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Young children working with geometric figures

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Abstract
This paper focuses on 20 young children spatial sense (3-4-years-old) from a public kindergarten, in Braga, Portugal. Three questions were addressed: 1) How do children understand some geometric figures? 2) How is children's spatial sense characterized, concerning geometric figures? 3) What specific geometry vocabulary do children learn? Qualitative methods using a case study approach were used to describe children's reactions when solving 10 tasks involving geometric figures and properties of these figures. Results suggests that children's ideas of particular figures can be improved when specific tasks and materials are used. These aspects are explained in the paper.

Key words: Geometric figures, kindergarten education, mathematics.

Theoretical framework

Mathematics is in our life and part of it, it helps us organize and look critically at the world around us. In addition, in order to reduce failure in the mathematics achievement, it is necessary to develop the "taste for mathematics and the rediscovery of mathematical relations and facts" (Bivar, Grosso, Oliveira & Timóteo, 2013, p. 2) since the early years of school.

Children's first experiences are of geometrical nature, when trying to understand the surrounding world, distinguishing an object from another, and when exploring the proximity of an object (Abrantes, Serrazina & Oliveira, 1999, p. 70). Particularly, official guiding documents for kindergarten such as those of the *Direção Geral de Inovação e Desenvolvimento Curricular* [DGIDC] (2010), also Silva, Marques, Mata and Rosa (2016), and Silva, Bacelar and Guimarães (1997) state that mathematics is present in children's play, in space and in time. Therefore, it is up to the teacher to question, encourage, provide, organize and combine meaningful materials and experiences to build ideas on the content area (DGIDC, 2010). For example, identify similarities and differences between objects and group them according to different criteria; understand that figure names (square, triangle, rectangle, and circle) apply regardless of their position or size.

Moreover, the Curricular Guidelines for Kindergarten Education allude to the need of the playful nature of the pedagogical process. The general principles of the document support the idea that this process should respect and value the physical characteristics of the child, their differences, and constitute a basis for new learning (Silva, Marques, Mata & Rosa, 2016). The use of games can assume a relevant role in this process. According to Dallabona and Mendes (2004), the seriousness and importance of contents to work with children is not lost when games are used, as play activities are indispensable for their development.

Specifically, the study of forms in space and spatial relationships is important in that it helps children relate mathematics to the real world (Abrantes et. al., 1999). Jones (2002) emphasizes the use of geometry in several areas of our everyday life: art, architecture, music involving geometric principles - symmetry, perspective, scale and orientation. Besides that, Jones (2002) confirms the idea that spatial reasoning is important in other curricular areas such as science, art, and technology. Developing this type of reasoning, different areas of knowledge can be connected, acting in an interdisciplinary way. Thus, one needs to know a bit more about the spatial reasoning of young children.

Piaget pioneered research into how young children learn about space and form. Piaget and Inhelder (1956) believed that the first notions of space are topological (perceiving open and closed relationships), and that only later children construct projective notions (relations between children and objects, points of view), euclidean notions and coordinates of space. Dina and Peter van Hiele studied the learning process of space and forms, and distinguished five levels of learning of geometry: Level I - referred to as visual level (visualization); Level II - descriptive level (analysis); Level III - that of logical relations and where geometry is included according to Euclid (Informal ordering / deduction); Level IV - studies the laws of logic (deduction); Level V - nature of the laws of logic (rigor) (van Hiele, 1986). Van Hiele's model suggests that students advance through thought levels in geometry.

In the study of children’s acquisition of geometric concepts and the development of geometrical thought, the van Hiele consider that the progress through the five levels is more dependent on instruction than on age or maturation. According to Hatfield, Edwards and Bitter (1997), the first three levels should focus on experiences in informal geometry during the elementary and middle school years. In Level I - referred to as visual level (visualization) - the child reasons about basic geometric concepts, such as simple shapes, by means of visual considerations. The child reacts to figures as a whole (for example, a square is a square because it looks like one). In Level II - descriptive level (analysis) – the child reasons about the geometric concepts by means of an informal analysis of the parts and attributes of relationships among the parts of a figures (for example, a square is a square because it has four equal sides and four right angles). In Level III - that of logical relations and where geometry is included according to Euclid (Informal ordering / deduction) – the child logically orders the properties of concepts, forms abstract definitions, and can distinguish between the necessity and sufficiency of a set of proprieties in determining a concept (for example, the definition of a square is dependent on properties that are related to other shapes – a square can be both a rectangle and a parallelogram). The kindergarten is the first moment in which informal geometry can be approached with the intentionality of prompting children’s geometrical thought.

Van Hiele (1986) asserted that children should have a wide variety of exploratory experiences to develop their spatial sense. To improve children's spatial sense in kindergarten, one must challenge them with activities that imply contact with manipulatives so that they can reflect on their experience. In this way, "children create dynamic mental images, expanding their repertoire and becoming accustomed to relate spatial knowledge to verbal and analytical knowledge" (Moreira & Oliveira, 2003, p. 99). Thus, it is essential to provide kindergarten children contact with activities that promote the development of their spatial sense.

This paper presents part of a research developed to understand the spatial sense of kindergarten children, namely their ideas about geometric figures. It addresses three central questions: 1) How do children understand some geometric figures? 2) How is children's spatial sense characterized, concerning geometric figures? 3) What specific geometry vocabulary do children learn?

Methods

Qualitative methods were used (Bogdan & Bicklen, 1991) in a case study approach (Yin, 1994) to have an insight of children's ideas of geometric figures. The participants were a group of 20 children (3 to 4 years-old) who attended a public kindergarten, in Braga, Portugal.

Children's reactions were observed in an intervention program focused on the identification of properties of geometric figures, and problem solving with these figures. The intervention program comprised 31 sessions, each lasting approximately 60 minutes, in which children were challenged to solve 10 tasks. All the tasks were related to geometric figures (squares, rectangles, triangles and circle).

During the intervention program children had the opportunity to develop their knowledge about geometric figures. In this program, children were given the opportunity to manipulate the geoboard, to prick paper with geometrical shapes (circles and triangles), and paper folding to produce squares and triangles, being stimulated to talk about their experiences, improving their mathematical communication. In task 1, children heard a story involving geometric shapes and talked about it in large group, referring to what they had learned with the story. In the following two tasks, children discovered the geometric figures in the "game of the elastic" and in the "game of the monkey". In the "game of rooster", children did pecking and built a board with the pieces of the game. In task 5, they saw a movie in which the houses were made of geometric figures (square, rectangle, circle, triangle) with a song about different geometric figures. In the end, children talked about it, developing their mathematical communication. In tasks 6 and 7, children continued to work with geometric figures, recognizing them in different positions, reproducing them in their geoboards, and then in the dotted paper. Children also built figures in the geoboard under some given conditions, such as "build a square with a stitch in the middle" or "build a square without stitches inside", and copy them to the dotted paper. In task 9, children used the Tangram to fill in a contoured picture, paving it. In task 10, the geoboard and the paper were used again to register some eventual improvement of children in solving problems with geometric figures.

During the intervention, children had moments of work in a large and small group, individually and in pairs. Through all these moments, children were supported by the researcher, giving them the opportunity to learn by action, challenging them to solve the problems by their own. In all the moments, children were free to interact with each other and with the researcher, one of the authors of this paper.

Data collection was carried out using photos, video records of children's performance when solving the tasks. Field notes were used by the researcher.

Results

In Task 1, the story "In the Land of Geometric Figures" (Mendes & Guedes, 2007), related to geometric figures, was read to children. At the end, children recounted it and looked for geometric figures (square, triangle, circle, rectangle) in the room. Then they registered by drawing and skirting the objects, when working in large group. In convergence with Matos and Gordo (1993), this activity helped to develop perceptual constancy as children had to recognize geometric figures in different positions, sizes, contexts and textures. As some children revealed difficulties recognizing geometric shapes, a song about geometric properties was brought to the room. With this song, children reviewed the circle and the number of sides of squares, rectangles and triangles.

In Tasks 2 and 3 games were used to explore the shape of the geometric figures, with the children. They played the "game of elastic" in large group (Figure 1), changing positions and changing figures (from a square to a rectangle, converting a square into a triangle), they also built and played the "game of the monkey" in large group (Figure 2), identifying squares and rectangles drew in the floor. During these tasks, children were asked to identify the geometric figures involved in the games. After these tasks, they were challenged to spontaneously initiate the discovery of these shapes during their daily routine and, for instance, by lunch time, squares were identified by them in the napkins in refectory.

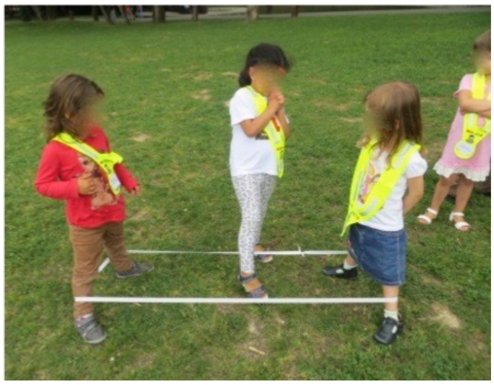


Figure 1. Children playing the "game of elastic".

Figure 2. Children playing the "game of the monkey".

Task 4 comprised the "game of rooster", made by children and played in pairs (Figure 3). In this game, children worked the perceptual constancy, as they identified with justification, the geometric figures (square, rectangle, triangle, circles), communicating mathematically. Then they painted them, working the visual motor coordination. Also in this task, they were required to perform the pecking, having to use their fine motor skills, that is, the ability to perform small movements with control and dexterity. This competence should be developed early, facilitating good results in writing and mathematics (McHale & Cermark, 1992). In the end, each child had his/ her own game of the rooster, and could play it with a colleague (Figure 3).

Figure 3. Build the game of rooster (pricking and painting of figures).

In the fifth task, the movie "The street of the forms", in which the houses were made of geometric figures (square, rectangle, circle, triangle) aimed to consolidate children's knowledge about geometric figures. The movie attracted children's attention and kept them motivated. In the end of the task, the group talked about the differences on shape of the squares, rectangles, circles and triangles. They also sang the song of the movie that talked about these geometric figures.

Task 6 aimed to address the concepts of 'inside and outside' and the properties of the geometric shapes, such as number of sides and size of the sides of rectangle and squares. The children were asked to recognize the relative position of a figure in relation to other. As suggested by the van Hiele's (1986) theory, children advance through levels of geometric thinking and the educator must conduct the following steps: information, directed orientation, explication, free orientation and integration. This was intended to help children with information, directed orientation and explication. In this sense, children were shown geometric figures in the geoboard (square and triangle), one inside the other, and they were asked to identify the figure and its relative position (inside and outside), still working the perception position (Figure 4). Transcription 1 shows a child's explanation when solving the task.

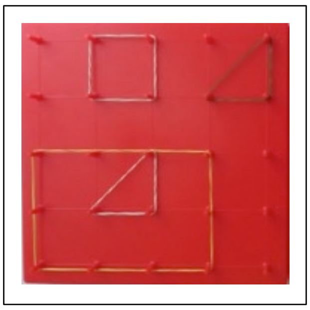


Figure 4. Problem presented in geoboard for large group discussion.

Researcher (R) - The green triangle is near by the rectangle
Child L – No, is not. It is near by the square.
R - What is the color of the triangle inside the rectangle?
Child L - White.
R – Is it right?
All children – Yes!!!
R - The square is inside the rectangle. [Repeat again]. Is it true or a lie?
Child V- No. It’s a lie!
R – So, what shape is inside the rectangle?
Child V - The triangle.
R - The square is outside the rectangle. Is it true or a lie?
Child O – It’s a lie.
R – So, where is the square?
Child O – [Stands up and points out to the square in the geoboard].
R - The green triangle is equal to the white triangle.
Child B – No!
R – Why not?
Child L - Because the white triangle is not equal to the green triangle.
R – Would you explain this better, please?!
Child L - Because one is green and the other is white.

Transcription 1. Large group discussion of the problem presented in the geoboard.

Transcription 1 shows that these children of 3- and 4-years-old are able to identify triangles, rectangles and squares, and are able to identify the relative position of the triangles and squares.

The task 7 aimed to understand if children could recognize geometric figures in different positions and sizes (perceptual constancy). An origami was built with the researcher’s help starting from a paper square sheet. Children were asked to identify this geometric figure. Then they were asked to fold it diagonally with the opposite vertices, making them to obtain a triangle. At this moment, the triangle was turned and children identified it by counting "1, 2, 3 little nozzles". The researcher asked for the name of those little nozzles, but only a few children answered and many had difficulty in saying “*vértice*”, the Portuguese word for vertex. After the folds, they had to draw: on one side a geometric figure, and on the other what they liked the most (Figure 5).

Figure 5. Drawings of triangles and rectangles made by two different children.

This task was a playful way of identifying and representing geometric figures, in a small piece of paper. This task with the origami allowed children to identify and represent the geometric figures in a small area and without any mesh (unlike the geometric figures in the dotted paper), but it was also an opportunity to draw and identify numbers. During this activity, all children were able to construct their game and most of them made accurate representations of the geometric figures (Figure 5). In addition, they all managed to copy the numbers (Figure 6), and one child even drew all the numbers on his/her own, almost perfectly.

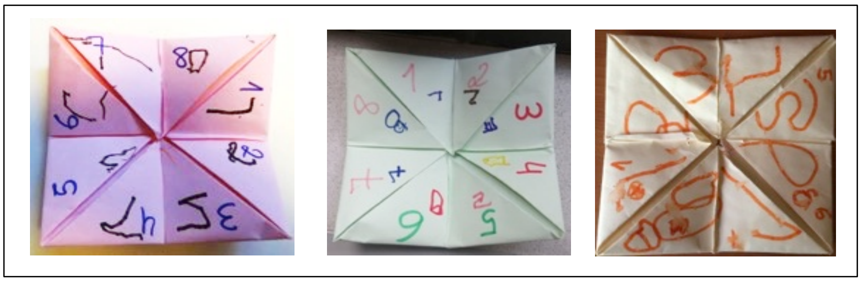


Figure 6. Example of written numbers produced by children.

The contrast between tasks 1 and 5 and tasks 6 and 7, in relation to the properties of the geometric figures (number of vertices, number of sides, four equal sides), allowed us to perceive that, as van Hiele (1986) suggested, a child who is at Level I (visual) is able to learn geometric vocabulary and can reproduce it. However, it does not recognize straight angles or parallel opposite sides, but is able to recognize rectangles, squares, and triangles by their appearance and position. Some of these children could recognize by experimentation, observation, measurement or drawing, that the opposite sides of a rectangle are equal, but still cannot perceive that the square is a special rectangle. There are also children who cannot reproduce geometric figures.

To solve problems with geometric figures, a geoboard was given to each child, and children were free to manipulate it. In task 8, children looked at the given figures represented on the geoboard and copy them on their own; they reproduced in their geoboard figures given on the dotted paper; build figures in the geoboard when conditions were given, such as "build a square with a stitch in the middle" or "build a square without stitches inside"), and copy them to the dotted paper. Some children found this task difficult, not responding to all requests. For example, one child had a lot of difficulties and could only accomplish the task on her geoboard after seeing the researcher doing it. Another child quickly identified the geometric figure and the number of stitches inside of it, but found difficult to manipulate the elastics in the geoboard. In relation to the condition of create a rectangle without stitches inside, there were children who did it quickly, after having constructed a square without stitches inside (Figure 7).

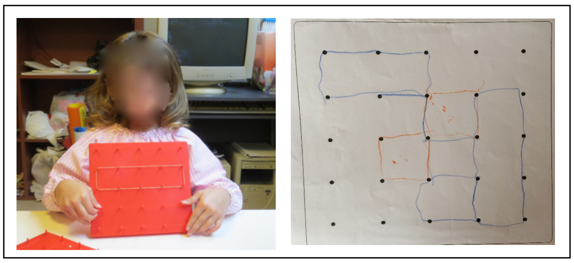


Figure 7. Child built a rectangle without stitches inside and reproduces it in dotted paper.

To build a square with a stitch in the centre was more difficult. Children performed differently, some children were quick to make proposals and others felt unable to do it. This aspect was evident in the geoboard reproductions; some children demonstrate not to perceive the way of drawing in the dotted paper (Figure 8), and others did it ignoring the location of the figure in the geoboard (Figure 9).

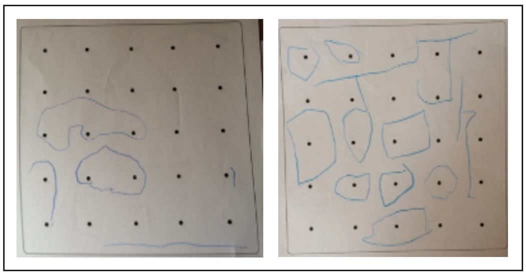


Figure 8. Absence of noticing how to draw geometric figures in dotted paper.

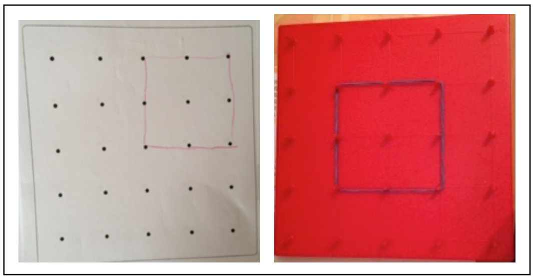


Figure 9. Child's reproduction on the dotted paper compared to the geoboard.

There were other children who represented a square occupying all points of the geoboard when they were shown the dotted paper. Throughout the week, the tasks were chosen to allow them to explore the geoboard longer before they were asked for the planned points. Children were enthusiastic about the task and they were quite absorbed by its resolution, without any distractions. With this task, perceptual constancy, visual motor coordination and visual memory were developed. Visual motor coordination is the "ability to coordinate vision with body movements" (Del Grande, 1990, p.14) and used it when drawing and painting figures on the dotted paper. In the early years, a child needs to make a great mental and motor effort to control their movements, and problem solving on the geoboard can help them to develop this ability by promoting hand-eye coordination. In this sense, and since there were observed several levels of performance among children of this study, it was important to promote the learning of the concepts of geometric figures and fine motricity, to improve their constructions on the geoboard and their representation in the dotted paper.

In task 9, the tangram pieces were given to children and they were asked to fill in a contoured picture - paving. In a second moment, a figure was given only with the outer line and they were asked to fill them with geometric figures seeking to develop the aforementioned capacities. The development of deep figure perception and visual discrimination are aspects that must be worked out during kindergarten education because they help to develop the spatial sense (Moreira & Oliveira, 2003). For Moreira and Oliveira (2003), perception lies deep in the child's ability to identify a given figure in a complex background, and can develop when the child is asked to make a construction with the tangram pieces. In turn, according to the same authors, the child has visual discrimination if is able to identify similarities or differences between objects, in this case, between geometric figures.

This task with the tangram surpassed expectations as all children were able to complete the contoured figures (Figure 10). With this task, they also had the opportunity to learn a new word in Portuguese "*paralelogramo*" - parallelogram – but found it difficult. As the best learning is practice in which learners hear and identify the signifier and meaning of the word, this was also an important moment to develop oral language.

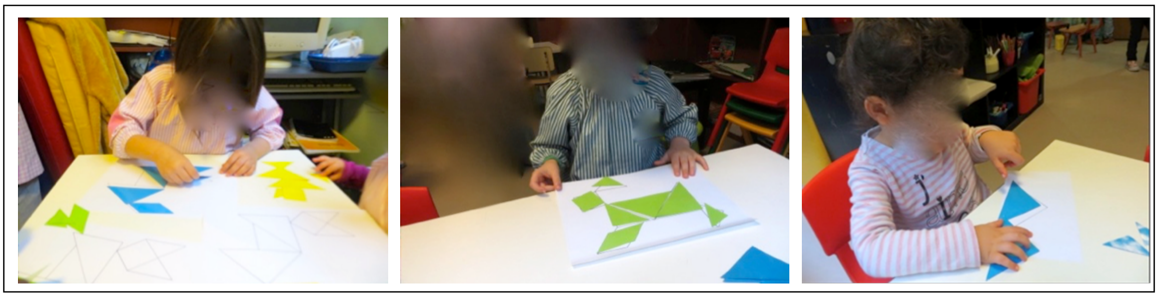
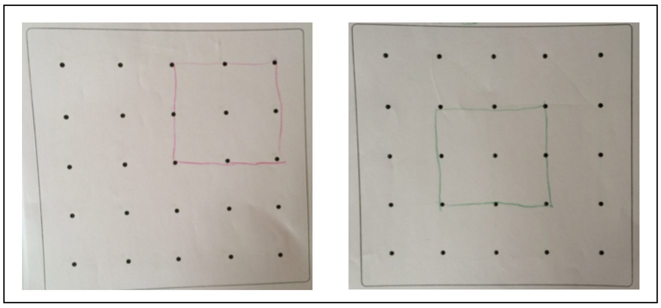


Figure 10. Children filling out contoured figures with Tangram.

In task 10, the geoboard was again used to register some improvement of children in solving problems with geometric figures. Some children immediately recognized the geoboard by naming it. With this task it was possible to verify that their fine motor skills were improved because they already manipulated the elastics better. They also recognized all the geometric figures presented, regardless of the position in which they appeared, even if the geoboard was rotated in front of them, revealing acquired perceptual constancy. In addition, children identified properties of geometric figures ("three nozzles" for the triangle, "four nozzles" for the square, "longer" for the rectangle, and "lazy" or "equal to the moon" for the circle), from the work done previously in other tasks on the properties of these figures.

In the translation process of the representation from the geoboard to the dotted paper, most of children also improved. Children were asked to make a square with a stitch in the centre and when passing to the dotted paper an improvement was noticeable. The child A could already draw on the correct position of the dotted paper after observing the geoboard (Figure 11), and this was an improvement as this child was ignoring its position in the beginning of the intervention. Another child showed the closure in the presented drawings, which also seems to be the result of the work carried out in the spatial notions - drawings of children (Figure 12).
Figure 11. Child's drawing on the dotted paper in the beginning (left) and end (right) of the intervention.



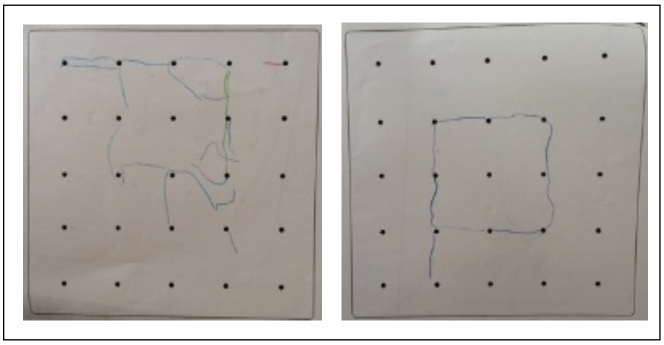


Figure 12. Child drawing at the beginning (left) and end (right) of the intervention.

Another group of 5 children continued to make dotted paper productions that were very different from the geoboard constructions presented to them (Figure 13).

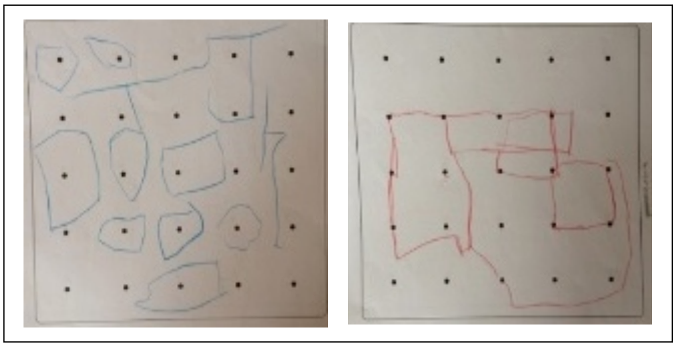


Figure 13. A child's production at the beginning (left) and end (right) of the intervention.

It is considered that this task helped children to develop their other cross-curricular skills of mathematics – problem solving, mathematical reasoning and communication. In the tasks presented in this intervention, children had the opportunity to think to solve the given problems, to identify the differences when they did not do it correctly, and to use the mathematical communication in the explanation of their resolutions. In this sense, and in agreement with (Bishop, 1980), there is evidence that developing informal knowledge of geometry in early childhood education can be quite beneficial because beyond stimulating positive ideas about mathematics, namely geometry, this knowledge is useful for children's daily lives.

Final notes

This research gives evidence that the manipulatives are fundamental in the transition from concrete to abstract in that they appeal to various senses, and are used by children as physical support in a learning situation. Therefore, it is very pertinent to use materials such as the geoboard and tangram. However, it is important to remember that only the use of materials does not guarantee effective and meaningful learning, because what is most important in the teaching and learning of mathematics is the mental activity to be developed in and by children. In addition, and because mathematics can also be considered as a form of communication, it is essential that the explorations carried out in this area can also be an opportunity for children to communicate their ideas. In this sense, group activities are extremely important as they enable the child to learn to work with peers and, of course, to communicate. Moreover, mathematical communication favours concentration, enriches the child's spatial vocabulary, and helps the child to develop visualization skills (Alves & Gomes, 2012).

Children of this study were able to identify properties of the geometric figures and to learn geometric vocabulary. Moreover, this study suggests that children had acquired aspects of visual perception (e.g. Del Grande, 1990; Frostig, Horne & Miller, 1994) as visual motor coordination when drawing on the dotted paper. The deep figure perception seemed to be acquired with the use of tangram and visual memory during activities with geoboard. Visual discrimination suggested to be gained by finding similarities and differences between each other's solutions in the geoboard. Children also showed that they could find congruent figures in different positions, and could identify the triangles, even those that did not have the vertex of the centre facing upwards, which according to Matos and Gordo (1993) suggests that children perceive the position in space. It was also verified that they recognized all the geometric figures presented, independently of the position in which they appeared, revealing, according to Matos and Gordo (1993), acquired perceptual constancy. They also identified properties of the geometric figures ("three nozzles" for the triangle, "four nozzles" for the square, "longer" for the rectangle, and "lazy" or "equal to the moon" for the circle), having been able to distinguish several figures.

For the work of solving problems with geometric figures, it was noticed that some children were able to use geometric vocabulary and reproduce figures. In addition, and converging with the ideas of Moreira and Oliveira (2003), children of this study were able to construct a rectangle with elastics in a geoboard, and given a figure, they could reproduce it. They also could recognize (by experimentation, observation, measurement, or drawing) that the opposite sides of the rectangle are the same. Not surprisingly, they still could not perceive that the square is a special rectangle, neither that this figure had right angles or parallel opposite sides (Crowley, 1987). A few other children did not seem to have reached this level because, among others, they could not draw geometric figures on the dotted paper.

Throughout this process, it was also noted that the fine motor skills of children improved during the intervention. Children's greatest difficulties were in manipulating the geoboard and understanding the dotted paper: some children did not do these tasks alone, or did so without finding a plausible solution, because it was the first time they dealt with this type of task.

Thus, the spatial sense seems to be able to be promoted with specific tasks on spatial notions and geometric figures, provided that their work is well planned, challenging for children, and appropriate materials are used. Nevertheless, more research is needed on these issues in order to be able to generalize results and characterize deeply children's abilities to understand the geometrical figures recommended in the mathematics for the kindergarten.

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