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Finite element model with imposed slip surfaces for earth mass safety evaluation

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Abstract: The study of earth masses requires numerical methods that provide the quantification of the safety factor without requiring detrimental assumptions. For that, equilibrium analysis can perform fast computations but require assumptions that limit its potentiality. Limit analysis does not require detrimental assumptions but are numerically demanding. This work provides a new approach that combines the advantage of both the equilibrium method and the limit analysis. The defined hybrid model allows probabilistic analysis and optimization approaches without the assumption of interslice forces. It is compared with a published case and used to perform probabilistic studies in both a homogeneous and a layered foundation. Analyses show that the shape of the density probability functions is highly relevant when computing the probability of failure, and soil elasticity hardly affects the safety of factor of the earth mass.

Key words: slope stability; probabilistic methods; reliability; genetic algorithms; safety factor

1 Introduction

Slopes, earth dams, concrete dams, earth retaining structures, tunnels, underground excavations, and foundations of any kind are a particular type of structures in which the living loads are small when compared with the dead ones. These types of structures are very common in geotechnical engineering and are analysed by the limit analysis (based on the plasticity theory and FEM method), the method of slices (based on the limit equilibrium theory), and other methods such as the rigid finite element method RFEM and the slip-line method. A review of all these methods can refer to Ref. [1].

With respect to the limit analysis, a statically admissible stress field distribution or a cinematically admissible failure mechanism is assumed and then an objective function is optimized with respect to a limited number of parameters in order to define the limit load. This method was used by several researchers [2−8]. If a static approach is used, and if a statically admissible stress field exists (SLOAN [9] stated that a statically admissible stress field is admissible if it satisfies the stress boundary conditions, the equilibrium equations, and the yield condition, i.e., the stresses must lie inside or on the yield surface in stress space), it can be ascertained that the applied load is less than the collapse load [10].

Consequently, the static approach provides a lower bound value. On the other hand, if a kinematic approach is used, a cinematically admissible velocity field is searched to have external plastic power dissipation higher than the internal one. In this situation, the applied load can be defined as the load under which there exists a statically admissible stress field [10]. This means that an upper bound is defined. The disadvantages in limit analysis are therefore to determine the lower and upper bounds values for the loads as close as possible to the collapse one, and the required computational effort to compute stress and velocity field distribution. However, since limit analysis is based on the FEM method and the plasticity theory, it can be applied to geotechnical structures of arbitrary geometry and complex load conditions. It can also model the non-linear soil behaviour and the post-failure behaviour of slopes [11].

Regarding the method of slices (based on the limit equilibrium theory), it postulates that the slope might fail by a mass of soil sliding along a failure surface. This method was used by several researchers [12−14]. Several approaches exist with different assumptions on the shape of the slip surfaces (circular or not) and the interslice forces. In all of them, failure does not occur if the shear strength is not fully mobilized all the way along the failure surface and all slices of the sliding mass are in

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static equilibrium. The method is characterized by the division of the sliding mass into slices, with forces acting on the base of the slice (normal and shear forces), the interslice forces, and the self-weight of the soil slice. All these forces must be in equilibrium. However, the interslice forces lines of action and/or magnitudes are unknown at the outset of the analysis. Also, since the number of equilibrium equations is always less than the number of unknowns, assumptions have to be made, which can lead to a wrong estimation of the safety factor. For the general method of slices, two main approaches can be defined: the first considers the equilibrium equations written in terms of displacements/forces, while in the second one, the equations are written in terms of stresses/strains. But in both situations, assumptions are made with respect to interslice behaviour, which in some approaches do not comply with all equilibrium conditions. The most well known methods of this classical type of approach are the ordinary method of slices, Bishop's method, force equilibrium methods, Morgenstern and Price's method, Janbu's generalised procedures of slices, and Spencer's Method. These methods must be seen as being able to provide an upper limit for the safety factor.

According to JIANG and MAGNAN [10], when using the method of slices, the following disadvantages exist.

1) The method cannot provide a correct value of the safety factor because of the assumptions involved in the forces on each slice.

2) The stress outside the assumed failure surface may violate the yield criterion.

3) A circular failure surface is not suitable for geotechnical structures with strong heterogeneity.

4) A shear failure assumption may not be valid when soil is modeled with complex criterion functions of non-linear form [15].

Some of these disadvantages in equilibrium methods come from the division of the sliding mass into slices [2], which requires further assumptions regarding interslice force directions. These assumptions are artificial and the main characteristic that distinguishes one limit equilibrium approach from another. For example, the ordinary method of slices does not consider forces between slices, while Bishop's method does (even if the energetic dissipation induced by relative movements between the slices is not considered in both methods). The use of limit analyses is then advantageous with respect to the equilibrium methods [2], since no assumption is needed about the shape or location of the failure surface (it occurs through the zones within the soil mass which are unable to resist to the applied shear stresses); the concept of slices is not used (which implies that there is no need for assumptions about interslice forces); and it is able to monitor progressive failure. However, despite the advantages of the limit analyses approach, the method of slices is still frequently used due to its simplicity and low computational efforts requirements, making it ideal to use with optimization algorithms. Also, engineers are often sceptical of the need of accurate methods like limit analysis, mainly due to the poor quality of soil properties often available and non-linear analysis is harder to justify in practice because there is usually a significant increase in complexity which may require the help of a modelling specialist. Moreover, the definition of a failure surface is not always possible in limit analysis.

Independent of the method used to evaluate the safety factor of earth masses, it is well known that soil properties are ruled by uncertainty due to inherent spatial variability and/or scarcity of representative data. Also, since the classical FEM method is a deterministic technique that does not deal with the stochastic nature of design parameters, it is not possible to guarantee that a design based on deterministic analysis using averaged values of the soil parameters will perform successfully. To overcome these limitations, engineers can use probabilistic analysis [16−19]. In geotechnical modelling, homogeneous soil layers are often considered, and each shear strength parameter may be described by a given probability distribution function, being the Gaussian commonly used. RUSSELLI [20] points out that little data are known about the skewness coefficient (it describes the degree of asymmetry of a distribution function) which implies the need of further testing. Nevertheless, the analysis performed showed that a Gaussian distribution may be assumed for the friction angle, but for the effective cohesion, a skewness value ranging from 1.7 to 4.0 was obtained [20]. Consequently, a lognormal distribution may better represent effective cohesion. Since the soil probabilistic data (i.e., mean value μ and standard deviation σ for all mechanical parameters) are usually very difficult to obtain in practice, and parameters found in similar soils may be used. PHOON and KULHAWY [21] presented data for a sand and a clay layers and found values for the coefficient of variation (σ/μ) in the range of 5% to 15% for the effective friction angle. This result was also obtained by HARR [22]. Different values were also suggested, namely 20% [22] and 40% for a particular clay layer [23], and 50% for Frankfurt clay [24].

To compute the probability of failure P_f, it is necessary to determine the expected mean value and the standard deviation of the performance function, i.e., the safety factor in the present situation. Methods such as Taylor's series, point estimation methods, and Monte-Carlo simulation are available for calculating the mean and standard deviation of the performance function.

Once the mean value and the standard deviation of the performance function were determined, the reliability index β (assumed to be the number of standard deviations by which the expected value of a normally distributed performance function exceeds zero), can be quantified in the standard procedure of reliability analysis [23−24]. The probability of failure can then be calculated using the cumulative distribution function of the standard normal distribution evaluated at $-\beta$, or as the integral of the probability density function of the safety factor in the critical domain. Both Taylor's series and point estimation methods are simple to use but do not simulate the random variation of the variables, and consequently their accuracy depends very much on various approximations. On the other hand, Monte-Carlo method simulates the random variation of the variables, but requires a huge number of simulations if there are low failure probabilities (often ten thousand simulations [25]). Since the number of repeated finite element analyses is significant (and therefore difficult to use with limit analysis models), the cost of the probability analysis is very high. To overcome this difficulty, a probabilistic search algorithm can be used [25]. The use of a search algorithm can also be used with other purposes, namely to evaluate the sliding mass in the method of slices.

Initially, pattern search schemes were used to identify circular or logarithmic spiral critical slip surfaces. However, these approaches often fail to capture the critical failure mode for non-homogeneous slopes [26], since for slopes with complex profiles the function of the safety factor is normally non-smooth and/or non-convex. Consequently, it may have multiple minima with respect to the location of the slip surface over the solution domain. Modern optimization techniques have been recently employed to overcome this limitation. MCCOMBIE and WILKINSON [12] applied a genetic algorithm (GA) to search for the global minimum safety factor in slope stability analysis, using Bishop's method, and showed that the GA can perform better than some traditional methods, such as pattern search schemes or a brute-force approach (which require the analysis of a very large number of possibilities).

2 Method

As shown in the previous section, the method of slices, even being more imperfect than the limit analysis method, is still commonly used in research when probabilistic analysis is intended. In this work, it was considered advantageous to define a model that incorporates the advantage of the FEM method (to remove the necessity of using slices), with an imposed slip surface (to avoid undefined slip surfaces), but that is still able to perform probabilistic analysis (to attend soil properties variability). In this case, the use of a global constrained optimization technique is desirable to increase model efficiency.

The model here described consists in a limit equilibrium model. Since only kinematic admissible slip surfaces are used (circular), the model provides an upper bound solution of the safety factor, and because numerous slip surfaces are tested, it is expected that the safety factor is near the real one.

Globally, two domains can be defined in the model (Fig. 1). The first one, simulated by the classical FEM method, is able to deal with multilayer soils, and does not require assumptions with respect to interslice forces. The mesh is generated automatically by Gmsh [27], and its input file and execution are controlled by the MATLAB code. In this domain, it is assumed that none of the triangular linear elements that compose the mesh reaches plasticity. The linear elastic behaviour is defined by the elastic parameters E and ν, respectively the elastic modulus and Poisson ratio of a certain foundation layer. The forces generated by the self-weight of the soil are computed using a standard gravity procedure (function of the unit weight γ) involving integrals over each element. The number of nodes and elements of the mesh cannot be predefined since the model uses a GA to search the critical slip surface, and consequently the mesh is automatically redefined during the optimization process. Even if FEM can identify all the plastification nodes we decide to pre-impose a slip surface as a boundary condition, assumed as the second domain, since the former method may not produce a clear surface. Although a large number of failure criteria exist for modelling the strength of soil, the Mohr-Coulomb criterion will be considered in this work since it is the most common in geotechnical practice. No hardening and/or softening is considered about the slip surface since stability analysis is related to the analysis of force and strength instead of displacement. The Mohr-Coulomb failure criterion (Eq. (1)) is used to define the elasto-plastic response of domain 2. On each node of the

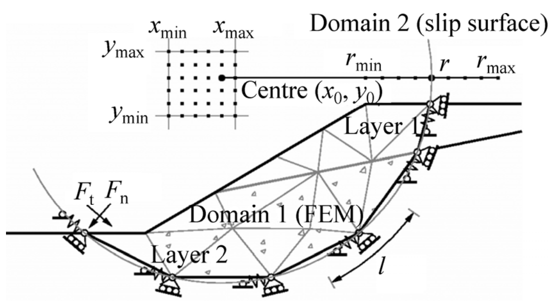

Fig. 1 Model domain decomposition and physical parameters

FEM mesh that belongs to the slip surface, the normal displacement is restricted by a simple support, and the tangential displacement is controlled by a spring (Fig. 1). The spring response is limited by Mohr-Coulomb criterion (Eq. (2)) with $F_{t;max}$ representing the maximum plastic shear force that each spring can support, F_n representing the normal force applied to each simple support (provided by the FEM calculation), c' representing the effective cohesion, ϕ' representing the friction angle, and l representing the length of the slip arc affected to each spring. If a negative value of F_n is provided by the FEM method (i.e., the force points to the centre of the slip surface), then a zero value is used to compute F_t. The spring response F_t cannot overcome the value of $F_{t;}$ (as imposed by the failure criterion), but since F_n is initially unknown, an iterative process has to be used to limit the value of F_t. This process allows spring response redistribution. In this context, $F_{s;l}$ is defined as the local safety factor of each spring (Eq. (3)).

$$\sigma_1(1+\sin\phi')-\sigma_3(1-\sin\phi')=2c'\cos\phi' \tag{1}$$

$$F_{t;max}=c'l+F_n\tan\phi' \tag{2}$$

$$F_{s;l}=\frac{F_{t,max}}{F_t} \tag{3}$$

An initial solution is computed to determine F_n and F_t (Eq. (4)), where k_s is the stiffness of each spring (Eq. (5)), δ_t is the displacement along the slip surface, and G is the shear modulus (Eq. (6)). From Eq. (5), it is possible to simulate shear distribution and soil stratigraphy along the slip surface during the iterative determination of F_t, leading to higher efforts in the less deformable soil layers.

$$F_t=k_s\delta_t \tag{4}$$

$$k_s=Gl \tag{5}$$

$$G=\frac{E}{2(1+v)} \tag{6}$$

If all $F_{s;l}$ values are greater than one (none of the springs reaches plasticity) or lesser than one (sliding mass is in a rupture situation), spring response redistribution is not necessary. Otherwise, the stiffness of the spring with the lowest value of $F_{s;l}$ is reduced by a percentage D_k. This spring response redistribution is repeated until the stop condition is reached, that is, all $F_{s;l}$ values are above or below one (Fig. 2).

The global safety factor of the slope $F_{s;g}$ (Eq. (7)) is defined as the number of nodes along the slip surface (N).

$$F_{s;g}=\frac{\sum_{i=1}^{N}F_{t;max}^{(i)}}{\sum_{i=1}^{N}F_{t}^{(i)}} \tag{7}$$

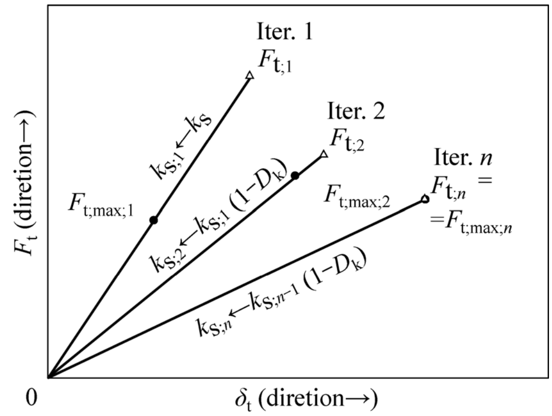

Fig. 2 Spring response redistribution attending Mohr-Coulomb failure criterion

Regarding the spatial variability of soil parameters, they were modelled based on the Monte-Carlo method, since it is able to simulate random variation of the variables and it can be applied not only to linear, but also to non-linear performance functions, as it is the case of the safety factor of earth masses. The random modelled parameters were the effective cohesion c' and the friction angle ϕ' (the unit weight γ is considered deterministic), assumed with lognormal and Gaussian probability distributions, respectively. The probability density function of the random variable F_s (in this work with the same meaning of $F_{s;g}$) can be considered as lognormal or as Gaussian, being the second one more commonly used. In this work, both of them were implemented.

Assuming, for the sake of clarity, x as a continuous random variable (in this work the safety factor F_s), and f as its probability density function, it is possible to determine the mean value (Eq. (8)), the standard deviation (Eq. (9)) and the skewness coefficient (Eq. (10)).

$$\mu(x)\int_{-\infty}^{\infty}xf(x)\mathrm{d}x \tag{8}$$

$$\sigma(x)=\left[\int_{-\infty}^{\infty}(x-\mu(x))^2f(x)\mathrm{d}x\right]^{\frac{1}{2}} \tag{9}$$

$$v(x)=\left[\int_{-\infty}^{\infty}\frac{(x-\mu(x))^3}{\sigma(x)^3}f(x)\mathrm{d}x\right]^{\frac{1}{2}} \tag{10}$$

If f follows a Gaussian probability distribution, the reliability index β is given by Eq. (11) [22], and if f follows a lognormal probability distribution, β is given by Eq. (12) [28], with $\overline{\mu}(x)$ and $\overline{\sigma}(x)$ given by Eqs. (13) and (14), respectively.

$$\beta=\frac{\mu(x)-1}{\sigma(x)} \tag{11}$$

$$\beta = \frac{\overline{\mu}(x)}{\overline{\sigma}(x)} \tag{12}$$

$$\overline{\mu}(x) = \ln(\mu(x)) - \frac{(\overline{\sigma}(x))^2}{2} \tag{13}$$

$$\overline{\sigma}(x) = \left[\ln\left(1 + \left(\frac{\sigma(x)}{\mu(x)}\right)^2\right)\right]^{\frac{1}{2}} \tag{14}$$

The probability of failure P_f is given by Eq. (15), where g is a function of the random variables and expresses the failure condition (in this work $g(x)=F_s-1$).

$$P_f = \int_{g(x)\le 0} f(x)\mathrm{d}x \tag{15}$$

Finally, the model takes advantage of a GA to search for the critical sliding mass. GA solve optimization problems by mimicking the principles of biological evolution, repeatedly modifying a population of individual points using rules modelled on gene combinations in biological reproduction. Due to its random nature, the genetic algorithm is a powerful tool to find the global solution, it allows solving unconstrained, bound-constrained, and general optimization problems, and it does not require the functions to be differentiable or continuous. This work uses the GA implemented in the Global Optimization Toolbox of MATLAB, that allows the definition of population size, number of elite children, crossover fraction, migration among subpopulations (using ring topology), and bounds, linear, and non-linear constraints for an optimization problem.

Two objective functions were considered: the safety factor and the probability of failure. The design variables in all cases were the centre (x_0; y_0) and radius r of the slip surface, which can range in the intervals [x_{min}; x_{max}], [y_{min}; y_{max}], and [r_{min}; r_{max}], respectively. The methods used to minimize the safety factor F_s and to maximize the probability of failure P_f are presented in Figs. 3 and 4, respectively. They combine spring response redistributions, probabilistic analysis, and an optimization search of the critical slip surface. The entire calculation was performed in MATLAB.

3 Case studies

To test the proposed model, namely its capacity to remove interslice assumptions, slip surfaces obtained by LIN et al [3] with a limit analysis were used. The model was built in FLAC^{3D} for a homogeneous soil slope, with 20 m of slope height, 45º of slope angle, being composed of 816 square elements and 1176 nodes. Since the size of the model affects the result in limit analysis, it was defined large enough to reduce the size effect, with a

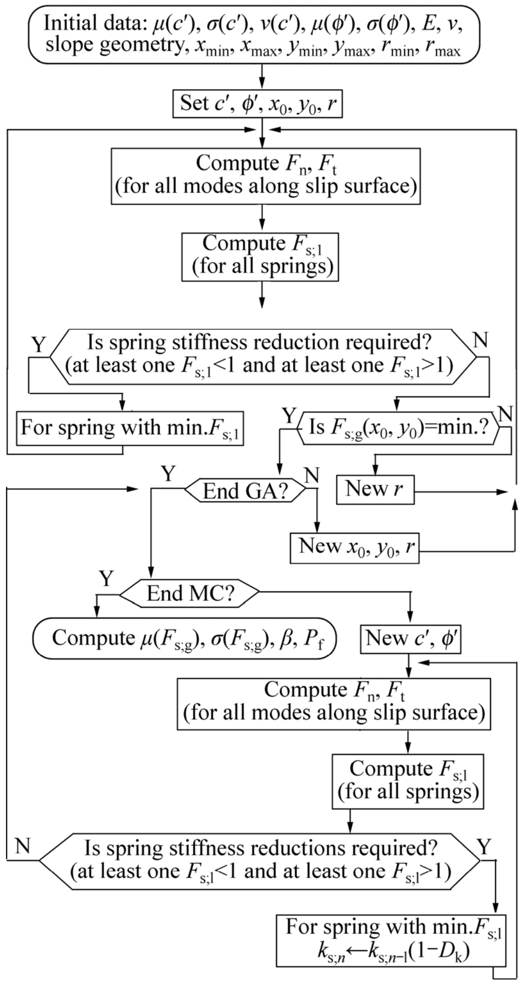

Fig. 3 Schematic representation of methodology used to find minimum safety factor F_s (Monte-Carlo is used to compute P_f afterwards)

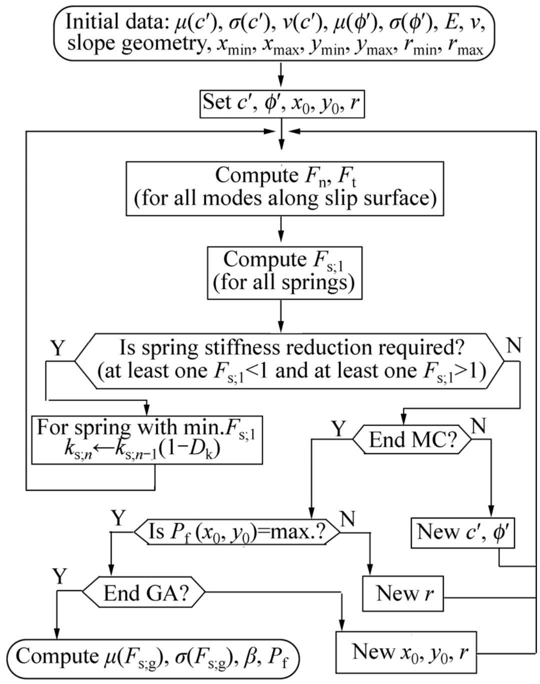

Fig. 4 Schematic representation of methodology used to find maximum probability of failure P_f

length from slope toe to the left boundary of 30 m, length from slope vertex to the right boundary of 55 m, and a length from slope toe to the bottom boundary of 20 m (that is equal to the slope height). The numerical model was fixed in both horizontal and vertical directions on the bottom boundary, only in the horizontal direction on the left and right boundaries, and was left free on the upper boundary. LIN et al [3] used a multiple a multiple slip surface searching method to find the relationship between the safety factor, the slip surface, and the soil geotechnical parameters effective cohesion and effective friction angle (the soil geotechnical parameters unit weight, elastic modulus, and Poisson ratio were fixed with the values of γ=25 kN/m^3, E=10 MPa, and ν=0.3, respectively). These geometric, geotechnical and simulations results were used to define four distinct "Case studies", described below.

Regarding the tests with variable effective cohesion, the effective friction angle ϕ' was fixed at 17º by LIN et al [3]. The effective cohesion c' ranged from 4.2 kPa to 268.8 kPa, being the slip surfaces defined as those closest to the unitary safety factor given in Fig. 5. By a polynomial interpolation of the safety factors found by LIN et al [3], it was estimated that an effective cohesion of 38 kPa leads to the unitary safety factor, being this the value used for comparison. These data define the "Case study 1". Then, for the tests with variable effective friction angle, the effective cohesion was fixed at 42 kPa by LIN et al [3]. The friction angle ranged from 1.75º to 44.37º, being the slip surfaces closest to the unitary safety factor given in Fig. 6. By using again a polynomial interpolation, it was estimated that an effective friction angle of 15º leads to the unitary safety factor. These data define the "Case study 2".

To perform the probabilistic analysis, no data were available by LIN et al [3], which required the assumption of the coefficient of variation, probabilistic distribution shape, and skewness coefficient. Cohesion was assumed with a lognormal distribution with a mean value of $\mu(c')$= 42 kPa, a coefficient of variation of $\sigma(c')/\mu(c')$=20 %, and a skewness coefficient $\nu(c')$=2.85, while the friction angle was assumed with a Gaussian distribution, with mean value of $\mu(\phi')$=17º and a coefficient of variation of $\sigma(\phi')/\mu(\phi')$=5% (Fig. 7). Cohesion is, in fact, represented by a standard lognormal probability distribution shifted by c'_0, which depends on the values of $\mu(c')$, $\sigma(c')$ and $\nu(c')$, and is given implicitly by Eq. (16).

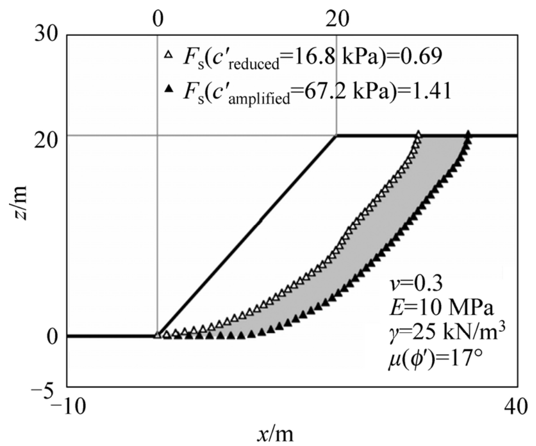

Fig. 5 Critical slip surfaces obtained using method proposed by LIN et al [3] (grey area localizes slide surface with unitary safety factor for "Case study 1")

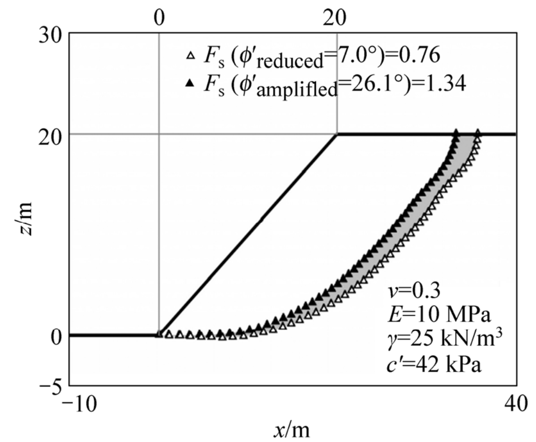

Fig. 6 Critical slip surfaces obtained using method proposed by LIN et al [3] (grey area localizes slide surface with unitary safety factor for "Case study 2")

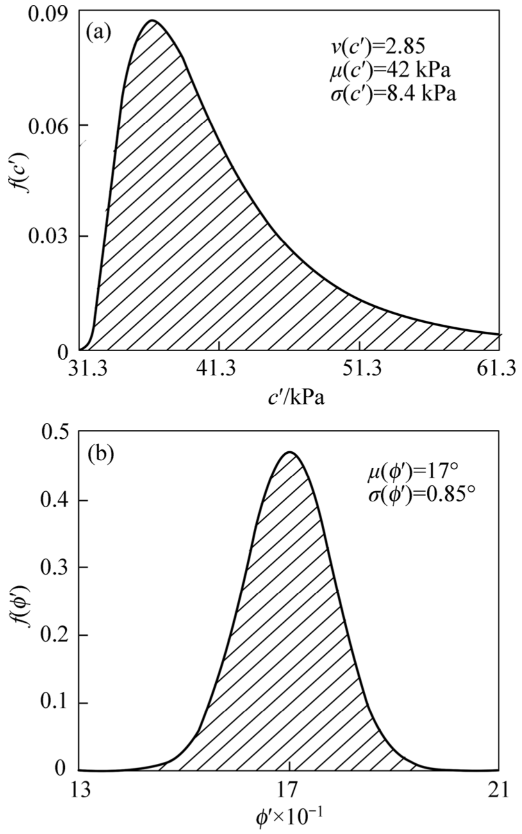

Fig. 7 Probability distributions functions assumed according to limits proposed by RUSSELLI [20] and used on Monte-Carlo method

$$v(c')=3\frac{\sigma(c')}{\mu(c')-c'_0}+\left(\frac{\sigma(c')}{\mu(c')-c'_0}\right) \quad (16)$$

The cohesion associated to a certain cumulative probability was obtained by the MATLAB lognormal probability density function, which requires as input the mean value $\mu_{\ln(c')}$ (Eq. (17)) and the standard deviation $\sigma_{\ln(c')}$ (Eq. (18)), both of them associated to a normal distribution and adding c'_0 on the function output.

$$\mu_{\ln(c')}=\ln(\mu(c')-c'_0)-\frac{1}{2}\ln\left[1+\left(\frac{\sigma(c')}{\mu(c')-c'_0}\right)^2\right] \quad (17)$$

$$\sigma_{\ln(c')}=\left[\ln\left(1+\left(\frac{\sigma(c')}{\mu(c')-c'_0}\right)^2\right)\right]^{\frac{1}{2}} \quad (18)$$

These data are associated with "Case study 3" and "Case study 4" (Fig. 8). The first one considers a homogeneous foundation while the latter considers a situation with a layered foundation.

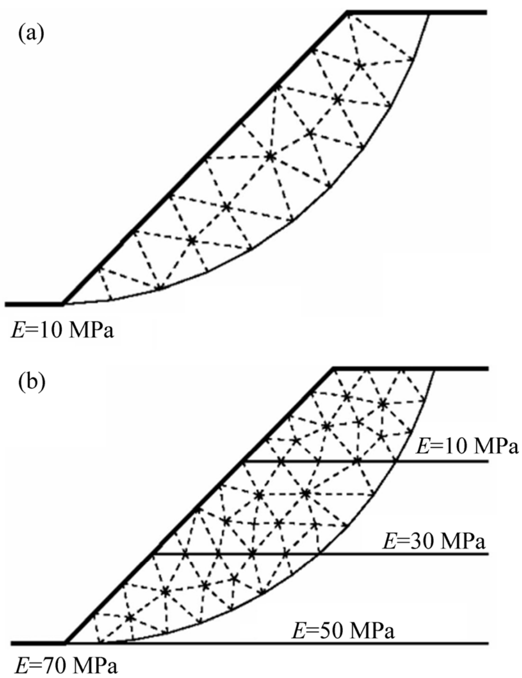

Fig. 8 Homogeneous (a) and layered (b) foundations

4 Results and discussion

The ability of the developed model to provide an accurate response was evaluated in six items, to assess if it is able to:

1) Quantify the safety factor of earth masses with the same precision as the limit analysis method;

2) Reduce computational effort by using a GA algorithm to determine the critical slip surface;

3) Identify the influence of layered foundation (non-constant elasticity);

4) Perform statistical analysis by using Monte-Carlo method and quantify the probability of failure;

5) Compare the influence of the shape of the density probability function when evaluating the probability of failure;

6) Compare the critical slip surface obtained with the maximization of the probability of failure instead of the minimization of the safety factor as the objective function.

The analysis started with three parametric studies carried out to quantify the required number of elements along the slip surface, the spring stiffness variation D_k, and the number of iterations for the Monte-Carlo technique. For the case studies with a homogeneous foundation, it was assumed that around 10 segments along the slip surface would allow an accurate definition of the slip surface geometry, while in the case study with a layered foundation around 16 segments were assumed (assumed enough as illustrated in Fig. 8). To define the stiffness variation D_k, all four case studies were initially studied with a very low D_k value (0.1%) and then the corresponding safety factor was compared to those obtained after doubling the value of D_k. The limit value of D_k was assumed as being the one that results in a safety factor with a maximum difference of 0.05 in comparison with that obtained for D_k=0.1%. In all case studies analysed, the worst scenarios produced a D_k value of 6:6%, being this used for all subsequent analysis. The last parametric study was the definition of the number of iterations for the Monte-Carlo simulations (n_{MC}) to analyse "Case study 3" and "Case study 4". Considering the value 0.025 as the acceptable variation for the safety factor and taking into consideration the data presented in Fig. 9, it was possible to define the value $n_{MC}=10^2$.

A slip surface is characterized by its centre (x_0; y_0) and its radius r, the design variables of the problem under study. The initial idea was to use a GA to find them. However, during the study, it was found that, for a given slip surface centre (x_0; y_0), the radius associated with the minimum safety factor could be evaluated by algorithms less precise but faster than GA, as shown in Fig. 10. Taking advantage of the unique minimum and smooth evolution of the function $F_s=f(r)$, the unconstrained non-linear minimization MATLAB function *fminsearch* was used to quantify the critical radius (Figs. 3 and 4 show that inside each iteration of the GA algorithm, the nonlinear minimization algorithm is executed until the critical radius is found with a tolerance of 0.5 m). However the centre of the slide surface was found with the GA implemented in MATLAB. It required the definition of two stop conditions: the maximum number of iterations (20) and the maximum number of stalls of the best solution (5).

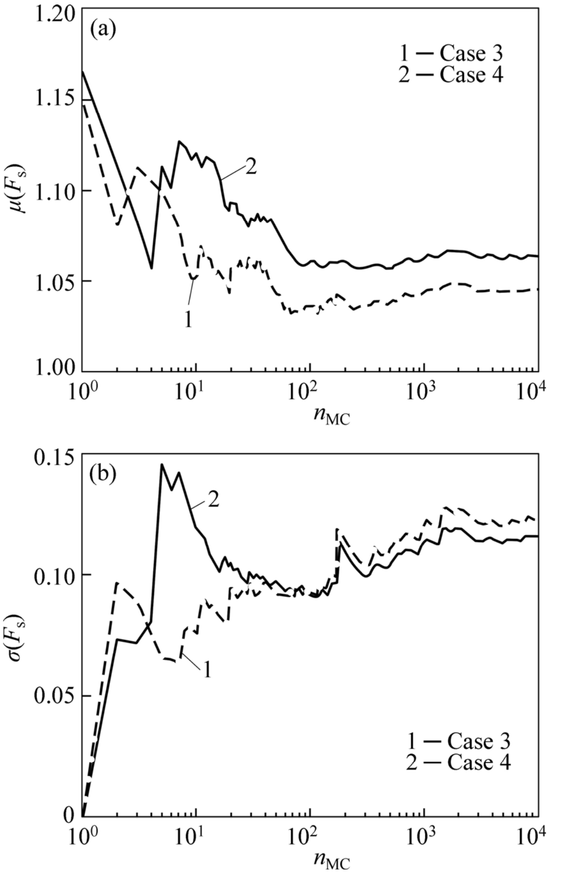

Fig. 9 Parametric study carried out to determine n_{MC}: (a) Evolution of mean value of safety factor; (b) Evolution of standard deviation of safety factor

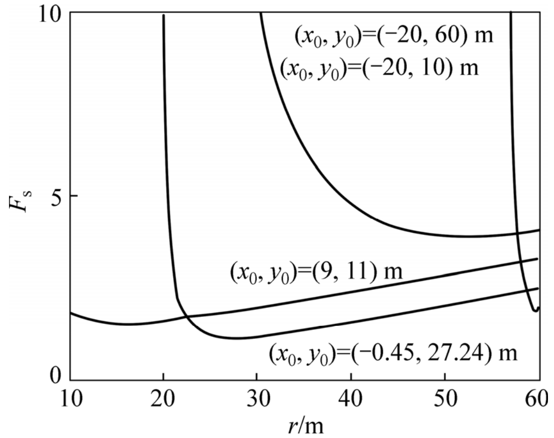

Fig. 10 Radius evolution for "Case study 3": Smooth evolution of safety factor with respect to radius variable with no local minima

The initial population was randomised using uniform distributions defined on the given bounds and composed of ten pairs (x_0; y_0). Children were composed of the best solution of the previous population and the remaining nine elements were chosen randomly following a Gaussian distribution around the best solution. No mutation children were used. The initial standard deviation was assumed equal to 20% of the initial population range, leading to standard deviations of 7 m and 10 m (limits [−20; 15] m for x and [10; 60] m for y), respectively. The standard deviation shrinks linearly as function of the number of iteration till 0 in the twentieth iteration.

In order to test the ability of the model to solve the limitation regarding the assumptions involved in the forces on each slice (common to all equilibrium methods), so as to guarantee the definition of a critical sliding surface, and also to confirm that the proposed method is so accurate as the limit analysis approach, the results obtained by LIN et al [3] were compared in "Case study 1" and "Case study 2". Brute-force analyses were performed to quantify the evolution of the safety factor and critical radius of the slip surface as function of the centre of slip surface (Figs. 11 and 12). From these analyses, it was observed that the model is able to reproduce with high accuracy those results given by LIN et al [3]. A unitary safety factor was found for a circular

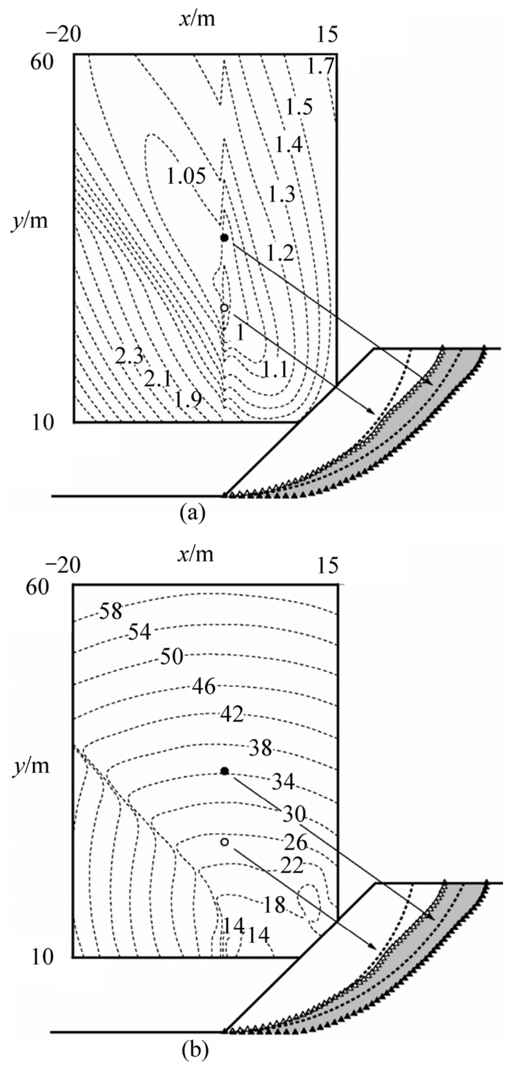

Fig. 11 Brute-force for "Case study 1": ● F_s=1.00 (unitary safety factor trough a slide surface enclosed in area defined by LIN et al [3]) and ○ F_s=0.93 (minimum value for safety factor): (a) Level-sets for variable F_s; (b) Level-sets for variable r

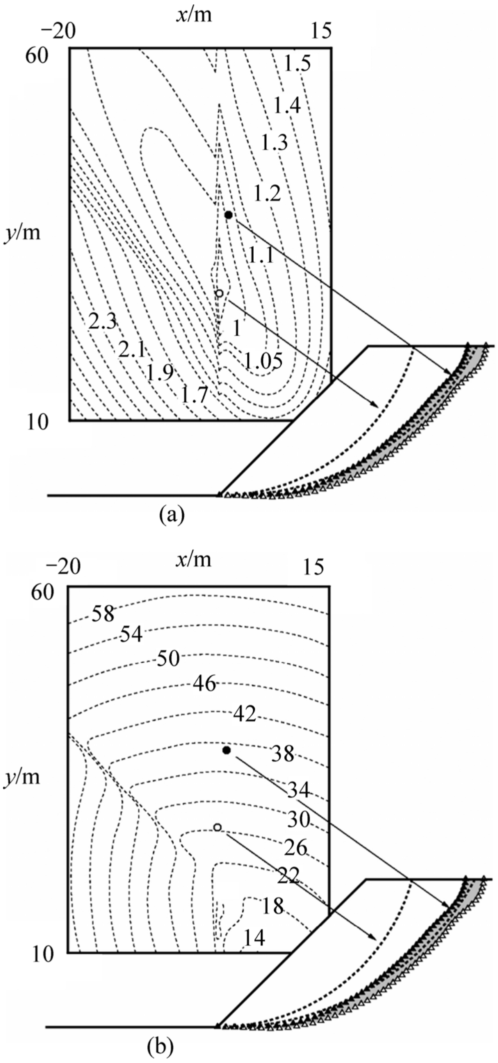

Fig. 12 Brute-force for "Case study 2": ● F_s=1.00 (unitary safety factor trough a slide surface enclosed in area defined by LIN et al [3]) and ○ F_s=0.92 (minimum value for safety factor): (a) Level-sets for variable F_s; (b) Level-sets for variable r

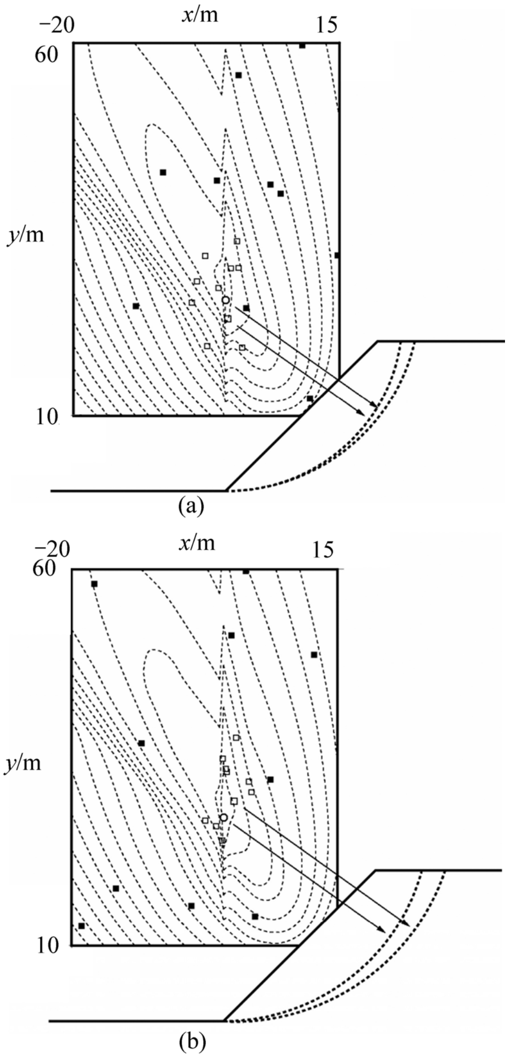

Fig. 13 Comparison between brute-force and GA solutions: (a) "Case study 1": ○ best F_s by brute-force (0.93); □ Best F_s by AG (0.92); ▪ Initial population; ▫ Final population; (b) "Case study 2": ○ Best F_s by brute-force (0.92); □ Best F_s by AG (0.95); ▪ Initial population; ▫ Final population

slip surface enclosed in the area defined by LIN et al [3] (grey areas in Fig. 11), and even low values were recorded. These results certify the hypothesis that the hybrid model is able to reproduce the safety factor with the same order of accuracy as the limit analysis method in those cases where a circular failure surface is expected.

The brute-force results were also used to test the efficiency of the GA search (Fig. 13). Even if computational time depends on the system, it was possible to state that all brute-force analysis were associated to very higher efforts when compared with those of GA analysis (GA computational time was only around 5% of the brute-force computational time). The use of the GA algorithm was found necessary, even if the evolution of the safety factor looks smooth (Fig. 13). This is due to numerical imprecision, that create several local minima. The solution founded by GA was always near the one obtained with the brute-force analyses. Improved results could be found with a higher number of stall iterations. However, the improved precision was considered not relevant (lower than 0.05) when compared with the raise of computational time consumption.

The proposed model, that was as accurate as the limit analysis approach presented by LIN et al [3], was used to quantify the relevance of the elastic parameter E in the evaluation of the safety factor F_s. Thus, "Case study 4" (Fig. 14) was studied and the critical slip surface found by means of a brute-force analysis and GA. All layers were defined with the same cohesion and friction angle (c'=42 kPa and ϕ'=17°), since it was intended to study the influence of soil elasticity. As in the case of homogeneous foundation, where the influence of the elasticity is known to be null, it was also found that

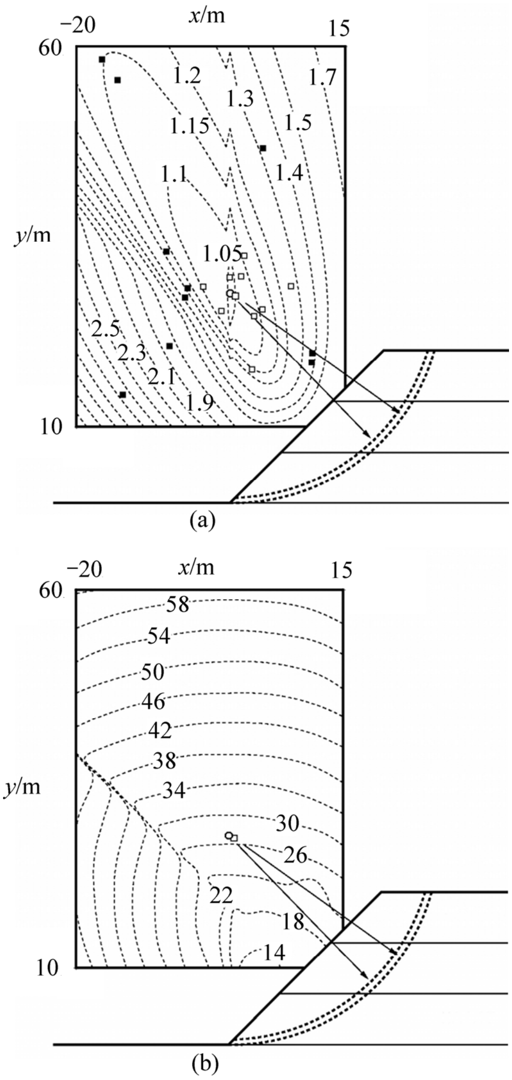

Fig. 14 Comparison between brute-force and GA solutions: (a) Level-sets for the variable F_s: ○ Best F_s by brute-force (1.03); □ Best F_s by AG (1.06); ▪ Initial population; ▫ Final population; (b) Level-sets for variable r: ○ Best F_s by brute-force (1.03); □ Best F_s by AG (1.06)

the elasticity parameter was not relevant for the evolution of the safety factor. In fact, no significant change was detected in the shape of the evolution of the safety factor as function of the centre of the slip surface (Fig. 14), neither in the value of the minimum safety factor. The use of the GA search methodology was also performed without the need of any adjustment. Summing up, it was concluded that in the studied situation ("Case study 4"), the elastic parameter E is not relevant for the analysis and may be neglected.

Regarding the statistical analysis, the influence of the shape of the density probability function was tested using "Case study 3" and "Case study 4". The probabilistic distribution functions were, as previously mentioned, the Gaussian for the friction angle and the lognormal for the cohesion. The slip surfaces were those obtained with GA searches for homogeneous (x_0; y_0; r)=(−0.45; 27.24; 27.24) m and layered (x_0; y_0; r)=(0.35; 27.37; 27.38) m studies. From the interpretation of those results, presented in Fig. 15, a lognormal distribution function produces a better adjustment for the statistical distribution of the safety factor F_s in both cases. It was also observed that the probability of failure using a Gaussian distribution function is not recommended. During these analyses, it was defined that the use of a lognormal distribution probability function for cohesion must be applied with the same distribution that the safety factor distribution.

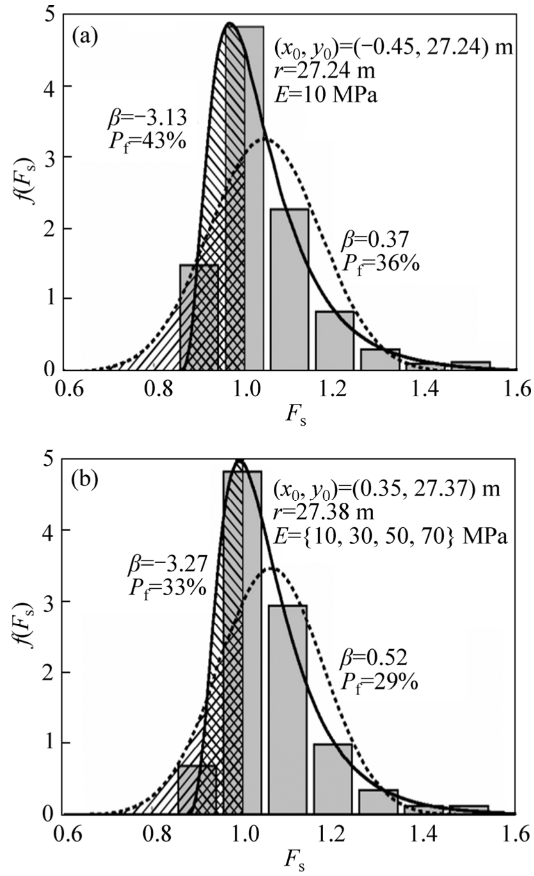

Fig. 15 Safety factor probabilistic density distribution (PDF): (a) "Case Study 3"; (b) "Case Study 4"

The methodology proposed in Fig. 4 was then used in "Case Study 3", where the probability of failure P_f assumed a lognormal probability distribution function in order to evaluate the last defined objective. The aim is to find the critical slip surface that maximizes the probability of failure P_f, and compare it with minimization of the safety factor F_s study. In this situation, a brute-force analysis was not easy, since the GA search (Fig. 16) required around one week to find the critical slip surface. In order to improve assurance in the results, the only stop criterion was the maximum number of iterations (defined equal to 20). From this analysis, it was found that the probability of failure is highly

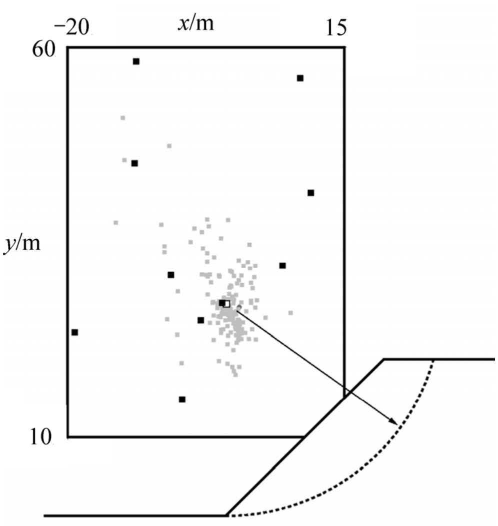

Fig. 16 Genetic algorithm probability of failure search for "Case Study 3": ■ initial population; ■ amid population; □ final population; $P_f = 47\%$; $F_s = 1.05$

dependent on the minimum value for the safety factor, and the methodology proposed in Fig. 4 (maximization of the probability of failure) is not advantageous, mainly due to the heavy computational effort. Consequently, the methodology proposed in Fig. 3 (minimization of the safety factor) should be preferred.

5 Conclusions

1) The proposed model is able to reproduce the safety factor with the same order of accuracy of the limit analysis method in those cases where a circular failure surface is expected.

2) The implementation of a genetic algorithm technique to reduce computational effort is shown to be an effective tool.

3) The existence of a layered foundation with non-constant elastic modulus appeared not to be relevant for the result obtained.

4) The objective function "minimization of the safety factor" can be consider better (at this preliminary stage of development) than the "maximization of the probability of failure" since the results found are very similar.

5) If it is important to analyse the probability of failure, a lognormal probability distribution function should be considered.

6) As a prospective work, the definition of non-circular slip surfaces is desirable, as well as the use of optimization tools to reduce the computational effort associated with Monte-Carlo method.

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