An application to Galicia pollution data and a model for preferential sampling

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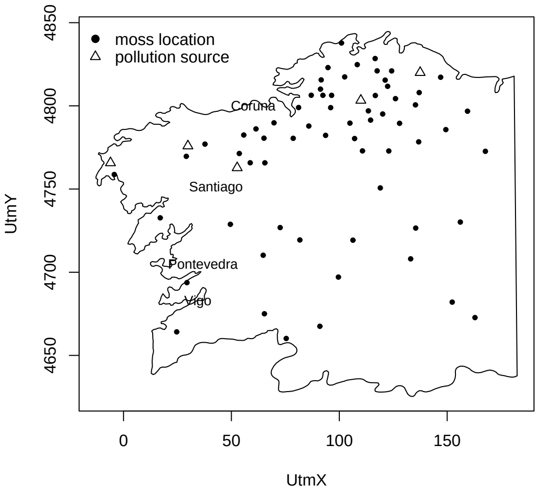
1995 Galicia pollution data - Cr, Ni and Pb



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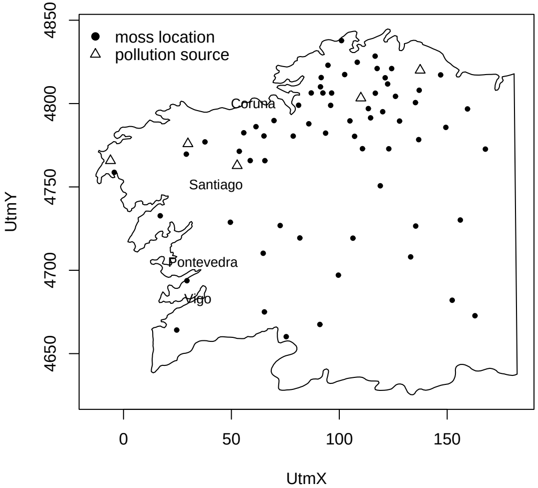
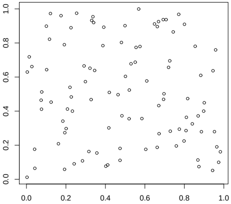


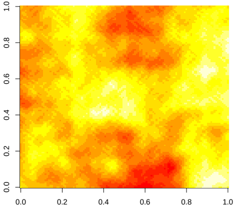
Formal tests of Monte Carlo point to rejection of $H0$: non-PS for Ni and Pb.

The traditional geostatistical model

observed data $(y, \mathbf{x})$



unobserved data s



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$y_1, \dots, y_n$ are measurements associated with these locations

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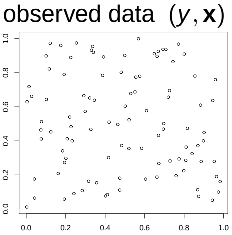

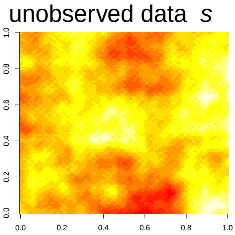

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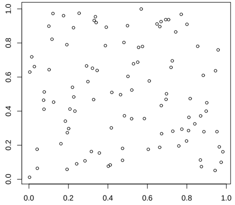

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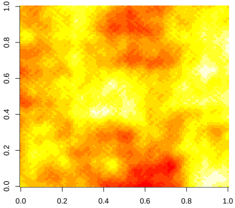

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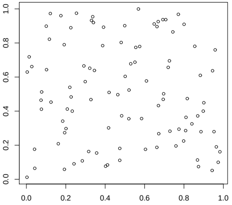

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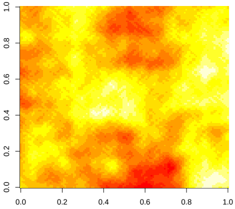

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2. Point process P of sample locations $\mathbf{x}$
3. Measurement process $Y \equiv Y(\mathbf{x_i})$ (noisy version of S)

Motivation

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What happens if these assumptions fail?

- preferability issue
- clustering issue

A variogram robust to clusters

$$2\widehat{\gamma}(u) = \frac{\sum_i \sum_j w_{ij}(u)[Y(\mathbf{x_i}) - Y(\mathbf{x_j})]^2}{\sum_i \sum_j w_{ij}(u)}$$

- $w_{ij}(u) = \frac{1}{\sqrt{n_i \times n_j}} \times K\left(\frac{u - \|\mathbf{x_i} - \mathbf{x_j}\|}{h}\right)$, where $n_i = \sum_k I_{\{\|\mathbf{x_i} - \mathbf{x_k}\| \leq \delta\}}$
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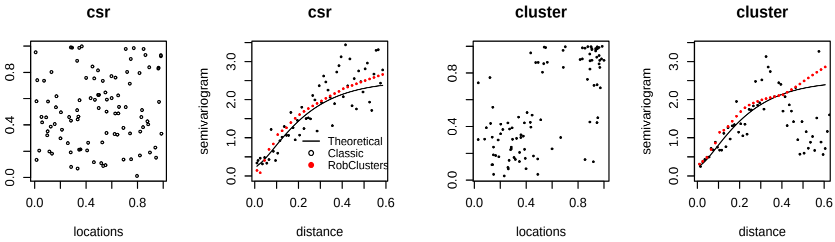

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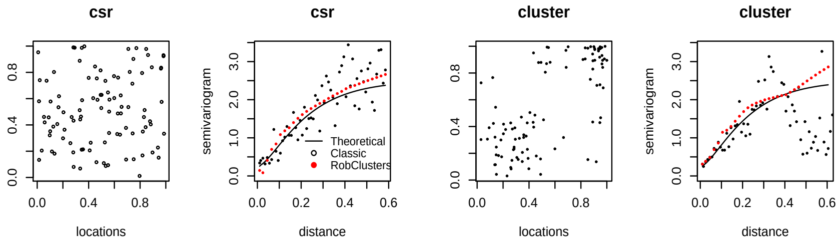

This new variogram is proven to enjoy good properties, such as asymptotic unbiasedness and consistency.

Model-based approach for preferential sampling

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Log-Gaussian Cox processes – $[P \mid S]$

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1. Cox process – *Diggle(2003)*:
   Useful to model aggregated spatial point patterns where the aggregation is due to a stochastic environmental heterogeneity.
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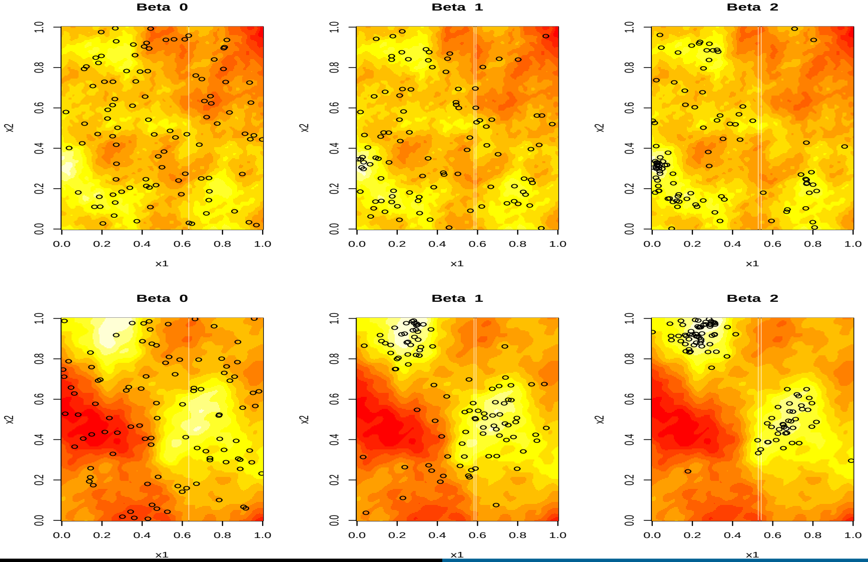
$$P \mid S \sim \mathit{Poisson}\,(\exp\{\alpha + \beta\, S(\mathbf{x})\})$$

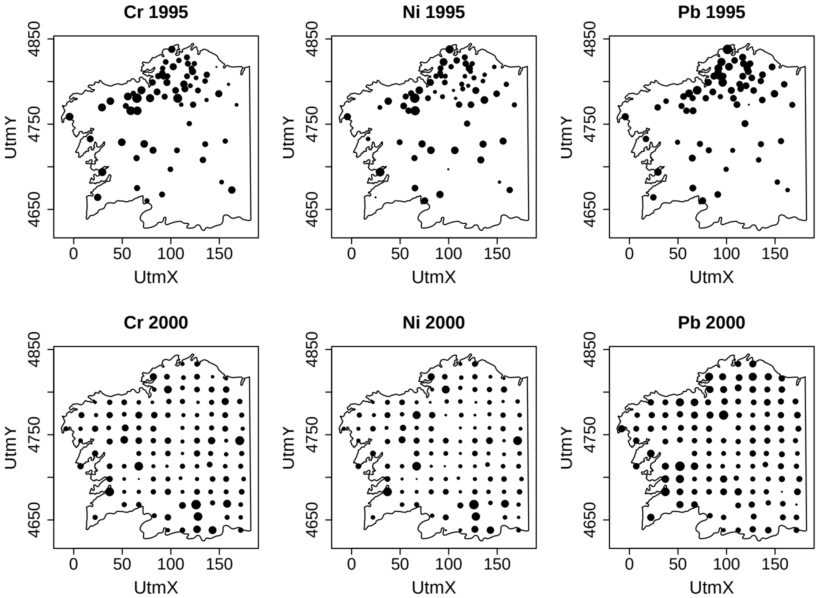
where $|\beta|$ identifies the *degree of "preferability"*, i.e., it measures how much the "intensity" depends on S

Influence of β on sample locations ($\alpha = 0$)

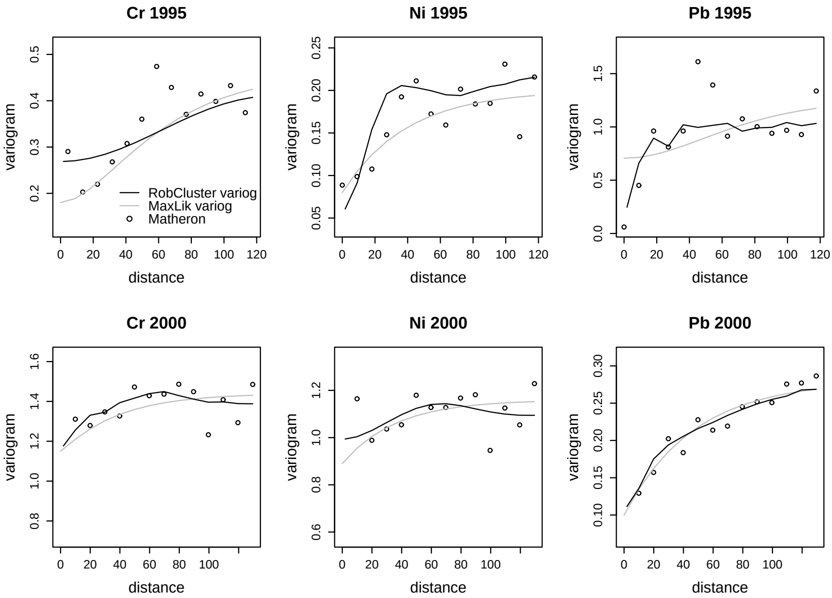
Given two distinct Gaussian fields $S(.)$



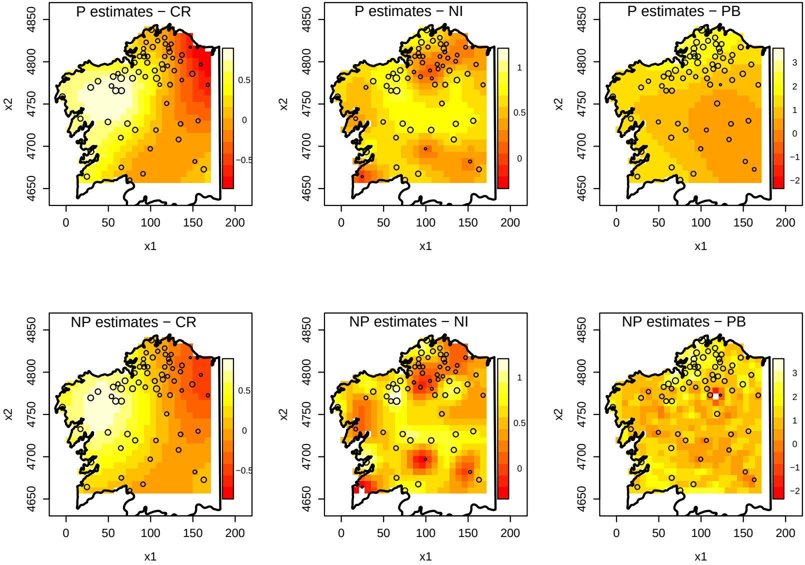
1995 versus 2000 Galicia pollution data



Variogram estimation for Cr, Ni and Pb data



1995 kriging — Parametric versus Non-Parametric



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Fundamental idea behind CV:

- estimate the concentration measurement $Y(\mathbf{x})$ at each sample point $\mathbf{x_i}$ from neighbouring data $Y_j = Y(\mathbf{x_j}),\ j \neq i,$ as if $Y_i = Y(\mathbf{x_i})$ were unknown.

$$MSE = \frac{1}{n}\sum_{i=1}^{n}(Y_i - \widehat{Y}_{-i})^2 \quad \text{and} \quad MSSE = \frac{1}{n}\sum_{i=1}^{n}\frac{(Y_i - \widehat{Y}_{-i})^2}{\widehat{\sigma^2_{-i}}}$$

Main conclusions

		MSE			MSSE		
		Cr	Ni	Pb	Cr	Ni	Pb
1995	P	0.265	0.187	0.798	1.167	1.292	0.996
	NP	0.247	0.172	0.801	0.837	1.305	1.032
2000	P	1.738	1.331	0.217	1.036	1.028	1.012
	NP	1.672	1.276	0.217	0.974	0.978	0.986

With respect to MSE, smaller values are normally associated with the NP approach, reinforcing the advantages of the NP variogram.

Note that MSE is sensible to data poorly *explained* by their neighbors, which possibly explains higher values for Cr and Ni in 2000.

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With respect to MSSE, note that these values are closer to 1 in 2000. We think such results, mainly those for Ni, support the need for a model-based approach to preferential sampling.