Numerical Experiments with a Modified Regularization Scheme for Mathematical Programs with Complementarity Constraints

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Abstract

On this paper we present a modified regularization scheme for Mathematical Programs with Complementarity Constraints. In the regularized formulations the complementarity condition is replaced by a constraint involving a positive parameter that can be decreased to zero. In our approach both the complementarity condition and the nonnegativity constraints are relaxed. An iterative algorithm is implemented in MATLAB language and a set of AMPL problems from MacMPEC database were tested.

Keywords: complementarity constraints, regularization scheme, SQP

1 Introduction

Mathematical Programs with Complementarity Constraints (MPCC) is an exciting application of nonlinear programming techniques. These kind of constraints may come in the form of a game, a variational inequality or as stationary conditions of an optimization problem. The main applications areas are Engineering and Economics [Ferris and Pang, 1997], [Outrata et al., 1998]. They are so pervasive in these areas because the concept of complementarity is tantamount with the notion of system equilibrium. They are very difficult to solve as the usual constraint qualifications necessary to guarantee the algorithms convergence fail in all feasible points [Chen and Florian, 1995]. This complexity is caused by the disjunctive nature of the complementarity constraints, from a geometric point of view, its feasible region is not convex nor generally connected.

There have been proposed some nonlinear approaches to solve MPCC, starting with the smoothing scheme [Facchinei et al., 1996] and [Fukushima and Pang, 1999], the regularization scheme [Scholtes, 2001], the modified relaxation scheme [Lin and Fukushima, 2005] and [Demiguel et al., 2005] where a two-sided relaxation scheme is presented. While on this work we focus on a regularization scheme to solve MPCC, there are other nonlinear approaches. For example the penalty approaches [Hu and Ralph, 2004], [Ralph and Wright, 2004], [Monteiro and Meira, 2011], [Monteiro and Rodrigues, 2011] and the "elastic mode" for nonlinear programming in conjunction with a sequential quadratic programming (SQP) algorithm [Anitescu, 2005]. We also emphasize the work [Raghunathan and Biegler, 2003] that uses interior point methods. In [Fletcher et al., 2006], SQP is guaranteed, to under relatively mild conditions, quadratically converge near a stationary point.

This paper is organized as follows. Next section defines the MPCC problem and presents our modified regularization scheme. Some concepts related to the optimality conditions are presented in Section 3. The implemented algorithm in MATLAB environment is detailed in Section 4. Numerical experiments to test the algorithm, are presented in Section 5. Some conclusions and future work ideas are carried out in Section 6.

2 Problem definition

We consider Mathematical Program with Complementarity Constraints (MPCC) as:

$$\begin{array}{ll} \min\limits_{x} & f(x) \\ \text{subject to} & g(x) \geq 0,\ h(x) = 0, \\ & 0 \leq G(x) \perp H(x) \geq 0, \end{array} \tag{2.1}$$

where $f : \mathbb{R}^n \to \mathbb{R}, g : \mathbb{R}^n \to \mathbb{R}^m, h : \mathbb{R}^n \to \mathbb{R}^p, G : \mathbb{R}^n \to \mathbb{R}^q, H : \mathbb{R}^n \to \mathbb{R}^q$ are all twice continuously differentiable functions. The notation $G(x) \perp H(x)$, means that $G(x)^T H(x) = 0$, due the complementarity nature of the complementarity constraints or $G_i(x) = 0$ or $H_i(x) = 0$, $i = 1, \ldots, q$. One attractive way of solving (2.1) is to replace the complementarity constraints by a set of nonlinear inequalities, such as $G_i(x)H_i(x) \leq 0$, and then solve the equivalent nonlinear program (NLP):

$$\begin{array}{ll} \min\limits_{x} & f(x) \\ \text{subject to} & g(x) \geq 0,\ h(x) = 0, \\ & G(x) \geq 0,\ H(x) \geq 0, \\ & G_i(x)H_i(x) \leq 0, \\ & i = 1, 2, \ldots, q. \end{array} \tag{2.2}$$

The major difficulty in solving (2.2) is that its constraints fail to satisfy the Mangasarian Fromovitz constraint qualification (MFQC) at any feasible point [Scholtes, 2001].
In this paper, we propose the following modified regularization scheme to solve the problem (2.1):

$$\begin{array}{ll} \min\limits_{x} & f(x) + \rho t \\ \text{subject to} & g(x) \geq 0,\ h(x) = 0, \\ & (G_i(x) + t)(H_i(x) + t) \geq t^2 \\ & (G_i(x) - t)(H_i(x) - t) \leq t^2 \\ & i = 1, 2, \ldots, q, \end{array} \tag{2.3}$$

where ρ is a parameter to penalize the relaxation parameter t. This reformulation has less constraints than problem (2.2). Figure 1 represents the feasible region of the complementarity constraints for the relaxation scheme proposed.

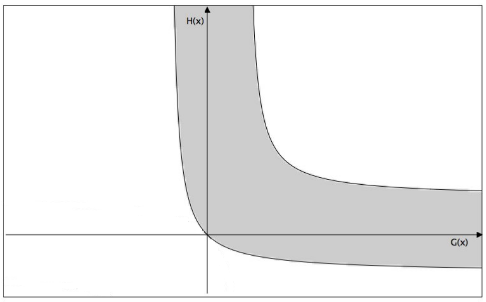

Figure 1: Feasible region of the complementarity constraints.

The novelty of our approach is the combination of [Lin and Fukushima, 2005] and [Kadrani et al., 2009] strategies also penalizing the regularization parameter t. This is similar to the so-called elastic mode, but in our work the complementarity constraints are maintained as constraints of problem. In the elastic mode strategy, the complementarity constraints are removed from the set of constraints and are included in the penalty function as a penalty term.

3 Optimal issues

Recent developments [Anitescu et al., 2006] show that a SQP method with an elastic mode is used to solve MPCC and there is a relationship between strong stationarity defined by [Scheel and Scholtes, 2000] and the Karush-Kuhn-Tucker (KKT) points. This relationship establishes convergence of SQP methods for MPCC formulated as NLP. Concepts like, stationarity, constraints qualification (LICQ - linear independence constraint qualification) and second order conditions (SOSC - second order sufficient condition) of the MPCC problem, will be defined in terms of the relaxed nonlinear program (RNLP) for (2.1) as follows:

$$\begin{array}{ll} \min\limits_{x} & f(x) \\ \text{subject to} & g(x) \geq 0,\ h(x) = 0, \\ & G_i(x) \geq 0,\ i \in I_G \setminus I_H, \\ & H_i(x) \geq 0,\ i \in I_H \setminus I_G, \\ & G_i(x) \geq 0,\ H_i(x) \geq 0,\ i \in I_G \cap I_H, \end{array} \tag{3.1}$$

where I_g, I_G and I_H are the following active sets at the point x^* feasible for (2.1):

$$\begin{array}{l} I_g = \{i \in \{1,2,\ldots,m\} | g_i(x^*) = 0\}, \\ I_G = \{i \in \{1,2,\ldots,q\} | G_i(x^*) = 0\}, \\ I_H = \{i \in \{1,2,\ldots,q\} | H_i(x^*) = 0\}. \end{array}$$

For a general constraints vector $v \in \mathbb{R}^n$ we consider a matrix ∇v containing the constraints gradients along the columns. The optimality concepts follow [Ralph and Wright, 2004] and the corresponding proves can be consulted in this work. There are several kinds of stationarity defined for MPCC problem, among them, the strong stationarity is the following one.

Definition 1. A point x^* that is feasible for (2.1) is strongly stationary if $d = 0$ ($d \in \mathbb{R}^n$) solves the following linear program:

$$\begin{array}{ll} \min\limits_{d} & \nabla f(x^*)^T d \\ \text{subject to} & g(x^*) + \nabla g(x^*)^T d \geq 0,\ h(x^*) + \nabla h(x^*)^T d = 0, \\ & \nabla G_i(x^*)^T d = 0,\ i \in I_G \setminus I_H, \\ & \nabla H_i(x^*)^T d = 0,\ i \in I_H \setminus I_G, \\ & \nabla G_i(x^*)^T d \geq 0,\ \nabla H_i(x^*)^T d \geq 0,\ i \in I_G \cap I_H. \end{array} \tag{3.2}$$

Combining the optimality conditions for (3.2) with the feasibility conditions for x^*, we obtain:

$$0 = \nabla f(x^*) - \sum_{i \in I_g} \lambda_i^* \nabla g_i(x^*) - \sum_{i=1}^{p} \mu_i^* \nabla h_i(x^*) - \sum_{i \in I_G} \tau_i^* \nabla G_i(x^*) - \sum_{i \in I_H} \nu_i^* \nabla H_i(x^*) \tag{3.3}$$

$$\begin{array}{l} 0 = h_i(x^*),\ i = 1,2,\ldots,p, \\ 0 = g_i(x^*),\ i \in I_g, \\ 0 < g_i(x^*),\ i \in \{1,2,\ldots,m\} \setminus I_g, \\ 0 \leq \lambda_i^*,\ i \in I_g, \\ 0 = G_i(x^*),\ i \in I_G, \\ 0 < G_i(x^*),\ i \in \{1,2,\ldots,q\} \setminus I_G, \\ 0 = H_i(x^*),\ i \in I_H, \\ 0 < H_i(x^*),\ i \in \{1,2,\ldots,q\} \setminus I_H, \\ 0 \leq \tau_i^*,\ i \in I_G \cap I_H, \\ 0 \leq \nu_i^*,\ i \in I_G \cap I_H. \end{array} \tag{3.4}$$

Definition 2. The MPCC-LICQ is satisfied at the point x^* if the following set of vectors is linear independent:

$$\{\nabla g_i(x^*) | i \in I_g\} \cup \{\nabla h_i(x^*) | i = 1,2,\ldots,p\} \cup \{\nabla G_i(x^*) | i \in I_G\} \cup \{\nabla H_i(x^*) | i \in I_H\}.$$

The linear independence constraint qualification (LICQ) is satisfied for RNLP (3.1).

For a strongly stationary point x^* and for some $(\lambda_i^*, \mu_i^*, \tau_i^*, \nu_i^*)$ satisfying (3.3-3.4) one defines the following sets:

$$\begin{array}{l} I_g^+ = \{i \in I_g | \lambda_i^* > 0\}, \\ I_g^0 = I_g \setminus I_g^+, \\ J_G^+ = \{i \in I_G \cap I_H | \tau_i^* > 0\}, \\ J_G^0 = (I_G \cap I_H) \setminus J_G^+, \\ J_H^+ = \{i \in I_G \cap I_H | \nu_i^* > 0\}, \\ J_H^0 = (I_G \cap I_H) \setminus J_H^+. \end{array}$$

The set $\bar{S}$ of normalized critical directions for the RNLP (3.1) is defined as follows ($s \in \mathbb{R}^n$):

$$\bar{S} = \{s | \|s\|_2 = 1\} \cap \{s | \nabla h(x^*)^T s = 0\} \cap \{s | \nabla g_i(x^*)^T s = 0, i \in I_g^+\} \cap \{s | \nabla g_i(x^*)^T s \geq 0, i \in I_g^0\} \cap$$

$$\cap \{s | \nabla G_i(x^*)^T s = 0, i \in I_G \setminus I_H\} \cap \{s | \nabla G_i(x^*)^T s \geq 0, i \in J_G^0\} \cap \{s | \nabla G_i(x^*)^T s = 0, i \in J_G^+\} \cap$$

$$\cap \{s | \nabla H_i(x^*)^T s = 0, i \in I_H \setminus I_G\} \cap \{s | \nabla H_i(x^*)^T s \geq 0, i \in J_H^0\} \cap \{s | \nabla H_i(x^*)^T s = 0, i \in J_H^+\}.$$

The set of normalized critical directions S^* for (2.1) is:

$$S^* = \bar{S} \cap \{s | \min(\nabla H_i(x^*)^T s, \nabla G_i(x^*)^T s) = 0, i \in J_G^0 \cap J_H^0\}$$

This is obtained by enforcing the additional condition that either $\nabla H_i(x^*)^T s = 0$ or $\nabla G_i(x^*)^T s = 0$, $i \in J_G^0 \cap J_H^0$.

Definition 3. Let x^* be a strongly stationary point. The MPCC-SOSC holds at x^* if there is $\sigma > 0$ such that for every $s \in S^*$, there are multipliers $(\lambda_i^*, \mu_i^*, \tau_i^*, \nu_i^*)$ satisfying (3.3-3.4) such that

$$s^T \nabla_{xx}^2 L(x^*, \lambda_i^*, \mu_i^*, \tau_i^*, \nu_i^*) s \geq \sigma.$$

The RNLP-SOSC holds at x^* if for every $s \in \bar{S}$, there are $(\lambda_i^*, \mu_i^*, \tau_i^*, \nu_i^*)$ such that (3.3) holds.

As theoretical support, we summarized some known results based on [Ralph and Wright, 2004] concerning constraint qualifications and first and second order optimality conditions of MPCC. In the same work some properties of regularization schemes are presented: estimating distance between their solutions and the MPCC optimum, boundedness of their Lagrange multipliers and local uniqueness of their solutions. Some alternative regularized formulations are also studied. A penalty approach similar to the elastic mode from [Anitescu et al., 2006] is also analyzed.

Based on these ideas and on [Lin and Fukushima, 2005] and [Kadrani et al., 2009], a computational implementation of a modified regularized scheme was developed. Details of the corresponding algorithm are in next section.

4 Algorithm details

An algorithm was implemented (Algorithm 1) to iteratively solve problem (2.3) for specific values of t and ρ, with $t \to 0$ and $\rho \to \infty$. This algorithm has two iterative procedures, the inner one is performed by `fmincon` routine from MATLAB Optimization toolbox, that uses the SQP method.

Algorithm 1 Modified regularization scheme

Take initial values x_0, $t_0 > 0$, $\rho_0 > 0$ and tolerances ϵ_1, ϵ_2.
for $k = 0, 1, 2, \ldots$ **do**
  Solve the minimization problem (2.3) with x_k, t_k and ρ_k obtaining x_{k+1}.
  if $\|\nabla L(x_{k+1}, \ldots)\| \leq \epsilon_1$ and $\|f(x_{k+1}) - f(x_k))\| \leq \epsilon_2$ **then**
    STOP.
  else
    $\rho_{k+1} = r_1 \rho_k,\ r_1 > 1.$
    $t_{k+1} = r_2 t_k,\ 0 < r_2 < 1.$
  end if
end for

To evaluate the stop criterium in the algorithm, we consider the following equality in the solution x^*:

$$\nabla L(x^*, \delta, \gamma, \xi, \zeta) = \nabla f(x^*) - \sum_{i=1}^{m} \delta_i \nabla g_i(x^*) - \sum_{i=1}^{p} \gamma_i \nabla h_i(x^*) - \sum_{i=1}^{q} \xi_i \nabla \Phi_{t,i}(x^*) + \sum_{i=1}^{q} \zeta_i \nabla \Psi_{t,i}(x^*)$$

where for $i = 1, 2, \ldots q$ and $x \in \mathbb{R}^n$,

$$\Phi_{t,i}(x) = (G_i(x) + t)(H_i(x) + t) - t^2$$

$$\Psi_{t,i}(x) = (G_i(x) - t)(H_i(x) - t) - t^2,$$

we have,

$$\nabla\Phi_{t,i}(x) = (G_i(x) + t)\nabla H_i(x) + (H_i(x) + t)\nabla G_i(x),$$

$$\nabla\Psi_{t,i}(x) = (G_i(x) - t)\nabla H_i(x) + (H_i(x) - t)\nabla G_i(x).$$

The Lagrange multipliers δ, γ, ξ and ζ are an output of the `fmincon` routine from MATLAB. The tolerances used in the stop criterium are $\epsilon_1 = \epsilon_2 = 10^{-4}$. We consider $r_1 = 10$ and $r_2 = 0.1$ in the parameters update scheme. The initial choices, $t_0 = 0.25$ and $\rho_0 = 1$ were considered. Next section reports the numerical results using 70 test problems.

5 Numerical results

This section describes the experiments with an implementation of our modified regularization scheme for problem (2.1). The computational experiments were made on a *2.26 GHz Intel Core 2 Duo* with 8GB of RAM, MAC OS 10.6.8 operating system. The MATLAB version used was 7.11.0 (R2010b). The `fmincon` routine is connected to the modeling language AMPL [Fourer and Kernighan, 1993] by a MATLAB mex interface. The test problems in Table 1 are from MacMPEC database [Leyffer, 2000].
In order to evaluate and compare the performance of the Algorithm 1, we implemented another regularized formulation (Algorithm 2) proposed in [Lin and Fukushima, 2005], using the same conditions. Table 2 reports the numerical results of both algorithms. The first column indicates the test problem, next six columns refer to Algorithm 1: f^* shows the final objective function value, $\|\nabla L\|$ presents the norm of the Lagrangian function of problem (2.3), `int` presents the number of internal iterations performed by the `fmincon` routine from Matlab, `ext` shows the number of external iterations, the last two columns report the number of function evaluations and the exit status, respectively. The last six columns present the corresponding results from Algorithm 2. Table 3 summarizes the exit status of the algorithms.

Table 1: Test problems

Problem	n	m	p	q	Problem	n	m	p	q
bar-truss-3	35	6	28	6	gnash16	13	8	4	8
bard1	5	3	1	3	gnash17	13	8	4	8
bard3	6	2	3	1	gnash18	13	8	4	8
bard1m	6	3	1	3	gnash19	13	8	4	8
bard3m	6	4	1	3	hs044-i	20	10	4	10
bilevel3	11	4	6	3	jr1	2	1	0	1
dempe	3	1	1	1	jr2	2	1	0	1
desilva	6	2	2	2	kth1	2	1	0	1
df1	2	3	0	1	kth2	2	1	0	1
ex9.1.1	13	5	7	5	kth3	2	1	0	1
ex9.1.2	8	2	5	2	nash1a	6	2	2	2
ex9.1.3	23	6	15	6	nash1b	6	2	2	2
ex9.1.4	8	2	5	2	nash1c	6	2	2	2
ex9.1.5	13	5	7	5	nash1d	6	2	2	2
ex9.1.6	14	6	7	6	portfl-i-1	87	12	13	12
ex9.1.7	17	6	9	6	portfl-i-2	87	12	13	12
ex9.1.8	11	4	5	3	portfl-i-3	87	12	13	12
ex9.1.9	12	5	6	5	portfl-i-4	87	12	13	12
ex9.1.10	11	4	5	3	portfl-i-6	87	12	13	12
ex9.2.1	10	4	5	4	qpec1	30	20	0	20
ex9.2.2	9	4	4	3	qpec2	30	20	0	20
ex9.2.4	8	2	5	2	ralph1	2	1	0	1
ex9.2.6	16	6	6	6	ralph2	2	1	0	1
ex9.2.7	10	4	5	4	ralphmod	104	100	0	100
ex9.2.8	6	2	3	2	scholtes1	3	1	0	1
ex9.2.9	9	3	5	3	scholtes2	3	1	0	1
flp2	4	2	0	2	scholtes3	3	1	0	1
flp4-1	80	60	0	30	scholtes4	3	1	0	1
flp4-2	110	110	0	60	scholtes5	3	1	0	1
flp4-3	140	170	0	70	scale1	2	1	0	1
flp4-4	200	250	0	100	scale2	2	1	0	1
gnash10	13	8	4	8	scale3	2	1	0	1
gnash11	13	8	4	8	scale5	2	1	0	1
gnash12	13	8	4	8	stackelberg1	3	1	1	1
gnash15	13	8	4	8	taxmcp	15	11	3	11

The algorithms present similar behaviour with respect to the internal and external iterations, to the function evaluations and to the solution accuracy. However Algorithm 2 does not solve eight problems, whereas Algorithm 1 only fails in three problems. Some of them are ill-posed, for instance, `ralph1`, `ralphmod` and `scholtes4`. The solutions obtained by both algorithms are similar to the ones reported in MacMPEC database with good accuracy.

6 Conclusions and future work

An iterative algorithm in MATLAB language to solve MPCC was implemented. The algorithm aims to compute a local optimal solution joining a modified regularization scheme and the SQP strategy. The algorithm is still in an improvement phase but some conclusions can already be taken: the promising

Table 2: Results of Algorithm 1 and Algorithm 2

Problem	f^*	$\|\nabla L\|$	int	ext	nfe	flag	f^*	$\|\nabla L\|$	int	ext	nfe	flag
bar-truss-3	10166,512	1,36E-05	39	5	1689	1	10166,5519	2,57E-05	61	5	2606	1
bard1	17	1,71E-07	25	5	210	1	16,9999952	2,46E-07	13	3	114	1
bard3	-12,679	1,49E-15	3	2	38	1	-12,678711	1,16E-06	3	2	38	1
bard1m	17	6,77E-08	21	5	208	1	16,9999952	1,90E-06	17	3	162	1
bard3m	-12,679	1,28E-07	19	5	187	1	-12,678722	1,33E-07	17	3	157	1
bilevel3	-12,679	1,04E-07	31	5	469	1	-12,678752	8,77E-08	29	5	438	1
dempe	28,25	2,47E-07	53	5	358	1	28,2500113	2,82E-07	45	2	311	1
desilva	-1	2,71E-08	3	2	38	1	-1	1,75E-08	3	2	38	1
df1	0	2,98E-08	5	2	28	1	0	1,49E-08	103	100	429	0
ex9.1.1	-13	2,82E-08	57	5	925	1	-13,000062	4,22E-08	50	5	813	1
ex9.1.2	-6,25	1,67E-08	23	4	257	1	-6,2500625	1,26E-08	14	4	167	1
ex9.1.3	-29,2	1,05E-05	46	6	1294	1	-29,20002	9,05E-07	39	3	1047	1
ex9.1.4	-37	4,00E-08	48	4	550	1	-37,000078	9,10E-09	44	4	494	1
ex9.1.5	-1	1,33E-08	62	6	1064	1	-1,0000003	3,90E-06	52	7	885	1
ex9.1.6	-49	7,57E-07	90	4	1573	1	-49,00035	3,73E-08	84	4	1558	1
ex9.1.7	-26	7,03E-05	69	6	1432	1	-26,000065	9,99E-08	58	6	1249	1
ex9.1.8	-3,25	3,01E-16	7	2	115	1	-3,25	0,00E+00	9	3	141	1
ex9.1.9	3,111	3,20E-09	80	6	1444	1	3,11108333	1,08E-07	80	5	1385	1
ex9.1.10	-3,25	3,01E-16	7	2	115	1	-3,25	0,00E+00	9	3	141	1
ex9.2.1	17	1,56E-07	25	5	355	1	16,9999952	1,28E-06	14	3	201	1
ex9.2.2	100	2,60E-06	40	5	490	1	99,9998557	5,67E-06	37	5	457	1
ex9.2.4	0,5	1,53E-08	25	6	295	1	0,49999994	9,69E-07	15	4	186	1
ex9.2.6	-1	3,34E-08	17	6	374	1	-1,0000001	1,40E-06	12	4	268	1
ex9.2.7	17	1,56E-07	25	5	355	1	16,9999952	1,28E-06	14	3	201	1
ex9.2.8	1,5	3,52E-09	11	5	123	1	1,49999997	2,90E-09	9	4	93	1
ex9.2.9	2	7,68E-10	10	2	131	1	2	0,00E+00	9	2	119	1
flp2	0	5,03E-07	19	2	124	1	0	1,53E-07	56	17	415	1
flp4-1	0	1,32E-25	2	2	326	1	0	0,00E+00	4	4	571	1
flp4-2	0	6,17E-25	2	2	446	1	0	0,00E+00	3	3	558	1
flp4-3	0	7,02E-16	2	2	566	1	0	0,00E+00	3	3	708	1
flp4-4	0	8,01E-25	2	2	806	1	0	0,00E+00	3	3	1008	1
gnash10	-230,824	3,39E-07	27	5	475	1	-230,82343	1,47E-07	14	2	238	1
gnash11	-129,913	2,19E-07	28	5	490	1	-129,91193	3,60E-06	17	3	297	1
gnash12	-36,933	2,30E-07	29	5	505	1	-36,933107	3,59E-06	15	3	267	1
gnash15	-354,7	4,00E-07	41	5	685	1	-256,38936	2,90E-07	69	8	1322	1
gnash16	-241,442	4,22E-07	44	5	737	1	-129,91192	3,70E-05	87	9	2000	1
gnash17	-90,749	2,45E-07	52	5	855	1	-36,933107	1,02E-05	78	9	1981	1
gnash18	-25,698	4,25E-07	58	5	941	1	-25,698216	1,01E-06	29	4	497	1
gnash19	-6,117	4,42E-07	62	6	1018	1	-	-	-	-	-	-
hs044-i	15,618	1,88E-06	39	6	984	1	15,617653	4,24E-06	36	5	897	1
jr1	0,5	2,64E-07	29	6	138	1	0,4999975	2,14E-08	30	6	142	1
jr2	0,5	1,83E-07	27	6	129	1	0,49999988	4,63E-07	18	4	88	1
kth1	0	4,71E-16	3	2	18	1	-3,548E+15	1,41E+00	124	100	821	0
kth2	0	1,63E-08	5	2	26	1	0	1,07E-08	4	2	19	1
kth3	0,5	1,76E-07	26	6	122	1	0,49999994	5,20E-08	22	4	101	1
nash1a	0	2,87E-07	6	1	55	1	0	2,36E-07	19	5	181	1
nash1b	0	1,32E-07	17	2	150	1	0	2,36E-07	21	5	196	1
nash1c	0	4,68E-07	19	2	167	1	0	2,36E-07	20	5	188	1
nash1d	0	5,24E-07	16	2	142	1	0	2,40E-07	21	5	190	1
portfl-i-1	0	2,55E-07	12	2	1244	1	1,424E-05	1,26E-06	43	9	4379	1
portfl-i-2	0	7,72E-07	11	2	1156	1	1,4573E-05	2,54E-07	37	9	4004	1
portfl-i-3	0	6,17E-07	11	2	1155	1	0	6,10E-07	11	2	1155	1
portfl-i-4	0	6,54E-07	11	2	1156	1	0	3,85E-07	10	3	1066	1
portfl-i-6	0	1,75E-06	13	2	1334	1	2,3417E-06	3,86E-07	40	10	4112	1
qpec1	80	1,72E-07	8	2	318	1	80	8,64E-07	9	2	350	1
qpec2	45	6,33E-07	62	6	2143	1	44,9999	3,51E-06	61	6	2110	1
ralph1	0	0	52	7	229	2	-6,475E-07	0,00E+00	309	100	1536	0
ralph2	0	2,88E-07	38	5	167	1	-1E-10	9,10E-07	544	100	2476	0
ralphmod	-683,033	6,69E-05	282	10	31101	2	-683,03302	8,49E-05	182	10	20040	1
scholtes1	2	4,55E-08	9	2	53	1	2	5,43E-08	8	2	44	1
scholtes2	15	1,33E-07	12	2	61	1	15	1,19E-07	8	2	49	1
scholtes3	0,5	1,68E-08	39	6	177	1	0,49999994	5,77E-08	33	4	145	1
scholtes4	0	0	58	7	318	2	-3,694E-07	0,00E+00	312	100	1960	0
scholtes5	1	1,46E-06	30	6	176	1	0,999995	1,46E-06	30	6	176	1
scale1	1	2,69E-06	36	10	212	1	0,99999925	2,23E+00	123	100	2626	0
scale2	1	1,75E-06	24	6	145	1	0,99999988	1,49E-06	17	4	94	1
scale3	1	3,67E-05	35	8	258	1	1	1,49E-04	131	100	930	0
scale5	99,999	4,08E-05	34	6	209	1	99,9999875	4,60E-05	27	4	146	1
stackelberg1	-3266,672	3,75E-06	16	4	96	1	-3266,6725	2,11E-06	16	4	96	1
taxmcp	0,819	1,29E-08	31	6	623	1	0,81870494	8,65E-07	29	8	591	1

Table 3: Algorithms exit flags

flag	status
0	Terminated by iteration limit (100)
1	Found an optimal solution of MPCC
2	`fmincon` stopped because step length to small (10^{-6})

numerical results present good accuracy of the solutions when compared with the ones provided from the MacMPEC test problem database. The implemented approach proved to be competitive when compared with other regularized scheme proposed in literature.

As future work, it is intended to study what kind of stationary point is achieved by this regularization scheme. Another future work idea is to implement other schemes to update the regularization and penalization parameters, based on a feasibility test.

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