Adaptive Linearizing Control of Bioreactors

Eugénio Ferreira
Univ. of Minho, Dept. of Biological Engng. - IBQF
BRAGA, PORTUGAL

Sebastião Feyo de Azevedo Univ. of Porto, Dept. Chem. Engng. - Systems & Robotics Institute
PORTO, PORTUGAL

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SUMMARY

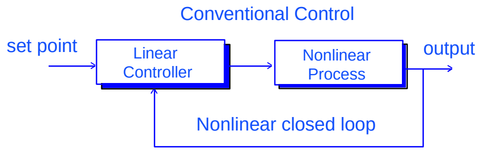
Synthesis of SISO and MIMO adaptive linearizing controllers for the regulation of bioreactors.

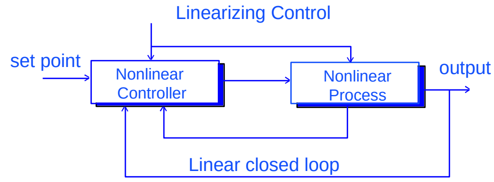
Case study: Baker's yeast fed-batch fermentation process.

Adaptive feature: on-line estimation of process time-varying parameters.

The adaptive algorithm proposed enforces a desired and pre-set 2nd order convergence dynamics.

Adaptive Linearizing Control





Linearizing Control Design

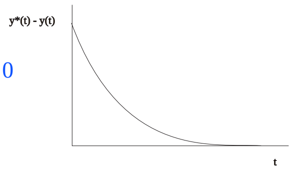
3-step procedure (Bastin & Dochain):

Step 1: derive a I/O model $$\frac{d^\delta y}{dt^\delta} = f_0(t) + u(t)\,f_1(t)$$

Step 2: select a *stable linear reference model* of the tracking error (y*-y)

$$\sum_{j=0}^{\delta} \lambda_{\delta-j} \frac{d^j}{dt^j}\left[y^*(t) - y(t)\right] = 0$$

λ_o=1



Step 3:Compute the control action such that the I/O model matches the reference model

$$u(t) = \frac{1}{f_1(t)}\left[-f_o(t) + \sum_{j=0}^{\delta-1} \lambda_{\delta-j} \frac{d^j}{dt^j}\left[y*(t) - y(t)\right] + \frac{d^\delta y*}{dt^\delta}\right]$$

Baker's Yeast Metabolic Pathways

Oxidative Growth on glucose (*respiration*):

Glucose + Oxygen $\longrightarrow$ Biomass + CO_2 + water

$$k_1 S + k_5 O \xrightarrow{\mu_s^o} X + k_7 C$$

Redutive Growth on glucose (*fermentation*):

Glucose $\longrightarrow$ Biomass + Ethanol + CO_2 + water

$$k_2 S \xrightarrow{\mu_s^r} X + k_8 C + k_3 E$$

Oxidative Growth on ethanol (*respiration*):

Ethanol + Oxygen $\longrightarrow$ Biomass + CO_2 + water

$$k_4 E + k_6 O \xrightarrow{\mu_e^o} X + k_9 C$$

BAKER's YEAST PROCESS
***Adaptive Regulation of ETHANOL* (*SISO*)**

RF:

$$\frac{dE}{dt} = -DE + k_3\varphi_2$$
$$OTR = k_5\varphi_1$$
$$DS_e = k_1\varphi_1 + k_2\varphi_2$$

R:

$$\frac{dE}{dt} = -DE - k_4\varphi_3$$
$$OTR = k_5\varphi_1 + k_6\varphi_3$$
$$DS_e = k_1\varphi_1$$

order reduction (using sugar and O_2 dynamics) $\Rightarrow$ $\varphi_1, \varphi_2, \varphi_3$ function of OTR and DS_e

RF: $$\frac{dE}{dt} = \frac{k_3}{k_2} DS_e - \frac{k_1 k_3}{k_2 k_5} OTR - DE$$

R: $$\frac{dE}{dt} = \frac{k_4 k_5}{k_1 k_6} DS_e - \frac{k_1 k_4}{k_1 k_6} OTR - DE$$

I/O reduced Model (1st order):

$$\frac{dE}{dt} = -\theta_2 OTR + \theta_3 DS_e - DE$$

1st order linear reference model of the tracking error:

$$\frac{d\left(E^* - E\right)}{dt} + \lambda\left(E^* - E\right) = 0$$

Baker's Yeast - State Model

$$\frac{d}{dt}\begin{bmatrix} X \\ S \\ E \\ O \\ C \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ -k_1 & -k_2 & 0 \\ 0 & k_3 & -k_4 \\ -k_5 & 0 & -k_6 \\ k_7 & k_8 & k_9 \end{bmatrix} \begin{bmatrix} \mu_s^o \\ \mu_s^r \\ \mu_e^o \end{bmatrix} X - D \begin{bmatrix} X \\ S \\ E \\ O \\ C \end{bmatrix} + \begin{bmatrix} 0 \\ DS_{in} \\ 0 \\ OTR \\ 0 \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ CTR \end{bmatrix}$$

Model RF: Respiro-fermentative state

$$\frac{d}{dt}\begin{bmatrix} X \\ S \\ E \\ O \\ C \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ -k_1 & -k_2 \\ 0 & k_3 \\ -k_5 & 0 \\ k_7 & k_8 \end{bmatrix} \begin{bmatrix} \mu_s^o \\ \mu_s^r \end{bmatrix} X - D \begin{bmatrix} X \\ S \\ E \\ O \\ C \end{bmatrix} + \begin{bmatrix} 0 \\ DS_e \\ 0 \\ OTR \\ 0 \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ CTR \end{bmatrix}$$

Model R: Respirative state

$$\frac{d}{dt}\begin{bmatrix} X \\ S \\ E \\ O \\ C \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ -k_1 & 0 \\ 0 & -k_4 \\ -k_5 & -k_6 \\ k_7 & k_9 \end{bmatrix} \begin{bmatrix} \mu_s^o \\ \mu_e^o \end{bmatrix} X - D \begin{bmatrix} X \\ S \\ E \\ O \\ C \end{bmatrix} + \begin{bmatrix} 0 \\ DS_e \\ 0 \\ OTR \\ 0 \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ CTR \end{bmatrix}$$

Control law: $$D = \frac{\lambda\left(E^* - E\right) + \hat{\theta}_2 OTR}{\hat{\theta}_3 S_e - E}$$

Parameter adaptation law (SODE):

$$\frac{d\hat{\theta}}{dt} = -\Gamma\phi\left(y^* - y\right)$$

$$\tau_j^2 \frac{d^2\hat{\theta}_i}{dt^2} + 2\zeta_j \tau_j \frac{d\hat{\theta}_i}{dt} + \hat{\theta}_i = \theta_i$$

$$\frac{d\hat{\theta}_2}{dt} = \frac{1}{\tau^2 OTR(t)}\left(E^* - E(t)\right) \qquad \frac{d\hat{\theta}_3}{dt} = -\frac{1}{\tau^2 D(t) S_e}\left(E^* - E(t)\right)$$

with $$\lambda = 2\frac{\zeta}{\tau} - \frac{1}{OTR(t)} \frac{dOTR(t)}{dt}$$

Adaptive Regulation of ETHANOL and Dissolved OXYGEN (MIMO)

order reduction (using sugar and CO2 dynamics):

RF

$$\frac{d}{dt}\begin{bmatrix} O \\ E \end{bmatrix} = -D\begin{bmatrix} O \\ E \end{bmatrix} + \begin{bmatrix} -k_5 & 0 \\ 0 & k_3 \end{bmatrix}\frac{1}{\tilde{k}_{RF}}\begin{bmatrix} -k_8 & k_2 \\ k_7 & -k_1 \end{bmatrix}\begin{bmatrix} DS_e \\ CTR \end{bmatrix} + \begin{bmatrix} OTR \\ 0 \end{bmatrix}$$

R

$$\frac{d}{dt}\begin{bmatrix} O \\ E \end{bmatrix} = -D\begin{bmatrix} O \\ E \end{bmatrix} + \begin{bmatrix} -k_5 & -k_6 \\ 0 & -k_4 \end{bmatrix}\frac{1}{\tilde{k}_{R}}\begin{bmatrix} k_9 & 0 \\ -k_7 & k_1 \end{bmatrix}\begin{bmatrix} DS_e \\ CTR \end{bmatrix} + \begin{bmatrix} OTR \\ 0 \end{bmatrix}$$

generically:

$$\frac{d}{dt}\begin{bmatrix} O \\ E \end{bmatrix} = -\begin{bmatrix} \theta_1 \\ \theta_3 \end{bmatrix}CTR + \begin{bmatrix} 1 \\ 0 \end{bmatrix}OTR + \begin{bmatrix} \theta_2 S_e - O \\ \theta_4 S_e - E \end{bmatrix}D$$

Reference model of the tracking error:

$$\frac{d}{dt}\begin{bmatrix} (O^* - O) \\ (E^* - E) \end{bmatrix} + \begin{bmatrix} \lambda_1 & \lambda_2 \end{bmatrix}\begin{bmatrix} (O^* - O) \\ (E^* - E) \end{bmatrix} = 0$$

Control Laws:

$$\begin{bmatrix} OTR \\ D \end{bmatrix} = \begin{bmatrix} 1 & -\frac{(\theta_2 S_e - O)}{(\theta_4 S_e - E)} \\ 0 & \frac{1}{(\theta_4 S_e - E)} \end{bmatrix}\left\{\begin{bmatrix} \lambda_1 (O^* - O) \\ \lambda_2 (E^* - E) \end{bmatrix} + \begin{bmatrix} \hat{\theta}_1 \\ \hat{\theta}_3 \end{bmatrix}CTR\right\}$$

Conclusions

$$P = \frac{X_f V_f - X_0 V_0}{(V_f - V_0)t_f}$$

$$R = \frac{X_f V_f - X_0 V_0}{\int_o^{t_f} FS_e dt + S_0 V_0 - S_f V_f}$$

	Productivity g/L/h	*Yield* gX/gS
SISO	**0.34**	**0.425**
MIMO	**0.54**	**0.460**

✗ Parameter Adaptation laws

2nd order dynamics: 2 tuning parameters with physical meaning (τ, ζ)

✗ Model based approach

✗ Reduced models

adequate for the synthesis of adaptive control laws

Parameter Adaptation laws (SODE):

$$\frac{d\theta_1}{dt} = -\frac{1}{\tau_1^2 CTR(t)}\left(O^* - O(t)\right) \qquad \frac{d\theta_3}{dt} = -\frac{1}{\tau_2^2 CTR(t)}\left(E^* - E(t)\right)$$

with

$$\lambda_1 = 2\frac{\zeta_1}{\tau_1} - \frac{1}{CTR(t)}\frac{dCTR(t)}{dt} \qquad \lambda_2 = 2\frac{\zeta_2}{\tau_2} - \frac{1}{CTR(t)}\frac{dCTR(t)}{dt}$$