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Application of Markov chains in the prediction of rock slopes degradation

Application des chaines de Markov dans la prévision de la dégradation de talus rocheux

J.Tinoco[*1], S. Sanches[2] , T. Miranda[1] , A. Neves[2] , M. Pinheiro[1] , A. Ferreira[2] and A.G. Correia[4]

[1] *Institution for Sustainability and Innovation in Structural Engineering/School of Engineering/University of Minho, Guimarães, Portugal*
[2] *Ascendi, Matosinhos, Portugal*
[*] *Corresponding Author*

ABSTRACT In this paper it is proposed a degradation model for rock slopes, which will integrate a global tool for sustainable management of road infrastructures, currently under development. The proposed model is based on a continue-time Markov chain and predicts the evolution of a Slope Quality Index (SQI) over time. Taking advantage of the memoryless property of Markov models, the proposed model is able to predict the next state only based on the current state, not depending on the sequence of events that preceded it. In addition, the effect of maintenance actions is also considered in the prediction of the state condition of a given slope. The achieved results, which represent a first iteration towards the development of a predictive model for rock slope degradation processes during its exploration phase, show that the use of Markov chains can lead to reliable results.

RÉSUMÉ Dans cet article, il est proposé un modèle de dégradation des talus rocheux, qui intégrera un outil global et durable de gestion des infrastructures routières, actuellement en développement. Le modèle proposé est basé dans une chaîne de Markov de temps continue et prévoit l'évolution d'un indice de la qualité du talus (SQI) avec le temps. Prenant en compte la propriété sans mémoire des chaines de Markov, le modèle proposé est capable de prédire l'état suivant utilisant seulement son état actuel, il ne dépende pas de la séquence des événements qui l'ont précédé. En plus, l'effet des actions de maintenance est également pris en compte dans la prédiction de l'état d'un talus. Les résultats obtenus, qui représentent une première itération à l'élaboration d'un modèle de prévision pour le processus de dégradation des talus rocheux au cours de sa phase d'exploration, montrent que l'utilisation des chaînes de Markov peut guider à des résultats fiables.

1 INTRODUCTION

Portugal, as most of the developed countries, has nowadays a fairly complete road network (Neves 2005). These transportation infrastructures, composed by a set of elements that can be classified as bridges, road pavements, retaining walls, slopes and electronic equipments, are necessary to ensure good mobility conditions. In order to guarantee all security and mobility conditions of the road network, it is important to make available a set of tools to support the maintenance strategies of each one of these elements. Although for some of them maintenance and conservation strategies based on degradation models already exist, for others, particularly for rock slopes, almost nothing has been done so far.

In order to overcome this drawback, namely in terms of degradation and maintenance models for rock slopes, in this paper a theoretical approach is proposed based on Markov chains (Knill 2009). The proposed model has the particularity of being able to be applied during the exploration phase of the slope and requires only basic parameters. The main reasons be-hind the choice of Markov chains for rock slope degradation modelling are, on one hand, due to its successful application in different knowledge domains, namely in the study of bridge degradation (Sobreiro 2011) and, on the other hand, due to some of its particular and useful properties, namely the memoryless property. According to this property the future behaviour of a given slope depends only upon the actual condition state and not on the sequence of events that preceded it (Yang et al. 2009). Thus, given the lack of historical data about rock slopes degradation processes, this property is of great importance for this study. The proposed model is based on a slope quality index (SQI) that is determined during the exploration phase of the infrastructure (Pinheiro et al. 2014), which range from 1 to 5, and predicts its evolution over time. The proposed model will integrate a tool that is currently in development under the project SustIMS (Sustainable Infrastructure Management System), aiming a sustainable and georeferenced management of road infrastructures. For the maintenance model it is considered the improvement in performance and the deterioration delay effect, as well as the reduction effect in degradation ratio of different maintenance and repair actions currently applied to rock slopes.

It should be also highlighted the innovation of the proposed model for rock slopes degradation prediction over time, as well as the maintenance model able to incorporate the impact of different actions. Indeed, so far and within our knowledge, there are no models able to predict the condition state of a given rock slope over time during its exploration phase, particularly based on a SQI that only requires basic information.

2 MARKOV MODEL

As stated, in this work a degradation model for rock slopes is proposed based on a Markov process. A Markov chain is a particular stochastic process with discrete states (where the in-dependent parameter, usually time, can be discrete or continuous). Stochastic processes have been extensively applied for modelling the deterioration of infrastructures over time such as bridges, pavement or wastewater systems, due to the randomness that characterizes a degradation process (Baik et al. 2006; Kobayashi et al. 2010). According to Mishalani & Madanat (2002) there are two degradation models in discrete state processes: time based models and state based models. In a state based model (adopted in the present work), it is defined the probability of a slope to stay in the same condition state or to move to the next state, in a predefined period of time Δt. In the present work we adopted a continuous time Markov process since the intervals between inspections Δt are not regular or similar for all slopes.

Yang et al. (2009) proposed the definition of a transition intensity matrix Q, defined as:

$$\frac{\partial}{\partial t}P = P \times Q \Rightarrow P = \exp(Q \times \Delta t) \tag{1}$$

The intensity matrix, Q, represents the instantaneous probability of transition between state i and state j (independently of Δt) and is directly correlated with any transition probabilities matrix P.

Keep in mind that the slope degradation is a natural and continuous process, it is impossible to improve its condition state without any intervention. Therefore, if a classification system contemplates 5 different condition states, then the matrix Q presents the format as shown in equation 2, meaning that during the deterioration process, in each infinitesimal time interval, the slope can only advance between adjacent condition states.

$$Q = \begin{bmatrix} -\theta_1 & \theta_1 & 0 & 0 & 0 \\ 0 & -\theta_2 & \theta_2 & 0 & 0 \\ 0 & 0 & -\theta_3 & \theta_3 & 0 \\ 0 & 0 & 0 & -\theta_4 & \theta_4 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{2}$$

The degradation model here proposed for rock slopes was implemented in the statistic software R (Core Team 2009) using the *msm* pack-age (Jackson 2011).

3 SYNTHETIC DATA

A database containing information related to the condition state of different rock slopes over time is a key element for the development of a degradation model, namely following a Markovian approach. However, since currently data related to the condition state of the rock slopes over time does not exist, a synthetic database was compiled aiming to simulate its degradation process as well as possible. For this task it was considered all the experience of the team involved in the SustIMS project related to the maintenance and conservation activities of rock slopes. The different criteria adopted for the synthetic database compilation are listed below.

- **Number of slopes**: 200;

- **Classification system for slope condition state**: $C_1 \rightarrow$ Very Good; $C_5 \rightarrow$ Very Bad;
- **Slope construction year**: between 01/01/1965 and 31/12/2005. The slope construction year was defined according to a lognormal distribution ($\mu = 0$ and $\sigma = 0.3$);
- **Initial condition state**: it was defined that after its construction a rock slope is in a condition state C_1, C_2 or eventually in C_3. Therefore, the initial condition state was defined according to a exponential distribution ($\mu = 1$);
- **Inspections frequency**: for the number of inspections that each slope was submitted during its lifetime, the following law was adopted: $N_{insp} = -0.005t^2 + 0.708t + 0.0871 \pm e$, where t is the age of the slope (years) at the moment of the inspection and e is a random error that follows a normal distribution $N(0,2)$.
- **Condition state over time**: the condition state of the rock slopes over time was defined considering that the mean time periods in states C_1 to C_4 are 14; 9; 7 and 4 years respectively. These values were affected by a random error according to a normal distribution $N(0,2)$.

Figure 1 illustrates the mean time period (and respective 95% level confidence intervals according to a t-student distribution) for states C_1 to C_4. The small deviations observed are due to the random error e that was introduced.

In order to perform a better characterization of the synthetic database, Table 1 summarizes the number of times that a state *r* is followed by a state *s*.

4 DEGRADATION MODEL

4.1 Model design

As previously mentioned, a markovian approach was adopted for the development of a degradation model for rock slopes that only takes into account its actual condition state. Moreover, considering the characteristics of the maintenance routines applied in this type of slopes, i.e., the fact that the inspections frequency is much higher than the time that these slopes take to move to the next state, it was defined that in a given moment a rock slope can only move between two consecutive condition states or stay in the same state. This implies that the matrix Q will have the structure presented in equation 2, respecting this way the condition of continuous degradation.

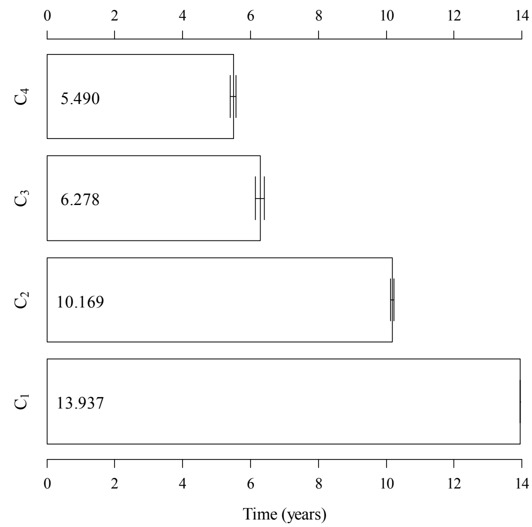

Figure 1. Experimental mean time period rock slopes measured in the synthetic database.

Table 1. Summary of the number of transitions from state *r* to state *s*.

From/To	C_1	C_2	C_3	C_4	C_5
C_1	1130	123	0	0	0
C_2	0	942	180	0	0
C_3	0	0	348	164	0
C_4	0	0	0	228	134

An initial estimate of matrix Q was obtained through the calculation of the θ_i values based on the Jackson (2011) approach:

$$\theta_i = \frac{n_{ij}}{\sum \Delta t_i} \quad (3)$$

where n_{ij} represents the number of elements that moved from state i to j and Δt_i is the time between observations where the initial condition state is i. In the calculation of matrix Q using equation 3 it is assumed that the data represent the exact transition moment in the Markov process (Jackson 2011). However, this situation hardly happens. However, assuming that the data represent the exact transition moment, the matrix Q calculated by equation 3 will correspond to the exact maximum likelihood estimates V. Thus, through the application of equation 3 the initial values of the matrix Q were estimated, which was then optimized in order to consider that the transitions take place at unknown occasions in between the observation times. At the end, the developed Markov model for rock slope degradation prediction is characterized by the following matrix Q:

$$Q = \begin{bmatrix} -0.0751 & 0.0751 & 0 & 0 & 0 \\ 0 & -0.0997 & 0.0997 & 0 & 0 \\ 0 & 0 & -0.1480 & 0.1480 & 0 \\ 0 & 0 & 0 & -0.1679 & 0.1679 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (4)$$

The calculation of the likelihood used during the optimization process of the matrix Q was defined through the following expression:

$$V = \sum_{slope=1}^{N} \sum_{transition=1}^{M} \ln(P_{ij}) = -1471 \quad (5)$$

where M is the number of transitions observed in slope s, N is the number of analyzed slopes and P_{ij} is the probability of occurrence of observed transition, as predicted by the Markov model, according to equation 1.

Matrix Q (see equation 4), which represents the proposed model for rock slopes degradation prediction, can now be used to quantify the transition probabilities for a given period of time. For instance, if we would like to estimate the transition probabilities for a period of 10 years, we will obtain the probability transition matrix presented in Table 2. From its analysis, we can see that for a slope that actually is in a condition state C_1, 10 years from now the probability of that slope being in condition state C_3 or higher is around 21%.

Table 2. Probability transition matrix for a period of 10 years based on the proposed model for rock slop degradation process.

From/To	C_1	C_2	C_3	C_4	C_5
C_1	0.4721	0.3141	0.1290	0.0549	0.0299
C_2	0	0.3690	0.2917	0.1865	0.1527
C_3	0	0	0.2276	0.3054	0.4669
C_4	0	0	0	0.1865	0.8135
C_5	0	0	0	0	1.0000

4.2 *Model assessment*

Equation 6 compares the total number of different condition states between two consecutive observations (observed and predicted by the model) (Sobreiro 2011). From its analysis, we can see that the observed and predicted values are very close, which means a good quality of the proposed model.

$$\begin{bmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \end{bmatrix}_{obs} = \begin{bmatrix} 1130 \\ 1065 \\ 564 \\ 392 \\ 656 \end{bmatrix} \Rightarrow \begin{bmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \end{bmatrix}_{pred} = \begin{bmatrix} 1129 \\ 1066 \\ 565 \\ 389 \\ 658 \end{bmatrix} \quad (6)$$

Assuming now the prediction of the slope degradation as a classification problem, the calculation of the confusion matrix (Hastie et al. 2009) can give us an overview of the quality of the proposed model. The global precision of the model is 84%. Additionally to the global performance, some other statistics can be calculated for each one of the classes (condition states), namely: Sensitivity (*Sen*) and Specificity (*Spe*) (Hastie et al. 2009). In a perfect model, both *Sen* and *Spe* will present a unit value. Table 3 summarizes the values of *Sen* and *Spe* for each one of the classes (condition states) showing once again the good performance of the proposed model.

Table 3. Summary of *Sen* and *Spe* values.

From/To	C_1	C_2	C_3	C_4	C_5
Sen	1.00	0.88	0.68	0.58	0.80
Spe	0.95	0.93	0.95	0.96	1.00

In order to illustrate graphically the performance of the proposed model, Figure 2 shows the relationship between the experimental values of the conditions state and the predicted ones according to the proposed model. Additionally, it is also shown the mean value and the standard deviation for each one of the 5 condition states, from which can be seen that the proposed model has a very interesting precision. However, some dispersion is also observed, except for condition state C_1.

The identification of the time period, according to the model is also an important aspect in model performance assessment. Therefore, the mean time period for each condition state (T_i) was calculated using equation 7.

Figure 3 shows the mean time period according to the proposed model. From its analysis we can see that there is a high proximity with the values observed in the database used in this work to fit the model (see Figure 1). This observation underlines once again the good performance of the proposed model for rock slope degradation prediction. The main difference between Figures 1 and 3 is related with the higher dispersion observed in the last one.

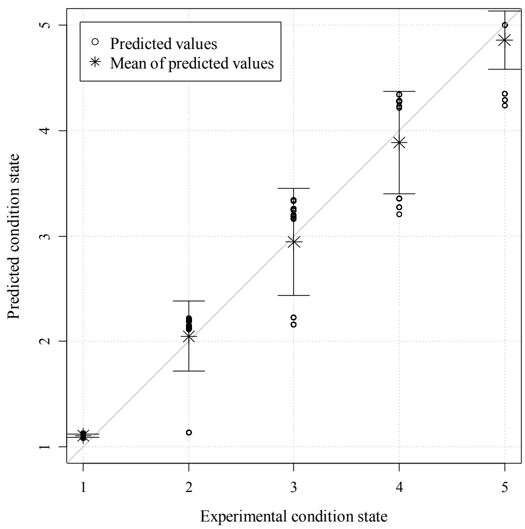

Figure 2. Relationship between experimental values of condition state and predicted.

$$T_i = \frac{1}{\theta_i} \tag{7}$$

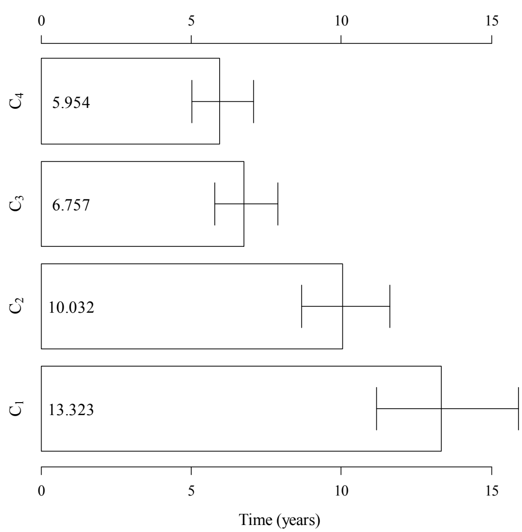

Figure 3. Mean time period in each condition.

5 MAINTENANCE MODEL

After the development of the degradation model it is important to understand the impact in the degradation curve of the application of maintenance and repair actions. Accordingly, we started by compiling a list with the different maintenance and repair actions that currently are applied to rock slopes and grouped them into three groups: improvement actions, delay actions or reduction actions. Improvement actions are those that when applied improve the slope performance, at the time of their application by a certain value. Delay actions holds the degradation process for a specified period of time, and reduction actions reduce the degradation ratio during a specific period of time. After that, we quantified their impact in the slope performance and modelled their effects over the degradation curve.

Let us assume that an action is to be applied at instant t_i, that at this time the slope performance is within the range of applicability of such action and that the condition state of the element can be determined through equation 1. At instant t_i, just before the application of the action, the condition state is defined through the following condition vector, C_t:

$$C_t = \begin{bmatrix} C_1 & C_2 & C_3 & C_4 & C_5 \end{bmatrix} \tag{8}$$

If, based on expert judgment, it is known that the maintenance action improves the condition state to C_{target}, then the components of the updated condition state vector are given by:

$$C^*_{t,i} \begin{cases} C_{t,i} & if\ i < C_{target} \\ \sum_{j=C_{target}}^{n} C_{t,j} & if\ i = C_{target} \\ 0 & if\ i > C_{target} \end{cases} \tag{9}$$

If we are applying a delay action that causes a delay in deterioration for a period of time after its application, it is assumed that the performance remains unchanged. This can be modelled considering that, during the effect of the action, the transition intensity matrix Q is an identity matrix. If an action causes a reduction in the deterioration ratio for a period of time after its application, the new condition state during that period can be calculated using equation 10, where the degradation ratio is reduced by a factor (RDR):

$$C_f = C_i \times \exp(Q \times \Delta t \times (100 - RDR)) \tag{10}$$

Figure 4 shows the impact in the degradation curve of applying a combination of these three type of actions (maintenance scenario), which are improvement, delay and reduction actions. Particularly, it was applied a reduction action after 5 years of the slope construction and then periodically every 10 years during 15 years, followed by a delay action when the slope was 25 years old and finally an improvement action after 40 years from the slope construction, which was then periodically applied every 10 years.

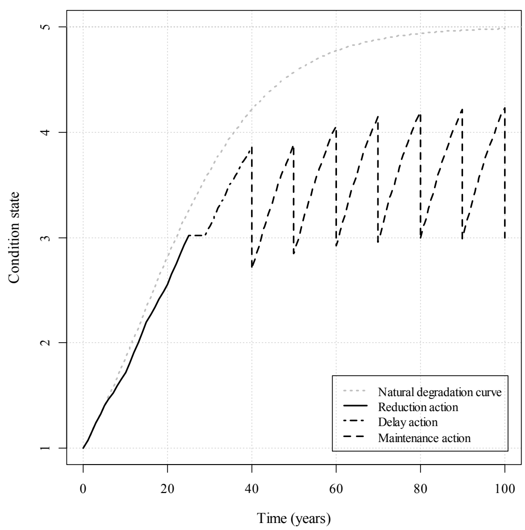

Figure 4. Impact in the degradation curve of applying a combination of three types of actions.

6 CONCLUSIONS

In this work, a theoretical approach for rock slope degradation prediction was presented. The model is based on Markov chains and predicts the evolution of a slope quality index (SQI) over time, which is defined during the exploitation phase of the slope and only requires basic information.

Based on the obtained results, the global performance of the proposed model is very good. Indeed, the model was able to predict with high accuracy the time period for each condition state observed in the database. Moreover, the proposed model showed a good capacity to predict correctly the slopes degradation over time. However, it should be stressed that the model was developed based on a synthetic database, which may be influencing its performance. Additionally, it was also proposed a maintenance model that is incorporated in the degradation model, particularly the effect of improvement, delay and reduction actions.

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