Paper delivered at

1st CLE Colloquium for Philosophy and Formal Sciences

Campinas, 21-23 March 2013

Accepted for publication in the Proceedings of the

Conference

The Sumerian ternary system and the concept of number

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Abstract

It is well known that Sumerians and Babylonians used a numeration system of base 12 and 60. We still have influence of that system in our nowadays counting of the hours of a day, twelve plus twelve, each hour has 60 minute and each minute 60 seconds. Also the circle has 360 degrees.

What is not so well known is that the Sumerians in an earlier period used a ternary system of numeration; the first notice about that system is in Thureau-Dangin (1928). Later Diakonoff (1983) described it in good details and recently, Balke (2010) studied the system and described it with more precision. Still the use of this system and the concept of number involved are open questions. I will answer to those problems making a formalization of the system and showing how it could be related to a cosmogonic design.

1. Numeral systems.

8	18	28	38	47	58
9	19	29	39	48	59
10	20	30	40	50	

The main Babylonian and Sumerian counting system was a sexagesimal system. It means that all numbers were expressed in a system of base 60. We still have remaining of that system in our everyday common life, like our time system, where an hour has 60 minutes, a minute has 60 seconds and the number of days of the year is 364 or 365. Of course 364 or 365 is not in the system of base 60 but it is due to inaccuracy the old system understanding that a year had 360 days. If we want to be properly accurate we should describe the Babylonian system as a quinary-vigesimal system. In that system we have special symbols for numerals 1, 60, 5 and 20. Those numbers are represented in the following table:

The '1' is called diš or aš, and '60' ĝešda. As long with '5' called ía and '20' called niš. In the sexagesimal system 60 is represented again by 𒁹, 61 is represented by 𒁹 𒁹, that could be confused with 𒈫, but the meaning is extracted from the context. According to Balke[1] the sexagesimal system is consistent.

2. The ternary system.

The ternary system is a special case among the Sumerian counting systems. Why three? That is a question that we probably do not be able to answer but we can try to approach a possible answer. It is found in very few texts, and well preserved and attested in one or two of those texts[2]

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[1] Balke, 2009, 694-8

[2] For a discussion on those texts see Diakanoff, 90-2, where he offers a comparison to similar counting systems in other cultures.

Balke[3] there are evidences that that system was used for incantations. We also know that it was developed between the 3rd and the 2nd Millennium. It was used in parallel with the 'common' quinary-vigesimal system.

One thing that held our attention is the fact that the names for those numerals recall verbal actions and that the names present dialectal characteristics. Diakanoff observation of the system is that it is formed by two numerals gi (also pi) and peš. In table 2, we find also merga, tah, taka$_4$ and gi$_4$, but, it seems that the most common names used for the numerals are gi and peš, additional it is used the verbal root bala meaning 'passed', for example '1 passed 3' means 4, and will be transliterated as peš-bala-gi[4].

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[3] Balke, 2010, p. 46.

[4] For the meaning of those Sumerian words, see Balke, p.50.

1	gi, bi, merga	To return, to repeat
2	tah	To add, to do something twice.
3	peš	To enlarge, to make thrice.
	bala	'passed'

The name for the first numeral is associated a verbal root 'to return' or 'to repeat', giving us the idea of start again. Number two is associated with 'to add' in the sense of making something twice and number three is linked to the verb 'to enlarge', making it larger.

According to Diakanoff[5] any combination of gi, peš and bala, that explains the evidence as collected by Balke[6] in the following table:

[5] Diakanoff, 1983, p. 90.

[6] Balke, o. c., p.46

| Cardinal				
1	gi	merga	gi	bi
2	tah	$taka_4$-a-bi	gi_4-bi	bi-bi
3	peš	peš	gi_4 -peš	peš
4	peš-gi	peš-bala	peš-gi	peš-bi
5	peš-bala-gi	peš-bala-gi_4		peš-bi-bi
6	peš-bala-gi_4-gi_4	peš-bala-gi_4-gi_4	peš-bala-gi	peš-peš
7	peš-peš-gi	peš-peš-gi_4		peš-peš-bi
8				peš-peš-bi-bi
9				peš-peš-peš
10				peš-peš-peš-bi
11				peš-peš-peš-bi-bi
12				peš-peš-peš-peš

3. A geometrical model.

As a way to understand better the meaning of the words used to describe the numerals in the ternary system, and in the hope of getting another possible use of that system, I describe here a geometrical model.

If we think that gi is a verbal root meaning to return or to repeat, we can interpret it as a loop, representing it in from the point of view of graph theory as:
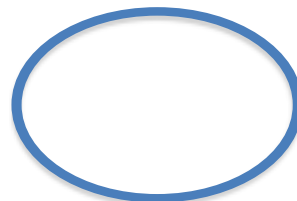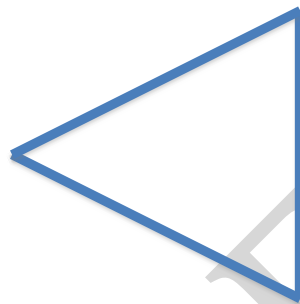
That is also just a dot, as it corresponds to the edge (e, e).

To add something, the meaning of the verbal root tah, will induce the idea of a line, linked by to dots.
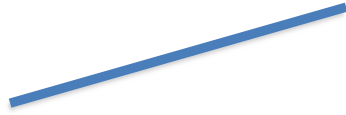
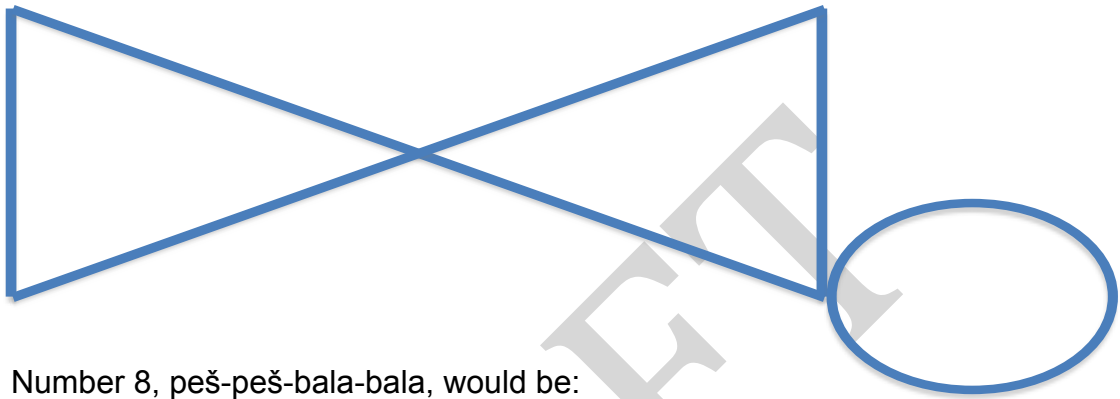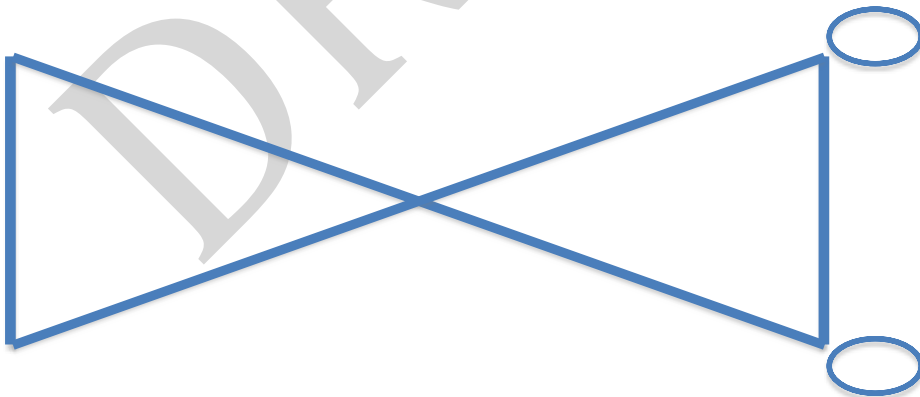Finally to expand, meaning of the root peš, will take us to a triangle.

In this way, we have a geometrical model for the ternary system of numeration.

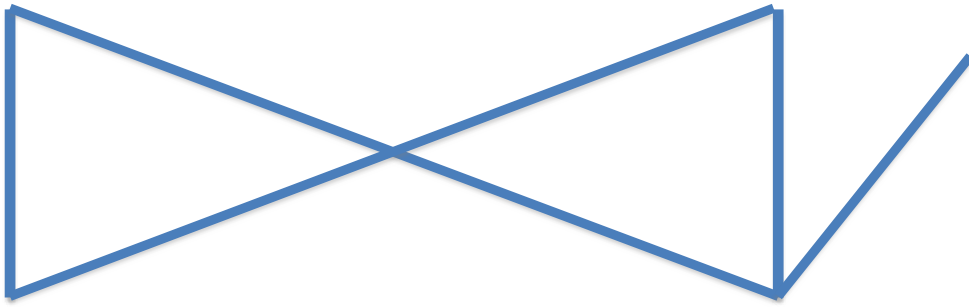Suppose now that you want to represent peš-peš-bala, that is number 7.
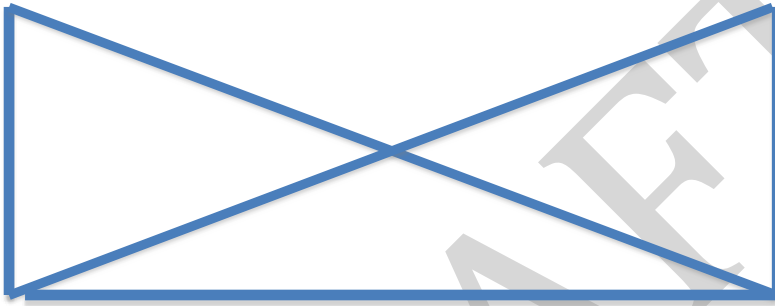
We will end with the following figure:
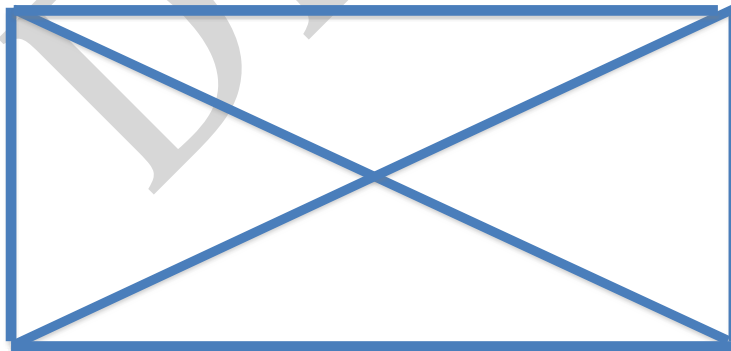

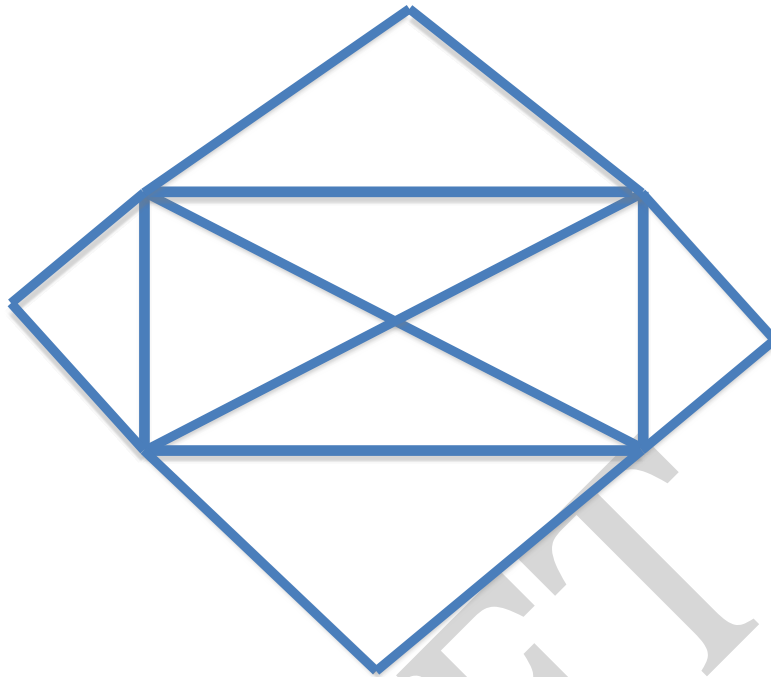Number 8, peš-peš-bala-bala, would be:
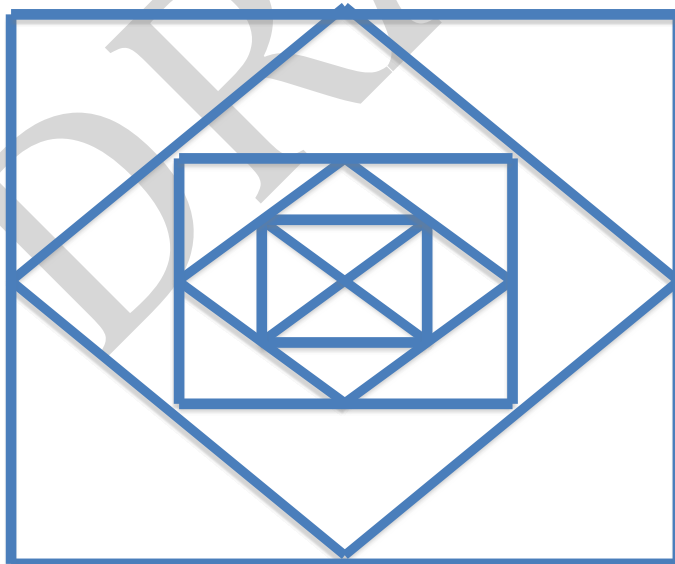

or

A representation of number 9, would be


Number 12 will be represented as:

Number 24 should look like:

Finally, number 60 will be represented as:

The idea of describing the ternary system with a geometrical model[7] is linked to the fact that the Sumerian-Babylonian geometry was based in the triangle, associated with a ternary division of the cosmos. The patterns obtained are similar to ones found in Babylonian tablets. Still, I am not saying that the Sumerian-Babylonians have thought in this way, it is just a model that shows resemblance with geometric patterns as they appear in some tablets, like the one showed here[8]:

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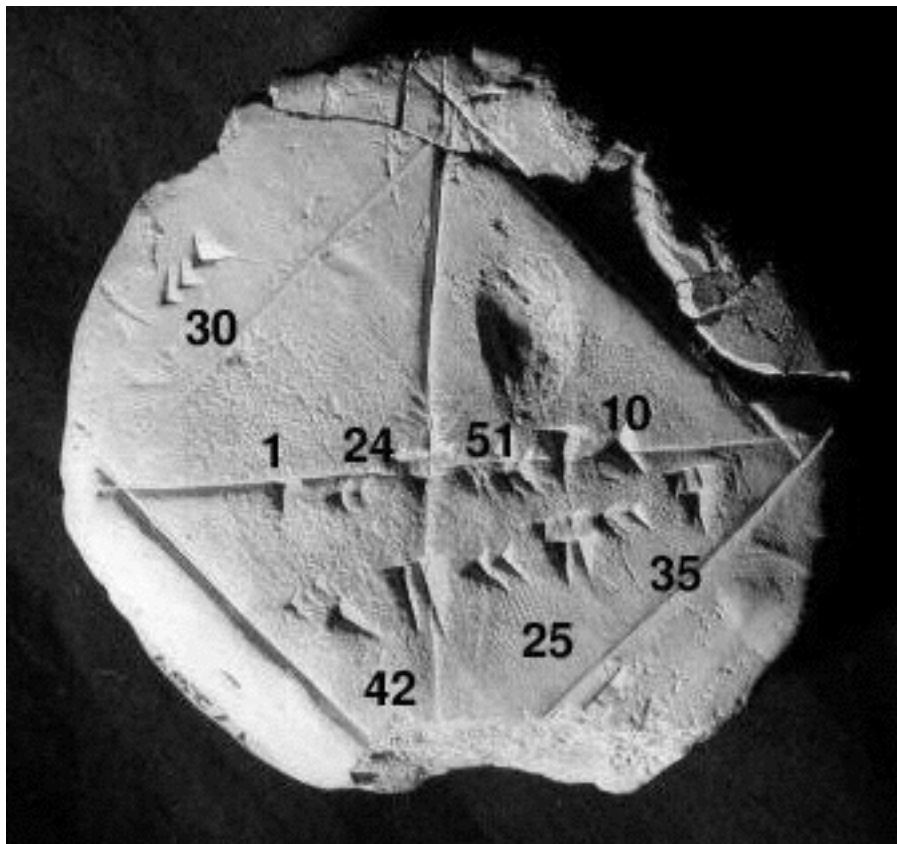
[7] On the use of mathematical patterns to depict the cosmos by the Sumerian, see for example Horowitz (2010).

[8] A black and white rendition of Bill Casselman (mailto:cass@math.ubc.ca) photograph of the Yale Babylonian Collection's *Tablet YBC 7289* (c. 1800–1600 BCE), showing a Babylonian approximation to square root of 2 in the context of Pythagorean theorem for an isosceles triangle. From http://www.math.ubc.ca/~cass/Euclid/ybc/ybc.html.

4. Other associations of number three to the Sumerian and Assyrian-Babylonian cosmogony.

In the Sumerian-Assyrian-Babylonian astrology the heaven was divided in three parts[9]: the northern hemisphere belonging to Enlil, the southern hemisphere belonging to Enki and the line of intersection of both were under the rule of Anu, who had the task of keep the equilibrium between both deities. They use to associate numbers to deities, for example the number 40 was associated to Enlil, who will be called Ea in the Hurrian-Hittite mythology. They also associated stars or constellations[10] to the gods. To Ea was associated the stars of Ea and the stars of the field or piece of land, that is the constellation of Pegassus.

Number three is a number that appear in many different cosmogonies. We know that in the beginning of several cosmogonic relates it is possible to find three entities, three gods, or three different objects that are responsible is somehow for the becoming of the universe. In the Rig Veda it is very common the set of ternary gods or a god with three faces or three characteristics. As we have shown before in Hesiod's *Theogony* there are three main objects in the beginning of the Cosmos, they are Chaos, Gaia and Ouranos. If I want to explain the use and reason for a ternary counting system using a cosmogonic relate I should refer to the Enûma Eliš, where Mummu, T'iamat and Apsu appear as primordial deities.[11]

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[9] The division of the world or part of it in three sections is common to several cultures. In the *Il.* 14.187-93 the cosmos is divided in three parts, each one ruled by one of the sons of Kronos. The cosmos is also divided in three parts in the Vedic hymns, as discussed in Macdonell, 2002 [1898], p. 9.

[10] The determinative, i. e., a symbol opposed to the name of a constellation, like DINGIR was opposed to the name of a deity, for constellation was MUL. The Pleiades, that was considered a constellations of constellations, or a cluster of constellations was called ^{MUL}MUL, and it was written as AN (✳) three times. The original symbol for AN was a star and it is the name of the god of heavens, almost equivalent to the Greek Ouranos. On respect to the names of the stars, see, for example, Horowitz (2005).

[11] I am preparing a paper that includes this topic. I should clarify here that it is not totally accepted that Mummu should be considered as another entity different from T'iamat. A good discussion can be found in Heidel (1948).

5. Conclusions.

The ternary system was used in a parallel way with the quinary-vigesimal and the sexagesimal systems. That system could be used for religious and incantation processes. It could be also related to the creational process of the gods and with the main primordial deities. All those hypothesis need future research to be confirmed, but one thing the we can affirm is that the conception of number for the Sumerian-Babylonians was beyond the simple process of counting, not only for an almost confirmed link to a mythical-religious thought but because its adequacy to a geometrical model and the use in simultaneous of different numeral systems.

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