A kinematic limit analysis approach for masonry buildings: in- and out-of-plane failure mechanisms

Gabriele Milani
Department of Engineering, University of Ferrara, Italy
gmilani@ing.unife.it

Paulo B. Lourenço
Department of Civil Engineering, University of Minho, Azurém, Guimarães, Portugal
pbl@civil.uminho.pt

Antonio Tralli
Department of Engineering, University of Ferrara, Italy
atralli@ing.unife.it

ABSTRACT Historical and existing masonry structures have usually an inadequate resistance to horizontal actions. Furthermore, historical city centres present high vulnerability under horizontal loads and this is mostly due to the absence of adequate connections between the various parts [1] [2]. This characteristic leads to overturning collapses of the perimeter walls under seismic loads and combined in- and out-of-plane failures. Even if limit analysis is not sufficient for a full structural analysis under seismic actions, since it does not provide displacements at failure, it can be used in order to have a quick estimation of both collapse loads and failure mechanisms [3].

In this paper, the micro-mechanical model presented by the authors in [4] and [5] for the limit analysis of respectively in- and out-of-plane loaded masonry walls [6] is utilized for the 3D analysis of entire buildings. In the model, admissible and equilibrated polynomial stress fields are imposed in order to estimate the macroscopic masonry failure surface. Finally, such surface is implemented in a FE kinematic limit analysis code and an example of technical relevance is discussed in detail.

1. INTRODUCTION

The evaluation of the ultimate load bearing capacity of entire masonry buildings subjected to horizontal loads is a fundamental task for the design of brickwork structures. Furthermore, many codes of practice, as for instance the recent Italian O.P.C.M. 3431 [7], require a static non linear analysis for existing masonry buildings, in which a limited ductile behavior of the elements is taken into account.

On the other hand, homogenization techniques can be used for the analysis of large scale structures. In this case, in fact, both mechanical properties of constituent materials and geometry of the elementary cell are taken into account only at a cell level, so allowing the analysis of entire buildings through standard FE codes. Furthermore, the application of homogenization theory to the rigid-plastic case requires only a reduced number of material parameters and provides important information at failure, as for instance limit multipliers and collapse mechanisms [8].

In this paper, the micro-mechanical model presented by the authors in [4] and [5] for the limit analysis of respectively in- and out-of-plane loaded masonry walls is utilized in presence of coupled membrane and flexural effects. In the model, the elementary cell is subdivided along its thickness in several layers. For each layer, fully equilibrated stress fields are assumed, adopting polynomial expressions for the stress tensor components in a finite number of sub-domains. In this way, linearized homogenized surfaces in six dimensions for masonry in- and out-of-plane loaded are obtained. Such surfaces are then implemented in a FE upper bound limit analysis code for the analysis at collapse of entire 3D structures.

In Section 2 the FE upper bound approach is presented, whereas in Section 3 a meaningful structural example is treated in detail [9]. The reliability of the proposed model is assessed through comparisons with results obtained by means of standard non-linear FE approaches.

2. THE FE KINEMATIC LIMIT ANALYSIS APPROACH

The upper bound approach developed in this paper is based both on the formulation of Sloan and Kleeman [10] for the in-plane case and on the formulation of Munro and Da Fonseca [11] for out-of-plane actions.

Both formulations use three noded triangular elements with linear interpolation of the velocity field inside each element. In addition, for the in-plane case discontinuities of the velocity field along the edges of adjacent triangles are introduced.

For each element E, three velocity unknowns per node i, say w_{xx}^{i} , w_{yy}^{i} and w_{zz}^{i} are introduced. For each interface between coplanar adjacent elements, four additional unknowns are introduced ($\Delta \mathbf{u}^{I} = [\Delta v_1 \; \Delta u_1 \; \Delta v_2 \; \Delta u_2]^T$), representing the normal (Δv_i) and tangential (Δu_i) in-plane jumps of velocities (with respect to the discontinuity direction) evaluated on nodes of the interface. For a continuum under in-plane loads three equality constrains representing the plastic flow (obeying an associated flow rule) are further introduced for each element. The reader is referred to [4] for a detailed discussion on the procedure used for obtaining a linear approximation of the failure polytope (with m hyper-planes) in the form $S^{\text{hom}} \equiv \mathbf{A}^{in}\mathbf{\Sigma} \leq \mathbf{b}^{in}$.

For what concerns out-of-plane actions, following Munro and Da Fonseca [11], out-of-plane plastic dissipation occurs only along each interface I between two adjacent triangles R and K.

Total external power dissipated can be written as $P^{ex} = \left(\mathbf{P}_0^T + \lambda \mathbf{P}_1^T\right)\mathbf{w}$, where $\mathbf{P}_0$ is the vector of (equivalent lumped) permanent loads, λ is the load multiplier, $\mathbf{P}_1^T$ is the vector of (lumped) variable loads and $\mathbf{w}$ is the vector of assembled nodal velocities. As the amplitude of the failure mechanism is arbitrary, a further normalization condition $\mathbf{P}_1^T \mathbf{w} = 1$ is usually introduced. Hence, the external power becomes linear in $\mathbf{w}$ and λ, i.e. $P^{ex} = \mathbf{P}_0^T \mathbf{w} + \lambda$.

After some elementary assemblage operations, a simple linear programming problem is obtained (analogous to that reported in [10]), where the objective function consists in the minimization of the total internal power dissipated.

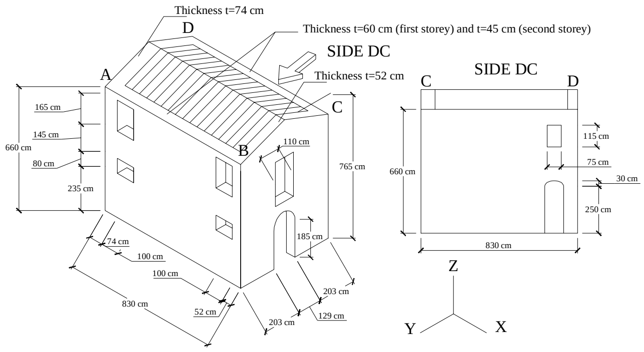
3. EXAMPLES

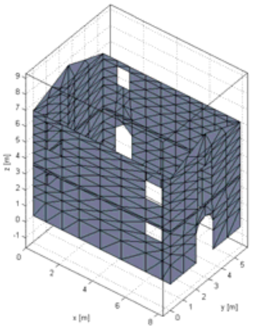
In this section, a 3D FE limit analysis on an ancient masonry building is presented. The model is an adaptation of a real house analyzed by De Benedictis et al. in [9] within a survey of the entire Ortigia center (Italy) coordinated by Giuffrè [3]. The building has two storeys and it is assumed, for the sake of simplicity, that its plan is rectangular, with dimensions 8.30 × 5.35 m.

Vertical load is constituted by walls self weight and permanent and accidental loads of the first floor and of the roof. In Figure 1, a three dimensional representation of the model is reported.

The building presents a rocking collapse mechanism of the BC façade [9], due to the absence of interlocking with its perpendicular walls. A restoration intervention is proposed in [9] in order to improve interlocking between perpendicular walls and floors stiffness, so aiming at a global failure mechanism.

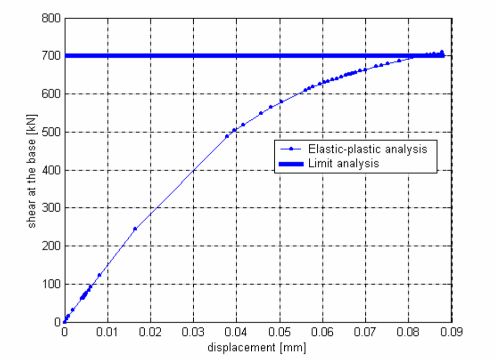
In the simulation here presented, only the building after the restoration intervention proposed in [9] is taken into consideration. Masonry bricks are assumed of dimensions 46 × 14 × 22 cm. In the homogenized FE limit analysis model, for joints reduced to interfaces a pure Mohr-Coulomb failure criterion with friction angle $\Phi = 30°$ and cohesion $c = 0.01 N/mm^2$ is adopted, in order to represent the very low tensile strength of masonry, whereas blocks are supposed infinitely resistant.

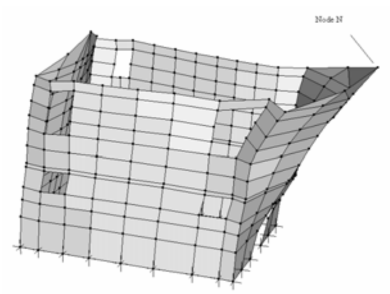




-a -b

Figure 1: Ancient masonry house case study. –a: geometry. –b: mesh used for the limit analysis (1576 triangular elements).





-a -b

Figure 2: Shear at the base – node N displacement curve. -b: deformed shape at collapse, standard FE procedure.

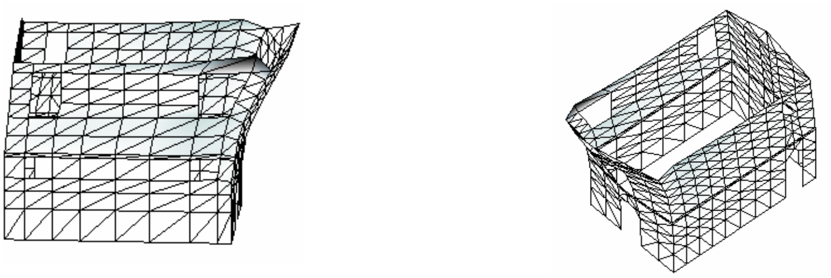


Figure 3: Deformed shape at collapse, homogenized limit analysis approach.

The results obtained with the homogenized FE limit analysis model (i.e. base shear at failure and failure mechanism) are compared with a standard FE elastic-perfectly plastic analysis performed by means of a commercial code. The analysis is conducted using a mesh of 324 four noded plate elements supposing masonry isotropic with a pure Mohr-Coulomb failure criterion ($c = 0.01 N/mm^2$ and $\Phi = 30°$). The kinematic FE homogenized limit analysis gives a total shear at the base of the building of $701 kN$, in good agreement with the results obtained with the standard FE procedure ($710 kN$), Figure 2. Good agreement is also found comparing the deformed shapes at collapse provided by the two models, Figure 2-b and Figure 3. Finally it is worth noting that the proportionality coefficient (i.e. the ratio between horizontal load at failure and vertical loads) obtained with the proposed homogenization model is equal to 0.36, in good agreement with that found in [12] (0.38).

4. ACKNOWLEDGMENTS

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