^{o*} = \frac{v^2}{8(v+2\lambda-\gamma)}.$$

For the weak market segment each firm chooses an advertising level ϕ^{r*} *given by*

$$\phi^{r*} = \frac{v-\gamma}{4\lambda}\frac{v+4\lambda-2\gamma}{4\lambda+2(v-\gamma)}, \tag{11}$$

and firm i'*s expected profit in this market segment is equal to:*

$$E\pi_i^{r*} = \lambda\left(\frac{v-\gamma}{8\lambda}\frac{v+4\lambda-2\gamma}{v+2\lambda-\gamma}\right)^2.$$

Proof. See the Appendix.

Assumption 2. *In order to guarantee that the conditions in Corollary 2 are met, in the remainder of the analysis we shall assume that* conditions *(A1) and (A2') hold, where (A2') is defined as follows:*

$$\text{(A2'):}\ \max\left\{\frac{(5v-9\gamma)^{1/2}(v-\gamma)^{1/2}-(v-\gamma)}{8},\frac{\gamma\left(\sqrt{2}+1\right)}{4}\right\}\leq\lambda\leq\frac{(v-3\gamma)(v-\gamma)}{8\gamma}.$$

Corollary 3. *When firms use targeted advertising as a price discrimination device, under Assumptions 1 and 2, each firm advertises less to its own (strong) market than to the rival's market, i.e.,* $\phi^{o*}<\phi^{r*}$.

Proof. See the Appendix.

To understand the practical implications of the previous result, consider, for instance, the case of a coffee shop (or any other store) that is running a location based targeted advertising campaign in a specific predefined area (geofence) as a way to create awareness for a new offer and drive consumers to take actions locally. The model suggests that this coffee shop will send more ads to consumers located closer to the competitor shop (weak market) than to consumers who are located near its shop (strong market).

Corollary 4. *From Proposition 5 and condition (A1) in Assumption 1, it follows that* $F_i^r(v-\gamma)=\frac{\phi^{o*}}{\phi^{r*}}\left(\frac{v-\gamma}{v}\right)$. *As* $\phi^{o*}<\phi^{r*}$, $F_i^r(v-\gamma)<1$. *Thus,* F_i^r *has a mass point at* $(v-\gamma)$ *equal to:*

$$m^r=1-\frac{\phi^{o*}}{\phi^{r*}}\left(\frac{v-\gamma}{v}\right).$$

Proof. See the Appendix.

In light of Corollary 4, it can be said that each firm uses a "Hi-Lo" pricing strategy in the rival's strong market. To squeeze more surplus from its weak captive customers, it charges the highest price $v-\gamma$, with probability m^r.[29] However, in order to poach the selective customers in its weak market, it occasionally quotes a low price. From Corollary 4 we can observe that m^r is increasing in ϕ^{r*}, and decreasing in ϕ^{o*}. When, say, firm i decreases the advertising reach in its strong market, the size of firm j's group of weak captive customers increases, because less consumers become aware of both firms. Thus, by decreasing the advertising effort in its strong market, firm i induces firm j to play less aggressively in this segment of the market.

Corollary 3 states that firms advertise more in its weak market than in its strong market. This finding is in contrast with Galeotti and Moraga-González (2008); Iyer, et al. (2005) and Brahim, et al. (2011) who find the reverse. In our framework each firm has an incentive to strategically reduce the advertising intensity to be targeted to its strong market as a way to induce the rival to play less aggressively in this market. To better understand the intuition

[29] Note that for the quadratic advertising cost function $F_i^r(v-\gamma)=\frac{4\lambda}{4\lambda+v-2\gamma}$. As $v>2\gamma$, it is always true that $F_i^r(v-\gamma)<1$. Therefore, $m^r=\frac{v-2\gamma}{4\lambda+v-2\gamma}$

behind this advertising strategy, it is useful to take into account the firms' equilibrium conditions regarding advertising decisions. Consider the case of segment i. In this segment the level of ϕ_i^{o*} is implicitly given by

$$\frac{1}{2}v - \phi_i^{o*}(v-\gamma) = A_{\phi_i^{o*}}(\phi_i^{o*}). \tag{12}$$

From (12) it follows that firm i's advertising reach in its strong market, ϕ_i^o, is independent of firm $j's$ advertising intensity in this market (ϕ_j^r). In contrast, firm j's equilibrium advertising decision in segment i (its weak market) is implicitly given by the following condition:

$$\frac{1}{2}(v-\gamma) - \frac{1}{2}\phi_i^o(v-\gamma) = A_{\phi_j^r}(\phi_j^{r*}). \tag{13}$$

The previous condition highlights the strategic substitutability between ϕ_j^r and ϕ_i^o. The lower is ϕ_i^o the higher is ϕ_j^r. Thus when firm i decreases ϕ^o, it induces firm j to increase ϕ_j^r due to the strategic substitutability property. Additionally, we have seen that a decrease in ϕ_i^o induces firm j to play less aggressively in prices in firm i's strong market. When ϕ_i^o decreases, both $p_{j\min}^r$ and the mass point m^r increase. Thus, by reducing ϕ_i^o, firm i increases the probability of firm j charging $v-\gamma$, and serving only its group of captive consumers in segment i. Although firm i ends up with less captive consumers than firm j in its strong market, the increase in m^r, increases the probability of firm i also serving all the selective consumers in its strong market.

Proposition 6. *With perfect targeting and price discrimination:*

(i) each firm wins the group of selective customers in its own market with a *probability equal to* $\tau \in [0,1]$ *where*

$$\tau = \int_{p_{\min}^r}^{v-\gamma}\left(\int_{p_{\min}^o}^{p_B+\gamma} f^o(p^o)\,dp^o\right) f(p^r)\,dp^r + m^r.$$

The quadratic technology yields:

$$\tau = \frac{4\lambda(v+4\lambda-\gamma)}{\gamma}\left(\frac{1}{v+4\lambda-2\gamma} - \frac{1}{\gamma}\left(\ln\frac{v+4\lambda-\gamma}{v+4\lambda-2\gamma}\right)\right) + \frac{5}{4\lambda+5}.$$

(ii) For the quadratic technology, the expected price targeted to own and the rival's customers are, respectively:

$$E^o(p) = \frac{(v+4\lambda-2\gamma)}{v}\left((v-\gamma)\frac{\gamma - p_{\min}^r \ln p_{\min}^r}{p_{\min}^r} - (\gamma - (\ln(v-\gamma))(v-\gamma))\right)$$

$$E^r(p) = \frac{4\lambda(v+4\lambda-\gamma)\left(vp_{\min}^o \ln\left(\frac{v}{p_{\min}^o}\right) - \gamma(v-p_{\min}^o)\right)}{p_{\min}^o(v-\gamma)(v+4\lambda-2\gamma)} + \frac{(v-\gamma)(v-2\gamma)}{v+4\lambda-2\gamma},$$

where $p_{\min}^o = (\gamma + p_{\min}^r)$.

Proof. See the Appendix.

Corollary 5. *When the marginal advertising cost λ increases:*
(i) ϕ^o and ϕ^r decrease;
(ii) the minimum price in the equilibrium support of both c.d.f $F^r(p)$ and $F^o(p)$ increases.
(iii) the mass point m^r decreases.

Proof. See the Appendix.

For the quadratic advertising cost function and the numerical example where $v = 7$ and $\gamma = 1$, assumption (A2') is satisfied when $\max\{0.81125, 0.60355\} \leq \lambda \leq 3$. Figure 1 plots the expected price targeted to old and the rival's customers as a function of advertising costs.

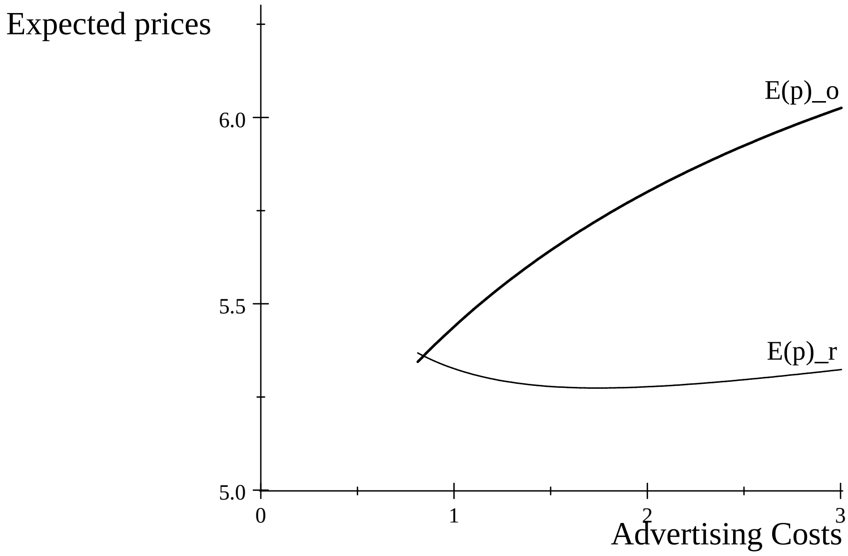

Figure 1: Expected Prices

For this numerical example it is interesting to note that when advertising is not too cheap, on average firms target ads with better deals to consumers in the weak market than to those consumers in the strong market. This result confirms the usual finding that price discrimination leads firms to charge more to their own customers than to their rival's customers. The numerical example considered shows however that it may happen that ads targeted to a firm's weak market quote on average a higher price than ads targeted to the firm's strong market. When advertising is too cheap each firm sends increasingly more ads to consumers in the rival's strong market, which increases the group of its weak captive consumers (see figure 5) and with a higher probability it will decide to focus on this group of captive consumers, offering them the price $v - \gamma$. This increases the average price to be quoted to group of captives in the weak market.

5 Competitive effects of targeted advertising

This section investigates how targeted advertising and price discrimination affect the equilibrium outcomes—i.e., advertising intensities, prices and profits. To perform comparative statics, we have to resort to a numerical analysis. We use the quadratic advertising cost function and assume that $v = 7$ and $\gamma = 1$. In this case assumption (A2') is satisfied when $\max\{0.81125, 0.60355\} \leq \lambda \leq 3$.

Effects on Advertising We have seen that with targeted advertising firms advertise less intensively in its own market than in the rival's market (i.e., $\phi^{o*} < \phi^{r*}$). With mass advertising firms choose an intensity of advertising to the whole market, which in equilibrium is equal to ϕ^m. Figure 2 depicts advertising intensities for different values of the marginal advertising cost λ.

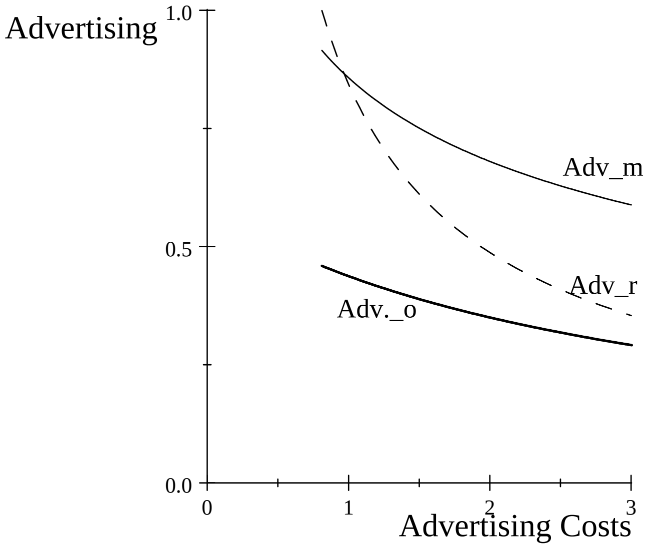

Figure 2: Advertising intensities

Figure 3 shows that in the case of mass advertising technologies, advertising expenditures are monotonically increasing with λ, while with targeted advertising, advertising expenditures are monotonically decreasing with λ. It also shows that when λ is high enough total advertising costs with targeted advertising can be below its counterparts with mass advertising. The finding that total advertising costs with targeted advertising are below those with mass advertising is also obtained in Brahim, et al. (2011) and Iyer, et al. (2005), and it can be partly explained by the fact that all of these models consider quadratic additive targeted advertising technologies. However, it is important to stress that our analysis reveals that the additive property is not sufficient to originate lower total advertising costs with targeted advertising. If λ is sufficiently low Figure 3 shows that in our model firms can spend more on advertising when they use targeted advertising than when they use mass advertising.

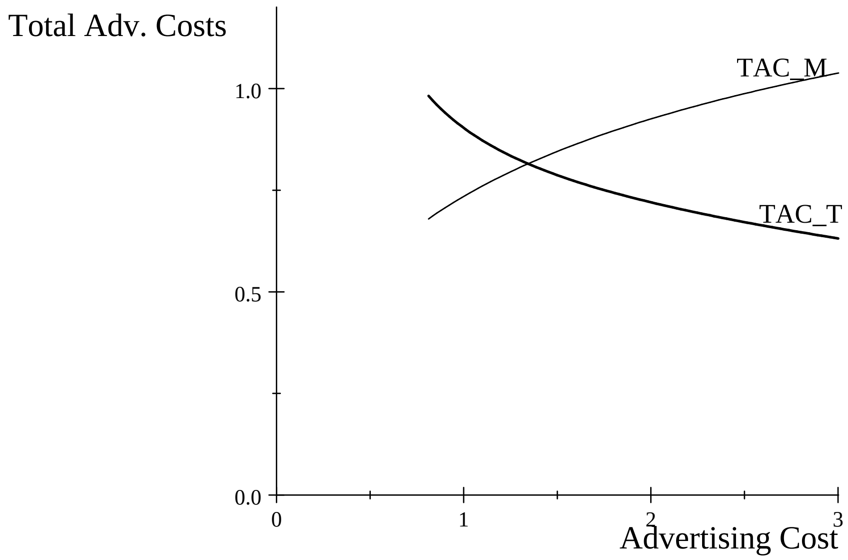

Figure 3: Total advertising costs

Effects on market segmentation Regarding the effects of targeted advertising on market segmentation it is worth noting that when advertising is not too cheap, for the numerical example considered, more consumers are informed with mass advertising than with targeted advertising. This implies that as advertising becomes more expensive more consumers are left out of the market with targeted advertising. Figure 5 shows that the group of selective consumers with targeted advertising is also below its mass advertising counterpart. Additionally, the fraction of consumers who are only aware of the less preferred firm is higher under targeted than under mass advertising. This means that, in our framework, targeted advertising might lead to a less efficient shopping to some consumers. This result will be useful to understand the negative effects of targeted advertising on consumer welfare discussed in section 6. This finding is in contrast with Brahim et al. (2011) in which targeted advertising always leads to a more efficient shopping for all consumers. The rationale for our result lies on the firms' optimal pricing strategies when price discrimination is feasible. More precisely it lies on firms' strategic incentives to reduce the advertising intensity in the strong market as a way to induce the rival to price less aggressively in this market.

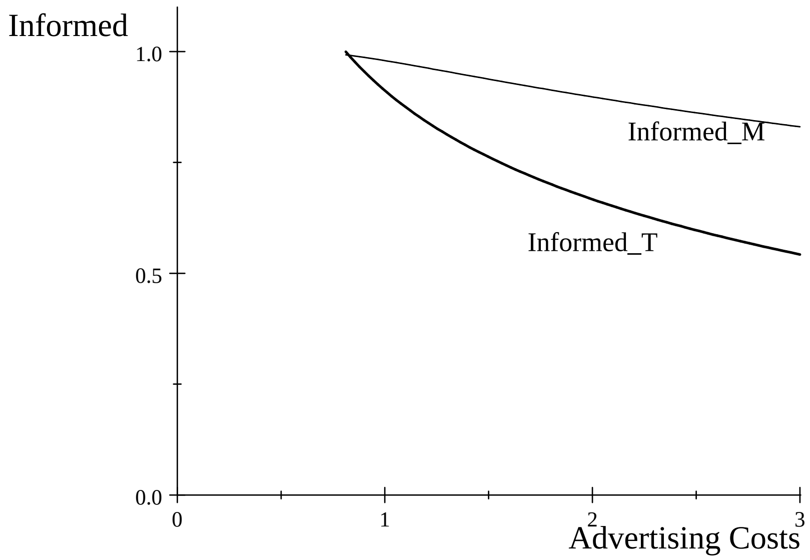

Figure 4: Informed consumers

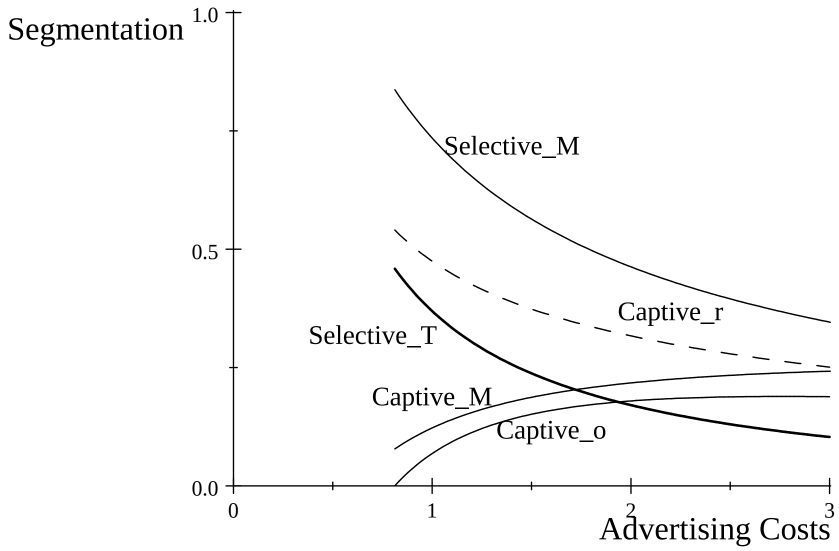

Figure 5: Market segmentation

Effects on Prices Even though it is not possible to establish a general stochastic ordering between F^m, F^r and F^o. Figures 6-8 plot these distribution functions for the parameters identified above.

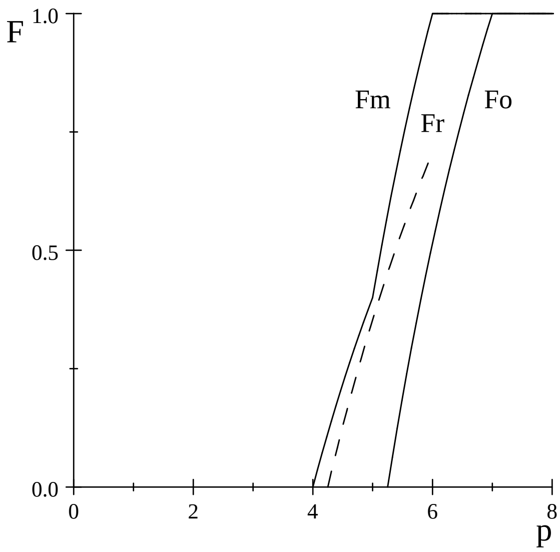

Figure 6: Distribution Functions ($\lambda = 3$)

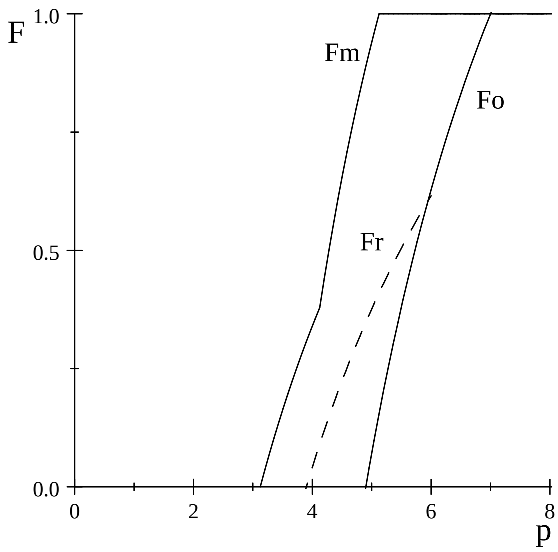

Figure 7: Distribution Functions ($\lambda = 2$)

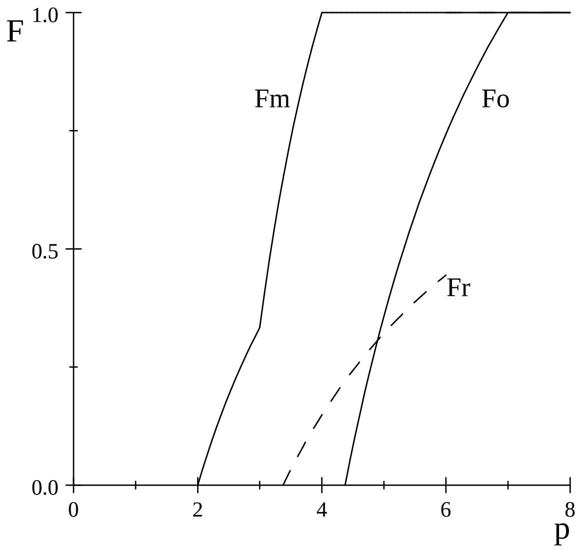

Figure 8: Distribution Functions ($\lambda = 1$)

For the numerical example considered, the figures show that F^m is stochastically dominated by F^r and F^o. This suggests that average prices with targeted advertising and price discrimination can be, *above* the average price with mass advertising and no discrimination.[30] This is an interesting result because it challenges the usual finding that in a competitive setting price discrimination generally reduces prices in all segments of the market (e.g. Thisse and Vives (1988), Fudenberg and Tirole (2000)). The reason is that in our framework targeted advertising

[30] It is important to stress that the same pattern was obtained for other values of λ satisfying assumptions (A1) and (A2'). Details are available from authors upon request.

softens price competition and allows firms to raise prices. These figures also suggest that the stochastic dominance of F^r and F^o over F^m is expected to be more significant as advertising becomes cheaper.

Effects on profits From the equilibrium solutions and assuming that conditions (A1) and (A2') are satisfied we have that each firm's profits with targeted and mass advertising are respectively given by

$$E\pi^t = \left(\frac{2\lambda + v - \gamma}{2\lambda}\right) A\left(\phi^{o*}\right) + A\left(\phi^{r*}\right)$$

and

$$E\pi^m = A\left(\phi^{m*}\right).$$

Proposition 7. *Taking into account that condition (A2') is satisfied, targeted advertising and price discrimination boost each firm's profits when λ is such that the following implicit condition is satisfied*

$$2\lambda\left[1 - \chi^*\left(\lambda, v, \gamma\right)\right] + (v - \gamma) > 0,$$

with $\chi^*\left(\lambda, v, \gamma\right) = \frac{(\phi^{m*})^2 - (\phi^{r*})^2}{(\phi^{o*})^2}$.

Proof. See the Appendix.

Figure 9 plots each firm's profits with targeted advertising (Profit_T) and with mass advertising (Profit_M) for the numerical example considered. For this example, we see that profits with targeted advertising and price discrimination are above their mass advertising counterparts when $\lambda < 2.4605$, while the reverse happens when $\lambda > 2.4605$.

Proposition 7 highlights that price discrimination by means of targeted advertising does not necessarily lead to the classic prisoner dilemma result that arises in models with full informed consumers. A common finding in models with (i) full informed consumers, (ii) best-response asymmetry and (iii) all firms engaging in price discrimination, is that equilibrium profits decrease with price discrimination (e.g. Chen (1997), Villas-Boas (1999), Fudenberg and Tirole (2000), Esteves (2010), Gehrig, et al. (2012)). In contrast, this paper shows that price discrimination can boost profits even when conditions (ii) and (iii) hold.

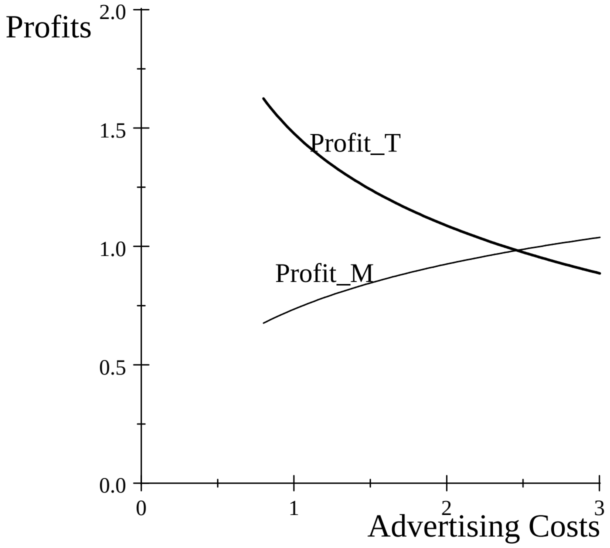

Figure 9: Profits

Proposition 7 suggests that in our framework equilibrium profits with targeted advertising are above their mass advertising counterparts for not too high advertising costs, the reverse happens for high advertising costs. This result is in contrast with Iyer, et al. (2005) who argue that targeting always increases firms' profits. The reason is that in their model targeting is always oriented to consumers who have a strong preference for the firms' product. Our result is also different from Brahim, et al. (2010) who show that when firms advertise to their strong and weak market, profits with targeted advertising are always above those with mass advertising. Although in their model price discrimination is not permitted, they argue that if price discrimination were introduced in their framework the negative effects of targeted on profits would still be present (p. 686).

For the numerical example considered, it is possible to say something about the impact of advertising costs on equilibrium profits. First, Figure 9 shows that with mass advertising and uniform pricing, firms benefit from increases in advertising costs. This confirms a well-known result in the literature on informative advertising: firms' profits may increase with advertising costs (e.g. Grossman and Shapiro (1984), Stahl (1994), Esteves (2009)). In general, whilst an increase in advertising costs has a negative *direct effect* on profits, there is, as well, a *strategic effect*: as advertising costs increase, firms respond with less advertising, and prices go up. When the strategic effect dominates, profits may increase with advertising costs. If we start with a situation in which ϕ^m is low (λ is high), additional advertising is more likely to increase the fraction of captive customers than the fraction of selective customers. It turns out that the probability of reaching an uninformed buyer is high and, then, firms have more incentives to focus on the group of captive consumers, thereby quoting high prices. However, as λ becomes increasingly smaller and firms advertise more, the reverse happens.

Second, Figure 9 shows that with targeted advertising and price discrimination, firms are always better off with decreases in advertising costs. This establishes that when price discrimination through targeted advertising is allowed, profits and advertising costs move in opposite directions. Following the previous reasoning, this suggests that the direct effect is stronger than the strategic effect. A decrease in λ, increases ϕ^r and the group of firm j's captive consumers in market i increases. Consequently, firm j plays less aggressively in firm i's strong market with a positive effect on both firms' profits. Under no discrimination, a reduction in advertising costs tends to push firms to price more aggressively leading to lower average prices. Expressed differently, without discrimination, an increase in λ has the strategic effect of reducing the group of selective consumers, which avoids a more aggressive behavior, thereby increasing the firms' profits. In contrast, with price discrimination, a decrease in λ has the strategic effect of increasing the rival firm's captive group of consumers which increases the probability of the rival firm charging the highest price $v - \gamma$. This acts to soften price competition and therefore to increase profits.

6 Welfare analysis

This section investigates the welfare effects of price discrimination enabled by informative targeted advertising. In our set-up, total welfare can be written as $v-$"expected disutility cost"$-$advertising costs.

Consider first the mass advertising/no discrimination case. Recall that customers' gross benefit when buying a certain good can be given by $v-$"expected disutility cost", where the latter is equal to γ, when the consumer buys the least preferred good; and zero, when the consumer buys the most preferred good. In the social optimal solution, consumers would buy from the most preferred firm, in order to obtain a gross benefit of v (and minimize the expected disutility cost). Taking into account the equilibrium market segmentation with mass advertising, we have that the firms' captive consumers in their strong market always buy efficiently. In contrast, captive consumers who are only aware of the least preferred product always buy inefficiently because they incur the disutility cost γ. Finally, regarding the group of selective consumers, they buy efficiently when firms share them equally, which occurs with probability q^m. With the remaining probability, $1 - q^m$, all the selective consumers buy from the same firm, which means that half of them buy inefficiently. Accordingly, in the case of mass advertising/no discrimination, total welfare can be represented as:

$$W^m = \phi^m (1 - \phi^m) (2v - \gamma) + v (\phi^m)^2 - \frac{\gamma}{2} (1 - q^m) (\phi^m)^2 - 2A(\phi^m). \quad (14)$$

Expected consumer surplus is $ECS^m = W^m - E\pi^m_{ind}$. As in equilibrium $E\pi^m = A(\phi^m)$, it follows that $E\pi^m_{ind} = 2A(\phi^m)$. Thus,

$$ECS^m = \phi^m (1 - \phi^m) (2v - \gamma) + v (\phi^m)^2 - \frac{\gamma}{2} (1 - q^m) (\phi^m)^2 - 4A(\phi^m) \quad (15)$$

Look now at the targeted advertising/price discrimination case. Selective consumers buy efficiently when each firm wins the group of selective consumers in its strong market, which occurs with probability τ. Regarding the segments of captive consumers, those that buy the most preferred product obtain a gross utility of v, while those consumers who are only aware of the less preferred brand obtain a gross utility of $v-\gamma$. Accordingly, with targeted advertising overall welfare is given by:

$$W^t = v\phi^o\left(1-\phi^r\right) + \left(v-\gamma\right)\phi^r\left(1-\phi^o\right) + v\phi^o\phi^r\tau + \left(v-\gamma\right)\left(1-\tau\right)\phi^o\phi^r - 2A\left(\phi^o\right) - 2A\left(\phi^r\right)$$

which simplifies to

$$W^t = v\phi^o + \left(v-\gamma\right)\phi^r\left(1-\phi^o\right) - \gamma\phi^o\phi^r\left(1-\tau\right) - 2A\left(\phi^o\right) - 2A\left(\phi^r\right). \tag{16}$$

Expected consumer surplus is equal to $ECS^t = W^t - E\pi^t_{ind}$. From $E\pi^t = \left(\frac{2\lambda+v-\gamma}{2\lambda}\right)A\left(\phi^{o*}\right) + A\left(\phi^{r*}\right)$ it follows that

$$E\pi^t_{ind} = \left(\frac{2\lambda+v-\gamma}{\lambda}\right)A\left(\phi^{o*}\right) + 2A\left(\phi^{r*}\right).$$

This yields:

$$ECS^t = v\phi^o\left(1-\phi^r\right) + \left(v-\gamma\right)\phi^r\left(1-\phi^o\right) + \left(v-\gamma+\tau\gamma\right)\phi^o\phi^r - \left(\frac{v+4\lambda-\gamma}{\lambda}\right)A\left(\phi^o\right) - 4A\left(\phi^r\right). \tag{17}$$

Figure 10 plot overall welfare and expected consumer surplus for the numerical example considered. It shows that consumers' surplus and overall welfare decrease when we move from mass advertising/no discrimination to targeted advertising/price discrimination.

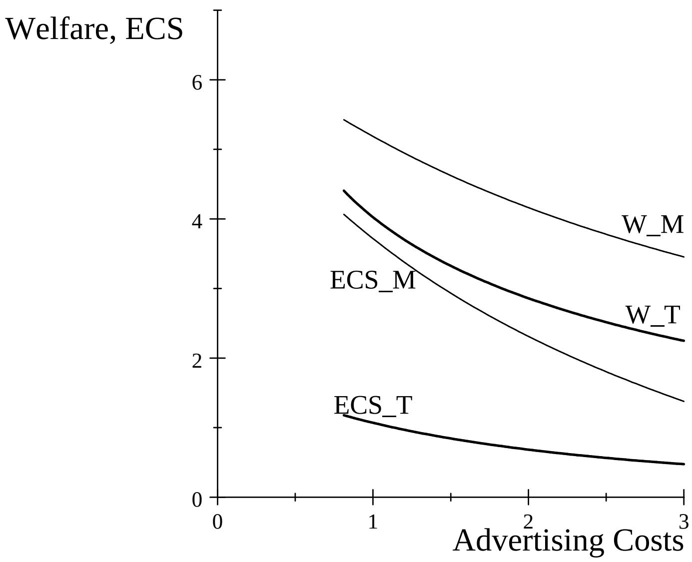

Figure 10: Welfare and Consumer Surplus

At least for the numerical example considered, when advertising is not too expensive targeted advertising and price discrimination can boost industry profits at the expense of social welfare and consumer welfare. When advertising costs are high, targeted advertising and price discrimination are bad for profits as well as for consumers' and overall welfare. The result that price discrimination can benefit industry profits and hurt consumers departs from the general presumption of Chen (2005), according to whom "price discrimination ... is by and large unlikely to raise significant antitrust concerns. In fact, as the economics literature suggests, such pricing practices in oligopoly markets often intensify competition and potentially benefit consumers." (p. 123).

The welfare results obtained highlight the importance of taking into account different forms of market competition when public policy tries to evaluate the welfare effects of price discrimination in competitive settings. This suggests that competition authorities should be particularly vigilant with regards to targeted advertising and price discrimination in industries wherein firms have nowadays the tools to personalise their ads and pricing offers.

7 Conclusions

This paper has investigated the effects of price discrimination by means of targeted advertising in a duopolistic market in which advertising plays two major roles: it is used by firms as a way to transmit relevant information to otherwise uninformed consumers, and it is used as a price discrimination device. Two advertising and pricing strategies were studied in the paper: a mass advertising/non-discrimination strategy and a targeted advertising/price discrimination strategy.

Under mass advertising firms choose an intensity of advertising to the entire market and all the ads announce the same price. When price discrimination by means of targeted advertising is used, firms choose different levels of advertising to each market segment, and ads targeted to different segments quote a different price.

The paper offers a contribution to the literature on competitive price discrimination and to the literature on targeted informative advertising. It has shown that firms advertise less intensively to its strong market than to its weak market. This finding is in contrast with Galeotti and Moraga-González (2008), Iyer et al. (2005) and Brahim et al. (2011) who find the opposite result. The reason is that in our framework each firm has an incentive to strategically reduce the advertising intensity to be targeted to its strong market, as a way to induce the rival to play less aggressively. We also find that targeted advertising may lead to less efficient shopping to some consumers. This result differs from Brahim et al. (2011), in which targeted advertising always leads to a more efficient shopping for all consumers.

The stylized model addressed in this paper has also provided new insights to the literature on price discrimination based on customer recognition. If firms need to invest in advertising to create awareness for their products, we find that prices with mass advertising (non-discrimination) can be, on average, below those with targeted advertising. This is an interesting finding as it challenges the usual result that price discrimination reduces all segment prices. We also find that price discrimination by means of targeted informative advertising does not necessarily lead to the classic prisoner dilemma result arising in models with full informed consumers and exhibiting best-response asymmetry. Our analysis reveals that, at least when advertising is not too expensive, each firms' profit with targeted advertising and price discrimination is above its non-discrimination counterpart.

The finding that profits with targeted advertising can be above their mass advertising counterparts for not too high advertising costs, and below those levels for high advertising costs departs from Iyer, et al. (2005) who argue that targeting always increases firms' profits. The reason is that in their model targeting is always oriented to consumers who have a strong preference for their product. Our result also departs from Brahim et al. (2011) who show that profits with mass advertising are always above the targeting profits when firms advertise to their strong and weak markets.

Finally, another relevant contribution of the paper was to investigate the welfare effects of targeted advertising with price discrimination in comparison to the mass advertising/no-discrimination case. We showed that, at least when advertising costs are not too high, price discrimination by means of targeted advertising can boost industry profits at the expense of consumer surplus and welfare. Thus, the paper has highlighted the importance of taking into account different forms of market competition when public policy tries to evaluate the profit and welfare effects of price discrimination and targeted advertising.

In light of the above, this paper has tried to contribute to the debate on the economic implications of targeted advertising and pricing with customer recognition. Notwithstanding the model addressed in this paper is far from covering all complex aspects of real markets, it has tried to offer a closer approximation of reality, where the quantity and quality of consumer-specific information that firms have been using to implement their advertising and pricing strategies is increasingly improving thanks to the advances in information technologies.

Appendix

Proof of Proposition 1. Look first at case *(i)*. Suppose (v, v) is an equilibrium in pure strategies. In this case firm i serves only the captive and the selective consumers who belong to its strong market, obtaining a profit equal to $\frac{1}{2}v\phi_i - \lambda\phi_i^2$. When (v, v) constitutes an equilibrium in pure strategies, the corresponding equilibrium advertising level is equal to

$$\phi_i = \frac{v}{4\lambda},$$

where we must impose

$$\lambda > \frac{v}{4} \tag{18}$$

to guarantee an interior solution in the advertising choice. Equilibrium profits for each firm are then equal to $(\pi_i^v)^* = \frac{1}{16}\frac{v^2}{\lambda}$. Any price greater than v is not part of an equilibrium strategy since at such a price no consumer is willing to buy from the firm. Any price lower than v but greater than $v - \gamma$ gives firm i the same market share but reduces its profit and so it is dominated by v. If firm i deviates and chooses $p_i^d = v - \gamma$, the firm starts selling its good to the group of captive consumers in its weak market. Firm i's profits are then

$$\pi_i^{d,v-\gamma} = (v - \gamma)\left(\phi_i\left(1 - \phi_j\right) + \frac{1}{2}\phi_i\phi_j\right) - \lambda\phi_i^2,$$

where $\phi_j = \frac{1}{4}\frac{v}{\lambda}$, since firm i takes as given firm j' s decisions (pricing and advertising). The deviation advertising reach would then be equal to $\phi_i^{d,v-\gamma} = \frac{(8\lambda-v)(v-\gamma)}{16\lambda^2}$ as long as $v < 8\lambda$. This condition guarantees $\phi_i^{d,v-\gamma}$ is positive and it always holds under (18). The condition $\gamma > \frac{(v-4\lambda)^2}{v-8\lambda}$ would guarantee $\phi_i^{d,v-\gamma}$ is smaller than 1. For $v < 8\lambda$, the RHS of the last condition is always negative and therefore the condition is always true. Thus, when $\lambda > \frac{v}{4}$, firm i's deviation profit is equal to $\pi_i^{d,v-\gamma} = \frac{1}{\lambda}\left(\frac{(v-\gamma)(v-8\lambda)}{16\lambda}\right)^2$. Comparing $\pi_i^{d,v-\gamma}$ and $(\pi_i^v)^*$, we obtain the following no-deviation condition:

$$\lambda < \frac{v\,(v-\gamma)}{4\,(v-2\gamma)}. \tag{19}$$

Accordingly, the conditions $\lambda > \frac{v}{4}$ and $\lambda < \frac{v(v-\gamma)}{4(v-2\gamma)}$ are necessary for (v, v) to be a Nash Equilibrium in Pure Strategies. Considering the two conditions simultaneously, it follows that when $v < 2\gamma$, (v, v) is a pure strategy equilibrium if $\lambda > \frac{v}{4}$. When $v > 2\gamma$, (v, v) is a pure strategy equilibrium in prices if $\frac{v}{4} < \lambda < \frac{v(v-\gamma)}{4(v-2\gamma)}$.

Finally, it remains to study if instead of deviating to $v-\gamma$, firm i would be interested in decreasing its price even further, serving not only all the captive consumers but also all selective consumers, by setting $p_i = v-\gamma-\varepsilon > 0$. In that case, firm i's profits are equal to $(v-\gamma-\varepsilon)\,\phi_i - \lambda\phi_i^2$, leading firm i to choose an advertising intensity equal to $\frac{v-\gamma-\varepsilon}{2\lambda}$ as long as $\lambda > \frac{v-\gamma-\varepsilon}{2}$.

The deviation profits are equal to $\pi_i^{d,v-\gamma-\varepsilon} = \frac{1}{4}\frac{(v-\gamma-\varepsilon)^2}{\lambda}$. As long as $v > 2\gamma$, it is always possible to find a sufficiently small ε for which $\pi_i^{d,v-\gamma-\varepsilon} > (\pi_i^v)^*$, which means that the deviation from v to $v-\gamma-\varepsilon$ is always profitable when $v > 2\gamma$. In contrast, when $v < 2\gamma$, such deviation is never profitable. When $v < 2\gamma$, we always have $\frac{v-\gamma-\varepsilon}{2} < \frac{v}{4}$ and therefore condition $\lambda > \frac{v-\gamma-\varepsilon}{2}$ always holds as long as $\lambda > \frac{v}{4}$.

Accordingly, (v, v) is an interior Nash Equilibrium in pure strategies iif $v < 2\gamma$ and $\lambda > \frac{1}{4}v$.

Addressing now case *(ii)*, suppose $(v-\gamma, v-\gamma)$ is an equilibrium in pure strategies. In this case, firm i serves all its captive consumers as well as the selective consumers in its strong market. Firm i's profit for a given advertising intensity ϕ_i write as $(v-\gamma)\left(\phi_i - \frac{\phi_i\phi_j}{2}\right) - \lambda\phi_i^2$, and firms' equilibrium advertising reach is then given by $\phi_i^{v-\gamma} = 2\frac{v-\gamma}{v-\gamma+4\lambda}$as long as

$$\lambda > \frac{v-\gamma}{4}. \tag{20}$$

The equilibrium profits are $(\pi_i^{v-\gamma})^* = 4\lambda\frac{(v-\gamma)^2}{(v+4\lambda-\gamma)^2}$. If firm i deviates to a higher price it must be to price v. In this case, firm i only sells to consumers in its strong market (captive and selective). Profits are given by $\frac{1}{2}v\phi_i - \lambda\phi_i^2$ and the deviation advertising reach is equal to $\frac{v}{4\lambda}$, which is an interior solution when (20) holds. Deviation profits are equal to $(\pi_i^{d,v})^* = \frac{1}{16}\frac{v^2}{\lambda}$.

Firm i does not have incentives to deviate to price v if $(\pi_i^{d,v})^* < (\pi_i^{v-\gamma})^*$, or equivalently, $\lambda > \frac{v(v-\gamma)}{4(v-2\gamma)}$ for $v > 2\gamma$. Note that the previous condition on λ is more restrictive than (20).

Consider next a deviation to a lower price. If firm i deviates and chooses $p_i^d = v-2\gamma-\varepsilon > 0$ it can also attract the group of selective consumers belonging to its weak market. In this case, firm i's profit writes as $(v-2\gamma-\varepsilon)\,\phi_i - \lambda\phi_i^2$, and firm i's deviation advertising reach is then given by $\phi_i^{d,v-2\gamma-\varepsilon} = \frac{v-2\gamma-\varepsilon}{2\lambda}$. Deviation profits are equal to $\pi_i^{d,v-2\gamma-\varepsilon} = \frac{(v-2\gamma-\varepsilon)^2}{4\lambda}$.

The deviation to $v-2\gamma-\varepsilon$ is unprofitable if $\pi_i^{d,v-2\gamma-\varepsilon} < (\pi_i^{v-\gamma})^*$, or equivalently $\lambda > \frac{v-\gamma}{4}\frac{v-2\gamma}{\gamma}$.

Thus, $(v-\gamma, v-\gamma)$ is an equilibrium in pure strategies if:

$$\lambda > \max\left\{\frac{(v-2\gamma)(v-\gamma)}{4\gamma}, \frac{v(v-\gamma)}{4(v-2\gamma)}\right\}.$$

In case *(iii)*, the conditions above do not hold and the game has no symmetric pure strategy equilibrium in prices. ■

Proof of Proposition 2. Suppose that firm j selects a price randomly from the c.d.f $F_j(p)$. For the sake of simplicity write $F_i(p) = F_j(p) = F(p)$. Suppose further that the support of the equilibrium prices is $[p_{\min}, p_{\max}]$. When firm i chooses any price that belongs to the equilibrium support of prices, and firm j uses the c.d.f $F(p)$, firm i's expected profit is always equal to a constant, which is denoted k^m minus advertising costs. When firm i charges price $p_i \leq v-\gamma$, firm i's expected profit, denoted $E\pi_i$, is

$$E\pi_i = p\phi_i\left(1-\phi_j\right) + p\phi_i\phi_j\left[1-\frac{1}{2}F_j\left(p+\gamma\right)-\frac{1}{2}F_j\left(p-\gamma\right)\right] - A(\phi_i)$$

In a MSNE we must observe that:

$$p_i\left[\phi_i\left(1-\phi_j\right)+\phi_i\phi_j\left[1-\frac{1}{2}F_j\left(p_i+\gamma\right)-\frac{1}{2}F_j(p_i-\gamma)\right]\right] - A\left(\phi_i\right) = k^m - A\left(\phi_i\right),$$

from which we obtain

$$F_j\left(p_i+\gamma\right)+F_j(p_i-\gamma) = 2-\frac{2k^m}{\phi_i\phi_j p_i}+\frac{2\left(1-\phi_j\right)}{\phi_j} \tag{21}$$

Suppose that p_1 is such that $p_1-\gamma = p_{\min}$ and p_2 is such that $p_2+\gamma = p_{\max}$. Then, $\forall p \leq p_1$, $F\left(p-\gamma\right)=0$ and $\forall p \geq p_2$, $F\left(p+\gamma\right)=1$.Using (21) it follows that

$$\forall p \leq p_1 \Rightarrow F_j\left(p_i+\gamma\right) = 2-\frac{2k^m}{\phi_i\phi_j p_i}+\frac{2\left(1-\phi_j\right)}{\phi_j}$$

and

$$\forall p \geq p_2 \Rightarrow F_j(p_i-\gamma) = 1-\frac{2k^m}{\phi_i\phi_j p_i}+\frac{2\left(1-\phi_j\right)}{\phi_j}$$

Thus,

$$\forall p \leq p_1 \Rightarrow F\left(p\right) = 2-\frac{2k^m}{\phi_i\phi_j\left(p-\gamma\right)}+\frac{2\left(1-\phi_j\right)}{\phi_j}$$

Similarly,

$$\forall p \geq p_2 \Rightarrow F(p) = 1-\frac{2k^m}{\phi_i\phi_j\left(p+\gamma\right)}+\frac{2\left(1-\phi_j\right)}{\phi_j}$$

Now it remains to show that $p_1 = p_2$. Suppose first that $p_2 < p_1$. Then, $\forall p \in [p_2, p_1]$ it follows $F\left(p-\gamma\right)=0$ and $F\left(p+\gamma\right)=1$ thus $p\left[\phi_i\left(1-\phi_j\right)+\frac{1}{2}\phi_i\phi_j\right] = k^m$.

Assume now that $p_2 > p_1$ and take $p \in [p_1, p_2]$ s.t. (21) holds. $\exists\widetilde{p}$ s.t. $\widetilde{p}-\gamma = p_L < p_1$ and $\widetilde{p}+\gamma = p_H > p_2$.

Since $p_L < p_1$ and $p_H > p_2$, it follows that

$$F\left(\widetilde{p}\right) = 2 - \frac{2}{\phi_i\phi_j}\left[\frac{k^m}{\left(\widetilde{p}-\gamma\right)} - \phi_i\left(1-\phi_j\right)\right]$$

and

$$F(\widetilde{p}) = 1 - \frac{2}{\phi_i\phi_j}\left[\frac{k^m}{\widetilde{p}+\gamma} - \phi_i\left(1-\phi_j\right)\right].$$

From the continuity of F it must be true that

$$2 - \frac{2}{\phi_i\phi_j}\left[\frac{k^m}{\left(\widetilde{p}-\gamma\right)} - \phi_i\left(1-\phi_j\right)\right] = 1 - \frac{2}{\phi_i\phi_j}\left[\frac{k^m}{\widetilde{p}+\gamma} - \phi_i\left(1-\phi_j\right)\right],$$

from which it follows that there is a unique positive value of $\widetilde{p}$ given by $\widetilde{p} = \sqrt{\frac{4k^m\gamma}{\phi_i\phi_j} + \gamma^2}$.

Since this must hold $\forall p \in [p_1, p_2]$ and they cannot all be equal it must be the case that $p_1 = p_2$. Since $p_1 = p_{\min} + \gamma$ and $p_2 = p_{\max} - \gamma$ it follows that $p_{\min} + \gamma = p_{\max} - \gamma$ or equivalently $p_{\max} - p_{\min} = 2\gamma$.

Let p be the price of firm i, then given that $p_{\max} - p_{\min} = 2\gamma$, it follows that for any $p \leq v - \gamma$, $F(p)$ is equal to

$$F(p) = \left\{\begin{array}{ccc} 0 & if & p < p_{\min} \\ 1 - \frac{2}{\phi_i\phi_j}\left(\frac{k^m}{p+\gamma} - \phi_i\left(1-\phi_j\right)\right) & if & p_{\min} \leqq p \leqq p_{\max} - \gamma \\ 2 - \frac{2}{\phi_i\phi_j}\left(\frac{k^m}{p-\gamma} - \phi_i\left(1-\phi_j\right)\right) & if & p_{\max} - \gamma < p \leqq p_{\max} \\ 1 & if & p > p_{\max} \end{array}\right\} \tag{22}$$

As in the symmetric MSNE, $\phi_i = \phi_j = \phi$, the c.d.f $F(p)$ can be written as:

$$F^m(p) = \left\{\begin{array}{ccc} 0 & if & p < p_{\min} \\ 1 - \frac{2}{(\phi^m)^2}\left(\frac{k^m}{p+\gamma} - \phi^m\left(1-\phi^m\right)\right) & if & p_{\min} \leqq p \leqq p_{\max} - \gamma \\ 2 - \frac{2}{(\phi^m)^2}\left(\frac{k^m}{p-\gamma} - \phi^m\left(1-\phi^m\right)\right) & if & p_{\max} - \gamma < p \leqq p_{\max} \\ 1 & if & p > p_{\max} \end{array}\right\},$$

From $F\left(p_{\min}\right) = 0$ and $F\left(p_{\max}\right) = 1$ it follows that

$$1 - \frac{2}{\phi_i\phi_j}\left(\frac{k^m}{p_{\min}+\gamma} - \phi_i\left(1-\phi_j\right)\right) = 0 \Longleftrightarrow \frac{2k^m}{\phi_i\phi_j + 2\phi_i\left(1-\phi_j\right)} - \gamma = p_{\min}$$

and

$$2 - \frac{2}{\phi_i\phi_j}\left(\frac{k^m}{p_{\max}-\gamma} - \phi_i\left(1-\phi_j\right)\right) = 1$$

Thus we obtain that:

$$p_{\max} = \frac{2k^m}{2\phi_i - \phi_i\phi_j} + \gamma \tag{23}$$

By continuity, for $p = p_{\max} - \gamma = \frac{2k^m}{2\phi_i - \phi_i\phi_j}$, it must be true that:

$$\frac{2\phi_i\phi_j\left(k^m\right)^2}{\left(2\phi_i - \phi_i\phi_j\right)^2} - 2\gamma k^m - \frac{\phi_i\phi_j}{2}\gamma^2 = 0$$

yielding:

$$k^m = \frac{\gamma\phi_i}{2\phi_j}\left(2-\phi_j\right)^2\left[1+\sqrt{1+\phi_j^2\left(2-\phi_j\right)^{-2}}\right]. \tag{24}$$

Thus,

$$p_{\max} = \gamma\frac{2-\phi_j}{\phi_j}\left[1+\sqrt{1+\phi_j^2\left(2-\phi_j\right)^{-2}}\right]+\gamma. \tag{25}$$

The expected profit of firm i, $E\pi_i = k^m - A(\phi_i)$ is equal to

$$E\pi_i = \frac{1}{2}\phi_i\left(p_{\max}-\gamma\right)\left(2-\phi_j\right) - A(\phi_i).$$

For $0 < \phi_i < 1$ (interior solution), each firm's advertising equilibrium level with mass advertising is obtained by maximizing $E\pi_i$ in order to ϕ_i. From the first order condition, the interior solution is given by[31] $\frac{\partial k^m}{\partial \phi_i} = A_{\phi_i}(\phi_i)$ which under symmetry writes as

$$\frac{1}{2}\left(p_{\max}-\gamma\right)\left(2-\phi^m\right) = A_\phi\left(\phi^m\right). \tag{26}$$

■

Proof of Corollary 1. Considering the quadratic advertising technology, $A(\phi^m) = \lambda(\phi^m)^2$, the equation (26) writes as:

$$\frac{1}{2}\left(p_{\max}-\gamma\right)\left(2-\phi^m\right) = 2\lambda\phi^m. \tag{27}$$

Substituting $p_{\max}$ by (25) and solving for ϕ^m, we obtain the equilibrium advertising level given by:

$$\phi^m = 2\frac{\left(\gamma^2+8\lambda\gamma\right)^{1/2}}{\left(\gamma^2+8\lambda\gamma\right)^{1/2}+4\lambda}. \tag{28}$$

Note that equation (27) had an additional solution, given by $\phi = 2\frac{\sqrt{\gamma^2+8\lambda\gamma}}{\sqrt{\gamma^2+8\lambda\gamma}-4\lambda}$. However, it can be easily seen that such solution cannot define the optimal advertising level in an interior solution since it would always lead to $\phi > 1$. Simple algebra shows that ϕ^m in equation (28) defines the interior optimal advertising level as long as $\lambda \geq \frac{\sqrt{2}+1}{4}\gamma$, which is always the case under condition (A2) in Assumption 1.

Plugging the optimal value of ϕ^m in equation (25), we get:

$$p_{\max} = \left(\gamma^2+8\lambda\gamma\right)^{1/2}+\gamma,$$

and, for $v > 2\gamma$, we have $p_{\max} \leq v-\gamma$ under under condition (A2) in Assumption 1, in particular when $\lambda \leq \frac{(v-\gamma)(v-3\gamma)}{8\gamma}$ and $v > 2\gamma$.

Analogously, replacing ϕ^m by (28) in equation (24) , we obtain

$$k^m = \frac{8\lambda\gamma\left(8\lambda+\gamma\right)}{\left[4\lambda+\left(\gamma^2+8\lambda\gamma\right)^{1/2}\right]^2}.$$

[31] Note that the second order condition is satisfied. It is given by $-A_{\phi\phi}(\phi^m) < 0$, which is always true, given our assumptions with respect to the advertising technology.

Substituting the resulting values for $p_{\max}$ and k^m in the c.d.f function (22), we obtain $F^m(p)$ equal to:

$$\left\{ \begin{array}{ccc} 0 & if & p < \left(\gamma^2 + 8\lambda\gamma\right)^{1/2} - \gamma \\ 4\lambda\left[\left(\gamma^2 + 8\lambda\gamma\right)^{-1/2} - \frac{1}{p+\gamma}\right] & if & \left(\gamma^2 + 8\lambda\gamma\right)^{1/2} - \gamma \leq p \leq \left(\gamma^2 + 8\lambda\gamma\right)^{1/2} \\ 1 - 4\lambda\left[\frac{1}{p-\gamma} - \left(\gamma^2 + 8\lambda\gamma\right)^{-1/2}\right] & if & \left(\gamma^2 + 8\lambda\gamma\right)^{1/2} \leq p < \left(\gamma^2 + 8\lambda\gamma\right)^{1/2} + \gamma \\ 1 & if & p \geq \left(\gamma^2 + 8\lambda\gamma\right)^{1/2} + \gamma \end{array} \right\}.$$

Finally, substituting the optimal advertising choice (28) in the equilibrium expected profits, $E\pi^m = \phi^m A_\phi(\phi^m) - A(\phi^m)$, equal to $E\pi^m = \lambda(\phi^m)^2$, we obtain:

$$E\pi^m = \frac{4\lambda\left(\gamma^2 + 8\lambda\gamma\right)}{\left[(\gamma^2 + 8\lambda\gamma)^{1/2} + 4\lambda\right]^2}$$

as stated in Corollary 1. To end the proof, it remains to verify that the domain $\frac{\gamma(\sqrt{2}+1)}{4} \leq \lambda \leq \frac{(v-3\gamma)(v-\gamma)}{8\gamma}$ in Corollary 1 is not empty. For that to be the case, we must observe $v > \left(\sqrt{2}+3\right)\gamma$.■

Proof of Proposition 3 Let $q \in [0,1]$ represent the probability with which each firm serves its group of selective customers. Because the model is symmetric both firms have the same support of prices. Then q can be written as:

$$q = 1 - 2\int_{p_{\min}+\gamma}^{p_{\max}} \left(\int_{p_{\min}}^{p_A - \gamma} f(p_B)\, dp_B\right) f(p_A)\, dp_A$$

from which we obtain:

$$q^m = 1 + \frac{8(k^m)^2}{(\phi^m)^4}\left[\frac{1}{(p_{\max}-\gamma)^2} - \ln\left(\frac{(p_{\max}-\gamma)^2}{p_{\max}(p_{\max}-2\gamma)}\right)\right].$$

Replacing k^m, ϕ^m and $p_{\max}$ by the equilibrium values described in Corollary 1, we obtain the result in Proposition 3. ■

Proof of Proposition 4. Look at firm $i's$ strong market. In this case $p_i^o \leq v$ and $p_j^r \leq v-\gamma$. Suppose that $(v, v-\gamma)$ is an equilibrium in pure strategies. Consider when indifferent consumers choose the firm they prefer. In this case firm i serves both the captive and selective consumers in its strong market while firm j serves its captive consumers in this market. Firm i's profits in this market are:

$$E\pi_i^o = v\frac{\phi_i^o}{2} - A(\phi_i^o). \tag{29}$$

Firm j's profits in the rival's market (i.e. firm i's strong market), denoted $E\pi_i^r$, is given by:

$$E\pi_j^r = (v-\gamma)\frac{\phi_i^r(1-\phi_j^o)}{2} - A(\phi_i^r). \tag{30}$$

Taking into account firm i's price and advertising decisions, if firm j deviates to $p_j^r = v - \gamma - \varepsilon$ it poaches all the selective consumers in market i. The profit from deviation would be equal to

$$\pi_j^{d,v-\gamma-\varepsilon} = (v - \gamma - \varepsilon)\frac{\phi_i^r}{2} - A(\phi_i^r).$$

$$(v - \gamma - \varepsilon)\frac{y}{2} - (v - \gamma)\frac{y(1-x)}{2} > 0$$

Thus, it is always possible to find a sufficiently small ε for which $\pi_j^{d,v-\gamma-\varepsilon} > E\pi_j^r$.■

Proof of Proposition 5. Here we prove that there is a symmetric equilibrium in mixed strategies in prices for an interior pure strategy equilibrium in advertising. As we focus on symmetric MSNE in prices, the c.d.f. are such that $F_A^o(p) = F_B^o(p) = F^o(p)$, and $F_B^r(p) = F_A^r(p) = F^r(p)$. Accordingly, for the sake of simplicity, with no loss of generality, we restrict our attention to firms' decisions in segment a, obtaining $F_A^o(p) = F^o(p)$ and $F_B^r(p) = F^r(p)$.

Given firms' pricing and advertising strategies targeted to segment a, firm A's expected profit in this segment, denoted $E\pi_A^o$ is equal to

$$E\pi_A^o = p_A^o\left[\frac{1}{2}\phi_A^o(1-\phi_B^r) + \frac{1}{2}\phi_A^o\phi_B^r \Pr(p_A^o < p_B^r + \gamma)\right] - A(\phi_A^o)$$

or equivalently,

$$E\pi_A = p_A^o\left\{\frac{1}{2}\phi_A^o(1-\phi_B^r) + \frac{1}{2}\phi_A^o\phi_B^r\left[1 - F_B^r(p_A^o - \gamma)\right]\right\} - A(\phi_A^o),$$

Similarly firm B's expected profit in segment a, denoted $E\pi_B^r$, is

$$E\pi_B^r = \frac{1}{2}p_B^r\left\{\phi_B^r(1-\phi_A^o) + \phi_B^r\phi_A^o\left[1 - F_A^o(p_B^r + \gamma)\right]\right\} - A(\phi_B^r).$$

Note that the minimum price firm B is willing to charge even when it is assured of getting the entire segment of selective customers in market a should satisfy the following condition

$$\frac{1}{2}p_{B\min}^r\phi_B^r = \frac{1}{2}(v-\gamma)\phi_B^r(1-\phi_A^o),$$

from which we obtain:

$$p_{B\min}^r = (v-\gamma)(1-\phi_A^o). \tag{31}$$

Given that firm B would never want to price below $p_{B\min}^r$, it is a dominated strategy for firm A to price below $p_{B\min}^r + \gamma$. Thus, the support of equilibrium prices for firm A is $[p_{B\min}^r + \gamma, v]$ while for firm B is $[p_{B\min}^r, v-\gamma]$. As usual in a MSNE each firm must be indifferent between charging any price in the support of equilibrium prices.

For firm B we must observe that for any $p_{B\min}^r \leq p_B^r \leq v - \gamma$:

$$\frac{1}{2}p_B^r\phi_B^r\left\{(1-\phi_A^o) + \phi_A^o\left[1 - F_A^o(p_B^r + \gamma)\right]\right\} = \frac{1}{2}(v-\gamma)\phi_B^r(1-\phi_A^o)$$

which simplifies to

$$F_A^o(p+\gamma) = \frac{1}{\phi_A^o}\left[1 - \frac{(v-\gamma)(1-\phi_A^o)}{p}\right] \text{ or, } F_A^o(p) = \frac{1}{\phi_A^o}\left[1 - \frac{(v-\gamma)(1-\phi_A^o)}{p-\gamma}\right]$$

where $F_A^o(v) = 1$. Note also that $F_A^o(p_{B\min}^r + \gamma) = 0$. In this way

$$F_A^o(p) = \left\{ \begin{array}{ccc} 0 & if & p \leq p_{B\min}^r + \gamma \\ \frac{1}{\phi_A^o}\left[1 - \frac{(v-\gamma)(1-\phi_A^o)}{p-\gamma}\right] & if & p_{B\min}^r + \gamma \leq p \leq v \\ 1 & if & p \geq v \end{array} \right\}.$$

Analogously, for firm A we must observe that for any $p_{B\min}^r + \gamma \leq p_A^o \leq v$:

$$E\pi_A^o = \frac{1}{2}p_A^o\phi_A^o\left\{(1-\phi_B^r) + \phi_B^r\left[1 - F_B^r(p_A^o - \gamma)\right]\right\} = \frac{1}{2}\left(p_{B\min}^r + \gamma\right)\phi_A^o \tag{32}$$

This simplifies to

$$F_B^r(p-\gamma) = \frac{1}{\phi_B^r}\left(1 - \frac{p_{B\min}^r + \gamma}{p}\right) \text{ or, } F_B^r(p) = \frac{1}{\phi_B^r}\left(1 - \frac{p_{B\min}^r + \gamma}{p+\gamma}\right). \tag{33}$$

Plugging (31) in (33), we obtain

$$F_B^r(p) = \frac{1}{\phi_B^r}\left[1 - \frac{v\left(1-\phi_A^o\right) + \gamma\phi_A^o}{p+\gamma}\right]$$

Note that $F_B^r(p_{B\min}) = 0$. Thus, the corresponding distribution is

$$F_B^r(p) = \left\{ \begin{array}{ccc} 0 & if & p \leq p_{B\min}^r \\ \frac{1}{\phi_B^r}\left(1 - \frac{v\left(1-\phi_A^o\right)+\gamma\phi_A^o}{p+\gamma}\right) & if & p_{B\min}^r \leq p \leq v-\gamma \\ 1 & if & p \geq v-\gamma \end{array} \right\}. \tag{34}$$

From (32) and (31) it follows that the expected profit obtained by firm A in market a when it charges any price in the support of equilibrium prices is equal to:

$$E\pi_A = \frac{1}{2}\phi_A^o\left[v - (v-\gamma)\,\phi_A^o\right] - \lambda\left(\phi_A^o\right)^2.$$

The profit-maximizing advertising intensity is obtained from the condition $\frac{\partial E\pi_A}{\partial \phi_A^o} = 0$, which implies that:

$$\frac{v}{2} - (v-\gamma)\,\phi_A^o = A_{\phi_A^o}\left(\phi_A^o\right), \tag{35}$$

since the SOC hold under our assumptions about the advertising technology.

Firm $i's$ equilibrium profit in its strong market is equal to:

$$E\pi_i^{*o} = \phi_i^o A_{\phi_i^o}\left(\phi_i^o\right) + \frac{1}{2}\left(\phi_A^o\right)^2(v-\gamma) - A\left(\phi_A^o\right). \tag{36}$$

To obtain the optimal advertising level ϕ_B^r, recall that firm B's expected profit in the MSNE is equal to

$$E\pi_B = \frac{1}{2}(v-\gamma)\,\phi_B^r(1-\phi_A^o) - A\left(\phi_B^r\right).$$

As the second order condition $\frac{\partial^2 E\pi_A}{\partial \phi_A^{o2}} < 0$ is always met, the following first order condition defines firm B's optimal advertising level in segment a

$$\frac{\partial E\pi_B}{\partial \phi_B^r} = 0 \Rightarrow \frac{1}{2}(v-\gamma)\left(1-\phi_A^o\right) = A_{\phi_j^r}\left(\phi_j^{r^*}\right) \Leftrightarrow \frac{1}{2}p_{j\min}^r = A_{\phi_j^r}\left(\phi_j^{r^*}\right). \tag{37}$$

Firm $i's$ equilibrium profit in its weak market is equal to:

$$E\pi_i^{*r} = \phi_i^{*r} A_{\phi_i^{*r}}\left(\phi_i^{*r}\right) - A\left(\phi_i^{*r}\right).$$

■

Proof of Corollary 2. Considering the quadratic advertising technology, the profit maximizing condition for ϕ_A^o (condition (35)) writes as:

$$\frac{v}{2} - (v-\gamma)\,\phi_A^o = 2\lambda\phi_A^o, \tag{38}$$

yielding:

$$\phi_A^o = \frac{v}{2\left[2\lambda + (v-\gamma)\right]},$$

as long as $\lambda > -\frac{v-2\gamma}{4}$, which always holds under assumption (A1). Substituting ϕ_A^o in the expressions of $p_{B\min}^r$ and $F_A^o(p)$, we obtain:

$$p_{B\min}^r = (v-\gamma)\,\frac{v-2\gamma+4\lambda}{2v-2\gamma+4\lambda},$$

and firm $i's$ equilibrium profit in its strong market is equal to:

$$E\pi_i^{*o} = \frac{1}{8}\frac{v^2}{v+2\lambda-\gamma}.$$

If we plug $p_{B\min}^r$ on the equilibrium condition for ϕ_B^r we obtain

$$\phi_B^r = \frac{v-\gamma}{4\lambda}\frac{v+4\lambda-2\gamma}{2v+4\lambda-2\gamma}, \tag{39}$$

which constitutes an interior solution as long as $\lambda > \frac{(v-\gamma)^{1/2}\left[(5v-9\gamma)^{1/2}-(v-\gamma)^{1/2}\right]}{8}$. Accordingly, firm $i's$ equilibrium profit in its weak market is given by

$$E\pi_i^{*r} = \frac{1}{\lambda}\left(\frac{v-\gamma}{8\lambda}\frac{v+4\lambda-2\gamma}{v+2\lambda-\gamma}\right)^2.$$

To finish the proof, it remains to analyze the conditions under which the MSNE prevails. The previous analysis shows that under (A1) the MSNE in prices leads to interior solutions for the advertising intensity if and only if $\lambda > \frac{(5v-9\gamma)^{1/2}(v-\gamma)^{1/2}-(v-\gamma)}{8}$. Combining this assumption with (A2), we obtain:

$$\max\left\{\frac{(5v-9\gamma)^{1/2}(v-\gamma)^{1/2}-(v-\gamma)}{8}, \frac{\gamma\left(\sqrt{2}+1\right)}{4}\right\} \leq \lambda \leq \frac{(v-3\gamma)(v-\gamma)}{8\gamma},$$

which together with assumption (A1) define the conditions under which Corollary 2 holds.■

Proof of Corollary 3. Considering the equilibrium advertising levels, simple algebra shows that $\phi_i^{o*} < \phi_i^{r*}$ as long as $\lambda < \frac{1}{4}\frac{(v-\gamma)(v-2\gamma)}{\gamma}$. When $\lambda = \frac{1}{4}\frac{(v-\gamma)(v-2\gamma)}{\gamma}$, we have $\phi_i^{o*} = \phi_i^{r*}$; when $\lambda > \frac{1}{4}\frac{(v-\gamma)(v-2\gamma)}{\gamma}$, we have $\phi_i^{o*} > \phi_i^{r*}$. Accordingly, under conditions (A1) and (A2') we always observe that $\lambda < \frac{1}{4}\frac{(v-\gamma)(v-2\gamma)}{\gamma}$ therefore it is always true that $\phi_i^{o*} < \phi_i^{r*}$.■

Proof of Corollary 4. From (34) in the Proof of Proposition 5, it follows that $F_i^r(v-\gamma) = \frac{\phi^{o*}}{\phi^{r*}}\left(1-\frac{\gamma}{v}\right)$. Substituting the optimal advertising intensities in Proposition 5, we obtain $F_i^r(v-\gamma) = \frac{4\lambda}{v+4\lambda-2\gamma} < 1$ for $v > 2\gamma$, which is always the case under (A1). ■

Proof of Proposition 6. Each firm serves its group of selective customers at p^o with probability given by $\tau \in [0,1]$:

$$\tau = \int_{p^r_{\min}}^{v-\gamma} \left(\int_{p^o_{\min}}^{p_B+\gamma} f^o\left(p^o\right) dp^o \right) f\left(p^r\right) dp^r + m^r,$$

or equivalently:

$$\tau = \int_{p^r_{\min}}^{v-\gamma} \left(\int_{p^o_{\min}}^{p_B+\gamma} -\frac{\left(v-\gamma\right)\left(x-1\right)}{x\left(p-\gamma\right)^2} dp \right) \left(\frac{x\gamma - v\left(x-1\right)}{y\left(p+\gamma\right)^2} \right) dp_B + m^r$$

which is equivalent to:

$$\tau = \frac{v-\gamma}{\phi^{r*}v} \left(1 - \frac{1}{\phi^{o*}}\right) \frac{\phi^{o*}\left(v-\gamma\right) - v}{\gamma^2 p^r_{\min}} \left(\left(v - p^o_{\min}\right)\gamma + v\left(\gamma - p^o_{\min}\right) \ln\left(\frac{p^o_{\min}}{p^r_{\min}} \frac{v-\gamma}{v} \right) \right) + m^r$$

since $p^o_{\min} = p^r_{\min} + \gamma$. Substituting ϕ^{r*}, ϕ^{o*}, $p^r_{\min}$ and $p^o_{\min}$ by the equilibrium values obtained in Proposition 5, the result in part (i) of Proposition 6 follows.

We next prove part (ii). For the quadratic technology and from F^o and F^r defined in Proposition 5 it is straightforward to find that:

$$E^o(p) = \int_{p^r_{j\,\min}+\gamma}^{v} pf^o(p)dp = \int_{p^r_{\min}+\gamma}^{v} \frac{p\left(v-\gamma\right)\left(v+4\lambda-2\gamma\right)}{v\left(p-\gamma\right)^2} dp$$

$$E^r(p) = \int_{p^r_{\min}}^{v-\gamma} pf^r(p)dp + (v-\gamma)m^r = \int_{p^r_{\min}}^{v-\gamma} \frac{4pv\lambda}{\left(p+\gamma\right)^2\left(v-\gamma\right)} \frac{v+4\lambda-\gamma}{v+4\lambda-2\gamma} dp + (v-\gamma)m^r$$

It is easy to obtain that

$$E^o(p) = \frac{1}{vp^r_{\min}} \left(v-\gamma\right)\left(v+4\lambda-2\gamma\right)\left(\gamma - p^r_{\min}\ln p^r_{\min}\right) - \frac{1}{v}\left(v+4\lambda-2\gamma\right)\left(\gamma - \left(\ln\left(v-\gamma\right)\right)\left(v-\gamma\right)\right),$$

which simplifies to

$$E^o(p) = \frac{\left(v+4\lambda-2\gamma\right)}{v} \left(\left(v-\gamma\right) \frac{\gamma - p^r_{\min}\ln p^r_{\min}}{p^r_{\min}} - \left(\gamma - \left(\ln\left(v-\gamma\right)\right)\left(v-\gamma\right)\right) \right)$$

and

$$\begin{aligned} E^r(p) &= \frac{4\lambda\left(v+4\lambda-\gamma\right)\left(p^r_{\min}\gamma - v\gamma + \gamma^2 + v\gamma\ln v - vp^r_{\min}\ln\left(p^r_{\min}+\gamma\right) - v\gamma\ln\left(p^r_{\min}+\gamma\right) + vp^r_{\min}\ln v\right)}{\left(p^r_{\min}+\gamma\right)\left(v-\gamma\right)\left(v+4\lambda-2\gamma\right)} \\ &\quad + \frac{(v-\gamma)(v-2\gamma)}{v+4\lambda-2\gamma}, \end{aligned}$$

which simplifies to

$$E^r(p) = \frac{4\lambda\left(v+4\lambda-\gamma\right)\left(vp^o_{\min}\ln\left(\frac{v}{p^o_{\min}}\right) - \gamma\left(v - p^o_{\min}\right)\right)}{p^o_{\min}\left(v-\gamma\right)\left(v+4\lambda-2\gamma\right)} + \frac{(v-\gamma)(v-2\gamma)}{v+4\lambda-2\gamma}.$$

This completes the proof. ■

Proof of Corollary 5. Consider first result *(i)*. From Corollary 2, it follows that $\phi^o = \frac{v}{2[2\lambda+(v-\gamma)]}$, and $\phi^r = \frac{v-\gamma}{4\lambda}\frac{v+4\lambda-2\gamma}{2v+4\lambda-2\gamma}$, which are both decreasing on λ since $\frac{\partial\phi_A^o}{\partial\lambda} = -\frac{v}{(v+2\lambda-\gamma)^2} < 0$ and $\frac{\partial\phi_B^r}{\partial\lambda} = -\frac{1}{8}(v-\gamma)\frac{4v\lambda-3v\gamma+8\lambda^2+2\gamma^2-8\lambda\gamma+v^2}{\lambda^2(v+2\lambda-\gamma)^2} < 0$. The sign of $\frac{\partial\phi_B^r}{\partial\lambda}$ depends on the sign of the polynomial $\Omega = 4v\lambda - 3v\gamma + 8\lambda^2 + 2\gamma^2 - 8\lambda\gamma + v^2$

Note that, under condition (A2'):

$$\max\left\{\frac{(5v-9\gamma)^{1/2}(v-\gamma)^{1/2}-(v-\gamma)}{8}, \frac{\gamma\left(\sqrt{2}+1\right)}{4}\right\} \leq \lambda \leq \frac{(v-3\gamma)(v-\gamma)}{8\gamma}$$

we have $8\gamma\lambda \leq (v-3\gamma)(v-\gamma)$, or equivalently,

$$8\gamma\lambda \leq v^2 - 4v\gamma + 3\gamma^2. \tag{40}$$

Note also that, Ω can be re-written as follows

$$\begin{aligned}\Omega &= 4v\lambda - 3v\gamma + 8\lambda^2 + 2\gamma^2 - 8\lambda\gamma + v^2 \\ &= 4v\lambda - 4v\gamma + v\gamma + 8\lambda^2 + 3\gamma^2 - \gamma^2 - 8\lambda\gamma + v^2\end{aligned}$$

or equivalently:

$$\begin{aligned}\Omega &= \left[v^2 - 4v\gamma + 3\gamma^2 - 8\lambda\gamma\right] + 4v\lambda + v\gamma + 8\lambda^2 - \gamma^2 \\ &= \left[v^2 - 4v\gamma + 3\gamma^2 - 8\lambda\gamma\right] + 4v\lambda + \gamma(v-\gamma) + 8\lambda^2\end{aligned}$$

The term in brackets is positive under Assumption (A2'), as shown in condition (40). The other terms are also positive since $v > \gamma$. Therefore $\frac{\partial\phi_B^r}{\partial\lambda} = -\frac{1}{8}(v-\gamma)\frac{\Omega}{\lambda^2(v+2\lambda-\gamma)^2} < 0$

Regarding result *(ii)*, we have that $p_{B\min}^r = (v-\gamma)\frac{v-2\gamma+4\lambda}{2v-2\gamma+4\lambda}$ (see Corollary 2), with $\frac{\partial p_{B\min}^r}{\partial\lambda} = \frac{1}{2}(v-\gamma)\frac{v+4\lambda-2\gamma}{v+2\lambda-\gamma} > 0$ and therefore, the minimum price in the equilibrium support of both c.d.f $F^r(p)$ and $F^o(p)$ is increasing in the marginal advertising cost λ.

Finally, concerning result *(iii)*, from Corollary 4 it follows that $m^r = \frac{v-2\gamma}{4\lambda+v-2\gamma}$, with $\frac{\partial m^r}{\partial\lambda} = -4\frac{v-2\gamma}{(v+4\lambda-2\gamma)^2} < 0.$■

Proof of Proposition 7. From the equilibrium solutions and assuming that (A1) and (A2') are satisfied, each firm's profits with targeted and mass advertising are given by

$$E\pi^t = \left(\frac{2\lambda+v-\gamma}{2\lambda}\right)A(\phi^{o*}) + A(\phi^{r*})$$

and

$$E\pi^m = A(\phi^{m*}),$$

respectively. From $E\pi^t - E\pi^m > 0$ we have:

$$\left(\frac{2\lambda+v-\gamma}{2\lambda}\right)A(\phi^{o*}) + A(\phi^{r*}) - A(\phi^{m*}) > 0$$

from which we obtain

$$\left(\frac{2\lambda+v-\gamma}{2\lambda}\right) > \frac{(\phi^{m*})^2 - (\phi^{r*})^2}{(\phi^{o*})^2}$$

Denoting $\chi^*(\lambda, v, \gamma) = \frac{(\phi^{m*})^2 - (\phi^{r*})^2}{(\phi^{o*})^2}$, the previous inequality can be written as:

$$2\lambda + v - \gamma > 2\lambda\chi^*(\lambda, v, \gamma)$$

or, equivalently,

$$2\lambda [1 - \chi^*(\lambda, v, \gamma)] + v - \gamma > 0.$$

Otherwise, $E\pi^m > E\pi^t$. If $2\lambda [1 - \chi^*(\lambda, v, \gamma)] + v - \gamma = 0$ then $E\pi^t = E\pi^m$.■

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