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Working Paper Series

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NIPE WP 03/ 2017

NÚCLEO DE INVESTIGAÇÃO EM POLÍTICAS ECONÓMICAS

UNIVERSIDADE DO MINHO

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URL:
http://www.nipe.eeg.uminho.pt/

«This work was carried out within the funding with COMPETE reference nº POCI-01-0145-FEDER-006683 (UID/ECO/03182/2013), with the FCT/MEC’s (Fundação para a Ciência e a Tecnologia, I.P.) financial support through national funding and by the ERDF through the Operational Programme on "Competitiveness and Internationalization – COMPETE 2020 under the PT2020 Partnership Agreement»

Behavior-Based Pricing under Imperfectly Informed Consumers*

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January 2017

Abstract

This paper is a first look at the dynamic effects of BBPD in a horizontally differentiation product market, where firms need to invest in advertising to generate awareness. When a firm is able to recognize customers with different purchasing histories, it may send them targeted advertisements with different prices. We show that in comparison to no discrimination, firms reduce their advertising efforts, charge higher first period prices and lower second period prices. As a result of that in contrast to the profit and consumer welfare results obtained under full informed consumers, we show that BBPD boosts industry profits at the expense of consumer welfare.

JEL classification: D43, L40, M37

1 Introduction

In many markets firms need to invest in advertising to create awareness for products, prices and special offers. The informative view of advertising claims that the primary role of advertising is to transmit information about (new) products' existence and/or price to otherwise uninformed consumers. When firms and consumers interact more than once, firms can gather information about the "reach" of their advertising campaign and learn the identity of consumers that come to know about their products. Firms can also collect information about the consumers'

*This research has been financed by Portuguese public funds through FCT (Fundação para a Ciência e a Tecnologia) within the project PTDC/IIM-ECO/2280/2014. Any errors are our own responsability.

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past shopping behavior. When firms realise that some consumers do not buy from them currently, they can use this information to price differently towards their own and their rival's previous customers. This form of price discrimination, termed behavior-based price discrimination (henceforth BBPD) or price discrimination by purchase history or dynamic pricing, is now widely observed in many markets. Such pricing strategies have been adopted by web retailers, supermarkets, telecom companies, banks, restaurants and many others.

The literature on BBPD has addressed issues related to price offers based on information revealed by consumers' purchase history.[1] However, with the exception of Esteves (2009a) and De Nijs (2013)[2] the literature has hitherto focused on the assumption that there is no role for advertising and that consumers are fully informed (e.g. Chen (1997) and Fudenberg and Tirole (2000)). Specifically, Esteves (2009a) and De Nijs (2013) depart from this assumption by investigating the competitive and welfare effects of BBPD in an informative advertising model under the homogeneous product assumption.

This paper extends Esteves (2009a) to a product differentiation market. The main goal is to evaluate the dynamic effects of BBPD when two firms endogenously segment the market into captive (partially informed) and selective (fully informed) customers by investing in informative advertising. We investigate *how* the permission of price discrimination affects: (i) the firms' pricing and advertising strategies and (ii) the level of profits and consumer welfare. We also look at the implications of BBPD in markets with imperfectly informed consumers in comparison to the case where consumers are fully informed- the Fudenberg and Tirole (2000) model.

The paper considers a two period model with two horizontally differentiated firms competing for ex-ante anonymous consumers with stable exogenous preferences across periods who can buy from a firm only if they receive an advertising message from it. In the first period firms have no information to engage in price discrimination. Because prices can change faster than consumers' awareness, in the second period, the level of awareness is constant and firms can only change prices. Advertising plays a dual role. On the one hand, it generates consumer heterogeneity in awareness of the firm's existence and prices. On the other hand, by collecting information about

[1] For a comprehensive survey on behavior-based price discrimination see Fudenberg and Villas-Boas (2007) and Esteves (2009b).

[2] De Nijs (2013) builds on Esteves (2009a) with one key modelling difference. While Esteves (2009a) assumes that firms make their advertising and first-period price decisions simultaneously, De Nijs (2013) consider a sequential timing in period 1. BBPD is employed in period 2.

the "reach" of their advertising, firms learn the identity of informed consumers who bought from them in the past and send later advertising messages (henceforth ads) with different prices to their own and to the rival's previous customers. Although in the recent past it was difficult for sellers to reconnect and communicate with "lost" customers and entice them back,[3] nowadays it is possible for advertising agencies (e.g. DoubleClick, Tacoda, ValueClick Media) to offer their clients the possibility to identify those visitors that were in their websites but did not buy the first time and reconnect subsequently with those potential consumers in order to encourage them to return and purchase. *The New York Times* (August, 29 and May, 16, 2010) reveals that this marketing practice, called *retargeting* is becoming increasingly common especially in online markets, such as retailing, travel, real estate and financial services.[4] Retargeting is based on the following main idea. Once a potential customer is aware of a firm's website (e.g. through normal advertising channels) and visits it, a cookie is passed to the consumer's browser that records his behaviour on the site and identifies him as either a nonpurchaser or a customer that bought from the firm. Then, at a determined time, old customers and rival's consumers are retargeted with messages specific to them.[5]

Within this theoretical framework, some novel results are obtained. In comparison to no-discrimination, BBPD in our setting boosts industry profits and harm consumers. This finding challenges the "traditional" view that such pricing practices in oligopoly markets often intensify competition and potentially benefit consumers. We also highlight that the ability of firms to engage in behavior-based price discrimination can have a significant impact on the firm's advertising strategies. A relevant contribution of the paper is to highlight that in comparison

[3] There are, of course, situations that motivate the present analysis where sellers have a way to communicate with current and potential consumers. There are for instance websites that ask consumers to register and their email may be one of the requirements, allowing the subsequent interactions. In the catalog industry, where firms rely on lists of names to advertise their products, it is also possible for sellers to identify different types of consumers and send them special offers.

[4] This marketing practice is also referred to as behavioral retargeting, remarketing or remessaging. For more on retargeting see, for instance, www.retargeter.com.

[5] Consider the following retargeting example. A consumer goes to an online shoe retailer and leaves the site without making a purchase. Then by utilizing a retargeting technology, the shoe retailer can catch the consumer the next time (when he's visiting a news site, perhaps). By visiting a site, a consumer has let that site know he is interested in the product and retargeting helps the advertiser entice the consumer to return and buy its product (e.g. receive 10 percent off if you buy today).

to the no-discrimination case, the permission of BBPD leads firms to strategically reduce their advertising choices in period 1 as a way to induce a softer pricing behavior in period 2.

Hence, for competition policy our analysis suggests that it is important to taking into account different forms of market competition when evaluating the profit and welfare effects of BBPD.

Related literature This paper is mainly related to two strands of the literature. It is related to the literature on competition with informative advertising (e.g. Butters (1977), Grossman and Shapiro (1984) and Stahl (1994)) in which rather than assuming that the information structure of consumers is exogenous, it is assumed that sellers can influence the consumers' information by investing in advertising. Specifically, it is assumed that a potential consumer cannot be an actual buyer unless firms invest in advertising. While Butters (1977) and Stahl (1994) look at competition in a homogeneous product market, Grossman and Shapiro (1984) look at the firms' advertising and price decisions in a product differentiation market. This paper is also related to the stream of research looking at the strategic effects of advertising in sequential games where firms first invest in advertising and, then, compete in prices (e.g. Ireland (1993), McAfee (1994) and Roy (2000)).[6] The main difference is that here we develop a model, where firms compete simultaneously at advertising and prices in the initial period and, if permitted, engage in BBPD in the next stage of the game.

The paper is also related to the literature on competitive BBPD where firms engage in price discrimination based on information about the consumers' past purchases. Like other forms of price discrimination, BBPD can have antitrust and welfare implications. While in the switching cost approach purchase history discloses information about exogenous switching costs (e.g. Chen (1997) and Taylor (2003)), in the brand preference approach purchase history discloses information about a consumer's exogenous brand preference for a firm (e.g. Villas-Boas (1999), Fudenberg and Tirole (2000)). A common finding in this literature is that BBPD tends to intensify competition and potentially benefit consumers (Chen (2005)). Behavior-based pricing tends to intensify competition and reduce profits in duopoly models where the market exhibits best response asymmetry,[7] when (i) all firms have the required information to engage

[6] An interesting contribution of Roy (2000) is to assume that firms can target consumers on the basis of their address (i.e. their location on a Hotelling framework). For other important contributions on targeted pricing in competitive settings see, for instance, Chen and Iyer (2002) and Iyer, et al (2005).

[7] Following Corts (1998), the market exhibits best response asymmetry when one firm's "strong" market is the

in price discrimination, (ii) consumer preferences are fixed across periods and (iii) consumers are fully informed (e.g. Chen (1997), Villas-Boas (1999), Fudenberg and Tirole (2000), Taylor (2003), Esteves (2010)).

Some authors have recently explored new avenues in the literature on BBPD. Chen and Pearcy (2010), for instance, look at BBPD under the assumption of correlated preferences across time. They show that if there is sufficiently strong dependence between preferences, BBPD reduces industry profits and increases consumer surplus. In contrast, under weak dependence they show that BBPD increases industry profits and reduces consumer surplus.[8] This paper enriches the literature on BBPD following the avenue of relaxing the assumption of perfectly informed consumers.[9] In so doing, we show that the use of BBPD in markets with informational differentiation among consumers (due to the firms' advertising decisions) can act in favour of industry profits at the expense of consumer welfare. A closely related paper is Esteves (2009a) which considers behaviour-based price discrimination in a homogeneous product market when initially the set of consumers who can buy the product is determined by advertising. Due to the homogeneous product assumption, the price equilibrium is in mixed strategies and only one of the two firms, namely the high priced firm in period 1, will have information to price discriminate in period 2. In comparison to no discrimination, Esteves (2009a) shows that BBPD might benefit all competing firms when advertising costs are such that firms advertise less under discrimination. By extending Esteves (2009a) to a product differentiation setting, new results are obtained. We will show that both firms will have the required information to engage in price discrimination, and even in this case, in comparison to no discrimination, firms will reduce their advertising efforts which translates into higher profits.

The rest of this paper is organized as follows. Section 2 sets out the model. Section 3 analyses the equilibrium advertising and pricing strategies when price discrimination is permitted. Section

other's "weak" market. In BBPD models there is best-response asymmetry because each firm regards its previous clientele as its strong market and the rival's previous customers as its weak market.

[8]Esteves and Reggiani (2014a) look at BBPD when demand is not inelastic; Esteves (2014b) extends the literature on BBPD allowing firms to employ retention strategies as a way to avoid losing part of the old customers willing to switch, and Esteves and Vasconcelos (2015) look at mergers when BBPD is permitted.

[9]Some authors have looked at competitive price discrimination when there is imperfect information on the firms' side: under static price discrimination (e.g. Chen et al (2001), Liu and Serfes (2004) and Esteves (2014)) and under BBPD (Colombo (2016)).

4 presents two benchmarks, the case where price discrimination is not allowed and the case where price discrimination is permitted but consumers are fully informed consumers. The competitive effects of BBPD are discussed in Section 5. The welfare effects of price discrimination are addressed in Section 6. Section 7 concludes and an appendix collects the proofs that were omitted from the text.

2 The model

There are two periods, 1 and 2, and two firms, A and B, launching a new differentiated non-durable good, which is produced at a constant marginal cost, assumed to be zero without loss of generality. The firms are located at the extremes of the unit interval, and consumers are uniformly distributed along this interval. The location of a consumer $\theta \in [0,1]$ represents his relative preference for firm B over A and remains fixed for both periods of consumption. The parameter t measures how much a consumer dislikes buying a less preferred brand. Although consumers are endowed with preferences over products, it is assumed that they are initially uninformed about the existence and the price of the goods. Like in Stahl (1994) a potential consumer cannot be an actual buyer unless firms invest in informative advertising.[10] The role of advertising is to convey information about the product existence and its price. There is a large number of consumers, with mass normalized to one, who desire to buy at most one unit of the good in each period. Each consumer has a reservation value v for the product, which is assumed to be large enough such that any informed consumer always buys one unit of the product.

The game proceeds as follows. In the first-period, firms choose advertising intensities and prices simultaneously and non-cooperatively. The advertising messages of each firm contain truthful[11] and complete information about the existence of its product and price. Firm i chooses its advertising level, ϕ_i, and its price, p_i, $i = A, B$. Because the number of consumers is normalized to unit, $\phi_i \in [0,1]$ can be interpreted as the share of consumers who receive ads from firm i. After firms have sent their ads independently, a proportion ϕ_i and ϕ_j of consumers is reached, respectively, by firm i and j advertising campaign. Therefore, the potential demand of firm i is made of a group of captive customers, namely $\phi_i\left(1-\phi_j\right)$, and a group of selective

[10] Implicitly it is assumed that for new products search costs are prohibitively high.

[11] This is guaranteed by the FTC regulation that prohibits advertisers from making false and deceptive statements about their products (see www.ftc.gov/bcp/conline/pubs/buspubs/ad-faqs.htm).

customers, namely $\phi_i \phi_j$.[12] A selective consumer buys from the firm offering him the highest surplus. A captive consumer is willing to buy the product as long as he gets a non-negative surplus.

Advertising has a long-run nature. In period 1, advertising creates awareness (and also informs about prices). Because prices can change faster than consumers' awareness, we assume that in period 2 the level of awareness is constant and firms can only change prices.

In period 1 price discrimination is unfeasible because firms have no information about consumers' types. However, in a repeated interaction, by collecting information about the "reach" of its advertising and about the informed customers' past behavior, a firm might be able to learn whether a previous informed consumer is an actual buyer or rather a customer that bought from a rival before. Note that as it is assumed that any informed consumer always buys, when say firm i realizes that a specific consumer receives one of its ad and decides not to buy product i in period 1, then it must be the case that this consumer also receive an ad from the rival which means that it is a selective consumer with a preference for the rival. However, it is important to stress that although each firm i has the ability identify the selective consumers who bought from the rival in period 1, it cannot distinguish within the group of their own informed previous consumers those who are captive and those who are selective with a preference for product i.

When a firm achieves this type of learning, in period 2 it may have incentives to entice the group of selective consumers previously buying from the rival to switch, by offering them a better deal. As said, we assume that in the second period firms can identify and reach the same consumers with no additional cost.[13] Thus, being price discrimination permitted in period 2, the firms are constrained to reach the same consumers but they can choose different prices to their own customers and to the rival's selective customers. Firm i selects a pair of second period prices, $\{p_i^O, p_i^R\}$, where p_i^O is firm i's price targeted to its own customers and p_i^R is firm

[12] The remaining consumers, in proportion $(1-\phi_i)(1-\phi_j)$, receive no ad from either firm, are uninformed and excluded from the market.

[13] There are several examples where firms can identify consumers that received their initial ads. When firms advertise their products through an advertising network they can have access to a retargeting technology through which it is easy for them to communicate with those customers that received their ads in the first period. Finally, firms may identify consumers at subsequent moments because in a first interaction they asked consumers to register. In this case, their email may be one of the requirements allowing sellers to subsequently communicate on a one-to-one basis.

i's price targeted to the group of selective consumers previously buying from the rival. We can think of second-period prices being quoted via private and personalized offers (e.g. retargeted ads, e-mail, mobile sms, creation of targeted websites, and so on). Firms and consumers have a common discount factor $\delta \in [0,1]$. Each firm maximizes its discounted profits, and each consumer maximizes his discounted utility.

Advertising technology Advertising is a costly activity for firms and conveys information about product existence and price. The advertising technology is exogenously given and the same for both firms. The cost of reaching a fraction ϕ of consumers is given by the function $A(\phi)$. This function increases at an increasing rate, which formally can be written $\frac{\partial A}{\partial \phi} = A_\phi > 0$ and $\frac{\partial^2 A}{\partial \phi^2} = A_{\phi\phi} > 0$. The latter condition means that it is increasingly more expensive to inform an additional customer or likewise, to reach a higher proportion of costumers. Additionally, there are no fixed costs in advertising, i.e., $A(0) = 0$. As the quadratic technology proposed in Tirole (1988) has the advantage of being extremely simple to manipulate algebraically, whenever a functional form is needed, we will assume that $A(\phi) = \frac{a\phi^2}{2}$. As in the present model there is a large number of consumers, normalized to one, a can be identified with the cost per ad.[14]

3 Equilibrium analysis

As usual we solve the game working backward from the second period.

Second-period pricing Assume that first period prices are $\{p_A^1, p_B^1\}$. Look first at the behavior of a captive consumer who is only aware of firm A. He buys from A as long as $p_A^1 + t\theta \leq v$. Similarly, if the consumer is captive to firm B he buys from B as long as $p_B^1 + t(1-\theta) \leq v$.

Look next at the behavior of a selective consumer. At first-period prices $\{p_A^1, p_B^1\}$ there is a cutoff $\theta^* \in [0,1]$ such that a fully informed consumer located at θ^* is indifferent between buying from A and B. With no loss of generality consider the group of selective consumers who bought from A in period 1, i.e., those located at $[0, \theta^*]$. Given the observed second period prices $\{p_A^O, p_B^R\}$ some of them might be willing to switch. Specifically, the indifferent consumer between

[14] Butters (1977) and Grossman and Shapiro (1984) propose other technologies with the same mathematical properties.

buying again from A at price p_A^O and switch to B at price p_B^R is located at θ_A^2 such that

$$\theta_A^2 = \frac{p_B^R - p_A^O + t}{2t}. \tag{1}$$

Consumers located at the left of θ_A^2 buy again from A, and those located at the right of θ_A^2 switch to firm B. Regarding the group of selective consumers previously buying from B, those located in the interval $[\theta^*, 1]$, the indifferent consumer between buying again from B at price p_B^O and switch to A at price p_A^R is located at θ_B^2, where

$$\theta_B^2 = \frac{p_B^O - p_A^R + t}{2t}. \tag{2}$$

Therefore, given firm i's second-period prices $\{p_i^O, p_i^R\}$, its second-period profit is $\pi_i^2 = \pi_i^O + \pi_i^R$, $i = A, B$, such that

$$\pi_A^O = p_A^O \phi_A \left[(1 - \phi_B) + \phi_B \left(\frac{p_B^R - p_A^O + t}{2t} \right) \right];$$

$$\pi_A^R = p_A^R \phi_A \phi_B \left(\frac{p_B^O - p_A^R + t}{2t} - \theta^* \right);$$

$$\pi_B^O = p_B^O \phi_B \left[(1 - \phi_A) + \phi_A \left(\frac{p_A^R - p_B^O + t}{2t} \right) \right];$$

$$\pi_B^R = p_B^R \phi_A \phi_B \left(\theta^* - \frac{p_B^R - p_A^O + t}{2t} \right).$$

Firm i chooses p_i^O in order to maximize π_i^O and p_i^R in order to maximize π_i^R. The first-order conditions yield[15]

$$p_A^O = \frac{t\left[4 + \phi_B (2\theta^* - 3)\right]}{3\phi_B} \text{ and } p_A^R = \frac{t\left[2 + \phi_A (1 - 4\theta^*)\right]}{3\phi_A}; \tag{3}$$

$$p_B^O = \frac{t\left[4 - \phi_A (2\theta^* + 1)\right]}{3\phi_A} \text{ and } p_B^R = \frac{t\left[2 + \phi_B (4\theta^* - 3)\right]}{3\phi_B}. \tag{4}$$

Therefore, firm A and B's second period profits are, respectively,

$$\pi_A^2 = \frac{t\left[\phi_A^2 (4 + \phi_B (2\theta^* - 3))^2 + \phi_B^2 (2 + \phi_A (1 - 4\theta^*))^2\right]}{18\phi_A\phi_B}, \tag{5}$$

and

$$\pi_B^2 = \frac{t\left[\phi_B^2 (4 - \phi_A (2\theta^* + 1))^2 + \phi_A^2 (2 + \phi_B (4\theta^* - 3))^2\right]}{18\phi_A\phi_B}. \tag{6}$$

[15] It is straightforward to see that the second-order conditions are also satisfied.

First-period pricing and advertising decisions Consider next first-period pricing and advertising decisions. If first-period prices lead to a cutoff θ^* the selective consumer located at θ^* is indifferent between buying from firm A in period 1 at price p_A^1 and then buying from B in period 2 at the poaching price p_B^R, or buying from B in period 1 at price p_B^1 and then buying from A at the poaching price p_A^R. At an interior solution we must observe:

$$v - p_A^1 - t\theta^* + \delta\left(v - p_B^R - t\left(1-\theta^*\right)\right) = v - p_B^1 - t\left(1-\theta^*\right) + \delta\left(v - p_A^R - t\theta^*\right). \tag{7}$$

From equations (3) and (4) we obtain:

$$\theta^* = \frac{t\delta\left(2\phi_B - 2\phi_A + \phi_A\phi_B\right) + 3\phi_A\phi_B\left(t - p_A^1 + p_B^1\right)}{2t\phi_A\phi_B\left(\delta+3\right)}. \tag{8}$$

Now consider the equilibrium choices of p_A^1 and p_B^1. At an interior solution, firm i's overall objective function is given by:

$$\underset{p_i,\phi_i}{Max}\Pi_i = \pi_i^1 + \delta\pi_i^2.$$

where π_A^2 is defined in (5) and

$$\pi_A^1 = p_A^1\phi_A\left[\left(1-\phi_B\right) + \phi_B\theta^*\right] - A\left(\phi_A\right). \tag{9}$$

As the game is symmetric we are looking for a symmetric subgame perfect nash equilibrium such that $p_A^1 = p_B^1 = p^1$, $\phi_A = \phi_B = \phi^*$ and $\theta^* = \frac{1}{2}$.

Proposition 1. *When price discrimination is permitted there is a SPNE in which:*
(i) Each firm selects an advertising reach, denoted $\phi^* \in [0,1]$, implicitly defined by:

$$A_\phi\left(\phi^*\right) = \frac{t\left(2-\phi^*\right)^2}{2\phi^*} - \frac{\delta t\left(2-\phi^*\right)}{6}. \tag{10}$$

(ii) Each firm chooses first and second period prices equal to

$$p^1 = t\left(1+\frac{\delta}{3}\right)\left(\frac{2-\phi^*}{\phi^*}\right), \tag{11}$$

$$p^O = \frac{2t}{3}\left(\frac{2-\phi^*}{\phi^*}\right), \tag{12}$$

$$p^R = \frac{t}{3}\left(\frac{2-\phi^*}{\phi^*}\right). \tag{13}$$

(iii) First and second period profits are, respectively:

$$\pi_1^* = \frac{t}{2}\left(2-\phi^*\right)^2\left(1+\frac{\delta}{3}\right) - A\left(\phi^*\right),$$

$$\pi_2^* = \frac{5}{18} t \left(2 - \phi^*\right)^2 . \tag{14}$$

Thus, each firm overall profit is equal to

$$\Pi^* = \frac{t}{18} \left(2 - \phi^*\right)^2 (8\delta + 9) - A\left(\phi^*\right). \tag{15}$$

Proof. See the Appendix.

Proposition 1 highlights that as in other models of BBPD firms charge lower prices to the rival's previous customers than to their own customers. Additionally, it shows that consumers face lower prices in the second-period than in the first-period. This is the all-out competition result that tends to occur in markets exhibiting best-response asymmetry. Note also that less advertising in period 1 has a positive effect both on first and second period prices $\left(\frac{\partial p^k}{\partial \phi} < 0, \text{ with } k = 1, O, R\right)$. Clearly, less advertising in period 1 increases the informational differentiation, reduces the elasticity of demand and allows firms to raise prices. When firms reduce their advertising reach they compete less aggressively in prices in both periods because less consumers will be aware of both firms. This strategic reasoning will be important to understand the impact of BBPD on the firms' advertising choices in period 1.

Using equation (10) and the fact that for the quadratic technology $A_\phi\left(\phi^*\right) = a\phi^*$ it is straightforward to prove corollary 1.

Corollary 1. *When firms use the quadratic technology, i.e., when* $A(\phi) = \frac{a}{2}\phi^2$ *there is a SPNE in which:*

(i) Each firm selects an advertising reach equal to $\phi^* = \frac{2}{1+\frac{\delta}{6}+\sqrt{\frac{\delta^2}{36}+\frac{2a}{t}}}$ *where* $0 < \phi^* < 1$ *as long as* $a > t\left(\frac{3-\delta}{6}\right)$.[16]

(ii) Each firm chooses first and second period prices respectively equal to:

$$p^1 = \frac{(\delta+3)\left(t\delta + \sqrt{t^2\delta^2 + 72at}\right)}{18}, \tag{16}$$

$$p^O = \frac{t\delta + \sqrt{t^2\delta^2 + 72at}}{9}, \tag{17}$$

$$p^R = \frac{t\delta + \sqrt{t^2\delta^2 + 72at}}{18}. \tag{18}$$

[16] It is important to stress that there is also a relation between a and v. Specifically, given the equilibrium prices and advertsing level we nust impose that consumer surplus is positive, thus given v there is a upward limit on a. In a numerical example this implies that v should be high enough in relation to a.

(iii) Overall equilibrium profit per firm are equal to:

$$\Pi^* = \frac{1}{\left(1 + \frac{\delta}{6} + \sqrt{\frac{\delta^2}{36} + \frac{2a}{t}}\right)^2} \left[\frac{t\,(8\delta + 9)}{18} \left(\frac{\delta}{3} + 2\sqrt{\frac{\delta^2}{36} + \frac{2a}{t}} \right)^2 - 2a \right]$$

Note that when $a \leq t\left(\frac{3-\delta}{6}\right)$ then $\phi^* = 1$, and the results under full information would be obtained (see section 4.2).

4 Benchmarks

Before proceeding we present next two benchmark models. We first consider the case where price discrimination is not permitted in period 2. Then we consider the case where price discrimination is permitted and consumers are perfectly informed about the firms' existence (i.e., $\phi = 1$). The analysis in the latter case is based on Fudenberg and Tirole (2000).

4.1 No discrimination

Consider that price discrimination is for any reason not permitted. In the first-period, firms choose advertising intensities and prices simultaneously and non-cooperatively. In the second period, firms are forced to set the same prices. This means that, once prices are publicly announced through advertising in period 1, they must remain for the entire duration of the game. I will use this benchmark case to evaluate the competitive and welfare effects of price discrimination with advertising. Firm i profit is equal to:

$$\pi_i = (1+\delta)\, p_i \phi_i \left[\left(1 - \phi_j\right) + \phi_j \left(\frac{p_j - p_i + t}{2t} \right) \right] - A\left(\phi_i\right)$$

Let the superscript *nd* identify the no-discrimination case. Following a similar approach as in Tirole (1988) it is straightforward to obtain Proposition 2.[17]

Proposition 2. *In the benchmark case without price discrimination there is a symmetric SPNE in which:*

[17] The results derived in Tirole (1988) are obtained for the special case of $\delta = 0$.

(i) Each firm selects an advertising reach, denoted $\phi^{nd} \in [0,1]$, implicitly defined by:

$$A_\phi\left(\phi^{nd}\right) = \frac{t(\delta+1)\left(2-\phi^{nd}\right)^2}{2\phi^{nd}}. \tag{19}$$

(ii) The price for the two periods of consumption is equal to

$$p^{nd} = \frac{t\left(2-\phi^{nd}\right)}{\phi^{nd}}. \tag{20}$$

(iii) Each firm's profit is

$$\Pi_i^{nd} = \frac{t(1+\delta)\left(2-\phi^{nd}\right)^2}{2} - A\left(\phi^{nd}\right).$$

Proof. See the Appendix.

With no-discrimination industry profits are

$$\Pi_{ind}^{nd} = (1+\delta)\,t(2-\phi^{nd})^2 - 2A\left(\phi^{nd}\right) \tag{21}$$

aggregate welfare is

$$W^{nd} = (1+\delta)\left[v\phi^{nd}\left(2-\phi^{nd}\right) - \frac{t}{4}\left(\phi^{nd}\left(4-3\phi^{nd}\right)\right)\right] - 2A\left(\phi^{nd}\right), \tag{22}$$

and consumer surplus equals:

$$CS^{nd} = (1+\delta)\left[v\phi^{nd}\left(2-\phi^{nd}\right) + \frac{t}{4}\left(16+\phi^{nd}\left(12-\phi^{nd}\right)\right)\right]. \tag{23}$$

From equation (19) for the specific case of the quadratic technology it is straightforward to obtain Corollary 2.[18]

Corollary 2. *When the advertising technology is $A(\phi) = \frac{a\phi^2}{2}$ there is a symmetric SPNE in which:*

(i) Each firm selects an advertising reach of

$$\phi^{nd} = \frac{2}{1+\sqrt{\frac{2a}{t(1+\delta)}}} \tag{24}$$

[18] As expected when $\delta = 0$ we obtain the static results presented in Tirole (1988).

with $0 < \phi^{nd} < 1$ *as long as* $a > \frac{t(\delta+1)}{2}$,[19] *and a price equal to*

$$p^{nd} = \sqrt{\frac{2at}{1+\delta}}. \tag{25}$$

(ii) Each firm's overall equilibrium profit equals

$$\Pi_i^{nd} = \frac{2a}{\left(1 + \sqrt{\frac{2a}{t(1+\delta)}}\right)^2}. \tag{26}$$

Proof. See the Appendix.

4.2 BBPD with perfectly informed consumers

Now we assume that consumers are fully informed about products' existence and prices. The analysis here is similar to Fudenberg and Tirole (2000) with consumer preferences uniformly distributed. Following Fudenberg and Tirole (2000) it is straightforward to establish Proposition 3. Let the superscript f identify the case with full informed consumers.

Proposition 3. *When price discrimination is permitted there is a symmetric SPNE in which:*

(i) First-period equilibrium prices are $p^{1,f} = t\left(1 + \frac{\delta}{3}\right)$.

(ii) Second-period equilibrium prices are $p^{O,f} = \frac{2}{3}t$ *and* $p^{R,f} = \frac{1}{3}t$.

(iii) Each firm overall profit equals

$$\Pi_i^f = \frac{1}{18}t\left(8\delta + 9\right). \tag{27}$$

Industry profit is

$$\Pi_{ind}^f = \frac{1}{9}t\left(8\delta + 9\right), \tag{28}$$

overall welfare equals

$$W^f = v\left(1+\delta\right) - \frac{1}{4}t - \frac{11\delta}{36}t, \tag{29}$$

and consumer surplus is given by

$$CS^f = v\left(1+\delta\right) - \frac{5}{4}t - \frac{43}{36}t\delta. \tag{30}$$

[19] For the quadratic technology when $a \leq \frac{t(\delta+1)}{2}$ then $\phi^{nd} = 1$.

As expected, the prices in Fudenberg and Tirole (2000) are a special case of those defined in Proposition 1 when $\phi = 1$. Further, it is straightforward to see that the prices under BBPD with full informed consumers are below their counterparts when consumers are imperfectly informed. In other words, $p^{1,f} < p^1$; $p^{O,f} < p^O$ and $p^{R,f} < p^R$.

5 Implications of price discrimination

This section investigates how the permission of price discrimination affects the equilibrium outcomes−i.e., advertising intensity, prices and profits−in markets where consumers are initially uninformed and firms need to invest in advertising to create awareness for their products.

Proposition 4. *Regardless of the advertising technology considered:*

(i) Firms advertise less under BBPD than under no-discrimination, i.e., $\phi^* < \phi^{nd}$.

(ii) The following relationship between first period, second period and non discrimination prices holds: $p^R < p^O < p^{nd} < p^1$.

Proof. See the Appendix.

Proposition 4 sheds light on the dynamic effects of price discrimination on firms' advertising and price decisions. Part (i) shows that BBPD has a significant effect on firms' advertising strategies. Specifically, it shows that the permission of BBPD leads firms to reduce their advertising efforts.

The result of less advertising due to price discrimination should be compare to less/more advertising in Esteves (2009a). Under the homogenous product approach, Esteves (2009a) shows that depending on whether advertising costs are high or low, firms may advertise more or less with discrimination, respectively. In this model only the high price firm in period 1 has information to price discriminate and so price discrimination raises the second-period profit of the discriminating firm. When discrimination is permitted each firm has a dynamic incentive to become the discriminating firm and to induce the non-discriminating firm to play less aggressively in the subsequent period. While the former goal is achieved by pricing strategically high in period 1, the latter goal is achieved by choosing a first-period advertising intensity that strategically increases the non-discriminating firm's captive segment.

By extending Esteves (2009a) to a product differentiation framework we show that the permission of price discrimination leads always to less advertising in period 1. In our framework first period equilibrium price is in pure strategies and both firms have information to engage in price discrimination in the next period. Firms take into account that more advertising creates a bigger common market which leads to more aggressive price behavior in period 2 due to the best response asymmetry feature of the market. On the other hand, firms also take into account that in period 2 they have no information to recognize in their base of own customers those who are captive and those who are selective. The higher is the group of captive customers the higher will be p^O and so p^R. This suggest that firms have a strategic incentive to reduce the first-period advertising intensity because by reducing the size of the group of fully informed consumers, they induce a softer pricing behavior in period 2 and so doing they reduce the negative effects of price discrimination.

Hence, the paper highlights that in comparison to the no-discrimination case, the permission of price discrimination leads firms to strategically reduce their advertising intensities in period 1 as an attempt to soften price competition in the subsequent period. This in turn also induces firms to play less aggressively in period 1.

Next we discuss the implications of BBPD on the prices paid by different types of consumers in both periods. As in other models of BBPD (e.g., Fudenberg and Tirole (2000)), in comparison to uniform pricing, under price discrimination consumers are overcharged in the first period but then strong competition leads to reduced prices in the second period. The reduction is more pronounced for the rival's previous customers that need to be encouraged to buy their less favorite good.

In comparison to the case where consumers are fully informed, consumers pay higher prices in both periods under imperfect information; and prices will be higher as firms advertise less intensively in period 1. Less advertising in period 1 increases the informational differentiation, reduces the elasticity of demand and allows firms to raise prices. Additionally, the fact that in period 2 firms cannot distinguish a captive from a selective previous customer also contributes to soften price competition in this period, allowing second-period discriminatory prices to be above their full information counterparts.

Regarding first-period prices, in comparison to Fudenberg and Tirole (2000), here there is an additional effect explaining why first period price with BBPD is further above the non-discrimination counterpart. Like in Fudenberg and Tirole because consumers correctly anticipate that they will be offered lower second-period prices, first period demand is less elastic allowing firms to raise first period prices. Additionally, the existence of imperfect awareness further reduces the elasticity of demand and allows firms to further raise prices in the beginning of the game. Indeed, note that at the symmetric equilibrium in Fudenberg and Tirole the elasticity of demand is equal to:

$$\varepsilon_1 = -\frac{\partial D_1}{\partial p_A^1}\frac{p_1}{D_1} = \frac{p_1}{t\left(\frac{\delta}{3}+1\right)} \tag{31}$$

while in the present model it equals:

$$\varepsilon_1 = -\frac{\partial D_1}{\partial p_A^1}\frac{p_1}{D_1} = \frac{\phi p_1}{t\left(\frac{\delta}{3}+1\right)(2-\phi)}. \tag{32}$$

Thus the lower is the intensity of advertising, the lower is ε_1 and so the higher are the first period prices. (Obviously, when $\phi = 1$ expression (32) is equal to expression (31)). This explains why $p^{nd} < p^{1,f} < p^1$.

Proposition 5. *Regardless the advertising technology considered behavior-based price discrimination boosts the firms' overall profit.*

Proof. See the Appendix.

In comparison to uniform pricing, although price discrimination has a negative effect on second-period profits (through lower prices in period 2), it has a positive effect on first-period profit (through higher prices in period 1). As the increase in first period profit more than compensates the decrease in second-period profit, there is a net positive effect on overall profits.

The finding that BBPD can boost profits is a relevant contribution of this model and challenges the traditional view that firms are worse off when engaging in behavior-based price discrimination practices. In fact, a standard result in the literature on BBPD is that it is generally the case that overall profits decrease when firms engage in price discriminate. In markets with best-response asymmetry this tends to occur when (i) all firms have the required information to engage in price discrimination, (ii) consumer preferences are fixed across periods and (iii) consumers are fully informed (e.g. Chen (1997), Villas-Boas (1999), Fudenberg and Tirole (2000), Esteves (2010)). An important contribution of the present model is to show that the use of BBPD in markets with informational differentiation among consumers (due to the firms' advertising decisions) can act in favour of industry profits.

Since price and advertising decisions are affected by the advertising cost, profits are also affected. We observe that the signal of the effect of a on profits is the same with and without price discrimination. Specifically, we find that profits increase as advertising becomes more costly, $\left(\frac{\partial \Pi}{\partial a} > 0\right)$. This is a well-known result in the literature on informative advertising (e.g. Grossman and Shapiro (1984), Stahl (1994)). In general, whilst an increase in advertising costs has a negative "direct effect" on profits, under competition there is also a "strategic effect", as advertising costs increase, firms respond with less advertising, permitting prices to rise. When the strategic effect dominates, profits may increase with advertising costs. Higher advertising costs lead to lower shares of informed consumers and so to a lower segment of selective customers. Demand becomes less elastic and prices move upwards. When price discrimination is permitted, the strategic effect is stronger, firms respond with lower levels of advertising which translates into higher profits.

If we depart from a situation where a is such that ϕ is high (a is low), less advertising is more likely to increase the fraction of captive customers (range of a where $\phi^* < 0.5$) than the fraction of selective customers. In this case the probability of reaching an uninformed buyer is high, firms have more incentives to focus on the group of captive consumers, they quote high prices and profits increase.

It is interesting to note that when advertising costs are such that $\phi^* > 0.5$, the share of selective consumers is higher than the share of each firm's captive consumers. As advertising costs increase the share of selective consumers is smaller allowing firms to compete less aggressively under price discrimination. Because in period 2 firms are not able to distinguish in their group of own customers the selective and captive consumers, they have less incentives to reduce the price targeted to their own previous customers when they face a higher share of captive consumers. As prices are strategic complements, when p_i^O moves upwards the same happens to p_j^R. Therefore, higher advertising costs has the strategic effect of increasing the group of captive customers, thereby softening price competition under BBPD. Additionally, a higher share of captive consumers also induces firms to adopt a softer behavior in period 1, which translates into higher prices and profits.

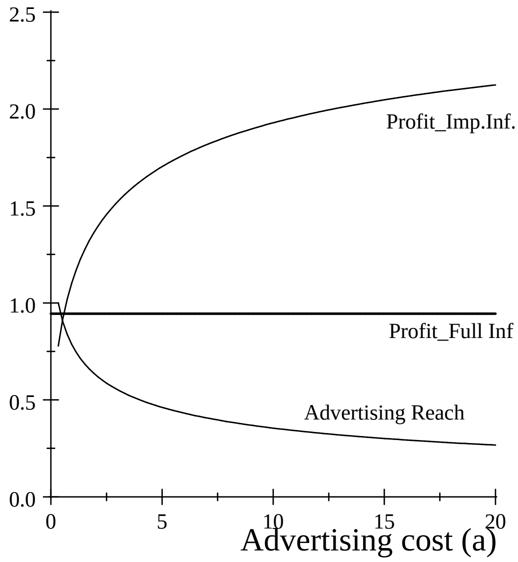
The model predicts that firms are expected to benefit the most from behavior-based price discrimination practices in industries characterized by high advertising costs. It can be said that a high a acts as collusive device because it induces firms to reduce the their advertising efforts and helps them sustaining higher profits. Additionally, the model predicts that restrictions on advertising and industry lobby in favour of these restrictions in contexts where price discrimination is permitted would act in favour of profits at the expense of consumers.

Next we discuss the profit implications of BBPD in markets where consumers are fully informed in comparison to the case where, for instance due to the firms' advertising decisions, consumers are imperfectly informed, in the sense that some are captive to one of the firms while others are fully informed (switchers). As first and second period prices with BBPD under imperfect information are above their full information counterparts, profits under imperfect information can be above the perfect information level. Using equations (15) and (27) it is straightforward to see that $\Pi^* > \Pi^f$ if $\frac{t}{18}(2-\phi^*)^2(8\delta+9) - A(\phi^*) > \frac{t}{18}(8\delta+9)$, which simplifies to

$$\frac{t(8\delta+9)}{18}\left[(2-\phi^*)^2 - 1\right] > A(\phi^*).$$

Note that the expression in brackets is positive and decreasing with ϕ. As $\frac{\partial \phi^*}{\partial a} < 0$ then as a increases, ϕ^* decreases and it is more likely to find that $\Pi^* > \Pi^f$. Thus, the positive effect of informational differentiation on profits needs to be higher than the cost of advertising.

Figure 1 plots each firm profit when it employs BBPD in the case where consumers are fully informed (Profit_Full Inf.) and in the case where firms need to invest in advertising to generate awareness (Profit_Imp.Inf.). Profits are plotted as a function of the advertising cost a. In the same figure, with a different interpretation of the vertical axis, we also plot the equilibrium level of advertising $\phi^* \in (0,1)$ as a function of a. The figure is plotted for the quadratic advertising technology, $t = 1$, $\delta = 1$ and for the range where $\phi^* < 1$ is defined, i.e., $a > t\left(\frac{3-\delta}{6}\right)$.



For the numerical example presented we find that $\Pi_i^f < \Pi_i^*$ as long as $a > 0.5816$ (or, $\phi^* < 0.88575$). This suggests that as long as advertising costs are not too low then $\Pi^* > \Pi^f$. In this case when we depart from BBPD with full information to BBPD with imperfect information we find that the positive impact of higher prices on profits more than compensates the negative impact of advertising costs. Due to the strategic effects of advertising, higher advertising costs translate into less advertising, more market power and higher firms' profits. Thus, the model predicts that in comparison to the benchmark case with fully informed consumers (e.g. Fudenberg and Tirole (2000)), behavior-based price discrimination can increase industry profits as long as advertising costs are not too low.

Therefore, the profit results obtained highlight the importance of taking into account the different forms of market competition when competition policy agencies try to evaluate the effects of price discrimination in competitive settings. This suggests that price discrimination strategies should not be considered in isolation. There are interactions between price discrimination and other marketing strategies such as the advertising decisions that need to be taken into account.

6 Welfare analysis

A recurrent policy question is whether to restrict price discrimination policies. This section evaluates the impact of price discrimination on consumer and social welfare. To simplify the analysis, throughout this section it is assumed that $\delta = 1$. Given the equilibrium solutions derived in Proposition 1, it is straightforward to obtain that welfare in period 1 and 2, is respectively given by

$$w_1^* = v\phi^* (2 - \phi^*) - \frac{t}{4} (\phi^* (4 - 3\phi^*)) - 2A (\phi^*),$$

$$w_2^* = 2\phi^* (1 - \phi^*) \left(v - \frac{t}{2}\right) + v\phi^{*2} - \frac{t}{36} \left(11\phi^{*2} - 8\phi^* + 8\right).$$

Note that if $\phi^* = \phi^{nd}$ overall welfare in period 1 would be equal to its no discrimination counterpart. Thus, overall welfare equals

$$W^* = 2v\phi^* (2 - \phi^*) + \frac{1}{9} t \left(13\phi^{*2} - 16\phi^* - 2\right) - 2A (\phi^*). \tag{33}$$

As equilibrium industry profit is $\Pi^*_{ind} = \frac{34t}{18} (2 - \phi^*)^2 - 2A (\phi^*)$, consumer surplus equals:

$$CS^* = 2v\phi^* - \frac{2}{9} t \left(2\phi^{*2} - 26\phi^* + 35\right). \tag{34}$$

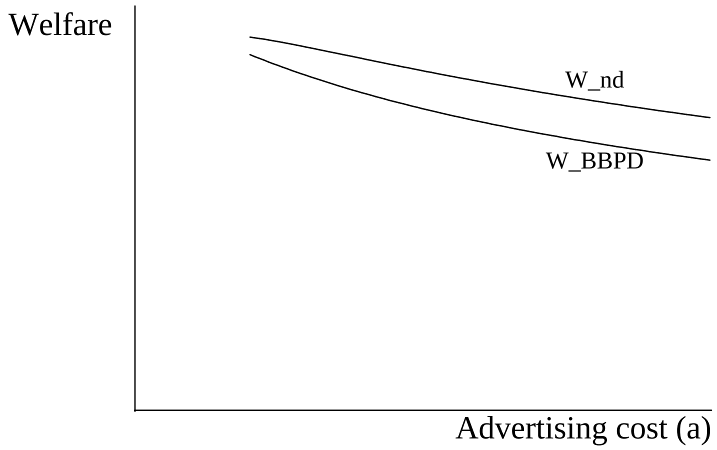
Taking into account equation (23) we can establish the following result.

Proposition 6. *In comparison to no discrimination, regardless of the advertising technology considered, BBPD boosts industry profits and reduces consumer surplus.*

Proof. See the Appendix.

Proposition 6 has important implications regarding the profit and consumer welfare effects of BBPD. In sharp contrast to the usual finding that price discrimination in competitive settings can benefit consumers at the expense of industry profits, our model predicts the reverse: price discrimination based on purchase history benefits industry profits and harms consumers.

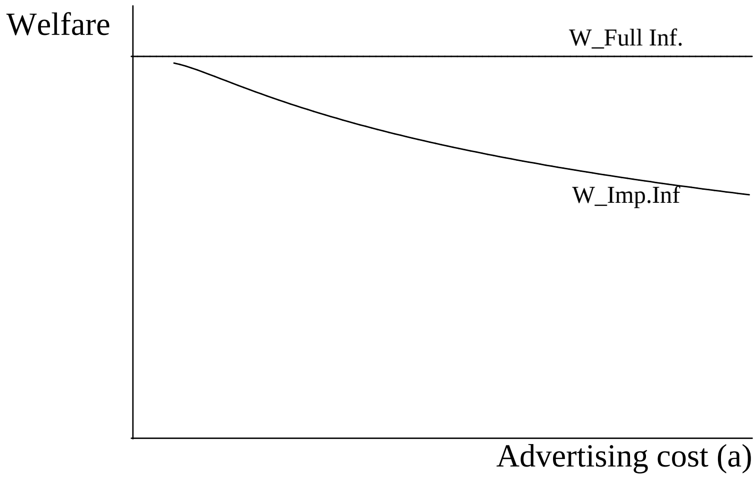
Regarding overall welfare, although we cannot formally prove whether it is higher under uniform pricing or under BBPD, Figure 2 plots aggregate welfare for the quadratic advertising technology, $t = 1$, $\delta = 1$ and for the range where a is such that both ϕ^* and ϕ^{nd} are defined which implies that $a > t$. We also take into account that v should be high enough in comparison to a such that all consumers entering the market get a non-negative surplus.



The numerical example presented shows that like in Fudenberg and Tirole (2000) welfare falls when firms engage in price discrimination based on purchase history. Here apart from the negative impact of BBPD on welfare due to more inefficient switching there is also the negative impact of less advertising when firms employ BBPD which translates into a smaller share of consumers entering the market. As aforementioned, the model highlights the importance of investigating the welfare effects of price discrimination in markets where consumers' awareness is determined by the firms' advertising decisions, which in turn can be affected by the possibility of engaging in price discrimination practices.

While consumer surplus increases at the expense of industry profits when BBPD is permitted and consumers are fully informed, the reverse happens in our framework. The model predicts that advertising choices might help firms to introduce imperfect information into the market which may act to soften price competition and boost profits when BBPD is permitted. In contrast to the relation between industry profits and advertising costs, consumer surplus and welfare are higher in markets with lower advertising costs (more advertising). From equation (34) it is straightforward to see CS^* increases with ϕ^* ($\frac{\partial CS}{\partial \phi} > 0$).

Compare next our consumer surplus and welfare results with those under full informed consumers. Using equations (30) and (34), it is straightforward to prove that $CS^* < CS^f$. As mentioned, although BBPD benefits consumers under perfect information, the same does not occur when consumers are imperfectly informed. In fact while BBPD under full information boosts consumer surplus at the expense of industry profits, the reverse happens in our framework. Figure 3 plots aggregate welfare for the quadratic advertising technology, $t = 1$, $\delta = 1$ and for the range where a is such that both ϕ^* is defined which implies that $a > \left(\frac{3-\delta}{6}\right)$. We also take into account that v should be high enough in comparison to a such that all consumers entering the market get a non-negative surplus. The numerical example shows that that overall welfare falls when BBPD is employed under a lower share of informed consumers.



The numerical example presented shows that welfare falls when we depart from BBPD with full informed consumers to the case where consumers are imperfectly informed due to the firms' advertising decisions. The reduction on welfare is expected to be stronger as advertising costs increase because the share of informed consumers who can enter the market falls. Hence, our model highlights that in comparison to the case of full informed consumers, BBPD under imperfectly informed consumers can benefit industry profits at the expense of consumer surplus and overall welfare. This is likely to be the case in industries with high advertising costs.

7 Conclusions

The economics literature on oligopoly price discrimination by purchase history is relatively new and has focused mostly on markets with perfectly informed consumers. With the exception of Esteves (2009a) and De Nijs (2013) the possibility of firms using advertising as a way to transmit relevant information to otherwise uninformed consumers has not been considered. The present article differs from the previous ones because we now consider duopoly competition with horizontal differentiation rather than duopoly competition with homogenous goods.

Our analysis challenges the traditional view that firms are worse off and consumer can be better off when firms have information to engage in behavior-based price discrimination practices. In comparison to no discrimination, we show that the impact of BBPD on profits and consumer surplus is the reverse of the profit and consumer welfare results derived in models with full informed consumers. In other words, we show that BBPD boosts industry profits and harms consumers.

Additionally, we also show that the permission of BBPD leads firms to strategically reduce their advertising efforts. Hence, the model predicts that firms are expected to benefit the most from behavior-based price discrimination practices in industries characterized by high advertising costs. The model also predicts that restrictions on advertising and industry lobby in favour of these restrictions in contexts where price discrimination is permitted would act in favour of profits at the expense of consumers.

In light of the above, this paper has tried to contribute to the ongoing debate on the economic implications of BBPD. For competition policy agencies, the profit and consumer results obtained highlight the importance of taking into account the different forms of market competition when evaluating the effects of price discrimination in competitive settings. This suggests that price discrimination strategies should not be considered in isolation. There are interactions between price discrimination and other marketing strategies such as the firms' advertising decisions that need to be taken into account. It is obvious that the specificity of each market plays an important role in the conclusions derived. A special limitation of the stylized model addressed in this paper is the assumption that advertising is the consumers' sole source of information. Although this assumption may at first sight seem odd in the context of online markets, it helped us to isolate the effects of price discrimination on the advertising decisions of firms. Evidently, while in new product markets this assumption might not be very restrictive, in other markets it might be inadequate. Allowing consumers to obtain information through advertising and costly search could be a natural extension, bringing new insights to the analysis.[20]

[20] Nevertheless it is important to stress that as along as there is some proportion of captive consumers we expect that our qualitative results should be obtained.

A Proofs

Proof of Proposition 1: Consider first the case of firm A. Its overall profit is equal to $\Pi_A = \pi_A^1 + \delta\pi_A^2$. As a remark notice that we are assuming the the first period cutoff is equal to

$$\theta^* = \frac{t\delta\left(2\phi_B - 2\phi_A + \phi_A\phi_B\right) + 3\phi_A\phi_B\left(t - p_A^1 + p_B^1\right)}{2t\phi_A\phi_B\left(\delta+3\right)}.$$

The derivative of overall profit with respect to p_A^1 can be written as:

$$\frac{d\pi_A^1}{dp_A^1} + \frac{d\pi_A^2}{d\theta^*}\frac{d\theta^*}{dp_A^1} = 0$$

where

$$\frac{d\pi_A^1}{dp_A^1} = \frac{3t\phi_A + t\delta\phi_B - \frac{3}{2}t\phi_A\phi_B - 3\phi_A\phi_B p_A + \frac{3}{2}\phi_A\phi_B p_B - \frac{1}{2}t\delta\phi_A\phi_B}{t\left(\delta+3\right)}$$

$$\frac{d\pi_A^2}{d\theta^*} = \frac{8}{9}t\phi_A - \frac{8}{9}t\phi_B - \frac{10}{9}t\phi_A\phi_B + \frac{20}{9}t\theta^*\phi_A\phi_B$$

$$\frac{d\theta^*}{dp_A^1} = -\frac{3}{6t+2t\delta}.$$

From $\frac{d\pi_A^1}{dp_A} + \frac{d\pi_A^2}{d\theta^*}\frac{d\theta^*}{dp_A} = 0$ it follows that:

$$\begin{aligned} 0 \;=\; & \frac{1}{t\left(\delta+3\right)}\left(3t\phi_A + t\delta\phi_B - \frac{3}{2}t\phi_A\phi_B - 3\phi_A\phi_B p_A + \frac{3}{2}\phi_A\phi_B p_B - \frac{1}{2}t\delta\phi_A\phi_B\right) \\ & - \frac{1}{t\left(\delta+3\right)^2}\left(4t\phi_A - 4t\phi_B - 2t\delta\phi_A + 2t\delta\phi_B - 5\phi_A\phi_B p_A + 5\phi_A\phi_B p_B\right) \end{aligned}$$

and so, firm A's best-response price function with respect to p_B^1 is:

$$p_A^1 = \frac{\left(1+\delta\right)t\left(10\phi_A + 2\delta\phi_B\right) + t\phi_B\left(8 - 9\phi_A\right) - t\delta\phi_A\phi_B(\delta+6) + \phi_A\phi_B p_B^1\left(3\delta - 1\right)}{\phi_A\phi_B\left(8+6\delta\right)}.$$

Looking now at the FOC with respect to ϕ_A, $\frac{\partial\Pi_A}{\partial\phi_A} = 0$ we have:

$$\frac{d\pi_A^1}{d\phi_A} + \frac{d\pi_A^2}{d\theta^*}\frac{d\theta^*}{d\phi_A} = 0.$$

Using the fact that

$$\frac{d\pi_A^1}{d\phi_A} = -\frac{p_A\left(3\phi_B\left(t + p_A^1 - p_B^1\right) - 6t + t\delta\phi_B\right)}{6t+2t\delta} - A_{\phi_A},$$

$$\frac{d\theta^*}{d\phi_A} = -\frac{\delta}{\phi_A^2\left(\delta+3\right)}$$

and

$$\frac{d\pi_A^2}{d\theta^*}\frac{d\theta^*}{d\phi_A} = \frac{2\delta}{3\phi_A^2\left(\delta+3\right)^2}\left(2t\left(2-\delta\right)\left(\phi_B-\phi_A\right)+5\phi_A\phi_B\left(p_A^1-p_B^1\right)\right).$$

it is straightforward to find firm A's best-response with respect to ϕ_j :

$$A_\phi\left(\phi_A\right) = \frac{2\delta\left(2t\left(\phi_A-\phi_B\right)\left(\delta-2\right)+5\phi_A\phi_B\left(p_A^1-p_B^1\right)\right)}{3\phi_A^2\left(\delta+3\right)^2} - \frac{p_A^1\left(-6t+\phi_B\left(3t+3p_A^1-3p_B^1+t\delta\right)\right)}{2t\left(\delta+3\right)}.$$

Symmetric expressions hold for firm B's best-response functions. Since we are looking for a symmetric equilibrium it must be the case that $p_A^1 = p_B^1 = p^1$ and $\phi_A = \phi_B = \phi^*$. It follows that

$$p^1 = t(1+\frac{\delta}{3})\left(\frac{2-\phi^*}{\phi^*}\right)$$

and

$$A_\phi\left(\phi^*\right) = \frac{p^1\left(6-\phi^*\left(3+\delta\right)\right)}{2\left(\delta+3\right)}$$

therefore,

$$A_\phi\left(\phi^*\right) = \frac{t\left(2-\phi^*\right)\left(6-\phi^*\left(3+\delta\right)\right)}{6\phi^*}$$

which simplifies to

$$A_\phi\left(\phi^*\right) = \frac{t\left(2-\phi^*\right)^2}{2\phi^*} - \frac{\delta t\left(2-\phi^*\right)}{6} \tag{35}$$

Next prove (ii). Replacing $\theta^* = \frac{1}{2}$and using the fact that $\phi^* = \phi_A = \phi_B$ and taking that $p_A^O = p_B^O = p^O$ and $p_A^R = p_B^R = p^R$ it is straightforward to find that

$$p^O = \frac{2t}{3}\left(\frac{2-\phi^*}{\phi^*}\right) \text{ and } p^R = \frac{t}{3}\left(\frac{2-\phi^*}{\phi^*}\right).\blacksquare$$

Proof of Proposition 2: As first period price decisions are valid for the two periods assuming that each firm discounts future profits using a common discount factor, $\delta \in (0,1)$, firm i profit is equal to:

$$\pi_i = (1+\delta)\left[p_i\phi_i\left(1-\phi_j\right)+p_i\phi_i\phi_j\left(\frac{p_j-p_i+t}{2t}\right)\right] - A\left(\phi_i\right)$$

In period 1 firms simultaneously choose prices and advertising levels. Each firm goal is to solve the following maximization problem:

$$\max_{p_i,\phi_i}\left\{(1+\delta)\left[p_i\phi_i\left(1-\phi_j\right)+p_i\phi_i\phi_j\left(\frac{p_j-p_i+t}{2t}\right)\right] - A\left(\phi_i\right)\right\}.$$

From the FOC we obtain:

$$p_i = \frac{1}{2\phi_j}\left(2t - t\phi_j + \phi_j p_j\right) \tag{36}$$

we obtain the FOC with respect to ϕ_i :

$$p_i\left(1+\delta\right)\left(1-\phi_j+\phi_j\left(\frac{p_j-p_i+t}{2t}\right)\right) = A_\phi(\phi_i). \tag{37}$$

Given the symmetry of the model, from equation (36) we obtain

$$p^{nd} = \frac{t\left(2-\phi^{nd}\right)}{\phi^{nd}},$$

and the equilibrium level of advertising is implicitly defined by

$$\frac{\left(1+\delta\right)t\left(2-\phi^{nd}\right)^2}{2\phi^{nd}} = A_\phi(\phi^{nd}),$$

Equilibrium profits are

$$\Pi^{nd} = \left(1+\delta\right)\frac{t\left(2-\phi^{nd}\right)^2}{2} - A\left(\phi^{nd}\right).$$

This completes the proof.■

Proof of Proposition 4: Given the advertising equilibrium solutions with no discrimination and with BBPD, respectively equal to:

$$A_\phi\left(\phi^{nd}\right) = \frac{t\left(\delta+1\right)\left(2-\phi^{nd}\right)^2}{2\phi^{nd}} \tag{38}$$

$$A_\phi\left(\phi^*\right) = \frac{t\left(2-\phi^*\right)^2}{2\phi^*} - \frac{\delta t\left(2-\phi^*\right)}{6} \tag{39}$$

it follows that the right hand side of both expressions is decreasing in ϕ and both would be equal when $\phi \in \{2,3\}$, which is not possible given the domain of ϕ. Thus, it is straightforward to see that for $\phi \in [0,1]$ the right-hand side of equation (38) is always higher than the right hand side of equation (39). Since the left hand side of both expressions is the same, then it is always the case that $\phi^* < \phi^{nd}$. This completes the proof of part (i), i.e., that $\phi^{nd} > \phi^*$.[21]

[21] For the quadratic technology ϕ^{nd} is defined as long as $a \geq t$, while ϕ^* is defined when $a \in R\backslash\left\{\frac{2t}{3}\right\}$. As long as $a \geq t$ is is always the case that $\phi^{nd} > \phi^*$.

Now we look at the behavior of the second period prices, when we move from no discrimination to discrimination. Notice that from proposition 1 we have $p^O = \frac{2t}{3}\left(\frac{2-\phi^*}{\phi^*}\right)$ and $p^R = \frac{t}{3}\left(\frac{2-\phi^*}{\phi^*}\right)$. From proposition 2 we have $p^{nd} = t\left(\frac{2-\phi^{nd}}{\phi^{nd}}\right)$. From $p^O < p^{nd}$ we obtain:

$$\frac{2t}{3}\left(\frac{2-\phi^*}{\phi^*}\right) < t\left(\frac{2-\phi^{nd}}{\phi^{nd}}\right)$$

We already know from part (i) that $\phi^{nd} > \phi^*$. Thus it is always true

$$\begin{aligned}
\frac{2t}{3}\left(\frac{2-\phi^*}{\phi^*}\right) &< t\left(\frac{2-\phi^{nd}}{\phi^{nd}}\right) < t\left(\frac{2-\phi^*}{\phi^*}\right) \\
\left(\frac{2-\phi^*}{\phi^*}\right)\left(\frac{1}{3}-1\right) &< 0.
\end{aligned}$$

Since $\phi^* \in]0,1]$ the previous expression is always negative. This proves that $p^O < p^{nd}$. Similarly, we can prove that $p^R < p^{nd}$. We obtain that

$$\begin{aligned}
\frac{t}{3}\left(\frac{2-\phi^*}{\phi^*}\right) &< t\left(\frac{2-\phi^{nd}}{\phi^{nd}}\right) < t\left(\frac{2-\phi^*}{\phi^*}\right) \\
\left(\frac{2-\phi^*}{\phi^*}\right)\left(\frac{1}{3}-1\right) &< 0.
\end{aligned}$$

Since $\phi^* \in]0,1]$ the previous expression is always negative. This completes the proof that $p^R < p^{nd}$. Now we prove that $p^1 > p^{nd}$. Using the fact that $p^1 = t(1+\frac{\delta}{3})\left(\frac{2-\phi^*}{\phi^*}\right)$ and $p_1^{nd} = t\left(\frac{2-\phi^{nd}}{\phi^{nd}}\right)$ consider as an hypothesis that $p^1 < p^{nd}$. We have that

$$\begin{aligned}
t(1+\frac{\delta}{3})\left(\frac{2-\phi^*}{\phi^*}\right) &< t\left(\frac{2-\phi^{nd}}{\phi^{nd}}\right) < t\left(\frac{2-\phi^*}{\phi^*}\right) \\
\left(\frac{2-\phi^*}{\phi^*}\right)(1+\frac{\delta}{3}-1) &< 0 \\
\left(\frac{2-\phi^*}{\phi^*}\right)\frac{\delta}{3} &< 0.
\end{aligned}$$

Since $\phi^* \in]0,1]$ and $\delta > 0$ then the previous inequality is always false. This it is always true that $p^1 > p^{nd}$ for any $\phi \in]0,1]$.■

Proof of Proposition 5: Each firm overall profit with and without discrimination is respectively equal to:

$$\begin{aligned}
\Pi^* &= \frac{t}{18}(2-\phi^*)^2(8\delta+9) - A(\phi^*) \\
\Pi^{nd} &= \frac{t}{2}\left(2-\phi^{nd}\right)^2(1+\delta) - A\left(\phi^{nd}\right)
\end{aligned}$$

From $\Pi^* > \Pi^{nd}$ it follows that:

$$\underbrace{A\left(\phi^{nd}\right) - A\left(\phi^*\right)}_{(+)} > \frac{t}{2}\left(2-\phi^{nd}\right)^2(1+\delta) - \frac{t}{18}\left(2-\phi^*\right)^2(8\delta+9).$$

Taking into account that $\phi^{nd} > \phi^*$, it is always true $A\left(\phi^{nd}\right) - A\left(\phi^*\right) > 0$. Therefore, if $\frac{t}{2}\left(2-\phi^{nd}\right)^2(1+\delta) - \frac{t}{18}\left(2-\phi^*\right)^2(8\delta+9) < 0$ the previous condition is always true. This yields:

$$\begin{aligned}\left(\frac{2-\phi^n}{2-\phi^*}\right)^2 &< \frac{8\delta+9}{9\delta+1}\\ \frac{2-\phi^n}{2-\phi^*} &< \sqrt{\frac{8\delta+9}{9\delta+1}}\end{aligned}$$

As $\phi^{nd} > \phi^*$ it is always true that $\left(\frac{2-\phi^n}{2-\phi^*}\right) < 1$. On the other hand, for $0 < \delta < 1$, it is straightforward to see that $\sqrt{\frac{8\delta+9}{9\delta+1}} > 1$. Therefore, $\frac{2-\phi^n}{2-\phi^*} < \sqrt{\frac{8\delta+9}{9\delta+1}}$ is always true. This completes the proof that $\Pi^* > \Pi^{nd}$.$\blacksquare$

Proof of Proposition 6: From

$$CS^* = 2v\phi^*\left(2-\phi^*\right) + \frac{2}{9}t\left(-2\phi^{*2} + 26\phi^* - 35\right)$$

and

$$CS^{nd} = 2v\phi^{nd}\left(2-\phi^{nd}\right) + \frac{t}{2}\left(-\phi^{nd^2} + 12\phi^{nd} + 16\right)$$

it follows that $CS^* < CS^{nd}$:

$$\underbrace{2v\phi^*\left(2-\phi^*\right)}_{(1)} + \underbrace{\frac{2}{9}t\left(-2\phi^{*2} + 26\phi^* - 35\right)}_{(2)} < \underbrace{2v\phi^{nd}\left(2-\phi^{nd}\right)}_{(3)} + \underbrace{\frac{t}{2}\left(-\phi^{nd^2} + 12\phi^{nd} + 16\right)}_{(4)}$$

Note that $\phi(2-\phi)$ is an increasing function of ϕ, $\forall\phi \in [0,1]$. Additionally as $v > 0$ and $\phi^{nd} > \phi^*$, then $2v\phi^{nd}\left(2-\phi^{nd}\right) > 2v\phi^*\left(2-\phi^*\right)$. Thus $(3) > (1)$. Compare now (2) and (4). Suppose that $(2) < (4)$, this yields

$$\frac{2}{9}t\left(-2\phi^{*2} + 26\phi^* - 35\right) < \frac{t}{2}\left(-\phi^{nd^2} + 12\phi^{nd} + 16\right)$$

It is straightforward to see that $\forall\phi^*, \phi^{nd} \in [0,1]$, the left-hand side expression is always negative while the right-hand side is always positive. Thus, it is always true that $(2) < (4)$, implying that $(1) + (2) < (3) + (4)$. This completes the proof that $CS^* < CS^{nd}$.$\blacksquare$

References

Armstrong, M. (2006), "Recent Developments in the Economics of Price Discrimination," *in Advances in Economics and Econometrics: Theory and Applications: Ninth World Congress of the Econometric Society,* ed. by R. Blundell, W. Newey, and T. Persson. Cambridge University Press, Cambridge, UK.

Butters, G. (1977), "Equilibrium Distribution of Sales and Advertising Prices." *Review of Economic Studies*, 44, 465-491.

Chen, Y. (1997), "Paying Customers to Switch." *Journal of Economics and Management Strategy,* 6, 877-897.

Chen, Y., C. Narasimhan and Zhang, Z. (2001), "Individual marketing with imperfect targetability." *Marketing Science,* 20, 23-41.

Chen, Y. (2005), "Oligopoly Price Discrimination by Purchase History." In: *The Pros and Cons of Price Discrimination*, The Swedish Competition Authority, Stockholm, pp. 101-130.

Chen, Y. and Iyer, G. (2002), "Consumer Addressability and Customized Pricing." Marketing Science, 21, 197-208.

Chen, Y. and Pearcy, J. (2010), "Dynamic Pricing: when to Entice Brand Switching and when to Reward Consumer Loyalty," *Rand Journal of Economics*, 41, 674-685.

Chen, Y., and Zhang, Z. (2009), "Dynamic Targeted Pricing with Strategic Consumers." *International Journal of Industrial Organization*, 27, 43-50.

Colombo, S. (2016), "Imperfect behaviour-based price discrimination." *Journal of Economics and Management Strategy,* forthcoming.

Corts, K. (1998), "Third-Degree Price Discrimination in Oligopoly: All-Out Competition and Strategic Commitment." *RAND Journal of Economics*, 29, 306-323.

De Nijs, R. (2013), "Information Provision and Behaviour-Based Price Discrimination." *Information Economics and Policy,* 25, 32-40.

Esteves, R.B. (2009a), "Customer Poaching and Advertising." *Journal of Industrial Economics*, vol 57 (1), 112-146.

Esteves, R.B. (2009b), "A Survey on the Economics of Behaviour-Based Price Discrimination," Working paper, WP-NIPE 5/2009.

Esteves, R.B. (2010), "Pricing with Customer Recognition," *International Journal of Industrial Organization*, 28, 669-681.

Esteves, R.B. and Reggiani, C. (2014), "Elasticity of Demand and Behaviour-Based Price Discrimination," *International Journal of Industrial Organization,* 32, 46-56.

Esteves, R.B. (2014a). "Price discrimination with private and imperfect information." *Scandinavian Journal of Economics*, 116, 766-796.

Esteves, R.B. (2014b), "Behaviour-Based Price Discrimination with Retention Offers," *Information Economics and Policy*, vol 27, 39-51.

Esteves, R.B. and Vasconcelos, H. (2015), "Price Discrimination under Customer Recognition and Mergers," *Journal of Economics and Management Strategy,* 24, 523–549.

Fudenberg, D. and Tirole, J. (2000), "Customer Poaching and Brand Switching." *RAND Journal of Economics,* 31, 634-657.

Fudenberg, D. and Villas-Boas, M. (2007), "Behaviour-Based Price Discrimination and Customer Recognition," In *Handbook on Economics and Information Systems*, ed. by T. Hendershott. North-Holland, Amsterdam.

Grossman, G., and Shapiro, C. (1984), "Informative Advertising with Differentiated Products." *Review of Economic Studies*, 51, 63-81.

Ireland, N. (1993), "The Provision of Information in a Bertrand Oligopoly." *The Journal of Industrial Economics*, 41, 61-76.

Iyer, G., Soberman, D. and Villas-Boas, M. (2005), "The Targeting of Advertising." *Marketing Science,* 24, 461-476.

Liu, Q. and Serfes, K. (2004), "Quality of Information and Oligopolistic Price Discrimination." *Journal of Economics and Management Strategy,* 13, 671–702.

McAfee, R. P. (1994). "Endogenous Availability, Cartels, and Merger in an Equilibrium Price Dispersion." *Journal of Economic Theory*, 62, 24-47.

Roy, S. (2000), "Strategic Fragmentation of a Market." *International Journal of Industrial Organization,* 18, 1279-1290.

Stahl, D. (1994), "Oligopolistic Pricing and Advertising." *Journal of Economic Theory*, 64, 162-177.

Taylor, C. (2003), "Supplier Surfing: Competition and Consumer Behavior in Subscription Markets." *RAND Journal of Economics*, 34, 223-246.

Thisse, J., and Vives, X. (1988), "On the Strategic Choice of Spatial Price Policy." *American Economic Review,* 78, 122-137.

Tirole, J. (1988), *The Theory of Industrial Organization.* Cambridge. The MIT Press.

Villas-Boas, M. (1999), "Dynamic Competition with Customer Recognition." *RAND Journal of Economics,* 30, 604-631.

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