Documentos de Trabalho

Working Paper Series

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NIPE WP 02/ 2017

NÚCLEO DE INVESTIGAÇÃO EM POLÍTICAS ECONÓMICAS

UNIVERSIDADE DO MINHO

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URL:
http://www.nipe.eeg.uminho.pt/

«This work was carried out within the funding with COMPETE reference nº POCI-01-0145-FEDER-006683 (UID/ECO/03182/2013), with the FCT/MEC's (Fundação para a Ciência e a Tecnologia, I.P.) financial support through national funding and by the ERDF through the Operational Programme on "Competitiveness and Internationalization – COMPETE 2020 under the PT2020 Partnership Agreement»

Personalized Pricing with Targeted Advertising: Who are the Winners? *

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Abstract

This paper investigates who wins and who loses when firms depart from a mass advertising/uniform pricing strategy (benchmark model) to a targeted advertising/price discrimination one. We characterize the equilibrium outcomes in both settings. Then, we address the competitive and welfare effects of personalized pricing with targeted advertising by comparing the results obtained under this business strategy to the ones arising under the mass strategy. We show that *all* segment consumers are expected to pay higher average prices under the personalized advertising/pricing strategy. We also show that the personalized strategy might be a winning strategy for firms. The overall welfare effects of the personalized strategy are ambiguous. However, even when the personalized strategy boosts overall welfare, consumers might all be worse-off. Thus the paper gives support to concerns that have been raised regarding the firms' ability to adopt personalized strategies to boost profits at the expense of consumers.

1 Introduction

The recent innovation in information technologies has drastically changed firms' marketing and communication strategies, allowing firms to send personalized information (including personalized price offers) to their customers. On the one hand, it is now much easier for firms to gather, store and process consumer-specific data, which increases their ability to segment consumers with different profiles. On the other hand, firms are now able to deliver timely, targeted and local informative/ advertising content in an unprecedented way.[1]

*We thank Noriaki Matsushima for detailed comments on an earlier version of this manuscript. This research has been financed by the European Regional Development Fund through COMPETE 2020 – Programa Operacional Competitividade e Internacionalização (POCI) and by Portuguese public funds through FCT (Fundação para a Ciência e a Tecnologia) within the projects PTDC/IIM-ECO/2280/2014 and POCI-01-0145-FEDER-006890. Any errors are our own responsability.

[1] A particularly important development allowing for personalized communication between firms and their customers is the generalized use of smartphones and other portable devices (e.g tablets or smartwatches). The statistical portal Statista refers that *"For 2016, the number of smartphone users is forecast to reach 2.08 billion"*. See http://www.statista.com/statistics/330695/number-of-smartphone-users-worldwide/ [Date of access:19 au-

Firms may now easily design real-time pricing strategies in which consumers get "special discounts" and other advantages depending on their " *location via mapping software, their browser and search history, whom and what they "like" on social networks like Facebook, the songs and videos they have streamed, their retail purchase history, the contents of their online reviews and blog posts."*[2] For example, in the case of the airline industry, the adoption of personalized pricing strategies has become quite frequent, to an extent that the IATA Resolution 787 *"establishes certain goals ...including capability to provide personalized pricing offers to consumers who shop for air transportation."*[3]

However, we are far from reaching consensus on the desirability of this type of practices. For instance, some senators in the USA are skeptical about IATA Resolution 787 since *"airlines could use that information to charge frequent business travelers higher prices."*[4] This discussion illustrates how the new forms of price discrimination such as behavior-based price discrimination, location-based pricing and other strategies involving personalized prices are challenging conventional wisdom regarding the welfare effects of competitive price discrimination. Both scholars and practitioners are now participating on a lively debate regarding the *pros* and *cons* of price discrimination through new technologies allowing for personalized communication between firms and their customers.

In this respect, the Office of Fair Trading (2010)[5] has argued that *"In the online environment, price targeting may be much less transparent, which may mean that consumers do not shop around sufficiently or find it harder to compare prices. The use of online tracking also raises the same privacy objections as targeted advertising."* Shiller (2013) also highlights some negative effects of personalized pricing. He simulates counterfactual environments in which Netflix adopts first degree price discrimination and finds out that *"some consumers pay twice as much as others do for the exact same product".* More recently, the Financial Times (2017) refers to the antitrust challenges raised by pricing strategies relying on customized algorithms.[6]

In contrast, Chen (2005) argues that *"Price discrimination under customer recognition ... is by and large unlikely to raise significant antitrust concerns. In fact, as the economics literature suggests, such pricing practices in oligopoly markets often intensify competition and potentially benefit consumers."* Along the same lines, the Executive Office of the President of the United States (2015)[7] argues that *"Economic reasoning suggests that differential* [personalized] *pricing,*

gust 2016]. These devices allow firms to gather considerable amounts of information on consumers' behavior and daily activities. Moreover, they allow firms to communicate with their customers anywhere at any time in a very effective way.

[2] In "Big Data and Differential Pricing", February 2015, Executive Office of the President of the United States, available online at https://www.whitehouse.gov/sites/default/files/docs/Big_Data_Report_Nonembargo_v2.pdf

[3] See https://www.transportation.gov/airconsumer/iata-resolution-787-order-show-cause [Accessed online 18 August 2016]

[4] See https://verdict.justia.com/2015/05/13/personalized-pricing-in-the-air-why-consumers-should-be-wary-of-a-new-airline-pricing-proposal [Accessed online 19 August 2016]

[5] In Office of Fair Trading, OFT1231, 2010.

[6] Financial Times (January, 2017) refers that "existing antitrust laws, premised on human intent and action, may be inadequate to prevent companies from abusing their market power in the digital era, some experts say". See: https://www.ft.com/content/9de9fb80-cd23-11e6-864f-20dcb35cede2 [Date of access: 26th January 2017]

[7] In "Big Data and Differential Pricing", February 2015, Executive Office of the President of the United States,

whether online or offline, can benefit both buyers and sellers... Thus, we should be cautious about proposals to regulate online pricing – particularly if we believe that online markets are particularly competitive."

In this paper, we contribute to this debate by providing an analytical study on the competitive effects of personalized pricing through targeted informative advertising. We first analyze a benchmark model in which firms only have access to mass communication technologies and are, therefore, confined to engage in uniform pricing strategies. Then, we shed some light on the competitive effects of personalized ads and pricing by comparing the (price and welfare) outcomes in this benchmark model to the ones arising in a model of personalized communication (with targeted ads and personalized price offers, as in Esteves and Resende, 2016).[8] While economists have long been concerned in understanding the profit and welfare effects of price discrimination and advertising separately, little is known about the competitive and welfare effects of price discrimination enabled by targeted advertising (such combination might be very important when firms adopt strategies based on personalized communication with their customers).

The paper aims at filling in this literature gap by investigating who may be the losers and the winners when firms implement price discrimination strategies through personalized informative ads. More precisely, we address the following research questions: What are the price and welfare effects of personalized pricing through targeted advertising? Can firms sustain higher prices and obtain greater profits by combining personalized ads with price discrimination strategies? Do consumers benefit from targeted ads enabling personalized pricing?

In order to address these questions, we propose a static game of duopoly competition based on Esteves and Resende (2016). There are two firms launching two *new* differentiated products. As in Stahl (1994), firms need to invest in advertising to generate demand awareness. By investing in advertising, firms endogenously segment the market into captive (partially informed) and selective (fully informed) customers. The model exhibits best-response asymmetry (Corts, 1998): each firm has a strong and a weak market segment (a firm's strong market includes consumers with a stronger preference for that firm, whereas its weak market includes consumers with a stronger preference for the rival market).[9] More precisely, the set of potential buyers is assumed to be composed of two distinct segments of equal mass. As in Shilony (1977), Raju *et al.* (1990), Esteves (2010) and Esteves and Resende (2016), everything else the same, consumers in segment i prefer product i over product j to a certain extent (measured by a brand awareness

available online at https://www.whitehouse.gov/sites/default/files/docs/Big_Data_Report_Nonembargo_v2.pdf

[8] In the paper we only compare our benchmark model in which firms use mass advertising (and therefore they do not have the technical means to engage in price discrimination) to a situation in which firms can personalize both ads and price offers at the same time. We do not compare the outcomes of our benchmark model to a set-up in which firms use targeted advertising and uniform pricing (which is also presented in Esteves and Resende, 2016) because the empirical evidence suggests that firms are actually using customized communication channels (e.g. targeted informative ads) as a price discrimination device (see the previous examples in the text). Therefore the most interesting welfare analysis requires the comparison of the benchmark of mass communication (with mass advertising and uniform pricing) to a set-up of personalized communication (with targeted ads and price discrimination).

[9] In the literature of price discrimination, a market is designated as "strong" if in comparison to uniform pricing a firm wishes to increase its price there. The market is said to be "weak" if the reverse happens.

parameter). Accordingly, consumers may end up buying their least preferred product, provided its price is sufficiently low *vis-à-vis* the competing product.

We depart from Esteves and Resende (2016) regarding the features of firms' advertising and pricing strategies. In our baseline model firms use a mass advertising strategy in which they choose an advertising to the entire market and all ads must have the same content. Accordingly, price discrimination is technically unfeasible (all the ads are identical, posting a similar price). We will then analyze the price, profit and welfare effects of personalized pricing through targeted ads by comparing our findings to Esteves and Resende (2016), who consider a model with targeted advertising and price discrimination, without providing any insights on the welfare outcomes or the price effects of customized information.

The stylized model addressed in this paper offers new insights to the literature on price discrimination based on customer recognition. First, we find that both firms will compete in both market segments only when the advertising costs are not excessively high and the consumers' willingness to pay for the products is sufficiently high *vis-à-vis* the brand awareness parameter.

Second, when both firms compete in both market segments, we find that average prices with mass communication (mass advertising and uniform pricing) are *below* their counterparts with personalized communication, regardless of the market segment. Consequently, we show that firms are able to take advantage of the interplay between price discrimination and targeted advertising to sustain higher prices. More precisely, we find that expected profits with targeted advertising and price discrimination are *above (below)* their non-discrimination counterparts, provided consumers' willingness to pay for the goods is high (low) enough and/or the advertising is not too expensive (cheap). Our results depart from the classic prisoner dilemma result that arises in models of price discrimination (without advertising) in a set-up with fully informed consumers and exhibiting best-response asymmetry (e.g. Thisse and Vives, 1988). Our results also depart from Iyer *et al.* (2005) who argue that targeting always increases the firms' profits; and from Brahim *et al.* (2010) who show that profits with mass advertising are always above the targeting profits, provided that firms advertise both to their strong and to their weak markets.

Finally, we find that it is not possible to make general predictions regarding the welfare effects of price discrimination through targeted ads since the expected welfare may either increase or go down when firms move from a set-up with mass communication (mass advertising/uniform pricing) to a set-up with personalized communication (targeted advertising/ price discrimination). However, even if welfare goes up when firms run targeted advertising campaigns with personalized price offers, firms may be the only ones benefiting from this strategy. On the contrary, consumers may end up being worse-off than in the case of mass advertising/ uniform pricing since firms take advantage of the interplay between targeted ads and price discrimination to induce inefficient shopping and relax price competition.

Related literature This paper is mainly related to two strands of the literature, namely the literature on competition with informative advertising and the more recent literature on price discrimination based on customer recognition.

Following the seminal work of Butters (1977), a vast literature has investigated competition with informative advertising (e.g. Stahl, 1994 or Grossman and Shapiro, 1984). The literature on competitive targeted advertising is relatively recent.[10] The works of Iyer *et al.* (2005), Galeotti and Moraga-Gonzalez (2008) and Brahim *et al.* (2010) are key contributions to the understanding of the economics of targeting advertising, namely in respect to firms' pricing and advertising decisions. Iyer *et al.* (2005) develop a model in which firms face two types of consumers: captive consumers (who are loyal to one of the firms and always buy from their favorite firm regarding the price of the other good) and shoppers (who always buy the less expensive good). They show that targeted advertising leads to higher profits regardless of whether firms have or not the ability to adopt price discrimination strategies.

Galeotti and Moraga-Gonzalez (2008) study firms' advertising and pricing strategies in a market with a homogeneous good, where market segmentation is based on consumer attributes that are completely unrelated to tastes. The paper compares market outcomes under mass advertising with uniform pricing and targeted advertising with price discrimination. Assuming *ex-ante* that one market segment is more profitable than the other one, the paper shows that the possibility of market segmentation may lead to positive profits within an otherwise Bertrand-like setting.[11]

Brahim *et al.* (2010) develop an extended version of Grossman and Shapiro (1984) with targeted advertising. The authors also show that targeted advertising does not always have a positive effect on firms' profits. In particular, when firms advertise in both market segments, targeted advertising may reduce equilibrium profits.

This paper is closely related to Esteves and Resende (2016). We enrich their work in two ways. First, we assess welfare outcomes in the model with targeted advertising and price discrimination. Second, we compare the price and welfare outcomes in their model to the ones arising in a model of uniform pricing and mass advertising, which allow us to characterize the strategic effects arising from the interplay between targeted advertising and price discrimination.

Finally, the paper is also related to the literature on competitive price discrimination. It is related to those models where, in the terminology of Corts (1998), the market exhibits *best-response asymmetry.* In these models profit will typically decrease when firms use price discrimination. A useful model for understanding the profit effects of price discrimination in markets with best-response asymmetry is by Thisse and Vives (1988). There are two firms located at the extremes of the segment $[0, 1]$. Consumers are uniformly distributed in the line segment

[10] See for example, Bergemann and Bonatti (2011), Chandra (2009); Gal-Or and Gal-Or (2005); Gal-Or and Gal-Or (2006); Gal-Or, Gal-Or, May and Spangler (2008).

[11] Esteban et al. (2006) and Esteban and Hernandez (2007) study targeted advertising in an oligopolistic market with vertical differentiation. The authors show that the possibility of targeting advertising with price discrimination may lead to permanent segmentation of the market. From a welfare perspective, targeted advertising is shown to have a positive effect on consumers' surplus and social welfare. Other papers studying targeted advertising include Roy (2000), Gal-Or and Gal-Or (2006); Gal-Or, Gal-Or, May and Spangler (2008). Roy (2000) studies optimal advertising choices when firms can target consumers on the basis of their address (i.e. their location on a Hotelling framework). Gal-Or and Gal-Or (2006) study the competitive effects of targeted advertising when a single media content distributor delivers advertising messages on the behalf of firms. Gal-Or, Gal-Or, May and Spangler (2008) deal with the issue of imperfect advertising tailoring, studying to which extent an advertiser should allocate resources to increase the quality of its targeting.

and firms can observe the location (or brand preference) of each individual consumer and price accordingly. They show that price discrimination intensifies competition, so that *all* prices fall as well as profits. Price discrimination based on customer recognition in a static model has also been examined by Bester and Petrakis (1996). Other authors have analyzed price discrimination under customer recognition in a dynamic setting where firms recognize consumers after the first-period interaction. This form of price discrimination has been termed as Behavior-Based Price Discrimination (BBPD) or price discrimination based on purchase history (e.g. Chen, 1997, Villas-Boas, 1999, Fudenberg and Tirole, 2000, Taylor, 2003, Esteves, 2010 and Gehrig *et al.* 2011, 2012).[12] In all of these approaches, profits fall down with price discrimination. More recently, a few works have stressed that profits may increase with price discrimination. In static settings due to firms' heterogeneity (e.g. Chen and Zhang (2002)), or multi-dimensional product differentiation (e.g. Esteves, 2009b). In dynamic settings, profits may increase with BBPD due to imperfect correlated preferences across time (Chen and Pearcy, 2010) and Shin and Sudhir, 2010); and consumers' fairness concerns (Esteves and Mahmood, 2017). In the previous works there is no role for advertising since consumers are perfectly informed. Esteves (2009a) departs from this hypothesis by looking at a set-up with informative advertising in which firms need to invest in advertising to generate demand and inform consumers about prices. However, Esteves (2009a) considers that consumers are ex-ante identical regarding their preferences for the products, whereas the present paper assumes that consumers are ex-ante heterogenous.

The rest of this paper is organized as follows. Section 2 describes the main ingredients of the benchmark model with mass advertising and uniform pricing. Section 3 briefly reviews the results of Esteves and Resende (2016) in a set-up with targeted ads and price discrimination. Then, we proceed to the analysis of the competitive effects of personalized communication (regarding equilibrium prices, market segmentation and profits). Section 4 focuses on the welfare effects of competitive price discrimination through targeted advertising. Finally, Section 5 concludes. The Appendix collects all the proofs.

2 Mass Advertising

2.1 The model

Consider a market with two firms, $i = A, B$. Each firm is launching a new good, produced at a constant marginal cost, which is assumed to be zero without loss of generality. Firms need to invest in informative advertising[13] to generate demand, meaning that consumers are only aware of the existence and the price of the new products when exposed to firms' ad campaigns. There is a unit mass of consumers with a common reservation price v. Each consumer buys at most a single unit of either good A or B. The set of consumers is divided in two segments with equal size: segment a and segment b. Consumers in segment i prefer product i over the other product

[12] For other recent papers on price discrimination and customer recognition see also Chen and Pearcy (2010), Ghose and Huang (2006), Shy and Stenbacka (2012).

[13] Implicitly it is assumed that search costs for new products are prohibitively high. See Bagwell (2007) for a survey on the economics of advertising.

by a degree equal to $\gamma > 0$, which can be interpreted as the degree of consumers' preference towards its favorite brand.[14] Hence, as in Shilony (1977), Raju *et al.* (1990), Esteves (2010) and Esteves and Resende (2016), we propose a model with best-response asymmetry, in which each firm *potentially* has a weak and a strong segment of consumers.

In our baseline model firms only have access to mass advertising technologies and therefore they are confined to uniform campaigns in which all the ads present exactly the same price to all the consumers in the market (regardless of whether they have a weak or a strong preference for their good).

We look for firms' optimal advertising and pricing decisions in the context of a *simultaneous non-cooperative* game. More precisely, the problem of firm i consists in choosing an optimal advertising reach, ϕ_i, and the corresponding uniform pricing strategy p_i, $i = A, B$. Advertising is a costly activity. Let $A(\phi_i) = \lambda\eta(\phi_i)$ denote the advertising cost of firm i when it reaches a fraction ϕ_i of consumers, with $A(0) = 0$; $A_{\phi_i} > 0$ and $A_{\phi_i\phi_i} \geq 0$.[15] Whenever a specific functional form is needed, we will use the quadratic advertising technology proposed in Tirole (1988), namely $A(\phi_i) = \lambda\eta(\phi_i) = \lambda\phi_i^2$ in order to guarantee that our results can be compared to the ones obtained by Esteves and Resende (2016) in a model with targeted advertising. This specification was also used in other models like Galeotti and Moraga-Gonzalez (2008) or Brahim *et al.* (2011).

After firms have sent their ads independently, a proportion ϕ_i and ϕ_j of customers is reached by firm i and j's advertising, respectively. Thus, in each market segment, firm i has a fraction $\phi_i(1-\phi_j)$ of captive customers, who are only aware of its product and a fraction $\phi_i\phi_j$ of selective consumers, who are fully informed about the existence and the price of both products. There is also a fraction of consumers $(1-\phi_i)(1-\phi_j)$ that remains uninformed.

Consider first the case of selective consumers. In segment a they compare the net utility of purchasing good A at price p_A, $v - p_A$, with the net utility of purchasing good B at price p_B, $v - \gamma - p_B$. For $p_i \leq v - \gamma$, $i = A, B$, a selective consumer in segment a buys good A if and only if $p_A - \gamma \leq p_B$. Analogously, a selective consumer in segment b buys good A if and only if $p_A + \gamma < p_B$. Similar reasoning is applied to obtain the conditions under which different types of selective consumers buy good B. A captive consumer to firm A in segment a, buys its good if $p_A \leq v$. If instead the captive consumer belongs to segment b, he/she only buys good A when $p_A \leq v - \gamma$.

[14] Note that the modelling options are compatible with other interpretations for γ. Indeed, it could also be interpreted as a search cost, a transportation cost (as in Shilony, 1977) or a switching cost.

[15] Subscripts denote partial derivatives. These assumptions on the specification of the advertising cost have the following implication: when ϕ_i goes up it becomes increasingly more expensive to inform an aditional consumer. This is a standard assumption in the informative advertising literature (e.g. Butters, 1977 or Tirole, 1988).

Firm i's demand, $D_i(p_i, p_j)$, when firms use a mass advertising technology is given by:

$$D_i(p_i,p_j) = \begin{cases} 0 & if & p_i > v \\ \frac{1}{2}\phi_i\left(1-\phi_j\right) & if & p_i \in (v-\gamma, v] \wedge p_j + \gamma < p_i \\ \frac{1}{2}\phi_i\left(1-\phi_j\right) + \frac{1}{2}\phi_i\phi_j & if & p_i \in (v-\gamma, v] \wedge p_j + \gamma \geq p_i \\ \phi_i\left(1-\phi_j\right) & if & p_i \leq v-\gamma \wedge p_j + \gamma < p_i \\ \phi_i\left(1-\phi_j\right) + \frac{1}{2}\phi_i\phi_j & if & p_i \leq v-\gamma \wedge p_j - \gamma \leq p_i \leq p_j + \gamma \\ \phi_i\left(1-\phi_j\right) + \phi_i\phi_j & if & p_i \leq v-\gamma \wedge p_i < p_j - \gamma \end{cases}$$

Firm i's expected profit is equal to $E\pi_i = p_i D_i(p_i, p_j) - A(\phi_i)$.

2.2 Equilibrium Analysis

We now study firms' equilibrium price and advertising strategies. Our analysis is focused on the *symmetric* Nash equilibrium. Given the rival's strategies, ϕ_j and p_j, the problem of firm i consists of choosing the advertising intensity, ϕ_i, and the price, p_i that maximize its expected profit. The next Proposition analyzes the conditions under which our price-advertising game has an interior Nash Equilibrium in pure strategies (PSNE), considering the quadratic advertising technology $A(\phi_i) = \lambda\phi_i^2$. We denote by p^* and ϕ^m firms' optimal choices in the symmetric equilibrium, with $p_i = p_j = p^*$ and $\phi_i = \phi_j = \phi^m$.

Proposition 1 *(i) When $v \leq 2\gamma$ and $4\lambda \geq v$, there is an interior PSNE. The equilibrium price is $p^* = v$ and the interior equilibrium advertising intensity is $\phi^m = \frac{v}{4\lambda}$.*

(ii) When $v > 2\gamma$ and $4\lambda \geq (v-\gamma)\max\left\{\frac{v}{v-2\gamma}, \frac{v-2\gamma}{\gamma}\right\}$, there is an interior PSNE. The equilibrium price is $p^* = v-\gamma$ and the interior equilibrium advertising intensity is $\phi^m = \frac{2(v-\gamma)}{v-\gamma+4\lambda}$.

Proof. See the Appendix.

Proposition 1 shows that the game with mass advertising and uniform pricing has an equilibrium in pure strategies provided that advertising costs are not too low. The features of the equilibrium depend on the magnitude of the brand awareness parameter (γ) *vis-à-vis* the reservation price (v). When $v \leq 2\gamma$, products are very differentiated in the eyes of consumers and firms use pricing strategies to fully separate their markets (in this setting, firms are unable to separate markets through customized information). Each firms acts as a monopolist in the respective strong market segment, charging a price equal to v. When $v \leq 2\gamma$ and advertising costs are low $4\lambda < v$, the full segmentation equilibrium described before can also be sustained. However, there is a corner advertising solution, with $\phi^m = 1$. In that case, all consumers are informed about the existence of their favorite product and all of them are buying efficiently (but their net surplus is zero since both firms are charging a price equal to v).[16]

[16] The Proof of this result is available from the authors upon request.

When the brand awareness parameter is not too high, in the sense that $v > 2\gamma$, firms are interested in serving the weak market as well. More precisely, in case *(ii)* of Proposition 1, each firm serves all the informed consumers in its strong market and the captive consumers in its weak market. Although both firms are active in both markets, the two segments remain separated to some extent (in the sense that firms do not compete for selective consumers in the weak market). In this solution, firms cannot charge a price above $v-\gamma$ (otherwise they would not serve captive consumers in the weak market), which means that firms are unable to fully extract consumers' surplus in the strong segment. For $v > 2\gamma$, the revenue loss in the strong market (where firms are giving away an amount γ for each unit sold to their customers) is more than compensated by the revenue increase resulting from extra sales to the *captive* consumers in the weak market, provided that advertising costs are not too low, i.e. $\lambda \geq \left(\frac{v-\gamma}{4}\right)\max\left\{\frac{v}{v-2\gamma}, \frac{v-2\gamma}{\gamma}\right\}$. When the advertising cost goes below this threshold, the result described in part *(ii)* of Proposition 1 is no longer a SPNE because firms start to be interested in competing for all the selective consumers in the weak market. Indeed, for low λ, both firms tend to advertise more (in both market segments),[17] increasing the fraction of selective consumers. The set of captive consumers in the weak market becomes rather small and therefore the strategy relying on sales to captive consumers in such segment becomes less attractive. The following Lemma shows that when $v > 2\gamma$ and $\lambda < \left(\frac{v-\gamma}{4}\right)\max\left\{\frac{v}{v-2\gamma}, \frac{v-2\gamma}{\gamma}\right\}$, there is no PSNE. However, we will see later that there exists an equilibrium in mixed-strategies.

Lemma 1 *When $v > 2\gamma$ and $4\lambda < (v-\gamma)\max\left\{\frac{v}{v-2\gamma}, \frac{v-2\gamma}{\gamma}\right\}$ there is no symmetric interior PSNE and firms will have incentives to compete for all the selective consumers in the market.*

Proof. See the Appendix.

The intuition behind Lemma 1 is the following. If advertising is cheap enough (small λ) and γ is not too high, firms may poach selective consumers in the weak market at a relatively low cost (informing consumers is not too expensive as λ is low and the discount needed to entice consumers to buy their least preferred product is not too large since γ is low). Thus, as long as v is sufficiently high, the existence of a positive fraction of selective consumers with a preference for the rival firm creates a tension between each firm's incentives to price low to attract consumers in its weak market and the firm's incentives to price high to extract rents from consumers in its strong market. In equilibrium each firm follows a mixed pricing strategy as an attempt to prevent the rival from systematically predicting its price, which in turn makes undercutting less likely.

From now on, we shall concentrate our analysis on the most interesting case in which firms compete for all consumers in the market, which occurs when $p \leq v-\gamma$ and Lemma 1 holds.

[17] Later on, we will demonstrate this result from an analytical point of view.

Proposition 2 below characterizes the symmetric interior Nash equilibrium in mixed strategies(MSNE).

Suppose that firm j randomly selects a price from the c.d.f (cumulative distribution function), $F_j(p)$. In a symmetric MSNE, both firms adopt the same pricing strategy, thus, for the sake of simplicity write $F_i(p) = F_j(p) = F(p)$. As we look at the domain $p \leq v - \gamma$, then all captive consumers will always buy the product from the firm they know. In the case of selective consumers, when firm i charges price p, three events are relevant. First, if $p_j > p + \gamma$ firm i captures the whole group of selective consumers in the market. This occurs with probability $[1 - F(p+\gamma)]$ and yields a total revenue equal to $p\phi_i$. Second, firm i captures no selective consumer if $p_j < p - \gamma$. This happens with probability $F(p-\gamma)$. In this case, firm i's revenue only stems from the sales to its captive consumers, being equal to $p\phi_i(1-\phi_j)$. Finally, each firm serves its group of strong selective consumers (who are all buying efficiently) if $p - \gamma < p_j < p + \gamma$. This event occurs with probability $[F(p+\gamma) - F(p-\gamma)]$ and yields a total revenue equal to $p\left[\phi_i(1-\phi_j) + \frac{1}{2}\phi_i\phi_j\right]$.

Firm i's expected profit, given the rival's decisions, can be written as follows:

$$E\pi_i = p\phi_i(1-\phi_j) + p\phi_i\phi_j\left[1 - \frac{1}{2}F(p+\gamma) - \frac{1}{2}F(p-\gamma)\right] - A(\phi_i).$$

In an MSNE, any price chosen from a firm's equilibrium price support should generate the same expected profit, which will be denoted by k^m.

Proposition 2. *When firms adopt a mass advertising campaign with uniform pricing, there is a symmetric interior MSNE with both firms competing for the selective consumers, such that:*

(i) each firm's price is randomly chosen from the c.d.f

$$F^m(p) = \left\{\begin{array}{ccc} 0 & if & p < p_{\min} \\ 1 - \frac{2}{(\phi^m)^2}\left[\frac{k^m}{p+\gamma} - \phi^m(1-\phi^m)\right] & if & p_{\min} \leqq p \leqq p_{\max} - \gamma \\ 2 - \frac{2}{(\phi^m)^2}\left[\frac{k^m}{p-\gamma} - \phi^m(1-\phi^m)\right] & if & p_{\max} - \gamma \leqq p < p_{\max} \\ 1 & if & p \geqq p_{\max} \end{array}\right\} \tag{1}$$

with

$$p_{\max} = \frac{2k^m}{\phi^m(2-\phi^m)} + \gamma \leq v - \gamma \text{ and } p_{\min} = p_{\max} - 2\gamma, \tag{2}$$

and

$$k^m = \frac{\gamma}{2}(2-\phi^m)^2(1+\Lambda), \tag{3}$$

with $\Lambda = \sqrt{1 + \left(\frac{\phi^m}{2-\phi^m}\right)^2} > 1$.

(ii) Each firm chooses an advertising reach $\phi^m \in (0,1)$, *implicitly given by:*

$$\frac{1}{2}(p_{\max} - \gamma)(2-\phi^m) = A_\phi(\phi^m). \tag{4}$$

(iii) Each firm earns an overall expected profit of

$$E\pi^m = \phi^m A_\phi(\phi^m) - A(\phi^m). \tag{5}$$

Proof.

See the Appendix.

Proposition 2 provides a characterization of the symmetric and interior MSNE for a general specification of the advertising cost function $A(\phi)$, with $A_\phi > 0$ and $A_{\phi\phi} \geq 0$.[18] The interior equilibrium in Proposition 2 has a few interesting features. To start, equilibrium decisions are not directly affected by the consumers' willingness to pay, v.[19] This is a standard result in models of uniform price competition with differentiated products (e.g. Hotelling model). The following Corollary discusses the impact of γ and λ on the equilibrium advertising intensity ϕ^m, under fairly general assumptions regarding the mass advertising cost function.

Corollary 1 *For $v > 2\gamma$, the equilibrium mass advertising levels are increasing with the brand awareness parameter γ. On the contrary, an increase in the marginal cost of advertising $A_\phi(\phi)$ leads to a reduction in ϕ^m.*

Proof. See the Appendix.

The second result in Corollary 1 is quite intuitive. As advertising becomes more expensive, firms advertise less. The impact of the brand awareness parameter is less straightforward. When γ goes up (but remains sufficiently low to guarantee the inequality $v > 2\gamma$), consumers in the weak market need a greater discount to be enticed to buy their least preferred good. When both firms are competing for all consumers in both segments (as in Proposition 2), they try to offset the negative price effect just described by making more sales in the weak market, which requires a greater investment in informative advertising, implying $\frac{\partial\phi^m}{\partial\gamma} > 0$.

It is worth noting that when both firms increase ϕ^m, the set of selective consumers becomes larger. Hence, not surprisingly, the maximum price in the support of the equilibrium price distribution moves to the left when ϕ^m goes up. In other words,

$$\frac{\partial p_{\max}}{\partial\phi^m} = -\gamma\left(\frac{2-\phi^m}{\phi^m}\right)^2 \frac{(\Lambda+1)\Lambda}{(\phi^m)^2 + 2(1-\phi^m)} < 0.$$

Therefore, the effect of γ on the expected profit is *a priori* ambiguous. On the one hand, γ has a direct positive effect on prices and profits (see (3)). On the other hand, it increases ϕ^m, which will have a negative effect on average prices.

[18] Note that this equilibrium characterization does not cover the case of costless advertising, in which the equilibrium woukd be $\phi^m = 1$. In that situation, the additional cost of doing an additional ad (zero) would always be lower than the corresponding additional revenue $\frac{1}{2}(p_{\max} - \gamma)(2 - \phi^m) > 0$ (since $\phi^m \leq 1$ and $p_{\max} > \gamma$ as in equation (2)). When advertising is costless, the equilibrium prices would then range from $p_{\min} = \sqrt{2}\gamma$ and $p_{\max} = (\sqrt{2}+2)\gamma$.

[19] Nonetheless, it is worth noting that v indirectly affects firms' profit in a very important way since consumer's reservation price v plays a key role in the delimitation of the domain in which the results in Proposition 2 may arise. This occurs both through the condition $v > 2\gamma$ (which determines firms' incentives to induce or not full market segmentation in equilibrium) and through the impact of v on the critical $\lambda - value$ in Lemma 1.

In order to obtain analytical insights and disentangle the effects described above, we need to specify the functional form of the advertising technology. In the remainder of the paper, we will consider the quadratic advertising technology $A(\phi) = \lambda\phi^2$. Besides the tractability advantages, such functional form has been used in previous works (E.g. Brahim *et al.,* 2011 or Galeotti and Moraga-Gonzalez, 2008) and it allows us to directly compare the results of this paper with the ones obtained by Esteves and Resende (2016) in a model of targeted advertising and price discrimination.

In Lemma 2, we characterize the interior MSNE, with $\phi^m < 1$. Afterwards, Corollary 2 briefly describes the equilibrium outcomes in the corner solution with $\phi^m = 1$. It is worth noting that in the last case, our model will reproduce the equilibrium outcomes obtained by Esteves (2010) in a model with full information. Thus, we will concentrate on the interior equilibrium outcomes, in which some consumers remain uninformed, thereby departing from the full information set-up of Esteves (2010). In particular, in Lemma 2, we assume that the conditions (A1) and (A2) hold in order to guarantee the existence of an interior MSNE in which both firms are competing for selective consumers.[20]

Lemma 2. *Let $A(\phi) = \lambda\phi^2$ and assume*

$$v \geq \left(\sqrt{2}+3\right)\gamma \tag{A1}$$

$$\frac{\gamma\left(\sqrt{2}+1\right)}{4} \leq \lambda < \left(\frac{v-\gamma}{4}\right)\min\left\{\frac{v-3\gamma}{2\gamma}, \frac{v}{v-2\gamma}\right\}. \tag{A2}$$

In the MSNE described in Proposition 2, each firm chooses an advertising reach $\phi^m \in (0,1)$, equal to

$$\phi^m = \frac{2\left(\gamma^2+8\lambda\gamma\right)^{1/2}}{\left(\gamma^2+8\lambda\gamma\right)^{1/2}+4\lambda}. \tag{6}$$

The support of the distribution of equilibrium prices is such that:

$$p_{\max} = \left(\gamma^2+8\lambda\gamma\right)^{1/2}+\gamma \text{ and } p_{\min} = p_{\max}-2\gamma$$

and the cdf $F^m(p)$ is equal to:

$$F^m(p) = \begin{cases} 0 & \text{if} \quad p < \left(\gamma^2+8\lambda\gamma\right)^{1/2}-\gamma \\ 4\lambda\left[\left(\gamma^2+8\lambda\gamma\right)^{-1/2}-\frac{1}{p+\gamma}\right] & \text{if} \quad \left(\gamma^2+8\lambda\gamma\right)^{1/2}-\gamma \leq p \leq \left(\gamma^2+8\lambda\gamma\right)^{1/2} \\ 1-4\lambda\left[\frac{1}{p-\gamma}-\left(\gamma^2+8\lambda\gamma\right)^{-1/2}\right] & \text{if} \quad \left(\gamma^2+8\lambda\gamma\right)^{1/2} \leq p < \left(\gamma^2+8\lambda\gamma\right)^{1/2}+\gamma \\ 1 & \text{if} \quad p \geq \left(\gamma^2+8\lambda\gamma\right)^{1/2}+\gamma \end{cases} \tag{7}$$

Firms get an overall expected profit $E\pi^m = A(\phi^m) = \frac{4\lambda\left(\gamma^2+8\lambda\gamma\right)}{\left[\left(\gamma^2+8\lambda\gamma\right)^{1/2}+4\lambda\right]^2}$.

[20] More precisely, the RHS inequality in condition (A2) guarantees that in equilibrium both firms compete for all selective consumers, with $p_{\max} \leq v-\gamma$. Condition (A1) and the LHS inequality of condition (A2) guarantee an interior advertising equilibrium, with $\phi^m < 1$.

Proof. See the Appendix.

Corollary 2. *When $2\gamma < v < \left(\sqrt{2}+3\right)\gamma$ or when $v > \left(\sqrt{2}+3\right)\gamma$ and $\lambda < \frac{\gamma\left(\sqrt{2}+1\right)}{4}$, all consumers are informed about the existence of both products (i.e. $\phi^m = 1$). In equilibrium, the support of the firms' optimal prices is $\left[\sqrt{2}\gamma, \left(\sqrt{2}+2\right)\gamma\right]$ and firms obtain an expected profit equal to $k^m = \left(\frac{1}{2}\sqrt{2}+\frac{1}{2}\right)\gamma - \lambda > 0$.*

Proof. See the Appendix.

Corollary 2 shows that, when λ is sufficiently low, in equilibrium all the consumers are fully informed. As mentioned earlier, in this case we recover the results obtained by Esteves (2010). Lemma 2 shows that new results may arise when firms need to invest in advertising to generate consumers' awareness. These new results depend on two key parameters: γ and λ. These parameters are related to two distinct sources of market power: product differentiation (through the brand awareness parameter, γ) and the existence of a fraction of consumers who remain uninformed about the existence of the rival product (as a result of firms' advertising decisions, which crucially depend on λ). The following Corollary points out how equilibrium prices are affected by γ and λ.

Corollary 3. *The equilibrium expected price is increasing both with γ and with λ, with:*
(i) $\frac{\partial p_{\max}}{\partial\gamma} = \frac{4\lambda+\gamma+\sqrt{\gamma^2+8\lambda\gamma}}{\sqrt{\gamma^2+8\lambda\gamma}} > 0$ *and* $\frac{\partial p_{\min}}{\partial\gamma} = \frac{4\lambda+\gamma-\sqrt{\gamma^2+8\lambda\gamma}}{\sqrt{\gamma^2+8\lambda\gamma}} > 0$;
(ii) $\frac{\partial p_{\max}}{\partial\lambda} = 4\frac{\gamma}{\sqrt{\gamma^2+8\lambda\gamma}} > 0$ *and* $\frac{\partial(p_{\max}-p_{\min})}{\partial\lambda} = 0$.

Proof. Follows directly from Lemma 2.

From Corollary 3 follows that equilibrium expected profits must be increasing with γ and λ.[21] It is worth noting that when λ increases, advertising is more expensive, which is of course bad for profits (*direct effect*). However, when λ goes up, firms reduce the intensity of advertising and the set of selective consumers is narrower. As a result, firms relax price competition (in order to extract more surplus from captive consumers), leading to an increase in expected prices, which obviously has a positive impact on prices (*strategic effect*). The next Lemma shows that, under conditions (A1) and (A2), the expected profits with mass advertising are increasing with λ, meaning that the strategic effect dominates the direct one (this result frequently arises in models with informative advertising, e.g. Grossman and Shapiro, 1984, Sthal, 1994 or Esteves, 2009a).

Lemma 3. *Under conditions (A1) and (A2), the equilibrium expected profit is increasing with advertising costs, $\frac{\partial E\pi^m}{\partial\lambda} > 0$.*

[21] Coming back to the discussion following Corollary 1, this allows us to conclude that, for the quadratic advertising technology, the positive direct effect of γ on prices and profits more than compensates its indirect negative effect on prices, as a result of the increase in the equilibrium advertising levels, $\frac{\partial\phi^m}{\partial\gamma} = \frac{8\lambda(4\lambda+\gamma)}{16\lambda^2\sqrt{\gamma^2+8\lambda\gamma}+8\lambda\gamma(8\lambda+\gamma)+\left(\gamma^2+8\lambda\gamma\right)^{\frac{3}{2}}} > 0$.

Proof. See the Appendix.

Finally, in order to analyze the welfare properties of the equilibrium in which firms randomize prices, we need to investigate whether selective consumers are buying efficiently in equilibrium. To this end, we need to compute the probability of firms sharing evenly the group of selective consumers, so that each firm is serving the group of selective consumers in the corresponding strong market.

Proposition 3. *Each firm serves its group of strong selective customers with probability* $q \in [0,1]$ *which is equal to*

$$q = 1 + 2\frac{(\phi-2)^2}{\phi^2} - \frac{8k^2}{\gamma^2\phi^4}\ln\left(\frac{(p_{\max}-\gamma)^2}{p_{\max}(p_{\max}-2\gamma)}\right)$$

For the quadratic advertising technology this probability is equal to:

$$q = 1 - 32\lambda^2\left[\frac{1}{\gamma^2}\ln\left(\frac{\gamma}{8\lambda}+1\right) - \frac{1}{\gamma(8\lambda+\gamma)}\right]$$

Proof. See the Appendix.

3 Competitive effects of customized ads and prices

3.1 Targeted advertising and price discrimination

This section analyzes the competitive effects generated by the combination of price discrimination and targeted advertising (personalized communication). We compare the equilibrium outcomes obtained in the previous section (in which firms choose an advertising intensity, ϕ^m, and the corresponding price, p, to the whole market) to the equilibrium outcomes described by Esteves and Resende (2016) in a model of targeted advertising and personalized pricing. Before moving to this comparison, we briefly recall some modelling options and some results obtained in Esteves and Resende (2016). They consider that firms may choose different advertising intensities to be targeted to the strong and to the weak market segments (ϕ_i^o and ϕ_i^r, respectively).[22] They may also choose different prices to each market segment (p_i^o, p_i^r). The advertising technology is additive separable, with $A(\phi_i^o, \phi_i^r) = \eta(\phi_i^o)^2 + \eta(\phi_i^r)^2$.

The next Remark reproduces the results in Esteves and Resende (2016) when firms use a quadratic advertising technology, $A(\phi_i^o, \phi_i^r) = \lambda(\phi_i^o)^2 + \lambda(\phi_i^r)^2$. As already mentioned, we are focusing our attention on the domain in which both firms have incentives to serve all the consumers, regardless of the market segment they belong (in other words, unless explicitly mentioned, we will assume that the conditions (A1) and (A2) resulting from Proposition 1 and Lemma 1 hold). Accordingly, we will restrain our attention to the results of Esteves and Resende (2016) for the case where v is sufficiently high.

[22] In their notation, the advertising intensity ϕ_i^o represents firm i's advertising intensity in its *own* strong market, whereas the r in ϕ_i^r stands for firm i's advertising intentity in the *rival*'s strong market (i.e. firm i's weak market).

Remark 1. *Let $v \geq 2\gamma$ and consider a quadratic target advertising technology. There is a symmetric Nash equilibrium in which*

$$\phi^{o*} = \frac{v}{4\lambda + 2(v-\gamma)} \tag{8}$$

$$\phi^{r*} = \begin{cases} \frac{v-\gamma}{4\lambda}\frac{v+4\lambda-2\gamma}{2v+4\lambda-2\gamma} & \text{if} \quad \lambda \geq \frac{v-\gamma}{8}\left(\sqrt{\frac{5v-9\gamma}{v-\gamma}}-1\right) \\ 1 & \text{if} \quad \lambda < \frac{v-\gamma}{8}\left(\sqrt{\frac{5v-9\gamma}{v-\gamma}}-1\right) \end{cases} \tag{9}$$

Regarding the pricing decisions, for $0 < \phi^{k*} < 1$, $k = o, r$, in the strong market each firm i, $i = A, B$ chooses a price randomly from the distribution $F^o(p)$

$$F^o(p) = \begin{cases} 0 & if \quad p \leq p^r_{j\min} + \gamma \\ 2\frac{2\lambda+v-\gamma}{v} - (v-\gamma)\frac{v+4\lambda-2\gamma}{v(p-\gamma)} & if \quad p^{r*}_{j\min} + \gamma \leq p \leq v \\ 1 & if \quad p \geq v \end{cases}, \tag{10}$$

with $p^r_{\min} = \frac{1}{2}(v-\gamma)\frac{v+4\lambda-2\gamma}{v+2\lambda-\gamma}$. In the weak market, each firm chooses a price randomly from the distribution $F^r(p)$ given by

$$F^r(p) = \begin{cases} 0 & if \quad p \leq p^r_{j\min} \\ 4\lambda\frac{2(v+2\lambda-\gamma)-v\frac{v+4\lambda-\gamma}{p+\gamma}}{(v-\gamma)(v+4\lambda-2\gamma)} & if \quad p^r_{j\min} \leq p \leq v-\gamma \\ 1 & if \quad p \geq v-\gamma \end{cases}. \tag{11}$$

$F^r(p)$ has a mass point at $v-\gamma$ with a density equal to:

$$m^r = 1 - \frac{\phi^{o*}}{\phi^{r*}}\left(\frac{v-\gamma}{v}\right) = \frac{v-2\gamma}{v-2\gamma+4\lambda} \tag{12}$$

The expected profit (considering both segments) is equal to:

$$E\pi^t = \left(\frac{2\lambda+v-\gamma}{2\lambda}\right)A(\phi^{o*}) + A(\phi^{r*})$$

Proof. See the Proof of Proposition 1 in Esteves and Resende (2016) and let $A(\phi^o_i, \phi^r_i) = \lambda(\phi^o_i)^2 + \lambda(\phi^r_i)^2$.

It is important to stress that Esteves and Resende (2016) did not present any insights on the welfare results in their model. Thus, in order to look at the welfare effects of personalized pricing through targeted ads, we first need to compute consumer surplus and overall welfare in the model of Esteves and Resende (2016). Therefore, we need to identify what is the probability that selective consumers buy their favorite product in equilibrium. Later on, this will allow us to account for the welfare effects of inefficient shopping by selective consumers.

Proposition 4 *With perfect targeting and price discrimination, each firm wins the group of selective customers in its own market with a probability equal to* $z \in [0,1]$ *where*

$$z = \left(\frac{v-\gamma}{\gamma^2 v \phi^{r*}}\right) p_{\min}^{o*} \left(\gamma + v\left(\ln \frac{(v-\gamma)\, p_{\min}^{o*}}{v p_{\min}^{r*}}\right) \frac{\phi^{o*}-1}{\phi^{o*}}\right) + m^r$$

For the quadratic technology, this probability is equal to:

$$z = \frac{4\lambda+\gamma}{\gamma} - 4\lambda\left(\ln \frac{v+4\lambda-\gamma}{v+4\lambda-2\gamma}\right)\frac{v+4\lambda-\gamma}{\gamma^2}. \tag{13}$$

Proof. See the Appendix.

Note that the relationship between the probabilities z (in Proposition 4) and q (in Proposition 3) is not straightforward. In the following figure, the dashed line illustrates the value of the probability q, for different advertising costs (and we set $\gamma = 1$). Note that in the case of mass advertising, q is independent of v. The other functions display z as a function of the advertising cost λ, for different values of v. The lines are depicted for $\gamma = 1$ and $v = \{5, 7, 10, 100\}$, with the probability of serving the selective consumers in the strong market being increasing with v. The figure suggests that for sufficiently high v and sufficiently high λ, the probability that selective consumers buy efficiently is higher under uniform pricing than under price discrimination. It also suggests that q is less responsive to the advertising cost λ than z. With personalized pricing through targeted ads, when λ is goes down, ϕ_i^o will increase. As a result, the set of potential who are potentially captive to the rival (weak) firm becomes narrower, which intensifies price competition. Firm i becomes more likely to win in its turf, i.e. z goes up when λ goes down.[23]

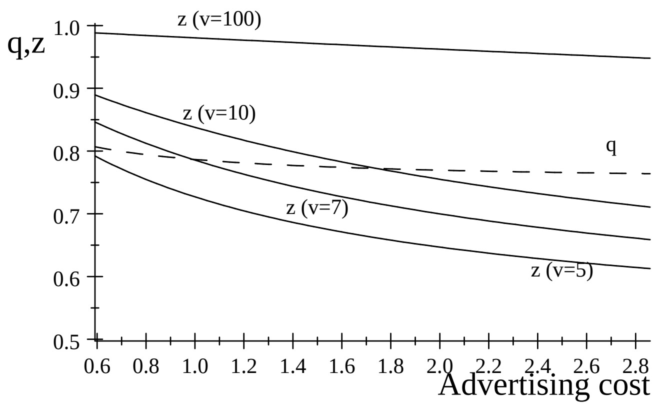

Fig 1. Prob serving selective consumers in strong market

3.2 Price effects

This section characterizes the price effects of personalized pricing with targeted ads. Proposition 5 shows that, under conditions (A1) and (A2), personalized price offers result into *higher* expected prices in both market segments. This is an interesting result because it challenges the

[23] Since this strategic effect does not arise in the model with mass advertising, we get that q is less responsive to λ than z.

usual finding that price discrimination in a competitive setting generally intensifies competition and reduces the prices to all segments of the market (see Thisse and Vives, 1988 or Fudenberg and Tirole, 2000, among many others). This result gives support to concerns that have been raised regarding the firms' ability to use data about consumers to adopt personalized pricing strategies and charge them higher prices.[24]

An important contribution of our analysis is to highlight that we should be cautious about proposals to regulate personalized pricing. Indeed, the presumption that markets are competitive is not a sufficient condition to avoid regulation. While under some market features competitive price discrimination might intensify competition and benefit consumers; the reverse might happen in other cases.

The rationale for this result lies on firms' ability to take advantage of the interplay between informative targeted ads and personalized prices. Esteves and Resende (2016) have already shown that, for $v \geq 2\gamma$ and $\lambda < \frac{v-\gamma}{4}\frac{v-3\gamma}{2\gamma}$,[25] firms strategically reduce the intensity of advertising targeted to the strong market (*vis-à-vis* the weak market) as a way to induce the rival to play less aggressively and sustain higher prices. Proposition 5 shows that the combination between personalized prices and targeted ads indeed leads to higher expected prices in both segments of the market because in the mass communication set-up, firms are no longer able to strategically use targeted ads to favorably segment the market in order to soften price competition).

Proposition 5. *Comparing equilibrium pricing behavior under uniform versus personalized pricing, we find that:*

(i) $p_{\min} \leq p^r_{\min}$ *and* $p_{\max} \leq p^r_{\min}$

(ii) F^r *first-order stochastically dominates* F^m*, with* $F^m(p) > F^r(p)$.

(iii) F^o *first-order stochastically dominates* F^m*, with* $F^m(p) > F^o(p)$.

(iv) The expected price with uniform pricing is below the expected prices with personalized pricing in both market segments, i.e., $E(p^m) < E(p^o)$ *and* $E(p^o) < E(p^r)$.

Proof. See the Appendix.

The results in Proposition 5 are illustrated in the following figures, in which the stochastic dominance of F^r over F^m and the stochastic dominance of F^o over F^m is quite clear. The figures are drawn for $\gamma = 1$ and $v = 7$. In Figure 2, we have $\lambda = 2$, whereas in Figure 3. we have $\lambda = 1$, given that for $v = 7$ and $\gamma = 1$, Assumption (A2) implies $0.60355 \leq \lambda \leq \min\{2.1, 3\}$

[24] See https://verdict.justia.com/2015/05/13/personalized-pricing-in-the-air-why-consumers-should-be-wary-of-a-new-airline-pricing-proposal [Accessed online 19 August 2016]

[25] This region of parameters includes our domain defined by conditions (A1) and (A2)

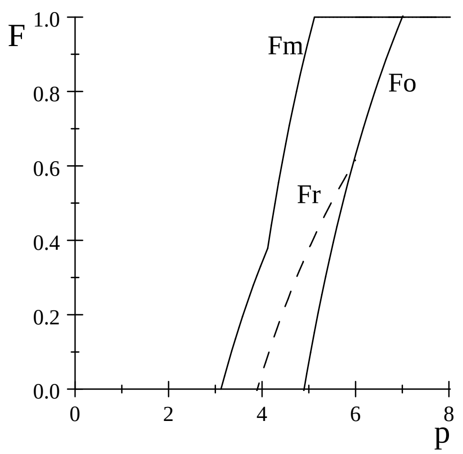

Fig 2. C.d.f. ($\lambda = 2$)

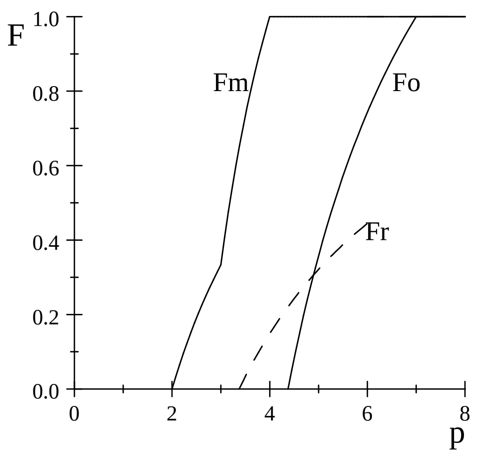

Fig 3: C.d.f. ($\lambda = 1$)

The figures also show that the dominance established in parts (ii) and (iii) of Proposition 5 appears to become more significant when λ is low. Indeed, when advertising costs go down (λ goes down), $p_{\max}$ and $p_{\min}$ go down by the same amount (recall that price dispersion is independent of λ, since $p_{\max} - p_{\min} = 2\gamma$). In a set-up with non-customized ads, average prices are necessarily lower when λ goes down. When firms target both ads and prices simultaneously, the reduction in λ translates into a reduction of the minimum price in the support of the cdf distribution and price dispersion is greater because the maximum price in the distribution remains unchanged (recall that $p^r_{\max} = v - \gamma$ and $p^o_{\max} = v$, meaning that the highest price in each segment is independent of the advertising cost λ). Consequently, when λ goes down, the stochastic dominance of F^r (and F^o) over F^m becomes more significant. This is also the case, when v goes up. As mentioned in Section 2, the distribution of prices under mass advertising is independent of v,[26] whereas under targeted advertising and price discrimination, an increase in v moves both F^r and F^o to the right.

3.3 Advertising effects

Let us now focus on the effects of targeted ads (with personalized price offers) on firms' optimal advertising decisions. It turns out that the comparison of equilibrium advertising intensities under mass and targeted advertising is rather complex. Although the quadratic cost specification favors higher advertising rates in a scenario of targeted advertising than in a scenario of mass advertising,[27] the interplay between advertising and prices makes the comparison between ϕ^m and (ϕ^r, ϕ^o) much more intricate. The figures below present two different configurations of equi-

[26] In the mass communication model (as in other models of price competition between differentiatied products, such as the Hotelling model), firms' profitability only depends on the degree of product differentiation, γ The consumers' willingness to pay does not affect profits since price competition among the firms will dissipate any rents they could possibly get when, *ceteris paribus*, consumers' reservation price v goes up (for the same γ).

[27] Note that similar advertising additive separable cost specifications have been used in previous literature dealing with targeted advertising, namely Galeotti and Moraga-Gonzalez (2008) and Brahim et al. (2011).

librium market segmentation.[28] When v is sufficiently low (see Figure 4), we get $\phi^m > \phi^r > \phi^o$. Everything else the same, when v gets larger, firms advertise more under customized information (namely in the weak market), whereas the equilibrium advertising level under mass communication remains unchanged (recall that ϕ^m is independent of v). Accordingly, as illustrated in Figure 5, for sufficiently large values of v, we obtain that $\phi^r > \phi^m$ (see Lemma 4). For large $v - values$, the comparison between ϕ^m and ϕ^o depends on the value of λ (see Lemma 5). We get $\phi^r > \phi^m > \phi^o$, provided advertising costs are sufficiently low. Differently, when both v and λ are large enough, we obtain $\phi^r > \phi^o > \phi^m$. For high v and low λ, we obtain $\phi^r > \phi^m > \phi^o$ (see Figure 5).

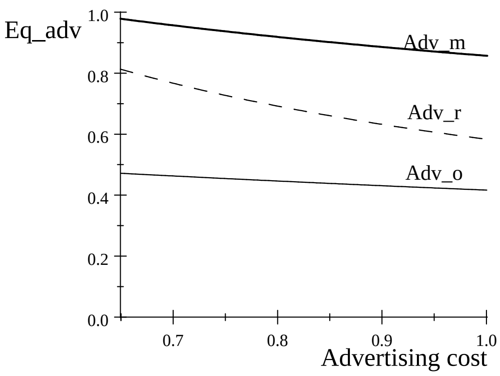

Fig 4. Eq adv $(v = 5, \gamma = 1)$

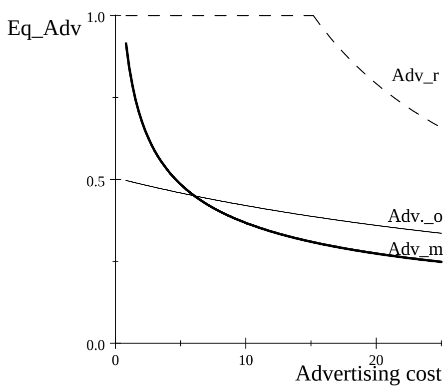

Fig 5. Eq. adv $(v = 100, \gamma = 1)$

Lemma 4 and Lemma 5 below provide further details on the relationship between ϕ^m, ϕ^o and ϕ^r.

Lemma 4. *Firms advertise more intensively in the weak market under a targeting strategy than under a mass strategy, $\phi^r > \phi^m$, when*

$$4\lambda (v - \gamma)(v + 4\lambda - 2\gamma) + \sqrt{\gamma^2 + 8\lambda\gamma}\left((v - \gamma)(v + 4\lambda - 2\gamma) - 16\lambda (v + 2\lambda - \gamma)\right) < 0. \quad (14)$$

Accordingly:

(i) If v is high enough, i.e., if

$$v > 6\lambda + \frac{3}{2}\gamma + \frac{1}{2}\sqrt{272\lambda^2 + 24\lambda\gamma + \gamma^2}, \quad (15)$$

it is always true that $\phi^r > \phi^m$.

(ii) On the contrary, when v is small enough, $v < \frac{(3\sqrt{2}+\sqrt{6}-2)\gamma}{2(\sqrt{2}-1)} = 5.6639\gamma$, each firm advertises more with mass advertising than with targeted advertising in both market segments. In this case, it follows that $\phi^m > \phi^r > \phi^o$.

[28] Figure 4 depicts optimal advertising levels as a function of λ, for $\gamma = 1$ and $v = 5$. We consider $0.65 < \lambda < 1$, which corresponds to the domain defined by conditions (A1) and (A2) for $v = 5$ and $\gamma = 1$. The thin line depcits ϕ^o, the dashed line represents ϕ^r and the thick line represents ϕ^m, with $\phi^m > \phi^r > \phi^o$, $\forall\lambda$. Figure 5 is drawn for a high value of v, with $v = 100$ and $\gamma = 1$, implying $0.603\,55 < \lambda < 25.255$.

Proof. See the Appendix.

Lemma 4 points that a sufficiently high value of v constitutes a sufficient condition to have $\phi^r > \phi^m$ in (i) and $\phi^r < \phi^m$ in (ii). For other $v-values$, the comparison between ϕ^r and ϕ^o depends on λ. It is worth noting that the critical $v-$threshold in (15) is increasing in λ. This suggests that it becomes easier to obtain $\phi^r > \phi^m$ when advertising costs are sufficiently low (this is in line with the findings of Esteves and Resende (2016) who argue that firms' incentives to adopt aggressive targeted advertising strategies arise for low advertising costs).

Lemma 5. *The fraction of consumers who receive informative ads about their favorite product is higher under mass advertising than under target advertising,* $\phi^m > \phi^o$ *when:*

(i) regardless of the value of v*, the advertising cost is sufficiently small, i.e.,*

$$\lambda < \frac{3}{4}\gamma\left(\sqrt{10}+3\right) = 4.621\,7\gamma, \text{ or}$$

(ii) the advertising cost is high, $\lambda > \frac{3}{4}\gamma\left(\sqrt{10}+3\right)$ *but* v *is not too large, with:*

$$v < 4\left(2\lambda-\gamma\right)\frac{3\gamma\left(8\lambda+\gamma\right)+4\lambda\sqrt{\gamma^2+8\lambda\gamma}}{\left(16\lambda^2-9\gamma\left(8\lambda+\gamma\right)\right)}.$$

Proof. See the Appendix.

From Lemma 4 and Lemma 5, we conclude that it is not possible to make unambiguous predictions regarding the features of equilibrium market segmentation under mass versus targeted advertising strategies. According to part (i) in Lemma 4, when v is sufficiently low (resulting in $\phi^m > \phi^r > \phi^o$) there will necessarily be more informed (and more selective) consumers under mass than under personalized communication. We will see later that in this domain, the use of targeted advertising messages (with personalized price offers) may indeed be welfare detrimental, specially to consumers. On the one hand, the combination between personalized pricing and targeted ads allows firms to relax price competition, resulting in higher prices in both segments of the market vis-à-vis the benchmark with uniform pricing and mass advertising. On the other hand, more consumers remain uninformed (and out of the market) under customized than under mass information, meaning that there is less efficient shopping in the former case (since the set of selective consumers is narrower with customized than with mass information; and the probability q is above z, as illustrated in Figure 1). This finding is in contrast with Brahim *et al.* (2011) in which targeted advertising always leads to a more efficient shopping for all consumers.

In other sub-domains, equilibrium market segmentation may have different features. For example, when v and λ are sufficiently large, we obtain $\phi^r > \phi^o > \phi^m$, so that both the fraction of informed and the fraction of selective consumers are larger under a targeted advertising strategy than under a mass advertising one. However, it is important to stress that this does not necessarily mean that targeted advertising is welfare-enhancing because: *(i)* more consumers are captive to the least preferred firm than to the favorite one; and *(ii)* the expected prices are greater under targeted advertising than under mass advertising (see Proposition 5).

3.4 Profit effects

Let us now analyze how price discrimination through targeted advertising affects the firms' equilibrium profits. Under conditions (A1) and (A2), we have $E\pi^t = \left(\frac{2\lambda+v-\gamma}{2\lambda}\right) A(\phi^{o*}) + A(\phi^{r*})$ and $E\pi^m = \phi^m A_\phi(\phi^m) - A(\phi^m)$. Therefore, we can established the following proposition.

Proposition 6. *The combination of targeted advertising and price discrimination boosts each firm's profits when λ is such that the following implicit condition is satisfied*

$$2\lambda(1-\chi^*(\lambda,v,\gamma)) + (v-\gamma) > 0, \text{ with } \chi^*(\lambda,v,\gamma) = \frac{(\phi^m)^2 - (\phi^r)^2}{(\phi^o)^2}.$$

Proof. See the Appendix.

Proposition 6 highlights that in comparison to mass advertising and uniform pricing, expected profits may be higher when firms employ a strategy of targeted advertising with personalized pricing offers. The magnitude of consumers' willingness to pay (v) plays a key role in this comparison. While expected profits under personalized pricing through targeted advertising are increasing with v, the expected profits with mass information are independent of v. Provided v is sufficiently large we find that $E\pi^t > E\pi^m$ because price discrimination allows firms to set larger prices in their strong turf.

Accordingly, price discrimination by means of targeted advertising does not necessarily lead to the classic prisoner dilemma result that generally arises in models with fully informed consumers. In the literature, price discrimination is in general bad for profits in models with: (i) fully informed consumers, (ii) best-response asymmetry and (iii) all firms engaging in price discrimination (e.g. Thisse and Vives, 1988; Chen, 1997; Villas-Boas, 1999; Fudenberg and Tirole, 2000, Esteves, 2010; Gehrig, *et al.,* 2012). Our analysis suggests that the interplay between targeted advertising and price discrimination may actually boost industry profits even when conditions (ii) and (iii) hold.

Figure 6 plots each firm's expected profits with price discrimination through targeted advertising (Profit_T) and the profits with mass advertising (Profit_M) for $\gamma = 1$ and for different values of v, (respectively, $v = 3 + \sqrt{2}; 5; 7$). The figure illustrates the results in Proposition 6, showing that $E\pi^t$ (dashed lines) can only be below $E\pi^m$ for sufficiently low values of v.[29]

When v is small enough (provided conditions (A1) and (A2) hold), we get $E\pi^t > E\pi^m$ only when advertising costs are sufficiently low. Indeed, for low values of λ, Esteves and Resende

[29] As v gets larger, profits will be always greater with target than with mass communication. The case $v = 7$ in Figure 6 illustrates this type of outcome: as v goes up, $E\pi^t$ will move upwards, whereas $E\pi^m$ remains unchanged.

(2016) argue that firms are able to strategically choose advertising in order to relax the intensity of price competition. On the contrary, we find that under mass communication, price competition is very intense (recall that in Section 2, price competition intensifies as λ goes down, leading to lower expected profit).

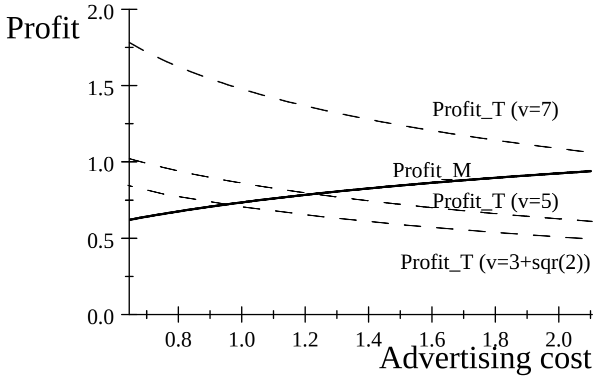

Fig 6. Profits

4 Welfare analysis

This section investigates the welfare effects of combining targeted advertising and price discrimination in comparison to mass advertising and uniform pricing. We start by presenting the welfare functions for both advertising/pricing strategies considered, namely mass advertising/no discrimination and targeted advertising/personalized prices.

Consider first the mass advertising and uniform pricing case. Recall that customers' gross benefit when buying a certain good can be given by $v-$"expected disutility cost", where the latter is equal to γ, when the consumer buys the least preferred good; and zero, when the consumer buys the most preferred good. In the social optimal solution with full information, consumers would buy from the most preferred firm, in order to obtain a gross benefit of v (and minimize the expected disutility cost). We obtain that firms' captive consumers in their strong market always buy efficiently. In contrast, captive consumers who are only aware of the least preferred product always buy inefficiently (incurring the disutility cost γ). Finally, selective consumers buy efficiently when firms share them equally, which occurs with probability q. With the remaining probability, $1-q$, all the selective consumers buy from the same firm, which means that half of them buy inefficiently. Accordingly, in the case of mass advertising/no discrimination, total expected equilibrium welfare can be represented as:

$$EW^{m} = \phi^{m}\left(1-\phi^{m}\right)\left(2v-\gamma\right) + v\left(\phi^{m}\right)^{2} - \frac{\gamma}{2}\left(1-q\right)\left(\phi^{m}\right)^{2} - 2A(\phi^{m}) \tag{16}$$

where ϕ^m is evaluated at the equilibrium value obtained in section 2. The quadratic advertising technology yields:

$$EW^{m} = 2\Xi\frac{4\lambda-\Xi}{\left(4\lambda+\Xi\right)^{2}}\left(2v-\gamma\right) + \left(v - 2\lambda - \left(\frac{16\lambda^{2}}{\gamma}\ln\frac{\gamma+8\lambda}{8\lambda} - \frac{16\lambda^{2}}{\left(8\lambda+\gamma\right)}\right)\right)\left(\frac{2\Xi}{\Xi+4\lambda}\right)^{2}, \tag{17}$$

with $\Xi = \sqrt{\gamma^2 + 8\lambda\gamma}$. The expected consumer surplus, denoted by ECS^m is simply $ECS^m = EW^m - E\pi^m_{ind}$, where $E\pi^m_{ind}$ stand for industry profits. The quadratic advertising technology yields $E\pi^m = A(\phi^m)$, implying $E\pi^m_{ind} = 2A(\phi^m)$ and $ECS^m = EW^m - 2\lambda(\phi^m)^2$.

As mentioned before, Esteves and Resende (2016) do not provide a welfare analysis. Therefore, we need to compute the welfare functions for the case where firms engage in targeted advertising and price discrimination. In their model, the selective consumers in each market segment will buy efficiently when each firm wins the group of selective consumers in its strong market, which in this case occurs with probability z. Regarding captive consumers, those who buy the most preferred product obtain a gross utility of v, while those consumers who are only aware of the less preferred product obtain a gross utility of $v - \gamma$. Accordingly, overall welfare with targeted advertising and price discrimination, denoted by EW^t is equal to:

$$v\phi^o(1-\phi^r) + (v-\gamma)\phi^r(1-\phi^o) + \phi^o\phi^r\left[vz + \frac{v(1-z)}{2} + \frac{(v-\gamma)(1-z)}{2}\right] - 2A(\phi^o) - 2A(\phi^r),$$

which simplifies to:

$$EW^t = v\phi^o + (v-\gamma)\phi^r(1-\phi^o) - \gamma(1-z)\frac{\phi^o\phi^r}{2} - 2A(\phi^o) - 2A(\phi^r),$$

where ϕ^o and ϕ^r are given by (8) and (9), respectively. For the quadratic advertising technology, we have:

$$EW^t = \left\{ \begin{array}{c} \frac{1}{2}\frac{v^2}{(v+2\lambda-\gamma)^2}(v+\lambda-\gamma) + \frac{1}{32}(v-\gamma)^2\frac{(v+4\lambda-2\gamma)^2}{\lambda(v+2\lambda-\gamma)^2} \\ -\frac{1}{8}v(v-\gamma)\frac{v+4\lambda-2\gamma}{(v+2\lambda-\gamma)^2}\left(\left(\left(\ln\frac{v+4\lambda-\gamma}{v+4\lambda-2\gamma}\right)\frac{v+4\lambda-\gamma}{\gamma} - 1\right)\right) \end{array} \right\}. \tag{18}$$

Let ECS^t represent expected consumer surplus with targeted advertising and prices. It follows that $ECS^t = W^t - E\pi^t_{ind}$. The quadratic advertising technology yields:

$$ECS^t = v\frac{v-\gamma}{(v+2\lambda-\gamma)^2}\left(\frac{v}{4} - \frac{v+4\lambda-2\gamma}{8\gamma}\left(\ln\left(\frac{v+4\lambda-\gamma}{v+4\lambda-2\gamma}\right)(v+4\lambda-\gamma) - \gamma\right)\right) \tag{19}$$

The comparison of welfare outcomes under mass and targeted advertising strategies is quite intricate due to the complexity of the expressions obtained in (17) and (18). Although we are able to define the condition on parameters λ, γ and v, for which targeted advertising and prices is welfare improving by imposing $W^t > W^m$, with W^t given by (18) and W^m given by (17), it is not possible to find the close-form solution to the previous inequality. Nevertheless, the following figures allow us to shed some light on the welfare implications of price discrimination through targeted ads (in comparison to the benchmark of mass advertising with uniform pricing).

Figures 7 and 8 represent W^t (dashed lines) and W^m (solid lines) as a function of the cost of advertising, λ.[30] We set $\gamma = 1$ and we consider different values of v, respectively a low value ($v = 7$) in Figure 7; and a high value ($v = 100$) in Figure 8.

Comparing the two figures, it is possible to conclude that moving from a strategy of mass advertising/no discrimination to a strategy of targeted advertising and personalized price offers

[30] In all the figures, the domain of λ is represented such that: (i) conditions (A1) and (A2) hold; (ii) $\phi^r < 1$). In the case of $v = 7, \gamma = 1$, these conditions imply $0.81 \leq \lambda \leq 2.1$. In the case of $v = 100, \gamma = 1$, we get $15.18 \leq \lambda \leq 25.26$.

might be welfare improving or not, depending on the domain of the parameters under consideration. The figures suggest that targeted advertising/price discrimination is welfare detrimental when v is sufficiently low. Specifically, figure 7 shows that when $v = 7$, we always have $W^t < W^m$. The rationale behind this result is related to the strategic effects entailed by the interplay between targeted ads and prices. When firms are able to targeted specific market segments, they will strategically reduce advertising levels (at least in the strong market) to sustain higher prices. As a result, targeted information generates two detrimental effects: (i) there are less informed consumers (and less selective consumers) in this case than under mass communication; (ii) more selective consumers may end up buying inefficiently than under mass communication.

As v increases, we find that under targeted advertising and price discrimination firms have less incentives to leave consumers uninformed. Thus, for sufficiently high values of v, the set of informed consumers (and selective consumers) is larger under targeted advertising than under mass advertising. This effect has a positive impact on welfare. Figure 8 shows that, when $v = 100$, $W^t > W^m$ for all values of λ. In this figure we are considering a $v - value$ that is sufficiently high to guarantee that $\phi^r > \phi^o > \phi^m$. In this scenario, some consumers who receive targeted ads, would not be informed about the products in the mass communication set-up (and therefore they would be excluded from the market).

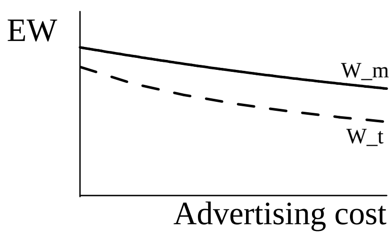

Fig7. Welfare for low v

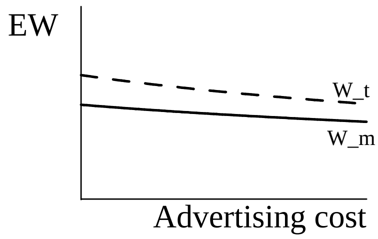

Fig 8. Welfare for high v

Look next at the effect of targeted advertising and pricing on consumer welfare. It is important to stress that the condition $W^t > W^m$ does not not necessarily mean that consumers overall benefit from customized information since expected prices are higher with personalized pricing than with uniform pricing. Indeed, Figure 9 plotted for $\gamma = 1$ shows that when v is high ($v = 100$) even when we have $W^t > W^m$, due to the reasons described above, regardless of λ, consumers are overall worse off since $ECS^m > ECS^t$.

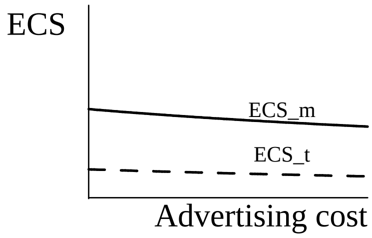

Fig 9. Cons. Surplus for high v

The comparison between Figures 8 and 9 shows that firms can indeed take advantage of price discrimination through targeted ads, because they may be able to increase their profits (see also Figure 6) at the expense of consumer welfare.

From a theoretical point of view, we cannot make general predictions on the welfare effects of price discrimination through targeted advertising. This suggest that who wins or who loses when firms depart from a mass advertising/pricing strategy to a targeted one might depend on the features of the market under consideration (e.g., level of advertising costs, consumer willingness to pay, brand preferences). Therefore, this constitutes an empirical matter and it is important to take into consideration specific market features when assessing the welfare effects of customized communication between firms and consumers.

Our numerical analysis suggests that a personalized advertising/pricing strategy will be welfare detrimental when v is low, while it shall have a positive welfare effect when v is high. However, it is worth noting that such welfare increase may only benefit firms (who get higher profits). Consumers may end up being worse-off both because they are paying a higher price for the product and a considerable fraction of them is buying inefficiently.

In light of these findings, our paper highlights the importance of ongoing scrutiny of markets in which firms have the tools to send personalized ads and personalized price offers to their consumers (e.g. online markets or markets in which firms use mobile *apps* to send personalized messages to consumers and control their consumption features, such as location or past consumption history). Our analysis suggests that the combination of personalized ads and prices may be welfare detrimental, especially for consumers. According to our results, this outcome is more likely in markets with (i) low advertising costs (which is actually frequent in the case of online advertising); and (ii) relatively high degree of product differentiation, implying γ is high in relation to v.

Finally, since our analysis shows that price discrimination through targeted advertising might boost industry profits at the expense of consumer welfare, an important implication for competition policy is that the decision of allowing or not firms to quote personalized prices through targeted ads should be sensitive to the welfare criterion under consideration. If total welfare is the criterion adopted by the competition authorities to evaluate these business practices then targeted advertising and price discrimination should not be blocked as long as $W^t > W^m$. If consumer surplus is instead the competition authority welfare standard, as it is the case in most antitrust jurisdictions, then $W^t > W^m$ is not a sufficient condition to allow firms to engage in targeted ads and prices. In this case competition authorities should take into account that consumers might all pay higher prices under targeted advertising and price discrimination which might lead to $ECS^t < ECS^m$ even when $W^t > W^m$.

5 Conclusions

This paper has investigated the effects of the combination of personalized pricing and targeted (informative) advertising in a competitive set-up. We have built on the model of Esteves and Resende (2016) who study optimal pricing and advertising strategies when firms use a targeted advertising technology that allows them to send different price offers to each market segment. We compared their pricing and advertising equilibrium with the one that would arise in a benchmark model with mass advertising and uniform pricing (in which firms are unable to send personalized content and therefore they do not have any tool to engage in price discrimination).

We find that price discrimination through targeted advertising actually leads to higher expected prices in both segments of the market since each firm will strategically reduce advertising in its strong market as an attempt to invite the rival to price less aggressively in this segment. This is an interesting finding as it challenges the usual result that price discrimination reduces all segment prices. In light of this strategic effect, we obtain that price discrimination by means of targeted informative advertising does not necessarily lead to the classic prisoner dilemma result arising in models with full informed consumers and exhibiting best-response asymmetry (Thisse and Vives, 1988 and the subsequent literature). When advertising is not too expensive, each firms' profit with targeted advertising and price discrimination is above its non-discrimination counterpart, even when v is low.

In what concerns equilibrium market segmentation, we find that it is not possible to establish general predictions regarding the relative size of the set of informed and selective consumers with mass and targeted advertising. We find that for sufficiently small (large) $v - values$, there will be more (less) informed and selective consumers with mass than with customized advertising messages. Interestingly, in contrast with previous literature (e.g. Brahim *et al.*, 2011), we find that targeted advertising combined with price discrimination does not always lead to a more efficient shopping for all consumers. The justification is two-folded. First, when firms run customized ads, firms will strategically advertise more intensively in the weak market than in the strong market, which means that more consumers will be captive to their least preferred product, buying inefficiently. Second, expected prices are higher under targeted information than under mass communication.

Finally, we have shed some light on the welfare effects of price discrimination through personalized ads. We have first characterized the equilibrium welfare outcomes in the set-up proposed by Esteves and Resende (2016), whose analysis neglected welfare issues. Then, we compared those outcomes to the ones arising in the model with mass advertising and uniform pricing. We find that this welfare comparison is quite intricate and it constitutes a relevant empirical question, since targeted-advertising may be welfare improving or not, depending on the features of the market in which firms participate (namely in what concerns the consumers' willingness to pay, v, the brand preference parameter, γ and the advertising cost λ). Our analysis reveals that personalized pricing/ ads will be welfare detrimental when v is low, while they shall have a positive welfare effect when v is high. However, it is worth noting that such welfare increase may only benefit firms (who get higher profits). Consumers may end up being worse-off both because they are paying a higher price for the product and a considerable fraction of them is buying inefficiently.

In light of these findings, our paper highlights the importance of ongoing scrutiny of markets in which firms have the tools to send personalized ads and personalized price offers to their consumers (e.g. online markets or markets in which firms use mobile *apps* to send personalized messages to consumers and control their consumption features, such as location or past consumption history). Our analysis suggests that the combination of personalized ads and prices may be welfare detrimental, especially for consumers. According to our results, this outcome is more likely in markets with (i) low advertising costs (which is actually frequent in the case of online advertising); and (ii) relatively high degree of product differentiation, implying γ is high in relation to v.

From a public policy perspective, our findings suggest that the assessment of welfare effects of price discrimination must take into consideration other forms of competition besides prices (e.g. personalized ads). Otherwise, we might be missing some important welfare effects resulting from the interplay between firms' different actions (like the interplay between targeted ads and prices occurring in the present paper).

The model addressed in this paper tried to offer a closer approximation of reality where the quantity and quality of consumer-specific information that firms have been using to implement their advertising and pricing strategies is increasingly improving thanks to the advances in information technologies. However, the model is obviously far from covering all complex aspects of real markets and more research is needed to have a better picture on the welfare effects of firms' personalized strategies in online markets. In our future research, we would like to address other important aspects of these markets such as the issues related to consumer search behavior, privacy concerns or the business strategy of intermediary advertising platforms and big data companies.

Appendix

Proof of Proposition 1. Look first at case *(i)*. Suppose (p_i, p_j) equal to (v, v) is an equilibrium in pure strategies. In this case firm i serves only the captive and the selective consumers who belong to its strong market, obtaining a profit equal to $\pi_{(v,v)} = \frac{1}{2} v\phi_i - \lambda\phi_i^2$. The optimal advertising level is equal to $\phi_i = \frac{v}{4\lambda}$, corresponding to the advertising intensity that maximizes $\pi_{(v,v)}$. When

$$4\lambda > v \tag{20}$$

we have an interior solution, with $\phi^m = \frac{v}{4\lambda}$. For lower advertising costs, we would have a corner solution with $\phi^m = 1$.

In the interior solution, firms' equilibrium profit is equal to $(\pi_i^v)^* = \frac{1}{16}\frac{v^2}{\lambda}$. Any price greater than v is not part of an equilibrium strategy since at such a price no consumer is willing to buy from the firm. Any price lower than v but greater than $v-\gamma$ gives firm i the same market share but reduces its profit and so it is dominated by v. If firm i deviates and chooses $p_i^d = v - \gamma$, the firm starts selling its good to captive consumers in its weak market, obtaining a profit equal to:

$$\pi_i^{d,v-\gamma} = (v-\gamma)\left(\phi_i\left(1-\phi_j\right) + \frac{1}{2}\phi_i\phi_j\right) - \lambda\phi_i^2,$$

where $\phi_j = \frac{1}{4}\frac{v}{\lambda}$, since firm i takes as given firm j's decisions (pricing and advertising). The deviation advertising reach would then be equal to $\phi_i^{d,v-\gamma} = \frac{1}{16}\frac{(v-\gamma)(8\lambda-v)}{\lambda^2}$ with $\phi_i^{d,v-\gamma} > 0$, under (20). The condition $\gamma > \frac{(v-4\lambda)^2}{v-8\lambda}$ would guarantee $\phi_i^{d,v-\gamma} < 1$. For $v < 8\lambda$, the RHS of the last condition is always negative and therefore the condition is always true. Firm i's deviation profit is therefore equal to $\pi_i^{d,v-\gamma} = \frac{1}{\lambda}\left(\frac{(v-\gamma)(v-8\lambda)}{16\lambda}\right)^2$. Comparing $\pi_i^{d,v-\gamma}$ and $(\pi_i^v)^*$, we obtain the following no-deviation condition:

$$-\frac{\Gamma\Omega}{256\lambda^3} > 0, \tag{21}$$

with $\Gamma = (v(v-\gamma) + 4\lambda(2\gamma - 3v))$ and $\Omega = (v(v-\gamma) - 4\lambda(v-2\gamma))$. For $\gamma < v < 2\gamma$, we have $\Omega > 0$. In that case, the non-deviation condition (21) requires $\Gamma < 0$, or, equivalently,

$$\lambda > \frac{v}{4}\frac{v-\gamma}{3v-2\gamma}, \tag{22}$$

which is always true under condition (20).

When $v > 2\gamma$ and $\lambda < \frac{v(v-\gamma)}{4(v-2\gamma)}$, we still have $\Omega > 0$. In that case, condition (22) remains necessary to preclude deviations from v to $v-\gamma$. Thus, the following no-deviation condition (for $v > 2\gamma$) must hold:

$$\frac{v}{4} < \lambda < \frac{v(v-\gamma)}{4(v-2\gamma)},$$

which is a non-empty interval. Finally, for $v > 2\gamma$ and $\lambda > \frac{v(v-\gamma)}{4(v-2\gamma)}$, we have $\Omega < 0$. In that case, the non-deviation condition (21) requires $\Gamma > 0$, or equivalently, $\lambda < \frac{v(v-\gamma)}{4(3v-2\gamma)}$, which is incompatible with $\lambda > \frac{v(v-\gamma)}{4(v-2\gamma)}$ since $\frac{v(v-\gamma)}{4(v-2\gamma)} > \frac{v(v-\gamma)}{4(3v-2\gamma)}$.

Summing up, firm i will not have incentives to deviate from v to $v-\gamma$ if (i) $\gamma < v \leq 2\gamma$, and $\lambda > \frac{v}{4} > \frac{v}{4}\frac{(v-\gamma)}{(3v-2\gamma)}$;or (ii) $v > 2\gamma$ and $\frac{v}{4} < \lambda < \frac{v(v-\gamma)}{4(v-2\gamma)}$.

Now, it remains to study if instead of deviating to $v-\gamma$, firm i would be interested in decreasing its price even further in order to poach selective consumers in the weak market, with $p_i = v - \gamma - \varepsilon > 0$. In that case, firm i's profits are equal to $(v-\gamma-\varepsilon)\,\phi_i - \lambda\phi_i^2$, leading firm i to choose an advertising intensity equal to $\frac{v-\gamma-\varepsilon}{2\lambda}$ as long as $\lambda > \frac{v-\gamma-\varepsilon}{2}$.

The deviation profits are equal to $\pi_i^{d,v-\gamma-\varepsilon} = \frac{1}{4}\frac{(v-\gamma-\varepsilon)^2}{\lambda}$. Comparing this profit with the equilibrium profit $(\pi_i^v)^* = \frac{1}{16}\frac{v^2}{\lambda}$, it is easy to see that for $v > 2\gamma$, it is always possible to find a sufficiently small ε for which $\pi_i^{d,v-\gamma-\varepsilon} > (\pi_i^v)^*$, which means that the deviation from v to $v-\gamma-\varepsilon$ is always profitable when $v > 2\gamma$. When $v < 2\gamma$, such deviation is never profitable.

Considering both types of deviation (to $v-\gamma$ and to $v-\gamma-\varepsilon$), it is possible to conclude that $p^* = v$ and ϕ^m is an interior Nash Equilibrium in pure strategies iff $v < 2\gamma$ and $4\lambda > v$, which proves case (i) in Proposition 1.

Let us now address case *(ii)*. Suppose (p^*, p^*) equal to $(v-\gamma, v-\gamma)$ is an equilibrium in pure strategies. In this case, firm i serves all its captive consumers as well as the selective consumers in its strong market. Firm i's profit for a given advertising intensity ϕ_i write as $(v-\gamma)\left(\phi_i - \frac{\phi_i\phi_j}{2}\right) - \lambda\phi_i^2$, and firms' equilibrium advertising reach is then given by $\phi^m_{p^*=v-\gamma} = 2\frac{v-\gamma}{v-\gamma+4\lambda}$,provided that advertising costs are sufficiently low (so that firms do not choose to inform all consumers in the market), i.e.:

$$4\lambda > v-\gamma. \tag{23}$$

The equilibrium profits are $(\pi_i^{v-\gamma})^* = 4\lambda\frac{(v-\gamma)^2}{(v+4\lambda-\gamma)^2}$. If firm i deviates to a higher price it must be to price v. In this case, firm i only sells to consumers in its strong market (captive and selective). Deviation profits are given by $\frac{1}{2}v\phi_i - \lambda\phi_i^2$ and the deviation advertising reach is equal to $\frac{v}{4\lambda}$, which is an interior solution when (20) holds. Deviation profits are equal to $(\pi_i^{d,v})^* = \frac{1}{16}\frac{v^2}{\lambda}$.

Firm i does not have incentives to deviate to price v if $(\pi_i^{d,v})^* < (\pi_i^{v-\gamma})^*$, or equivalently,

$$(\pi_i^{d,v})^* - (\pi_i^{v-\gamma})^* < 0 \Leftrightarrow \frac{\Omega\Phi}{16\lambda\left(v+4\lambda-\gamma\right)^2} < 0$$

with $\Phi = (12v - 8\gamma)\lambda + \left(v^2 - v\gamma\right) > 0$ and $\Omega = \left(v\left(v-\gamma\right) - 4\lambda\left(v-2\gamma\right)\right)$. We have seen before that $\Omega > 0$ for $\gamma < v < 2\gamma$ or $v > 2\gamma$ and $\lambda < \frac{v(v-\gamma)}{4(v-2\gamma)}$. Therefore, in that region of parameters the firm has incentives to deviate to a higher price v. On the contrary, when $v > 2\gamma$ and $\lambda > \frac{v-\gamma}{4}\frac{v}{v-2\gamma} > \frac{v-\gamma}{4}$, the firm prefers to set the price $v-\gamma$ instead of v.

Let us consider next a deviation to a lower price. When firm i deviates to $p_i^d = v-2\gamma-\varepsilon$ (with $p_i^d > 0$ for $v > 2\gamma$), it sells to all the consumers that are informed about its product (including the group of selective consumers that belongs to its weak market). In this case, firm i's deviation profit writes as $(v-2\gamma-\varepsilon)\phi_i - \lambda\phi_i^2$, and firm i's deviation advertising reach is then given by $\phi_i^{d,v-2\gamma-\varepsilon} = \frac{v-2\gamma-\varepsilon}{2\lambda}$. Deviation profits in this scenario are equal to $\pi_i^{d,v-2\gamma-\varepsilon} = \frac{(v-2\gamma-\varepsilon)^2}{4\lambda}$.

This deviation is unprofitable if

$$\pi_i^{d,v-2\gamma-\varepsilon} - (\pi_i^{v-\gamma})^* < 0 \Leftrightarrow \frac{\vartheta\varsigma}{4\lambda\left(v+4\lambda-\gamma\right)^2} < 0, \tag{24}$$

with $\vartheta = \left((v-2\gamma)(v-\gamma) - 4\lambda\gamma\right)$ and $\varsigma = 4\lambda\left(2v-3\gamma\right) + (v-2\gamma)(v-\gamma) > 0$, for $v > 2\gamma$. Condition (24) is equivalent to $\vartheta < 0$, i.e. $4\lambda > \frac{(v-2\gamma)(v-\gamma)}{\gamma}$. Therefore, the pair of prices $(v-\gamma, v-\gamma)$ and the corresponding advertising level $2\frac{v-\gamma}{v-\gamma+4\lambda}$ are a PSNE iff firms do not have incentives to increase nor decrease their prices, which requires $v \geq 2\gamma$ and $4\lambda > (v-\gamma)\max\left\{\frac{v}{v-2\gamma}, \frac{v-2\gamma}{\gamma}\right\}$. This proves part (ii) in Proposition 1.■

Proof of Lemma 1. Part *(ii)* in Proposition 1 shows that the game has an interior PSNE for $v > 2\gamma$ and $4\lambda \geq (v-\gamma)\max\left\{\frac{v}{v-2\gamma}, \frac{v-2\gamma}{\gamma}\right\}$. Thus, in order to show that the game has no symmetric PSNE, it remains to check that the game has no corner PSNE in which all consumers are informed about the existence of the product and firms set a price equal to $v-\gamma$. Suppose $(\phi^m)^* = 1$ and $p_i = p_j = v - \gamma$. In this case, all consumers are selective and the equilibrium would exhibit full market segmentation, with firms serving consumers in the respective strong market only, getting profits equal to $\frac{v-\gamma}{2} - \lambda$.

This could never be an equilibrium: under full information, each firm would be better off it charged a price v, as this would keep the level of sales unchanged but the firm would extract more profit from each customer.■

Proof of Proposition 2. Suppose that firm j selects a price randomly from the c.d.f $F_j(p)$. Suppose further that the support of the equilibrium prices is $[p_{\min}, p_{\max}]$. When firm i chooses any price that belongs to the equilibrium support of prices, and firm j uses the c.d.f $F_j(p)$, firm i's expected equilibrium profit does not depend on the price and it is always equal to a constant k^m minus advertising costs. More precisely, for $p_i \leq v - \gamma$, in a MSNE we must have:

$$p_i\left[\phi_i\left(1-\phi_j\right)+\phi_i\phi_j\left[1-\frac{1}{2}F_j\left(p_i+\gamma\right)-\frac{1}{2}F_j(p_i-\gamma)\right]\right]-A\left(\phi_i\right)=k^m-A\left(\phi_i\right) \Longleftrightarrow \tag{25}$$

$$F_j\left(p_i+\gamma\right)+F_j(p_i-\gamma)=2-\frac{2k^m}{\phi_i\phi_j p_i}+\frac{2\left(1-\phi_j\right)}{\phi_j} \tag{26}$$

Suppose that p_1 is such that $p_1 - \gamma = p_{\min}$ and p_2 is such that $p_2 + \gamma = p_{\max}$. Then, $\forall p \leq p_1$, $F(p-\gamma) = 0$ and $\forall p \geq p_2$, $F(p+\gamma) = 1$. Using (25) it follows that $\forall p \leq p_1 \Rightarrow F_j(p_i+\gamma) = 2 - \frac{2k^m}{\phi_i\phi_j p_i} + \frac{2(1-\phi_j)}{\phi_j}$ and $\forall p \geq p_2 \Rightarrow F_j(p_i - \gamma) = 1 - \frac{2k^m}{\phi_i\phi_j p_i} + \frac{2(1-\phi_j)}{\phi_j}$. Thus, in a symmetric MSNE with $F_i(p) = F_j(p) = F(p)$, we have $\forall p \leq p_1 \Rightarrow F(p) = 2 - \frac{2k^m}{\phi_i\phi_j(p-\gamma)} + \frac{2(1-\phi_j)}{\phi_j}$ and $\forall p \geq p_2 \Rightarrow F(p) = 1 - \frac{2k^m}{\phi_i\phi_j(p+\gamma)} + \frac{2(1-\phi_j)}{\phi_j}$.

Now it remains to show that $p_1 = p_2$. Suppose first that $p_2 < p_1$. Then, $\forall p \in [p_2, p_1]$ it follows $F(p-\gamma) = 0$ and $F(p+\gamma) = 1$, implying $p\left[\phi_i\left(1-\phi_j\right) + \frac{1}{2}\phi_i\phi_j\right] = k^m$. This corresponds to the profit obtained when each firm sells their product to the set of captive consumers, sharing the group of selective consumers, who buy efficiently. Assume now that $p_2 > p_1$ and take $p_{\max} - p_{\min} \leq 2\gamma$, a condition that will hold in the MSNE as we will later demonstrate. Take $p \in [p_1, p_2]$ s.t. (25) holds. Then, $\exists \widetilde{p}$ s.t. $\widetilde{p} - \gamma = p_L < p_1$ and $\widetilde{p} + \gamma = p_H > p_2$.

Since $p_L < p_1$ and $p_H > p_2$, it follows that

$$F\left(\widetilde{p}\right)=2-\frac{2}{\phi_i\phi_j}\left[\frac{k^m}{(\widetilde{p}-\gamma)}-\phi_i\left(1-\phi_j\right)\right] \text{ and } F(\widetilde{p})=1-\frac{2}{\phi_i\phi_j}\left[\frac{k^m}{\widetilde{p}+\gamma}-\phi_i\left(1-\phi_j\right)\right].$$

From the continuity of F it must be true that

$$2-\frac{2}{\phi_i\phi_j}\left[\frac{k^m}{(\widetilde{p}-\gamma)}-\phi_i\left(1-\phi_j\right)\right]=1-\frac{2}{\phi_i\phi_j}\left[\frac{k^m}{\widetilde{p}+\gamma}-\phi_i\left(1-\phi_j\right)\right],$$

implying the existence of a unique positive value of $\widetilde{p}$ given by $\widetilde{p} = \sqrt{\frac{4k^m\gamma}{\phi_i\phi_j} + \gamma^2}$.

Since this must hold $\forall p \in [p_1, p_2]$ and they cannot all be equal it must be the case that $p_1 = p_2$. Since $p_1 = p_{\min} + \gamma$ and $p_2 = p_{\max} - \gamma$ it follows that $p_{\min} + \gamma = p_{\max} - \gamma$ or equivalently $p_{\max} - p_{\min} = 2\gamma$.

Let p be the price of firm i, then given that $p_{\max} - p_{\min} = 2\gamma$. In this symmetric MSNE, $\phi_i = \phi_j = \phi$, the c.d.f $F^m(p)$ can be written as:

$$F^m(p) = \left\{ \begin{array}{ccc} 0 & if & p < p_{\min} \\ 1 - \frac{2}{(\phi^m)^2}\left(\frac{k^m}{p+\gamma} - \phi^m\left(1-\phi^m\right)\right) & if & p_{\min} \leqq p \leqq p_{\max} - \gamma \\ 2 - \frac{2}{(\phi^m)^2}\left(\frac{k^m}{p-\gamma} - \phi^m\left(1-\phi^m\right)\right) & if & p_{\max} - \gamma < p \leqq p_{\max} \\ 1 & if & p > p_{\max} \end{array} \right\} \tag{27}$$

From $F\left(p_{\min}\right) = 0$ and $F\left(p_{\max}\right) = 1$ it follows that

$$1 - \frac{2}{\phi_i\phi_j}\left(\frac{k^m}{p_{\min} + \gamma} - \phi_i\left(1 - \phi_j\right)\right) = 0 \Longleftrightarrow p_{\min} = \frac{2k^m}{2\phi_i - \phi_i\phi_j} - \gamma$$

$$2 - \frac{2}{\phi_i\phi_j}\left(\frac{k^m}{p_{\max} - \gamma} - \phi_i\left(1 - \phi_j\right)\right) = 1 \Longleftrightarrow p_{\max} = \frac{2k^m}{2\phi_i - \phi_i\phi_j} + \gamma \tag{28}$$

By continuity, for $p = p_{\max} - \gamma = \frac{2k^m}{2\phi_i - \phi_i\phi_j}$, we must have $\frac{2\phi_i\phi_j(k^m)^2}{\left(2\phi_i - \phi_i\phi_j\right)^2} - 2\gamma k^m - \frac{\phi_i\phi_j}{2}\gamma^2 = 0$. Hence

$$k^m = \frac{\gamma\phi_i}{2\phi_j}\left(2 - \phi_j\right)^2\left[1 + \sqrt{1 + \phi_j^2\left(2 - \phi_j\right)^{-2}}\right], \tag{29}$$

and

$$p_{\max} = \gamma\frac{2 - \phi_j}{\phi_j}\left[1 + \sqrt{1 + \phi_j^2\left(2 - \phi_j\right)^{-2}}\right] + \gamma, \tag{30}$$

which is independent of firms' own advertising decisions.

Since $p_{\max} - \gamma = \frac{2k^m}{2\phi_i - \phi_i\phi_j}$, the expected profit of firm i, $E\pi_i = k^m - A\left(\phi_i\right)$ is equal to

$$E\pi_i = \frac{1}{2}\phi_i\left(p_{\max} - \gamma\right)\left(2 - \phi_j\right) - A\left(\phi_i\right).$$

In the interior solution, $\phi_i \in (0,1)$, each firm's advertising equilibrium level with mass advertising is obtained by maximizing $E\pi_i$ with respect to ϕ_i. From the first order condition, the interior solution is given by[31] $\frac{\partial k^m}{\partial\phi_i} = A_{\phi_i}\left(\phi_i\right)$ which under symmetry writes as

$$\frac{1}{2}\left(p_{\max} - \gamma\right)\left(2 - \phi^m\right) = A_\phi\left(\phi^m\right), \tag{31}$$

implying that equilibrium profits can be written as $E\pi_i = \phi^m A_\phi\left(\phi^m\right) - A\left(\phi^m\right)$.■

[31] Note that the second order condition is satisfied. It is given by $-A_{\phi\phi}\left(\phi^m\right) < 0$, which is always true, given our assumptions with respect to the advertising technology.

Proof of Corollary 1 In the symmetric equilibrium, we have $\phi_i = \phi_j = \phi^m$. Introduce this in (30) and then plug the resulting expression in the FOC concerning optimal advertising choices (4):

$$\gamma\left(\frac{(2-\phi^m)^2}{2\phi^m}\right)\left(1+\sqrt{1+\left(\frac{\phi^m}{2-\phi^m}\right)^2}\right) = A_\phi\left(\phi^m\right), \tag{32}$$

Note that the RHS does not depend on γ. Thus, for a given ϕ^m, an increase in γ only shifts upwards the marginal revenue associated with ϕ^m (denoted by MgR_ϕ, which is a decreasing function of ϕ^m). Therefore when γ goes up, ϕ^m also goes up (since the shift upwards of MgR_ϕ leads to an upwards movement along the marginal cost of advertising and$A_\phi > 0$, $A_{\phi\phi} \geq 0$).

Analogously, a reduction in the marginal cost of advertising moves this curve downwards, generating a downward movement along the MgR curve and an increase in the equilibrium advertising level (recall that the MgR is decreasing with ϕ^m). ■

Proof of Lemma 2. Considering the quadratic advertising technology, $A\left(\phi^m\right) = \lambda\left(\phi^m\right)^2$, the equation (31) writes as:

$$\frac{1}{2}\left(p_{\max}-\gamma\right)\left(2-\phi^m\right) = 2\lambda\phi^m. \tag{33}$$

Substituting $p_{\max}$ by (30) and solving for ϕ^m, we obtain the equilibrium advertising level[32] given by:

$$\phi^m = 2\frac{\left(\gamma^2+8\lambda\gamma\right)^{1/2}}{\left(\gamma^2+8\lambda\gamma\right)^{1/2}+4\lambda}. \tag{34}$$

Simple algebra shows that ϕ^m in equation (34) defines the interior optimal advertising level as long as $\lambda > \frac{\sqrt{2}+1}{4}\gamma$, otherwise $\phi^m = 1$. Plugging the optimal value of ϕ^m by (34) in equation (29) , we obtain $k^m = \frac{8\lambda\gamma(8\lambda+\gamma)}{\left[4\lambda+(\gamma^2+8\lambda\gamma)^{1/2}\right]^2}$ and $E\pi^m = A\left(\phi^m\right) = \frac{4\lambda\left(\gamma^2+8\lambda\gamma\right)}{\left((\gamma^2+8\lambda\gamma)^{1/2}+4\lambda\right)^2}$.

Similarly, replacing ϕ^m in equation (30), we get $p_{\max} = \left(\gamma^2+8\lambda\gamma\right)^{1/2}+\gamma$, and it can be easily checked that, for $v > 2\gamma$, the condition $p_{\max} \leq v-\gamma$ requires

$$\lambda < \frac{1}{8}\left(v-\gamma\right)\frac{v-3\gamma}{\gamma}. \tag{35}$$

When λ is above the threshold in (35), the results in Proposition 2 do not hold as the demand structure would be different from the one considered in the derivation of this Proposition. Let us now compare (35) to the thresholds on λ already pointed out in Lemma 1, i.e. $\lambda < \frac{v-\gamma}{4}\max\left\{\frac{v}{v-2\gamma}, \frac{v-2\gamma}{\gamma}\right\}$. First, note that $\frac{1}{8}\left(v-\gamma\right)\frac{v-3\gamma}{\gamma} < \left(\frac{v-\gamma}{4}\right)\frac{v-2\gamma}{\gamma}$ for any $v > \gamma$. Second, note that $\frac{1}{8}\left(v-\gamma\right)\frac{v-3\gamma}{\gamma} > \left(\frac{v-\gamma}{4}\right)\frac{v}{v-2\gamma}$ if $v > 6\gamma$.

Accordingly, the interior solution holds when

$$\frac{\sqrt{2}+1}{4}\gamma < \lambda < \frac{v-\gamma}{4}\min\left\{\frac{v-3\gamma}{2\gamma}, \frac{v}{v-2\gamma}\right\}$$

[32] Note that equation (33) had an additional solution, given by $\phi^m = 2\frac{\sqrt{\gamma^2+8\lambda\gamma}}{\sqrt{\gamma^2+8\lambda\gamma}-4\lambda}$. However, it can be easily seen that such solution cannot define the optimal advertising level in an interior solution since it would always lead to $\phi^m > 1$.

The previous condition is not empty if the two following conditions hold:

(i) $\frac{\sqrt{2}+1}{4}\gamma < \frac{v-\gamma}{4}\frac{v}{v-2\gamma}$, which requires $\kappa(v) = \gamma^2\left(2\sqrt{2}+2\right) + v^2 - v\gamma\left(\sqrt{2}+2\right) > 0$. This second-degree polynomial above attains a minimum at $v = \gamma + \frac{1}{2}\sqrt{2}\gamma$, with $\kappa\left(\gamma + \frac{1}{2}\sqrt{2}\gamma\right) = \left(\sqrt{2}+\frac{1}{2}\right)\gamma^2 > 0$, meaning that condition condition (i) holds.

(ii) $\frac{\sqrt{2}+1}{4}\gamma < \lambda < \frac{v-\gamma}{8}\frac{v-3\gamma}{\gamma}$, which requires $v > \gamma\left(\sqrt{2}+3\right)$.

In summary, when $v > \gamma\left(\sqrt{2}+3\right)$ and $\frac{\sqrt{2}+1}{4}\gamma < \lambda < \left(\frac{v-\gamma}{4}\right)\min\left\{\frac{v-3\gamma}{2\gamma}, \frac{v}{v-2\gamma}\right\}$,we have an interior equilibrium with the features described in Proposition 2.■

Proof of Corollary 2. When $2\gamma < v < \gamma\left(\sqrt{2}+3\right)$ or when $v > \gamma\left(\sqrt{2}+3\right)$ and $\lambda < \frac{\sqrt{2}+1}{4}\gamma$, we have a corner solution in which $\phi^m = 1$. Replacing this advertising value in the values of $p_{\max}$ and k^m in (30) and (29), we obtain the results in Corollary 2.■

Proof of Lemma 3 The impact of λ on $E\pi^m$ is given by

$$\frac{\partial E\pi^m}{\partial\lambda} = -4\frac{4\lambda\gamma^2 - \left(\gamma^2+8\lambda\gamma\right)^{\frac{3}{2}}}{48\lambda^2\sqrt{\gamma^2+8\lambda\gamma} + 64\lambda^3 + 12\lambda\gamma^2 + 96\lambda^2\gamma + \left(\gamma^2+8\lambda\gamma\right)^{\frac{3}{2}}}.$$

We have $\frac{\partial E\pi^m}{\partial\lambda} > 0$ if

$$4\lambda\gamma^2 - \left(\gamma^2+8\lambda\gamma\right)^{\frac{3}{2}} < 0. \tag{36}$$

Note that the LHS is decreasing with λ, with the first derivative with respect to λ being equal to: $4\gamma\left(\gamma - 3\sqrt{\gamma^2+8\lambda\gamma}\right) < 0$. Therefore, to show that $4\lambda\gamma^2 - \left(\gamma^2+8\lambda\gamma\right)^{\frac{3}{2}} < 0$, it is sufficient to show that the polynomial is negative in the smallest value of $\lambda = \frac{\gamma\left(\sqrt{2}+1\right)}{4}$. For this value of λ, the LHS in (36) is equal to $\left(-69\sqrt{2}-98\right)\gamma^3 < 0$, which guarantees that $\frac{\partial E\pi^m}{\partial\lambda} > 0$, when $\lambda > \frac{\gamma\left(\sqrt{2}+1\right)}{4}$.■

Proof of Proposition 3 Let $q \in [0,1]$ represent the probability with which each firm serves its group of selective customers. Since the model is symmetric both firms have the same support of prices. Then q can be written as:

$$q = 1 - 2\int_{p_{\min}+\gamma}^{p_{\max}}\left(\int_{p_{\min}}^{p_A-\gamma} f(p_B)\,dp_B\right) f(p_A)\,dp_A$$

Note that $p_A < p_{\max}$, which implies $p_A - \gamma < p_{\max} - \gamma = p_{\min} + \gamma$, which means that

$$\int_{p_{\min}}^{p_A-\gamma} f(p_B)\,dp_B = F_B(p_A-\gamma) = \frac{2}{\phi^m} - 1 - \frac{2k^m}{p_A\left(\phi^m\right)^2},$$

Note also that when computing the first integral, we have $p_A > p_{\min} + \gamma$, which means we must obtain $f(p_A)$ from the third branch in the cdf, with $f(p_A) = 2\frac{k^m}{\phi^2(p_A-\gamma)^2}$.Thus, we have $\int_{p_{\min}+\gamma}^{p_{\max}}\left(\int_{p_{\min}}^{p_A-\gamma} f(p_B)\,dp_B\right) f(p_A)\,dp_A = -\frac{(\phi-2)^2}{\phi^2} + \frac{4k^2}{\gamma^2\phi^4}\ln\left(\frac{(p_{\max}-\gamma)^2}{p_{\max}(p_{\max}-2\gamma)}\right)$ and, in the MSNE:

$$q = 1 - \frac{8k^2}{\gamma^2\left(\phi^m\right)^4}\ln\left(\frac{\left(p_{\max}-\gamma\right)^2}{p_{\max}\left(p_{\max}-2\gamma\right)}\right) + 2\frac{\left(\phi^m-2\right)^2}{\left(\phi^m\right)^2},$$

In the case of the quadratic technology, we obtain
$q = 1 - 32\lambda^2 \left[\frac{1}{\gamma^2}\ln\left(\frac{1}{8\lambda}\gamma + 1\right) - \frac{1}{\gamma^2 + 8\lambda\gamma}\right]$.■

Proof of Proposition 4. Each firm serves its group of selective customers at p^o with probability given by $z \in [0,1]$:

$$z = \int_{p^r_{\min}}^{v-\gamma}\left(\int_{p^o_{\min}}^{p_B+\gamma} f^o(p^o)\, dp^o\right) f(p^r)\, dp^r + m^r,$$

From the Proof of Proposition 1 in Esteves and Resende (2016) follows that $\int_{p^o_{\min}}^{p^r+\gamma} f^o(p^o)\, dp^o = F^o(p^r+\gamma) = \frac{1}{\phi^{o*}}\left(1 - \frac{(v-\gamma)(1-\phi^{o*})}{p^r}\right)$ and $f(p^r) = \frac{v-(v-\gamma)\phi^{o*}}{\phi^{r*}(p^r+\gamma)^2}$. Thus:

$$\int_{p^r_{\min}}^{v-\gamma}\left(\frac{1}{\phi^{o*}}\left(1 - \frac{(v-\gamma)(1-\phi^{o*})}{p^r}\right)\frac{v-(v-\gamma)\phi^{o*}}{\phi^{r*}(p^r+\gamma)^2}\right) dp^r \Rightarrow$$

$$\Rightarrow \quad z = \left(\frac{v-\gamma}{\gamma^2 v\phi^{r*}}\right) p^{o*}_{\min}\left(\gamma + v\left(\ln\frac{(v-\gamma)p^{o*}_{\min}}{vp^{r*}_{\min}}\right)\frac{\phi^{o*}-1}{\phi^{o*}}\right) + m^r.$$

The value of z under the quadratic specification for the advertising cost function is obtained by replacing $p^o_{\min}, p^r_{\min}$, ϕ^{o*} and ϕ^{r*} in (13). ■

Proof of Proposition 5. (i) The condition $p_{\min} < p^r_{\min}$ requires $-\frac{1}{4}\frac{X(\lambda)}{(v+2\lambda-\gamma)^2} < 0$,with

$X(\lambda) = -128\gamma\lambda^3 + 8(v-\gamma)\left(2(v-7\gamma)\lambda^2 + \left(-4v\gamma + 2\gamma^2 + v^2\right)\lambda\right) + (v-2\gamma)(v+2\gamma)(v-\gamma)^2$

Accordingly, $p_{\min} < p^r_{\min} \Leftrightarrow X(\lambda) > 0$. Note that $X(\lambda)$ is a third-degree polynomial on λ such that: (i) $X(\lambda)$ has at most three roots; (ii) $\lim_{\lambda\to-\infty} X(\lambda) = +\infty$; (iii) $X(0) = (v-\gamma)^2(v-2\gamma)(v+2\gamma) > 0$; and (iv) $X'(0) = 8(v-\gamma)\left(-4v\gamma + 2\gamma^2 + v^2\right) > 0$ for $v \geq \left(\sqrt{2}+3\right)\gamma$. From (i)-(iv), it follows that $X(\lambda)$ has exactly one positive root (since $\lim_{\lambda\to+\infty} X(\lambda) = -\infty$). Note also that $X\left(\frac{v-\gamma}{4}\left(\frac{v-3\gamma}{2\gamma}\right)\right) = \left(-v\gamma - \gamma^2 + v^2\right)(v-\gamma)^2 > 0$, implying that the positive root of $X(\lambda)$ must be larger than $\frac{v-\gamma}{4}\left(\frac{v-3\gamma}{2\gamma}\right)$. Consequently, given condition (A2), we must have $X(\lambda) > 0$ for $\lambda < \left(\frac{v-\gamma}{4}\right)\min\left\{\frac{v-3\gamma}{2\gamma}, \frac{v}{v-2\gamma}\right\}$, which implies $p_{\min} < p^r_{\min}$.

The condition $p_{\max} < p^r_{\max} = v - \gamma$ follows directly from the fact that we are restricting our analysis to the domain of parameters in which firms compete for all selective consumers in the market, implying $p_{\max} < v - \gamma = p^r_{\max}$.

(ii) We have just shown that $p_{\min} \leq p^r_{\min}$ and $p_{\max} \leq p^r_{\min}$. Moreover, we know that $F^r(p)$ has a mass point at $p = v-\gamma$; and both cdf are increasing with p. To show that $F^m(p) > F^r(p)$, $\forall p$, it is sufficient to show that this is the case when $p = \left(\gamma^2 + 8\lambda\gamma\right)^{1/2}$, which is the price for which the slope of F^m changes (it increases). Since

$$\begin{aligned} F^m\left(\left(\gamma^2+8\lambda\gamma\right)^{1/2}\right) &= \frac{4\lambda\gamma}{\sqrt{\gamma^2+8\lambda\gamma}\left(\gamma+\sqrt{\gamma^2+8\lambda\gamma}\right)} \\ F^r\left(\left(\gamma^2+8\lambda\gamma\right)^{1/2}\right) &= \frac{4\lambda\left(2\sqrt{\gamma^2+8\lambda\gamma}(v+2\lambda-\gamma) - (v-\gamma)(v+4\lambda-2\gamma)\right)}{(v-\gamma)\left(\gamma+\sqrt{\gamma^2+8\lambda\gamma}\right)(v+4\lambda-2\gamma)}, \end{aligned}$$

when $2\sqrt{\gamma^2+8\lambda\gamma}\,(v+2\lambda-\gamma) < (v-\gamma)(v+4\lambda-2\gamma)$, we have $\left(\gamma^2+8\lambda\gamma\right)^{1/2} < p^r_{\min}$, which would necessarily imply that $F^m(p) > F^r(p), \forall p$.

Otherwise, we need to compute $K(v) = F^m\left(\left(\gamma^2+8\lambda\gamma\right)^{1/2}\right) - F^r\left(\left(\gamma^2+8\lambda\gamma\right)^{1/2}\right)$, with $K(v) > 0$ iff

$$(\gamma+\Xi)\,v^2 + ((4\lambda-3\gamma)\,\Xi - \gamma\,(12\lambda+5\gamma))v + (\gamma\,(2\gamma-4\lambda)\,\Xi - 4\gamma\,(4\lambda+\gamma)\,(2\lambda-\gamma)) > 0,$$

$\Xi = \sqrt{\gamma^2+8\lambda\gamma}$. It can be shown that the polynomial $K(v)$ is increasing[33] with v, with

$$K'(v) = \sqrt{\gamma^2+8\lambda\gamma}\,(2v+4\lambda-3\gamma) + \gamma\,(2v-12\lambda-5\gamma) > 0$$

Hence, if $K(v) > 0$ for the lowest values of v in our domain, it will be positive in the whole domain defined by conditions (A1) and (A2). In this respect, it is worth noting that the condition $\lambda < \left(\frac{v-\gamma}{4}\right)\left(\frac{v-3\gamma}{2\gamma}\right)$ can be re-written as $-\frac{1}{8}\frac{-4v\gamma+3\gamma^2-8\lambda\gamma+v^2}{\gamma} < 0$, implying

$$v > 2\gamma + \sqrt{\gamma\,(8\lambda+\gamma)},$$

with $K\left(2\gamma + \sqrt{\gamma\,(8\lambda+\gamma)}\right) = 4\lambda\gamma^2 > 0$.

The result that $K(v) = F^m\left(\left(\gamma^2+8\lambda\gamma\right)^{1/2}\right) - F^r\left(\left(\gamma^2+8\lambda\gamma\right)^{1/2}\right) > 0$, together with result (i) in this Proposition and the fact that $F^r(p)$ has a mass point at $p^r_{\max}$ (whereas F^m has not) implies $F^m(p) > F^r(p)$ for any price in the support of $F^m(p)$, which proves point (ii) in Proposition 5.

(iii) Since $p^o_{\min} = p^r_{\min} + \gamma$, and $p_{\min} < p^r_{\min}$, we have that $p_{\min} + \gamma < p^r_{\min} + \gamma = p^o_{\min}$. Since $p_{\min} + \gamma = \left(\gamma^2+8\lambda\gamma\right)^{1/2}$, this means that $F^m\left(\left(\gamma^2+8\lambda\gamma\right)^{1/2}\right) > F^o\left(\left(\gamma^2+8\lambda\gamma\right)^{1/2}\right) = 0$. Taking into consideration that $p_{\max} < p^o_{\max}$ (since $p_{\max} \leq v-\gamma$ and $p^o_{\max} = v$) and the fact that F^o has no mass point, being equal to 1, only when $p \geq v$, then $F^m(p) > F^o(p), \forall < p < \left(\gamma^2+8\lambda\gamma\right)^{1/2} + \gamma$, implying result (iii) in Proposition 5.

(iv) Since $p_{\min} \leq p^r_{\min}$ and $p_{\max} \leq p^r_{\min}$ and $F^m(p) > F^r(p), \forall p$, then it must the case that $E(p^m) < E(p^r)$. The same applies mutatis mutandis, in the case of the firms' own strong market segment, with $E(p^m) < E(p^o)$.■

Proof of Lemma 4. The condition $\phi^m = \frac{2\left(\gamma^2+8\lambda\gamma\right)^{1/2}}{\left(\gamma^2+8\lambda\gamma\right)^{1/2}+4\lambda} > \phi^r = \frac{v-\gamma}{4\lambda}\frac{v+4\lambda-2\gamma}{2v+4\lambda-2\gamma}$ holds iff

$$4\lambda\,(v-\gamma)\,(v+4\lambda-2\gamma) + \sqrt{\gamma^2+8\lambda\gamma}\,((v-\gamma)\,(v+4\lambda-2\gamma) - 16\lambda\,(v+2\lambda-\gamma)) < 0. \tag{37}$$

If

$$((v-\gamma)\,(v+4\lambda-2\gamma) - 16\lambda\,(v+2\lambda-\gamma)) > 0, \tag{38}$$

[33] To prove this result note that the first term in $K'(v)$ is always positive, whereas the second one is positive if $v > 6\lambda + \frac{5}{2}\gamma$, which means that $K'(v) > 0$ for $v > 6\lambda + \frac{5}{2}\gamma$. Otherwise, $K'(v) > 0$ requires $Y(v) = 4\lambda v^2 + \left(4\lambda\,(4\lambda-\gamma)+\gamma^2\right)v + \left(16\lambda^3 - 40\lambda^2\gamma - 9\lambda\gamma^2 - 2\gamma^3\right) > 0$,which is always the case since $Y'(v) = 8v\lambda + 16\lambda^2 - 4\lambda\gamma + \gamma^2 > 0$ and, in the lowest possible value of $v = \left(3+\sqrt{2}\right)\gamma$, we have $Y\left(\left(3+\sqrt{2}\right)\gamma\right) > 0$, implying $K'(v) > 0$ even when $v < 6\lambda + \frac{5}{2}\gamma$.

the condition (37) is never possible, implying $\phi^r > \phi^m$. Solving the inequality (38) for v, we get the following sufficient condition to have $\phi^r > \phi^m$:

$$v > 6\lambda + \frac{3}{2}\gamma + \frac{1}{2}\sqrt{272\lambda^2 + 24\lambda\gamma + \gamma^2}$$

Note that this condition becomes easier to satisfy as advertising costs become lower. When $\lambda = \frac{(1+\sqrt{2})}{4}\gamma$, the condition becomes $v > 10.473\gamma$. It can also be shown that for very low values of v, we have $\phi^m > \phi^r$. In order to prove this result, first notice that for the lowest possible value of $\lambda = \frac{(1+\sqrt{2})}{4}\gamma$, we get $\phi^m = 1$. Thus, if $\phi^r < 1 \Leftrightarrow v < \frac{(3\sqrt{2}+\sqrt{6}-2)\gamma}{2(\sqrt{2}-1)} = 5.6639\gamma$, we have $\phi^m > \phi^r$ for $\lambda = \frac{(1+\sqrt{2})}{4}\gamma$. For $\lambda = \frac{v-\gamma}{4}\frac{v-3\gamma}{2\gamma}$ (which constraints the largest admissible values of λ), we still have $\phi^m > \phi^r$[34]. Since both advertising levels are decreasing with λ, it must be the case that $\phi^m > \phi^r$ for $\frac{(1+\sqrt{2})}{4}\gamma < \lambda < \frac{v-\gamma}{4}\frac{v-3\gamma}{2\gamma}$ and $v < \frac{(3\sqrt{2}+\sqrt{6}-2)\gamma}{2(\sqrt{2}-1)}$. ■

Proof of Lemma 5. The condition $\phi^m = \frac{2(\gamma^2+8\lambda\gamma)^{1/2}}{(\gamma^2+8\lambda\gamma)^{1/2}+4\lambda} > \phi^o = \frac{v}{4\lambda+2(v-\gamma)}$ holds iff

$$\begin{aligned} &4v\lambda + \sqrt{\gamma^2 + 8\lambda\gamma}\,(-3v - 8\lambda + 4\gamma) < 0 \Leftrightarrow \\ \Leftrightarrow\ &\left(16\lambda^2 - 9\gamma(8\lambda+\gamma)\right)v^2 - 24\gamma(8\lambda+\gamma)(2\lambda-\gamma)v - 16\gamma(8\lambda+\gamma)(2\lambda-\gamma)^2 < 0. \end{aligned}$$

For $\lambda < \frac{3}{4}\gamma\left(\sqrt{10}+3\right) = 4.6217\gamma$, the inequality always holds since all the terms in the LHS are negative. For larger values of λ, we need to solve the polynomial, obtaining[35]

$$v < \underline{v} = 4(2\lambda - \gamma)\frac{3\gamma(8\lambda+\gamma) + 4\lambda\sqrt{\gamma^2+8\lambda\gamma}}{\left(16\lambda^2 - 9\gamma(8\lambda+\gamma)\right)}$$

Summing up, for $v < \underline{v}$, we have $\phi^m > \phi^o$. For $v > \underline{v}$, we only have $\phi^m > \phi^o$ if $\lambda < 4.6217\gamma$.■

Proof of Proposition 6. For the quadratic advertising technology, we have $E\pi^t = \left(\frac{2\lambda+v-\gamma}{2\lambda}\right)A(\phi^{o*}) + A(\phi^{r*})$ and $E\pi^m = A(\phi^{m*})$. Thus $E\pi^t - E\pi^m > 0$ requires

$$\left(\frac{2\lambda+v-\gamma}{2\lambda}\right)A(\phi^{o*}) + A(\phi^{r*}) - A(\phi^{m*}) > 0 \Leftrightarrow \left(\frac{2\lambda+v-\gamma}{2\lambda}\right) > \frac{(\phi^{m*})^2 - (\phi^{r*})^2}{(\phi^{o*})^2}$$

Denoting $\chi^*(\lambda, v, \gamma) = \frac{(\phi^{m*})^2-(\phi^{r*})^2}{(\phi^{o*})^2}$, the previous inequality can be written as

$$2\lambda\left[1 - \chi^*(\lambda, v, \gamma)\right] + v - \gamma > 0.$$

If $2\lambda\left[1 - \chi^*(\lambda, v, \gamma)\right] + v - \gamma = 0$ then $E\pi^t = E\pi^m$. Otherwise, $E\pi^m > E\pi^t$.■

[34] Note that $\phi^m\left(\lambda = \frac{v-\gamma}{4}\frac{v-3\gamma}{2\gamma}\right) - \phi^r\left(\lambda = \frac{v-\gamma}{4}\frac{v-3\gamma}{2\gamma}\right)$ is equal to $2\gamma\frac{8v^2\gamma^2-13\gamma^4+6v\gamma^3-6v^3\gamma+v^4}{(v-3\gamma)(v-\gamma)(v+\gamma)\left(-2v\gamma-\gamma^2+v^2\right)}$, which is positive since the denominator is positive upon direct observation and the numerator is also positive, since it is increasing with v and it is positive for the lowest admissible value of v, $v = \left(3+\sqrt{2}\right)\gamma$.

[35] The polynomial has a second root on v but it is negative for $\lambda < \frac{3\left(\sqrt{10}+3\right)\gamma}{4}$.

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