

Employment protection and unemployment benefits: On technology adoption and job creation in a matching model*

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Abstract

We analyse the effects of different labour market policies – employment protection, unemployment benefits and payroll taxes – on job creation and technology choices in a model where firms are matched with workers of different productivity and wages are determined by ex-post bargaining. The model is characterised by two intertwined sources of inefficiency, namely a matching externality and a hold-up externality associated with workers’ bargaining strength. Results depend on the relative importance of the two externalities and on worker risk aversion. ‘Flexicurity’, meaning low employment protection and generous unemployment insurance, can be optimal if workers are sufficiently risk averse and the hold-up problem is relatively important.

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1 Introduction

The present paper investigates theoretically optimal labour market policies in a matching-type model of the labour market. More precisely, we set up an optimal taxation problem with three choice variables; an unemployment benefit, a tax that can regulate firing costs up or down, and a payroll tax (or subsidy) on labour earnings. In the matching model we employ, two sources of inefficiency interact. First, we envisage that some match-specific investment in technology is undertaken before a job opening is filled. When workers can capture quasi-rents from sunk investments, this influences incentives both for technology investments and job creation. The second source of inefficiency is the matching externality. The more entrepreneurs who decide to start up a firm, the lower is the probability that a given firm is matched with a worker.

A key purpose of our analysis is to investigate the optimality of so-called ‘flexicurity’ policies in a context with inefficiencies in technology choices and job creation, as described above. Among policy makers flexicurity has become something of a buzzword. Can one make relatively rigid labour markets in parts of Europe more prone to change, while still preserving the economic safety net which to some extent is absent in the US? The concrete idea is to build down employment protection, but at the same time provide economic security outside the original firm by supplying generous unemployment benefits and retraining support. Denmark and the Netherlands are countries where flexicurity purportedly exists, and these countries have over the last years experienced smaller unemployment problems than many other European economies. In the context of our theoretical framework, flexicurity is taken to mean a situation where unemployment benefits are combined with low (or zero) employment protection, while the payroll tax is used to balance public budgets.

It is not obvious that a flexicurity type package of policies would be sensible in the current setting. Technology investments are seen as (at least partly) firm-worker specific, for example due to training and adoption costs.¹ The presence of sunk and match-specific technology investments casts doubt on the extent to which a policy with lower firing costs, to induce mobility, is the correct one. Maintaining an existing relationship between firm and worker has value. But even

¹What we for simplicity dub as ‘technology’ investments can be seen as a mix of technology investments and firm-sponsored worker training necessary to employ the new technology. This creates a lock-in and a struggle for the division of rents. The model could have included training costs paid by the workers, but this would not change the main course of the analysis.

in this framework, some workers will not find employment. Some are unlucky and not matched, some lose their job because their productivity turns out to be very low. These workers demand unemployment insurance, and the more risk averse they are the more so. Unemployment benefits strengthen the outside option for workers and influence how rents are shared inside firms. The ex-post bargained wage is driven up, and more so the higher the bargaining strength of workers. This implies in turn that job creation suffers. This can, but must not, imply that optimal policy takes on a flexicurity shape – this depends on the specifics of the situation.

In the positive part of our analysis, we show that layoff taxes harm job creation but stimulate technology investments. Job creation is also harmed by unemployment benefits and payroll taxes, whereas the effect on technology investments is *a priori* indeterminate. In the welfare analysis, we start out by deriving the first-best solution and compare it with the ‘market solution’, where no policy is in place, in order to identify the various inefficiencies that create a scope for policy. We then analyse the optimal policy by performing numerical simulations for a parameterised version of the model. Three parameters turn out to be essential for the optimal policy package: (i) the matching elasticity, which measures the reduction in matching probability due to increased entry of firms, (ii) workers’ power to capture quasi-rent, and (iii) worker risk aversion. A high (low) matching elasticity relative to workers’ ex post bargaining power implies that the equilibrium entry of firms is too high (low) in the absence of policy, whereas high risk aversion increases the value of unemployment benefits as insurance against unemployment.

We show analytically that in the special case of no worker power, zero employment protection is never optimal.² The reason is that, in this case, the market solution yields too much entry of firms/jobs and therefore too little investment in technology behind each job. This creates a scope for employment protection as part of the optimal policy, since a positive layoff tax simultaneously increases technology investments and reduces the number of job openings. This suggests that flexicurity is never optimal if the bargaining power of workers is sufficiently low. Indeed, for the more general case of positive worker bargaining power, numerical simulations of the second-best solution confirm that a necessary condition for flexicurity to be an optimal

²We have chosen to model employment protection as a layoff tax. The alternative would have been severance pay. A main difference is that a layoff tax is paid to the authorities while severance pay is a transfer between private parties. Boeri, Garibaldi and Moen (2016) carefully investigate optimal severance pay, with a focus on the relationship between tenure and severance and also open the black box of judicial systems. These issues are abstracted from here.

policy is that worker bargaining power is sufficiently high relative to the matching elasticity, such that equilibrium entry is below (or sufficiently close to) the first-best level of entry in the absence of policy. If, in addition, workers are sufficiently risk averse, the optimal policy is a flexicurity policy where unemployment benefits are financed by payroll taxation and where the layoff tax is zero. In such a case, a positive layoff tax is welfare detrimental for two different reasons: it stifles entry and it increases income inequality between employed and unemployed workers.³

In an extension to the main model we also examine a particular version of a progressive payroll tax, where the tax is levied only on the part of the wage payments that exceed workers' outside options. In this case the payroll tax does not directly affect entry, dismissal rates and technology investments, which makes it possible to raise tax revenues in a non-distortive way. It turns out that this effectively makes the layoff tax redundant and therefore greatly enhances the scope for flexicurity as the optimal policy.

Many authors have investigated flexicurity – or the two constituent parts of that policy, employment protection and unemployment benefits – in frameworks different from the current matching framework.⁴ We would like to suggest a delineation between external and internal flexibility of the labour market. The original eurosclerosis debate painted a picture of lacking sectoral reallocation.⁵ This we would refer to as (lacking) 'external' flexibility. If the problem is that workers are unwilling to quit ailing industries in order (hopefully) to be reemployed in the sunrise part of the economy, lower employment protection combined with unemployment insurance would probably improve that economy's 'flexibility'. However, a very important source of productivity growth is the adoption of new technology in the industries where the workers already are employed. We dub this 'internal' flexibility. It is far less obvious that flexicurity

³Our focus is on specific investments, lock-in, rent-sharing, and ex post bargaining, so we employ a matching model where these elements are incorporated. Moen's competitive search model (1997) shows that it is possible to picture matching frictions in a manner that to a larger extent captures features normally associated with competitive equilibria. But, as said, for our purpose ex post bargaining rather than price-taking agents is essential for the argument.

⁴Literature on flexicurity include Andersen and Svarer (2007), Boeri and Garibaldi (2009), Cockx and van der Linden (2010), Boeri, Conde-Ruiz and Galasso (2012), and Andersen (2012).

⁵See Bentolila and Bertoli (1990).

policy will improve an economy's track record also when it comes to this type of flexibility.^{6,7} However, as already hinted at, flexicurity can actually improve welfare also in this setting, but only under specific conditions.

The received literature both on employment protection and unemployment insurance abounds.^{8,9} Much less attention has been paid to how these policy instruments should be combined. An exception is Blanchard and Tirole (2008). They study the optimal combination of employment protection and unemployment insurance in a matching model that is not fully dynamic in the sense that rematching after the termination of a given employment relationship is abstracted from.^{10,11} It is known to be notoriously difficult to study optimal policy in fully dynamic matching models without assuming linear utility.¹² But risk-neutrality and equal marginal utility of income for capital owners, employed workers and the unemployed also appear as an unattractive starting point for analysis.

We therefore use much the same model as Blanchard and Tirole (the version of it that includes ex-post wage bargaining), but extend it along two different dimensions: (i) by applying a matching function, which introduces a negative externality in the firm entry decision and leads, in general, to inefficient entry, (ii) by introducing endogenous technology, where the presence of ex-post bargaining creates a hold-up problem that leads to inefficient technology choices. These

⁶In previous work (Lommerud and Straume 2012) we investigated how flexicurity affected the adoption of labour-saving technology in a context with trade unions. In this set-up better technology makes some workers more productive while others lose their job. Good unemployment insurance softens the consequences of job loss and makes the union more favourably inclined towards technological change, and this is partly the reason why we find that flexicurity policy works well also in this context.

⁷The literature on technology investments and labor market frictions of various kinds include Dowrick and Spencer (1994), Ulph and Ulph (1998), Lommerud, Meland and Straume (2006), Kreckemeier and Nelson (2006) and Lommerud, Meland and Straume (2012).

⁸Work on sectoral reallocation and/or employment protection include Lazear (1990), Dixit and Rob (1994), Bertola and Rogerson (1997), Ljungqvist (2002), Bertola (2004), Rogerson (2005), Piccirilli (2010) and Karabay and McLaren (2011).

⁹Examples from the literature on unemployment insurance are Baily (1978), Shavell and Weiss (1979), Hansen and Imrohorgulu (1992), Hopenhayn and Nicolini (1997, 2009), Acemoglu and Shimer (2000), Chetty (2006), Hassler and Rodriguez Mora (2008), Landais, Michailat and Saez (2010) and Boeri and Macis (2010).

¹⁰The combined choice of employment protection and unemployment insurance is also studied by Anesi and De Donder (2013), but in a political economy context where the equilibrium policy package is a result of coalition formation.

¹¹Belot, Boone and van Ours (2007) also use a 'one-shot' matching model as ours to analyse employment protection. Decreuse and Garnier (2013) look at the effect of job protection and unemployment insurance in a matching framework, but the main focus is on the adaptability versus productivity of education investments. See also Wasmer (2006).

¹²Mortensen and Pissarides (2003) study the joint determination of a series of taxes and subsidies in a matching model, but resorted to the assumption of risk-neutral workers. Pissarides (2001) studies employment protection in a matching model with non-linear utility. Alvarez and Veracierto (2000) study payroll and layoff taxes and severance payments with non-linear utility, while results largely are given as numerical simulations.

additional inefficiencies imply that the first-best solution is unattainable in our model (apart from in the special case of no worker bargaining power). This stands in contrast to the Blanchard-Tirole analysis, where policy is motivated by the need to insure workers against unemployment. In their analysis, the optimal policy always imply a combination of unemployment insurance and layoff taxes. In contrast, we find that no employment protection can often be part of the optimal policy package, if firm entry is inefficiently low. Thus, in terms of policy implications, a key difference between the Blanchard-Tirole paper and the present paper is that we find a much larger scope for flexicurity as the optimal policy.

2 Model

Consider an economy that consists of n workers and a large number of entrepreneurs. We assume that workers are risk-averse and entrepreneurs are risk-neutral, with worker utility given by a strictly concave utility function $u(x)$, where x is income. An entrepreneur who decides to start up a firm has to pay k and, in addition, install costly technology that affects labour productivity. After deciding on the level of technology, each firm posts a vacancy for one worker. Firms and workers are matched according to a matching function $m(n, s)$, where s is the total number of job openings (firms). In order to make the matching function conceptually meaningful, we assume that $m(n, s) \leq \min\{n, s\}$. We also make the standard assumptions that m is increasing, concave and homogeneous of degree one in (n, s) . This implies that we can write $m(n, s) = s\rho(q)$ and $m(n, s) = n\gamma(q)$, where $q := n/s$ is the ratio of workers to jobs, and therefore an inverse measure of market tightness; $\rho(q) := m(n, s)/s$ is the probability that a job opening will be filled; and $\gamma(q) := m(n, s)/n$ is the probability that an unmatched worker will find a job. The properties of the matching function imply that $\rho'(q) > 0$, $\rho''(q) < 0$, and $\gamma'(q) < 0$, $\gamma''(q) > 0$.

If a firm and a worker are matched, the productivity of the match is given by

$$y = \phi + \varepsilon, \tag{1}$$

where $\phi > 0$ reflects the technology of the firm and ε reflects the productivity of the worker. Workers are heterogeneous with respect to their productivity, which is only revealed after the match is formed. More specifically, we assume that ε is randomly distributed on $[0, 1]$ with a

density function $f(\varepsilon)$ and a corresponding cumulative distribution function $F(\varepsilon) = \int_0^\varepsilon f(x) dx$. Since all workers draw their productivity from the same distribution, they are *ex ante* identical.

We consider the following sequence of events:

Stage 0: A welfare-maximising policy maker chooses the following policy variables: A layoff tax $c \geq 0$, a payroll tax/subsidy $t \leq 0$, and an unemployment benefit $b \geq 0$.

Stage 1: Each entrepreneur decides whether or not to pay k in order to start up a firm.

Stage 2: Each entrepreneur who decided to start a firm chooses how much to invest in technology. Achieving a technological level ϕ costs $(\psi/2)\phi^2$, where $\psi > 0$.

Stage 3: Workers and firms are matched according to $m(n, s)$. Those workers that are not matched become unemployed and receive unemployment benefit b .

Stage 4: Worker productivity is revealed and each firm decides whether to keep the worker or lay him off. At this stage, entry and technology costs are sunk. If the firm decides to break up the match, it has to pay the layoff tax c .¹³ A dismissed worker becomes unemployed and receives unemployment benefit b .

Stage 5: Each worker that is not laid off bargains with his or her employer over the wage rate, w , and production takes place.¹⁴

The model set-up requires that there is a real cost of separation if the worker is dismissed, since the value created by the technology investment is match-specific. This would be the case if there is a cost of initiating and adapting a worker to the technology in question, which is lost if the worker-firm match is ended.¹⁵ The technology investment is therefore best seen as a mix of technology investments and firm-related training costs paid by the firm.¹⁶

¹³In principle, one could add break-up costs other than the layoff tax, but the essential feature is that policy makers have access to an instrument that can regulate after-tax dismissal costs to what they want them to be.

¹⁴Notice that the firms have no incentives to offer wage contracts which include severance payments in case of dismissals. Since technology investments take place before workers are hired, such contracts would not be time consistent. After making the investment, the firms have nothing to gain from higher firing costs.

¹⁵The assumption that the technology investment is purely match-specific is made for analytical convenience without much loss of generality. The main mechanisms of the model would be unchanged if the technology investment had an alternative (but lower) value outside the match.

¹⁶Based on Swiss survey data, Blatter, Muehleman and Schenkeret (2012) find evidence of quite sizeable hiring costs (including the costs of adaption) for high-skilled workers. They find that such costs are typically higher in jobs with a higher skill requirement (for example, in jobs where more advanced technology is used) and that marginal hiring costs for high-skilled workers can amount to as much as half a year of wage payments.

Notice also that there are two different sources of unemployment in the model. In equilibrium, the unemployed consist of those workers who were not matched with a firm (the unlucky ones) and those who were initially matched but subsequently laid off (the less productive ones).

2.1 Wage bargaining

After productivity is revealed, each matched worker bargains with her employer over the wage rate. We consider a simple two-stage bargaining game suggested by Blanchard and Tirole (2008): In the first stage, the worker proposes a wage that the employer can either accept or reject. If the proposal is rejected, the wage is set in the second state by the worker (with probability β) or the firm (with the remaining probability). Being risk-averse, the worker will propose the highest wage that the firm is willing to accept. Being risk neutral, the firm is at most willing to accept a wage equal to the expected wage in case the game proceeds to the second stage. This wage is equal to the worker's outside option plus a share β of the rents from the match. In equilibrium, the worker will propose this wage and the firm will accept. Thus, the equilibrium wage is given by

$$w = (1 - \beta)(v + b) + \frac{\beta(y + c)}{1 + t}, \quad (2)$$

where $v > 0$ is the income that an unemployed worker can earn in the informal sector. The parameter $\beta \in (0, 1)$ can thus be thought of as representing the relative bargaining power of workers. The corresponding *ex post* profit for the firm from this match is given by

$$\pi = y - w(1 + t) = (1 - \beta)(y - (1 + t)(v + b)) - \beta c. \quad (3)$$

The main determinants of the bargained wage are intuitive. Higher unemployment benefits will improve workers' outside options and therefore lead to higher wages. For a given level of worker productivity, better technology will generate a larger surplus with a correspondingly higher wage (if workers have some bargaining power). On the other hand, a higher payroll tax will reduce the joint net surplus and cause a wage drop, although the wage drop is not large enough to prevent the firm's labour cost after tax from rising.

2.2 Dismissals

The worker and the firm will separate only if the joint net surplus from production is negative. Thus, a worker whose productivity satisfies the condition

$$y \geq \hat{y} := (1+t)(v+b) - c. \quad (4)$$

will not be laid off.¹⁷ The condition (4) defines a lower threshold in terms of worker productivity, given by

$$\hat{\varepsilon} := \max \{0, (1+t)(v+b) - c - \phi\}. \quad (5)$$

Hired workers with productivity $\varepsilon < \hat{\varepsilon}$ will be dismissed, while the remaining hired workers will be retained. Thus, $\hat{\varepsilon}$ determines the expected dismissal rate in the economy.

2.3 Technology choice

Before the firm is potentially matched with a worker, investments in technology are made. The optimal technological level is chosen to maximise expected profits:

$$\Pi^e = \rho(q) \left(\int_{\hat{\varepsilon}}^1 \pi f(\varepsilon) d\varepsilon - cF(\hat{\varepsilon}) \right) - k - \frac{\psi}{2} \phi^2, \quad (6)$$

where π is given by (3). For a given dismissal rate, the optimal level of technology, ϕ^* , is implicitly given by

$$\rho(q) (1 - \beta) (1 - F(\hat{\varepsilon})) - \psi \phi^* = 0. \quad (7)$$

For a given value of q , optimal choices of technology and dismissal rate are jointly determined by (5) and (7).

2.4 Job creation

At Stage 1 of the game, each entrepreneur decides whether or not to enter the market and start up a firm. We assume that the number of entrepreneurs is so large that there is no strategic interaction among them. Thus, firms enter the market until expected profits are zero. The free

¹⁷It is easily confirmed that the condition $\pi \geq -c$ is equivalent to the condition $w \geq b+v$. Thus, the firm and the worker will always agree on when to separate.

entry condition is given by

$$\Pi^e = \rho(q) \left(\int_{\widehat{\varepsilon}}^1 \pi f(\varepsilon) d\varepsilon - cF(\widehat{\varepsilon}) \right) - k - \frac{\psi}{2}\phi^2 = 0, \quad (8)$$

which determines q^* and thus s^* (recall that $q := n/s$). The full equilibrium is given by the triple $(\widehat{\varepsilon}^*, \phi^*, q^*)$ that simultaneously solves (5), (7) and (8).

3 Equilibrium effects of labour market policies

A policy maker can affect job creation, technology choices and dismissal rates by using the available tax instruments: c , b and t . The relationship between each tax instrument and the equilibrium outcomes is the following:¹⁸

Proposition 1 *A marginal increase in c leads to less job creation, higher technology investments and lower dismissal rates. A marginal increase in either b or t leads to less job creation, whereas the effects on technology investments and dismissal rates are indeterminate.*

The effects of tax policy on job creation are unambiguous. The negative relationship between firing costs and job creation is due to two factors. Higher firing costs mean that not only does it become more costly for the firm to lay off workers, but it also becomes more costly to retain them, since retained workers are in a better position to negotiate higher wages. Both effects contribute to reducing the expected profits of running a firm and therefore lead to less job creation. The effects of unemployment benefits and payroll taxes also work in the same direction. Higher unemployment benefits increase the wage and therefore reduce ex post profits. Higher payroll taxes reduce the bargained wage, but not enough to offset the reduction in profits. Thus, in both cases, job creation is hampered.¹⁹

The effects of tax policy on technology choices and dismissal rates can be decomposed into two different sub-effects: (i) direct effects for a given number of vacancies and (ii) the indirect effects via job creation as described above. The direct effects are clear-cut. Higher unemployment benefits or payroll taxes increase the bargained wage and therefore increase the level of worker

¹⁸All proofs in Appendix.

¹⁹A payroll subsidy will obviously have the opposite effect.

productivity that is necessary to make the match profitable, resulting in a higher dismissal rate. Higher firing costs also increase the bargained wage, but this effect is more than outweighed by the increase in the direct cost of dismissals, resulting in a lower dismissal rate. The effects on incentives for technology investments follow straightforwardly. A higher (lower) dismissal rate increases (reduces) the probability that technology investments are wasted. Thus, once an entrepreneur has sunk the cost of starting up a firm, any policy that increases (reduces) the expected dismissal rate will reduce (increase) the incentives for investing in technology in order to make the hired worker more productive.

However, the direct effects of tax policy on technology and dismissal choices can be reinforced or counteracted by indirect effects through more or less entry. More entry implies that the probability that a firm is matched with a worker goes down (there are more firms ‘competing’ for workers), which in turn implies that the expected benefits of technology investments are lower, which again implies that the optimal dismissal rate is higher. And *vice versa* in the case of lower entry. Higher unemployment benefits, payroll taxes or firing costs all lead to less entry, which results in a higher matching probability and therefore stronger incentives for technology investments. This implies that the positive direct effect of firing costs on technology investments is reinforced by less entry. On the other hand, the negative direct effects of unemployment benefits or payroll taxes on technology investments are counteracted by less entry, making the overall effects *a priori* ambiguous. Similarly, higher firing costs lead unambiguously to lower dismissal rates, whereas the effects of unemployment benefits or payroll taxes are indeterminate because of counteracting indirect effects. Notice, however, that the effects of a marginal increase in b are always qualitatively similar to the effects of a marginal increase in t (see the proof of Proposition 1 in the Appendix). For example, if a higher b increases incentives for technology investments, then so does a higher t .

3.1 Total employment

Although the effects of the various policy instruments on job creation are clear-cut, unambiguous effects on total employment do not automatically follow, since this also depends on dismissal

rates. Formally, total employment is given by

$$L = n\gamma(q)(1 - F(\hat{\varepsilon})). \quad (9)$$

Higher firing costs lead to less job creation but lower dismissal rates, implying that the effect on total employment is ambiguous. The employment effects of unemployment benefits and payroll taxes are also indeterminate, because of the ambiguous effects on dismissal rates.

3.2 Flexicurity

Our analysis can also shed some light on the effects of a labour market policy which has come to be known as *flexicurity*, combining low employment protection with a relatively generous income support to unemployed workers. In our model, a policy reform towards more flexicurity would correspond to a simultaneous reduction in c and increase in b . The results in Proposition 1 show that the two different legs of flexicurity have contrasting effects on job creation and that the overall effect on technology investments is, at best, mixed. Lower firing costs stimulate job creation, whereas higher unemployment benefits have the opposite effect. The effects on technology investments are either negative or mixed. Less employment protection has an unambiguously negative effect on technology choices, whereas higher unemployment benefits stimulate technology investments only if indirect effects (through less job creation) dominate the direct negative effect.

4 Welfare

We now turn to a welfare analysis, where we first derive the first-best solution and compare it to the equilibrium outcome derived in Section 2 in the absence of labour market policies (i.e., $t = c = b = 0$). We then proceed to characterise the second-best solution where the tax instruments are set to maximise expected social welfare.

4.1 The first-best solution

Suppose that revenues can be transferred in a lump-sum manner between firms and workers, and that the social planner can directly choose the income to employed and unemployed workers,

the dismissal rate, the technology level and the number of firms/jobs. Since expected profits are zero in equilibrium, social welfare is defined as expected worker utility. Defining the problem in per-worker terms, the first-best solution solves

$$\max_{w, b, \hat{\varepsilon}, \phi, q} W = \frac{\rho(q)}{q} [(1 - F(\hat{\varepsilon})) u(w) + F(\hat{\varepsilon}) u(v + b)] + \left(1 - \frac{\rho(q)}{q}\right) u(v + b) \quad (10)$$

subject to the resource constraint

$$\frac{\rho(q)}{q} \left[\int_{\hat{\varepsilon}}^1 (y - w) f(\varepsilon) d\varepsilon - F(\hat{\varepsilon}) b \right] - \frac{1}{q} \left(k + \frac{\psi}{2} \phi^2 \right) - \left(1 - \frac{\rho(q)}{q}\right) b = 0. \quad (11)$$

Denoting the Lagrangian multiplier by λ , the first-order conditions for an optimal solution are

$$(w) : \frac{\rho(q)}{q} (1 - F(\hat{\varepsilon})) u'(w) - \lambda \frac{\rho(q)}{q} (1 - F(\hat{\varepsilon})) = 0, \quad (12)$$

$$(b) : \left(1 - (1 - F(\hat{\varepsilon})) \frac{\rho(q)}{q}\right) u'(v + b) - \lambda \left(1 - (1 - F(\hat{\varepsilon})) \frac{\rho(q)}{q}\right) = 0, \quad (13)$$

$$(\hat{\varepsilon}) : \frac{\rho(q)}{q} [u(v + b) - u(w)] f(\hat{\varepsilon}) - \lambda \frac{\rho(q)}{q} (\phi + \hat{\varepsilon} - w + b) f(\hat{\varepsilon}) = 0, \quad (14)$$

$$(\phi) : \lambda \left[\frac{\rho(q)}{q} (1 - F(\hat{\varepsilon})) - \frac{\psi}{q} \phi \right] = 0, \quad (15)$$

$$(q) : \frac{1}{q^2} \left(\begin{array}{c} (\rho'(q) q - \rho(q)) (1 - F(\hat{\varepsilon})) (u(w) - u(v + b)) \\ + \lambda \left(\begin{array}{c} (\rho'(q) q - \rho(q)) \left(\int_{\hat{\varepsilon}}^1 (y - w) f(\varepsilon) d\varepsilon + (1 - F(\hat{\varepsilon})) b \right) \\ + \left(k + \frac{\psi}{2} \phi^2 \right) \end{array} \right) \end{array} \right) = 0. \quad (16)$$

From (12)-(16), we can characterise the first-best optimal solution as follows:

Proposition 2 *The socially optimal first-best solution is characterised by*

$$w = v + b, \quad (17)$$

$$\hat{\varepsilon} = v - \phi, \quad (18)$$

$$\phi = \frac{\rho(q)}{\psi} (1 - F(\hat{\varepsilon})), \quad (19)$$

$$(\rho(q) - \rho'(q)q) \int_{\hat{\varepsilon}}^1 (y - v) f(\varepsilon) d\varepsilon = k + \frac{\psi}{2} (\phi)^2. \quad (20)$$

The first-best levels of technology, dismissal rates and firm entry are determined by the standard rule of (expected) marginal benefits equal to marginal costs. Notice that, all else equal, more entry (which implies lower q) reduces the first-best level of technology, since more entry reduces the probability that a given firm is matched with a worker. Notice also that, since workers are risk averse, the first-best outcome has workers fully insured against unemployment. That is, workers receive the same utility regardless of whether they are unemployed or not, as indicated by (17).

4.2 Welfare properties of the no-policy equilibrium

In the absence of any labour market policies, how does the market equilibrium derived in Section 2 compare with the first-best outcome? Setting $b = c = t = 0$, a comparison of the first-best outcome with the equilibrium outcome given by (5), (7) and (8), reveals the following:

Proposition 3 *In the absence of labour market policies: If $\beta > 0$, technology investments are too low and dismissal rates are too high, given the level of firm entry. In addition, workers are underinsured. For given levels of technology investments and dismissal rates, firm entry is too high (low) if $\beta < (>) \eta$, where $\eta := \rho'(q)q/\rho(q)$.*

In the no-policy equilibrium, the first-best outcome is not achieved. If workers have some bargaining power ($\beta > 0$), they are able to capture parts of the surplus from production ex post. This reduces the expected marginal revenue of technology investments and therefore leads to a suboptimally low technology level in equilibrium. This, in turn, leads to an equilibrium dismissal rate that is suboptimally high. Furthermore, $\beta > 0$ implies that income is higher for the employed than for the unemployed, which means that workers are underinsured compared with the first-best outcome.

The above is true for any given level of firm entry. However, for a given level of technology (and thus dismissal rate), entry can be too high or too low compared with the first-best outcome. The direction of the inefficiency in the entrepreneurs' entry decisions depend on the size of workers' share of the surplus (β) relative to the matching elasticity (η). More entry reduces

the probability for each firm of being matched with a worker. Thus, the decision to enter the market imposes a negative externality on other firms, which, in isolation, leads to excessive entry from a viewpoint of social welfare. On the other hand, the hold-up problem caused by ex post bargaining creates an inefficiency in the other direction, with too weak incentives for entry. For a given level of technology, these two inefficiencies exactly nullify each other, yielding entry at the first-best level, if $\eta = \beta$, which is the well-known Hosios condition (Hosios, 1990). Otherwise (and still for a given value of ϕ), entry is too high (low) if η is higher (lower) than β . This implies that the first-best outcome is not achieved even in the case where workers have no bargaining power, which eliminates the hold-up problem. For a given level of entry, if $\beta = 0$, technology and dismissal choices are at the first-best level and workers are fully insured. However, entry is inefficiently high, since $\eta > 0$.

4.3 The second-best solution

In a second-best world, dismissal, technology and entry choices are decided by firms (and not by the welfare-maximising social planner), and these are given by, respectively, (5), (7) and (8). The policy maker can only influence these choices through his choices of the tax instruments c , t , and b . Without deriving the second-best solution, we can immediately observe that the first-best outcome cannot be achieved as long as $\beta > 0$. The reason is simply that $w > v + b$ for any β (cf. (2)), implying that full insurance against unemployment cannot be achieved by labour market policies as long as workers have some ex post bargaining power.

The second-best problem can be defined as

$$\max_{b,t,c} W = u(v+b) + \frac{\rho(q)}{q} \left[\int_{\hat{\varepsilon}}^1 [u(w) - u(v+b)] f(\varepsilon) d\varepsilon \right] \quad (21)$$

subject to

$$\frac{\rho(q)}{q} \left(t \int_{\hat{\varepsilon}}^1 w f(\varepsilon) d\varepsilon + cF(\hat{\varepsilon}) + (1 - F(\hat{\varepsilon}))b \right) - b = 0. \quad (22)$$

The first-order condition for an optimal choice of b is given by²⁰

²⁰Again, λ denotes the Lagrangian multiplier.

$$\begin{aligned}
0 &= \frac{\rho(q)}{q} \left(1 - \beta + \frac{\beta}{1+t} \frac{\partial \phi^*}{\partial b} \right) \int_{\widehat{\varepsilon}}^1 u'(w) f(\varepsilon) d\varepsilon + \left[1 - (1 - F(\widehat{\varepsilon})) \frac{\rho(q)}{q} \right] u'(v+b) \\
&+ \frac{\rho'(q)q - \rho(q)}{q^2} \left[\int_{\widehat{\varepsilon}}^1 (u(w) - u(v+b)) f(\varepsilon) d\varepsilon \right] \frac{\partial q^*}{\partial b} \\
&+ \lambda \left(\begin{aligned} &\frac{\rho(q)}{q} \left(t \left(1 - \beta + \frac{\beta}{1+t} \frac{\partial \phi^*}{\partial b} \right) (1 - F(\widehat{\varepsilon})) - (t(v+b) + b - c) f(\widehat{\varepsilon}) \frac{\partial \widehat{\varepsilon}^*}{\partial b} \right) \\ &+ \frac{\rho'(q)q - \rho(q)}{q^2} \left(t \int_{\widehat{\varepsilon}}^1 w f(\varepsilon) d\varepsilon + cF(\widehat{\varepsilon}) + (1 - F(\widehat{\varepsilon}))b \right) \frac{\partial q^*}{\partial b} \\ &- \left(1 - \frac{\rho(q)}{q} (1 - F(\widehat{\varepsilon})) \right) \end{aligned} \right). \tag{23}
\end{aligned}$$

Higher unemployment benefits have three different direct effects on expected utility – two positive and one negative – given by the first three terms in (23): (i) wages go up for the employed, even if higher b leads to less technology investments;²¹ (ii) income increases also for the unemployed; but (iii) job creation goes down, which reduces expected income and therefore utility. In addition, a marginal increase in b has the following budget effects: (i) wages increase, leading to higher tax revenues; (ii) if dismissal rates increase, this will lead to lower tax revenues if b and t are sufficiently large relative to c ; (iii) job creation goes down, which implies lower income and therefore less tax revenues; and, finally, (iv) higher unemployment benefits obviously have a direct cost.

The first-order condition for the optimal choice of c is given by

$$\begin{aligned}
0 &= \frac{\rho(q)}{q} \frac{\beta}{1+t} \left(1 + \frac{\partial \phi^*}{\partial c} \right) \int_{\widehat{\varepsilon}}^1 u'(w) f(\varepsilon) \\
&+ \frac{\rho'(q)q - \rho(q)}{q^2} \left[\int_{\widehat{\varepsilon}}^1 [u(w) - u(v+b)] f(\varepsilon) d\varepsilon \right] \frac{\partial q^*}{\partial c} \\
&+ \lambda \left(\begin{aligned} &\frac{\rho(q)}{q} \left(F(\widehat{\varepsilon}) + t \frac{\beta}{1+t} \left(1 + \frac{\partial \phi^*}{\partial c} \right) (1 - F(\widehat{\varepsilon})) - (t(b+v) + b - c) f(\widehat{\varepsilon}) \frac{\partial \widehat{\varepsilon}^*}{\partial c} \right) \\ &+ \frac{\rho'(q)q - \rho(q)}{q^2} \left(t \int_{\widehat{\varepsilon}}^1 w f(\varepsilon) d\varepsilon + cF(\widehat{\varepsilon}) + (1 - F(\widehat{\varepsilon}))b \right) \frac{\partial q^*}{\partial c} \end{aligned} \right). \tag{24}
\end{aligned}$$

Higher firing costs have two direct effects on expected utility – one positive and one negative –

²¹Notice that

$$\begin{aligned}
&1 - \beta + \frac{\beta}{1+t} \frac{\partial \phi^*}{\partial b} \\
&= \frac{(1 - \beta)(\psi - \rho(q) f(\widehat{\varepsilon}))}{\psi - \rho(q)(1 - \beta) f(\widehat{\varepsilon})} + \frac{\beta(1 - \beta)\rho(q)}{(\psi - \rho(q)(1 - \beta) f(\widehat{\varepsilon}))} \frac{(1 - F(\widehat{\varepsilon}))\psi\phi}{(k + \frac{\psi}{2}\phi^2)} > 0,
\end{aligned}$$

implying that a higher b has a positive net effect on wages.

given by the first two terms in (24): (i) higher c leads to higher wages, an effect that is reinforced by larger technology investments; (ii) higher c leads to less job creation and therefore increases the probability of unemployment. In addition, we have the following budget effects: (i) a direct increase in revenues; (ii) an increase in tax revenues because of higher wages; (iii) the effect of fewer dismissals on total revenues, which is positive if b and t are sufficiently large relative to c ; (iv) lower tax revenues because of less job creation.

Finally, the first-order condition for an optimally set payroll tax, t , is given by

$$\begin{aligned}
0 = & -\frac{\rho(q)}{q} \frac{\beta}{1+t} \int_{\hat{\varepsilon}}^1 \left(\frac{\phi + \varepsilon + c}{1+t} - \frac{\partial \phi^*}{\partial t} \right) u'(w) f(\varepsilon) d\varepsilon \\
& + \frac{\rho'(q)q - \rho(q)}{q^2} \left[\int_{\hat{\varepsilon}}^1 (u(w) - u(v+b)) f(\varepsilon) d\varepsilon \right] \frac{\partial q^*}{\partial t} \\
& + \lambda \left(\begin{aligned} & \frac{\rho(q)}{q} \left(\int_{\hat{\varepsilon}}^1 \left(w - \frac{t\beta}{1+t} \left(\frac{\phi + \varepsilon + c}{1+t} - \frac{\partial \phi^*}{\partial t} \right) \right) f(\varepsilon) d\varepsilon - (t(v+b) + b - c) f(\hat{\varepsilon}) \frac{\partial \hat{\varepsilon}^*}{\partial t} \right) \\ & + \frac{\rho'(q)q - \rho(q)}{q^2} \left(t \int_{\hat{\varepsilon}}^1 w f(\varepsilon) d\varepsilon + cF(\hat{\varepsilon}) + (1 - F(\hat{\varepsilon}))b \right) \frac{\partial q^*}{\partial t} \end{aligned} \right). \tag{25}
\end{aligned}$$

A higher payroll tax has two direct negative effects on expected utility, given by the first two terms in (25): (i) higher t leads to lower wages, an effect that might be reinforced by lower technology investments; (ii) higher t also leads to less job creation and therefore lower expected income. In addition, the budget effects are: (i) higher tax revenues, though this effect is dampened by lower wages; (ii) higher t might also lead to more dismissals, which reduces tax revenues if t and b are sufficiently large relative to c ; and, finally, (iii) less job creation, with a corresponding reduction in income and therefore tax revenues.

4.4 A parameterised model

Due to the complexity of the model, a further characterisation of the second-best solution requires that the model is parameterised. Suppose that worker productivity is distributed according to $\varepsilon \sim U[0, 1]$, implying $f(\varepsilon) = 1$ and $F(\varepsilon) = \varepsilon$, and suppose that the matching function has the following Cobb-Douglas form: $m = \alpha n^\sigma s^{1-\sigma}$, where $\alpha > 0$ and $\sigma \in (0, 1)$, which implies $\rho = \alpha q^\sigma$ and $\eta = \sigma$. Suppose also that workers' utility function is given by $u(x) = x^\mu$, $\mu \in (0, 1)$, which implies that μ is an inverse measure of the degree of risk aversion.

4.4.1 Special case: No worker bargaining power

Consider first the special case in which workers have no bargaining power; i.e., $\beta = 0$. The equilibrium wage is then $w = v + b$ regardless of the worker's productivity. Thus, employed and unemployed workers have the same income and there is no rationale for insurance against unemployment. Furthermore, the hold-up problem disappears, implying that, for a given number of firms, each firm has first-best incentives for technology investments. The only remaining inefficiency is related to the entry decision. For any $\sigma > 0$, there is too much entry, which – in the absence of policy – leads to underinvestment in technology and too many dismissals. However, since there is only one source of inefficiency, the first-best outcome can actually be achieved by an optimal tax policy package.

Applying our parameterisation to (18)-(20), the *first-best* solution, in terms of $\hat{\varepsilon}$, ϕ and ρ , is implicitly given by

$$\hat{\varepsilon} = v - \phi, \quad (26)$$

$$\rho(1 - \hat{\varepsilon}) - \psi\phi = 0, \quad (27)$$

and

$$\frac{(1 - \sigma)\rho}{2}(1 - \hat{\varepsilon})(2(\phi - v) + 1 + \hat{\varepsilon}) - k - \frac{\psi}{2}\phi^2 = 0. \quad (28)$$

Similarly, applying our parameterisation to (5), (7) and (8), the *equilibrium* values of $\hat{\varepsilon}$, ϕ and ρ are implicitly given by

$$\hat{\varepsilon} = (1 + t)(v + b) - c - \phi, \quad (29)$$

$$\rho(1 - \hat{\varepsilon}) - \psi\phi = 0, \quad (30)$$

and

$$\frac{\rho}{2}(1 - \hat{\varepsilon})((2(\phi - (1 + t)(b + v)) + 1 + \hat{\varepsilon})) - \rho c \hat{\varepsilon} - k - \frac{\psi}{2}\phi^2 = 0. \quad (31)$$

Assuming that the first-best solution is interior with respect to the dismissal rate (i.e., $\hat{\varepsilon} > 0$), the first-best solution can be implemented by setting one of the tax instruments at the level that induces first-best level of entry (for given values of ϕ and $\hat{\varepsilon}$) and by setting the two remaining tax instruments at levels which simultaneously induce the first-best level of dismissals and balance

the regulator's budget, which is given by

$$\frac{\rho}{q} (t(1 - \hat{\varepsilon})(v + b) + c\hat{\varepsilon} + (1 - \hat{\varepsilon})b) - b = 0. \quad (32)$$

The equilibrium level of technology will then automatically be at the first-best level, since technology choices depend on ρ and $\hat{\varepsilon}$ but do not depend directly on the tax parameters. The optimal tax policy is given by

$$t = \frac{\sigma\psi^2(1-v)^2(q-\rho)}{\sigma\psi^2\rho(1-v)^2 + 2qv(\psi-\rho)^2}, \quad (33)$$

$$b = \frac{\sigma\rho\psi^2(1-v)^2}{2q(\psi-\rho)^2}, \quad (34)$$

$$c = \frac{\sigma\psi^2(1-v)^2}{2(\psi-\rho)^2}, \quad (35)$$

where

$$\rho = \frac{\psi \left(4k + \psi(1-v)^2(1-\sigma) \right) - (1-v) \sqrt{\psi^3 \left(\psi(1-\sigma)^2(1-v)^2 - 8k\sigma \right)}}{4k + 2\psi(1-v)^2} \quad (36)$$

We see that $b > 0$ and $c > 0$ for all parameter values. Thus, the optimal policy always includes layoff taxes and unemployment benefits. Positive values of both these policy instruments contribute towards less entry, which is too high to begin with when $\beta = 0$. The optimal payroll tax is also positive if $q > \rho$, which implies $q > \alpha^{\frac{1}{1-\sigma}}$. Thus, the optimal payroll tax is positive if the first-best level of q is sufficiently high (which is equivalent to the first-best level of entry, s , being sufficiently low). This will be the case if the entry cost k is sufficiently high, for example. Otherwise, if the first-best level of entry is sufficiently high, the first-best solution is implemented with payroll subsidies in combination with layoff taxes and unemployment benefits.

Proposition 4 *If workers have no bargaining power, the first-best solution can be implemented by optimal tax policy. The optimal policy always includes strictly positive layoff taxes and unemployment benefits, in addition to a payroll tax (subsidy) if the first-best level of entry is sufficiently low (high).*

Perhaps the most striking feature of this result is that the optimal policy always includes a strictly positive layoff tax, which implies that flexicurity is never an optimal policy if workers have no bargaining power. More generally, this result suggests that a necessary condition for flexicurity to be an optimal policy is that workers' bargaining power is sufficiently high relative to the matching elasticity. This conjecture is confirmed below where we perform numerical simulations of the second-best solution for $\beta > 0$.

4.4.2 Numerical simulations of the second-best solution

In the more general case with $\beta > 0$, the optimal (second-best) solution can only be solved numerically. We perform numerical simulations of the optimal solution for three different regimes, where the matching elasticity (σ) is, respectively, (1) lower than, (2) equal to, or (3) higher than the workers' share of the surplus (β).²² Tables 1 and 2 show the first-best outcome and the no-policy equilibrium, respectively, for each of the three regimes.²³ Consistent with Proposition 3, we see that technology investments are too low and the dismissal rates too high in the no-policy equilibrium, whereas the amount of job creation (s) is too low in Regime 1 and too high in Regimes 2 and 3.²⁴

Table 1: First-best solution

	ϕ	$\hat{\varepsilon}$	s	L
(1) $\sigma = 0.4 < \beta$	0.096	0.004	1.743	0.834
(2) $\sigma = 0.5 = \beta$	0.12	0	1	0.6
(3) $\sigma = 0.6 > \beta$	0.168	0	0.573	0.480
Parameter values: $n = 1; v = 0.1; k = 0.12; \psi = 5; \alpha = 0.6; \beta = 0.5$				

²²The simulations are made using the software General Algebraic Modeling System (GAMS). Further details are available upon request.

²³Notice that, in Regimes 2 and 3, the first-best solution is a corner solution with zero dismissal rate.

²⁴Notice that $\beta = \sigma$ yields optimal entry only if we keep ϕ and $\hat{\varepsilon}$ constant. However, since the equilibrium technology investments and dismissal rates are, respectively, too low and too high, the threshold value of β , above which equilibrium entry is too low, is strictly higher than σ . Thus, entry is above the first-best level for $\beta = \sigma$.

Table 2: No-policy equilibrium

	ϕ	$\widehat{\varepsilon}$	s	L
(1) $\sigma = 0.4 < \beta$	0.053	0.047	1.191	0.635
(2) $\sigma = 0.5 = \beta$	0.053	0.047	1.150	0.613
(3) $\sigma = 0.6 > \beta$	0.053	0.047	1.124	0.599

Parameter values: $n = 1; v = 0.1; k = 0.12; \psi = 5; \alpha = 0.6; \beta = 0.5$

The corresponding second-best solution, where the tax instruments t , c and b are jointly set to maximise social welfare, is shown in Table 3.²⁵ For each of the three regimes, we present the second-best solution for different degrees of worker risk aversion (inversely measured by the parameter μ).²⁶ A noteworthy feature of the chosen parameterisation, with uniform productivity distribution and Cobb-Douglas matching technology, is that a marginal increase in either b or t (evaluated at the no-policy equilibrium) leads to more technology investments in equilibrium. Thus, the indirect effects of these tax policies (through less entry) dominate the direct ones.²⁷ This is important to keep in mind when interpreting the results.

There are three clear patterns emerging from the results displayed in Table 3.²⁸ First, whenever the second-best solution deviates from the no-policy equilibrium, the optimal policy stimulates technology investments and stifles job creation. This is (partly) caused by the positive effect of t and b on technology investments, which implies that any policy that gives stronger incentives for technology investments (which are too low in the no-policy equilibrium) leads to less entry.

²⁵Notice that we restrict b and c to be non-negative, which is consistent with their interpretations as, respectively, unemployment benefits and a layoff tax.

²⁶Notice that the degree of risk aversion does not affect the first-best solution, nor the no-policy equilibrium.

²⁷From the proof of Proposition 1 (see Appendix), $\partial\phi^*/\partial b > 0$ and $\partial\phi^*/\partial t > 0$ if

$$\frac{(1 - F(\widehat{\varepsilon}))\psi\phi}{k + \frac{\psi}{2}\phi^2} - f(\widehat{\varepsilon}) > 0.$$

Setting $F(\widehat{\varepsilon}) = \widehat{\varepsilon}$ and $f(\widehat{\varepsilon}) = 1$, and inserting the equilibrium values of ϕ and $\widehat{\varepsilon}$ from (??)-(??), yield

$$\frac{(1 - \widehat{\varepsilon}^*)\psi\phi}{k + \frac{\psi}{2}(\phi^*)^2} - 1 = 1 > 0.$$

²⁸Similar patterns emerge in all the numerical simulations we have tried with other parameter configurations.

Second, the scope for an active labour market policy (i.e., a policy package given by $(t, c, b) \neq (0, 0, 0)$) is larger if the matching elasticity is high relative to the workers' share of the surplus. In Regime 1 ($\sigma < \beta$), the number of firms is too low in the no-policy equilibrium, which implies that there is a welfare trade-off between stimulating technology investments and stimulating job creation. The inefficiency in technology choices can be reduced by setting positive values of at least two of the three tax instruments, such that the budget constraint holds, but only at the cost of increasing the inefficiency in job creation, and *vice versa*. As a result, the second-best solution might be achieved in the no-policy equilibrium. In our numerical simulations, this happens if the degree of risk aversion is sufficiently low. Otherwise, if workers are sufficiently risk-averse, the need for unemployment insurance implies that the optimal solution has a positive unemployment benefit, which is financed by a positive payroll tax. However, in Regimes 2 and 3 (in which $\sigma \geq \beta$), there is no welfare trade-off between job creation and technology investments, since firm entry is excessively high in the absence of policy. This creates a larger scope for active labour market policies, as each of the available policy instruments – payroll taxes, layoff taxes and unemployment benefits – will simultaneously reduce inefficiencies along two different dimensions: technology choices and job creation. In this case, the second-best solution is never achieved in the no-policy equilibrium.

Table 3: Second-best solution

	μ	t	c	b	ϕ	$\hat{\varepsilon}$	s	L
(1) $\sigma = 0.4 < \beta$	0.001	0.157	0	0.047	0.058	0.113	0.812	0.470
	0.25	0.096	0	0.035	0.056	0.092	0.919	0.518
	0.5	0.027	0	0.013	0.054	0.062	1.093	0.594
	0.75	0	0	0	0.053	0.047	1.191	0.635
	1	0	0	0	0.053	0.047	1.191	0.635
(2) $\sigma = 0.5 = \beta$	0.001	0.294	0	0.069	0.061	0.158	0.676	0.415
	0.25	0.240	0	0.063	0.060	0.142	0.731	0.440
	0.5	0.175	0	0.054	0.059	0.122	0.809	0.474
	0.75	0.095	0.005	0.036	0.057	0.087	0.909	0.522
	1	-0.001	0.029	0	0.059	0.012	1.005	0.594
(3) $\sigma = 0.6 > \beta$	0.001	0.398	0.043	0.087	0.083	0.136	0.460	0.380
	0.25	0.346	0.050	0.083	0.083	0.114	0.477	0.396
	0.5	0.283	0.059	0.076	0.083	0.083	0.501	0.417
	0.75	0.205	0.073	0.063	0.084	0.040	0.535	0.449
	1	0.127	0.082	0.045	0.081	0	0.604	0.490

Parameter values: $n = 1; v = 0.1; k = 0.12; \psi = 5; \alpha = 0.6; \beta = 0.5$

Third, higher risk aversion (a lower value of μ) moves the optimal policy package in the direction of flexicurity, with higher unemployment benefits and less employment protection. When workers are more risk averse, the need for insurance against unemployment is more accentuated. Put differently, the welfare gain of a policy that reduces income differences between employed and unemployed workers increases with the degree of risk aversion. For a worker with productivity ε , this income difference is given by

$$w(\phi^*, \varepsilon) - (b + v) = \beta \left(\frac{\phi + \varepsilon + c}{1 + t} - (b + v) \right). \quad (37)$$

It is evident from (37) that, for a given level of technology, the income gap between employed and unemployed workers decreases with b and t , and increases with c . Thus, a policy package that implies an increase in unemployment benefits and a reduction in firing costs – a policy change towards more flexicurity – offers more insurance to workers. The extra funds needed to finance such a policy shift are raised by increasing the payroll tax, which also, in itself, reduces the income gap in (37) and therefore provides even more insurance to workers. Defining flexicurity as a policy that combines $b > 0$ and $c = 0$, our numerical simulations show that flexicurity is the optimal policy if two conditions are met: (i) the matching elasticity is sufficiently low relative to workers’ bargaining strength, and (ii) the degree of worker risk aversion is sufficiently high. Somewhat simplistically put, the first condition rules out $c > 0$ as an optimal policy, whereas the second condition ensures that $b > 0$ is part of the optimal policy.

Finally, it should be noted that the positive effect of t on technology investments implies that the scope for payroll subsidies as part of the optimal policy is limited. A negative relationship between t and ϕ^* would have made payroll subsidies a potentially potent policy instrument in cases where job creation is too low. This is not the case, though, in our parametric example, where payroll subsidies increase the inefficiency of firms’ technology choices.

5 Extension: Progressive payroll taxation

Our main analysis is based on the assumption of linear tax schedules. Although such schedules are arguably easier to implement in practice, they are not necessarily optimal. In this extension we will briefly show how the optimal outcome can be improved by implementing a particular type of progressive payroll taxation, where the tax is levied only on the part of the wage that exceeds the workers’ outside option. Keeping the notation t for the marginal payroll tax rate, a firm hiring a worker at wage rate w must then pay taxes equal to $t(w - v - b)$.

Ex post bargaining now yields a wage

$$w = (1 - \beta)(b + v) + \beta \frac{y + t(b + v) + c}{1 + t} \quad (38)$$

and profits

$$\pi = (1 - \beta)(y - b - v) - c\beta. \quad (39)$$

A striking feature of this particular tax schedule is that profits do not depend on the payroll tax rate, since an increase in t is exactly offset by a reduction in the bargained wage, keeping ex post profits constant. The expected profits of starting up a firm, which determines equilibrium entry, is still given by (8), but where π is given by (39) instead of (3), and where the optimal dismissal rate, $\hat{\varepsilon}$, is given by

$$\hat{\varepsilon} := \max \{0, v + b - c - \phi\}. \quad (40)$$

For given dismissal and entry rates, the optimal level of technology is implicitly given by (7), as before. Notice that, since t does not affect ex post profits, it does not affect dismissal rates, technology choices and entry either. Thus, the only effect of the payroll tax is to reduce wage inequality (between workers of different productivity and between employed and unemployed workers).

In Table 4 we present numerical solutions of the optimal (second-best) policy for the same set of parameters as in the main analysis (in Table 3).²⁹ The simulation results reveal that there are, in qualitative terms, only two possible solutions: (i) setting t so high that payroll tax revenues are maximised and distributing these revenues as unemployment benefits, or (ii) no policy (i.e., $t = c = b = 0$). The former solution is optimal for all values of μ in Regimes 2 and 3. In these regimes, entry is too high in the absence of policy. A policy of collecting payroll tax revenues and distributing them as unemployment benefits then serves a dual purpose; it reduces income inequality and it also reduces entry (through a higher b). In the optimal solution, this policy is taken to the extreme, leading to a complete income equalisation between employed and

²⁹For space saving purposes, and to avoid too much repetition, we only report outcomes for $\mu \geq 0.5$, since, within each of the three regimes, all outcomes are equal for $\mu \in (0, 0.5]$.

unemployed workers and across workers of different productivity.³⁰

Table 4: Second-best solution with a progressive payroll tax

	μ	t	c	b	ϕ	$\hat{\varepsilon}$	s	L
(1) $\sigma = 0.4 < \beta$	0.5	max	0	0.146	0.064	0.183	0.520	0.331
	0.75	max	0	0.146	0.064	0.183	0.520	0.331
	1	0	0	0	0.053	0.047	1.191	0.635
(2) $\sigma = 0.5 = \beta$	0.5	max	0	0.164	0.065	0.199	0.543	0.354
	0.75	max	0	0.164	0.065	0.199	0.543	0.354
	1	max	0	0.164	0.065	0.199	0.543	0.354
(3) $\sigma = 0.6 > \beta$	0.5	max	0	0.181	0.067	0.214	0.560	0.374
	0.75	max	0	0.181	0.067	0.214	0.560	0.374
	1	max	0	0.181	0.067	0.214	0.560	0.374

Parameter values: $n = 1; v = 0.1; k = 0.12; \psi = 5; \alpha = 0.6; \beta = 0.5$

This solution is also the optimal one in Regime 1, but only if μ is sufficiently low. In this regime, entry is too low to begin with, which implies that a policy of using payroll tax revenues to finance higher unemployment benefits has both a positive and a negative effect on welfare. It reduces income inequality but brings equilibrium entry further away from the first-best level. Intuitively, the positive effect on income inequality outweighs the negative effect on entry if workers are sufficiently risk averse. In the extreme case of $\mu = 1$, there is no welfare gain of reducing income inequality and the optimal policy is no policy, which is similar to the result obtained in the main analysis for the same parameter configuration ($\sigma = 0.4$ and $\mu = 1$).³¹

A striking feature of the results in Table 4 is that $c = 0$ in all second-best policy outcomes. When the payroll tax is designed in such a way that tax revenues can be raised without affecting entry, dismissal rates or technology investments, the layoff tax simply becomes redundant as a policy instrument. Consequently, the scope for flexicurity is greatly enhanced. Apart from

³⁰Analytically, payroll tax revenues per retained worker are maximised at $\beta(y + c - b - v)$ for $t \rightarrow \infty$. Numerically, the software used to produce the simulations reported in Table 4 reports optimal values of t that ranges from 54655 to 241389, depending on the value of μ . In all cases, the equilibrium outcomes are identical down to a large number of decimal places.

³¹In Regime 1, our numerical simulations (not reported) show that the threshold level of μ , above which the optimal policy switches from maximising payroll tax revenues to setting $t = c = b = 0$, is somewhere between $\mu = 0.85$ and $\mu = 0.9$.

the case of $\sigma = 0.4$ and $\mu = 1$, the optimal policy is a flexicurity policy for all parameter configurations reported in Table 4. It is also easily confirmed that replacing a linear with a progressive payroll tax leads to higher expected welfare in the optimal policy solution (except when the optimal policy is no policy in both cases).

6 Concluding remarks

Governments influence the workings of the labour market in so many ways. Much work in economics has studied the impact of unemployment insurance and employment protection, but far less attention has been paid to how these instruments should be optimally combined. The so-called flexicurity literature often takes as its starting point a picture of the labour market where lacking sectoral reallocation is the problem.

We have studied quite a different model framework: entrepreneurs must pay costs just to open up a job and to install a level of technology at the workplace. Optimal policy depends crucially on parameter values, but we have indicated that flexicurity can actually be optimal also in this setting, perhaps counterintuitively. One should remember that even in a framework where sunk investments and rent sharing are important, there will still be some unemployed workers, and they demand economic security in the form of unemployment insurance outside an original firm rather than employment protection within such a firm. The specifics of the situation determines optimal policy, but this is the basic reason why flexicurity policy under some circumstances is optimal even in a model framework that is very different from one emphasizing the need for sectoral reallocation of workers.

One way to approach this is to look at how lower employment protection influences technology investments and job creation in the model. Lower dismissal costs increase the number of dismissals, and it also reduces the incentives to invest in technology which makes a given worker more productive. But the effect on job creation is different. Low firing costs make it less costly to lay off workers, but also less costly to retain them, as the bargaining position of the workers is weakened. Thus, although low firing costs are bad for technology investments, it is good for job creation. Indeed, if the hold-up problem is more important than the matching externality, and if workers are sufficiently risk averse, optimal policy will take on a flexicurity-flavoured shape.

Our model and analysis obviously rest on a number of assumptions, some of which are more crucial than others. One of our assumptions is that technology investments take place before firm-worker matches are formed. An alternative assumption would be that firms invest in technology after the match is formed but before productivity is revealed and wages are bargained. This would slightly change some of the mechanisms of our model. Importantly, this alternative assumption would make decisions on technology and dismissals independent of the number of jobs in the economy and thereby break the link between entry and technology decisions. The comparative statics effects of the various policy instruments (given by Proposition 1) would be the same as before, with the exception that the effect of higher payroll taxes or unemployment benefits on technology investments would be unambiguously negative. This would likely create a greater scope for payroll *subsidies* to be part of an optimal policy package. If worker bargaining power is high relative to the matching externality, such that job creation is too low, payroll subsidies financed by a layoff tax would stimulate technology investments and possibly also job creation, and would likely be part of the optimal policy, at least if the degree of worker risk aversion is sufficiently low.

Labour market models come in all forms and shapes. Questions on, more generally, how unemployment insurance, employment protection and payroll taxation should be combined, and more specifically, on the optimality of flexicurity, should be investigated in various model formats. We would be hard-pressed to argue that the model employed here is the only correct picture of the labour market, but we think it complements simpler models that focus on sectoral reallocation rather than sunk investments in ongoing relationships. At some point in time, it could be fruitful to develop models that highlighted both these aspects of the labour market.

Appendix

Proof of Proposition 1 The effect of tax policy on job creation is found by total differentiation of (8) only, since the Envelope Theorem eliminates effects that go through ϕ^* and $\widehat{\varepsilon}^*$.

The effects are given by

$$\frac{\partial q^*}{\partial c} = -\frac{\partial \Pi^e / \partial c}{\partial \Pi^e / \partial q} = \frac{\rho(q) (\beta (1 - F(\widehat{\varepsilon})) + F(\widehat{\varepsilon}))}{\rho'(q) \left(\int_{\widehat{\varepsilon}}^1 \pi f(\varepsilon) d\varepsilon - cF(\widehat{\varepsilon}) \right)} > 0, \quad (\text{A1})$$

$$\frac{\partial q^*}{\partial b} = -\frac{\partial \Pi^e / \partial b}{\partial \Pi^e / \partial q} = \frac{\rho(q)(1-\beta)(1+t)(1-F(\hat{\varepsilon}))}{\rho'(q)\left(\int_{\hat{\varepsilon}}^1 \pi f(\varepsilon) d\varepsilon - cF(\hat{\varepsilon})\right)} > 0, \quad (\text{A2})$$

$$\frac{\partial q^*}{\partial t} = -\frac{\partial \Pi^e / \partial t}{\partial \Pi^e / \partial q} = \frac{\rho(q)(1-\beta)(v+b)(1-F(\hat{\varepsilon}))}{\rho'(q)\left(\int_{\hat{\varepsilon}}^1 \pi f(\varepsilon) d\varepsilon - cF(\hat{\varepsilon})\right)} > 0. \quad (\text{A3})$$

For a given number of vacancies, the effects of tax policy on technology choices and dismissal rates are found by totally differentiating the system of equations given by (5) and (7), and using Cramer's Rule. The total effects, when taking into account that entry is endogenous, are given by³²

$$\frac{d\phi^*}{dc} = \frac{\partial \phi^*}{\partial c} + \frac{\partial \phi^*}{\partial q} \frac{\partial q^*}{\partial c} = \frac{\rho(q)(1-\beta)}{\psi - \rho(q)(1-\beta)f(\hat{\varepsilon})} \left(f(\hat{\varepsilon}) + \frac{\psi\phi(\beta(1-F(\hat{\varepsilon})) + F(\hat{\varepsilon}))}{\left(k + \frac{\psi}{2}\phi^2\right)(1-\beta)} \right) > 0, \quad (\text{A4})$$

$$\frac{d\phi^*}{db} = \frac{\partial \phi^*}{\partial b} + \frac{\partial \phi^*}{\partial q} \frac{\partial q^*}{\partial b} = \frac{(1-\beta)(1+t)\rho(q)}{\psi - \rho(q)(1-\beta)f(\hat{\varepsilon})} \left(\frac{(1-F(\hat{\varepsilon}))\psi\phi}{\left(k + \frac{\psi}{2}\phi^2\right)} - f(\hat{\varepsilon}) \right) \geq 0, \quad (\text{A5})$$

$$\frac{d\phi^*}{dt} = \frac{\partial \phi^*}{\partial t} + \frac{\partial \phi^*}{\partial q} \frac{\partial q^*}{\partial t} = \frac{\rho(q)(1-\beta)(v+b)}{\psi - \rho(q)(1-\beta)f(\hat{\varepsilon})} \left(\frac{(1-F(\hat{\varepsilon}))\psi\phi}{k + \frac{\psi}{2}\phi^2} - f(\hat{\varepsilon}) \right) \geq 0, \quad (\text{A6})$$

$$\frac{d\hat{\varepsilon}^*}{dc} = \frac{\partial \hat{\varepsilon}^*}{\partial c} + \frac{\partial \hat{\varepsilon}^*}{\partial q} \frac{\partial q^*}{\partial c} = \frac{-\psi}{\psi - \rho(q)(1-\beta)f(\hat{\varepsilon})} \left(1 + \frac{\psi\phi^2}{\left(k + \frac{\psi}{2}\phi^2\right)} \frac{F(\hat{\varepsilon}) + \beta(1-F(\hat{\varepsilon}))}{(1-\beta)(1-F(\hat{\varepsilon}))} \right) < 0, \quad (\text{A7})$$

$$\frac{d\hat{\varepsilon}^*}{db} = \frac{\partial \hat{\varepsilon}^*}{\partial b} + \frac{\partial \hat{\varepsilon}^*}{\partial q} \frac{\partial q^*}{\partial b} = \frac{(1+t)\psi}{\psi - \rho(q)(1-\beta)f(\hat{\varepsilon})} \left(\frac{k - \frac{\psi}{2}\phi^2}{k + \frac{\psi}{2}\phi^2} \right) \geq 0, \quad (\text{A8})$$

$$\frac{d\hat{\varepsilon}^*}{dt} = \frac{\partial \hat{\varepsilon}^*}{\partial t} + \frac{\partial \hat{\varepsilon}^*}{\partial q} \frac{\partial q^*}{\partial t} = \frac{(v+b)\psi}{\psi - \rho(q)(1-\beta)f(\hat{\varepsilon})} \left(\frac{k - \frac{\psi}{2}\phi^2}{k + \frac{\psi}{2}\phi^2} \right) \geq 0, \quad (\text{A9})$$

where, notice that, equilibrium existence requires $\psi > \rho(q)(1-\beta)f(\hat{\varepsilon})$. *Q.E.D.*

Proof of Proposition 2 Solving (12) and (13) for λ yields, respectively, $\lambda = u'(w)$ and $\lambda = u'(v+b)$, which implies $u'(w) = u'(v+b)$ and thus $w = v+b$. By inserting $w = v+b$ in (14)-(16), the conditions given in (18)-(20) follow straightforwardly. *Q.E.D.*

Proof of Proposition 3 In the absence of labour market policies, i.e., for $c = t = b = 0$, the

³²Notice that (7) and (8) have been used to simplify the expressions in (A4)-(A9).

equilibrium is given by

$$\phi = \frac{\rho(q)}{\psi} (1 - \beta) (1 - F(\widehat{\varepsilon})), \quad (\text{A10})$$

$$\widehat{\varepsilon} = v - \phi, \quad (\text{A11})$$

$$\rho(q) (1 - \beta) \int_{\widehat{\varepsilon}}^1 (y - v) f(\varepsilon) d\varepsilon = k + \frac{\psi}{2} \phi^2, \quad (\text{A12})$$

and the bargained wage is

$$w = v + \beta (y - v). \quad (\text{A13})$$

Comparing (A11) and (18) shows that, given the level of technology, equilibrium dismissal rates are at the first best level. Thus, equilibrium dismissal rates are too high (low) only if technology investments are too low (high). Further, a comparison of (A10) and (19) shows that, for a given level of q , equilibrium technology investments are below first-best if $\beta > 0$. Consequently, equilibrium dismissal rates are too high. Further, it follows from (A13) that $w > v$ whenever $\beta > 0$. Finally, for given levels of ϕ and $\widehat{\varepsilon}$, a comparison between equilibrium entry and first-best entry follows from a comparison of (A12) and (20). These two conditions can be written as, respectively,

$$\rho(q) (1 - \beta) = \frac{k + \frac{\psi}{2} \phi^2}{\int_{\widehat{\varepsilon}}^1 (y - v) f(\varepsilon) d\varepsilon} \quad (\text{A14})$$

and

$$\rho(q) - \rho'(q) q = \frac{k + \frac{\psi}{2} \phi^2}{\int_{\widehat{\varepsilon}}^1 (y - v) f(\varepsilon) d\varepsilon}. \quad (\text{A15})$$

For given levels of ϕ and $\widehat{\varepsilon}$, the RHS of (A14) and (A15) are identical. Notice also that the LHS of both (A14) and (A15) are increasing in q . It follows that the equilibrium level of q is above (below) the first-best level, implying that equilibrium entry is below (above) the first-best level, if $\rho(q) (1 - \beta) < (>) \rho(q) - \rho'(q) q$, which can be re-written as $\beta > (<) \eta$, where $\eta := \rho'(q) q / \rho(q)$. *Q.E.D.*

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