Universidade do Minho
Escola de Engenharia

Nabil BACHA

Decision support model for condition-based maintenance of manufacturing equipment

Modelo de apoio à decisão para a manutenção condicionada de equipamentos produtivos

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Doctoral Thesis for PhD degree in Industrial and Systems Engineering

Work performed under the supervision of
Prof.Dr. António Ismael Freitas Vaz

July, 2019

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"It is not possible to manage what you cannot
control and you cannot control what you cannot
measure!" (Peter Drucker)
"research creates more problems than it resolves".
"success*: the fact of getting or achieving wealth, respect, or fame*"

STATEMENT OF INTEGRITY

I hereby declare having conducted my thesis with integrity. I confirm that I have not used plagiarism or any form of falsification of results in the process of the thesis elaboration.

I further declare that I have fully acknowledged the Code of Ethical Conduct of the University of Minho.

University of Minho, 30/07/2019

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Signature: _________________

Abstract

Nabil BACHA. **Decision support model for CBM of manufacturing equipment.** 2019, 190 p. Doctoral thesis presented in industrial and systems engineering (thesis for the degree of doctor of philosophy, department of production and system. University of Minho-Guimarães.

Introduction: This thesis describes a methodology to combine Bayesian control chart and CBM (Condition-Based Maintenance) for developing a new integrated model. In maintenance management, it is a challenging task for decision-maker to conduct an appropriate and accurate decision. Proper and well-performed CBM models are beneficial for maintenance decision making. The integration of Bayesian control chart and CBM is considered as an intelligent model and a suitable strategy for forecasting items failures as well as allow providing an effectiveness maintenance cost. CBM models provides lower inventory costs for spare parts, reduces unplanned outage, and minimize the risk of catastrophic failure, avoiding high penalties associated with losses of production or delays, increasing availability**.** However, CBM models need new aspects and the integration of new type of information in maintenance modeling that can improve the results. ***Objective:*** The thesis aims to develop a new methodology based on Bayesian control chart for predicting failures of item incorporating simultaneously two types of data: key quality control measurement and equipment condition parameters. In other words, the project research questions are directed to give the lower maintenance costs for real process control. ***Method:*** The mathematical approach carried out in this study for developing an optimal Condition Based Maintenance policy included the Weibull analysis for verifying the Markov property, Delay time concept used for deterioration modeling and PSO and Monte Carlo simulation. These models are used for finding the upper control limit and the interval monitoring that minimizes the (maintenance) cost function. ***Result:*** The main contribution of this thesis is that the proposed model performs better than previous models in which the hypothesis of using simultaneously data about condition equipment parameters and quality control measurements improve the effectiveness of integrated model Bayesian control chart for Condition Based Maintenance.

Keywords: Bayesien Control Chart, CBM, Case studies, Monte Carlo Simulation, Optimization, Probability Theory, Renewal Theory, Stochastic Process.

Resumo

Nabil BACHA. **Modelo de apoio à decisão para a manutenção condicionada de equipamentos produtivos.** 2019, 190 p. Tese de Doutoramento em Engenharia Industrial e de Sistemas, Universidade de Minho. Guimarães.

Introdução: Esta tese descreve uma metodologia para combinar *Bayesian control chart* e CBM (Condition- Based Maintenance) para desenvolver um novo modelo integrado. Na gestão da manutenção, é importante que o decisor possa tomar decisões apropriadas e corretas. Modelos CBM bem concebidos serão muito benéficos nas tomadas de decisão sobre manutenção. A integração dos gráficos de controlo Bayesian e CBM é considerada um modelo inteligente e uma estratégica adequada para prever as falhas de componentes bem como produzir um controlo de custos de manutenção. Os modelos CBM conseguem definir custos de inventário mais baixos para as partes de substituição, reduzem interrupções não planeadas e minimizam o risco de falhas catastróficas, evitando elevadas penalizações associadas a perdas de produção ou atrasos, aumentando a disponibilidade. Contudo, os modelos CBM precisam de alterações e a integração de novos tipos de informação na modelação de manutenção que permitam melhorar os resultados.***Objetivos:*** Esta tese pretende desenvolver uma nova metodologia baseada *Bayesian control chart* para prever as falhas de partes, incorporando dois tipos de dados: medições-chave de controlo de qualidade e parâmetros de condição do equipamento. Por outras palavras, as questões de investigação são direcionadas para diminuir custos de manutenção no processo de controlo.***Métodos:*** Os modelos matemáticos implementados neste estudo para desenvolver uma política ótima de CBM incluíram a análise de Weibull para verificação da propriedade de Markov, conceito de atraso de tempo para a modelação da deterioração, PSO e simulação de Monte Carlo. Estes modelos são usados para encontrar o limite superior de controlo e o intervalo de monotorização para minimizar a função de custos de manutenção.***Resultados:*** A principal contribuição desta tese é que o modelo proposto melhora os resultados dos modelos anteriores, baseando-se na hipótese de que, usando simultaneamente dados dos parâmetros dos equipamentos e medições de controlo de qualidade. Assim obtém-se uma melhoria a eficácia do modelo integrado de *Bayesian control chart* para a manutenção condicionada.

Palavras-chave:*Bayesien Control Chart*, CBM, Estudo de caso, Optimização, Processos estocásticos, Simulação Monte Carlo,Teoria da probabilidade, Teoria da Renovação.

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Nomenclature

Symbol	Description
A	Event
A(t)	Availability
B	Event
C	Cost
C_{ij}	The expected cost of monitoring and false alarm
g(t)	The performance function
$\mathbf{E(CC)}$	Average long run expected cycle cost
$\mathbf{E(CL)}$	Average expected cycle length
$\mathbf{E(UT)}$	Average expected cycle length
$\mathbf{E[N(t)]}$	The expected number renewals
$\mathbf{F}_k(\mathbf{t})$	k-fold convolution of F(t)
$\boldsymbol{f(t)}$	Probability density function
$\boldsymbol{F(t)}$	Cumulative distribution function
$\boldsymbol{Ga(x;\beta(t),\lambda,\mu)}$	Probability density function of gamma distribution
g(t)	The performance function
$\mathbf{H}_{\boldsymbol{i}}$	Event i (outcome)
$\mathbf{h}(\boldsymbol{x})$	The hazard function
$\mathbf{H(t)}$	The renewal function
$\mathbf{h^*}$	Optimal interval monitoring
$\mathbf{L_1}$	First stage (from normal state to warning state)
$\mathbf{L_2}$	Second stage (delay time failure)
$\boldsymbol{M(t)}$	Maintainability

m(t)	Density distribution function of maintainability
M	Medium
N(t)	The total number of events
P(A)	Probability of event A
P(B)	Probability of event B
p_{ij}	Transition probability from current state i to next state j
P(A/B)	Probability of A given that B is true
P(B/A)	Probability of B given that A is true
$\mathbb{P}(t)$	Probability at time t
$\mathbf{P}(\mathbf{H}_i)$	The probability of an event H_i) will occur before the collection of new data
$\mathbf{P}(\mathbf{Y}/\mathbf{H}_i)$	Likelihood function (prior probability)
$\mathbf{P}(\mathbf{H}_i/\mathbf{Y})$	Posterior probability of any one of events (H_i)being true given data
$\|\mathbf{P}ij\|$	Transition matrix of Markov chain
$\mathbf{P}_k(\mathbf{t})$	Probability of that number of arrival event.
p^*	Upper control limit (posterior probability)
q(t)	The load effect
r(t)	The instantaneous resistance
R(t)	Reliability function
$\mathbf{S_k}$	The waiting time
SD	Standard Deviation
Sig	Signifance value
T	Time space
T'	Time to Failure
$\mathbf{T^2}$	Control chart Hotelling'
U	Space states
X_i	Inter-occurrence time

$\overline{X}$	X-bar control chart
$\mathbf{X}(\theta)$	Random variables
$\boldsymbol{Y_t}$	State at time t

λ (t)	The failure rate function
Ω	State space
$\boldsymbol{\Gamma}(.)$	The gamma function
$\boldsymbol{\beta}(\mathbf{t})$	The shape function
$\boldsymbol{\lambda}$	Scale parameter
$\boldsymbol{\mu}$	Location parameter
$\boldsymbol{r}$	Correlation Coefficient
θ	Parameters

Subscripts

0	Initial time
1	Normal state
2	Warning state
3	Failed state
i	Current state
j	Future state

Superscripts

k	Number of variable
prevmain	Preventive maintenance
monit	Monitoring
f	Failed state
Inspe	Inspection

List of abbreviations

Symbol	Description
ARL	Average Run Length
AHP	Analytic Hierarchy Process
CBM	Condition-Based Maintenance
CSMC	Continuous State Space and Discrete Time Space
CSMP	Continuous State Space and Continuous Time Space
DSMC	Discrete State Space and Discrete Time Space
DSMP	Discrete State Space and Continuous Time Space
DTM	Delay Time Modelling
EWMA	Exponentially Weighted Moving Average Chart
EM	Expectation Maximization Algorithm
FIT	Failure in time
HMM	Hidden Markov Model
IAEA	International Atomic Energy Agency
IEEE glossary	International Journal of Electronics and Electrical Engineering
ISO	International Standard organization
IDD	Independent Identically Distributed
MUCUSUM	Multivariate Cumulative Sum Control Chart
MLE	Maximum likelihood estimation
OEE	Overall Equipment Effectiveness
KMO	Kaiser-Meyer-Olkin
PCA	Principal Component Analysis
PHM	Propotional Hazard Model
PSO	Particl Swarm Otpimization
ROCOF	Rate of Occurrence of Failure
SMDP	Semi Markov Decision Process
SEOR	System Engineering and Operation Research

Chapter 1

General Introduction

This chapter will introduce the research topic of the thesis, and its motivation. It will then describe the scope and the objective of the thesis. The adopted methodology will be presented and justified. In addition, a description of the research design for this study was made. Finally, the outline of the thesis is considered.

Contents

1.1 Problem outline and motivation

Nowadays companies are surviving in a community that is characterized by an extensive international competition, fast and lasting industrial system development. Indeed, due to the fast development of industrial systems and productive paradigms, the industrial systems become more and more complicated requiring a high quality and reliability. In such environment, it implies that companies have high levels of competitiveness and maturity able to give better manage for their manufacturing resources and human capacities as well as manufactured equipment. A companies that can improve and manage its manufactured equipment, resources and human capacities can achieve competitive advantage over its competitors and improve its productivity. However, the most common challenge that companies are facing is making well maintenance decision.

In today's globalized worlds, manufacturing companies, nuclear plants and aircraft industry seek continuously to improve their managing manufactured performance requirements in order to ensure rapid growth and long term-job. Some typical key performance requirement can be associated with: operating cost, asset availability, and safety, lost time injuries, customer's satisfaction, maintenance cost, business costs, productivity, and number of environmental incidents, Overall Equipment Effectiveness (OEE) and asset utilization. Yet, in recent year, advancements in technology have allowed researchers and mangers to remain significant progress in addressing all of these compositely. In fact, it is evident that companies which can manage and improving its performance requirements can have a great impact on customer needs(e.g., quality, price and delivery), and generate a great value. This is illustrated in Figure 1.1

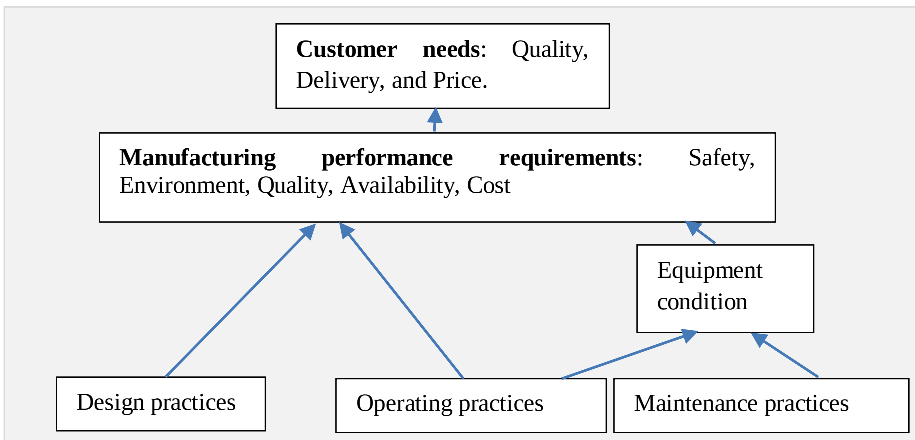


Figure 1.1-Managing manufacturing performance requirements to meet customer needs

In fact, for ensuring productivity production systems must have a high availability and well controlled, once the shutdown of production system during one day in companies can have significant economic consequences. Nevertheless, scientists and managers wondered about the effect of production interruptions originated by item's condition while a condition of an item plays a crucial role in the production systems. Hence, unreliability of item is mainly problem that may have a great influence on companies in terms of project budget, production scheduling and production quality.

Even though production system is still working, there could be one item or more that work under specific condition, the most important is how to deal with this problem which might trigger to terrible circumstances. A way to solve availability and reliability of item problem is to use

Maintenance function. The last is fundamental to ensure the availability of item, reducing production loss, maintain the inherent capability of an item, and increasing reliability. This can be attained through the development and implementation of a rigorously maintenance management and adopted best practices. Maintenance department in organization is typically the responsible for providing the roles, responsibilities, resources and procedures, maintenance process, used to carry out the maintenance program. Thus, maintenance practices are required to restore the item's function in order to maintain the productivity of the industry. Eventually, Maintenance function remains a powerful tools to make a huge contribution to the industry and well controlled the health state of an item, however the cost spent on maintenance activities became higher. Today, for the most part of practitioners and academic researchers are aware of is the fact the costs incurred to keep an item in good condition has become very costly. Hence, previous research has shown that over 70% of the total production cost can be spent on preventive maintenance (Amar et al., 2006). Further, as much as one-third of the maintenance cost is wasted due to the fact that's its unnecessarily incurred (e.g. inappropriate planning, overtime cost, useless preventive maintenance, maturity level.).Indeed, the effectiveness of company is mainly influenced by the maintenance role and impact on other working areas such as production, quality, production cost (Al-Najjar, 2007).

Since 1979, the maintenance expenses for many industries have increased by 10-15% per year (Wireman 1990).Eventually, preventive maintenance has become a major expense of many industrial companies. Close analysis of case studies shows that more efficient maintenance approaches such as Condition-based maintennce (CBM) are being implemented to handle the situation (Jardine et al., 2006). On another hand, the annual cost of maintenance (corrective and preventive) as a fraction of the total operating budget, can go up to 15% for manufacturing companies, 10%-40%for the mining industry, 20%-30% for chemical industry (Nguyen et al., 2008) and 40% for iron and steel industries(Chu et al., 1998). Therefore, the development of maintenance technologies has become more and more important for maintenance optimization. CBM or predictive maintenance utilizes appropriate condition monitoring techniques and maintenance technologies to increase the efficiency and profitability of industrial systems. It is considered an effective approach to improve availability and reduce maintenance cost (Saranga

& Knezevic 2001). Predictive maintenance is based on probabilistic fault prediction which takes into consideration the deterioration caused by incipient fault.

Obviously, the partial or total loss of item function may have impact in the quality of manufactured products, environment integrity and in the safety of production systems, once it can cause physical damages, the degradation of item affects product quality. In addition, it is in the interest of maintenance manager to know the relationship between product quality and health state of item. Therefore, the relationship between product quality and equipment performance has led to encourage practitioners and researchers to design integrated models that have shown to be an effective way to improve productivity and lowering cost (Mehrafrooz & Noorossana 2011; Bengtsson et al. 2010).Further, research is needed to clarify the relationship between quality product and health sate of item. Recently, however, awareness of the development of intelligent model as well as incorporating a new information has increased. To date, no models were found in the literature that use simultaneously data from both quality control and equipment's condition monitoring. In bridging this literature gaps, an effective model of maintenance decision making is proposed.

There is a rapidly growing literature on maintenance decision making, which indicates that integrated models offer economic benefit and showing an excellent result. Literature on maintenance decision making has focused almost exclusively on two main integrated model as follow: The proposed models in the literature used quality characteristics plotted in a control chart for maintenance planning. Another type of models use equipment parameters plotted in a control chart to take a decision about the necessity of a preventive maintenance action.

1.2 Research scope and objective

The performance and competitiveness of manufacturing companies depends on the reliability, its manufactured item' s availability and product quality. It doesn't depend on that only, but this is what we focus on in my research study. Based on these, to ensure that item are performing at the required level to meet production and quality goals, an accurate maintenance decision and appropriate performance appraisal system must be put in place. The level of health state of item indicated by likelihood of failure and deterioration degree is determined by comparing them to the threshold (specific value). To ensure that companies achieves the desired production system's performance and maturity, manufacturing companies entails to adopt new management maintenance practices. In general manner, the process of maintenance management involves: knowledge acquisition, asset maintenance strategy, asset maintenance work, maintenance support.

Furthermore, since equipment in a production system has become more advanced, more expensive, and may have a significant impact on production function and products quality, the cost spent in preventive maintenance has become higher and higher. Therefore, Preventive maintenance has become a major expense of many industrial companies. Maintenance function may lead to the success of a production quality, depending on its level of effectiveness and maturity. This effectiveness of maintenance is likely connected to the quality of product and is worthy to be studied in order to increase customer satisfaction.

Maintenance strategies have progressed from breakdown maintenance, to preventive maintenance, and then to CBM. Breakdown maintenance is reactive in nature while the action of repair or replacement is done only when equipment has already failed. Preventive maintenance is proactive in nature and consists of a set of tasks (replacements, adjustments, inspections and lubrications) in order to prevent catastrophic failures or to eliminate any degradation in equipment. If the deterioration level of items is correlated strongly with a control parameter, the decision about the realization of preventive maintenance operations can be based on system's condition, this is CBM(Rabbani et al., 2008). CBM is a set of maintenance actions performed based on real-time or near real-time assessment of equipment condition obtained from embedded sensors, external tests or measurements using portable equipment. The data obtained from condition monitoring helps maintenance managers to decide if maintenance is necessary or not by analyzing the actual condition of equipment (Jardine et al., 2006). Therefore, CBM is considered as an intelligent preventive maintenance and a suitable strategy for forecasting items failures. CBM models provides lower inventory costs for spare parts, reduces unplanned outage and minimize the risk of catastrophic failure, avoiding high penalties associated with losses of production or delays. The main goal of CBM models is to avoid unnecessary maintenance tasks by taking maintenance actions only when there is evidence of abnormal behaviors of an item (Jardine et al., 2006). CBM models can be more cost effective than time-based maintenance and is one of the least expensive and most effective strategies for proactive maintenance (Yang, 2003); (Ilangkumaran and Kumanan, 2009).

According to literature, several authors have recognized that using statistical control chart for maintenance planning leads to a significant benefit in term of reducing maintenance costs (Cassady et al., 2000; Jardine et al., 2006). Nowadays, several maintenance models using control charts for maintenance optimization has been extensively developed, and they can be classified into two different groups. The first one regards the use of control chart to monitor a process through some quality characteristic for maintenance planning; the second one regards the use of control chart to monitor the health state of equipment while in operation through a health parameter. Control charts might be considered an effective tool to indicate and detect the early signs of health state of a deterioration or malfunction of equipment. In this case, the control chart uses parameters values obtained from periodic condition monitoring of equipment, in order to decide whether maintenance action should be performed or not. However, that Bayesian control charts are been proved to be superior tools to control the process compared with the non-Bayesian charts (Yin, 2012). In fact, there is a direct relationship between equipment maintenance and product quality, products quality is often affected by the equipment health state. Ultimately, improving equipment performance would also enhance product quality (Cassady et al., 2000). Nowadays, in maintenance area decision-maker face challenge in term of appropriate and accurate decision. A proper and well-performed CBM models are beneficial for maintenance decision making. However, CBM models need new aspects and integrate a new type of information in maintenance modeling that can improve the results.

In recent years, a lot of works have been proposed for decision making in CBM implementation. However, there are notable gape in the literature with regards to relationship between quality control and item condition parameters, thus, on the basis of the information and literature currently available no work was found about the simultaneous use of data from quality control measurements and from condition item monitoring. Indeed, a better understanding the relationship between quality product and condition item monitoring makes condition based maintenance model proposed in this study an effective way will allow us to improve efficacy maintenance applications. The need to reduce uncertainty about maintenance decision making brought to the fore the need to use to use both type of data to provide information to decision maker in order to decide if a maintenance action should to be performed or not.

This present work based on mathematical model and optimization methodology, aim to find out the optimal values of parameters that minimize the hourly maintenance cost. In the present study, the issue under scrutiny is how to design an intelligent model for maintenance decision making. This research is therefore study impending failure of Atox Mill Equipment, and whether it is failed or not. The case study was applied in a Portuguese cement company (Secil-Outão). The study was conducted on Atox Mill Equipment, one of the most important equipment in cement industry. Therefore, in response to the purpose of this study, condition parameters data related to this equipment and quality control measurement of Raw Mill were gathered and statistically analyzed. The main aims of this research study is intend to highlight the problem refers to maintenance decision action. An added advantage of this study was provide further improvement about effectiveness of integrated model for condition based maintenance.

The intended purpose of this new design model under study is to provide an effectively maintenance cost and an accurate maintenance action. In order to achieve the research objective of this thesis, this study will provide an explanation as how to design new integrated model for condition based maintenance by completing the following tasks presented below:

- **Task 1: Literature review**
  - Description:
    - Gather all important researches from various scientific sources in order to obtain an overview of the state of the art of the research area.
    - Evaluate the relevance, value and sufficiency of the bibliography found.
    - Study and analyze the existing models.
    - Study the feasibility of application of theoretical knowledge and methods for the construction of the model.
- **Task 2: Design the integrated model**
  - Description:
    - Identification of the model parameters.

- Defining a framework that describes the behavior of the considered system as a basis for the construction of the model.
- Select the appropriate techniques for system behavior modeling such as stochastic approach mathematical theory and probability distribution.
- Defining the assumption to be considered for designing the model.
- Developing the mathematical model that attempt to explain the system behavior, to study the effects of different parameters and also to allow the prediction of future system behavior.

➢ **Task 3: Define the optimization methodology**

- Description:

  The objective of this task is to define the method to be used in order to find the values of the decision variables which lead to an optimal value of the objective function. This task involves the followings steps: (i) Define or select the optimization algorithm that has the best performance for the defined model. (ii) Design the flowchart of the optimization algorithm using standard flowchart symbols.

The optimization software more widely used to optimize multi-objective function subject to constraints is MATLAB. The optimization toolbox of MATLAB offers a collection of functions that extend its capability and allows computing, visualization and programming. The optimization toolbox includes routines for many types of optimization such as unconstrained nonlinear minimization, constrained nonlinear minimization, constrained linear least squares, in this study bound constraint continuous optimization was selected.

➢ **Task 4: Evaluation and validation of the model**

- Description:
  - Applying the designed model on a particular case in order to demonstrate its economic benefit.
  - Evaluation of the new model through results analysis in order to demonstrate the usefulness and effectiveness of the model that has been constructed.
  - Searching for errors in the model.

- **Task 5: Writing and refining dissertation**
  - Description:
    - Writing thesis.

1.3 Research approach adopted and methodology

This chapter outline the research approach adopted and methodology. The research methodology related with this study refer to quantitative approach. Data were collected through two ways: the first one is condition monitoring technique aimed to record the Atox Mill equipment's parameters and the second one portable measure equipment aimed to record the size of dust's particle.

Nowadays, maintenance area decision-maker face challenge in term of appropriate and accurate decision. A proper and well-performed CBM models are beneficial for maintenance decision making. However, CBM models need new aspects and integrate a new type of information in maintenance modeling that can improve the results. According to literature, a lot of works have been proposed for decision making in CBM implementation. However, no work was found about the simultaneous use of data from quality control and from equipment condition monitoring. Therefore, the mainly objective of this research work was envisaged to address the gap identified. Using simultaneous both type of data quality control and equipment condition monitoring may improve results as well as reducing decision maker 'error. The new model was developed in this work promising to reducing uncertainty about maintenance decision. In doing so, five fundamental questions are raised, this study is motivated by those questions which are to be the focus of this research project: (i) How to identify failure model (failure mechanism)? (ii) How to identify degradation model? (iii) How to estimate parameters model? (iv) How to build a cost model for a CBM application?,(v) How to design an optimal Bayesian control that involves optimal inspection. Likewise, research objective of this study was depending on research questions. Ultimately, the specific objectives are derived to provide focus for the research activities, in order to fulfill the specific aim of the research in a structured and scientific manner (see table 1.1).

Table 1.1- Relationship between research questions and research objectives

Research question	Research objective
How to identify failure model (failure mechanism)	to establish probability
How to identify degradation model	to establish transition probability between all possible state to another
How to estimate parameters model	to estimate posterior probability distribution
How tobuild a cost model for a CBM application	to estimate expected average cost per unit time
How to design an optimal Bayesian control that involves optimal inspection	to design optimization model for decision

As stated by Herbert J. Rubin (1983): "The objective of academic research, whether by sociologists, political scientists or anthropologists, is to try to find answers to theoretical questions within their respective field. In contrast, the objective of applied social research is to use data so that decisions can be made". The explanation of the research process that we used for this academic research is too important. The fundamental idea behind it is that description of research process can be provide for the reader sufficient information such as : (i) the readers need to know the reason why we select a specific method instead others, (ii) the readers need to know the whole steps, decision or choices made at different level of understanding about the research to be conducted, (iii) the reader need to know the general approach and goal of study, often the result of thesis stimulates new future work and fresh research questions, (iv) It is proof that we use a scientific method in a systematic manner in order to conduct research project, (v) through this section, my work's validity is judged.

Research strategies	Research question
Experiment, history and case study	How, Why
Action research	How?
Grounded history, ethnography, survey and archival analysis	Who, What, Where, How many, How much?

Table 1.2- Relationship between research questions and research strategies.

Research design can be viewed as plans and procedure for research that span the decision from board assumption to detailed methods of data collection and analysis (Cresweell, 2002). Indeed, the definition of research design will be considered as the general plan of how we will go about answering our research question. In another word, research design gives us a general plan for implementing the following: a research strategy, detail like whether the study will involve qualitative or quantitative approach, groups or individual, how many variables, interview or observation, case study or experiment, tools.

Conducting research process requires identifying (several terminology) the following research aspect: research method (e.g. qualitative or quantitative), research approach (e.g, deductive or inductive), and collecting data approach (structured response or unstructured), samples (small or large samples), research procedures (defining step by step a description of specific research study), and research strategies (a general approach to research determined by the kind of question that the research study searches to answer: test hypothesis or discover an idea), often the choice of research strategies depends mainly on the research questions to be answered and on research interest (figure 1.2). Nonetheless, the table below depicted a number of useful distinctions between research method and which is more adequate to different forms of research questions (Yin, 2002).

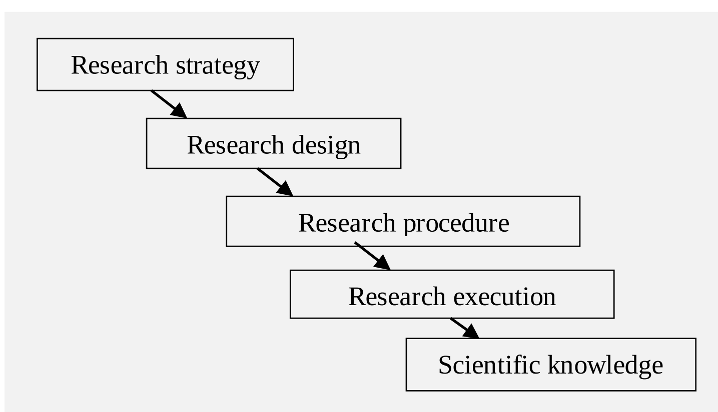


Figure 1.2- Scheme of the academic research.

Depending on the existing source related with CBM models, there are good reasons why the approach followed in this work is quantitative approach instead others (e.g., qualitative approach, mixed method approach). This approach is considered as a suitable and good design for this work for many factors such as: measure objective facts focus on variable, theory and data are separate, subject statistical analysis, researcher is not involved. The rationale for using quantitative approach in this study was to explore the relationship between dependent and independent variables as well as correlation and statistical descriptive between those variables.

Quantitative research can be twofold: the first one is known as experimental research (explanatory), and the second one can be viewed as a descriptive. The first one tests the accuracy of a theory by determining if the independent variable(s) causes an effect on the dependent variable, so that we will be able to illustrate the relationship between variables. Often, surveys, correlation studies, and measures of experimental outcomes are evaluated to establish causality within a credible confidence range. The second one measures the sample at a moment in time and simply describes the sample's demography (Lowhorn, 2007; Saunders et al., 2006). Taking into account the type of information needed and the nature of this work, deductive study is adopted in this work. Conducting research requires following a sequence of steps. In the quantitative approach viewpoint, there are essentially seven steps, Figure 1.3, illustrated how the study will be conducted.

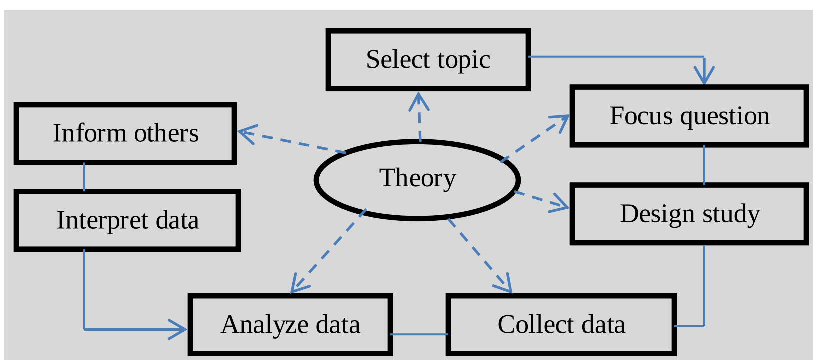


Figure 1.3- Steps in the quantitative research process.

The process of conducting a quantitative study begins with selecting research topic, in what concern to the nature of this work; the research topic can be addressed in this study is about decision support model for condition based maintenance of manufacturing equipment. Indeed, the research interests of this study are: to reduce maintenance cost and improving availability of the equipment. The second step is about defining research question which is considered as a crucial step, this because research topic is too broad for conducting this study.

The next steps are regrouped into four phases: design, development, implement, evaluate. Designing model step consists to develop the mathematical model that attempt to explain the system behavior, and to allow the prediction of future system behavior. This step involves identification of the model parameters; defining a framework that describes the behavior of the considered system as a basis for the construction of the model, select the appropriate techniques for system behavior modeling such as stochastic approach mathematical theory and probability distribution, defining the assumption to be considered for designing the model.

Define the optimization methodology step aims to define or select the optimization algorithm that has the best performance for the defined model, and also to design the flowchart of the optimization algorithm using standard flowchart symbols. Furthermore, the optimization software more widely used to optimize multi-objective function subject to constraints is MATLAB. The optimization toolbox of MATLAB offers a collection of functions that extend its capability and allows computing, visualization and programming. The optimization toolbox includes routines for many types of optimization such as unconstrained nonlinear minimization, constrained nonlinear minimization, constrained linear least squares and so on. In this research project, the optimization problem is formulated and solved in a Semi-Markov decision process framework, the objective is to minimize the long-run expected average cost as well as considering the availability maximization objective subject to an additional constraint that guaranteeing the occurrence of the true alarm signal. Thus, the aim is to find out the optimal values of parameters that minimize the hourly maintenance cost. Evaluation and validation of the model step is empirical inquiry based on case studies, involving quantitative data collection and analysis.

In this work applied research is considered, and applied for a specific research, the goal concerns about the application of research techniques, procedures, and methods for the purpose of enhancing understanding about a phenomena: predict impending failure of equipment. This applied research can be used to accomplish various aims such as: applying the designed model on a particular case in order to demonstrate its economic benefit, evaluation of the new model through results analysis in order to demonstrate the usefulness and effectiveness of the model that has been constructed, searching for errors in the model, testing the model within real world. Depending upon the desired outcome of the research and how to acquire the knowledge, there is two possibilities may choose between inductive or deductive approach, the research performed in this work follow a deductive approach, this approach move from general to specific often begins with theory for principally on testing the theory (figure 1.4). This theory gives formal and logical explanation of some events that include prediction of how things relate to one another (Zikmund, 2010).

The theory supported of this research supporting that the integrated model based on Bayesian control chart and CBM can be predict impending failure. That is used to pursue the next step which is hypothesis. The hypothesis offered by theory stating that decreasing maintenance cost can be made through integrated model based on Bayesian control chart and CBM. After formulating the hypothesis, so this hypothesis proposed in this work is proved in the context of industrial partner of this thesis-a large sized manufacturer of cement industry (Secil). Once the hypothesis offered by theory are confirmed, the theory is supported, however if the hypothesis is rejected, the theory is not supported, in this case it is necessary to re-evaluated or modify the theory. The decision about accepted or rejected such hypothesis mainly based on the result of examination the specific outcomes.

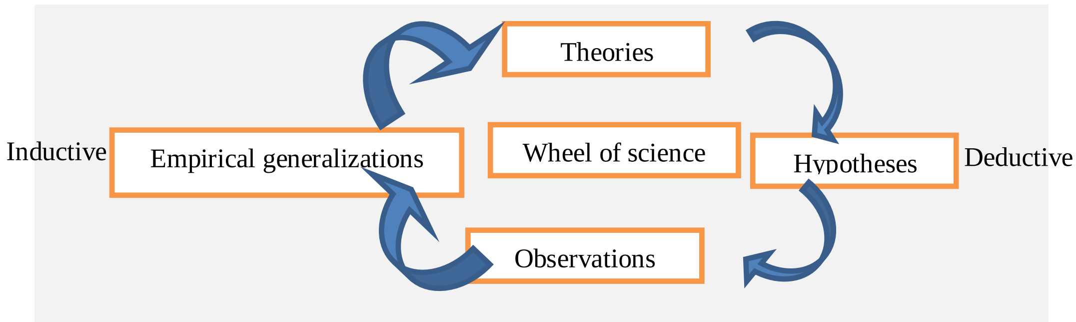


Figure 1.4-Induction and deduction approach (Walter Wallace, 1971).

According to literature there are essentially six major steps in the deductive process including: theory, hypothesis, data collection, findings, hypotheses confirmed or rejected, revision of theory (Bryman, 2006). The research processes adopted on this work is not strictly linear; it may flow in several directions before reaching the goal of this study. Figure 1.5 depicted overview about research methodology used in this study for answering the research questions, although, the series of decision about specific steps required to follow for completing the design study (adapted from Walter Wallace (1971)).

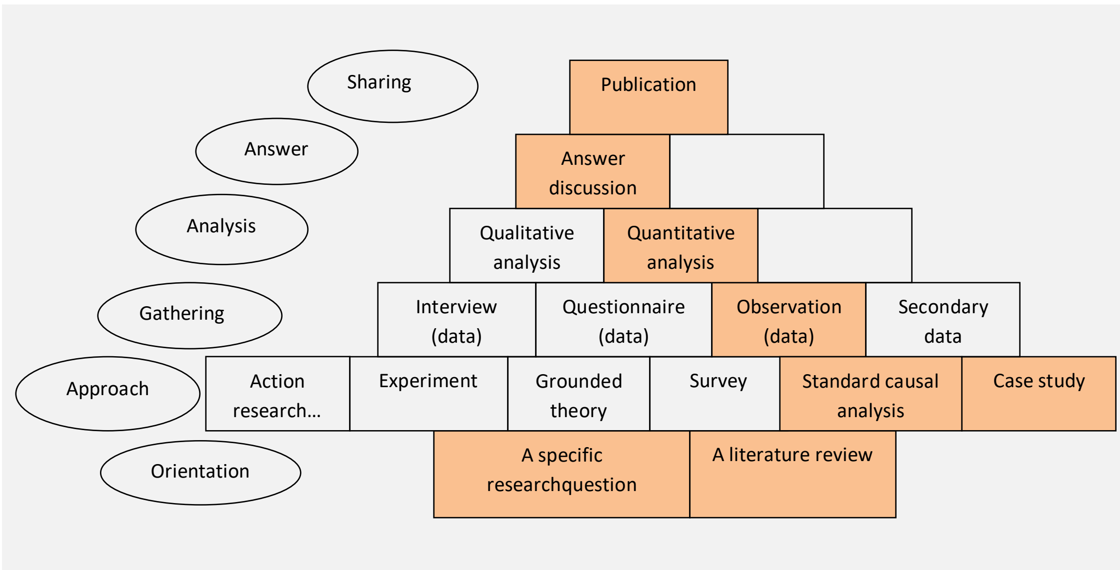


Figure 1.5- Depicted a scheme for general methodology of design any scientific research lead to scientific knowledge occurs with credibility, transferability, dependability.

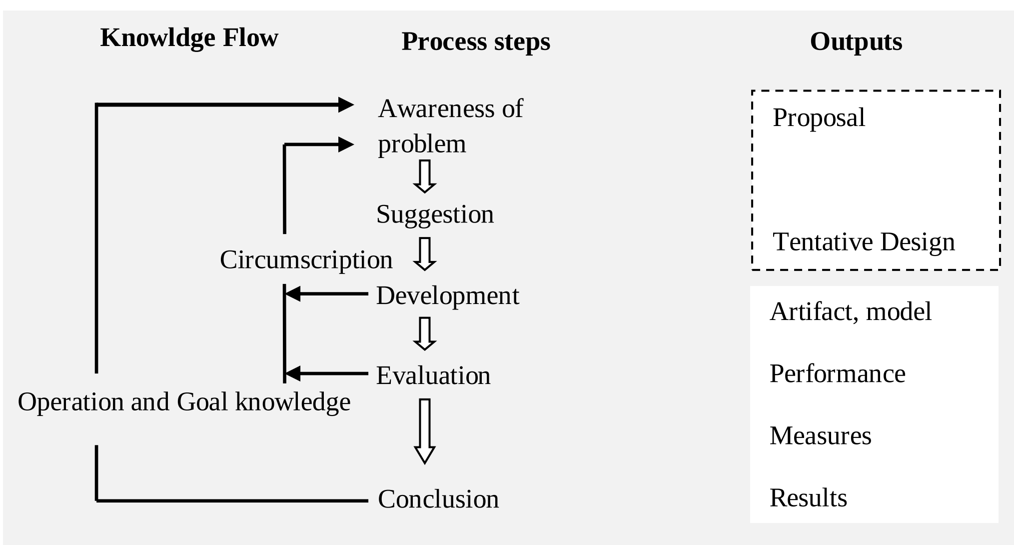


Figure 1.6- A scheme for general methodology of design science research (Vaishnavi and Jr., 2008).

This research project is an interactive process in which steps bled into each other, it applies one cycles of the steps, this research project uses scientific methods to transform idea into scientific knowledge, so results one article and many conferences. Fig 1.6 showed how to develop more sophisticated model by application (by involving) scientific method in order to ensure robust and better accuracy. The thesis aims to develop a new methodology based on Bayesian control chart for predicting failures of item incorporating simultaneously two types of data: key quality control and measured machine condition indicators. These observation can be fused by using Bayes theory to give a posterior probability estimate of the warning state which is unobservable (upper control limit), in this case the process is monitoring by plotting the posterior probability in control chart in which can be compared with a control limit to assess whether a full inspection is need or not. In this study, we review modelling approaches for system deterioration. An Matlab software for analyzing and estimate the transition probabilities in 3-State Models. State 1 and 2 are unobservable, represent normal and warning state, respectively. Only the failure state 3 is assumed to be observable. Although, failure modeling is important for designing such Bayesian control chart. The objective is to find a stopping rule under partial observations, minimizing the long-run expected average cost per unit time for a given sample size and sampling interval. An algorithm must be developed to find the optimal control limit and the minimum average cost. This approach are illustrated using real data obtained from condition monitoring technique collected at regular time epochs from Atox Mill used in cement industry (Secil Portugal). One of the main contributions of this a new approach is promote to give the lower maintenance cost for real process control.

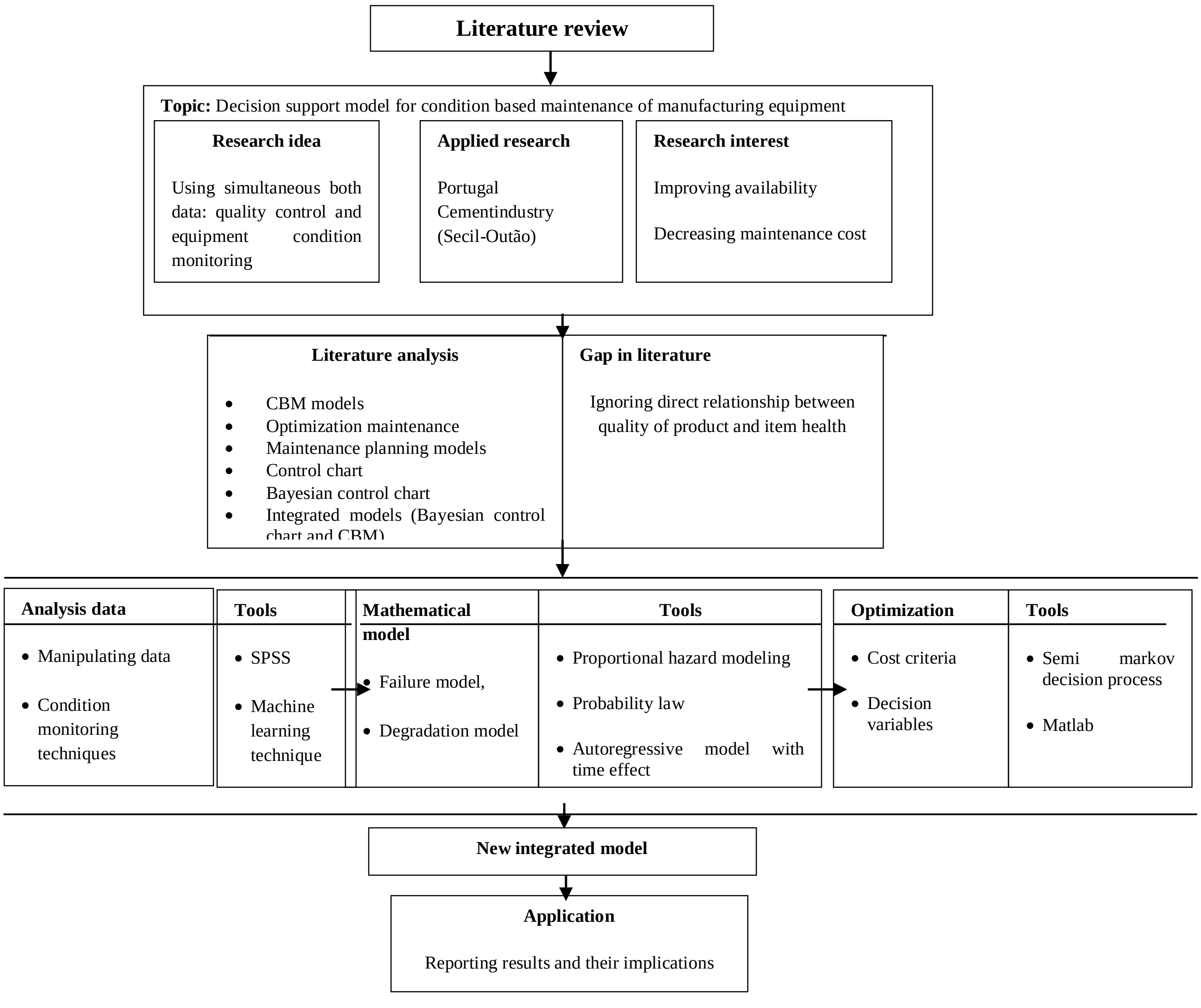


Figure 1.7- A scheme for research methodology of design this research project.

This research work is a multi-stage process that should be undertaken to achieve the goals set. The precise number of stages varies, but they usually include formulating and clarifying a topic, reviewing the literature, designing the research, collecting data, analyzing data, conclusion and finally writing up. Various models can be found in research textbooks such as hypothetico-deductive process, exploratory study, and inductive process.

To develop a decision making models, the scientific method includes a step to design the model. Furthermore, this research method fits with the objective of this project and, then, it will be followed for its development by addressing the following 3 steps:

- Literature review
    - Define the research question and objective.
    - Generate and refine the keywords.
    - Define the parameters related to the present work.
    - Understand the importance and purpose of the present research project based on a critical literature review.
    - Write a state of the art report.
- Design
    - Describe the system to be modeled and define its boundaries.
    - Analyze and describe the structure and requirements of the model.
    - Apply appropriate skills and theoretical knowledge to construct the model.
    - Evaluate the solution through training data.
- Validation:
    - Plan and design experiment.
    - Define the data collection process.
    - Perform experiment.
    - Record and analyze ongoing and final results.
    - Interpret the results and draw conclusions.

1.4 The structure general of the thesis

This thesis consists three parts are as follow: chapter 1 to 2 on general introduction and state of the art, it summarizes the main findings gathered from various sources (knowledge flux), chapter 3 on theory applied in CBM modeling, finally chapter 4 on the process of design model and conclusion (central design model section and output).

Part one: “knowledge flow and Research topic”

Chapter 1: This serves as an introduction to the reader in term of motivation, aims, scope, and research question and why these research area important, gape in literature and topic were. It also explains the research methodology that applies throughout the preparation of this study. The objective of the proposed work is presented in this part.

Chapter 2: This chapter provides some concept and background of the maintenance as well as summarizes the different type and models which are found in the literature. It also focuses on condition based maintenance models. It will then explain the importance of statistical control chart for condition based maintenance.

Part two:“A theory and tools applied for many practical problems”

Chapter 3: This chapter discuss the different stochastic processes and how they were applied to CBM modeling. Then, it will presents the important of Bayesian control chart in CBM and why it is the best tools for maintenance optimization as well as some practical application of that model in real world phenomena.

Part three: “Process of design and conclusions”

Chapter 4: This chapter presents research strategy on how we go about answered the research questions, as well as how we found out in response to these research questions.

Chapter 5: In this chapter, the practical application of the designed model on cement industry is discussed. The study case is about Atox Mill ’ equipment, this chapter also contains: introduce the company, data collected and analysis, techniques and algorithm that have applied for developed work, the analysis and results study in relation to the research questions are presented and discussed.

Chapter 6: This chapter draws general conclusion relative to this study, original contribution, significance of this work in maintenance field, and finally future research resulting from this work are provided.

Chapter 2

Literature review

This chapter presents a comprehensive state of the art that covers a short overview of theconcept key, terminology and characterizations related to maintenance. Although, it includes a board rang literature - related to CBM models of real problems. Several alternative maintenance models are available in the engineering literature with different ways which are hightlighed.The focus on this chapter will be the integrated model condition based maintenance and Bayesian control chart that are used in maintenance decision making.

Contents

2.1 Manufacturing Equipment Failure, Degradation, Fault, and Error

Machinery equipment has to keep at a high level of performance during their whole cycle life. However, it is difficult to the system run or has extremely high reliability without intervention of the people who works continuously to keep creating safety and keep continuing the system within the boundaries of tolerable performance (Cook, 2000). Each kind of system from manufacturing industry to aircraft system are dynamic feature and the behavior of item changing over time because of it is operating under certain stress or load in the real environment as well as the intervention of engineers and expert. The systems are inherently unavoidably of some of unwanted technical events such as degradation, failure, error, and fault. Consequently, when they will appear the results will be catastrophic as an accident or incident, even loss of working day and economic losses (e.g., failure of the server makes it hardly to serve the customer or user).

For avoiding all these technical events, system operation requires more robust system and management skills such as: designed safety system when new risks are being taken (O'Conner, 2002), intelligent model for prediction, new vision of maintenance, advanced condition monitoring technology, changing the behavior way, new regulation and rules, reducing degree of uncertainly, adapting a new technology in operating and controlling room, invest in staff experience for future. Therefore, in nature system operations are dynamic inclusive organization, human, and technical and a failure as well. The machinery equipment operation can be affect by four future technical events as follow:

Event 1-Degradation. The effect of this random event might be appeared as a failure where degradation process (degradation path) used as underlying to find out the occurrence of the failure. In fact, failure occurred when the amount of degradation beyond to the degradation or approximate level at which the failure would occur. Generally, an item is subject to degradation with time (age), usage (working hour) and obvious environment where it working. To assess reliability requires not only recorded the information of the failure time but also requires the amount of degradation because if, assess reliability only upon on failure time might be considered as difficult to assets. There are two ways to provide information or degradation data in such degradation measurement, the first one it is possible to measure physical degradation as a function of time (e.g., tire wear), the other one is considered as unobservable, in such degradation measurements can be result of monitoring performance degradation, e.g., power output (Meeker and Escobar, 1998).

Naturally, an item is subject to degradation that lose their utility over time, due to decay, damage or spoilage. For example, these are commonly found in electronic/electro-mechanic, pharmaceutical, chemical and goods(Wee and Widyadana 2013). The deterioration is described as an increasing stochastic process where the item has a threshold and it fails once the degradation level exceeds this identified threshold (Abdel-Hameed 2010). It's recognized that deterioration of item is a function of a set deterioration parameters. Some of examples of deterioration parameters are found in deformation level, are: corrosion level, diameter of shaft, temperature, depth of cut, and lubrication condition (Amari et al. 2006). It is recommended to model deterioration in terms of a time dependent stochastic process $\{X(t), t \geq 0\}$where $X(t)$ is a random quantity for all $t \geq 0$. A variety of stochastic process introduced for modeling the multi-stage of deterioration process or item can be found in literature(Singpurwalla 1995).

To develop such models for degradation process required start with deterministic description of the degradation process often in the form of a differential equation or system of differential equations. Then randomness can be introduced, as appropriate, using probability distribution to describe variability in initial conditions and model parameters like rate constants or material properties. However, the important challenge facing the degradation analysis is to find variables that are closely related to failure time and develop methods for accurately measuring these variables. Indeed, structural capacity deterioration is among the main causes of increasing failure probabilities of structural systems (Barone and Frangopol 2014; Makis, 2003).

Thus, the relationship between amount of degradation and failure time can be used to advantage in estimating reliability. However, when there is not a strong correlation between failures times and degradation, there may be not helpful by using degradation data instead of traditional censored failure-time data. There are two approach for modeling stochastic deterioration, we can use either a failure rate function or stochastic processes such as: Random Deterioration Rate, Markov Process, Brownian Motion With Drift, and Non-Decreasing Jump Process, Gamma Process (Noortwijk 2009).

Although, there are three following possible shape for degradation path: (i) linear degradation, (ii) convex degradation, (iii) concave degradation.

Event2-Failure. The term failure is commonly used in reliability engineering to designate the inability of an item, product or service to perform required functions on demand due to one or more defects (Bauer et al., 2006). The failure occurs in engineered system when the item stops performing its required function. In another word, failure occurs when item no longer can perform its intended function safely (Affonso, 2006). Indeed, Failure may refer to an event that can occur at any time and nearly at any place, it is always present by the nature own. The failure time is defined as the time when the actual degradation path crosses the critical level for the degradation path. Naturally, a failure occurs when the degradation level is found above the predetermined threshold or warning limit, (Cook, 2000). Therefore, the relationship between component failure and amount of degradation makes it possible to use degradation models and data to perform inference and prediction about failure time (Meeker and Escobar 1998).

There is a considerable amount of description the concept of failure on engineering books and papers. According to norm (NP EN 13306, 2001) failure is defined as a **s**tate of an item characterized by inability to perform a required function, excluding the inability during preventive maintenance or other planned actions, or due to lack of external resources. A failure is considered as the inability of any item to do what its users want it to do (Moubray, 1997). Blache and Shrivastava (1994), defined the failure as the termination of the ability of an item to perform a required function. Frawley (2002), described the failure as an event in which a previously acceptable product does not perform one or more of its required functions within the specified limits under specified condition. The Society of Automation Engineering's (SAE) “Reliability and Maintainability Guideline for Manufacturing Machinery and Equipment (M and E)” defines the failure as an event when equipment is not available to produce parts at specified conditions when scheduled or is not capable of producing parts or perform scheduled operations to specification. This is due of the following root causes: electrical, mechanical, environmental, thermal, physical. Therefore, it is necessary to perform an action while the failure occurs, under these circumstances failure of item can be a root cause of significant damage and health safety environment hazards (Tsang, 1995). In addition, there are other definition to the failure and characterization available in the engineering literature such as: Del Frate et al., (2011); Prasad et al., (1996); Tam and Gordon, (2009); Frate, (2013); Birolini (2007).

Generally, items may suffer two different types of failures as follows: The first one refer to soft failure (minor failure)mean there is a gradual loss of item's performance but neither total nor rapid, which result in item malfunctioning, on the whole, item is still considered to be in working state (the item is progressively wearing out). In that case, items continue to operate even if a minor failure occurred and has not yet been detected but under inferior conditions which incur a malfunction cost per time unit. Eventually, this failure can be identified at specified level of degradation and then corrected by repair. According to International Electro-technical Commission (IEC vocabulary), soft or a minor failure is also called a gradual failure.

The second one is called hard failure (major failure), which is non-repairable. In this case, the losses function of an item is total which renders the item completely out-of-function, and they need only replacement action. According to International Electro-technical Commission, failure is called as sudden failure while a sudden drop beyond the acceptable limits qualifies as a sudden failure. A major failure is fatal for the item, i.e. the item is no longer functional, and incurs a downtime cost per time unit until the item is replaced. In addition, an item which has malfunctions, due to an undetected minor failure, can still suffer a major failure with higher probability compared to a normally operating item of the same age. Both of these two type of failure are not directly observable and the actual items condition can only be detected through inspection. Hence, an items that have already suffered a minor failure are more likely to major failures where the probability of a occurrence major failure is higher in malfunctioning items then in normally operating ones (Meeker and Escobar 1997; Panagiotidou and Tagaras 2014).

Nevertheless, effectiveness of maintenance strategy is not focusing only on the profit but also on predict of the impending failure. Generally , in order to avoid failure three ways are required: the first one is a technical way action including an item failure prediction, reliability analysis and engineering, as well as inspections are periodically held to detect any failures and the inspected items are preventively maintained, repaired or replaced according to their condition (Panagiotidou 2014). The second one is human strategy by investing in knowledge, the last one refer to the variety of organizational, institutional, and regulatory defenses (e.g., policies and procedures, certification, work rules, team training) (Cook, 2000). However, failure changing constantly because of changing technology, work organization, and efforts to eliminate failures.

In addition, there are several good reasons why an item become failure, some of those reasons including: component failures, customer use/installation, condition field can also result in failure. Taking into those components, the consequence of failure can leading up to any significant catastrophic such as: process shutdown, damage property, and economic cost penalties. In terms of classification of failure, the last can be ranked according to its consequence such as: catastrophic, critical, major, and minor (Hammer, 1972).

According to literature, failure can be analyzed in terms of three main assumptions including: missing functionality, utilization context, item level. In order to determine the occurrence of failure defining the performance parameters for all functions, target levels, and acceptable limits are required. Generally, an item failure is often based not only in the age but also based on non-age related factors (e.g., Maintenance models is often based on non-age related factor).

In mathematical, failure probability of a system is defined as the probability of violating one or more limit states associated with the system failure modes. The performance function g(t) for a given failure mode is generally defined as:

$$g(t) = r(t) - q(t) \tag{2.1.}$$

Where r(t) and q(t) are the instantaneous resistance and load effect at the time instant t, respectively. Resistance and load are time-dependent random variables; for engineering systems, if no maintenance is considered, resistance is usually deteriorating over time, while loads are increasing (Barone & Frangopol 2014).

Event 3-Fault. According to the IEEE glossary, the fault can be viewed as the state of an item characterized by inability to perform a required function, excluding the inability during preventive maintenance or other planned action, or due to lack of external resources (e.g., short circuit, broken transistor, incorrect step, incorrect process, and incorrect data definition in a computer program). Furthermore, fault is totally different to failure; it can be interpreted as a physical defects, imperfections, or flaw that occurs as an erroneous state in hardware or software (software does not have age or wear out, it is a collection of instructions and code installed into the computer and cannot be touched or broken, hardware is any physical device, something that you are able to touch). For occurring failure requires multiple faults where, the fault state can be appeared only when at least one of the performance parameters will trespass the boundary (Cook, 2000).

In engineered system, the fault can be classified into three categories as follow: the first one is a permanent fault remains in existence indefinitely if no corrective action is taken (e.g., caused short circuit in transistor), the second one is a transient fault can appear and disappear within a very short period of time (e.g., caused by lightning), and the other one is an intermittent fault (e.g., weak solder joint) appear, disappears and then reappears repeatedly. Depending on fault cultures and professional categories (staff or line) failure has interpret upon various ways such as: failure with respect of each other's, e.g. Failed says production manager is when equipment stop working, failed says maintainer when consumption is higher, failed says safety officer if the leak creates a pool of oil on the floor in which people could slip.

Figure 2.1 shows variation of observed level of a performance parameters of an item versus time. Consider a path starting at $t_{0}=0$, where the observed performance satisfying target value, then a graduated performance starts deviate from the target value at some intermediate time t_1 which after some intermediate time, the observed performance exceeds the acceptable limits and then may be reaches the time t_2, at this time the item have to be in a fault state.

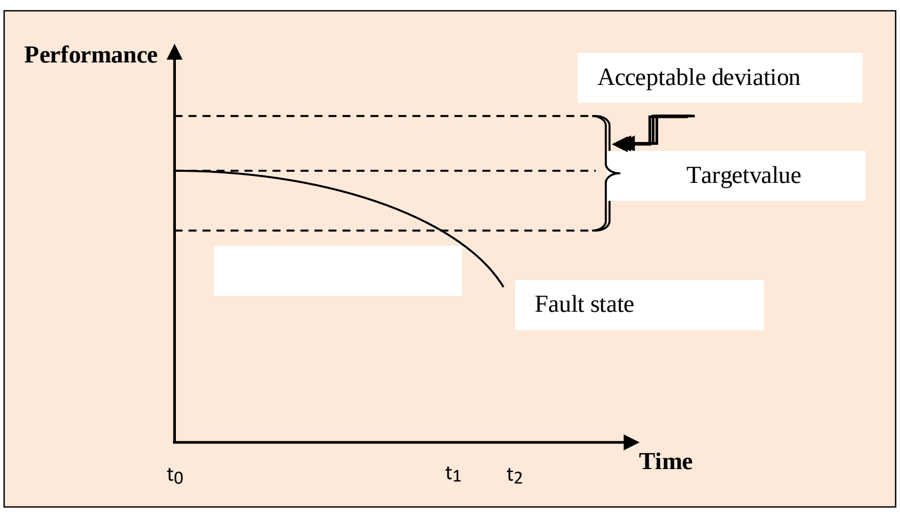


Figure 2.1- level of a performance variable of an item versus time (Rausand and Øien 1996).

In general, there are four origins of faults as follow: (i) specification mistakes refers to incorrect algorithms, incorrectly specified requirements (timing, power, environmental). (ii) Implementation mistakes such as: poor design, software coding mistakes, (iii) component defects such as: manufacturing imperfections, random device defects, components wear-outs, (iv) external factors such as: radiation, lightning, operator mistakes. Besides, it is very difficult analyze a system without assuming some fault models, because it is not easy to simulate faults or to design test procedure. In monitoring system, there are a set of defenses such as: (i) fault detection refers to identify that a fault has occurred, fault location, (ii) fault containment refers to prevent propagation of the fault, (iii) fault recovery refers to modify structure to remove faulty component.

Event 4-Error. Notion of error is relative and it depends on what the conceptually is applied, thus, although depending either on the framework interested such as: financial, statistic, decision or the culture of precision. For example: errors in the displayed data, radar error, compass error, error in interpretation, random error, password error. In engineered system, an error can be interpreted as a deviation from correctness or accuracy, thus, error is usually associated with incorrect value in the system state. In fact, scientist has to undertake to reduce and to avoid a potential error as much as possible regardless the field (e.g., organization, engineered system).

Therefore, the culture of error and environment should be developed in organization to change human behavior to find out: how to prevent error, what are the key role in organization to contribute an error, how to detect an error, how to reach understanding notion of the error and they must communication with only one way. Therefore, a methodology for identifying, predicting an errors, how to correct and error, required hazard analysis tools such as: HAZOP, CW, SHERPA,HAZAN. The prediction of human behavior in complex environments, assessment errors models (Power and Fox, 2014; Kletz, 1992; Kirwan, 1992). A schematic diagram showing all potential components of error:

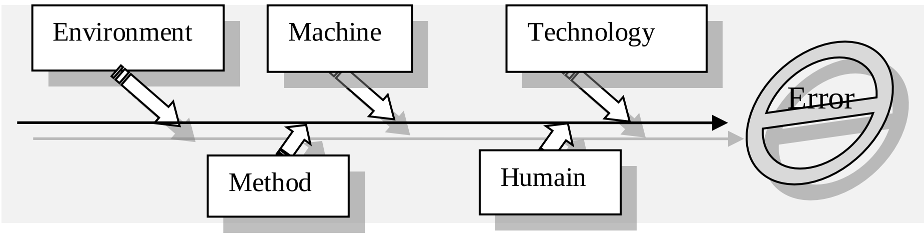


Figure 2.2-The five components of error.

Considering what have been mentioned before, obviously there are dependency between all these unplanned events (failure, fault, error, degradation), faults can result in error, error can lead to system failures, error are the effect of faults, failure are the effects of errors (Dubrova, 2013). A Figure 2.3 illustrate the relationship cause-effect between origins fault, error, failure, and fault state, and how they are impacting in a system and lead to the fault state.

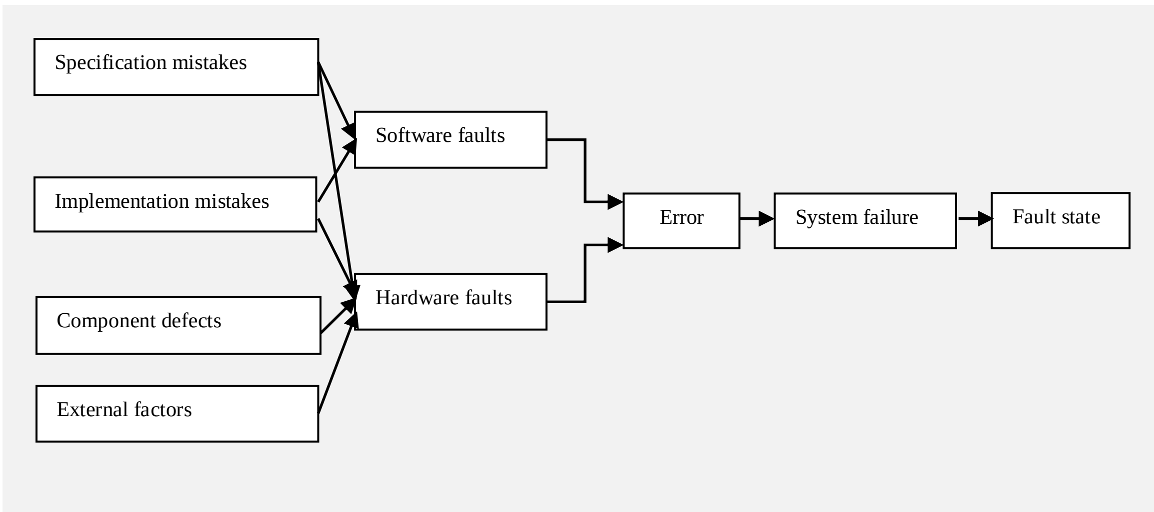


Figure 2.3-Cause effect relationship between fault, error, failure, and fault state.

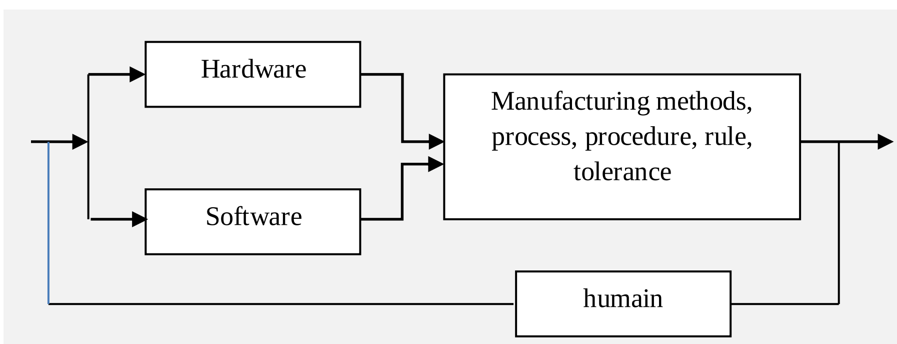


Figure 2.4-System processes.

Likewise, the sustainability of failures was made by researchers has played a key role in increasing the reliability and availability of system. Many engineers and researchers academic spend considerable capacity and resource in order to try to assure production with rate failure equal zero. A significant improvement has been made to develop a new methods and tools for understanding and avoiding failure, as results increasing the reliability and availability of systems which affect directly safety and economic operation of process such as shutdown or catastrophic accident especially for complex system (e.g., nuclear plant, aircraft, and power plant). Furthermore, an appropriate maintenance strategy can be considered as an efficient way to assure a satisfactory level of reliability during the useful life of items (Jardine et al. 2006; Barone & Frangopol 2014). Besides, an accurately modeling of degradation, life time, failure, risk, is major challenge facing for the engineering community. The source of uncertainties is not only related to the structural models but also to randomness inherent within natural phenomena. Although, a rational way to treat uncertainties arising from natural randomness, modeling, and prediction imperfections is to consider probabilistic approaches (Barone & Frangopol 2014). Figure 2.5 depicted Resistance, load effect and maintenance cost over time shows the effect of essential maintenance by assuming that the structural resistance is returned to its initial value after repair. Time delay for performing repair is not considered; therefore, the resistance is instantaneously increased at the repair time, and the cost of the maintenance intervention is concentrated at the same instant. It has to be noted that the load is, in general, increasing over time due, for example, to the increasing demand in terms of traffic load to which bridges are usually subjected. Such load is not affected by maintenance actions during the structural life-cycle.

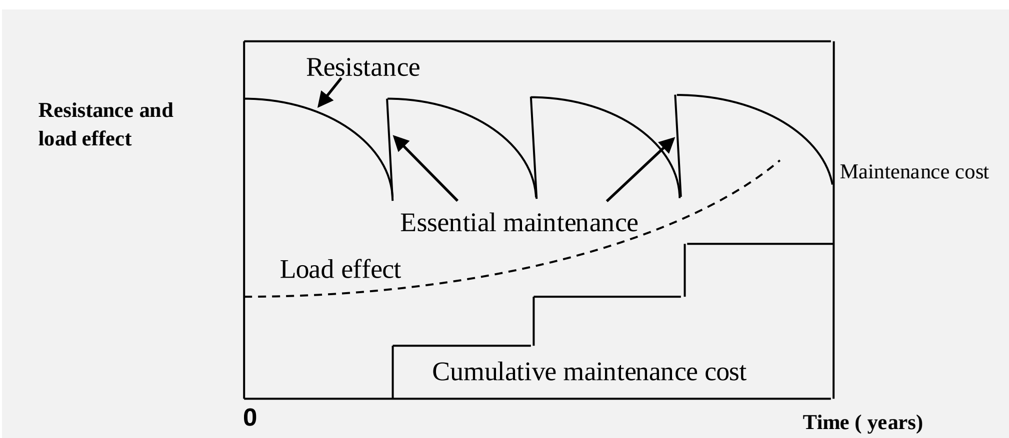


Figure 2.5-Resistance, load effect and maintenance cost over time (Barone & Frangopol 2014).

2.2 Dependability Measures: Reliability, maintainability and availability

Nowadays, dependability is becoming more important in the engineering area for both hardware and software system, this because due to the increasing a huge number of the system complex and automat programmable in manufacturing industry, nuclear power, aircraft. All of these system need to be operating within context of dependability. In modern community, researchers and engineers are making a great effort to improve efficiency maintenance cost through to enhance on the reliability, safety, availability, and maintainability in order to provide as possible as a safe operating climate. However, insufficient reliability of item generates high maintenance expense and unavailability of system or arresting of the service and eventual loss of market (Salata et al., 2014). Detecting the loss of the item's reliability is considered as an advanced warning that degradation has started. Thought, to detect this change in performance level through reliability measures ensure to forecast impending failure.

The maintenance is considered as one of the other ways that can increase the reliability of system. The lower reliability of an item have to decrease to the maximum by maintain and restoring the machinery item in which guaranteeing the highest efficiency and safety for their use. From mathematical point of view, the most commonly used definition of dependability measures is the following:

Reliability

There are many definitions of reliability that can be found in papers and books, thus, various ways that the concept reliability has been applied vary from industry to industry, and from user to user, as well as cycle life of items. According to Salute, et al. (2014) reliability of a system can be interpreted as the property of the system to keep in time interval the quality in some expected condition during its use. The reliability society of the institute of electrical and electronic and engineers (IEEE) considered the reliability as design engineering discipline which applied scientific knowledge to assure a product will perform its intended function for required duration with a given environment. Lloyd Contra (1993), views the reliability as product performance over time. The reliability is defined as the probability that items will perform its intended function under operating conditions, for a specified period of time. Indeed; improving reliability task is performed by improving product quality, unreliable product lead to not a high quality product (Meeker and Escobar, 1998).There is a strong relationship between improvement quality and increasing the reliability of system. The parameter obtained through the reliability theory must be monitored, because its play key role important in the continuity of system operating and perpetual service. Decreasing reliability of system below a specified level of performance can imperil the safety of customer and economic. The procedure must be following to increase the reliability are (Kane and Cepin, 2012):

- Improving the quality of every component that is part of the complex system.
- Formulating methods aiming at planning reliable systems and optimized methods of maintenance for those systems during their exertion (continuously).

Reliability is the analysis of failures, their causes and consequences. Reliability system is one of the most important characteristics of system or product quality. The assessment and analysis of system as a function item reliability in which it is necessary during the life cycle of item to predict the failure as early as possible. In addition, measures of component importance with respect to reliability provide information that is needed to develop effective strategies to improve system reliability (Meeker & Escobar 1997). It is considered as a crucial attribute have to be satisfied before considering other quality attributes. Once achieves high reliability through careful systematically monitoring all phases of the development process, i.e. design, materials, production, quality assurance efforts, ongoing maintenance, and a variety of related decisions and activities. All of these factors must be considered in determining the costs of production, purchase, and ownership of a product (Blischke and Murthy, 2003). They are three reliability methods with different aim and intend such as: (i) method to measure and predict failures, (ii) methods to accommodate failures, (iii) methods to prevent failures.

An item failure in the system cannot occur expect on a specific occasion when a set of specific data is put into the system under a specific condition of item. Item reliability is dependent on the input data and the internal condition of the item. Reliability is one feature of dependability among other such as: maintainability, availability, safety, therefore, generally depends on knowledge of a repair time distribution. A specific performance measures can be embedded into reliability analysis by the fact that if the performance measures is exceed a certain level, a failure can be occurred. Item reliability depends on component quality, manufacturing, robustness and quality of design which has high reliability and result minimum maintenance requirement.

Mathematically, Reliability is the probability that the system will perform its intended function under specified working condition for a specified period of time. The reliability function is defined as the probability of the system being in the set of up states U through the time interval$[0, t], t \geq 0$. The reliability function denoted by R (t), and it can be written as follow (Osaki, 2002):

$$\boldsymbol{R}(\boldsymbol{t}) = \mathbb{P}(Y_v \epsilon\ U\ for\ all\ v\epsilon\ [0, t]) \qquad (2.2).$$

The interval reliability is defined as the probability that the system is in the set of up states U through the time interval $[t, t + x]$, Y is state at time v with x, t≥0. The interval reliability can be written as follow:

$$\boldsymbol{R}(\boldsymbol{x}, \boldsymbol{t}) = \mathbb{P}(Y_v\ \epsilon\ U\ for\ all\ v\epsilon\ [t, t + x]) \qquad (2.3).$$

In another word, Mathematically, the reliability function R(t) is the probability that a system will be successfully operating without failure in the interval from time 0 to time t, where T' is a random variable representing the failure time or time-to-failure.

$$\boldsymbol{R}(\boldsymbol{t}) = P(T' > t), t \geq 0 \qquad (2.4).$$

Hence, the interval reliability is equal to the reliability, if $t = 0$, so from Eq(2.5), then R(x,0)=R(x). Otherwise, if $x = 0$, the interval reliability is equal to the availability, R(0,t)=A(t). The joint interval reliability is defined as the probability that the system is in the set of up states U throughout both $[t_1, t_1 + x_1]$ and $[t_2, t_2 + x_2]$ with $t_1, x_1, t_2, x_2 \geq 0$.

$$R(x_1, x_2, t_1, t_2) = \mathbb{P}(Y_v\ \epsilon U for all v\epsilon [t_1, t_1 + x_1] U [t_2, t_2 + x_2]) \qquad (2.5).$$

In addition, item reliability is derived from the following equation: Probability of item working plus probability of item failure. Therefore, Probability of item working =1-probabaility of item failure, where the probability of item working is the definition of reliability. Generally, there are two types of reliability computation:

(i) Reliability up to time t can be computed as:

$\mathrm{R(t)} = 1 - \text{probability of failure up to time t}$

$$=1-\int_0^t f(t)dt=1-F(t) \quad (2.6).$$

$\mathrm{F(t)}$: Is the cumulative probability up to time t, also called cumulative distribution function (cdf)

(ii) Reliability during an interval t_2-t_1 can be computed as:

$$R(t_2-t_1) = 1-\int_{t1}^{t2} f(t)dt \quad (2.7).$$

The failure probability, or unreliability can be expressed as follow:

$$F(t) = 1 - R(t) = P(T \le t) \quad (2.8).$$

Sometimes the probability of failure during an interval can be defined in terms of reliability. For example:

$$P(t_1, t_2) = F(t_2) - F(t_1) \quad (2.9).$$

Where , $F(t_2)$ =1-R (t_2), and $F(t_1)$ =1-R (t_1), then

$$P(t_1, t_2) = [1 - R\ (t_2)] - [1 - R\ (t_1)] \quad (2.10).$$

If the time-to-failure random variable T has a density function (t) , then

$$R(t) = \int_t^{\infty} f(x)dx \quad (2.11).$$

R(t), F(t) and f(t) are closely related to one another. If any of them is known, all the others can be determined.

Availability

Availability is defined as the probability that the system is in the set of up states U at the time instant t, $t \geq 0$. Different from the reliability that focuses on a period of time when the system is free of failures, availability concerns a time point at which the system does not stay at the failed state. Mathematically, the availability denoted by A (t), and can be computed as follow (Osaki, 2002):

$$A(t) = \mathbb{P}(\text{system is up or available at time instant t}) \quad (2.12).$$

Point of view mathematical, availability is defined as the probability that system is available at time t; however the reliability is the defined as the probability of system is working on a period of time.

The joint availability refers the probability of the system being in the set of up states U at both time instants t and $t + x$, witht, $x \geq 0$. The joint availibility can be written as follow:

$$A(x, t) = \mathbb{P}(Y_t, Y_{\underline{t+1}} \in U) \quad (2.13).$$

Where U is set of up states.

Generally, maintenance management aims to improve operational availability and functional availability. Operational availability is used to calculate how long the system is in operation in relation to intended operating time. It mainly shows the technical capability to keep on operating while maintenance is being carried out. Systems that must be taken out of operation for longer repair works have a lower level of operational availability than others. A system with low reliability but high operational availability indicates an efficient maintenance organization. Furthermore, functional availability is used to quantify the system's capability both to be in operation and at the same time maintain intended levels of function.

Maintainability

Maintainability is defined as the probability that a failed system will be restored to a functioning state within a given period of time when maintenance is performed according to prescribed procedures and resources. Maintainability analysis search to minimize downtime, reduce repair time, and as result reduce the maintenance cost. Some of inputs for maintainability come from reliability analysis such as lowers item failure rate over the long term, reduce item's warranty cost (Raheja and Allocco, 2005). In Mathematical point of view, the Maintainability denoted by *M(t)*, is defined as the probability that the failed system will be back in service by time:

$$\mathrm{M(t)} = \mathbb{P}\ (\mathrm{T} \leq \mathrm{t}) = \int_0^{\mathrm{t}} \mathrm{m(x)dx} \qquad (2.14).$$

Where, M(t) is the maintainability function, T denoted the time to repair or the total downtime, T follow a density function distribution denoted by m(t).

Generally, maintainability is used to calculate the probability that repair times will not exceed an acceptable level. Repair work usually involves the system being taken out of operation, so it is of interest to minimize repair times. Long repair times mean low maintainability (Myrefelt, 2004).

The commonly reliability terms based on methods and procedures for lifecycle predictions apply to specific item is the following:

a) **Mean time to failure (MTTF)**

The mean time to failure can be defined as an expected value of the lifetime before a failure occurs, and it's denoted by MTTF. It is one of many ways to evaluate reliability of items. Thus, it is used for non-repairable products. The expression of MTTF is given by:

$$\boldsymbol{MTTF} = \int_0^{\infty} t.f(t)dt = \int_0^{\infty} R(t)dt \qquad (2.15).$$

Example. If the lifetime distribution function follows an exponential distribution with parameter λ, that is, F (t) =$1 - \exp(-\lambda t)$, the MTTF is expressed as follow:

$$\boldsymbol{MTTF} = \int_0^\infty R(t)dt = \int_0^\infty \exp(-\lambda\, t)dt = \frac{1}{\lambda} \qquad (2.16).$$

b) Mean time between failure (MTBF)

Mean time between failures is known as the expected number of operating hours before a system fails, and it is denoted by MTBF. Mean time between failures is supported by the companies that are ISO certified, as functional as possible intended to achieve the goals of "zero defect" and "continual improvement". The value of MTBF can be computed as follow:

$$\textbf{MTBF}= \frac{1}{\text{Failure rate 1+failure rate 2+}\cdots\text{+failure rate n}} \qquad (2.17).$$

Where denominator is the failure rate of each component of the system.

c) Failure rate function

The failure rate function represents the changing rate in the aging behavior over the life of a population of components. The failure rate function, or hazard function, is very important in reliability analysis because it specifies the rate of the system aging. The instantly reliability of two identical items may provide the same reliability, but the failure rate function can be different. The definition of failure rate function is considered as the probability that an item of age t will fail in the small interval from tto $\mathrm{t} + \mathrm{dt}$. The failure rate function denoted by λ (t) and can be expressed as follow:

$$\boldsymbol{\lambda}\ \textbf{(t)}= \lim_{\Delta t \to 0} \frac{R(t)-R(t+\Delta t)}{\Delta t R(t)} = \frac{f(t)}{R(t)} \qquad (2.18).$$

When the failure distribution function follows an exponential distribution, the failure rate function is a constant (λ). For this reason the system does not have any aging property, e.g., software systems, electronic component. In this case, the failure rate function can be computed as follows:

$$\boldsymbol{\lambda}\ \textbf{(t)}= \frac{f(t)}{R(t)} = \frac{\lambda.\exp(-\lambda t)}{\exp(-\lambda t)} = \lambda \qquad (2.19).$$

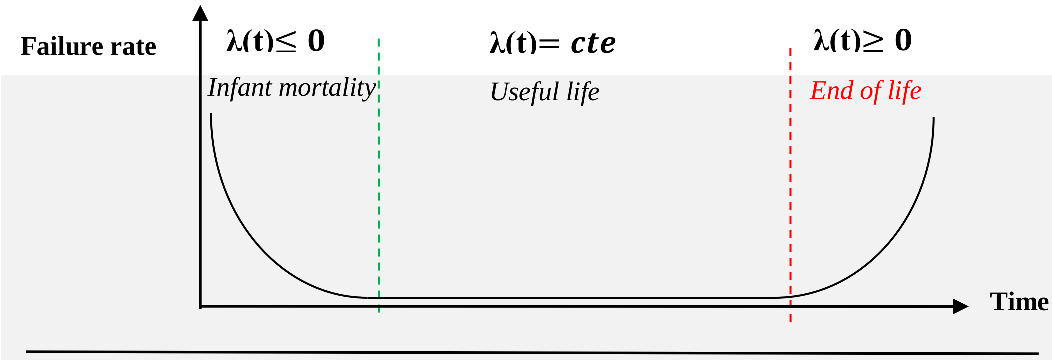


Figure 2.6-Bathtub curve of failure rate.

The bathtub curve failure rate is very important in reliability engineering and widely used by the academic researchers and practitioners (Figure 2.6). The shape of failure rate function is similar to the shape of the bath, for this reason the name is derived from the cross-sectional shape of a bathtub. The plot showing failure rate or hazard rate versus time, it describes item failure lifetime distribution over time which contributed to the bathtub curve. In reliability the term failure rate is either hazard rate which is the likelihood of failure of an un-repairable item or for the rate of occurrence failure, and denoted by ROCOF which is applied to repairable items (Lioyd and Condra, 1999). Indeed, the hazard rate entails three distinctive parts as follow: the first one is called as an infant mortality where the hazard rate decreasing (λ (t)≤ 0), the second part is known as useful life where the hazard rate is constant (λ (t)$= cte$), and finally the third part which known as an end of life or wearing out as the product exceeds its design lifetime, where the hazard rate increasing (λ (t)$\geq$ 0).

Therefore, the group known as AGREE (Advisory Group for the Reliability of Electronic Equipment) who was the first introduced the Bath-tube in reliability engineering in the 1950's, and discovered that the failure rate of electronic item had a pattern similar to the death rate of people in a closed system. Specifically, they noted that the failure rate of electronic components and systems follow the classical "bathtub" curve (Bazovsky 1961).

d) Mean Time To Repair (MTTR)

The mean time to repair (mean downtime) is the time needed to repair a failed item. Generally, MTTR is used to compute the value expected time for the restoration to replace a failed item. The value of MTTR is the time interval during which the system is in a state of unavailability (Blanchard and Lowery, 2005). The cost to repair an item increased, while the time to bring the system into normal state is longer, for this reason it is necessary to reduce the value of MTTR at significant level. MTTR can be interpreted as an indicator of performance of the maintenance service efficacy, especially from the point of view of the logistic organization (Ashrae, 1999).

Example. If the time to repair a failed items follows an exponential distribution with parameter μ, that is, F (t) =$1 - \exp(-\mu t)$, the MTTR is expressed as follow:

$$\boldsymbol{MTTR} = \int_0^\infty \exp(-\mu\, t) dt = \frac{1}{\mu} \qquad (2.20).$$

The mean time between failures (MTBF) is another important measure in repairable systems. This implies that the system has failed and has been repaired. Like MTTF and MTTR, MTBF is an expected value of the random variable time between failures.

Failure in Time (FIT)

Failure in time reports the number of expected failures per one billion hours of operation for a device. Failure in time is another way of reporting MTBF. It denoted by FIT, this term is used particularly by the semiconductor industry but is also used by component manufacturers.

When the failure rate and repair rate are constant, the lifetime distribution function $F(t) = 1 - exp(-\lambda t)$, and a maintainability function $V(t) = 1 - exp(-\mu t)$with parameter μ, then $MTTF = 1/\lambda$ and $TTR = 1/\mu$. The MTBF is the sum of MTTF and MTTR then, the steady-state availability = MTTF/MTTF+MTTR

2.3 Maintenance Function

Maintenance is defined as the combination of all technical and administrative actions, including supervision actions, intended to retain an item in, or restore it to a state in which it can perform a required function. This definition is also adopted by (Dekker, 1996), and by the maintenance standard EN13306: 2001. The maintenance function has attracted the attention of many practitioners and academics once it plays a key role in achieving productivity and cost effectiveness, as well as customer's satisfaction. It contributes to reduce equipment downtime, to improve quality and to ensure availability and safety (Muller et al. 2008). Kelly (1984) shows that maintenance organization has three essential components which are interconnected, as follows: resources (e.g., personnel, tools/equipment, spare parts), administration (e.g., manager, hierarchy, authority) and work planning and control (planning, feedback information). There are lot of articles and books that address the concept of maintenance in engineering and quality area (Moubray. 1997; Gulati, R. and Smith, R. 2009,). Garg and Deshmukh (2006a), narrowed down maintenance literature into six main-classes as follows: policies, information system, optimization models, techniques, scheduling, and performance measurement.

Engineering maintenance is an important sector of the economy. Each year U.S. industry spends well over $300 billion on plant maintenance and operation, and in 1997 the U.S. Department of Defense's budget request alone included $79 billion for operation and maintenance. Furthermore, it is estimated that approximately 80% of the industry dollars is spent to correct chronic failures of machines, systems, and people. However, the elimination of many of these chronic failures through effective maintenance can reduce the cost between 40 and 60% (Dhillon, 2002). Therefore, in many companies maintenance spending overcome 40% of the operating budgets. Several maintenance models can be provided for improving the life-cycle performance of a system. In general, maintenance models are important to ensure an item as possible as running well and they are the basis for maintenance policies.

2.4 Maintenance Types and Models

Generally, maintenance activities performed in equipment can be classified into two main categories: corrective maintenance and preventive maintenance. Preventive maintenance is further subdivided in systematic maintenance and CBM. This subdivision refers to what triggers maintenance activities. Current maintenance strategies have progressed from breakdown maintenance, to preventive maintenance, then to CBM managed by experts, and lately towards a futuristic view of intelligent predictive maintenance systems.

Firstly, corrective maintenance is reactive in nature and it refers to maintenance activities performed only after a system failure in order to restore its functionality. Corrective maintenance actions are usually referred to as repair or run-to-failure (NP EN 13306: 2001).

According to (Garg and Deshmukh 2006a) and (NP EN 13306:2001) preventive maintenance is proactive in nature and consists of a series of tasks (e.g., replacements, adjustments, inspections and lubrications), that are performed on plant equipment, machinery and systems by maintenance personnel before the occurrence of a failure. Preventive maintenance aims to protect them and to prevent or eliminate any degradation in their operating conditions. As was mentioned above, preventive maintenance is further subdivided in systematic maintenance and CBM which are defined respectively as follows:

Systematic maintenance also referred as time-based maintenance or block-based maintenance, consists on activities performed at predetermined intervals. These scheduled activities on equipment aim to ensure its correct functioning (NP EN 13306: 2001).

Nevertheless, if the deterioration level of components is correlated strongly with a control parameter, the decision about the realization of preventive maintenance operations can be based on system's condition, this is called Condition Based Maintenance (Bengston, 2007; Dileo et al. 1999). CBM is a set of maintenance actions performed based on real-time or near real-time assessment of equipment condition obtained from embedded sensors, external tests or measurements using portable equipment. In fact, the data obtained from condition monitoring techniques helps the maintenance manager to decide if maintenance is necessary or not by analyzing the actual condition of equipment (Jardine et al. 2006).

Several maintenance models have been proposed in literature to answer a question about maintenance optimization. They are suitable applied for both either for single-unit systems or multi-unit systems in which the different characteristics, advantages and disadvantages, different degree of maintenance (e.g., perfect, imperfect, and minimal) as well as with different type of maintenance (e.g., preventive, corrective, opportunistic which it is relevant only to multi-unit systems) are presented. Literature provides many papers and books have been devoted to the design several models that applied in the real systems. The book “mathematical theory of reliability" includes surveys maintenance models that aim to determine optimal decision to procure, inspect, and repair and/or replace deteriorating unit-system. (McCall, 1965) surveys maintenance scheduling policies for stochastically failing item. The author defines the maintenance policy for an uncertain distribution of times to failure and for known distribution of times to failure. Thomas (1986) describes some of the maintenance and replacement models that have been suggested for multi-item systems. (Valdez-Flores and Feldman 1989) survey preventive maintenance models, including optimization models for repair, replacement, and inspection of systems subject to stochastic deterioration. (Cho & Parlar 1991) survey the literature related to maintenance optimization and replacement models for multi-unit systems, known in literature optimal models can be classified into five main groups as follows: machine interference/repair models, group/block/cannibalization/opportunistic models, inventory/maintenance models, age replacement models and inspection/maintenance models. The authors also provide many possible ways to classify the literature related to maintenance optimization such as: information availability, single unit or multi-unit systems, time-event/action relationship, state-event/action relationship, model types, optimality criterion, method of solution and planning time horizon. Wang (2002), surveys maintenance policies for deteriorating systems. The authors summarize, classify and compare various existing maintenance policies for single-unit and multi-unit systems. The optimization of maintenance planning using mathematical modeling has been also increasingly reported in literature by several authors (Pierskalla and Voelker 1976; Valdez-Flores and Feldman 1989; Dekker and Scarf 1998; Dekker 1996; Garg and Deshmukh 2006a).

Among the most useful replacement models currently in popular use are the age replacement policy and the block replacement policy. The first one considers that the unit is replaced upon failure or at age T, and the second one considers that the unit is replaced upon failure and at predetermined time (T, 2T,...). Since then, models have been proposed based on these past too models. Barlow and Proschan (1965) proposed a periodic replacement policy, referred to the age replacement models with minimal repairs between replacements, and discussed the problem of determining an optimal preventive replacement time (age T) to minimize the long-run average expected cost per unit time over an infinite horizon. Barlow et al. (1963),developed a periodic maintenance model where a sequence of check times is specified in order to minimize the expected total costs. Berg and Cleroux (1982)extended the ordinary block replacement model proposed by Barlow et al. 1963), and provide the optimal block interval T between preventive maintenance which minimizes the long run expected cost. Since, numerous articles have been proposed in this area, including theories and practical applications that have appeared in journals and international conferences.

2.5 Condition based maintenance Approach

CBM policy is based inherently on prior approach. The idea of prior approach is to use a prior historical information item condition for solving the problem of prognosis of item health and maintenance decision making. CBM is based on sequence of steps able subsequently to asset and to monitor the item condition in machinery or system as well as performed an appropriate maintenance decision.

Therefore, CBM is a methodology that undertaking endeavors to predict incipient impending faults, setting up CBM enable to promote a more accurate decision maintenance action. In practical application, CBM decision is often including the following: no action, replacement, inspection, and preventive maintenance. CBM model performed based on a stochastic deterioration process (Butcher 2000; Jardine et al., 2006; NP EN 13306: 2001).

There are two main reasons to develop CBM program: (i) diagnostic of failures, (ii) prognostic of failures. The first one refers to posteriori approach which consists in the detection, isolation and identification specific components in the equipment that are failure of course after its occurrence as well as to determine the root cause of failure. The second one refers to priori approach that allows the prediction of a failure before its occurrence in order to avoid item breakdown or a potential accident as well as to determine whether a problem exists in equipment, how serious the problem is, and how long the equipment can run before its failure (Okumura 1997; Kothamasu and Huang 2007; Jardine et al. 2006). Indeed, the main goal of CBM policy is to avoid unnecessary maintenance tasks by taking maintenance actions only when there is evidence of abnormal behaviors of an item (Jardine et al. 2006). CBM policy allows better perform maintenance action. It has been considered more powerful than the traditional preventive maintenance such as block replacement maintenance, age replacement maintenance, breakdown maintenance, periodic maintenance.

The principal benefit of CBM program is the reduction of the maintenance cost by decreasing the unnecessary maintenance action, as well as the avoidance of penalty costs. The primary objectives of an optimized maintenance strategy program that includes predictive or condition based maintenance CBM are: improve system availability, improve equipment reliability, enhance equipment life, detect problems as they occur, save maintenance costs and reduce parts inventory (IAEA, 2007).

CBM has been applied in many areas since the mid 80's, e.g., wind turbine, military equipment and aircraft, and has benefited from the advances in monitoring, technology, operation research and signal processing techniques (Wang et al. 2012; Butcher 2000; Waeyenbergh & Pintelon 2002). Hence, CBM can be more cost effective than time-based maintenance and is one of the least expensive and most effective strategies for proactive maintenance. It allows reducing maintenance costs and increasing equipment uptime (Ilangkumaran & Kumanan 2009; Yang 2003). The impact of CBM on company's profitability has been reported in (Al-Najjar 2007).

CBM policy benefit from advanced condition monitoring techniques as a support technical for ensuring that critical item parameters will be monitored and collected, because when an event happened in the item, may be some of the item parameters subjected to change, and obvious result as a changing equipment performance (e.g. changes in vibration, changes in power usage, changes in operating performance, changes in temperatures, changes in noise levels, changes in chemical composition, increase in debris content and changes in volume of material). Furthermore, the advantage using condition monitoring techniques in CBM is because it has capability to provide to decision maker suitable and useful information about the health state of item. The information extracting from analyzing sample data collected promise to incipient fault detection and to warn about the future failure.

A modern system such as manufacturing industry, aircraft, and nuclear plant, as well as service businesses are characterized by their great quality, efficiencies, effectively, productivity. Nowadays, it‘s evident that robust item and advanced technology play primary role for manufacturing industry, aircraft, power plant, organization to continuously improve productivity, efficiency, quality, effectiveness, as well as to stay competiveness. However, an item is becoming more complex, more sensible, and expensive, thus, inventor technical, concept, design, and product development is becoming vital toward independency economy, which means an item has increased in importance and should be in full operational mode to avoid any extra losses (Bengtsson 2007).

In many practical applications (mechanical item), several studies are performed related to the historical item failure indicate that only about 15% to 20% of equipment failures are age-related, and the other 80% to 85% of equipment failures are based totally on the effects of random events that happen in the system (Amari et al. 2006). Hence, since a member is put in production line, it will undergo degradation overtime, the last were mainly occurs because of the effect of the random event. Modern systems and processes are characterized by their availability importance as result have to be aware of mustn't to assume the degradation is fatalism. Therefore, system engineer should be improved skills and model knowledge to be able to find out the solution about the different problem that facing everyone and everywhere. Maintenance has gained increasing importancefor technical process as a support function in order to reach comprehensive system integrity (e.g., improve to improve reliability, availability, maintenance and, quality product,and supply on-time deliveries). Nowadays, there is unanimity between researchers and managers to move from traditional preventive maintenance to intelligent preventive maintenance which is called CBM (Jardine et al. 2006). CBM policy has allowed a better extension of useful life, a reduction of through-life cost of item, an improvement of operational availability, increasing mission effectiveness, decreasing cost related spare part, and a reduction of the maintenance burden (Jardine et al. 2006; Gallasch et al. 2013).

Considering, the large amount of maintenance definition, characterizations, and policies on the literature review, CBM models does lead to a cost effective maintenance decision. Generally, CBM is performed based on the following: a better knowledge, stochastic algorithm, and sufficient method in an effective and systematic way for handling all kind of complex situations that arise in practical.

CBM decision-making yield one of the best maintenance management (Amari et al. 2006). Implementation an effective maintenance management required presence such characteristic as follow: knowledge of equipment failure mechanisms, causes, symptoms, detection and diagnostic procedures. The diagnosis procedure can be divided into five hierarchical: (i) Data acquisition: it is the basis of the procedure. Generally technical processes are already sensor equipped and this phase does not present any difficulty. (i) Data reduction: information about the process structure is diluted in the observed signals. In this phase, we want to extract a good fault indicator from measured signals. The efficiency of the detection diagnosis procedure is determined by this step,(ii)Detection: this operation gives the answer to the question is the system in a structural normal state or under fault. Given that the system environment is noisy, the tools for detection are of statistical nature and thus imply a risk of false detection or of missing a fault, (iv) Diagnosis: it performs the localization (sensor, actuator, process) and gives attributes (steady or unsteady fault, evolving or cataleptic fault) and the degree of severity. This step is not already active but asked only when the detection module indicates a fault, (v)Fault recovery: when a fault is appeared, what is the best action that the controller has to perform stages (Isermann, 2011).

The CBM decision is directly based on the observed state. CBM models focusing on data determined, data collected, data analyzed, data processed, and data modeled, (Jardine et al. 2006; Ahmad & Kamaruddin 2012). They are collected by using condition monitoring techniques, on real-time or near real-time, on-line or off-line (e.g., embedded sensors, external tests or measurements portable). The main contribution of data obtained by using condition monitoring techniques is to help the maintenance manager to decide if maintenance is necessary or not. Generally, CBM policy requires the following information: item variables history, age item, degradation until the moment of decision making. The history of the machine and the previous degradation pattern is important in determining the current and future operating condition of the machine. The health state of an item can be modeled not only by the failure models but also by the degradation model. The degradation and failure models are needed for optimizing the control limit policy (Banjevic et al. 2001).CBM policy can be considered as a new vision of maintenance management that it is necessitate for improving maintenance management (Martin 1994; Jiang 2011; Jardine et al. 2006; Ahmad and Kamaruddin 2012).

An intelligent preventive maintenance such CBM can be considered as a suitable strategy for forecasting the failure item and scheduled maintenance, it has direct impact on the budget of the company and it is being seen as the most significant economic benefits. Although, according to Amari et al., on (2006) some advantages of implementation CBM policy are: reduction in the total maintenance program cost, avoidance of very disruptive equipment outages, reduction of costly preventive maintenance activities when condition assessment shows no need of the scheduled maintenance, however, the advantage CBM policy not only reduces the amount of maintenance performed but also avoids maintenance-induced failures (Amari et al. 2006). Indeed, the majority of studies were presented in literature reveal that CBM policy is recommended and more attractive to use for engineering systems. CBM policy has become increasingly important in developing effective maintenance management reach to improve the reliability of systems as well as increase the system performance, even the complexity of systems increase. CBM policy has been considered as an intelligent maintenance policy uses to provide the health state of item that serves to predict the failure time or remaining useful life of item. In practical application, thus, it is becoming an effective maintenance policy typically for many engineered system, and has been raised, e.g., from public transportation systems, nuclear power plants, manufacturing systems, to aircraft systems. Therefore, it is recognized that an effective CBM requires the completion of three fundamental steps (see Figure 2.7):

- Data acquisition step (information collecting), to obtain relevant data about system condition, e.g., vibration, temperature, voltage/current, and oil composition;
- Data processing (information handling), to handle and analyze the data or signals collected in the first step;
- Maintenance-decision making, to recommend efficient maintenance policies.

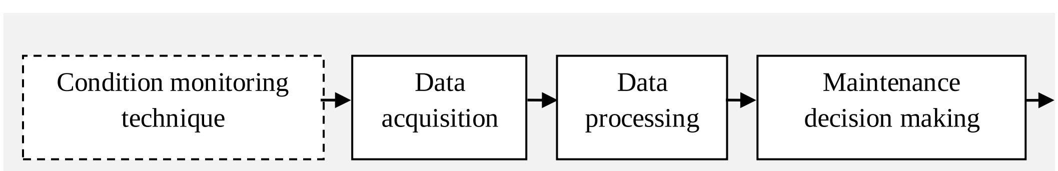


Figure 2.7- CBM approach (Jardine et al. 2006).

Obviously, selecting an effective maintenance policy is powerful strategy to improve the capability of decision making maintenance (Jardine et al. 2006). Researches, though a great solution to predict impeding failure and take decisions based on item condition is CBM. Furthermore, CBM requires definition failure and deterioration models (Jardine et al. 2006; Martin 1994). However, a major challenge facing researcher’s community is how to establish an intelligent CBM models in maintenance decision making taking into consideration the following features such as: reliability model, performance model, and optimization model. Probably these requirements are interpreted as an issue which can have a significant impact on effectiveness of condition based maintenance policy. This issue has gained particular importance over the last 10 years. Amari et al., (2006) simplified the solution to develop an effective CBM by involving the following steps:

- Identify failure mechanisms, causes, and detection and prevention methods.
- Identify deterioration model associated with the system. The model can be built using the knowledge of failure mechanism as well as the existing data related to failures. Further, deterioration model has been developed by using the following techniques: Hidden Markov Models, Gamma Process, Delay Time Concept, Data Mining Techniques, and Statistical Techniques.
- Determine the costs and effects associated with the various kinds of failures and maintenance actions.
- Develop an optimal CBM policy that involves optimal inspection schedules and the optimal maintenance decisions.

2.6 CBM versus traditional maintenance

During all the past decades, a lot of maintenance models have been extensively developed in the literature and they can be classified into two distinct kinds of maintenance policies: traditional maintenance policies and CBM policies. Maintenance policies can be applied for both single-unit systems and multi-unit systems (set of system with a number of sub-systems). The basic assumptions for single-unit-systems under all maintenance police are that the unit lifetime has increasing failure rate. Furthermore, maintenance policies for single-unit systems are more established, and are the basis for maintenance policies of multi-unit systems. However, various dependences in multi-unit systems render maintenance of a multi-unit systems differ from a single-unit system(e.g., economic dependence, failure dependence or correlated failure). For example failed item in the system may also cause failure to other item (Sarkar et al., 2011).

Sarkar et al. (2011) presented an overview, summarizes, classifies, and compares various existing traditional maintenance policies around 170 Authors. In the following some categories of maintenance policies: age replacement policy, block replacement policy, periodic preventive maintenance policy, failure limit policy, sequential preventive maintenance policy, repair cost limit policy, repair time limit policy, repair number counting policy, reference time policy, mixed age policy, group maintenance policy, opportunistic maintenance policy. The authors classified maintenance policies into two categories as follow: the first one is maintenance policies of single-unit systems such as: age-dependent preventive maintenance policy, periodic preventive maintenance policy, failure limit policy, sequential preventive maintenance policy, and repair time limit policy, repair number counting and reference time policy; and the second one is maintenance policies of multi-unit system such as: opportunistic maintenance policies, group maintenance policy. Further, they authors showed for each kind of these models different characteristics, advantages and disadvantage for each kind of these models. However, all these models do not take into account information collected by condition monitoring technique, further, they are based on the concept of the bathtub curve, where hazard rate increase in wear out phase. One the most maintenance policies for unit-system that have received much more attention in the literature are age-dependent preventive maintenance policy and periodic preventive maintenance policy. Furthermore, the periodic preventive maintenance policy is seemed more practical than the age-dependent preventive maintenance policy, because it does not require keeping records on unit usage (Sarkar et al., 2001). In traditional maintenance policies, two types of maintenance have been carried out: corrective maintenance and preventive maintenance (Amari et al., 2006).

CBM models is based on condition monitoring techniques. The channel information comes from condition monitoring techniques that have to take into consideration in order to contribute to develop models close to reality. Thus, effectiveness of a system depends on both the quality of its design as well as the proper maintenance actions to prevent it from failing (Sarkar et al., 2001). These condition monitoring data contain useful information concern the health state of item as well as will be useful into deterioration models and failure models. Currently, there is evidence that the impact of CBM have had an important effect on effectiveness of system and there is increasing interest by researchers to develop as much as possible CBM models.

A study compares between CBM and the traditional preventive maintenance (e.g., age replacement policy, time replacement policy) has been shown that the traditional preventive maintenance were not very encouraging, because the drawback of these models is essentially about the machinery failure forecast. According to literature review, CBM is an effective way of predictive maintenance, is based on actual item condition rather than on time or usage interval for determining how soon a failure will occur. In contrast traditional maintenance policies are based on age of equipment, time interval, usage interval, thus, chance of component failure depends entirely on the age. These models are based on the concept of the bathtub where hazard rate increases. According to Amari et al., (2006), an interesting study was made by United Airlines aircraft component and U.S. Navy indicates that only 3% to 4% of equipment failures can be explained using bathtub curve hazard rate. Although, a most important result several studies of failure equipment indicate that traditional maintenance policies is inappropriate in most cases because only about 15% to 20% of equipment failure are age related. The other 80% to 85% of equipment failures are based totally on the effects of random events that happen in the system. This means that failure is not dependent only on the age of the component, because varying levels of latent defects and impurities can exist. This leads to different rates or patterns of defect propagation, e.g. variation in raw material, power, loads, operator skills, maintenance activities, rigor environment, leadership, maintenance activities, floods, and earthquakes can all influence the failure mechanisms. Therefore, the failure propagation is a complex stochastic process not only depends on age but also depends on several other factors and events. All these reasons for equipment failure results as traditional maintenance policies are not considered optimal maintenance policies.

2.7 Application of Condition Monitoring Techniques in CBM Policy

According to NP EN 13306:2001, monitoring is defined as an activity, performed either manually or automatically, intended to observe the actual state of an item. Furthermore, monitoring is distinguished from inspection in that it is used to evaluate any changes in the parameter of the item with time, thus, monitoring may be continuous, over a time interval, or after a given number of operations. Further, condition monitoring techniques is considered as a major process of CBM policy. It used as a means enable to monitor parameters of condition in item such as: vibration, temperature, position. The popular condition monitoring techniques are used by practitioners and researchers to monitor the health state of an item as follow: vibration analysis, lubricant analysis, infrared thermograph, ultrasound emission, shock pulse analysis, thermal image analysis.

In practical application, condition monitoring methods are performed on-line or off-line. Off-line tests require the interruption of operation or shutdown (motor), while on-line methods provide diagnostics during the operation. There are some circumstances that off-line monitoring techniques have some advantageous, such as reduction in noise contamination, as well as load and speed repeatability. Indeed, the majority of condition monitoring addressed are on-line monitoring technique except induced voltage, motor circuit analysis and surge test (Marium al., 2011).

A condition monitoring technique uses an instrument and technique such as measuring instrument or sensors. The measuring instruments and sensors can provide the amount physical quantity. The measuring instruments have to transform physical quantity into extent or useable amount such as: pressure, temperature, velocity. The selection of the measuring instrument is based not only on the type of variable to be measured but also on the environment of the system where it is operating (e.g., observation represents the spectrometric readings of oil samples coming from engines or vibration data collected from motor). The major benefit provided by the CBM policy is the ability to incorporate prior information where a prior approach allows the prediction of a failure before its occurrence to avoid item breakdown or a potential accident (Jardine et al. 2006).

Considering condition monitoring technique as an underpinning process in CBM policy for the prognostic of failures where various sensors or measuring equipment at discrete or continuous time are able to provide parameters condition item. The success of condition based maintenance policy requires an advanced technology and suitable tools in order to achieve the ability to develop an accurate diagnostic model.

The physical process to be measured is in the left of the Figure (2.8), and the measured is represented by an observable physical variable X. Note that the observable variable X need not necessarily be the measured but simply related to the measured in some known way. For example, the mass of an object is often measured by the process of weighing, where the measured is the mass but the physical measurement variable is the downward force the mass exerts in the Earth’s gravitational field (Webster and Eren 2014).

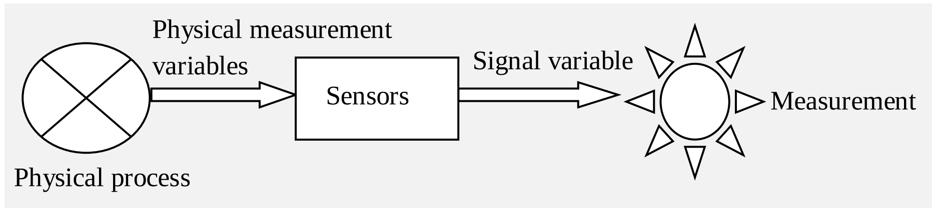


Figure 2.8- Measurement diagram

Hence, the acquisition of item condition information through a condition monitoring technique in CBM policy is considered as platform or database used to enable a future problem to be predicted, diagnosed and corrected before breakdowns or other serious consequences occur. Today, there are a large and growing variety of forms of condition monitoring techniques for machine condition monitoring such as: vibration monitoring, acoustic analysis, motor analysis technique, motor operated valve testing, oil analysis detection, thermograph, tribology, and process parameter monitoring, visual inspections. The condition monitoring techniques can be divided into main groups as follow:

The first one is measurement instrumentation which can be viewed as a device or equipment intended to record or measures a physical variable. In abstract term, the instrument is a device that transforms a physical variable of interest (the measured) into a form that is suitable for recording (the measurement). The second one is a sensor, which has the function of converting the physical variable input into a signal variable output (Webster and Eren 2014).

Vibration monitoring-Vibration analysis detects repetitive motion of a surface on rotating or oscillating machines. The repetitive motion may be caused by unbalance, misalignment, resonance, electrical effects, rolling element bearing faults, or many other problems. The various vibration frequencies in a rotating machine are directly related to the geometry and the operating speed of the machine. By knowing the relationship between the frequencies and the types of defects, vibration analysts can determine the cause and severity of faults or problem conditions. For example, on rotating machines vibration analysis monitors the following conditions:

- Cracks, pits, and roughness in rolling element bearing components.
- Unbalance of rotating machine parts,
- Shaft misalignment,
- Coupling problems,
- Bends, bows, and cracks in shafts,
- Excess sleeve bearing wear.
- Loose parts,
- Misaligned or damaged gear teeth,
- Deterioration caused by broken or missing parts,
- Deterioration caused by erosion and corrosion,
- Resonance of components.
- Electrical effects.

Oil analysis detection-It is non-destructive way to gauge the health of an engine by looking at what's in the oil. A way of performing oil analysis is based on collecting a volume of fluid from lubricated or hydraulic machinery for the purpose of oil analysis. Oil analysis was first used after World War II by the US railroad industry to monitor the health of locomotives. In 1946 the Denver and Rio Grande Railroad's research laboratory successfully detected diesel engine problems through wear metal analysis of used oils. In addition, integrating oil and vibration analysis can yield early detection and trending of numerous equipment problems.

Wear Particle Analysis-Wear particle analysis is a direct approach to visualizing damaging causes and effects taking place in lubricated machinery by capturing and viewing particles extracted from lubricating oil.

Thermograph-A thermal imaging camera is a reliable non-contact instrument which is able to scan and visualize the temperature distribution of entire surfaces of machinery and electrical equipment quickly and accurately.

2.8 CBM Optimization

During the past decades, CBM models have been widely spread generally in textbook and particularly in industrial. This mean CBM models is appropriate strategy and optimal maintenance policies compared with other models and for this reason CBM has long been researched by many authors. However, due to limitation existing in CBM models, many researchers be aware of that CBM needed towards on continuously developing and conducting advanced research in this important area, this because there is a powerful demand from company to improve system availability, operational safety, maintenance cost effectiveness, customer satisfaction, reduce failure frequency. According to Noortwijk (2009), a characteristic feature of optimizing maintenance is about an uncertainty such as deterioration, cost and availability, in maintenance management, the most important uncertainties are: the time to failure (lifetime), the rate of deterioration.

The primary objective of an optimized maintenance strategy is to: improve availability, reduce forced outages, improve reliability, enhance equipment life, reduce wear from frequent rebuilding, minimize potential problems in disassembly and reassembly detect problems as they occur, save maintenance costs, reduce repair costs, reduce overtime, and reduce parts inventory requirements. However, CBM policy is becoming increasingly important provided to maintenance optimize strategy as the complexity of systems increases. Recent advances in CBM and condition monitoring technologies have given arise to a number of prognostic models that attempt to forecast machinery health based on condition monitoring data (Heng 2009).

Dekker and Scarf (1998) illustrate various classifications of maintenance optimization models by analyzing 112 papers. The potential benefit by using advanced signal processing and artificial intelligence techniques is to develop a robust maintenance optimization.

Balakrishnan (1992) presented an application of simulation models to evaluate maintenance policies. For example: selected out of opportunistic, failure and block) for an automated production line in a steel rolling mill. Markovian probabilistic models for optimizing maintenance policy have also been discussed by Bruns (2002), Marquez and Heguedas (2002), Chiang and Yuan (2001) and Lam (1999) in great detail.

Several algorithms computations are applied to the CBM policy. The optimization theory involves the success of the CBM policy in order to establish optimal decision making maintenance. The role of optimization theory within integrated model design an is to estimate parameters' model as well as to compute the optimal control policy for maintenance optimization problem stochastic algorithm (e.g. genetic algorithm, simulation, Monte Carlo Markov Chain). Obviously, the optimization problem, regardless the applied field, is to find the best solution from entire feasible region such as control limit and sampling interval which must minimize or maximize the objective function, e.g. genetic algorithm, value-iteration algorithm (Chen and Trivedi, 2004), Semi Markov Decision Process (Jiang et al.,2011), linear programming (Taylor, 1996), Expectation Maximization algorithm, (McLachlan and Krishnan 2008), policy-iteration algorithm (Kim and Makis, 2009). Villiam Makis presented an algorithm seek to find the optimal control limit and the minimum average cost. Obviously, the preliminary step before the construction of expected cost or downtime models, it is necessary to estimate the values of the parameters that characterize the defect arrival and failure processes.

Generally most of CBM models in literature used two objective functions whose readers are familiar, and it will may be can classify into one which either (Barlow and Hunter, 1960; Biswas and Sarkar, 2000; Sarkar and Chaudhuri, 1999; Jiang, 2011): (i) minimize average expected cost per time unit, (ii) maximize average expected availability.

The first one is to find the optimal control chart control policy (e.g., control limit, sampling interval) that minimizes average expected cost per time unit. The idea is to derive explicit expression cost per time unit. From renewal theory, the expected average cost of the system is equal to the expected cycle cost divided by the expected cycle length. For example, the optimization problem based on Bayesian approach which can be expressed as follow (Wang, 2012):

$$\text{Min } z(h, p^*) = \frac{E(CC)}{E(CL)} \qquad (3.20)$$

Where, h: sampling interval, p^*: upper control limit range from [0,1], $E(CC)$: expected cycle cost, and $E(CL)$: expected cycle length.

The second one is to determine the optimal control chart parameters such as sampling interval and upper control limit or posterior probability (threshold) that maximizes average long run expected availability. From renewal theory, the average long run expected availability equal the expected system uptime incurred in one cycle divided by the expected cycle length. The cycle is completed when the system is brought back to the in normal state which known as good as new. The expression of average long run expected availability which can be generalized by (Jiang, 2011):

$$\text{Max } z(h, p^{*}) = \frac{E(UT)}{E(CL)} \tag{2.21}$$

Where, h: sampling interval, p^{*}: upper control limit range from $[0,1]$, $E(UT)$: expected system uptime if full inspection is initiated, and $E(CL)$: expected cycle length if the full inspection is initiated. Most of CBM models consider the first objective function (cost criterion) and few papers use the second optimization criteria (availability criterion). However, it is possible in optimization CBM models consider: the following optimization problem: (i) minimize average expected cost per time unit subject to constraint availability is satisfied or (ii) maximize average expected availability subject to maintenance cost is satisfied. The optimal maintenance policy must be based also on availability criteria. Considering cost and availability criteria in optimal maintenance policy needed to achieve the best operating performance (Sarkar, 2000). Further, maintenance time should be not negligible because this assumption makes availability modeling realistic and possible will result in realistic system reliability measures as well as the structure of system must be considered in order to obtain optimal system reliability performance and optimal maintenance policy. Other factors which may affect an optimal maintenance policy are illustrate in Figure 2.9 (adapted from (Sarkar, 2000)).

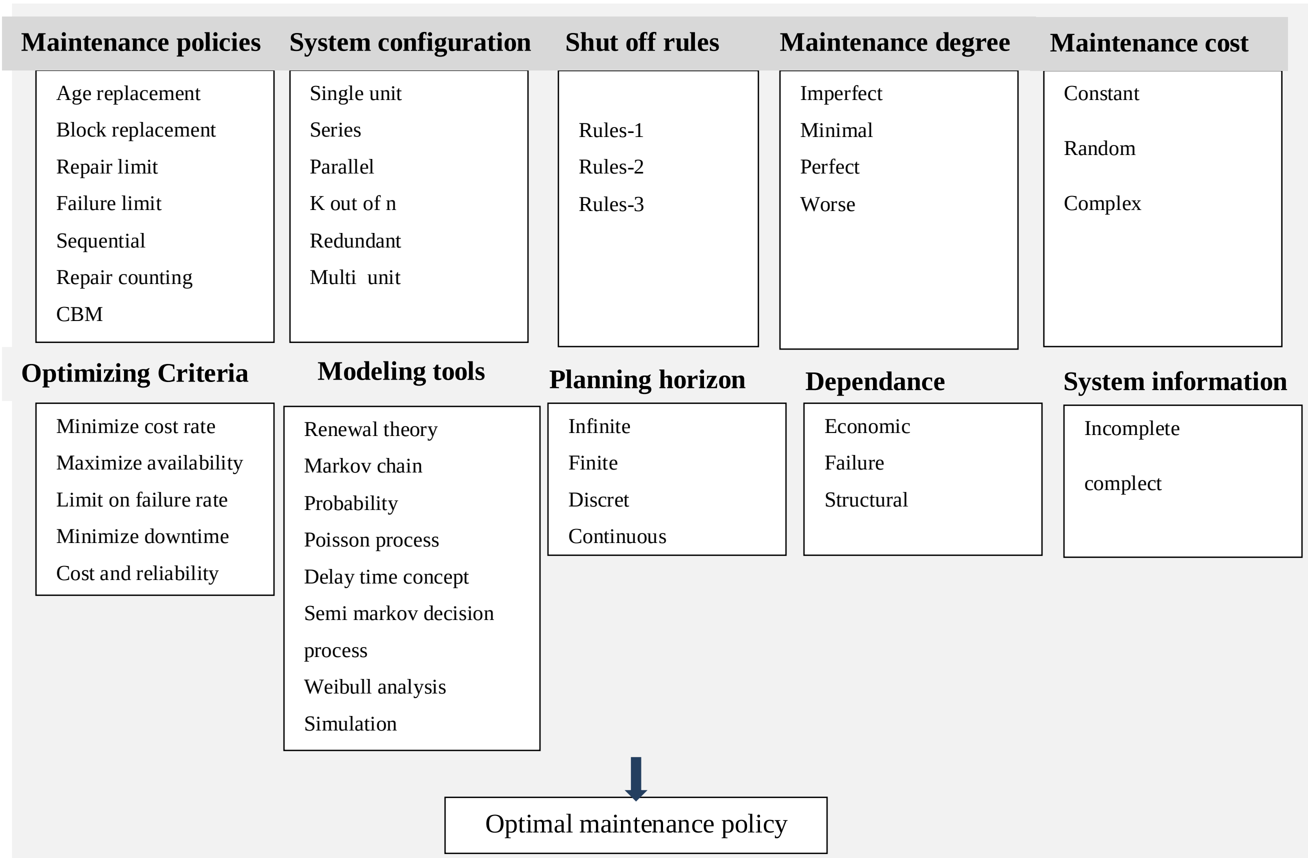


Figure 2.9- Maintenance policy and its influence factors.

2.9 CBM Models

Nowadays, the area of CBM has extensively interested by the numerous researchers and practitioners. Over the last twenty years, several kinds of CBM models have appeared in the maintenance literature. The success of the CBM policy is based on the ability to develop failure prediction models as accurately as possible by means failure model and degradation models as well as involvement an advanced technology and suitable tools. An accurate and efficient modeling for CBM policy represents a significantly challenge. According to Amari et al., (2006) CBM model include a stochastic process, a set of maintenance action and their effects, and a scheduled inspection policy that identifies the condition of deterioration.

McMillan and Ault (2008) evaluate the cost-effectiveness of CBM via Monte Carlo simulations. The authors' address the states of a wind turbine condition in more detail by considering the individual state of critical components like gearboxes and generators and they compare a six-month periodic maintenance policy with CBM. Sensitivity analysis shows that the benefit is dependent on wind profile, typical downtime duration and wind turbine sub-components replacement cost. Marseguerra, Zio and Podofillini (2002) proposed a model to find the optimal degradation level of multi-component system using genetic algorithm and Monte Carlo simulation. The predictive model describing the evolution of the degrading system is based on Markov Model and, Monte Carlo simulation and Genetic Algorithm is used to determine the optimal degradation level beyond which preventive maintenance has to be performed. The developed Markov model incorporates an intermediate state to represent component degradation behavior of a gearbox. Condition monitoring of equipment is used to evaluate the system state, and it is assumed that it equipment reveals exactly the degradation status of each turbine component.

Makis and Jiang (2003) present a model to determine the replacement policy that minimizes the long run expected average cost per unit time. The replacement problem is formulated as an optimal stopping problem with partial information and is transformed to a problem with complete information by applying the projection theorem to a smooth semi martingale process in the objective function. A dynamic equation is derived and analyzed in the piece wise deterministic Markov process stopping framework. Liu et al. (2012) presents a general framework for planning and optimizing CBM scheduling. The authors assume that the hazard function depends, not only on time, but also on the system state which degrades over time. The optimum threshold value triggering the maintenance is obtained by maximizing system's availability over its life cycle. Luce (1999) presents a study to improve the availability of production equipment by selecting the best maintenance management method. Corrective maintenance, systematic preventive maintenance and CBM costs are compared and the Weibull law is used to model time to failure. Barbera, el al., (1999) developeda CBM model with exponential failures and fixed inspection intervals for a two-unit system in series. The proposed model aims to minimize the long-run average cost of maintenance actions and failures and the optimal solution to this problem is obtained via dynamic programming. Pedregal and Carmen

(2009) provide an economic model for condition monitoring through vibration data of turbine driving a centrifugal. Saranga and Knezevic (2001) developed a mathematical model for reliability prediction of condition-based maintained systems using the concept of multiple relevant condition predictors in order to identify all the possible failure mechanisms that affect the system. Use of suitable relevant condition predictors for each failure mechanism allows identifying the equipment of degradation. The Markov Process is used to model the degradation. Jardine et al. (2003) presents an optimal policy for the interpretation of inspection data from a CBM program at a nuclear reactor station. Jamali et al. (2005) proposed an optimal periodic replacement strategy. A model is proposed to determine the preventive replacement interval and the threshold age for which the replacement is performed in order to maximize the steady state availability under a cost constraint and minimize the average total cost per unit of time on an infinite horizon while respecting the predefined availability threshold.

Since the proportional hazards model (PHM) was introduced in 1972 by Cox, several researchers proposed optimization models for CBM using proportional hazards model. The reliability is determined based on proportional hazards model and, based on a control limit policy, the maintenance cost is defined. Ghasemi el al., (2010) proposed methods to estimate the parameters of condition monitored equipment whose failure rate follows the Cox's time-dependent Proportional Hazards Model. The authors considered that the equipment's unobservable degradation state transition follows a Hidden Markov Model and used the Maximum Likelihood Estimation to estimate the parameters of the Hidden Markov Model and of the Proportional Hazards Model. Many works can be found in this area such as Lin et al. (2006), Makis et al. (2006), Vlok et al. (2002), Makis and Jardine (1992), Banjevic et al. (2001), and Gupa and Sirirat (2006). Adjengue, Yacount and Ilk (2007) used an expectation maximization algorithm for estimating the parameters of a CBM model. Jardine, Banjevic and Joseph (1999) also used expectation maximization for estimating the parameters of a CBM model and present an optimal maintenance program based on vibration monitoring of critical bearings on machinery in the food processing industry. Statistical analysis of vibration data is undertaken using the software package EXAKT to establish the key vibration signals that are necessary for risk estimation. The risk curve is identified using a proportional hazards model and cost data are then blended with risk to identify the optimal maintenance program.

It is recognized that maintenance optimization often uses operations research enabling to maximize profit or minimize cost. Cost functions depend on the reliability and maintainability characteristics of the system of interest and allow determining the decision variables values by its minimization. The parameters often considered are the cost of failure, the cost of downtime, the cost of corrective maintenance, the cost of preventive maintenance and the cost of system replacement (Cassady and Pohl, 2003). Optimal Maintenance strategies are often constructed using stochastic models and focus on finding an optimal inspection time or the optimal acceptable degree of system degradation before performing maintenance and/or replacement. Optimizing CBM depend on many factor, including modeling system degradation, modeling system reliability, choosing the appropriate performance measure, and the optimization of inspection schedules (Peng, Dong and Zuo,2010). Maintenance optimization models intend to optimize an objective function which allows determining the interval between inspections or maintenance intervention. Models can also be of help in determining effective and efficient schedules taking all kind of constraints into account (Dekker, 1996).The optimization methods used include linear and nonlinear programming, dynamic programming, Markov decision methods, decision analysis techniques, search techniques and heuristic approaches. Hundreds of articles appear every year in journals and international conferences such as Louit et al. (2011), Chen and Trivedi (2002), and Yam et al. (2001).

2.10 Control chart and optimization maintenance

In the past several decades, a lot of maintenance optimization appeared in literature review in different ways. The major method of optimization maintenance has been joined in recent year refers to control chart. It is vital that control chart becomes integrated with CBM to ensure a great result and successful rather than the classical maintenance (Jardine et al. 2006; Amari et al. 2006). It is evident that maintenance management associated with appropriate tools and specific skills ensure to improve a new vision of maintenance management. Furthermore, optimization of maintenance is based on intelligent prediction tools (Tian, 2011). In fact, the integrated maintenance models can be benefit in improving the system performance subject to many stresses (e.g., cost, availability, safety, quality products, satisfied customer) as much as possible. Implementing an efficient maintenance policy in maintenance management enables to improve system performance, productivity, and associated gain. According to literature, several maintenance models under control chart for maintenance optimization has been extensively developed, and they can be classified into two different group according to characteristic of collected data (Benerjee and Rahim, 1993; Ben-daya and Rahim, 2000, Lee and Rahim, 2001.etc.). The first one is Control chart used to monitor a process through some quality characteristic, the second one control chart is used to monitor the health state of equipment while in operation.

2.10.1 Statistical Process Control and Maintenance Planning

During the past 20-25 years, statistical quality control has proven to be extremely effective. The implementation of techniques such as statistical process control (control charts) and off-line quality control methods (Taguchi methods) have led to quality improvements that have resulted in industrial productivity increases in the neighborhood of 15-25% (Cassady et al., 2000). The economic design of control charts and the optimization of preventive maintenance policies have separately received a tremendous amount of attention in the quality and reliability literature over the years in an attempt to reduce the costs associated with manufacturing processes. Until recently, no proposal had been made to integrate these two fields and utilize the relationship between quality and equipment performance to improve the productivity of a manufacturing process (Yeunget al., 2008). According to Ben-Daya and Duffuaa (1995), quality is becoming a business strategy leading to success, growth and enhanced competitive position. Organizations with successful quality improvement programs can enjoy significant competitive advantages. Consequently, the role of equipment maintenance in controlling quantity, quality and costs is more evident and important than ever. To succeed in this new environment, equipment must be maintained in ideal operating conditions and must run effectively. Figure 2.10 shows the dependence between maintenance, quality and productivity.

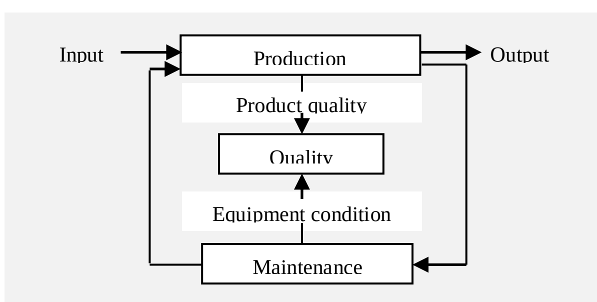


Figure 2.10- Production, quality and maintenance dependences (Ben-daya and Duffuaa, 1995).

In fact, control chart is a tool for statistical process control characterized by three parameters: sampling interval, sampling size, and control limit. To design control charts, three approaches have been used: economic design, statistical design, and economic-statistical design. Economic design was first proposed by Duncan (1956) which developed an economic model to be used for the selection of Shewhart x-bar control chart parameters under renewal theory. This model becomes a standard in this research area. Economic design intends to estimate system parameters that minimize the expected total cost which is considered the objective function. In statistical designs, constraints on average run length (ARL), or equivalently Type I and Type II error probabilities are considered. The advantage is that it requires no cost or system parameter estimation other than a specification of ARL's for particular shifts against which protection is desired. ARL is considered as an important performance measure for control chart design. In fact, ARL helps reducing the occurrence of false alarms as well as enable fast identification of the out-of-control condition. Furthermore, by assuring that shifts are signaled rapidly and false searches or improper adjustments are avoided, one can guarantee high quality products.

Economic-statistical design is a method proposed by Saniga (1989) which consists in an economic model with added constraints on ARL's (or equivalently Type I and Type II error probabilities). This last design requires the estimation of the same costs and system parameters as in economic design as well as the specification of desired ARL's and shifts against which protection is desired. The last design shows some important advantages such as statistical properties, more economical to use, and guarantee of high quality products. Many approaches and applications have been developed in the area of economic design of process control charts. Ho and Case (1994) shows a survey and brief summary of the economic designs published during the period from 1981 to 1991.

The economic design of control charts and the optimization of preventive maintenance policies are two research areas that have recently received a great deal of attention in the quality and reliability literature. Montgomery (1980) listed 51 references on the economic design of control charts. Keats et al. (1997) summarize some important works about the integration of statistical process control and preventive maintenance. Both of these research areas are focused on reducing the costs associated with the operation of manufacturing processes (Cassady et al., 2000). Tagaras (1988) was the first to propose an economic model that incorporates both process control and maintenance scheduling. The proposed model simultaneously optimizes the design parameters of control chart and the parameters of maintenance scheduling. Lesage and Dehombreaux, (2012) provide a methodological approach to evaluate the potential role of quality control in the improvement of maintenance policy. Ben-Daya and Duffuaa (1995) proposed two approaches for linking and modeling the relationship between quality and maintenance. The first approach is based on the idea that maintenance affects the failure pattern of equipment and the concept of imperfect maintenance should be used to model this relationship. Therefore, maintenance will also affect quality inspections. In the second approach, the deviation of product quality characteristics from their target value is reduced when maintenance is performed.

Furthermore, integrated models for the joint optimization of process control charts and maintenance operations enrich the existing literature about maintenance models. Many researchers investigated the relationship between quality and maintenance. Rahim and Banerjee (1993) developed an economical design of $\overline{X}$ control chart for quality control where the quality characteristic is dependent upon the age of equipment. It is considered that the equipment has an increasing failure rate, the times to failure follow a Gamma distribution and the sampling interval is not constant. Jenningst and Drake (1998) and Zhang and Berardi (1997) extended the work of Rahim and Banerjee (1993) and provided an economic statistical design model for the X-bar control chart considering that failure mechanism follows a Weibull distribution. Linderman, Mckone-Sweet and Anderson (2005) developed a model to demonstrate the economic benefit of integrating statistical process control and equipment maintenance. They demonstrate the usefulness of an adaptive maintenance policy where the scheduling of maintenance actions adapts to the stability of the process. Rahim (1994) propose a model to determine the optimal control chart design parameters and production quantity so as to minimize the expected total cost (the quality control cost and the inventory control cost) per unit time. Both uniform and non-uniform inspection schemes are considered. Rahim and Muhammad (2010) developed an integrated model for economic production planning, quality control, and preventive maintenance scheduling using a tabu-search algorithm to determine the optimal values of the model parameters. Ben-Daya (1999) developed an integrated model for the joint optimization of the economic production quantity, the economic design of $\overline{X}$-control chart and the optimal maintenance level, considering that the deteriorating process of in-control period follows a general probability distribution with increasing hazard rate.

Cassady et al. (2000) performed a preliminary investigation of using a $\bar{X}$ chart in conjunction with an age-replacement preventive maintenance policy. The authors used a simulation-based optimization approach in conjunction with a genetic algorithm to minimize the average cost per hour of the manufacturing process. They present a single numerical example to show that the combined policy can achieve greater productivity than either policy in isolation. Yeung et al., (2008) extended the work presented by Cassady et al. (2000). The objective of their research was to develop the most cost-effective policies that utilize the $\bar{X}$-control chart in conjunction with an age-based preventive maintenance policy to improve the performance of a manufacturing process. Cassady et al., (2000) used simulation-based optimization in conjunction with a genetic algorithm in order to evaluate such policies. In this work, the authors formulate a Partially Observable, Discrete-time Markov decision Process (PODMP) to evaluate the long-run expected cost and develop an algorithm to obtain the parameters values which minimizes long-run expected cost (the cost of inspection, maintenance and poor quality cost).

Ben-Daya and Rahim (2000) provide a model for incorporating the effects of preventive maintenance on quality control charts. The model allows the joint optimization of quality control charts (number of inspections, sample size, sampling intervals and control limit) and preventive maintenance level to minimize the total expected cost. Weibull shock model with increasing hazard rate is used to illustrate the effect of the maintenance level on quality control costs. Wu and Wang (2005) used an adaptive control chart to monitor process quality and compared it with static control chart and both are designed by the minimization of the expected long run cost per time unit. Chan and Wu (2009) present an integrated model for the joint optimization of quality inspection and maintenance that uses Cumulative Count Conforming Chart. Panagiotidou and Tagaras (2010) developed a model to integrate statistical process control and preventive maintenance of manufacturing equipment. The authors considered that time of shifts in the quality level, as well as the time to failure in both in-control state and out-of-control state are exponentially distributed random variables and the failure rate in the out of-control state is higher than the failure rate in the in-control state. Panagiotidou and Nenes (2009) developed an integrated model for quality and maintenance and considered that the time to the transition to an out-of-control state follows an exponential distribution. They applied an adaptive variable-parameter Shewhart control chart for process monitoring and compared their model with a fixed parameter control chart. Mehrafrooz and Nourossema (2011) developed an integrated model which is an extension of Panagiotidou and Nenes (2009) that joint both statistical control chart and maintenance to improve quality of products. The authors showed through experimental tests that the integrated model has better economic behavior than the planned maintenance model. The assumptions in this work and in Panagiotidou and Nenes (2009) are similar, but the main difference between the two models is that Panagiotidou and Nenes (2009) do not consider planned maintenance. Charongrattanasakul and Pousakul (2011) developed an integrated model of Statistical Process Control and planned maintenance using EWMA control chart and used genetic algorithm in order to find the optimal parameters values that minimize the hourly cost. In another work, Chen et al. (2011) show the details of the development of a model for the economic design of $\bar{X}$-control chart. In this model, it is considered that preventive maintenance restores the system to an "as good as new" condition. The social loss cost of the Taguchi's loss function is also considered in the objective function. Thus, the optimal solution is found using the algorithm defined by Rahim (1993). Pandey, Kulkarni and Vrat (2012) developed a methodology for simultaneous optimizing the design parameters of preventive maintenance and control chart incorporating the Taguchi loss function. The proposed model enables the determination of the optimal values of each of the four decision variables, i.e., sample size, sample frequency, control limit coefficient, and preventive maintenance interval that minimize the expected total cost per unit time. Nowadays, seen in this light, new aspects can be incorporated in integrated model and brings improvement in the results. More and more researchers have been discussing the relationship between quality and maintenance such as Kniele et al. (1989), Lee and Rahim (2001), Lochner (1987) and Tapiero (1986).

2.10.2 The Use of Control Chart for Condition Monitoring in CBM policy

Nowadays, research in CBM policy has been carried out by enormous of the researcher's community as the technological progress has been growth rapidly. CBM models appeared every year in technical literatures. In fact, technology, automation and complexity of the system increase, it must be aware of the complex system maintenance. Nowadays, in real life decision maker face tight appropriate and accurate decision. However, a proper and well-performed CBM policy promises helpful managerial insights to management in maintenance decisions making. CBM policy decision can provide a good idea of which decision should be selected; some of these CBM decisions can include a wide range of actions as follow: (i) Adjustments to the equipment. An adjustment can be a simple fine-tuning of a cam on a limit switch or involve the tuning of a boiler combustion control system to maximize fuel efficiency, (ii) Replacement of damaged or warn components, (iii) replacement of disposable component such as air, oil, or fuel filters, (iv) performance of an overhaul that aims to restore the equipment to as good as new condition.

However, the challenge and opportunities here is reside on developing new integrated models arises from using a selective appropriate tools and knowledge such as: the information collected through condition monitoring, parameter selection for monitoring condition, an appropriate stochastic process for predicting item failure, optimizing maintenance, optimizing organizational maintenance efficiency. Furthermore, the overall objective is to establish the applicability of these skills and knowledge together in which should be incorporated into an integrated approach and unifying framework in order to form an effective CBM strategy.

Control charts might be considered an effective tool to indicate and detect the early signs of health state of a deterioration or malfunction. In this case, the control chart uses parameters values obtained from periodic condition monitoring of equipment, in order to decide whether maintenance action should be performed or not. In many CBM applications, the true state of the system is unobservable and can only be inferred using an observation process which is stochastically related to the hidden state of the process. The model often used to represent this situation is the Hidden Markov Model (HMM) which is able to represent stochastic process.

Nowadays, the design of prediction model including control chart has been increased. The development of sophisticated technologies has contributed to the adoption of integrated models which have attracted the interest of many practitioners and researchers once it contributes to reduce maintenance cost, to increase lifetime of components, to increase reliability, to improve safety and decrease downtime. Over the last few years, several outstanding articles have been published addressing this area. This includes the article of Wu and Makis (2007) which proposes a model to determine the optimal control chart parameters that minimize the long-run average maintenance cost per unit time. An experimental test with real data was performed to make a computational comparison between the economic statistical design and the economic design of chi-square control chart and to demonstrate the effectiveness of the economic statistical design of chi-square control chart for CBM. Wang and Zhang (2008) presents an integrated model that uses an adaptive Shewhart average level chart methods and an auto-regression model to model the identification of the initiation point of a random defect (the adaptive Shewhart average level chart is adopted when the stochastic process is non-stationary and non-gaussian). An empirical experiment has been performed on a set of vibration data of rolling element bearings. Zhou and Lui (2011)designed a moving range control chart to predict failures using the information given by oil spectral analysis under Projection Pursuit approach (PP). The proposed method has been tested on a set of oil data of marine diesel. Louit et al. (2011) present a robust multivariate control charts for early detection of broken rotor bars in an induction motor fed by a voltage source inverter. Yu (2011) showed that the Locality Preserving Projections and Exponentially Weighted Moving Average chart (LPP-EWMA) is capable to recognize a slight degradation of bearing at early stage and clearly reveal the degradation propagation of bearing performance on its whole. Lampreia et al. (2012) developed a multivariate control chart Hotelling's T^2 in order to monitor the vibration of repairable systems using both independent data and auto-correlated data.

Recently, some review papers discussed the efficiency of Bayesian control chart for CBM and show wide success of this chart in this area. This includes the work provided by Jiang and Makis (2009) which consists of the design of a multivariate Bayesian control chart for CBM. The Bayesian control chart is a chart showing plotted values of the posterior probability that the hidden system is in the warning state given all past information. The authors prove that the Bayesian control chart is much more effective for fault detection than other charts such as Hotelling's T^2, MEWMA, and MCUSUM and, thus, the maintenance cost will be much lower in practice. Makis (2007) also developed a multivariate Bayesian control chart for process monitoring. Wang (2012) applied a safety constraint to the adaptive Bayesian CBM model presented by Flag et al. (2012). Yin (2008) presents an economic and economic statistical design of the Bayesian chart for CBM. Yin and Makis (2010) developed an optimization integrated model for the economic and economic statistical design of the multivariate Bayesian control chart. The paper shows that the economic statistical design of the multivariate Bayesian control chart performs better than the chi-square control chart.

Most optimal maintenance models in the literature use as optimization criterion the minimization of system maintenance cost rate but ignore availability. However, maintenance aims to improve system availability. Therefore, the optimal maintenance policy must be based not only on cost rate but also on reliability measures. It is important to note that, for multi-component systems, minimizing system maintenance cost rate may not imply maximizing the system reliability measures. Sometimes, when the maintenance cost rate is minimized the system reliability measures are so low that they are not acceptable in practice, since various components in the system may have different maintenance costs and different reliability importance (Wang and Pham, 1997). Therefore, to achieve the best operating performance, an optimal maintenance policy needs to consider both maintenance cost and reliability measures simultaneously. As mentioned above, optimizing an objective function in CBM consists in minimizing the long-run expected average cost per unit time (Jiang et al., 2001; Makis and Jiang, 2003) or in maximizing the long run expected average system availability per unit time (Barlow and Hunter, 1960; Sarkar and Chaudhuri, 1999; Biswas and Sarkar, 2000). A lot of published works propose cost models and few published works propose availability models. A recent article, Yu and Makis (2012), uses as objective function the availability. In another work, Jiang, Kim and Makis (2012) designed an optimal multivariate Bayesian control chart that maximizes the long-run expected average availability per unit time.

2.11 Limitation of existing CBM models

Amari et al., (2006) have shown that CBM models are considered as an optimal tool for maintenance management. However, CBM models is needed continuously to develop today more than ever, therefore, many drawback related for designing CBM models which needed better models and algorithm for handling all kinds of complex situation that arise in practical CBM decision-making. They have been shown in CBM models presence of six limitations as follows: **Inspection distribution-**The major limitation with existing CBM models is that the exponential distributed inspection interval does not help engineers to determine when to perform inspections. **Inspection model-**The existing CBM models assume that inspections intervals are independent of the system deterioration condition, which is an inefficient schedule. This leads to performing inspections unnecessarily when the system is functioning properly. Additionally, it leads to not performing inspections when there is a need. **Deterioration models-**The existing CBM models are limited to unidirectional, single-step deterioration that neither models the combined effects of various deterioration mechanisms nor the effects of random events such as floods and earthquakes. **CBM Decisions-**In existing models, only a few kinds of CBM decisions are considered: No action, Minor Maintenance, preventive maintenance. In reality, there can be several types of maintenance decisions such as refill the lubricant, replace the screws, align the bearing, and replace the bearing. **Effects of maintenance-**All existing CBM models support only the maintenance actions that reduce the current deterioration level. However, in some cases, the CBM actions may reduce the rate of deterioration instead of actual deterioration. For example, the wear propagation of a cutting tool can be reduced by changing the cutting depth, cutting speed, or the lubrication flow rate. **Optimization procedure-**Another major disadvantage of existing CBM models stems from their procedures for finding the optimal decisions. The existing models first find a closed-form cost function and then compute the cost for all possible combinations of parameters to find the minimal cost. This approach is feasible only for simple deterioration processes and a limited number of decision variables.

The work of Marcus Bengston (2007) suggested a solution about how CBM approach can be implemented in industrial setting as well as developed a method that can assist companies in their implementation effort. Further, the procedure has been divided in three phases: (i) feasibility test phase, (ii) analysis phase, (iii) implementation phase, (iv) assessment phase. However, in CBM implementation general enabling factor are required. Some of them focusing on the following: management support, education and training, good communication, and motivation. Generally, conditions necessary for a successful implementation process are typically ones of culture change and change management. The procedure for implementing CBM on industry is governed by the following:

- Requirement substantial efforts by all site personnel and management,
- The staff must have commitment to the process and its new technologies as well as their use, staff has to trust the training and the technology,
- Management must have the commitment to procure an adequate equipment,

- Management must have the commitment an adequate training for the all personnel
- The organization's support for implementation the CBM policy must be totally guided to achieve the goal and success predefined, management has to reinforce this expectation until to accomplish the end of implementation's procedure.
- The needed resources must be available during all steps of implementation the CBM policy, this includes the management support and attitude to trust and maintain it.
- Writing the formal description of the following techniques that should be included in a CBM project, technical environment, degree of development concerning computer, sensor technologies, and advance in skills and knowledge in prognostic technology as well as flexibility, maturity environment, professional and intellectual rigueur.
- The evolution and monitoring the process of implementation must be assessed during all cycle life of project and after exploitation as well.
- Increased awareness of CBM concepts and Service approaches
- Improved coordination of service initiatives, and additional advocacy in the form of policy, and guideline should be described the objective, goal, and procedure of the implementation CBM policy.
- Increase maturity level in the knowledge related methods and avoid obligatory all factor can be transform the environment into college.
- Culture change is necessary towards CBM framework, Leadership and manager should be able to manage the change that is faced by their organization, human behavior. Reduce the risk that CBM implementation has not considered by the companies.

Chapter 3

Stochastic Process Applied to CBM models

The purpose of this chapter is to define stochastic and deterministic concept as well as to select which approach will be used throughout this work. Lastly, it provides a description of some of the most common stochastic process approach in use today, thereby summarizing some works that use stochastic processes to develop CBM models with special emphasis on Markov chain, renewal process, delay time concept, and gamma processes.

Contents

3.1 Stochastic and deterministic concepts

Over many decades ,there has been an ever increasing interest by scientific community to study random phenomena in various fields of science engineering, due to the uncertainty of both system behavior and environment of the system itself. In general cases, the uncertainty means the lack of certainty about the behavior of the system. Those concept can be associated with a state of having limited knowledge where it is impossible to describe exactly the current state, the future state, or more than one possible outcome. In fact, such systems are subjected to change over time and evolved in time and space with random manner, and it has been extensively studied over the last three decades. Mathematical models of such systems are known as stochastic processes (Hoel et al., 1986; Bartlett, 1955).

In general cases, the majority of phenomena in industrial complex systems are non-deterministic. These systems are known as stochastic or random systems. It should be noted that, since the late 19's century more stochastic models than deterministic models have been developed (Medhi, 1982). Stochastic model is used when the analyzed system has at least one stochastic component, but it could have one or more deterministic components. Besides, deterministic models have no random or probabilistic component, and the entire input and output of system relation of the model is conclusively determined. A deterministic model is used in that situation where the result is established straight forwardly from a series of conditions. In a situation where a cause and effect relationship is stochastically or randomly determined, a stochastic model is used. It is clear that stochastic modeling is more complex than deterministic modeling in terms of data collection, processing, computational and run time. The system having stochastic element is generally not solved analytically.

In fact, different types of stochastic processes are widely used by researcher for system modeling. Therefore, many models rely on stochastic processes aims understanding, predicting, and controlling uncertain behaviors that are subjected to chance. Stochastic processes theory has attracted the attention of many researchers in a pervasive manner, particularly in the last three decades and it plays an essential role in many scientific fields: physics, medicine, oceanography, finance, chemistry, astronomy, communication and control theory, management science (Karlin and Taylor, 1975; Parzen, 1999). For example: Deterioration in structural material, maintenance decision making, congestion in telephone lines or road traffic, effect of air pollution on health, genetic determinants of diseases, stock market fluctuations, the phenomenon of carcinogenesis, waiting line analysis, and so on (Cox and Miller, 2001; Parzen, 1999).

Nowadays, stochastic processes are widely applied in industrial maintenance. It has been playing an important role in solving optimization maintenance problems (Wang, 2003; Noortwijk, 2009; Kallen and Noortwijk, 2005).

3.2 **Stochastic Processes**

Stochastic processes are used to describe the evolution of dynamic systems over time via random changes in accordance with probability theory. Over the last decades, many books devoted exclusively to stochastic processes have been published such as: Gallager, 2013; Singpurwalla, 1995; Osaki, 2002; Ross, 1996; Ross, 2000; Ross (2000); Karlin and Taylor (1975);Karlin and Taylor (1982); Bailey (1964); Cinlar (2013); Hoel et al., (1972); Parzen (1999).

Nowadays, there is in fact a great increasing in the application of random variables for all practical purposes in probability theory (Medhi, 1982). Furthermore, the probability theory and mathematical statistics are powerful tools used for describing and modeling stochastic events, variables, relations, systems or processes (Valdma, 2007). Many systems operate in dynamic environments. Processes have some random or stochastic elements involved in its structure and changing with time, therefore, they have explicit time dependence (Bartlett, 1978; Gheorghe, 1990). In a given time interval, a system or process can have more than one state. A state of system undergoes random changes over time, based on the present state, the evolution of the system is predicted (Gheorghe, 1990).Indeed, the state of system might be defined at any instant even the value of state variables of a system are unknown. However, due to the uncertainty related to many factors, it is impossible to define exactly the actual and future state of the process. This explains why it is required that the process state is represented by a random variable. Stochastic process models are capable to capture and analyze the inter-dependence of random variables, their change in time and limiting behavior by using probability law (Ross, 2000).

According to Gallager (2013), stochastic processes constitute a branch of the probability theory treating probabilistic systems that evolve in time. A probabilistic system is defined as a system that consists of at least one probabilistic component and may contain deterministic components(Valdam, 2007). Dynamic probabilistic systems are characterized by states, holding times in any given state and transitions among states (Gheorghe, 1990).

Stochastic processes can be defined mathematically in different ways. According to Ross (1996), a stochastic process $\Omega = \{X(\theta), \theta \in \Theta\}$is a collection of random variables$X(\theta)$ indexed by a parameter θ. That is for each θ belonging to the index set $\Theta, X(\theta)$ is a random variable which represents the state of the process at time θ.The index θ often referred as time, and $X(\theta)$ represents process state at time θ. If Θ is a set of integers (discrete or countable finite), representing specific time points, the stochastic process Ω is called discrete time stochastic process. In this case, it is usual that the subscript θ is replaced by n, and the stochastic process is normally denoted by $\{X_n\}$.Otherwise, if Θ is continuous or uncountable(interval or real line), the stochastic process Ω is called continuous time stochastic process. In this case, every continuous time stochastic process has probability density function. In addition, a continuous-time stochastic process $\{X(\theta), \theta \in \Theta\}$ is said to have independent increments if for all $\theta_0<\theta_1<\theta_2<\theta_3<\ldots<\theta_n$, the following random variables: $X(\theta_1)-X(\theta_0)$, $X(\theta_2)-X(\theta_1)$,…, $X(\theta_n)-X(\theta_{n-1})$ are independent. In this case, the general subscript θ can be replaced by t and change the notation slightly, writing X(t) or X_t rather than$X(\theta)$ (Karlin and Taylor, 1975; Cox and Miller, 2001; Breuer, 2007).

In spatial process, Θ is a vector, representing the location in space rather than time. The process defined by the collection of random variables $\{X_{(\mu,\upsilon)}\}$,at position (μ,υ), varies across a two-dimensional space. Processes that evolve in both time and space are said to be spatio-temporal processes (Breuer, 2007).Stochastic processes can be classified on the basis of the nature of their state space A, the index parameter Θ, and the dependence relations among the random variable X(t).In the case of state space A, if the state space A is discrete the sequence of random variables $\{X(\theta), \theta \in \Theta\}$ is said to be a discrete-state process. Otherwise, if the state space is continuous, then the sequence of random variables is said to be a continuous-state process. Therefore, there are four classifications of stochastic processes: (i) Stochastic process with discrete parameter and state space, (ii) Stochastic process with continuous parameter and discrete state space, (iii) stochastic process with discrete parameter and continuous state space, (iv)Stochastic process with continuous parameter and state space (Karlin and Taylor, 1975; Cox and Miller, 2001; Osaki 2002). In terms of stationary there are two distinct types of stochastic processes, those referred as stationary processes and the others referred as non-stationary processes. For stationary processes, the probability distribution of the number of events that occur in any interval remains the same as time progresses and depends only on the length of the time interval. The random mechanism producing the process is not changing, and thus, a stationary process has the property that the mean, variance and autocorrelation structure do not change over time. Non-stationary processes are also known as evolutionary processes and the probability distribution of the random variables does not remain the same over time.

A sample path of a stochastic process represents the number of events that occur in a specified interval [0, t]. A typical sample path is shown in Figure 3.1, given an interval [0, t], three events occur between 0 and t. The initial event occurs at time t_1, the second event occurs at time t_3, and the last event occurs at t_4 (Ross, 1996).

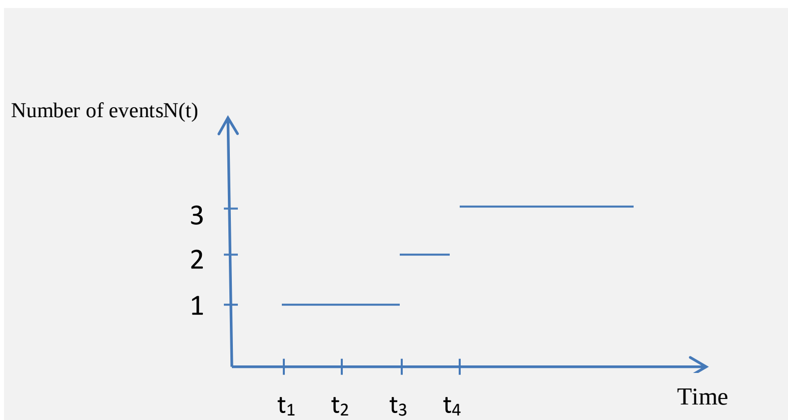


Figure 3.1. A sample path of X(t) during the interval [0, t] (Ross, 1996).

A counting process is viewed as a counting number of events occurring over time under study. A stochastic process {N(t), t≥0}is said to be a counting process if N(t) represents the total number of events that have occurred up to time t (number of birth per day, number of failure up to time t, number of light bulbs which were burn out during the interval size t, number of patient in emergency during night, number of internet disconnection during one months, number of arrived student in professor's office by the time t. In addition, a counting process is constant between events, and jumps one unit at each event time, and denoted by N(t) where *t* is time. Poisson process, binomial processes are an example of counting process.

3.2.1 Renewal theory and its application in maintenance

Renewal process is defined as an arrival process in which the inter-occurrence times between successive events are positive, independent, and identically distributed random variables (Markovich, 2007). The major reason for using the term "renewal" comes from the basic assumption that when the event of interest occurs, the behavior of the process starts as new (time zero), in the sense that the initial situation is reestablished. This means that, starting from this "renewal instant", the waiting time for the second occurrence of the event has the same distribution as the time needed for the first occurrence.

Renewal theory provides a theoretical framework that counts the occurrence of events in repeated independent trials (embedded renewal processes) for a specific stochastic process under study. In queuing process, the embedded events could be the arrivals of customers to a waiting line (queues) in order to receive a service. In inventory process, the embedded events could be the replenishment of stock when the inventory position drops to the reorder point or below it. In reliability problems, the embedded events could be the successive replacements of electric lights bulbs, or the successive occurrence of the failure of valves (Tijms, 2003).

Since renewal processes is an arrival process in interval time (0, t), it can be then specified in three ways: (i) by the joint distributions of the arrival epochs $S_1, S_2, \ldots$, (ii) by the joint distributions of the inter-arrival times $X_1, X_2, \ldots$ (iii) by the joint distributions of the counting process N(t); for $t \geq 0$.

Let $X_1, X_2, \ldots, X_k$ be a sequence of non-negative, independent random variables representing the inter-occurrence time between the *(k-1)*th and *k*th event (holding time). For any $k \geq 0$, S_k is called an arrival time sequence refer to as the successive instants while a specific event occurs (jump time), and the interval $[S_k, S_{k+1}]$ being called renewal interval. The waiting time until the occurrence of the *k*th event denoted by S_k is written as follow: $S_k = \sum_{i=1}^{k} X_i$, ($S_0 = X_0 = 0$). Associated with a renewal process there is a renewal counting process N(t) that tracks the total number of renewals in [0,t). For $0 < S_k \leq t$, N (t) is defined by:

$$N(t) = \max\{k \geq 0: S_k \leq t\} = \min\{k \geq 0: S_k \geq t\} \qquad (3.1).$$

As illustrated in Figure 3.2, the time up to the *n*th renewal is known as the time at which the *n*th renewal will occur. For example: the *3*th failure occurs at time $S_k = X_1+X_2+X_3$. Then, the number of renewals in time t_i is defined as the number of renewals occurring in interval (0, t].

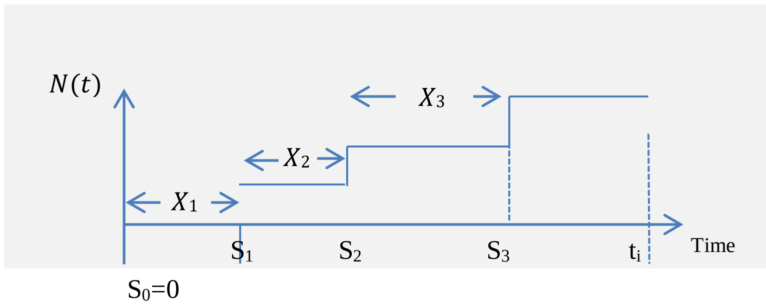


Figure 3.2. Path sample evolution of renewal process.

For a given interval time of size t, N (t) can be satisfying the following axioms (Tijms, 2002; Callagger, 2013; Karlin and Taylor, 1975):

If $s< t, then\ N(s) \leq N(t)$. For $s< t, N(t) - N(s)$equals the number of events in (s,t). Hence, for some k the renewal occur at time t only if $S_k= t$.

The inter-arrival times for the renewal process $\{N(t), t \geq 0\}$ having identical probability density function denoted by$f(t)$, if $F(t)$is the cumulative density function of $\{N(t), t \geq 0\}$ then:
$F(t) = P(X_k \leq t)$, if t equal zero $F(0) = 0$ elsewhere.

Therefore, the waiting time for the second occurrence of the event has the same distribution as the time needed for the first occurrence (Barbu and Limnios, 2007).

That is, if renewal process $\{N(t), t \geq 0\}$ equals to zero, with the property that:

$$\{N(t) = k\} = \{S_k \leq t, S_{k+1} > t\}, \text{For any } t \geq 0, k \geq 0. \quad (3.2).$$

A Renewal function constitutes the basic characteristic of an underlying renewal process, and it's uniquely determines the renewal process. Many probabilistic models and sequential analysis are based on renewal function estimation (Feller, 1971; Karlin and Taylor, 1975).

Given any interval of length t, the renewal function $\mathrm{H(t)}$ is defined as the expected number renewals in that interval. That is, for k=1,2,3...assuming that μ is the expectation denoted by $\mathrm{E[N(t)]}$ exist if : $0 < \mu < \infty$, then $\mathrm{H(t)}$ renewal function can be expressed directly in term of expectation by (Gallagar, 2013; Karlin and Taylor, 1975):

$$\mathrm{H(t)} = \mathrm{E[N(t)]} \quad (3.3).$$

$$\mathrm{E[N(t)]} = \sum_{\mathrm{k}=1}^{\infty} \mathrm{k\,P[N(t) = k]} = \sum_{\mathrm{k}=1}^{\infty} \mathrm{P[N(t) \geq k]} = \sum_{\mathrm{k}=1}^{\infty} \mathrm{P}\{S_{\mathrm{k}} \leq t\} = \sum_{k=1}^{\infty} \mathrm{F_k(t)} (3.4).$$

Where $\mathrm{F_k(t)} = \mathrm{P}\{S_{\mathrm{k}} \leq t\}$.

Given sample evolution of renewal process with two sequence S_k and S_{k+1}, then those two sequence having cumulative density function $F_k(t)$ and $F_{k+1}(t)$ respectively. F has mean and finite variance. For $\mathrm{k} \geq 1$, defining $\mathrm{F_k(t) = P(N(t) \geq k) = P(S_k \leq t)}$ Where $\mathrm{F_k(t)}$ is a k-fold convolution of F(t). Denoting T the random time, the renewal function H(t) is defined by relation (Chaudhry and Templeton, 1983; Frees, 1986):

$$\mathrm{H(t)} = \sum_{k=1}^{\infty} \mathrm{F^k}(t) \quad (3.5).$$

For $\mathrm{k} \geq 1$, let denote $\mathrm{P^k(t)}$ the probability of that number of arrival event in the interval $(0, t]$, so that:

$$\mathrm{P^k(t) = P(N(t) = k) = P(S_k \leq t \text{ and } S_{k+1} >} t) = \mathrm{F^{\,k}(t) - F^{\,k+1}(t)} \quad (3.6).$$

A renewal process can be classified into two types in term of time, one for discrete time, and the other for continuous time. These are called discrete time renewal process and continuous time renewal process respectively. A renewal process is said to be a discrete time renewal process if successive renewals are recorded in discrete time, which are indexed in the order: $t, 1t, 2t,$ ....Otherwise, renewal process is called a continuous time renewal process when the time of occurrence of an event is continuous. Continuous-time renewal processes can be approximated with discrete-time renewal processes (Weide andNoortwijk,2008). However, discrete-time renewal processes may not be simply approximated by continuous time renewal processes, especially when the counts are relatively small.

A renewal process is called a Poisson process, if the inter-arrival times random variables $\{X_1, X_2, \ldots, X_k\}$ are i.i.d. and have an exponential cumulative density function with a parameter known as the rate of process ($\lambda > 0$). A Poisson process is a special case of a renewal process, and is the only renewal process having independent and stationary increments as well as Poisson process is the only renewal process with the memoryless property (e.g., counting the number of components that are replaced during fixed time intervals). For any interval (0,t], λt is expected number of arrivals in that interval. The inter-arrival times having exponential distribution, The exponential distribution is the only distribution with the memoryless property, continuous probability density function of exponential distribution can be expressed by:

$$f(t) = \lambda e^{-\lambda t}, \quad \text{for } t \geq 0 \text{ and } \lambda > 0 \qquad (3.7).$$

A several authors have reported several methods for calculating the renewal function (Free, 1986, 1986, 1988; Baker 1993; Dohi et al., 2002; Nagai et al., 2000; Ross 1989; Markovich 2006; Schneider et al., 1990). There are two common computational methods for estimating renewal function. The first one refers to analytical computation method and the second one refers to numerical computation methods. Estimation renewal function with analytical computation method can be used by the following methods: phase renewal processes, gamma approximations, method based on equilibrium distribution, and in a specific case in which the knowledge of the distribution is incomplete and only the information on a few moments is available the bounds approach (upper and lower) may be useful to evaluate the renewal function. The most famous bounds for the renewal function have been derived by Barlow and Porschan (1965). Many numerical computation methods have been considered as tools for renewal function estimation: Laplace inversion technique, cubic splines algorithm, discretization algorithms, and approximation by rational functions. In estimation renewal function, numerical computation methods provide result more accuracy than the analytical computation methods and consequently involving in a great number of studies.

According to Osaki (2002), the mathematician William Feller (1906-1970) was considered the first who developed renewal theory, and the popular figures specializing in probability theory among other mathematicians. Feller (1968) proved the elementary renewal theory via Laplace transform methods. Laplace transform methods are considered the main mathematical tools to find the distribution function, mean, and variance of the number of renewals of a renewal process. The inter-arrival time distribution of a renewal process has Laplace transform. When the inter-renewal time distributions do not have Laplace transforms, the Padé method provides a good approximation (Chaudhry, 2013). In the 1960's, renewal theory has been developed primarily by the investigation of some general results in probability theory that connected with sums of independent non-negative random variables (Dohi et al., 2002). Most of the important results on renewal theory and its divergence were established by Smith (1958) and Cox (1962). A book was published by Feller (1968) covering a wide range of topics related to probability theory (e.g., renewal theory, Markov processes) and its application. Since then, renewal theory was first applied in reliability context as a technique to study complex systems. Applications are considered in study of some particular probability problems that connected with failure, and the analysis of components replacement problems. For example: using known distributions for the time to failure of each component to find the mean number of failures of the entire system in a given interval (Heyman and Sobel, 1982).

Renewal theory has a wide range of applied probability models, and is considered a powerful modeling tool in many applications: queuing analysis, inventory analysis, reliability analysis, telecommunication networks, and so on (Chaudhry, 2013). Therefore, renewal theory has played an important role in understanding and studying a lot of stochastic processes that occur randomly over time and which returns to a state probabilistically equivalent to the starting state (Heyman and Sobel, 1982; Tijms, 2003). Some of these application as follows: discrete event systems arising in queuing theory (Scarf et al., 1962), production and inventory control (Barlow and Porschan, 1975), design of communication systems, performance evaluation in computer science (Radner et al., 1967), and product warranty estimation (Gertslbakh, 1977), failure and maintenance of systems (Ascher and Feingold, 1984).

In many situations, stochastic processes have regenerative behavior; the process regenerates from time to time in a fixed interval or in a random time (Cox, 1962). Therefore, it is important for many problems that involve modeling the behavior of some processes to study only a single regeneration cycle. The time interval between two regeneration epochs is called cycle. A sequence of regeneration cycles constitutes what is known as a renewal process (Markovich, 2008; Gallagar, 2013).

In maintenance applications, renewal theory has proven its usefulness in estimating the long run average such as: availability, cost, or availability and cost in one realization of process. These embedded renewal epochs allowing separate the long term behavior of the process which can be studied through renewal theory from the behavior of the current process within a renewal period. For example : Renewal theory is used to estimate long run average cost per time unit, long run average availability per time unit, long run fraction of time the system spends in a given set of states (Weide and Noortwijk, 2008). Whenever, it is often assumed that a process starts afresh or as good as new means that the state returns always to the first state and starting at time zero while maintenance action brings a system back to its original condition as well as the cycle ended when the maintenance action bring the system in its normal state. (Barllowand Porschan, 1965; Noortwijk, 2009).

3.2.2 Markov Chain

A Russian Mathematician, Andrei Andreyevich Markov (1856–1922), was the first who introduced the concept of Markov chain in 1907 (Basharin, 1990). A.A.Markov considered that theoutcome of a given experiment can affect the outcome of the next experiment; this is known as a Markov chain process (Basharin et al., 2003).

A Markov chain is a random process represented by a sequence X_0, X_1,…of random variables having Markov property that the conditional distribution of X_{n+1} given X_n, X_{n-1},..., X_1, X_0 depends only on the present value (X_n), and is independent of the past (Cox and Miller, 2001). Markov chain aims to describe a process in which considering the probability of any particular future behavior of the process, when its present state is known, is not altered by additional knowledge concerning its past behavior this is called memoryless property (Markov property). Markov chains have been recognized as one of the great prominent tool in stochastic modeling interrelating the probabilities, states and transitions. It can be used to model a random system that changes states according to a transition rule that only depends on the current state. Letting consider $\{X_n, n=0,1,2,\ldots\}$a sequence belonging to the set of states with length n denoted by Ω recorded respectively at $t_1, t_2, \ldots, t_n$. In a Markov chain, if the chain is currently in state (E_i) then its moves to state (E_j) at the next step with a probability denoted by P_{ij}. Assuming $\{X(t), t \geq 0\}$ is a continuous-time stochastic process taking values on the discrete state space Ω. Let's T be a time space, $T \epsilon$ $[0, \infty)$. Mathematically, for any real numbera, $b \geq 0$ the process $\{X(t), t \geq 0\}$is a Markov chain process if satisfying the following expression (Karlin and Taylor, 1975):

$$P(a<X_t\leq b|X_{t_1}=x_1, X_{t_2}=x_2,\ldots,X_{t_n}=x_n)= P(a<X_t\leq bX_{t_n}=x_n) \qquad (3.8).$$

A Markov chain diagram is a schematic representation used to visualize Markov chain process. Markov chain diagram is entirely constructed by space states (nodes), its transitional probabilities (arcs), and initial probabilities. In a Markov chain diagram, the process moves successively from one state to another. Each move is called a step (Figure 3.1). If a transition is possible from state *i* to state *j*, the directed edge from node *i* to node *j* is labeled with the probability of going from state *i* to state *j* and denoted as $p_{ij}^{(n)} > 0$, in *n* steps. Consider setting Markov chain diagram might be there is an edge from possible state to itself that indicates the possibility that the process continues to occupy the same state in the next period. The process can remain in the same state with probability denoted by p_{ii}. Therefore, an absorbing Markov chain is a Markov chain where an absorbing state is accessible from any set of states. An absorbing state is a state that, once entered, can't go to another state(Ross, 2000;). In another word, A state Ei of a Markov chain is called absorbing if it is impossible to leave it (i.e., p_{ii} = 1). A Markov chain is absorbing if it has at least one absorbing state, and if from every state it is possible to go to an absorbing state (not necessarily in one step). In an absorbing Markov chain, a state which is not absorbing is called transient.

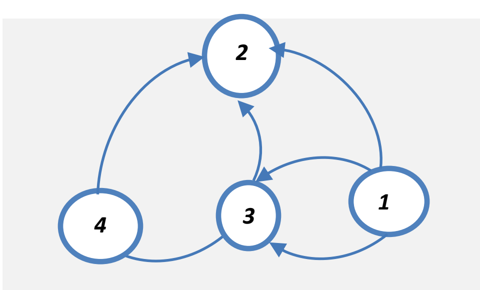


Figure 3.3. An absorbing Markov chain

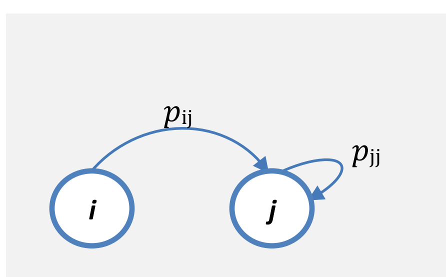


Figure 3.4. Absorbtion state*j*

Since, a Markov process is a stochastic process, there are four distinct types of Markov processes classified according to their state-space and time characteristic (see Table 3.1).

Table 3.1. The four types of Markov processes.

Type	**State space**	**Time**	**Name**
1	**D**iscrete	**D**iscrete	**D**iscrete **T**ime **M**arkov **C**hain
2	**D**iscrete	**C**ontinuous	**C**ontinuous **T**ime **M**arkov **C**hain
3	**C**ontinuous	**D**iscrete	**C**ontinuous **S**tate **S**pace **M**arkov **C**hain
4	**C**ontinuous	**C**ontinuous	**C**ontinuous **S**tate **S**pace **T**ime **M**arkov **C**hain

If a process is a considered discrete and continuous in term of state whereby in this situation Markov process can be classified into types: the first one is known as a Discrete State Space and Discrete Time and the second one is known as Discrete State Space and Continuous Time. A discrete-state and discrete-time Markov process is usually called the discrete time Markov chain (DTMC). The Discrete Time Markov Chain can be defined as a set of states with countable state space since the transition state occur in discrete time. It is represented by fixed transition probabilities. In Discrete Markov Chain the transitions occur at discrete times. The second type of Markov process is characterized by the discrete-state and continuous-time Markov process which very often called Continuous Time Markov chain (CTMC). The time spent in each state takes non-negative real values following exponential distribution and transitions can happen at any time where the transitions occur in a very short time interval. The continuous-time Markov chain is said to have stationary or homogeneous transition probabilities.

Therefore, if the states space is assumed to be continuous, this implies that Markov chain can be divided in two types(one a process is considered discrete, and the other process is considered continuous in time): The first one is called Continuous State Space and Discrete Time, and the second one is called Continuous State Space and Continuous Time. The first one is applied only if there are discrete changes in time in environment where the states of the system are continuous over a specified range. The second one is characterized by the process and states space of the system is both continuous in time. For example: Continuous Time Markov Chain is considered as one the most widely stochastic method used by mathematician and engineering. In particular, classes of Markov processes which are useful for modelling stochastic deterioration is refer to a discrete-time Markov processes, and having a finite or countable state space and continuous-time Markov processes with independent increments. Therefore, Brownian motion (Wiener process) with drift also called the Gaussian, the compound Poisson process, and the Gamma process are considered as a Continuous-Time Markov Processes with independent increments (Noortwijk, 2006). According to Shamshad et al., (2005) Markov chain of the first order is one for which each subsequent state depends only on the immediately preceding one. Markov chains of second or higher orders are the processes in which the next state depends on two or more preceding ones.

Example3.1: Consider that the health state of items has three different states normal, warning, and a failed denoted respectively by state 0, 1 and 2. Thus, the system subject to gradually deterioration, then the transition from normal state to failed state is usually hidden state representing warning state, the only observable states is called failed state, otherwise normal and warning state are hidden (non-observable) (Jiang et al., 2012). The process first move from state to another state, any state 0, 1, and 2 is accessible from any of the three states, but 1 is not accessible from state 2. The Markov chain diagram (transition diagram) is represented as follows:

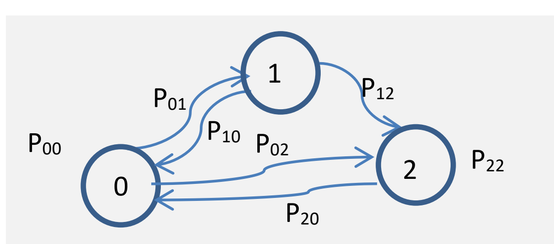


Figure 3.5. Markov Chain Diagram with three transition states.

The probability of reaching state *j* from the state *i* in two steps consists the probabilities of going from state *i* to any other possible state and then going from that step to *j*. Each move is called a step.

$P_{ij}^{(2)} = \sum p_{if} p_{fj}$, Where f is the set of all possible states (3.9).

If j is not accessible from i, which denoted as $P_{ij}^{(n)} = 0$for all n≥ 0.This means that the chain started from i never visits j. As illustrated in figure above a state is called absorbing state is a state that once entered cannot be left. However, an absorbing Markov chain is a Markov chain in which every state can reach an absorbing state. A Markov chain is an absorbing chain if satisfies the following axioms: if there is at least one absorbing state and it is possible to go from any state to at least one absorbing state in a finite number of steps. In an absorbing Markov chain, a state that is not absorbing is called a transient state.

Ina Markov chain, the transition matrix is a matrix that indicates the transition probabilities for a Markov chain to move from one state to another. Assuming $\{X_n\}$, n= 0,1,2,…be a sequence continuous time Markov chain, all existing number of transition probabilities between different states of two consecutive occasions provide a matrix as square array is called the matrix of transition probabilities, or the matrix transition of the process. In fact, the probability transition matrix is a tool for describing the Markov chains' behavior. Each element of the matrix represents probability of going from a specific condition to a next state, and is denoted by p (Shamshad et al., 2005). The transition matrix of Markov chain should satisfied the following conditions:

$p_{ij} \geq 0$, i, j = 0, 1, 2, … (3.10).

$\sum_{j=0}^{\infty} p_{ij} = 1$ (3.11).

The purpose of Markov chain on practical application aims to model the evolution of behavior of system that is considered random processes given in mathematically form, even if the initial condition is known, there are several directions in which the process may evolve. Suppose that the current state of process is in state *i*, then it moves to another state j this occur with probability p_{ij}.

According to diagram of Figure 3.1, the Markov chain is described as follows: the chain has three states, 5 transition probabilities. Thus, the symbol p_{ij} represents that the probability of transition from state *i* to state *j* in one step. For example p_{01}consists that the next state of the system in 1 depends only on the present state 0, not on the preceding states. The transition matrix of Markov chain denoted by P represented as follows:

$$P = \begin{bmatrix} p_{00} & p_{01} & p_{02} \\ p_{10} & p_{11} & p_{12} \\ p_{20} & p_{21} & p_{22} \end{bmatrix}$$

Considering the example illustrated in Figure3.1, $p_{21} = p_{22} = p_{11} = p_{11} = 0$.The chain started from state 2 never moving to state 1. There are two non-absorption states respectively, 1, and 2. The Markov process has stationary transition probabilities only when one-step transition probabilities ($P^{n,n+1}_{ij}$) are independent of the time variable. Mathematically, the one-state transition probability is denoted by $P^{n,n+1}_{ij}$, and is expressed by (Karlin and Taylor, 1975):

$$P^{n,n+1}_{ij} = P\{X_{n+1} = j / X_n = i\} \qquad n = 0,1,\ldots \qquad (3.12).$$

Let' assume *j* is accessible from *i*in *n* steps with probability $P^{n}_{ij} > 0$and *k* is accessible from *j in m* step with transition probability$P^{m}_{ij} > 0$, so that imply *k* is accessible from *i* in *m+n* steps with transition probability$P^{n+m}_{ij} > 0$. The m-step transition probabilities at time *n* is the probability of X_{n+m} will be in state *j*, given that X_n is in state *i*(Ross, 2000), so that:

$$P^{n,n+m}_{ij} = P\{X_{n+m} = j / X_n = i\}, \qquad n=,1,.. \qquad (3.13).$$

Thus, the corresponding m-step transition matrix at time n is denoted by P(n,n+m). The transition matrix should satisfy:

$$P(m,n) = P(m,l)\,P(l,n), \qquad m \leq l \leq n \tag{3.14}$$

Or, equivalently,

$$P_{ij}(m,n) = \sum P_{if}(m,l)P_{fj}(l,n), m \leq l \leq n \tag{3.15}$$

This equation is known as the Chapman-Kolmogorov equation (Ross, 2000).

Hidden continuous Markov chain (HMM) is often called a doubly stochastic process and it has Markov property. Hidden Markov model is a tool for representing probability distributions over sequences of observations. A Hidden Markov model is an extension of the Markov Model, in which it is assumed that the system unobservable aimed to explain the observed sequence whereby computing the probability of observation sequence. Therefore, the observations are probabilistic function of the hidden states (Rabiner and Juang, 1986). The state duration where the sojourn time in each state have exponential distribution which is sometimes not realistic in real-world application, is considered as the one of the most weakness modeling the process by HMM. The set of states are often considered discrete, this is an advantageous representation that allowing seeing the transition from state to another state. Hidden Markov model was first used in the research area of speech recognition (Rabiner, 1990), in computational molecular biology, artificial intelligence, and in a pattern recognition (Ghahramani, 2001). Rabiner (1990) presents an overview of Hidden Markov models including estimation procedures and inference technique. It is can be also used in theory and application for maintenance optimization, numerous paper have been published in literature using Hidden Continuous Markov Chain, for example: system diagnostics and fault detection (Lin and Makis, 2003; Makis and Jiang, 2003; Wang, 2006).

From a practical point of view, Markov chain modeling are widely used by the researchers around the world across various areas such as: reliability, maintainability, safety, environmental, finances, queuing systems, inventory systems, physics and biology system, medical decision making, recovery, relapse, and death due to disease. Therefore, many ideas connected to Markov chain and have property Markov as well. Markov chains have a direct relationship to other stochastic processes such as: Brownian motion, random walk process, gamma process, delay time concept, inverse Gaussian process. Markov processes have long been considered the most popular as stochastic processes for maintenance optimization problem.

3.2.3 Gamma Process

Gamma process is a stochastic process with independent non-negative random increments following a gamma distribution (e.g. the increments of degradation growth of an item). Gamma process is usually defined as follows. Let assume that $\mathrm{X} = \{\mathrm{X(t)}, \mathrm{t} \geq 0\}$ is a stochastic process taking a value on (0,∞), a random quantity X has a gamma distribution with shape function $\beta(t) > 0$, scale parameter $\lambda > 0$ and location parameter μif its probability density function is given by:

$$f\ (x;\beta(t),\lambda,\mu)= \frac{\lambda^{\beta(t)}}{\Gamma(\beta(t))}(x-\mu)^{\beta(t)-1}e^{-\lambda(x-\mu)}\text{ , for real } x \geq \mu,\ \beta(\mathrm{t}) > 0, \lambda > 0. \quad (3.16).$$

Gamma process shall be referred as $Ga(x;\beta(t),\lambda,\mu)$.

$\Gamma(.)$is the gamma function which is expressed by a convergent improper integral whenever $\beta(\mathrm{t}) > 0$:

$$\Gamma(\beta(\mathrm{t})) = \int_0^{\infty} t^{\beta(\mathrm{t})-1}\, e^{-t}\, dt \quad (3.17).$$

Therefore, Gamma process is considered as a special case of a pure-jump increasing Lévy process having an infinite number of jumps in finite time intervals. Thus, a Gamma process has direct connection with a Poisson process.

A gamma distribution is an increasing function of time and right continuous with the following proprieties: gamma distribution approaches the normal distribution when scale parameter becomes large. Therefore, gamma distribution has density only for positive real numbers.

Therefore the lower incomplete gamma function can be written as follows:

$$\Gamma_1(\beta(\mathrm{t})) = \int_0^{t} t^{\beta(\mathrm{t})-1}\, e^{-t}\, dt \quad (3.18).$$

Whereas, the upper incomplete gamma function can be written as follows:

$$\Gamma_2(\beta(t)) = \int_t^{\infty} t^{\beta(t)-1} e^{-t} dt \qquad (3.19).$$

The shape function $\beta(t)$ is a non-decreasing, right-continuous, real-valued function for $t \geq 0$, with$\beta(0) = 0$.

If μ = 0 and λ = 1, Gamma distribution is called standard gamma distribution denoted by Ga $(x, \beta(t), 1, 0)$:

$$f(x) = \frac{x^{\beta(t)-1}}{\Gamma(\beta(t))} e^{-x} \qquad (3.20).$$

If μ = 0, gamma distribution parameterized with a scale parameter and shape parameter, gamma distribution function can be written as follow:

$$f(x) = \frac{1}{\Gamma(\beta(t))} \lambda^{\beta(t)} x^{\beta(t)-1} e^{-\lambda x} \text{, for real } x \geq 0 \qquad (3.21).$$

An exponential distribution is considered as a special case of a gamma distribution, this mean that a gamma process with $\beta = 1$(and μ = 0), is commonly called to be an exponential distribution, where the parameter λ occurs as a scale factor.

A gamma process belongs to family of stochastic degradation processes (e.g. inverse gaussian process, delay time concept, Markov chain), and widely used to represent the progressive degradation process. As degradation is naturally uncertain and non-decreasing, and may arise in stochastic fashion due to wear, fatigue, corrosion, crack growth, erosion, consumption, creep, swell, gamma process can be applied to define degradation model (Paroissin and Salami, 2014; Abdel-hameed, 2010). Gamma process gives a proper model for random deterioration with time, thus, Gamma process is used to model the uncertainty in the time to failure (lifetime) and/or the rate of deterioration. Gamma processes are fitted to data on creep of concrete, fatigue crack growth, thinning due to corrosion, and corroded steel gates, deterioration process of coating.

According to Paroissin and Salami (2014), Gamma process is one of the most popular stochastic process among other to model degradation of device in reliability theory.

A continuous time stochastic degradation process $X = \{X(t), t \geq 0\}$ is a gamma process if it is satisfies the following properties (Mahmoodian and Alani, 2013; Noortwijk, 2009):

- A continuous-time Markov process $\{X(t), t \geq 0\}$.
- It is assumed that the process starts from a value 0 at time $t = 0$, X(0)=0 with probability equal to one.
- The result distribution of non-decreasing degradation jump stochastic process defines gamma process.
- X (t) random variable has independent increment.
- The mean value of the process increases linearly.
- Probability density function has explicit expression.
- Homogeneous in time.
- A gamma process is a stochastic process with independent increments: for $0 < s < t$, the distribution of $X(s + t) - X(s)$, follow Gamma distribution.
- Letting $\{X(t_2) - X(t_1)\}$ be the damage increment from time t_1 to t_2. The increment are non-negative quantity and having gamma distribution with shape function $[\beta(t_2)-\beta(t_1)]$ and scale parameter λ.

Statistical estimation methods and simulation technique are considered as two computational approaches for estimating unknown parameters of gamma model. Several methods of estimating parameters have been presented by Noortwijk (2009): Maximum-likelihood method, Least squares model, Method of Moment Method of Bayesian, Markov Chain Monte Carlo Simulation. The application of these methods for estimating the parameter of gamma distribution is demonstrated by using observed set data as well as applying a certain objective function.

Therefore, in term of stationarity various gamma processes have been discussed in the literature. The existing Gamma process used for deterioration prediction modeling can be classified into two categories asa stationary gamma process, non-stationary gamma process. A gamma process is called stationary gamma process if the shape function $\beta(t)$ is linear over time. Generally $\beta(t) = \alpha t^b$, when $b = 1$, the shape function become $\beta(t) = \alpha t$, in this case gamma process is linear and stationary. A Stationary gamma process with shape function $\beta(t) = t$ and scale parameter 1 will be called standard. Further, Gamma process having an increasing expected deterioration and linearly over time $E(X) = \lambda^{-1}\beta(t)$. Otherwise, if the shape function $\beta(t)$ is non-linear $\beta(t) = \alpha t^b (\alpha > 1$ and $b > 1)$ the increments follow non-stationary gamma process.

The cumulative density function, the survival function, and the hazard function, cannot be written in closed form, these defined as follows (Meeker and Escobar, 1998; Noortwijk, 2009):

Using probability density of gamma function to calculate cumulative density function, the cdf is:

$$\mathrm{F}(x) = \int_0^{\mathrm{x}} \frac{1}{\Gamma(\beta(\mathrm{t}))} \lambda^{\beta(\mathrm{t})} x^{\beta(\mathrm{t})-1} \mathrm{e}^{-\lambda x}\, \mathrm{d}x \text{ . For real } x \geq 0. \quad (3.25).$$

Then, the survival function R(t) (reliability function)represents the probability of surviving of an item beyond time t. For this distribution Reliability function is:

$$R(t) = 1 - \Gamma(\mathrm{t}) \quad (3.26).$$

Where,$\Gamma(\mathrm{t})$ represents the lower incomplete gamma function.

The hazard function represents the instantaneous failure rate. The hazard function is therefore:

$$\mathrm{h}(x) = \frac{\mathrm{f}(x)}{1-\mathrm{F}(x)} \quad (3.27).$$

$$= \frac{\frac{1}{\Gamma(\beta(\mathrm{t}))} \lambda^{\beta(\mathrm{t})} x^{\beta(\mathrm{t})-1} e^{-\lambda x}}{1-\int_0^{\mathrm{x}} \frac{1}{\Gamma(\beta(\mathrm{t}))} \lambda^{\beta(\mathrm{t})} x^{\beta(\mathrm{t})-1} \mathrm{e}^{-\lambda x}\, \mathrm{d}x} \quad (3.28).$$

The Hazard function is increasing when$\lambda > 1$, decreasing when$\lambda < 1$, and constant when $\beta(\mathrm{t}) = 1$

Leonhard Euler is considered the fonder of gamma function (1729) afterwards gamma function arises in many areas. Beginning of 1950’s, Moran was the first who applied gamma process in theoretical physics for modeling the water flow into a dam. Abdel-Hammed on 1975 is considered the first who adopted a gamma process for modeling the stochastic deterioration occurring randomly over time. The author developed a mathematical model based on gamma process for maintenance optimization. In the maintenance policy, the gamma process describes structural stochastic deterioration occurring over time. A survey of the application of gamma processes in maintenance was published by Noortwijk (2009).

Since their introduction in the area of reliability on 1975, several authors have provided a wide variety of inspection models for optimizing maintenance under the assumption of gamma deterioration process (Abdel-Hameed, 1975; Kallen and Noortwijk, 2006; Yang and Klutke, 2000).In reliability engineering and failure analysis, gamma process is used to model deteriorating process. This stochastic process has provided useful means for modeling time-dependent phenomena such as degradation, especially for reliability and maintenance analysis. A gamma process is useful as a model for the lifetimes of non-repairable systems. It has proven to be useful in determining optimal inspection and maintenance decisions (Noortwijk, 2009).

Generally, a gamma process has been considered straightforwardly as an appropriate and suitable tool for modeling a gradual damage monotonically increasing over time mainly due to mathematically tractable. In addition, a gamma process has the independent increments property and the assumption of a constant scale parameter (Guida and Pulcini, 2013; Barlow and Proschan, 1975; Noortwijk, 2007). Due to the advantageous using gamma process for describing degradation phenomena a wide range and diverse academic research have encouraged working on them in many areas of science such as physics and engineering (e.g. deterioration prediction of buildings, maintenance modeling, mathematical finance, credit derivatives models.).

Therefore, an advantage of modeling deterioration processes by gamma processes is that the required mathematical calculations are relatively straightforward and usually the degradation is monotonically accumulating over time, in reliability engineering and failure analysis, gamma process is used to model deteriorating process (Mahmoodian and Alani, 2013).

From practical point, a stochastic gamma process provide framework applicable for modeling deterioration in concrete pipes (Mahmoddian and Alian, 2013), modeling the material fatigue crack growth properties. In maintenance decision making, gamma process is used to detect the first time that process exceeds a random threshold, this process can be used as a model for the lifetime of a device or for the random time between two successive imperfect maintenance actions (Frenk and Nicolai, 2007).

3.2.4 Delay Time Concept

Delay time concept is a tool for modeling and optimizing plant inspection practices. This concept defines the failure process of an item as a two-stage failure process: The first stage represents a normal operating stage, starting from a normal state to an initial point that a fault can be identified, in which in this stage a fault become visible. The second stage represents the failure delay time, starting from the point of fault identification to a failure state. Further, in this stage a fault leads to an eventual breakdown (failure), these faults of an item will not appear as failures, but are present for a while before becoming sufficiently evident to be noticed and declared as failures (Christer et al., 1973).The term delay time refer to period between the epoch of fault first initiating and the epoch of a failure resulting from the fault. These two failure stages could follow any arbitrary continuous probability density function and will not necessarily be described by the same probability law (Jiang, 2013).

Every system built by humans is unreliable in the sense that it degrades with age and/or usage. A system is said to fail when it is no longer capable of delivering the designed outputs. Some failures can be catastrophic in the sense that they can result in serious economic losses, affect humans and do serious damage to the environment. Typical examples include the crash of an aircraft in flight, failure of a sewerage processing plant and collapse of a bridge. The degradation can be controlled, and the likelihood of catastrophic failures reduced, through maintenance actions, including preventive maintenance, inspection, condition monitoring and design-out maintenance. Corrective maintenance actions are needed to restore a failed system to operational state through repair or replacement of the components that caused the failure (Kobbacy and Murthy, 2008).

For instance, operation of many complex systems comprised of several pieces of equipment tend downtime from time to time and, when that happens, they need to be identifying and replacing or repairing faulty item before they cause a failure enable to improve system availability. In fact, the initial point of defect is very important to the set-up of an appropriate inspection interval (Wang, 2009; Jiang 2013). The delay time concept is depicted in Figure3.6. The occurrence of a fault in a system sometimes may not lead to an immediate system failure; in this case the system stays in a defective state which is known as failure delay time. The problem of time delays is typical for many physical or technical systems. In this context, failure delay time enable to provide the opportunity for item's preventive maintenance to be carried out in order to decide either the defective item may be replaced preventively or rectify the identified defects before failures occurrence.

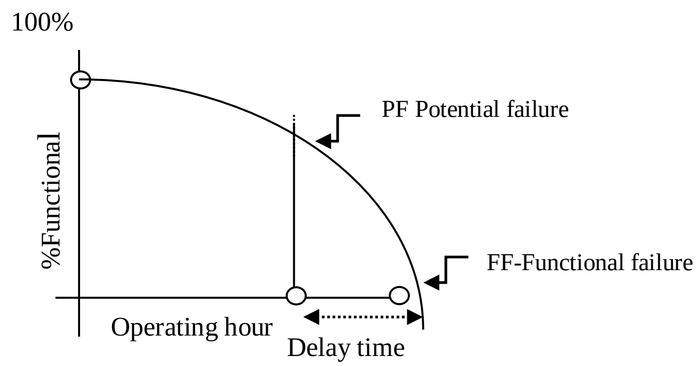


Figure 3.6. Potential failure and functional failures (Moubray,1997).

Delay time concept is often used for modeling two categories of engineered system. The first system is known as a single component that subject only to a single failure mode while delay time concept aims to modeling one defect, this model is known as component tracking model (Baker and Wang, 1992; Wang and Christer, 1997). In contrast, complexes system comprises many components and subject to many different failure modes (Christer et al., 1997; Christer and Waller, 1984; Pillay et al., 2007; Akbarov et al., 2008; Wang and Christer, 2003; Wang, 2009; Wang et al., 2010). In this situation, many defects can exist simultaneously as well as many failures can occur within interval between inspections, this is particularly important for the method using objective data (Wang et al., 2010). Defect arrivals from all components are grouped and modeled by a stochastic point process along delay time model, such as a Homogeneous Poisson Process (HPP) or Non-Homogeneous Poisson Process (NHPP) (Wang et al., 2010).

Delay time concept aims to provide modeling framework for describing the transition time from a normal state (potential failure) to a failed state (functional failure). Mathematically, the delay time modeling concept is used to formulate the transition probability from one state to another state (Christer et al., 2001; Wang, 2012).

Delay time concept and models based on it allowing to describe the underlying state transition process, this could be applied by analysis the failure processes of an item. In addition, Delay time concept is employed in maintenance modeling to capture the relationship between equipment performance and inspection intervention (Wang, 2011). The relationship between item failure and inspection interval can be explicitly modeled by delay time concept Wang (2012).

For estimating the delay time model parameters we need the collected maintenance data. Indeed, subjective and objective estimation are represented as two ways for estimating the delay time model's parameters. For the first one, expert judgment (failures and faults data) has been used to estimate the parameters of delay time concept (distribution function for both stages) (Wang, 2008; Christer and Redmond, 1990; Christer et al., 1998). However, opinion of expert includes uncertainty in which the estimation of parameters could not be a good estimate, indeed, model parameters have been estimated mainly from subjective data. For the second one, the origin of failures data and inspections are obtained from past records. Baker and Wang (1992) were the first provide an approach to estimate delay time parameters using objective data. However, the parameter estimation of the delay time concept modeling was solved by using simulation data, subjective data, or a combination of subjective data and objective data, mainly because there was rarely sufficient maintenance data to allow the use of fully objective data to solve it. The common method for estimating the parameters of delay time distribution have been reported in many papers and case studies are: Method of moment, Maximum likelihood function, least square method, Markov Chain Monte Carlo (MCMC).

Christer and Waller (1994) used subjective data to solve the DTM parameters, and Christer and Redmond (1990) studied the maximum likelihood function method of the subjective estimate of the DTM. Corresponding with the subjective estimation methods, Baker and Wang (1991, 1992) put forward a parameter estimation method founded on objective data. Christer and Wang (1995) then proposed a method that uses subjective and objective data to solve the DTM for multi-components in complex systems. Wang et al. (2007), used maximum likelihood for estimating the parameters model.

The general purpose of existing models based on delay time concept have been developed in the literature, this approach is enable to provide the rationale for technical system' inspection and preventive maintenance. Therefore, it has numerous applications and developed delay time concept in many case studies of industrial maintenance of production plant and for other problems regardless the area e.g. Christer and Waller (1984); Christer et al (1995); Desa (1995). Theoretical developments can be found in baker and Wang (1991); Baker and Christer (1994); Christer and Redmond (1990); Christer and Wang (1995).

Christer (1976) is considered the first who mentioned the concept of the delay time in a context of maintenance of building. The author exploits the idea of delay time for a fault in building structure, since then this concept was applied to several others maintenance optimization problems. Delay time concept was first applied to an industrial maintenance problem by Christer and Waller (1984). Christer (1992) also propose models of condition monitoring inspection with irregular inspection intervals based on delay time concepts. Another paper published by Christer et al. (2001) recognizes the robustness of the semi-Markov and delay time maintenance models to the Markov assumption. The authors present a prototype model of the industrial maintenance problem using the delay time concept. Many papers and case studies appeared in literature embracing delay time modeling of industrial asset inspection problems. The delay time model has been widely applied in CBM models (Jiang, 2013). Wang (2007) also presented a two-stage prognosis model in CBM.

Baker and Christer (1994) discussed the development of delay-time analysis in modeling engineering aspects of maintenance problems as well as a state of the art and future trends of this models. An extension of Christer' model has been made by Wang on (1992). In a thesis of Wang (1992), various models for condition monitoring inspection based on the delay-time concept, thus, many algorithms have been presented for condition monitoring inspection modeling.

Recently, Wang (2012a) designed a multivariate Bayesian control chart for CBM. The transition between states and the relationship between observed information and the state are not Markovian. However, a two-stage failure process characterized by the delay time concept is used to describe the underlying state transition process. Bayesian theory is used to compute the posterior probability of the underlying state, which is embedded in the simulation algorithm. The model was designed on the basis of the delay time analysis. The author presents various models for condition monitoring inspection. As the distribution of the delay time is important to delay time modeling, a new approach to estimate the delay time distribution is proposed. Numerical examples are presented to illustrate how models and algorithms are performed. Wang et al., (2010) used delay time model to determine the optimal inspection interval for a system with different components and failure modes. Numerical examples demonstrate the results of the proposed model. In this study, simulation is easier to run than the analytical or numerical counterpart. Monte Carlo simulation is used to obtain the optimal control chart parameters, which are the monitoring interval and the upper control limit. Another article Wang (2012b) presented an overview of the recent advances in delay-time-based maintenance modeling. Werbińska et al., (2015) demonstrate the applicability of the delay time concept to determine the optimal interval between inspections performance

Okumura (1997) presents a method for determining the discrete time points of inspection for a deteriorating single-unit system characterized by three states as follow: normal state, a defect state and a failed state. The transition of the states are described using a delay time model in which the transition time from normal state to a defect state and that from defect state to a failed state (delay time) are assumed to be independent. These two stages following an arbitrary probability density function. The author proposes a method for determining the inspection time vector which minimizes the long-run average cost per unit time. In another work, Aven and Castro (2009) proposed a methodology for determining an optimal inspection interval using delay time concept. The authors assume that the system has three states: the perfect functioning state, a defective state and the failure state. By using renewal theory the authors derive the expression of expected discounted cost per unit time as objective function subject to safety constraints. In this case, two safety constraints are considered: (i) the probability of at least one failure in the bounded interval should not exceed a fixed value; (ii) the fraction of time the system is in the defective state should not exceed a fixed limit. Wang et al., (2011) proposed an availability model and parameters estimation method for the delay time model.

Jiang (2013) reveals a close relationship between delay time concept and gamma process, and show how they are mutually converted means given the existing results from one model to analyze the other that can be approximately determined if it is more difficult to analyze the second model and to estimate their parameterization. For example: derive gamma process model from the delay time concept and vice versa. Indeed, the difference between gamma process and delay time concept is that in delay time concept the defect is physically identifiable without quantitative information while in gamma process, variety of degradation phenomena in engineering structure or components is identifiable by amount quantitative. Generally, the delay time concept needs less data than gamma process

Therefore, delay time modeling concept enables identified the presence of the failure when it occurs. For gamma process, the failure is defined and usually associated with a functional failure; the degradation level is quantitatively measured. Generally, the intersection between alarm limit and the level of degradation can be considered as potential failure also, the intersection between degradation level and the failure limit can be considered as functional failure. In addition, the time interval with the degradation level that is smaller than the alarm limit, and it can be considered as the normal phase in the concept delay time. Then, the time interval with degradation level that occurs between alarm limit and failure limit can be considered as the defective phase. Moreover, the availability of data takes into consideration the criteria selected among which model could be used as the delay time concept or gamma process.

3.3 Bayesian Probability Theory

3.3.1 Bayesian theorem

The Bayesian probability theory is a statistical approach and a direct application of the probability theory. It is originally ascribed to the statistician Thomas Bayes (1701-1761) who was known by having formulated a specific case of the theory that bears his name (i.e. The Bayes' theory). The principal characteristics of that theory it's explicit use of probability. Indeed, Bayesian theory describes the probability of an event based on the conditions that might be related to it. Letting A and B be two events where P(A) and P(B) are the probabilities of A and B, independent of each other. Bayes’ theory is stated mathematically as below:

$$P(A/B) = \frac{P(B/A)P(A)}{P(B)} \qquad (3.29).$$

In this formulate A and B are events.

- P(A) and P(B) are the probabilities of A and B without regard to each other.
- $P(A/B)$, a conditional probability, is the probability of A given that B is true.
- $P(B/A)$ is the probability of B given that A is true.

Therefore, we define the probabilities of A and B without regard to each other as follow:

$$\mathrm{P(B) = P(B/A)P(A) + P(B/Non\ A)P(Non\ A)} \qquad (3.30).$$

Bayesian theorem aims to discover the probability that A is true supposing that the new evidence is true also. This is a conditional probability where one preposition might be true. The sample space of conditional probability may contain subsets within which it is desirable to make separate probability statement. Furthermore, Bayes' rule mainly involves the manipulation of conditional probabilities to assess the posterior probability.

This theory provides an expression for the conditional probability of A given to B is true, that is similar to posterior probability. The last is commonly expressed in terms of the prior probability of B, prior probability of A, and the conditional probability of B given to A. This shows uncertainty about A after taking the prior data into account. Indeed, Bayesian theory provides results based on prior knowledge or results of previous model that can be used as information about the current model.

In practical application, Bayesian theory is often used for understanding, modeling and reasoning uncertainty about any variable or parameter based on statistical data analysis (Gelman et al., 2004). Therefore, it is refers to either to confirm the relative validity of hypothesis based on observed data, to quantify uncertainty in inferences, or to adjust the parameters of a specific model (Meel and Sieder, 2006; Gelman et al., 2014). The source of uncertainty consists either epistemic uncertainty due to lack of knowledge, that can be reduced by receiving further information or, refers to intrinsic chance variation in the system and cannot be resolved, except by direct observation (Damien et al., 2013).

Therefore, Bayes' rule is widely used in statistics, science and engineering. Moreover, it has been used in a wide variety of contexts, ranging from marine biology to the development of "Bayesian" spam blockers for email systems (e.g., model selection, probabilistic expert systems based on Bayes networks, statistical proof in legal proceedings, email spam filters (Rosenthal, 2005; Stone, 2013).However there uses are very different regarding the area. For example: in the science's philosophy, Bayesian theory refers to try to clarify the relationship between theory and evidence. For example, what is the probability that you actually have the disease? It depends on the accuracy and sensitivity of the test, and on the background (prior) probability of the disease. In reliability engineering, Bayesian theory refers to try to estimate the probability that the current system being in abnormal state given data.

Hence, posterior probability expression of random variable H_i given the data Y includes two prior beliefs, and likelihood function that must be specified as follows:(i) prior probability $P(H_i)$mean the probability of an event or outcome H_i will occur before the collection of new data (Y). In general, the past data and judgment of expert (subjective opinion) are considered the two way to define the prior information, (ii) The probability of observable evidence $P(Y)$ which is the total probability of an outcome that can be realized via several distinct events. Posterior probability is normally calculated by updating the prior probability using Bayes' theorem. In addition, likelihood function $P(Y/H_i)$ also is required. The last is defined as the probability of those observed outcomes (Y) given those event (Bernando and Smith, 2000; Meel and Seider, 2006; Wang, 2012).

In case where, alternative hypotheses (events) are mutually exclusive, we can compute the posterior probability of any one of them being true as follows:

$$P(H_i/Y) = \frac{P(Y/H_i)P(H_i)}{\sum_{j=1}^{n} P(Y/H_j)P(H_j)} \qquad (3.31).$$

By using law of total probability, the formula for calculating $P(Y)$is$P(Y) = \sum_{j=1}^{n} P(Y/H_j)P(H_j)$

In order, to simplify the above expression, posterior probability can be expressed as follows:

$$P(H_i/Y) = \frac{P(Y/H_i)P(H_i)}{P(Y)} \qquad (3.32).$$

Meeker and Escobar, (1998) showed that the most important thing influence Bayes' theory estimates is: The last is entirely depend entirely on prior assumption and ultimately it requires the possibility of making change about prior distribution. By the way it is necessary to check the effect that the changes that might have effect on final answers of interest.

3.3.2 Bayesian inference

The probabilities involved in Bayes' theory may have different interpretations, one of these interpretations is that the theory used directly as part of a particular approach to statistical inference. Bayesian inference is a method of statistical inference (e.g. Bayesian information criterion, Maximum a posteriori estimation) in which Bayes' rule is used to update the probability for a hypothesis as an acquired evidence. Bayesian update is particularly important in the dynamic analysis of a sequence of data. Bayesian inference has found application in a wide range of activities, including science, engineering, philosophy, medicine, and law. Therefore, Bayesian inference in the context of decision theory is closely related to subjective probability which is often called "Bayesian probability". It is this concept that provides a rational method for updating believes. Bayesian probability theory aims to provide a mathematical framework using observed data in order to perform inferences or reasoning such as prediction for a new observation. All prediction problems of the complexity of the observed phenomena are supported by Bayesian inference especially when is considered in a treatment or estimate of uncertainty. Bayesian linear regression, Bayesian estimator and approximate Bayesian computation are considered as computational methods rooted in Bayesian statistics. Bayesian inference is one famous technique in mathematical statistic that enable learning from experience (fixed data), it has proved to be more realistic prediction, most appropriate and coherent approach in mathematic statistics. Bayesian inference is closely related to subjective probability, often called Bayesian probability. Bayes's theory is the foundation of Bayesian inference in which Bayes' rule is used to update the probability for a hypothesis as evidence is acquired. Bayesian updating is particularly important in the dynamic analysis of a sequence of data.

There are three essential steps within application of Bayesian inference process: (i) specifying a probability model by setting up probability distribution for all observable and unobservable quantities in problems, (ii) estimating and interpreting posterior probability distribution and (iii) updating the compute of posterior probability of the process. For model based Bayesian inference, B is replaced within observation Y, A with event H.

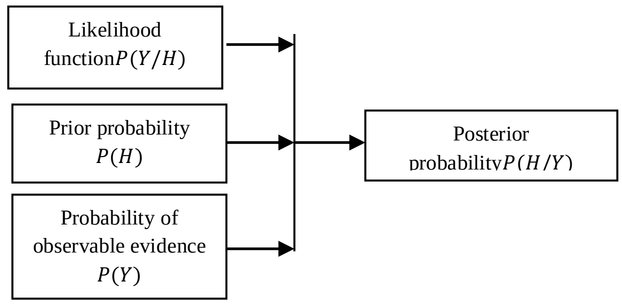


Figure 3.7. Bayesian method for making inference or prediction.

3.3.3 Bayesian application

Bayesian approach has become increasingly popular and successfully used by people to solve different problems, not only in academic research but also in industry, it has been seen by researchers as having most appropriate approach and can effectively be used in variety area such as: astrophysics, medicine, biology, neuroscience, finance, maintenance, public health, epidemiology, meteorology, strategic economic decision making, fault diagnosis, safety and risk analysis. Therefore, it will continue to receive great attention by researchers and practitioners, especially, recent high-speed computers have facilitated its use for many applications. The paper published over the past decade cover a wide range of problems in applying Bayesian probability theory to real work phenomena in general (e.g., industrial, engineering and human), and for maintenance optimization in particular, (Hodges, 1987; Berger and Pericchi, 2000).The main advantage of Bayesian theory is enable to provide a way for understanding several real world phenomena, and to make intelligent judgments, informed decisions, and inference data in the situations of uncertainty and variation (Gelman, 2004; Devore, 2009). Bayesian theory continues to arise in a wide variety of areas. In order to illustrate the application of Bayesian approach in maintenance area, the highlight of some of these applications in maintenance optimization problems bellow.

3.4 Building Bayesian Control Chart for CBM

Great advances in constructing Bayesian control chart for CBM have been made in the last ten years (Wang, 2012). Bayesian control chart represents a new vision for monitoring the processes over time. Bayesian theory can play key role for probability assessment of the system is in warning state with help prior knowledge of previous model, given fixed data. Bayesian control chart aims to monitor the posterior probability updated, which is considered as an cognitive ability shows if the deterioration level exceeds threshold, and it is considered as important information may then be useful by decision maker for predicting item's healthy state, remaining useful life, and selects an appropriate maintenance with lowest cost. Thus, it has been shown in the literature that Bayesian control chart for CBM is a more effective tool (Makis, 2008, Wang, 2012; Jiang et al., 2011). The posterior probability is probability of a system being in warning state, this probability may then be plotted on the Bayesian control chart. A signal is giving by the Bayesian control chart when the posterior probability exceeds the upper limit (fixed control limit). Despite, upper control limit, low control limit and central limit are require to estimate in traditional control chart. However, the emergence of a new numerical and statistical method into coherent manner can be useful and feasible for the Bayesian approach (Meeker and Escobar. 1998).

The process of design Bayesian control chart for CBM can be divided into the following three mains tasks: (i) estimating the values of the parameters that characterize the defect arrival, (ii) estimating the amount of degradation and failure processes, and (iii) formulating the structure of the expected cost and/or availability models. Bayesian control chart parameter can be expressed economically if the objective function is to minimize the cost and to maximize the availability.

While, it is often using stochastic processes technique (Markov chain, Gamma process, Delay time concept), and Bayesian theory to develop the posterior probability function of the underlying state given observed monitoring information history. These observations can be fused through Bayesian theory to give a posterior probabilistic estimate of the warning state which is often not directly observable. The Bayesian approach is searching to compute the optimal control limit by evaluate the posterior probability, while it is recognized that the lower control limit is equal to zero while optimal control limit have to be determined.

In general, many authors assume that process operational process into three states: two unobservable states (hidden state) either in control state (0) or out control state (1), while it will not be able to determine directly the behavior of state, otherwise the only no operational state in process refer to failure state (observable state) where the property failure of system can be directly identified. Bayesian control chart for CBM aims to monitor a health state of an item through observed data come from condition monitoring technique. Several sensors or sources provide information about the parameters of item condition. The suitable condition monitoring variables are available and called an observation. These observations can be fused through Bayesian theory to establish probabilistic relationship between observed data and health state of an item. This form is the basis of a Bayesian control chart through the calculation of the posterior probability of the warning state since then compared with specified upper control limit. Using Bayesian theory is able to estimate a posterior probability. The posterior probability is defined as the probability of the system being in warning state (random event) given the past information. A warning state is viewed as unobservable state. The posterior probability of the warning state is updated (new data), using Bayes's theorem, and then the new value of posterior probability plotted in chart in order to compare with a preset threshold level (upper control limit) in order to assess whether a full inspection is needed or not. Maintenance can then be carried out if indicated as necessary by the inspection. A characteristic feature using Bayesian control chart for CBM is that decision maintenance action must be performed under posterior probabilistic.

During the last past decades, Bayesian control chart for CBM has been appeared very faster in the literature. A recent developments about Bayesian control chart for CBM can be found in the following papers: Makis on (2008); Jiang and Makis on (2009); Jiang et al., (2011); Kim et al., (2011); Wang on (2012): developed a Bayesian control chart for monitoring a multivariate process mean in a long production run for a given sample size and sampling interval. The problem formulated as a stopping problem with partial information and applies the maximization technique (e.g., Aven and Bergman, 1986; Makis and Jiang 2003) to transform the problem into a stopping problem with an additive objective function, which is easier to analyze. From renewal theory, the long-run expected average cost per unit time is calculated for any policy as the expected cost of cycle divided by the expected cycle length, where a cycle is completed each time the process is stopped. The objective is to find a stopping rule under partial observations, minimizing the long-run expected average cost per unit time for a given sample size and sampling interval. An algorithm has been developed to find the optimal control limit and the minimum average cost. A numerical example was given to illustrate the effectiveness of proposed model.

In another similar study was made by Jiang and Makis (2009). The authors designed a multivariate Bayesian control chart for a CBM application. The deterioration system is described by the Markov chain. The system deterioration process is modeled as a 3-state hidden Markov process, with good, warning and failure states. In this paper, a new fault detection scheme is developed based on the average run length criterion. The Average run length (ARL) is an important performance measure for control chart design. Focus on using this particular measure will help to reduce the occurrence of the false alarms as well as enable fast identification of the out-of-control condition. This paper attempts to develop the multivariate Bayesian control chart design using posterior probability statistic and ARL representation in which fixed sampling interval is considered.

A numerical example reveals that the Bayesian control chart is much more effective for fault detection than the other charts; the maintenance cost will be much lower for real process control. Jiang et al., (2011) consider an availability maximization problem for partially observable systems subject to random failure instead the long run expected average cost per unit time. The statistical constrains can be expressed by the probability of true alarm. The statistical constraint has a powerful impact in improving the performance of control chart. The authors highlight the need powerful tools as Semi Markov Decision Process (SMDP) in order to develop an efficient computational algorithm. In this article availability maximization problem is equivalent to solving a parameterized system of linear equation. In order to find optimal solution, it is necessary to solve the linear equation, parameterized by control limit and sampling interval. The authors assume that the deterioration and failure of an item follows a continuous time homogeneous Markov chain. SMDP is a powerful tool in analyzing sequential decision processes with random decision epochs (Chen and Trivedi, 2004). It has applied in searching for the optimal maintenance policy for CBM. SMDP also known as Markov renewal programs, it was first introduced by Jewell and De Cani. Indeed, it is used in modeling stochastic control problems, and it is characterized by continuous-time Markov chains where the sojourn time in each state is a general continuous random variable, and it includes renewal processes. A stochastic control problem allows knowing the state of system at each instant of time.

The authors consider Bayesian control chart is far more effectiveness then age replacement policy which does not take into account the condition monitoring information. In other work Kim et al., (2011) provide a methodology for solving the problem about predicting failures of a partially observable failing system. The aim of this paper is to develop a multivariate Bayesian control chart for failure prediction using real multivariate spectrometric oil data coming from failing transmission units. Formulated optimization problem in this study consists to find the optimal value of the control limit and the sampling interval that systematically minimizes the long run expected average cost per time unit. The authors solve the problem optimization in the Semi-Markov Decision Process (SMDP) framework. The authors analyze real multivariate data from spectrometric analysis of oil samples collected at regular time epochs from transmission units of heavy hauler trucks used in mining industry as case study. The degradation system is modeled as a 3-state continuous time Markov chain, where two states unobservable represent good and warning operational states, and one observable state represents the failure state.

A vector autoregressive model was fitted to the healthy portions of each data history, and the residuals of the fitted model used as the observation process in the HMM framework. The parameters of HMM were estimated by EM algorithm. A cost-optimal Bayesian control chart constructed and applied as the fault prediction scheme. As conclusion, the authors suggest that the model presented can be applied to a wide range of deteriorating stochastic systems with multivariate condition monitoring data. According to the authors the lack of model designed is about stationary multivariate data histories. In practice, one may encounter data sets that exhibit non-stationary behavior. Multivariate Bayesian control chart unlike other methods such as Age-based, T-square, MCUSUM indicate that a Multivariate Bayesian control chart has the highest number of predicted failures and lowest total maintenance cost.

Wang on (2012) design a multivariate Bayesian control chart for real time CBM of complex systems. When complex systems are monitored, multi-observations from several sensors or sources may be available. However, the authors assume that the transition between states and the relationship between observed information and the state are not Markovian. The authors assumed that the transition between states was modeled by the concept delay time as well as the duration of both stage of delay time concept they are random. They are not necessary follow exponential distribution, so this means that the two stages failure processes are not Markovian. The delay time concept used to compute the transition probability from one state to another using the delay time modeling technique instead of using Markov chain modeling.

A strategy tends to combine the use of the delay time concept and the Bayesian theory to develop the posterior probability function of the underlying state given observed condition monitoring information history. This makes the paper different from others where a Markovian assumption is often used for state transition. Although, the two stage duration of the delay time concept respectively follow Exponential distribution and Weibull distribution. However, Monte Carlo simulation algorithm used to solve the optimization problem; Monte Carlo simulation is used to obtain the optimal control chart parameters including monitoring interval and the upper control limit. Moreover, the authors developed two kinds of simulation algorithms are: the first one is simulation algorithm for one renewal cycle based on the block-based monitoring policy, and the second one is simulation algorithm for one renewal cycle based on the age-based monitoring policy. The average run length is introduced within this study especially because it is considered as an important performance measure for control chart design. Therefore, it will must to reduce the occurrence of false alarm as well as the fast identification of the out of control. A Numerical example is given to illustrate the modeling idea proposed by authors. As results, the authors suggest needed more research to consolidate the model presented in this paper for practical application. Further, better posterior probability prediction for state identification in the case of Weibull than exponential based on the parameters used. The Weibull case also leads to a better cost outcome, but needs more frequently monitoring.

In majority of these papers, the authors provide a methodology for understanding step by step the important information and skills must be used to design Bayesian control chart for CBM. There is a rapidly growing literature on designing Bayesian control chart for CBM which highlight the benefit of Bayesian control chart in fault prediction and maintenance cost.

3.5 Bayesian Control Chart versus Traditional control Charts for CBM

According to Makis (2008), early contributions of the Bayesian process controls that not directly linked to a control chart design (implicitly deal with control chart design) were first studied by: Girshik and Rubin (1952), Eckles (1968), Bather (1963), Taylor (1965,1967), Ross (1971), since then many other papers use Bayesian process (Chen et al., 2004; Won and Modarres, 1998; Mehranbod et al., 2005; White, 1977; Wu, 2004; Meel and Seider, 2006). There are different models for maintenance optimization in which Bayesian control chart has been used (Tagaras 1994, 1996; Calabrese, 1995; Porteus and Angelus, 1997; Vaughan, 1993; Tagaras and Nikolaidis; 2002). According to literature, reveals there are two group of research in the area of adaptive statistical process control: the first one refers to the classical control chart framework (non-Bayesian control chart). The second one is related with the control policy by using a Bayesian approach, whereby the control policy is based on continuously updating the knowledge about the state of the process using Bayes' theorem (Tagaras and Nikolaidis, 2001). In the case of multiple observations, it is difficult if not impossible to place a limit on each of the multiple observations directly, since it is difficult to define what is warning or defective state based on each observation separately (Wang, 2008). Traditional control charts T-square, EWMA, CUSUM, X-bar chart are commonly used for CBM models. These models involve traditional control charts, and are used for control policy over time. The disadvantages of those models consists the use of only the current sample to calculate the value of the control statistic as well as they are not very effective for detecting small or moderate sized sustained shift in the process mean (Makis, 2008).

In comparative context, there are models better than others. A quantitative analysis is more common used by researchers to illustrate the effectiveness and difference between models. In this context, the difference between Bayesian control charts and the other models is demonstrated by either numerical example or real data. According to literature Bayesian control charts for CBM is considered as an optimal tool and perform much better for fault detection than the others traditional control charts (e.g., MUCUSUM, x-bar control chart, EWMA).Further, Bayesian control charts is economically effectiveness than others traditional control charts, it promote to give the lower maintenance cost for real process control (Jiang and Makis, 2009; Makis, 2008).

Bayesian control chart is based on posterior probability (upper control limit), in this case monitor the process by plotting the posterior probability on the chart in order to detect that the system is out control (warning state), then update this posterior probability after each sample, using Bayes's theory. In contrast, the strategy to monitor the process in tradition control charts is based on the following parameter's control chart: sample size, sampling interval, upper control limit and lower control limit. The optimal traditional control charts for CBM giving an alarm signal when the value of control statistic plotted in control chart is over upper control limit or down lower control chart. Bayesian control chart is an optimal tool for multivariate process control. However, multivariate control charts are not optimal because the auto and cross-correlation among multiple variables make these charts difficult to interpret all the information from the process data (Rui and Makis, 2009). A table (3.2) below summarizes various maintenance models that have been published in literature:

Table 3.2. Summary of previous integrated model studies

Study	State space	Stochastic process modeling	Methodology of making a design			Optimal Parameters models	Decision criteria &optimization technique
			Pdf Observation	**Transition probability**	**Posterior probability**		
1. **Jiang Rui, ,VilliamMakis (2012).**	3 states	Hidden Markov Process	Multivariate normal distribution	-Kolmogorov 's backward different equation -Laplace transform technique	- Bayes 's theory	-Monitoring interval -Upper control limit.	-cost criteria -Semi Markov decision process
2. Villiam Makis (2007).	3 states	Hidden Markov Process	Multivariate normal distribution	Markov chain	Bayes' theory	-Monitoring interval -Upper control limit.	-Cost criteria -Optimal stopping rule
3.Wenbing Wang (2007)	3 states	Delay time concept (non markovian chain)	-Weibull distribution Or -Exponential distribution	Delay time modeling concept	Bayes' theory	-Monitoring interval -Upper control limit.	-Cost criteria -Monte Carlo Simulation
4. Jong Kim, M., et al. (2011)	3 states	Hidden Continuous time Markov Chain process	Bivariate normal distribution	Expectation maximization algorithm	Bayes' theory	-Monitoring interval -Upper control limit.	-Availability criteria -Semi Markov Decision Process
5. Jiang et al., (2012).	3 states	Hidden Continuous time homogeneous Markov Process	Multivariate normal distribution	-Kolmogorov 's backward different equation -Laplace transform technique	Bayes' theory	-Monitoring interval -Upper control limit.	-Availability criteria -statistical constraint Semi Markov Decision Process

Chapter 4

Analysis and characterization the conceptual proposed model

This chapter will provide the thesis statement. The main aim of this chapter is to show how to answers the research question posed, where the objective is designing a new integrated model considering both data item's condition parameters and quality measurement control. The mathematical model for maintenance decision making was developed where the problem definition, assumption and notation was provided. The second part is devoted to the optimization problem resulting from the optimal parameters computation, furthermore, optimization approach is also presented.

Contents

4.1 Model description

In this research work, we consider a complex equipment Atox Mill with multiple channeled monitoring information. It is assumed that the state of the operational system is completely unknown unless an inspection is performed. The feature operational system can be in one of the following two states: normal and warning state indicated respectively by 1 and 2, which are not observable, only the failure state indicated by 3 is assumed to be observable. In this situation, the state of system cannot be observed directly and can be only estimated by using the observation provided by the system condition monitoring while the system being in non-operational state can be observed immediately. The observations coming from several sensors at each monitoring period are assumed to be independent. Establishing sample of observation measurement about Atox Mill equipment condition parameter and a key quality measurement condition monitoring can involve by using either various sensors such as micro-sensor, ultrasonic sensors, acoustic emission sensors or by portable measurement instrument specified by the framework and monitoring policy implementation (Secil industry).

The proposed model is designed under the assumption that the initial condition of system is on normal state. If the system is supposed to be in the warning state maintenance actions should be carried out to renew the system as good as new. A well organized and properly conducted intervention of technician is mandatory for the system reverts to its original condition.

According to the chosen methodology five systematic key steps should be used in a systematically sense in order to solve the research problem: (i) system monitoring acquisition, (ii) processing and cleaning data, (iii) expression of probability and parameters estimation (establishing stochastic relationship between observation process and system state), (iv) posterior probability estimation, (v) designing an optimal Bayesian control chart. Therefore, the process of application of this decision maintenance support within the system enables the decision maker to select an appropriate decision; to determine the effect of each possible decision; to solve problems about maintenance. Verifying if the true signal in the warning state given by Bayesian control chart, a full inspection initiated. However, if the signal is false the decision maker waits for a new observation. Based on plotting posterior probability updated in Bayesian control chart, the value of posterior probability updated in Bayesian control chart indicates the state of system if either in warning state or in normal state. The upper control limit is used in conjunction with the posterior probability updated. As mentioned in this study case, the motivation for the Bayesian theory is basically for the case where we have multiple variables.

In each step the available results needs to be used as information (input) for the next step. There are numerous techniques, tools, and algorithms in the literature that can be used for modeling and analyzing condition monitoring data. Other methods and algorithms that appear in the literature and have never been used in this area require testing to assess if they provide beneficial effect in term of efficient maintenance decision making.

System monitoring acquisition has been used for collecting and storing data, it is considered the first step towards a final maintenance decision making step. Prioritizing equipment condition monitoring target means to find out and to select in order of importance and which item has critical reliability. It is necessary to select type of data that can be related to the health condition of the target equipment. There are two independent sources of information to be collected S_1 and S_2. S_1represents data about item condition parameters and S_2 represents observable data about quality key measurement characteristic. The information collected about item condition parameters and quality key measurement characteristic are provided by condition monitoring techniques. These two random variables from system condition monitoring are considered independent.

Data cleaning step consists most important step in any data processing which enable for detecting, and correcting inaccurate, incorrect part of data, by using a set of tools, and then transform them into valuable information in order to provide data quality. This step supported by tools become necessary to solve any quality data problems because very often contain noise and error might be included on collected data.

Establish stochastically relationship between observation process and system condition: this step consists on studying and analyzing condition monitoring data. In fact, the observable vector process is stochastically related to the hidden states of the system. The specific objective is to define and use failure model and deterioration model of the system subject to random failures for an accurate assessment of prognosis. Besides, due to the uncertainty associated with the deterioration trend, the deterioration model can be developed by using stochastic processes techniques: Hidden Markov models, Delay time model, and Gamma process. In condition monitoring only the failure is observable. The degradation is gradual not sudden. Stochastic processes attempts to study a several stage that follows before it goes to a failure state. Several methods can be used in order to estimate parameters of models by using condition monitoring data(e.g. Expectation maximization algorithm, Markov Chain Monte Carlo, Maximum likelihood Estimation).

Posterior probability for the warning state step: where deterioration, failure model, and prior probability are defined enable us to quantifies the posterior probability. Bayes's theorem is used to quantify posterior probability; it shows an expression for the conditional probability of being in warning state given data.

Making decision: Making decision is most important for solving problem of maintenance, it is considered as knowledge process resulting in selecting better action under specific criteria. Once update posterior probability for the system being in warning state is above upper control limit P of the system representing sufficient information for decision making to take an optimal decision and solve problem of maintenance by selecting set of actions among several alternative possibilities. Making decision requires necessary knowledge of posterior probability which means conditional distribution of the uncertain event refer to warning state given two types of data. Using Bayes theory, the expression of posterior probability is expressed in term of probability density function and denoted by $P(X_t/S_1, S_2)$: For$t \geq 0$, let P be the probability that the process is in warning state at time t given the observations up to t, $X_t = 2$is anunobservable state of the process at time t and S_1, S_2 are observed data about condition equipment paramours and quality control measurement respectively.

$$P(x_t = 2 / S_1, S_2) = \frac{P(S_1, S_2 \cap x_t = 2)}{P(S_1, S_2)} \quad (4.1).$$

Assume that the (S_1, S_2) and $(X_t= 1)$ are random variables on a probability space. The probability that (S_1, S_2) would be observed whether or not was $(X_t= 1)$ true is denoted by $P(S_1)$, and $P(S_2)$. Hence, using Bayes theory the conditional distribution of $(X_t= 1)$ given the data provides the posterior probability density function of $(X_t= 1)$. The likelihood function for the available data and specified model given the warning state $(X_t= 1)$, is denoted by $P(S_1, S_2/ X_t= 1)$. The likelihood function refers to assessing the probability of the observed data (S_1, S_2) arising from the hypothesis $(X_t= 1)$. Prior probability is expressed in terms of a probability density function and denoted by $P(Xt= 1)$. The prior probability is defined as the probability that $(X_t= 1)$ in the absence of any information about (S_1, S_2). In general, the past data and/or judgment of expert or subjective opinion are considered the two way to define the prior information. The maintenance optimization is based on an objective function which corresponds to the long run average cost per time unit. The objective is to find an optimal value of the upper control limit and monitoring interval that minimizes the long run average cost per time unit, by using genetic algorithms then established software code developed in the Matlab. From renewal theory, the long-run expected average cost per unit time is calculated for any policy as the expected cost of cycle divided by the expected cycle length, where a cycle is completed each time the process is stopped, because it always returns to state 1, whether the assignable cause is present or not. Optimal Bayesian control chart aims to monitor the posterior probability that the system is in warning state. After each collected observation, the posterior probability (fixed control limit) is updated and plotted on the chart if its value is exceeding upper control limit. When the posterior probability exceeds a fixed control limit on the optimal Bayesian control chart, full system inspection is initiated. Every inspection is normally assumed to be perfect in the sense that it reveals the true state of the system without error.

In general, at every inspection epoch there are two decisions that have to be made. The first one decision is to determine that maintenance action to take, whether the system should be replaced or repaired to a certain state or whether the system should be left as is. The other decision is to determine when the next inspection is to occur.

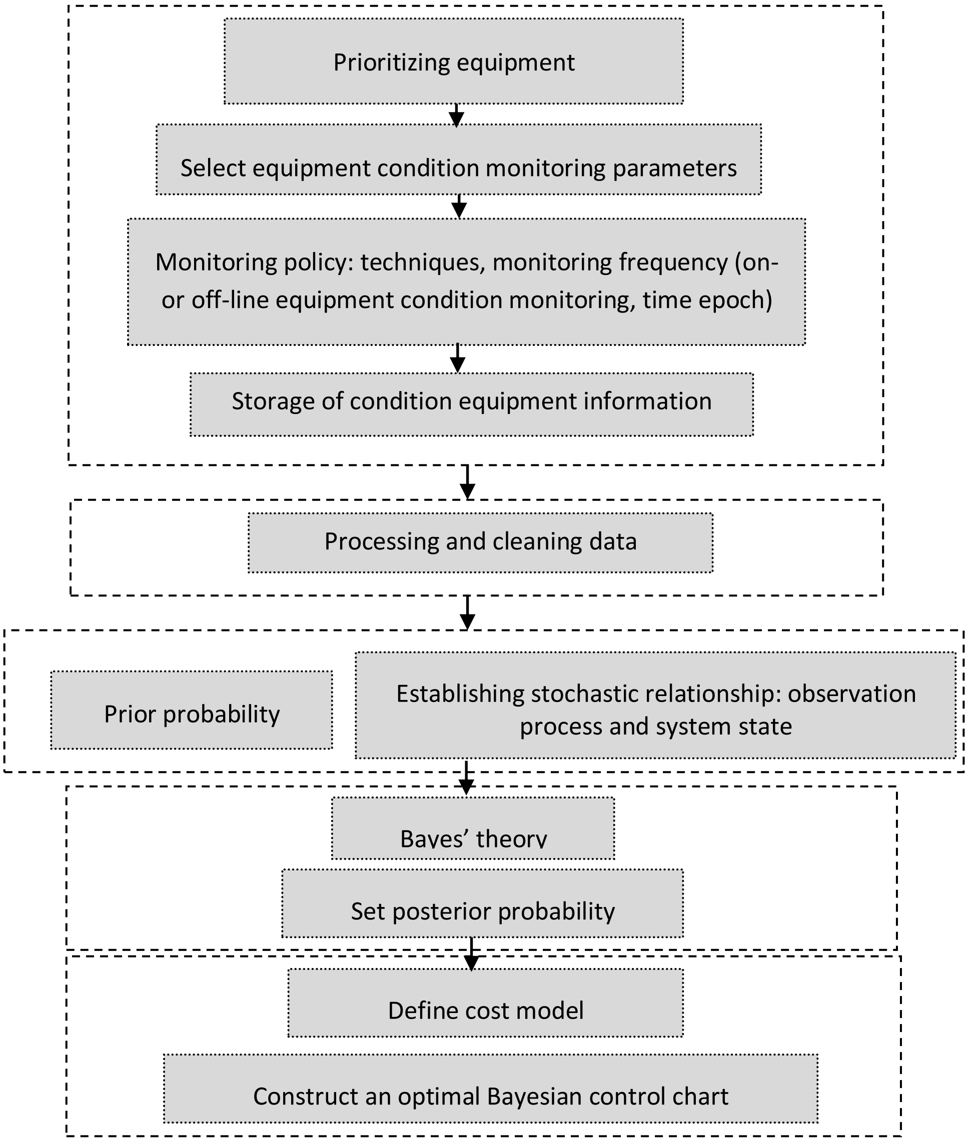


Figure 4.1- General maintenance integrated model architecture.

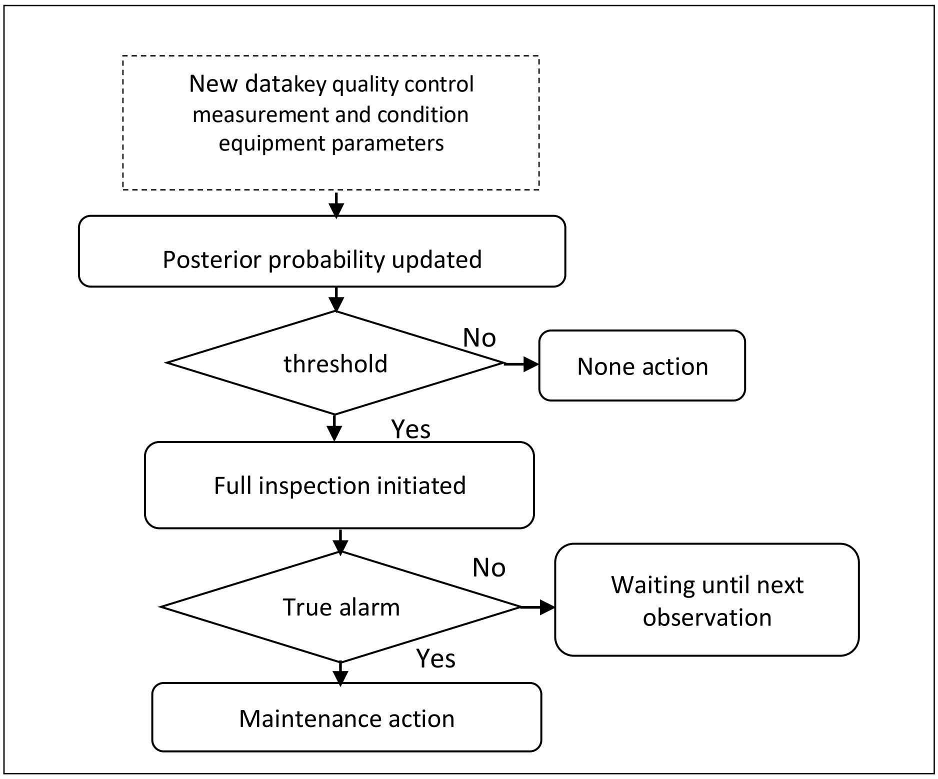


Figure 4.2-Flowchart maintenance decision making based on posterior probability.

4.2 Model assumption and notation

General assumptions which will be used on proposed model, are defined bellow:

1. Maintenance are assumed to be perfect.
2. Maintenance activity is carried out based on the result of maintenance decision support which is known as Condition-based maintenance
3. The system is monitored through perfect inspection.
4. Model for infinite time horizon has been chosen on proposed model (long-term behavior of subsystem under study.
5. After each intervention of technician the item back to the initial state as good as new.
6. The duration refer to tow stage of failure process (l_1, l_2) are independent.
7. The system under study can be in a healthy or unhealthy operational state, or in failure state is indicated respectively by 1,2, and 3
8. State 1 and 2 not observable, only the failure state indicated by 3 is assumed to be observable.
9. The state of the operational system is completely unknown unless an inspection is performed.
10. We assume that the system under study is in a healthy state at time 0, $P(X_0=0)=1$.

11. The observations coming from several sensors at each monitoring period are assumed to be independent
12. Given Bayes's theorem, posterior probability is update at each interval monitoring.
13. The observations coming from several sensors at each monitoring period are assumed to be randomly.
14. Multivariate Bayesian control chart is in place with upper control limit
15. Once the probability of unhealthy state known, Bayesian control chart given signal alarm that the system required to assess whether a full inspection is needed or not, as well perfect maintenance is carried out to renew the system.
16. In the beginning of study, the system start working in health state.
17. In term of probability, the relationship between observation and state are conditional.
18. Maintenance action is mandatory in order to change system state from unhealthy state to healthy state.
19. Once the system under study is critical in terms of performance, condition monitoring technique online is assumed to be necessary.
20. We are assuming data is sampled each 30 minutes.
21. We assume that the times spent on both restorations are negligible

4.3 Mathematical Model developed

4.3.1 A novel Condition-based maintenance description

As described previously in this thesis, in many real-phenomena decision makers would like to get an accurate information about the health state of an item (e.g., power plant, aircraft, industrial facilities, mines, power plants, spacecraft) in order to be able to predict the impending failure. Given the background, it should be noted that an effective Condition-based maintenance plays a key role here. These are our research contributions and they are applied to a real system.

This section advances a process for development mathematical model for maintenance decision making for unreliable systems (Atox Mill equipement). The study case for this study was Atox Mill 'equipment of cement industry. Indeed, applying the designed model on Atox Mill 'equipment in order to demonstrate its economic benefit. Insight and information about the health state of Atox Mill ' equipement are important to inform maintenance and production department in order to make an intelligent and integral decision, critical to executing successful maintenance strategy.

Having prepared and validate the data as described in the next chapter, the next step is to build a maintenance decision making's model for unreliable Atox Mill equipment by involving four mains parts: (i) the definition of failure model that described the stochastic relationship between observation process and system state (the probability density function of observation given state 3 equal to zero), (ii) description the deterioration model able to estimate the transition probabilities between discrete states (normal state and warning state), (iii) estimating expression of posterior probability based on Bayes theory and with failure and deterioration models, this process mainly builds upon Bayesian theory and renewal theory. In fact, he aim is to model the posterior probability distribution $P(x_t = 2 / S_1, S_2)$ given the continuous observed data about Atox Mill equipment condition parameter and a key quality measurement condition monitoring, (v) the definition of function with decision variables that need to be optimized, in this study the formulation of objective function upon renewal theory under availability criteria with two decision variables such as: upper control limit and sampling interval, in fact, the Bayesian control chart is then applied to monitor the health state of Atox Mill equipment .

The history of all data collected on-line from Atox Mill equipment, those data is used to calculate the posterior probability of the system being in a warning state. This probability can then be carried on Bayesian card. The lower limit of Bayesian control chart is zero and the upper control limit is a posterior probability (decision variable) to be determined. An alarm occurs when the posterior probability exceeds the upper control limit. When an alarm occurs, a complete inspection can be performed to see if the Atox Mill equipment is in warning state or not. However, the alarm can be false if the inspection found that the system is in normal state. If the inspection reveals that the system is in the warning state, the system is restored to normal state by proper maintenance. Once the system fails, the maintenance is carried out again to restore the system to the normal state.

4.3.2 Computational methodology for posterior probability

In this section we have showed that the method able to model the transition probabilities between discrete states and establishing the relationship between y_i^k and x_i, i =1,2; where *i* is the number of state and *k* is number of variables In the most of previous studies about the degradation modeling, researchers have often focused on the assumption that the process are Markovian or not, instead to demonstrate its fit with the process under study (the future state the process only depends on the current state with time between states follows exponential distribution). However, it is important to check if Markov property really holds or not. In this context, our work therefore to propose graphical approach focusing on the analysis of the Weibull distribution, the last were enables to provide information whether the process is Markovian or not, which will be describe later in more detail.

From results of the next chapter, it is possible to state that the degradation process are not Markovian, in another word the transition state has not Markov property where the Weibull distribution is not identical to the exponential distribution with shape parameters different from 1. It is commonly known that when the shape parameter is different form 1 the transition rate between states are not constant, because for this the transition state cannot be modeled by exponential distribution. Therefore, exponential distribution is the only continuous distribution with a memoryless property (Collgar, 2002). For this reason, Markov models are governed by exponential distribution. It is also important to note that, in this section we show how the delay time concept can be used for degradation modelling.

This approach have already been concretized (proposed) and we will do later with study case. Furthermore, failure data histories of failing Atox Mill ' equipment was gathered from 04/01/2015 to 31/12/2015, then we found the result about the Weibull distribution application in this situation.

In the development of mathematical model for maintenance decision making under study we need first to compute conditional probability for the system being in warning state (2) given observed variables Y_i, at time t_i $P(x_i / Y_i)$, the last was update when an new observation available and it is important to define it in order to compare with upper limit P*.

In the whole of developed work, (whereas) we have assumed that the state of Atox Mill equipment can be in operating state normal and warning state or non-operating state which called failed state, where state are random variables indicated by X, X={1,2,3}.

Considering the initial state of the Atox Mill equipment at $t_0$is in normal state (as good as new), where the probability of system being in normal state given observed variables at t_0=0 is certain and equal to 1, $P(x_0 = 1 / Y_0) = 1$, however, the probability of Atox Mill being in warning and failed state are impossible and respectively equal to: $P(x_0 = 2 / Y_0) = 0$, $P(x_0 = 3 / Y_0) = 0$,

Since the sum of the probabilities at time t_0,$P(x_0 = 1 / Y_0)$+ $(x_0 = 2 / Y_0) = 0$, $P(x_0 = 3 / Y_0) = 1$, the probability of Atox Mill being in warning and failed state are impossible and respectively equal 0, so $P(x_0 = 2 / Y_0) = 0$, $P(x_0 = 3 / Y_0) = 0$.

Therefore, a full inspection of Atox Mill is carried out when the new value of posterior probability being in warning state given observation Y= (Y_0, Y_1,…, $\boldsymbol{Y}_{i-1}$, Y_i) found above the defined upper control limit P*, where vector Y are 5 dimensional observations at t_i.It should be noted that Observed data consist two types: condition monitoring data about Atox Mill and data about quality characteristic measurement which is size of particle.

The mathematical statement that represents the posterior probability consists the calculation of conditional probability from prior probability, condition probability of unobserved state (warning) conditions on observed dataY_i is occurred. In another word, probability that the random variable X takes the value $x_i = 2$ (epoch monitoring = 30 min), given that Y= (Y_0, Y_1,.., Y_{i-1}, Y_i) is expressed by $P(x/y)$. This conditional probability is given by the conditional *pdf*, $f(x\ /\ y)$.

The most important and interesting question is what $P(x_i = 2\ /\ Y_i)$ is equal to. It is recognized that, computing the posterior from the prior with Bayes' theorem. The idea behind the equation bellow is mathematical formulation of updating probability. From Bayes' theorem we relate the two probability as follows:

$$P(x_i = 2/Y_i) = P(x_i = 2/y_i^k, k = 1, \dots, 5, Y_{i-1}) \quad (4.2).$$

$$= P(x_i = 2, Y_i) = P(Y_i) \quad (4.3).$$

$$P(x_i = 2\ /\ Y_i) = P((Y_i/x_i)P(x_i))/(P(Y_i)) \quad (4.4).$$

Where, $P(Y_i)$ is the marginal probability of Y_i and $P(x_i)$ prior probability of the Atox Mill ' equipment being in warning state (1) given $Y_{i-1}$was occurred at time i, to estimate prior probability require the use of the chain rule of total probability (two state are possible normal state and warning state), so we find:

$$P(x_i = 2\ /\ Y_{i-1}) = \sum_{x_{i-1}=1}^{2} p(x_i\ /x_{i-1}) p_{i-1}(x_{i-1}/\ Y_{i-1}) \quad (4.5).$$

The marginal probability, we can write this as:

$$P(Y_i) = p(y^k{}_i, k = 1, \dots, 5/Y_{i-1}) = \sum_{x_i=1}^{2} p(y_i, k = 1, \dots 5/x_i, Y_{i-1}) p(x_{i-1}/\ Y_{i-1}) \quad (4.6).$$

Using joint distribution and chain rule for conditional probability we can write above expression as follows:

$$P(Y_i) = \sum_{x_i=1}^{2} \prod_{k=1}^{5} p(y_i/x_i)\, p(x_i/\, Y_{i-1}) =$$

$$\sum_{x_i=1}^{2} \prod_{k=1}^{5} p(y_i/x_i) \sum_{x_{i-1}=1}^{2} p(x_i\,/x_{i-1}) p_{i-1}(x_{i-1}/\, Yi-1\,) \quad (4.7).$$

$$P(x_i = 2\,/\,Y_i) = \frac{\prod_{k=1}^{5} p(y_i\,/x_i\,) \sum_{xi-1=1}^{2} p(x_i\;/x_{i-1}) p_{i-1}(x_{i-1}/\,Y_{i-1})}{\sum_{x_i=1}^{2} \prod_{k=1}^{5} p(y_i\,/x_i\,) \sum_{x_{i-1}=1}^{2} p(x_i\;/x_{i-1}) p_{i-1}(x_{i-1}/\,Y_{i-1})} \quad (4.8).$$

To follow that, the posterior probability (Eq 4.8) requires identification failure model and degradation model. For the first one we need to compute conditional probability of observed data given normal or warning operational state, in this case we need to compute $P(y^k_i/x_i)$ which are stochastically related to warning state and normal state x_i can be found in normal state (1) or warning state (2), thus, the parameters estimation of this failure model is considered. Since, the distribution for the first case ($x_{i=1}$) is different from the probability distribution for the second case ($x_{i=1}$), four parameters must be estimated. We will do this later in the next chapter.

Since the parameters α_i, β_i(i belong to 1 or 2), stochastically the observed data at time:0, t_1, t_2, t_3,…related to the state process while the system is in operational states (state 1, state 2) via the following formulate:

$$P(y^k_i/x_i) = \begin{cases} f(y^k_{i;}\, \alpha_i,\ \beta_i), & x_i=1. \\ f(y^k_{i;}\, \alpha_i,\ \beta_i), & x_i=2. \end{cases} \quad (4.9).$$

For the second model we need modeled state transition, we need to compute $P(x_i/x_{i-1})$, where model parameters are estimated through using Maximum likelihood estimation.

Once the process has not Markov property, delay time concept use in the modeling of degradation process of Atox Mill ’ equipment, it provide a relevant framework for modeling degradation model. Furthermore, Delay time concept is a stochastic process used for state transition, where Atox mill equipment at any time can be occupied one of a set of discrete states randomly (e.g., normal, warning, failed), Delay time concept enables to modeling the degradation of Atox Mill ’s equipment, as a two-stage failure process. This models has a transition from new (1) to a defect state (2), then from defect state to failed state (3), which is called defective stage and time l_1 are randomly. The second stage is called failure delay time, the time l_2 are randomly, and quantities of interest are the transition probabilities. The stochastic behavior of Atox Mill 'equipment is directly described through the possible transition probability, the two stage of failure process displayed in Figure 4.3.

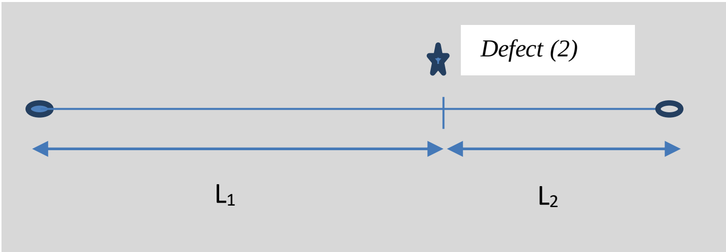


Figure 4.3- Tow stage of failure process.

Usually, real data typically consist of times of occurrence of transitions between the states and the types of transitions that occur. Following this consideration, the parameters of the model are estimated using the Maximum-Likelihood method (MLE) based on the observed data of Atox Mill 'equipement, through the application of MLE algorithm coded in Matlab software. For calculation posterior probability the parameters of failure model and degradation model must be calculated.

Considering the process are not Markovian where the past history of observed data is considered in calculating the transition probability. Furthermore, in analogies with definition of delay time concept, this model has the following transition states: p_{11}, p_{12}, p_{13}, p_{13}, p_{22}, p_{23}. As mentioned before, by definition transition probability $p_{ij}$is the probability of moving from state i (the current state) to the state j, so find:

***(a)* p_{11}:** the probability of being stay in state 1 (the current state).

***(b)* p_{12}:** the probability of moving from state 1 (the current state) to the state 2.

***(c)* p_{13}:** the probability of moving from state 1 (the current state) to the state 3.

***(d)* p_{22}:** the probability of being stay in state 2 (the current state).

***(e)* p_{23}:** the probability of moving from state 2 (the current state) to the state 3.

To formulate those transition probability may be estimated by conditional probability law

$$P(x_i/x_{i-1})=\int f(x_i/x_{i-1})=\int f(x_i, x_{i-1})/\int f(x_{i-1}). \qquad (4.10).$$

Then using Eq (4.10), the following estimation of transition probabilities is given by:

(a) $p_{11}=\frac{\int_{t_i}^{\infty} f_1(l_1)dl1}{\int_{t_{i-1}}^{\infty} f_1(l_1)dl1}$ (4.11).

(b) $p_{12}=\frac{\int_{t_{i-1}}^{t_i} f_1(l1) \int_{t_{i-h1}}^{\infty} f_2(l_2)dl_1dl_2}{\int_{t_{i-1}}^{\infty} f1(l_1)dl1}$ (4.12).

(c) $p_{13}=\frac{\int_{t_{i-1}}^{t_i} f_1(l_1) \int_0^{t_{i-h1}} f_2(l_2)dl_1dl_2}{\int_{t_{i-1}}^{\infty} f1(l1)dl1}$ (4.13)

(d) $p_{22}==\frac{\int_0^{t_{i-1}} f_1(l_1) \int_{t_{i-h1}}^{\infty} f_2(l_2)dl_1dl_2}{\int_0^{t_{i-1}} f_1(l_1) \int_{t_{i-1}}^{\infty} f_2(l_2)dl_1dl_2}$ (4.14).

(e) $p_{23}==\frac{\int_0^{t_{i-1}} f_1(l_1) \int_{t_{i-1-h1}}^{t_{i-h1}} f_2l_2dl_1dl_2}{\int_0^{t_{i-1}} f_1(l_1) \int_{t_{i-1-h1}}^{\infty} f_2(l_2)dl_1dl_2}$ (4.15).

4.3.3 Mathematical equation that describe the expected cycle cost per time unit.

This corresponds with what we mentioned earlier, failing of the equipment Atox Mill due to human errors, unreliability of component, these last were the first cause of intervention of a maintenance technician. The proposed models guaranteeing predict impending failure and reactivity of the technician for requests of intervention and its availability to realize an effective maintenance action, thus reducing the incidents and the consequent losses of time and financial ones. In this proposes effective methods to reduce the cost of maintenance. To manage this challenge, a mathematical model and optimized algorithm was performed for failing Atox Mill equipment in order to define with manner continuously the behavior of Atox Mill equipment. This section is generally one of the most important part in my research process where subsequently mathematical model we formulate the problem as optimization problem considering the cost criteria, which is based on the definition of a nonlinear objective function.

After formulating the mathematical model, cost criteria for the system was defined and developed.

In this optimization problem developed, the definition of the objective-function, the decision variables is necessary in order to reach a significant results. In this work, a most crucial steps in formulating a problem is the definition of objective function, a single objective function on an economic basis is considered, which will be describe in more detail.

Based on assumption that time horizon is infinite, in this case a renewal theory is required in order to compute the expected average cost per unit time, from renewal theory, the cost equation represents the expected cycle cost divided by the expected cycle length, and this is the convention in much of CBM optimization.

The objective is to a design a novel multivariate Bayesian control chart with lower maintenance cost. The optimization problem consists in the minimization the expected average cost per unit time. For that, bound constrained optimization problems used for optimizing an objective function that subject to bound constraints on the values of the variables. In mathematical terms. Continuous optimization problem consider two decision variables: (i) interval monitoring, (ii) upper control limit. However, the sample size (n)does not will be considered as decision variable, the number of sample size *n* equal to 5 variables in all study. In this optimization problem, the decision variables are two quantities that need to be determined in order to solve the problem, therefore, the problem is solved when the best values of the variables have been identified.

After defining clearly the decisions variables, and optimization condition such as upper bound and lower bound of interval monitoring for posterior probability, the next step we are looking for the best value of these continuous decision variables optimal sampling interval value and the upper control limit that minimize the average cost as an objective function.

As a general rule in much optimization theory, the function *f* is called objective function, in this maintenance application, the problem stated in term of minimization the cost problem which is to minimize a function $f: R^n \to R$, the objective function, over a specified set $C \subset R^n$, the feasible set. A feasible solution that minimizes the objective function is called an optimal solution. In mathematic term, the optimization problem formula that we are going to optimize as in the following equation:

$$\begin{cases} \min z_{(h,p)} \dfrac{E(CC)}{E(CL)} \\ Subject\ to \\ 0 < p \leq 1 \\ 0.05 \leq h \leq 30\, min. \end{cases} \qquad (4.16).$$

E(CC): Expected cycle cost

E(CL): expected cycle length.

This denotes the minimum value of the objective function $f = \frac{E(CC)}{E(CL)}$, when choosing p from the set of real numbers as condition of algorithm for posterior probability the lowed bound and upper bound respectively equal to 0.01 and 0.99, for the second decision variables which is the interval monitoring lower bound and upper bound respectively equal to 0.05 and 30 min. For constructing the expression of objective function first we will formulate the expected cycle length denoted by E(CL) and expected cycle cost denoted by E(CC), both expression will be described in more detail bellow. Here we want to refer to the paper of Wang (2012). This paper shows how to estimate E(CC) and E(CL).

a) Cycle cost

The mathematical expressions that define the expected cycle cost include the following costs of: renewal cost when the system is found to be defective by an inspection, cost for checking the true state of the system, and renewal cost if the system is failed, the expected cycle cost for Atox Mill equipment where $0 \leq t \leq L_1 + L_2$ is given by:

E(cycle cost)=E(cost/system fails before t) P(system fails before t)+E(cost/system doesn't fails before t) P(system doesn't fails before t).

$$E(CC)=\sum_{i=1}^{\infty}\sum_{j=1}^{i} r_1^{i-j}\,(1-r_1)\int_{t_{j-1}}^{t_j}(C_{ij}+C_{monit}+C_{inspec}+C_{maint})f_1\,(l_1)$$
$$\times[1-F_2\,(t_i-l_1)]\,dl_1\;+\sum_{i=1}^{\infty}\sum_{j=1}^{i} r_1^{i-j}\int_{t_{j-1}}^{t_j}(C_{ij}+C_f)f_1\,(l_1)$$
$$\times\,[F_2\,(t_i-l_1)-F_2\,(t_{i-1}-l_1)]dl_1 \qquad (4.17).$$

Where

$C_{ij} = (i-1)C_{monit} + r_0\,(i-j)C_{inspec}$:The expected cost of monitoring and false alarm inspection before a renewal at t_i or l_1, $l_1 \in [t_{i-1}, t_i]$, and the defect occurs at l_1, $l_1 \in [t_{j-1}, t_j]$.

r_0 : Conditional probability the system in stage 2 (P($x_i\,/Y_i) \geq p^*$).

r_1 : Conditional probability is in stage 1 (P($x_i\,/Y_i) < p^*$).

b) Cycle length

Resembling to the previous expression, the expected cycle length can be obtained as follows:

E(cycle length)= E(S/system fails before t) P(system fails before t)+E(S/system doesn't fails before t) P(system doesn't fails before t).

Where S: successive time that system consider as good as new.

$$\sum_{i=1}^{\infty} \sum_{j=1}^{i} t_i\, r_1^{i-j}\,(1-r_1) \int_{t_{j-1}}^{t_j} f_1\,(l_1\,)$$

$$\text{x } [1\text{-}F_2\,(t_i - l_1\,)]\; dl_1\; + \sum_{i=1}^{\infty} \sum_{j=1}^{i} r_1^{i-j} \int_{t_{j-1}}^{t_j} l\, f_1\,(l_1\,)\, f_2\,(l - l_1\,)]\; dl_1\, dl_1$$

(4.18).

Bythe infinite horizon, so long term average cost per time unit is equal to:

$$\frac{\text{E(cost/system fails before t)P (system fails before t)+E(cost/system doesn't fails before t) P(system doesn't fails before t)}}{\text{E(S/system fails before t) P(system fails before t)+E(S/system doesn't fails before t) P(system doesn't fails before t)}}$$

(4.19).

Where; E(CC) and E(CL) are approximate by Monte Carlo Simulation procedure.

4.4 Solving bound constraint non-linear optimization problem with PSO

In term of computational algorithm (technique), the stochastic algorithm used in this Bound constraint optimization problem called swarm particle algorithm (PSO). It is a stochastic algorithm that has an heuristic feature, this technique based mainly on population with two components such as: particle velocity and position. This PSO was implemented to reach global solution, and developing a code in Matlab. PSO has been successfully applied in many area such as: function optimization, fuzzy system control, and artificial neural. In this case, PSO was efficient global search algorithm to achieve optimal solution from all feasible solutions (population).

The search strategy may and often do find global optimal solutions, but they are not guaranteed to do so. Nonetheless, this methods are widely used, often finding very good solutions, and can be applied to nonlinear, complex problems. They are made up of three basic components: a set of variables, a fitness function to be optimized (minimize or maximize) and a set of constraints that specify the feasible spaces of the variables. The goal is to find the values of the variables that optimize the fitness function while satisfying the constraint. They are made up of three basic components: a set of variables, a fitness function to be optimized (minimize or maximize) and a set of constraints that specify the feasible spaces of the variables.

A PSO algorithm strategy usually starts with a feasible population, exploring all the solution near these points, then looking for a better one, and repeats the process if an improved point is found

(Thomas, David, & Leon, 2001).Compared with other stochastic algorithm, PSO has the following advantages:

* The algorithm is simple, there are not many parameters to be adjusted.
* The algorithm is powerful, PSO is much faster for above benchmark functions, and the above results also show that it can deal with many kinds of optimization problems with constraints.
* There is no predefined limit to the objective and constraints; it does not need to preprocess the objective and the constraints.

Figure 4.4, illustrate the PSO global algorithm for bound unconstraint optimization problem. For the local version, there is only one difference in the algorithm; instead of finding the gBest, each particle finds a neighborhood best (pBest) to update the new velocity.

```
   For each particle {
           Initialize particle
       }
           End
   (Estimate intensity
       Of a particle as objective)
       Do
       for each particle
       {Calculate fitness value
       }
       If the fitness value is better than pBest
       {set pbest= current fintess value
}
if pBest better than gBest
{set gBest= pBest
   }
   For each particle
   {
   -calculate velocity according to equation of velocity
   - Update particle position according to equation of position
   }
```

Figure 4.4- The global PSO Algorithm for unconstrained non-linear optimization problem.

According to Hu and Eberhart (2002), demonstrated that PSO is an efficient and general method to solve most unconstrained parameter optimization problems. The main steps in the process of a modelling a Bayesian control chart applications for condition based maintenance are schematically describes in Figure 4.5.

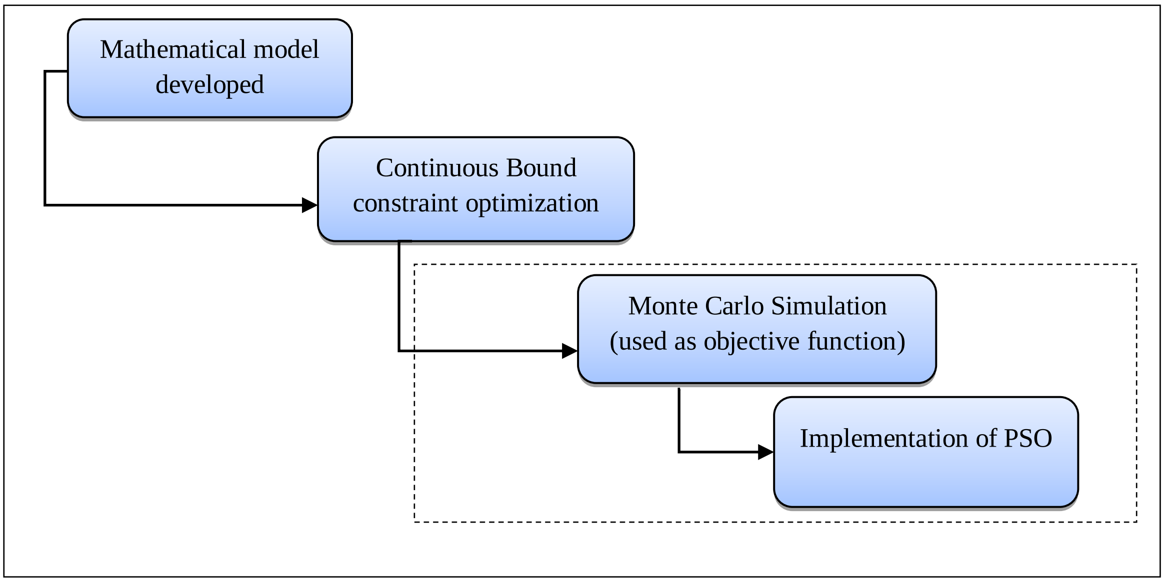


Figure 4.5-Process of the development of an optimal Bayesian control parameters.

Chapter 5

A Case study in Cement Industry

This chapter addresses a case study in cement industry in order to evaluate the effectiveness of the proposed model. This includes,introduction of Secil cement company, afterward a brief descriptive of Atox Mill equipment, then the schematic representation of the Atox Mill Equipment of cement industry. The main aim of this section is to report the results of the data analysis used to test a hypothesis and corrrelation. It was conducted a case study using a realdata gathered from Atox Mill . These data could then be analyzed withstatistical and descriptive methodsusing SPSS for analysis data. Although,Weibull analysis for verifying markov property was presented. The aim of this chapter is to implement and evaluate a novel Condition based maintenance model based on optimal multivariate Bayesian control chart.

Contents

5.1 Secil Cement Company

Secil is a Portuguese company specializing in cement product. The company was founded in 1930 and it is today one of Portugal’s largest supplier of cement producers. Secil offers a wide range of white and grey cement products, of different types and classes, produced 4 million tons of cement each year. Therefore, it is currently the second largest cement company in Portugal and biggest competitor Cimpor. Its Subsidiaries are: Secil-Outão, Cibra-Pataias and Maceira-Liz plants with an annual output of 4 million tons of cement, although, the activities of this company of cement are mainly located in Portugal, but also in other regions where the group is present. Additionally, Secil currently has 4 cement plants in different countries: Tunisia, Lebanon, Brazil, Angola and Cape Verde. In 2014, Secil employs approximately 1407 workers and has an annual value a revenue of about €429.60 million.

5.2 Cement manufacturing process

Cement is an inorganic, non-metallic substance with hydraulic binding properties, and is used as a bonding agent in building materials. It is a fine powder, usually gray in color that consists of a mixture of the hydraulic cement minerals to which one or more forms of calcium sulfate have been added (Greer et al., 1992). As shown in the figure 5.1, there are fourth mains process in the cement production process: (i) extraction and grinding of raw materials (acquisition and preparation raw material), (ii) milling raw, (iii) clinker production process, (iv) the storage and grinding of cement (Clinker milling). Each of these process components is described briefly below:

Step 1:”Extraction and grinding raw material”

The initial production step in cement manufacturing process is raw materials acquisition and handling. The raw materials used in the manufacturing of cement include limestone, marl (shal) and clay. These raw material are mined and crushed by grinders to the degree of fineness needed for the subsequent steps in the process (raw milling). Other supplementary materials such as sand and iron are sometimes added to in different proportions to obtain the desired composition of the feed to the cement production process. Therefore, these raw materials are obtained from quarries. After mixing and homogenization, the second stage is the milling raw.

Step 2:”Milling raw”

This process take place into Atox mill, the raw materials are taken from their storage locations and transported to Atox mill before being grinding and crushing raw material, the grinding produces a fine powder, which is called raw mill. The Atox raw mill uses pressure and shear generated between the rollers and the rotating table to crush and grind raw materials. Feed material is directed into the grinding table by the feed chute. The rotation of the grinding table accelerates the material towards the grinding track and passes it under the rollers. Partially ground material passes over the dam ring encircling the grinding table and into the hot gas stream coming from the nozzle ring. The main goal of this second step in cement manufacture is preparing kiln feed (raw mix).

Step 3:"Clinker production process"

Clinker production process is the main step in the cement production manufacturing. After preheater the raw mill into cyclone preheater and precalciner the resulting dry powder is fed to the rotary kiln through cyclone preheater. Cement rotary kiln is equipment for transforming the raw mill into clinker (for calcining cement clinker). The chemical reactions and physical process take place in a rotary kiln fired to temperature around 1450, at this temperature chemical reaction is produced and leads to the transformation the raw mill to clinker, this reaction chemical are known as "clinkerization" and the product obtained is called "clinker". The clinker is stored in silos before being grinding and storage of cement.

Step 4:" Grinding of clinker"

The clinker produced is grinding by using cement mill. At this step of cement production, the clinker transformed into the fine grey powder that is cement, the grinding clinker produce a cement. The clinker is the main constituent of most cements, however, gypsum is added to the clinker as well as other additives can also be used in the composition of cement. When cement are produced from the cement mill equipment, then they need to be conveyed, stored and reloaded too. Finally, cement is stored in silos before being packaged by a bagging machine into 25-35 kg bags or supplier to customer in bulk using tanker trucks. The detailed scheme of the process is presented in figure 5.1.

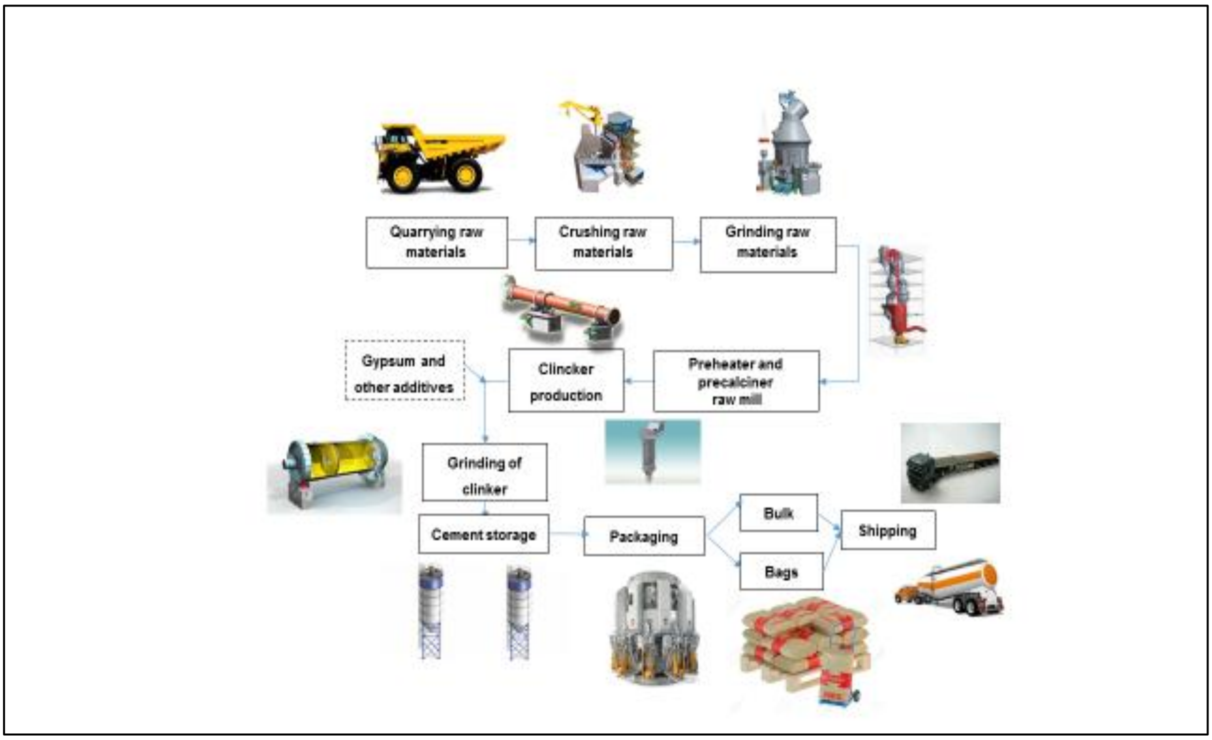


Figure 5.1-Simplified process flow diagram for Cement manufacturing.

5.3 Atox Mill functioning

Atox Mill vertical is a type of grinder used to grind materials into extremely fine powder for use in cement. Feed material is directed onto the grinding table by the feed chute. The rotation of the grinding table accelerates the material towards the grinding track and passes it under the rollers. Partially ground material passes over the dam ring encircling the grinding table and into the hot gas stream coming from the nozzle ring. Coarser material and bigger lumps drops through the nozzle ring and is eventually recirculated into the feed material inlet. A figure 5.2 bellow shows representation graphics of Atox Mill Equipment.

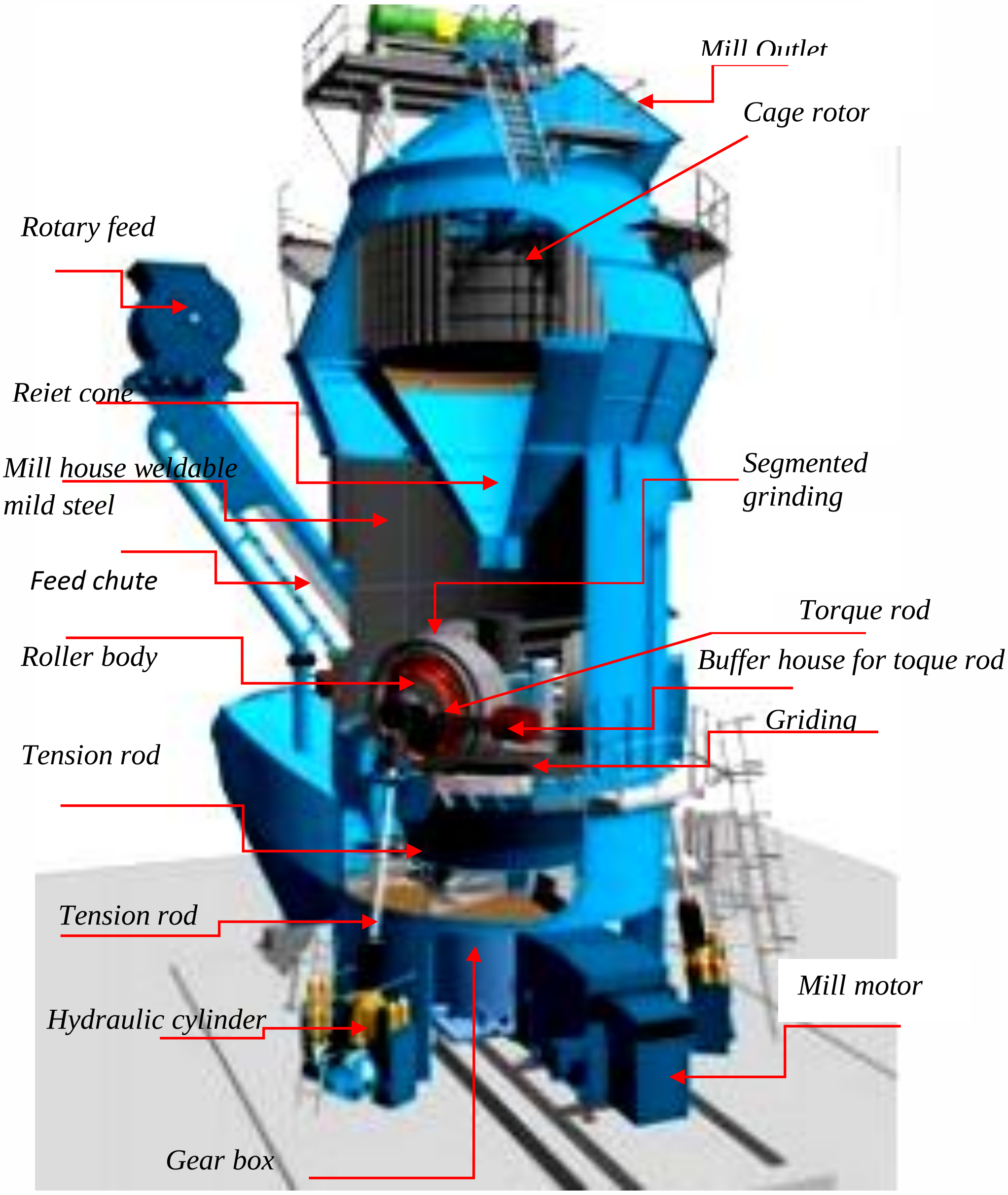


Figure 5.2- A Schematic representation of Atox Mill Equipment.

Figure 5.3- A grinding roller of Atox Mill Equipment.

The technical features of Atox Mill equipment are:

- Atox Mill Model (dimension): Atox 50
- Table Diameter: 5meters
- Number of rollers: 3units
- Rollers Diameter: 3meters
- Motor Power: 3000kW
- Motor Speed: 743rpm
- Motor Voltage: 6000V
- GearBox Ratio: 28,03

5.4 Quantitative data collection

In the cement industry, Atox Mill Equipment has been considered as the most important equipment due to its downtime cost, higher overall cost, production cost, and maintenance requirement are higher. In this context, elaboration and efficient strategies for prognostic of failure is mandatory requirement. The state of the Atox Mill during its function can be defined by one of the following modes: healthy, unhealthy operational state, or in failure state as listed in the last column of table 5.1. The process used for illustration of the application, our approach consist of grinding mill using Atox Mill equipment.

As it was mentioned previously, the principal technique used in this work was quantitate aspect in which intendsto satisfy the objectives of this study. Once the Atox Mill equipment is operational, multivariate observations that are related to the system state and quality control of particle' size are sampled through condition monitoring technique at discrete time points, in this study case the sample are taken each 30 minutes.

These observations could be used to derive quantitative estimates of parameters, testing hypothesis (normality and correlation), understanding the nature of multi-observation, and understanding the process under study, using packaging SPSS.

For the purposes of this research, the channel of information’s come from Atox Mill were used. Indeed, condition monitoring techniques online must be required in order to collect numerical data. In this case, many variables provide by sensors, the type of variables are quantitative and continuous (ratio data). In order to illustrate the proposed model, we consider a real Atox Mill data set, data are in the form of numbers and recorded at discrete time about half an hours during specified period from 04/02/2015 to 31/12/2015,

In the current research, the variables who were selected had special relationship with the health state of Atox Mill under study. Within this context, the data set consist of four variable about condition of this equipment, and one about quality control measurement of dust (output Atox Mill). Indeed, condition monitoring technique of the Atox Mill involving the taking of at regular time (half an hours) of a sample of the following continuous variables: temperature of filter, vibration of Atox Mill, power of motor, pression of Atox Mill, size of dust (output of Atox Mill). Data classifies as confidential any of the following data variables: temperature, pression, vibration, power motor, size particle as well as time failure when can appear in the thesis, the sample size is quite large (the current study involved 15160 samples).A table 5.1shows the variables types that come from multi- sensors on the Atox Mill equipment.

Table 5.1. Summary of variables types and its characteristic.

Label of variables	**Unit**	**Sample size**	**Maximum**
Temperature of filter	(ºC)	**15160**	**304.68**
Vibration	(mm/s)	**15160**	**5.30**
Power of motor	(kW/h)	**15160**	**2490.79**
Pression	(mbar)	**15160**	**63**
Size of dust	(R90µm)	**15160**	**11**

The main difficulty find on this work is to obtain practical (empirical) data. For that, it has been spend long time looking those data and finally we found it such as Secil Company aforementioned the company who provide us data. These observed data correspond the variables related with the health state of Atox Mill equipment and quality measurement of dust (output of Atox Mill). They are essential for building the model. However, these data are not directly used for estimating model parameters, because they are not complete and include missing values. In this context, to rectify one's inaccurate or incomplete data is of key importance in order to ensure the quality of the data processed. In this context, steps must be taken to ensure that inaccurate or incomplete data are deleted or corrected, in another word, it was necessary to correct them, with statistical tools and expert judgment (sample, the value missing). This steps was carried out jointly with the Secil' engineer.

It should be mentioned, that it is not easy to get the whole values of measured variables and the historical failure due to the absence culture of Condition-based maintenance policy and level of maturity of maintenance department. As mentioned before, this lack of information requires statistic method known as Principal Component Analysis (PCA), an appropriate tool than can be used in such situation. PCA aims at reducing a large set of variables to a small that still contains most of the information in the large set as results we can identified the variables that produces a lot of information about the behavior of Atox Mill equipment. The output of PCA illustrate that the vibration and the size of particle are two most important variables. Moreover, data provide by cement industry (Secil) were completed analyzed after collected with the aid of Microsoft Office Excel and Statistical Software SPSS.

This section is intended to draw conclusions from observed data by statistically analyzing it using packaging SPSS. Indeed, creating statistic analysis and graph such as: histogram, box-plot, frequency table, statistic indicator (e.g., central tendency indicators: dispersion indicator, mean, mode, variance) helps to understand a sense of distribution the variables that is in consideration, whether or not variables has significant effect, the relationship between two variables (e.g., means and standard deviations, test statistic, degrees of freedom, obtained value of the test, and the probability of the result occurring by chance indicated by *p-value*(Sig)).Statistics analysis is mandatory in my thesis with the goal to substantiate my findings. Indeed, in this section we provide the main steps for making descriptive and correlation analysis using SPSS packaging. A SPSS software is very common used in many statistical and econometric studies by many researchers.

This analysis of data process basically including: preparing, cleaning, transforming, and modeling in order to discovering useful information, reducing complexity of data and suggesting conclusion. Before we can analyze data they must be gathered, cleaned, and entered in SPSS in appropriate format and code. Generally, the following steps are used to figure out numerical and visual outputs: (i) Gather and code data (descriptive analysis, box plots or other graphs, z-value), (ii)Type of data (iii) Check data for normality if needed: Are the data normally distributed?, If No apply nonparametric analysis, if Yes apply parametric methods, observed measurement that come from a population that is normally distributed can usually be treated as parametric, (iv) correlation matrix, (v) Draw conclusions. As shown in Figure 5.4,the flowchart provide guideline about the steps must be performed, from data gathering to drawing conclusions in order to select the appropriate statistic methods (test selection process).

Descriptive statistics for the whole variables are given as mean (M) and standard deviation (SD). Normality of distribution was tested using Kolmogorov-Smirnov test at the 5% level of significance, then. Spearman's correlation analysis was used to know the strength and of correlation between the variables.

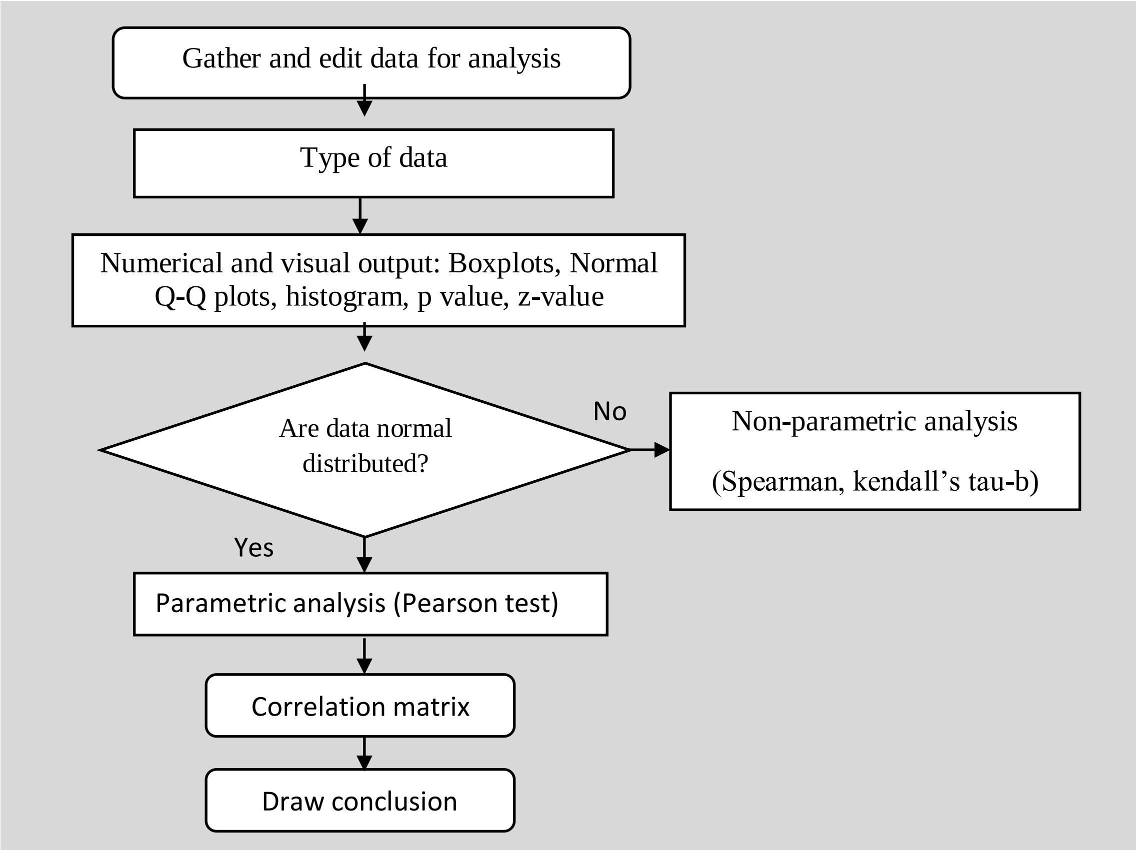


Figure 5.4-Flowchart depicted steps for descriptive and correlation analysis.

5.4.1 Descriptive and correlation analysis in multivariable

In order to choose the right type of correlation measure (parametric or non-parametric)Normality tests are used. The last is mandatory required steps to check whether data (dependent variables) are approximately normal distributed or not. It should be noted that, descriptive and correlation analysis were done using the 5% level of significance. It is recognized that hypothesis test is a procedure for deciding if a null hypothesis should be accepted or rejected in favor of an alternate hypothesis. A statistic is computed from an Atox Mill 's data (Annex 1) and is analyzed to determine if it falls within a preset acceptance region. If it does, the null hypothesis is accepted otherwise rejected.

When performing a test of statistical significance, it is useful convention to distinguish between i. The particular hypothesis that the research is seeking to examine; and ii. The logical antithesis of the research hypothesis (alternative hypothesis). The second of these items is commonly spoken as the null hypothesis, where the word "null" in this context has the meaning of "zero". A conventional symbolic notation is H_0, which is H for "hypothesis", with a subscripted zero, to denote that it is "null". The research hypothesis sometimes is spoken of as the experimental hypothesis or alternative hypothesis, and is denoted as H_1, In general, the null hypothesis is to the effect that the observed results will not significantly differ from expected values. The following numerical and visual output must be investigated:

- Statistical hypothesis testing consists diagnostic hypothesis tests for normality, testing the data against the null hypothesis that it is normally distributed, if significance level (α, or alpha),*p-value* less then alpha(p mean the probability of making a decision to reject the null hypothesis when the null hypothesis is actually true (a decision known as a Type I error, or "false positive determination"). The hypothesis test for normality statement can be written as follows:

  H_0 (Null hypothesis): attempt to show the data are normal,
  H_1 (Alternative hypothesis): attempts to show that the data are no Normal.

  In this study we consider confidence level of 95% refer to alpha equal to 0.05. Practically, if $p \leq 0.05$, we will reject the null hypothesis; then the data appear to be form a normal distribution, otherwise we will accept it.

- The skewness and kurtosis are a way to asses normality and must be computed in order to quantify how far from normality the distribution is in terms of asymmetry and shape. The formula for calculating the skewness standard z-value and Kurtosis standard z-value is given below: $\text{Skewness z} - \text{value} = \text{Skewness} \div \text{standart error(Skewness)}$ For data $Y_1, Y_2, \ldots, Y_N$, for i=1,…,N the formula for skewness is:

$$\text{Skewness} = \frac{\sum_{n=1}^{N}(Y_i-\overline{Y})^3/N}{s^2} \quad (5.1).$$

  $\text{Kurtosis z} - \text{value} = \text{kurtosis} \div \text{standart deviation (Kurtosis)}$, the formula for Kurtosis is:

$$\text{Kurtosis} = \frac{\sum_{n=1}^{N}(Y_i-\overline{Y})^{43}/N}{s^4} - 3 \quad (5.2).$$

  As the formula shows, Skewness z-value is the skewness measure divided by its standard error, kurtosis z-value is the kurtosis measure divided by its standard error. These two value should be somewhere in the span of $-1.96\ and + 1.96$, if we divide either score by its standard error and the result is greater than ±1.96, it suggests that data are not normal. Beside of these two condition that indicate that our data are approximately normally distributed or not, if the Kolmogorov-Smirnov test p-value above 0.05, the distribution can be considered normal, so we don't reject the null hypothesis. Table below shows the *p-value* for each variables (Table 5.1)

In addition of above elements, informal approach to testing normality such as visual inspection of the distribution data in graph may be needed: histogram, Normal Q-Q plots and Box-plots should usually indicate that our data are approximately normal distributed or not. According to literature, if the histogram indicates a symmetric, moderate tailed distribution, then the recommended next step is to do a normal probability plot to confirm approximate normality. If the normal probability plot is linear, then the normal distribution is a good model for the data (Normality Q-Q plot). In fact, Normal distribution has bell-shaped which mean symmetric histogram with most of the frequency counts bunched in the middle and with the counts dying off out in the tails. SPSS output for skewness and kurtosis tests from a sample of test scores and descriptive statics table is given in Annex 1.

5.4.2 Reporting the results of normality test in multivariable

For testing whether variables contains normally distributed data implies that the conditions mentioned in the previous section for each variable must be satisfying:

Testing if temperature filter's variables contains normally distributed data (mean=124.80, SD=48.198)?The results of the Kolmogorov-Smirnov ($p \leq .05$) and a visual inspection of their histogram (Figure 5.5), box plots(Figure 5.6) , and Normal Q-Q plots (Figure 5.7), showed that the exam score were not normal distrusted for temperature filter's variables, with a skewness 0.216 ($Sz = 10.8$) and kurtosis of 0.762 ($Kz = 19.05$).

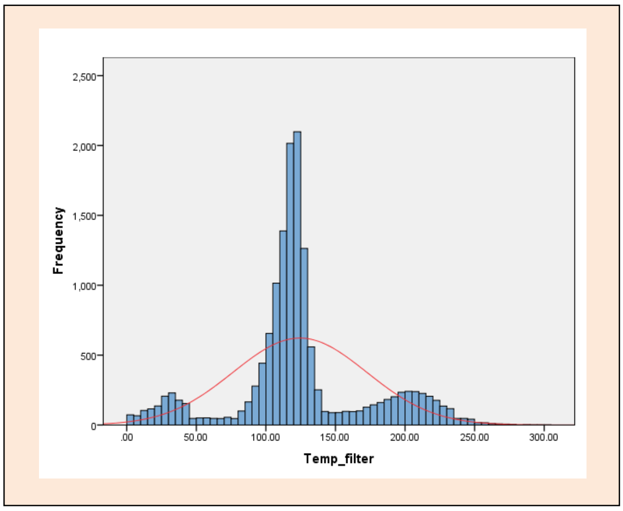


Fig 5.5- Edited Histogram of temperature of filter with normality plot (M=124.80, SD=48.198).

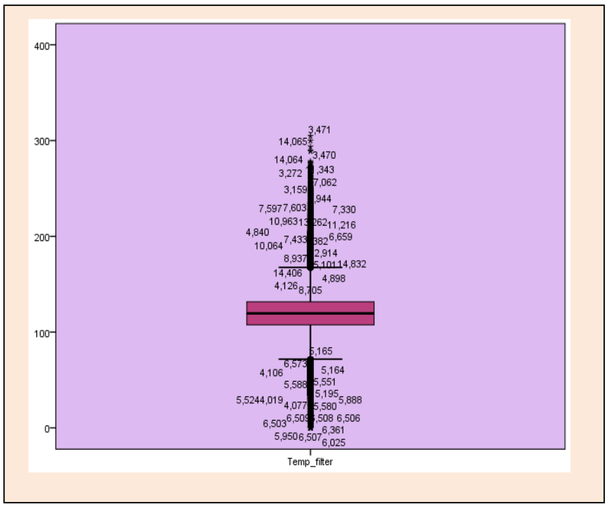


Fig 5.6- Boxplot shows feature statistical of temperature

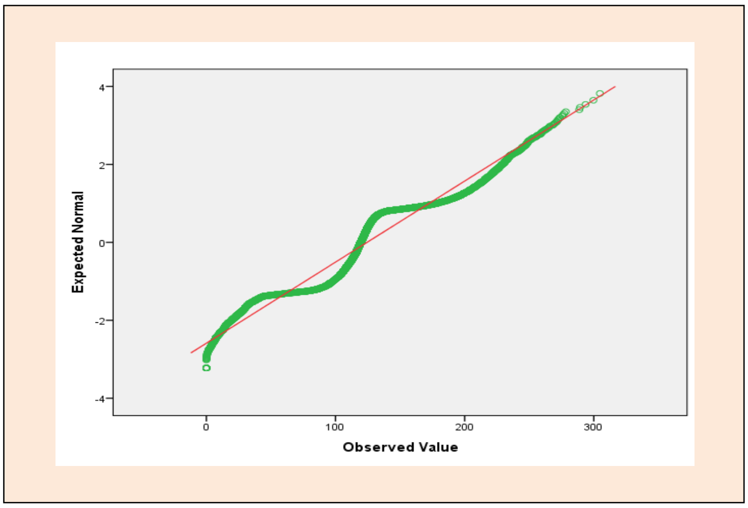


Fig 5.7-Normality Q-Q plot of filter's temperature.

Testing if power of motor variables contains normally distributed data (mean=1323.52, SD=962.594)? The results of the Kolmogorov-Smirnov test ($p = .000 \leq .05$) and a visual inspection of their histogram (Figure 5.8), Normal Q-Q plots (Figure 5.9) and box plots (Figure 5.10), showed that the exam score were not normal distrusted for power of motor' variables, with a skewness -2.343 ($Sz = -97.62$) and kurtosis of 9.363 ($Kz = 191.081$).

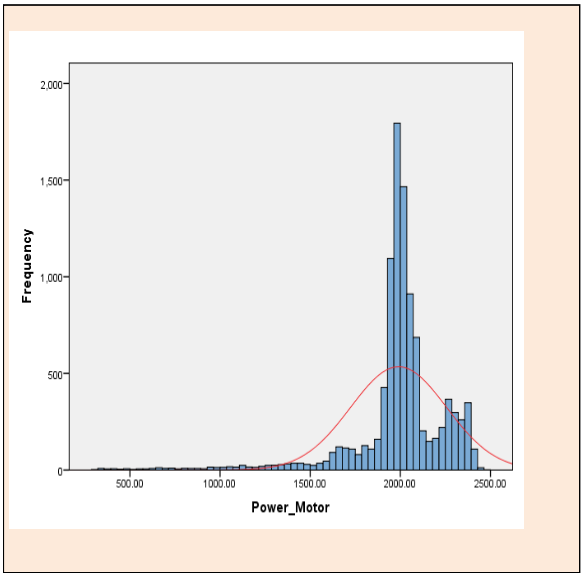


Fig 5.8- Edited Histogram of motor' power with normality plot (M=1986.40, SD=274.663).

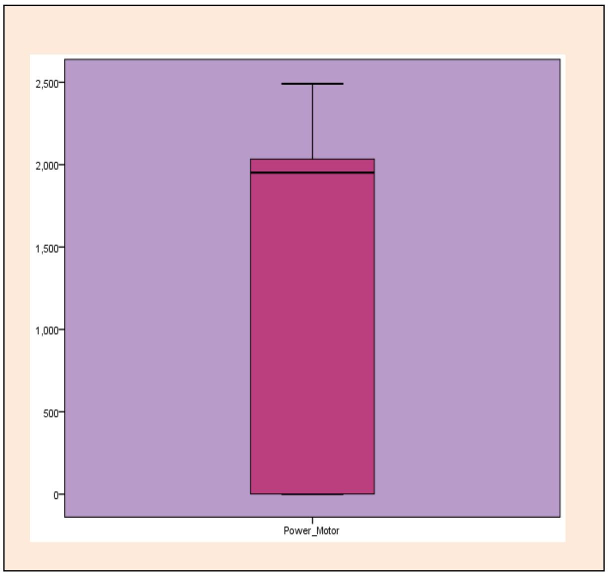


Fig 5.9- Boxplot shows features statistical of motor'power.

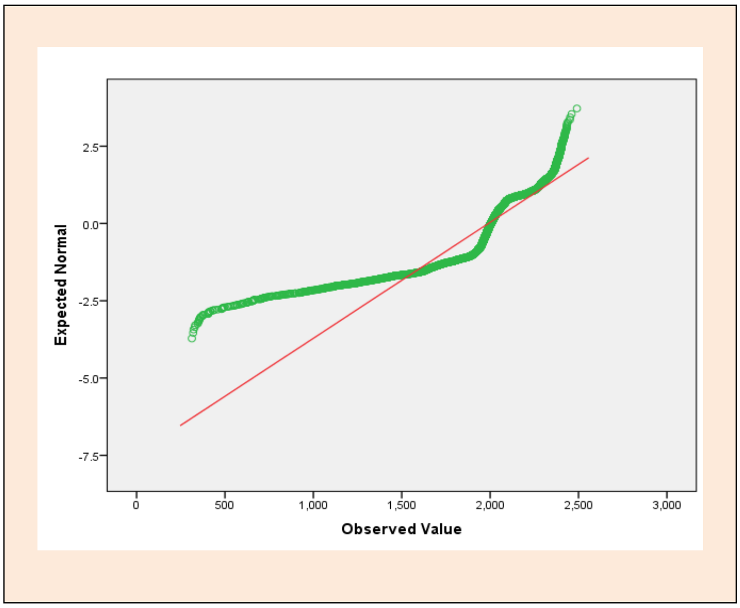


Fig 5.10- Normality Q-Q plot of motor's power.

Testing if Atox Mill ' pression variable contains normally distributed data (mean=33.29, SD=23.274)? The results of the Kolmogorov-Smirnov test ($p = .000 \leq .05$) and a visual inspection of their histogram (Figure 5.11), Normal Q-Q plots (Figure 5.12) and box plots (Figure 5.13), showed that the exam score were not normal distrusted for Atox Mill 'pression variables, with a skewness -4.136 ($Sz = -172.33$) and kurtosis of 23.24 ($Kz = 474.34$).

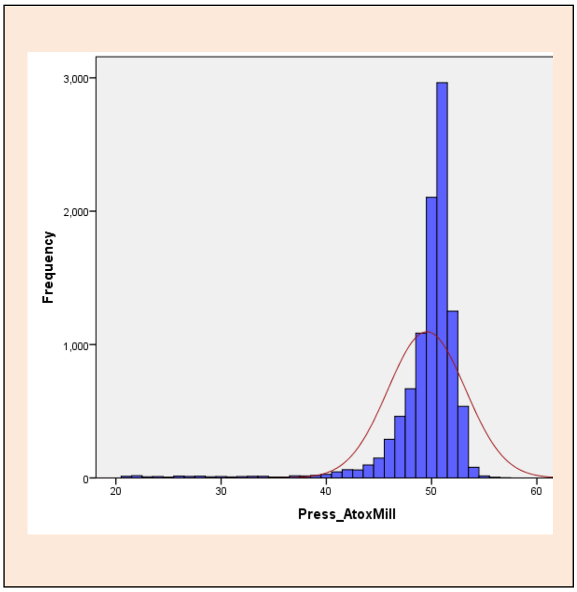


Fig 5.11- Edited Histogram of Atox' pression with normality plot (M=33.29, SD=23.274).

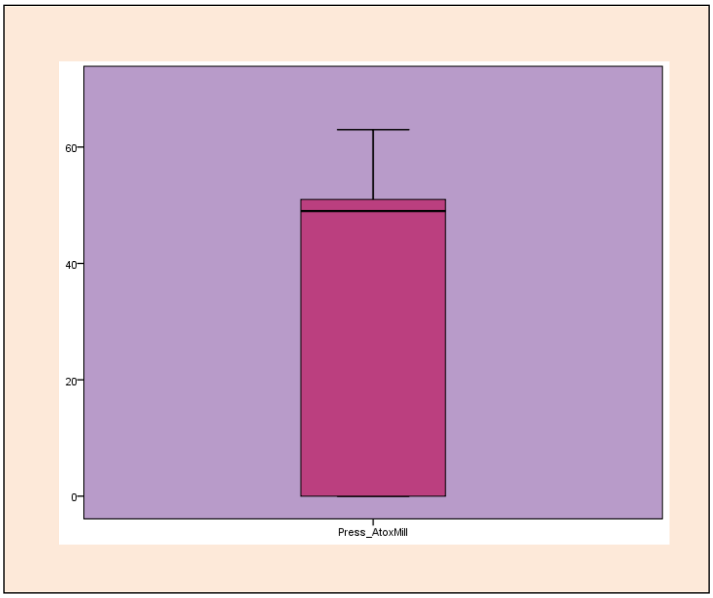


Fig 5.12- Boxplot shows features statistical of Atox'pression.

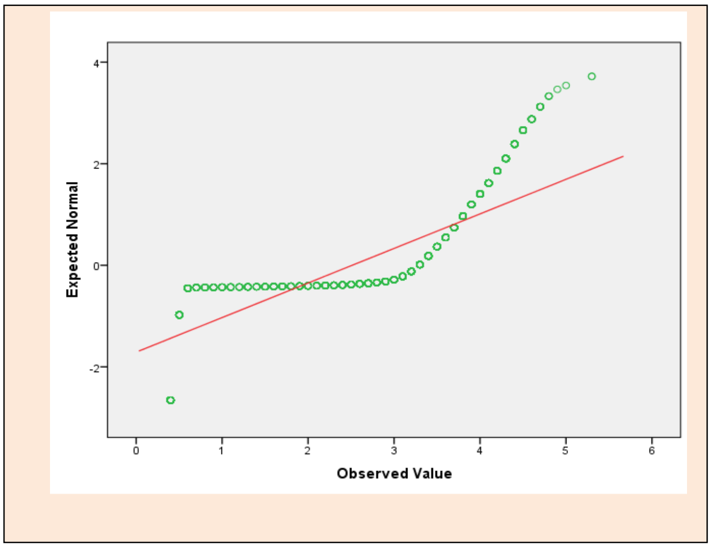


Fig 5.13- Normality Q-Q plot of Atox Mill 'pression.

Testing if particle 'size variable contains normally distributed data (mean=12.73, SD=.759)? The results of the Kolmogorov-Smirnov test ($p = .000 \leq .05$) and a visual inspection of their histogram (Figure 5.14), Normal Q-Q plots (Figure 5.15) and box plots (Figure 5.16), showed that the exam score were not normal distrusted for particle 'size variables, with a skewness 2.650 ($Sz = 106$) and kurtosis of 23.24 ($Kz = 290.77$).

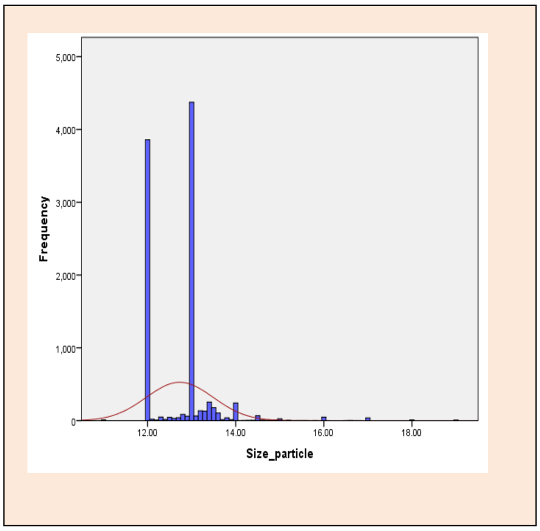


Fig 5.14- Edited Histogram of with normality plot (M=12.722, SD=.855).

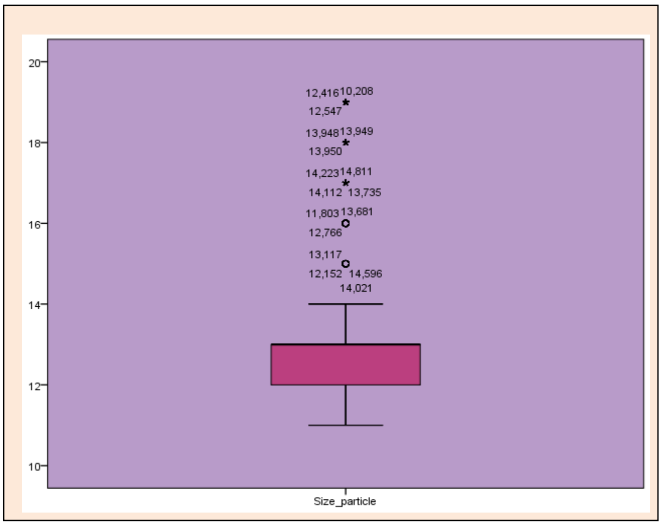


Fig 5.15- Boxplot shows features statistical of particle'size

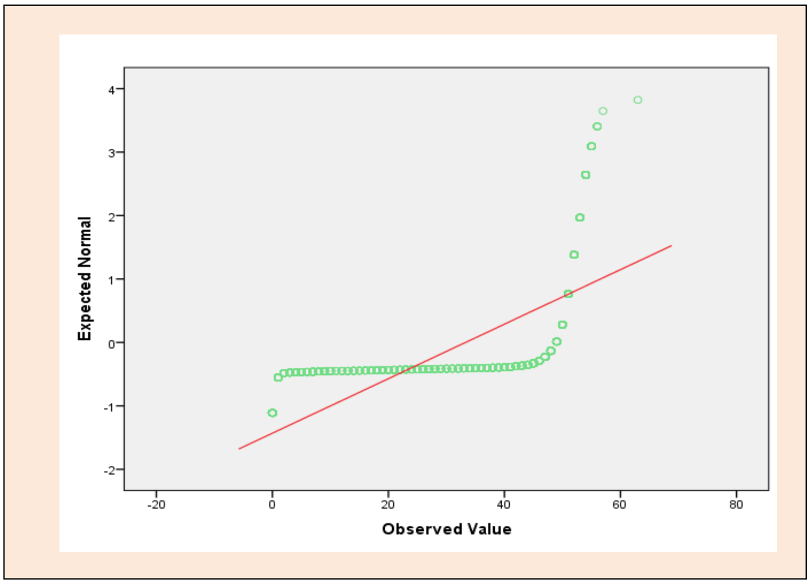


Fig 5.16- Normality Q-Q plot of particle 'size.

Testing if Vibration Atox Mill variable contains normally distributed data (mean=2.512, SD=1.469)? The results of the Kolmogorov-Smirnov test ($p = .000 \leq .05$) and a visual inspection of their histogram (Figure 5.17), Normal Q-Q plots (Figure 5.18) and box plots (Figure 5.19), showed that the exam score were not normal distrusted for vibration Atox Mill variable, with a skewness -0.530 ($Sz = -26.5$) and kurtosis of -1.524 ($Kz = -38.1$).

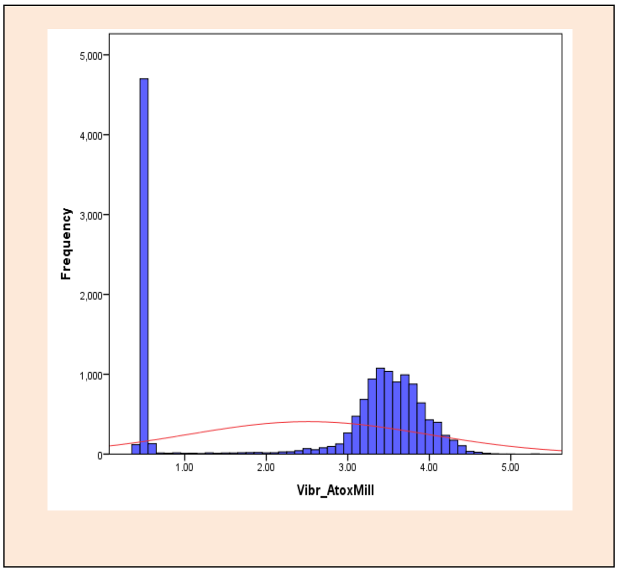


Fig 5.17- Edited Histogram of Atox Mill's vibration with normality plot (M=2.512, SD=1.469).

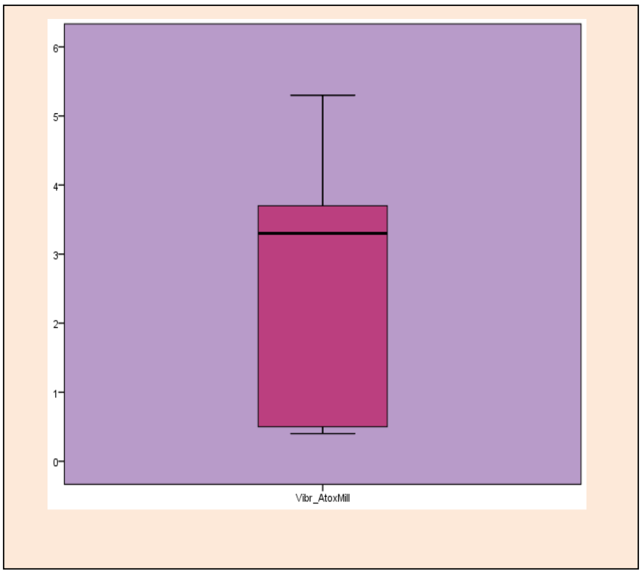


Fig 5.18- Boxplot shows features statistical of vibration

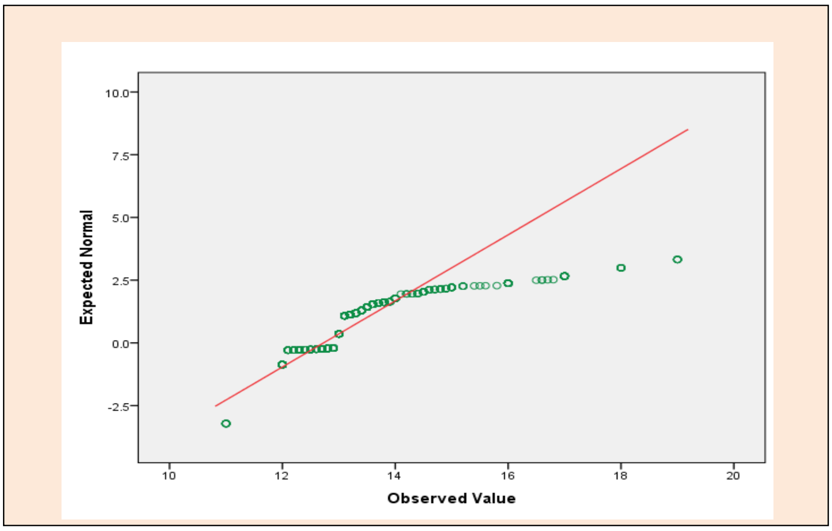


Fig 5.19- Normality Q-Q plot of Atox Mill 'vibration.

In summary, the five variables that are involving in this normality hypothesis test are not normal distributed. In this study, the null hypothesis states that these variables are normally distributed, against the alternative hypothesis that it is not normally distributed with significant level *p-value*=.05. In this study, the value of Sig (*p-value*) found smaller than significant level (*p-value*), so can reject the null hypothesis and conclude the data are not from a population with normal distribution.It means those variables are not normal distributed.

Table-5.2- Kolmogorov-Smirnov test (*p-value*).

Tests of Normality

	Kolmogorov-Smirnov[a]		
	Statistic	df	Sig.
Temp_filter	.191	15159	.000
Vibr_Atox Mill	.258	15159	.000
Power_Motor	.222	10096	.000
Press_Atox Mill	.237	10121	.000
Size_particle	.218	10047	.000

a. Lilliefors Significance Correction

SPSS output gives p-value (significance)

5.4.3 Reporting results of correlation analysis and dependences

In statistic field, correlation methods can be used to determine whether there is relationship between two or more variables, and its makes inference about the strength of the relationship. In fact, correlations between variables can be measured with the use of different coefficients such as: Pearson's coefficient, Spearman's rho coefficient, and Kendall's tau coefficient. These correlation coefficient is the most widely used, it measures the strength or weakness of the relationship between variables. Selected an appropriate correlation coefficient is based on result of normality test, So, when the variables are not normally distributed or the relationship between the variables is not linear, it may be more appropriate to use the Spearman rank correlation method instead Pearson's correlation method.

As mentioned before, the first step of the data analysis was to check the normality distribution of the compared data. Overall, the results indicate that distribution of the data was not normal distributed. The second step was to check whether there exist or not relationship between variables. Spearman's rank correlation coefficient (non-parametric) rank statistic was selected to test the correlation of the compared variables (measure of the strength of the association between two variables).

5.4.3.1 Correlation and dependences among variables

In this section, the analysis and correlation of Atox Mill'variables have been carried out. Based on previous section, Non parametric correlation analysis the association between variables are appropriate, In this case we should use Spearman's rho for correlation analysis (non-parametric method). Indeed, Spearman's rank correlation coefficients were computed to know the strength and association between variables. It should be noted that, scatter plot might uses to identify the type of association between the variables, the SPSS output gives us correlation coefficient, sig (*p-value*). The correlation coefficient indicated by (r_s) can take value from -1 to +1, +1 indicates a perfect positive linear relationship, and -1 indicates a perfect negative linear relationship. Zero value indicates the variables are uncorrelated and there is no linear relationship. The closer r_s is to zero, the weaker the association between the variables. Negative correlation coefficient refer to that the two variables move in the opposite direction from each other - as one goes up, the other goes down, however, the two variables move in the same direction.

In this study, the null hypothesis states is defined by "there is no correlation between two variables", against "the alternative hypothesis is there a correlation", at the 5% level of significance (*p*-value is .05). The correlation is considered significant when the *p-value* lower then .05 (null hypothesis were true), otherwise we don't reject the null hypothesis.

The general form of a null hypothesis (H_0) and alternative hypothesis (H_1) for a Spearman correlation is:

H_0: there is no correlation between the two variables in the population.

H_1: There is correlation between two variables in the population.

The significance test is investigating whether the null hypothesis was true or false. However, it is important to realize that the visual interpretation of scatter plots, Spearman correlation coefficients, is necessary in order to identify the strength that might exists between two variables. The value of Spearman correlation coefficients as well as the significant effect are shown in table 5.3.

Table 5.3- Non parametric correlation matrix among five variables from Atox Mill condition data.

Correlations

			Temp_filter	Vibr_AtoxMill	Power_Motor	Press_AtoxMill	Size_particle
Spearman's rho	Temp_filter	Correlation Coefficient	1.000	-.218**	-.157**	-.076**	-.023*
		Sig. (2-tailed)	.	.000	.000	.000	.023
		N	15159	15159	10096	10121	10047
	Vibr_AtoxMill	Correlation Coefficient	-.218**	1.000	.126**	.103**	.146**
		Sig. (2-tailed)	.000	.	.000	.000	.000
		N	15159	15159	10096	10121	10047
	Power_Motor	Correlation Coefficient	-.157**	.126**	1.000	.040**	-.038**
		Sig. (2-tailed)	.000	.000	.	.000	.000
		N	10096	10096	10096	10050	10027
	Press_AtoxMill	Correlation Coefficient	-.076**	.103**	.040**	1.000	.061**
		Sig. (2-tailed)	.000	.000	.000	.	.000
		N	10121	10121	10050	10121	10005
	Size_particle	Correlation Coefficient	-.023*	.146**	-.038**	.061**	1.000
		Sig. (2-tailed)	.023	.000	.000	.000	.
		N	10047	10047	10027	10005	10047

**. Correlation is significant at the 0.01 level (2-tailed).

*. Correlation is significant at the 0.05 level (2-tailed).

Table 5.2 shows the spearman's correlation coefficients to assess the relationship between the temperature of Atox Mill and other variables such as: power of motor, vibration, pression, size of particle, and their Sig (*p-value*). From table 5.2, it is possible to see that there is no correlation between temperature of Atox Mill and the following variables: power motor (r_s=.-157, Sig=.000), vibration (r_s=.-218, Sig=.000), pression (r_s=-.076, Sig=.000), and size of particle (r_s=-.023, Sig=.023), regarding the strength of the relationship this is a zero correlation. Since the *p-value* less then significant level (=.05), we reject the null hypothesis as well as the scatterplot showed that there is no linear relationship between temperature variables and others variables (Figure. 5.20).

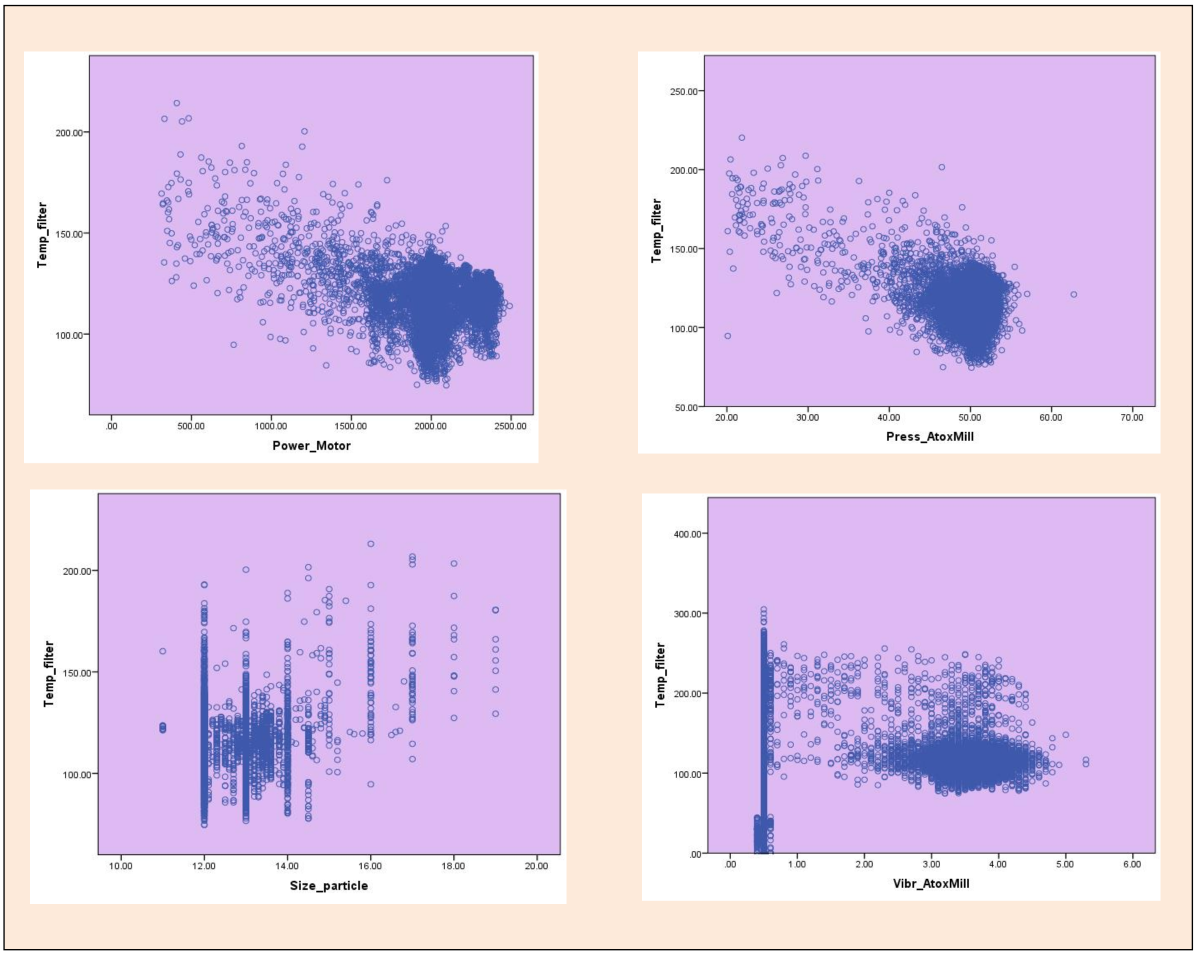


Figure.5.20- Scatterplot of filter ‘temperature against others variables

For the power of motor ‘variable, the results of analyses showed no significant correlations between the power of motor and temperature (*p-value*=.000), vibration (*p-value*= .000), pression (*p-value*=.000), and size of particle (*p-value*= .000). Regarding the correlation, there is no significant correlation occurs between power of motor and other variables: temperature (r=-0.157), vibration (r_s= .126), pression (r_s=.040), and size of particle (*p-value= -.038*), it should be noted that, in this case we don’t reject the null hypothesis. Thus, scatterplot depicted that there is no linear relationship between power of motor ‘variables and others variables (Figure 5.21).

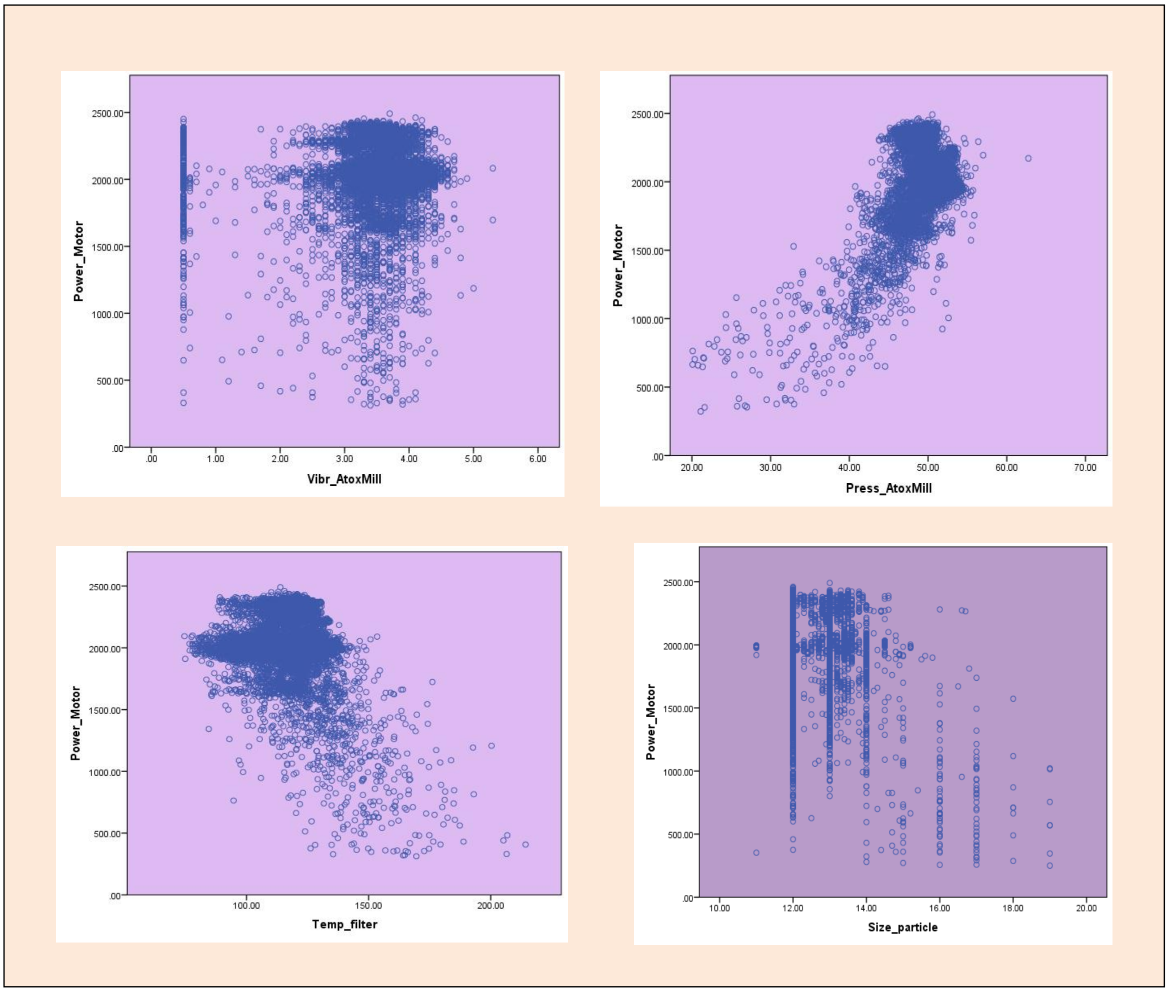


Figure. 5.21- Scatterplot of motor 'power against others variables

In the analyses on rejecting or accepting the null hypothesis regarding correlation between vibration 'variable and others variables, the results showed that there is near zero correlation between vibration and the following variables: temperature (r_s=-.218), power of motor (r_s=.126), pression (r_s=.103). However, there is positive correlation between size of particle and vibration (r_s=.146, *p-value*=.000), the two variables that move in the same direction, but regarding the strength of the relationship this is a weak correlation, a visual inspection of scatter plot illustrate that the two variables move in the opposite direction from each other as one goes up, the other goes down, but regarding the strength of the relationship this is a weak correlation. So, there is no relationship between vibrations and other variables expect size of particle, its weak relationship (Figure 5.22). It is possible to see in table 5.2 that, there is no significant correlation between vibration and all compared variables *(p-value*< .05).

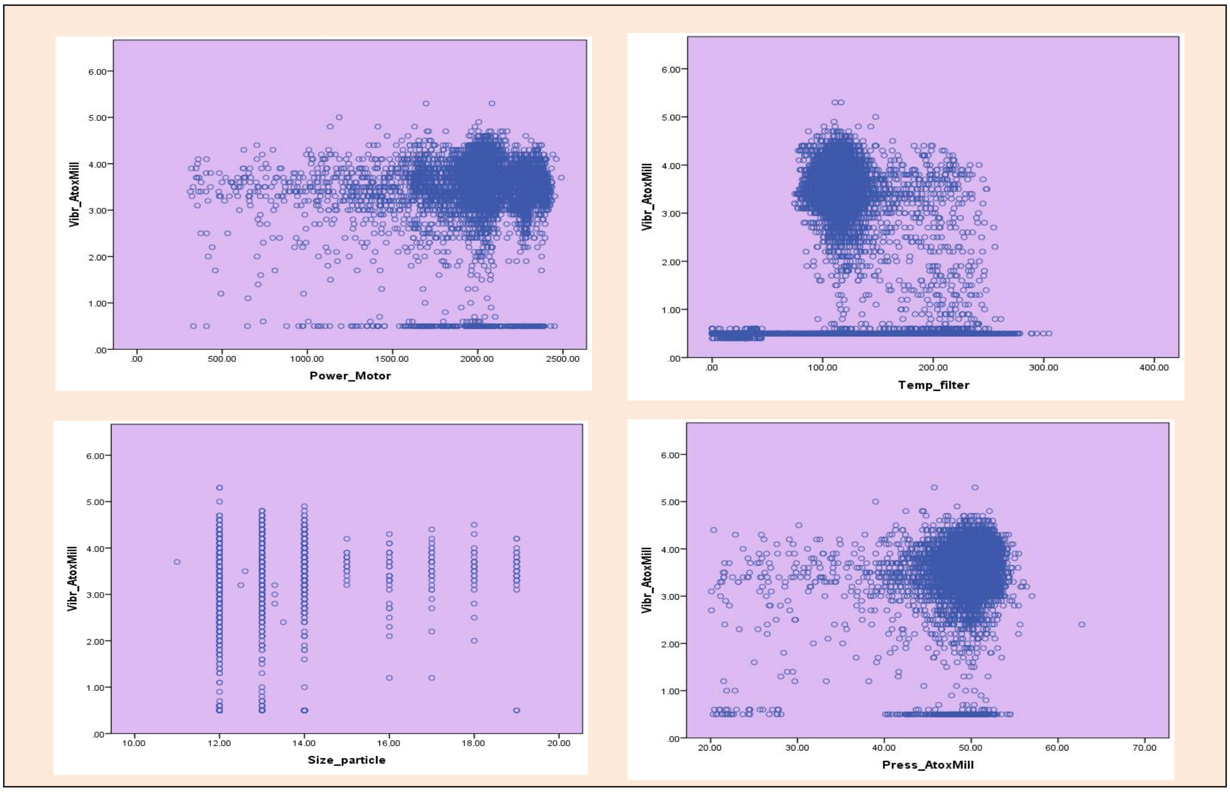


Figure 5.22- Scatterplot of vibration against others variables.

In the correlation analysis of pression and compared variables, a Pearson correlation coefficient was computed to assess the relationship between the pression and the following: temperature, power, vibration, and size of particle. There was a near zero correlation between the pression and the following variables: the temperature (r_s=-.076, *p-value*=.000), power of motor ($r_{s=}$*.040*, *p-value*=.007), vibration ($r_{s=}$*.103*, *p-value*=.000), size of particle (r_s=.061, *p-value*=.000). A scatterplot summarizes the results (Figure5.23). Overall, there was no correlation between pression and other variables (Figure5.23).

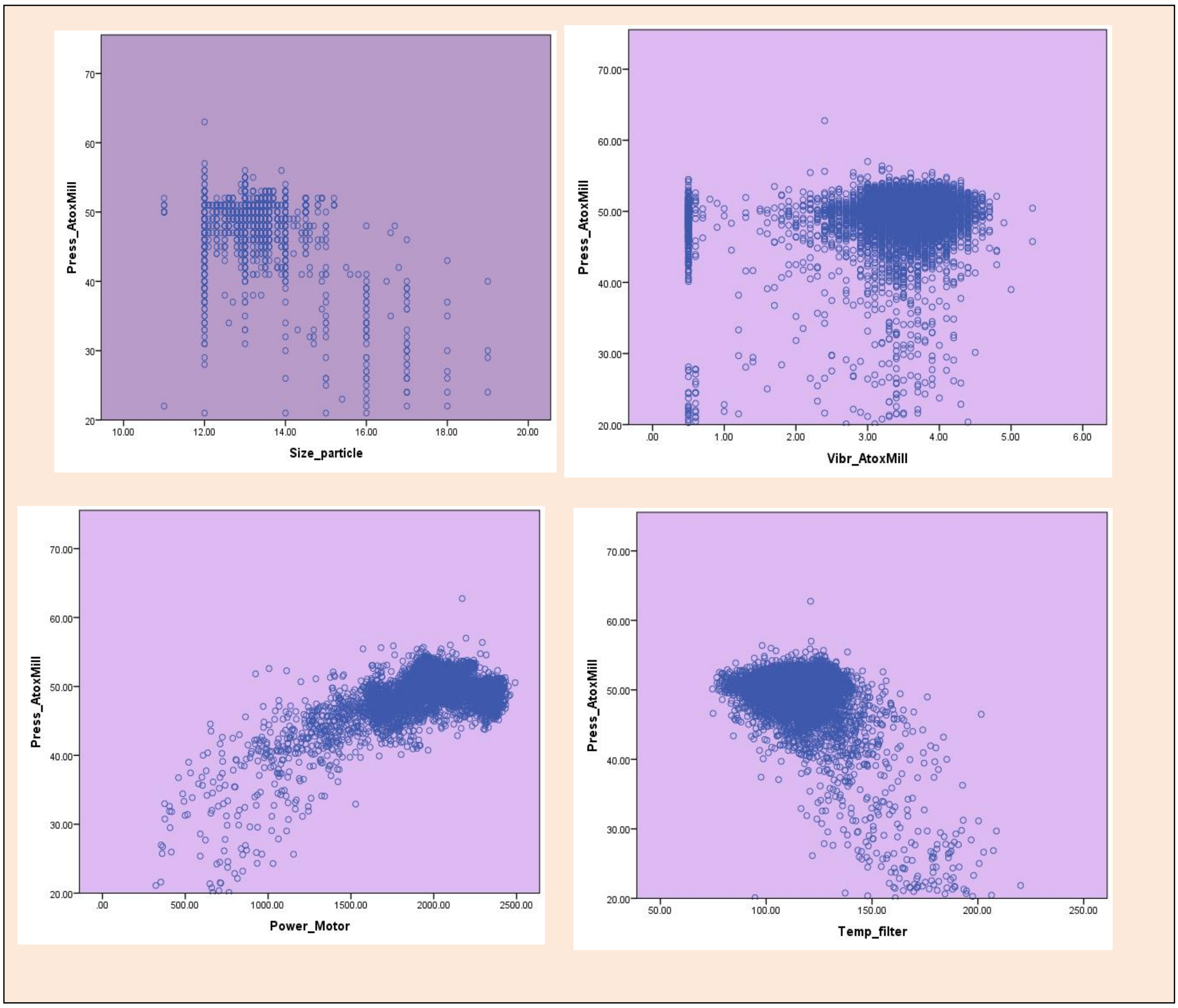


Figure 5.23- Scatterplot of pression' variable against others variables.

As shown in table 5.2, we can see there was not significant correlation between size of particle and the following variables: temperature (r_s=-.023, *p-value*=.000), power of motor (r_s=-.038, *p-value*=.005), pression (r_s=.061, *p-value*=.000), However, there is positive correlation between size of particle and vibration (r_s=0.146, *p-value*=.000), the two variables that move in the same direction, but regarding the strength of the relationship this is a weak correlation. Overall, there was no correlation between size particle and other variables, which was not statistically significant (Figure 5.24).

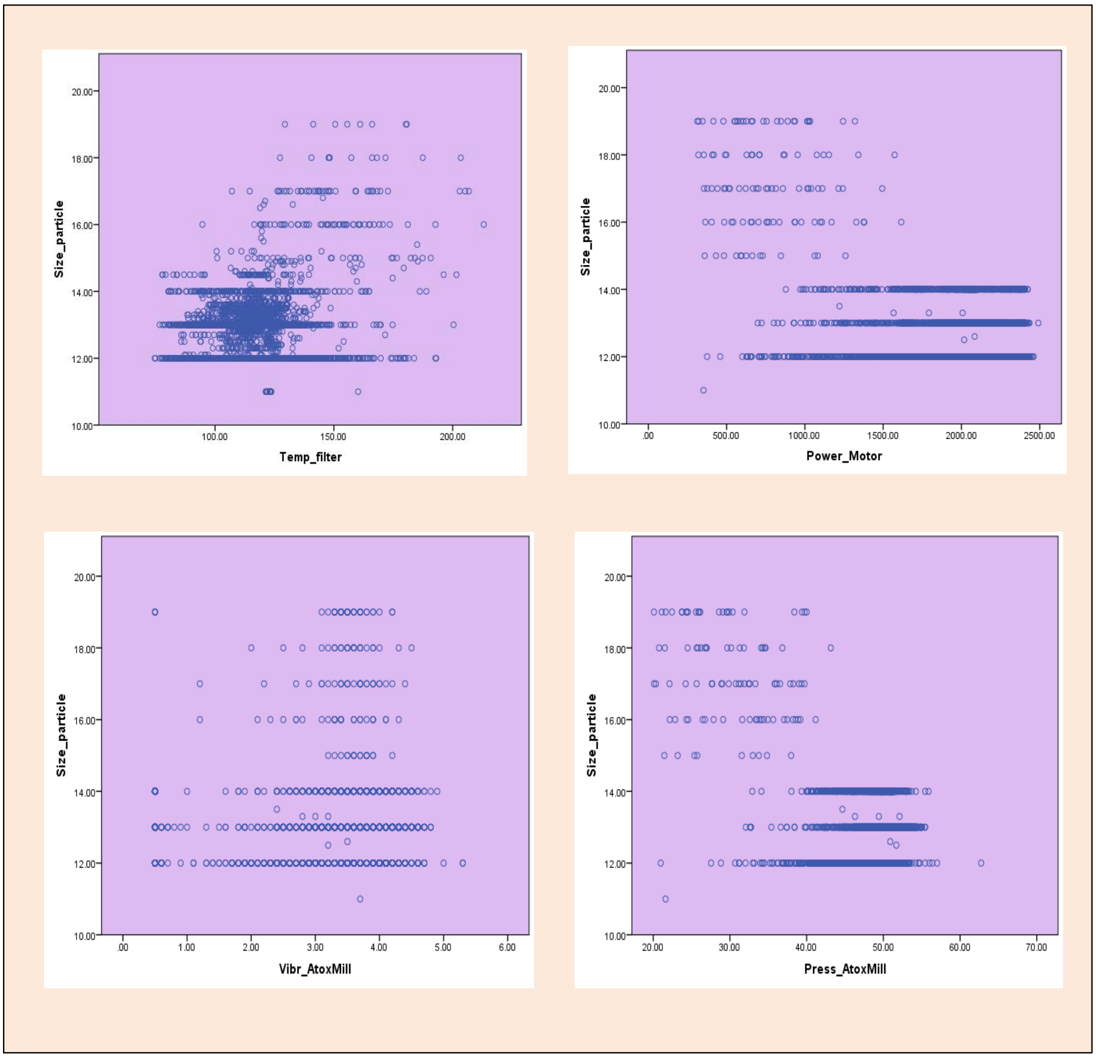


Figure 5.24- Scatterplot of particle' size against other variables

Analysis of the results of bivariate correlations among five variables, showed that the value of Sig found less than significant level (*p-value*=.05), all of the five compared dimensions are not significantly correlated ($p < 0.05$), from table 5.3 and scatter plots of bivariate correlations, we can tell that there are not significant correlation between the variables as well as no substantial correlation between the variables, for this reason we found that the null hypothesis were true and we can't reject it, as results there is no relationship between among variables. It means those variables are independent.

5.5 Principal component analysis for identifying the variability in data

In this section, principal component analysis (PCA) was conducted to identify the variability of data distribution for each variables and to determine which one has significant variance in data. Besides, statistical tests were also used to know whether there are any significant. For these purposes, PCA packaging in SPSS were used in this study. The Kolmogorov-Smirnov's normality test was used for this purpose, correlation matrix, scatterplot, component matrix.

The idea behind this method is that can help us to restructure our data, the way that the 'information contained' is measured is by considering the variability within and co-variation across variables, that is the variance and co-variance (i.e. correlation). Thus, we obtain a set of factors which summarize, as well as possible, the information available in the data. The advantageous of this statistic methods is to simply discover the linear combinations that reflect the most variation in the data. Secondly to discover if the original variables are organized in a particular way reflecting another a 'latent variable', thirdly we might want to confirm a belief about how the original variables are organized in a particular way. In another word, Results of PCA analysis for our data set are as follow:

From table 5.4, the absolute value of Kaiser-Meyer-Olkin Measure of Sampling Adequacy (KMO) is higher and classified as middling (.734), with significant level p-value less than .05, indicate we can procced the next step of PCA, so the PCA is really useful.

Table 5.4. KMO and Bartlett's test

Kaiser-Meyer-Olkin Measure of Sampling Adequacy.		.734
Bartlett's Test of Sphericity	Approx. Chi-Square	9268.310
	df	10
	Sig.	.000

From table 5.5 and figure 5.25, we can determine how many important components are present in the data. Indeed, the next table 5.5 shows the importance of each of the five principal components. Only the first two have eigen values over 1.00, and together these explain over 65.7% of the total variability in the data. The table presents the eigen values and percentage of variance. This decision is supported by the scree splot in figure 5.25.

Table 5.5. Total variance explained.

Total Variance Explained

Component	Initial Eigenvalues			Extraction Sums of Squared Loadings			Rotation Sums of Squared Loadings		
	Total	% of Variance	Cumulative %	Total	% of Variance	Cumulative %	Total	% of Variance	Cumulative %
1	2.284	45.672	45.672	2.284	45.672	45.672	2.114	42.283	42.283
2	1.002	20.032	65.704	1.002	20.032	65.704	1.171	23.421	65.704
3	.752	15.040	80.744						
4	.524	10.490	91.233						
5	.438	8.767	100.000						

Extraction Method: Principal Component Analysis.

The information on the table 5.5 can be visually seen on the figure 5.25 that we have to retain only two components, as shown. It can see inflection on the plot in the point two, as results two component should be retained.

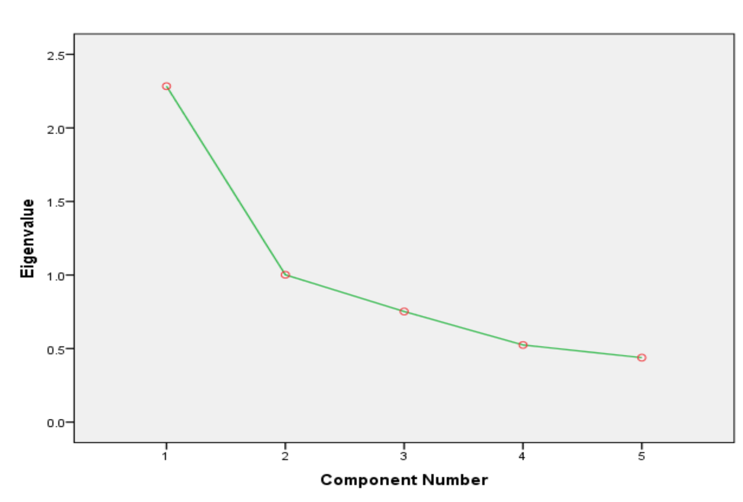


Figure 5.25- Scatterplot of Eignvalue versus component number.

Table 5.6 show the expected pattern, with high positive and high negative loadings on the first component.

Table 5.6. Component Matrix

	Component 1	Component 2
Temp_filter	-.578	-.312
Vibr_Atox Mill	.717	.344
Power_Motor	.816	-.021
Press_Atox Mill	.801	-.121
Size_particle	-.356	.878

Extraction Method: Principal Component Analysis.

a. 2 components extracted.

5.6 Weibull analysis for verifying Markov chain assumption with non-censored data

Generally, there are different approaches to model the transition probabilities. In this section, we are going to selecting which an appropriate stochastic processes used for degradation model of Atox Mill equipment. In this context, Weibull analysis is used for verifying whether the Markov property hold for the Atox Mill condition data with control quality measurement or not.

In order to test the Markov property we need to verify if the time to failure follow exponential distribution or not. As noticed in previous, Markov property (memoryless property)refer to that the transition probability from the present to future state don't depend on past states. The test based on the analysis of Weibull distribution for non-censoring data, the last means that we do know the exact time of an event (failed state), it is considered the most useful density distribution in reliability calculation. The distribution of Weibull is often used in the field of lifetime analysis, because of its flexibility as well as due the broad range of distribution shape that are included in Weibull distribution (e.g., Exponential, Rayleigh, Poisson &Binomial).The log Normal is not a member of the Weibull family. As stated above, the advantage of Weibull distribution is flexibility feature, for that it has become widely used in many application regardless the areas (Weather Forecasting, Extreme Value Theory, General Insurance, Survival Analysis, Hydrology, reliability engineering, Statistics Classes, Health for detecting cancer disease).

The formula for the probability density function of the general Weibull distribution is defined by:

$$f(x,a,b,c) = \begin{cases} \mathrm{b}a^{-1}(x-c)^{b-1}\, e^{-(\frac{x-c}{a})^{b}} & If\ x \in (0,\infty) \\ 0 & \text{Otherwise,} \end{cases} \qquad (5.1).$$

Where a is the scale parameter, b is the shape parameter and c is the location parameter. The case where b = 0 and c = 1 is called the standard Weibull distribution. In this section we consider c = 0, so the Weibull law is called the 2parameter Weibull distribution. The equation for the standard Weibull distribution reduces to:

$$g(x,a,b,0) = \begin{cases} \mathrm{b}a^{-1}(x)^{b-1}\, e^{-(\frac{x}{a})^{b}} & If\ x \in (0,\infty). \\ 0 & \text{Otherwise,} \end{cases} \qquad (5.2).$$

The Weibull distribution can be used in wide variety of situation and, dependent on the value of shape parameters a, the last can be used to approximate different distribution (Table 5.7).

Table 5.7- Weibull distribution approximation.

Shape parameters	**Approximate probability distribution**
b=1	Weibull distribution is identical to the exponential distribution
b=2	Weibull distribution is identical to the Rayleight distribution
b=2.5	Weibull distribution approximates the Lognormal distribution
b=3.6	Weibull distribution approximates the normal distribution
b=5	Weibull distribution approximates the peaked normal distribution

There are two ways adjustment of Weibull law: the first one is called graphical method, and the second one is known as numerical method such as: Least Squares Method, Maximum Likelihood Estimator. According to the accuracy and computing time The methods are used will be used to checking the Markov property is graphical method consist Weibull probability plot, the idea is based on equation of a straight line by plot F (t) versus t, where Ln((ln (1/(1-F(t))) function the two parameters of Weibull law and generally used Benard's equation (Median Rank) =((i-0.3)/(N+0.4)) to estimate the F(t) where N equal to 235 (number of failed state) and *i* represents failure order. The slope of equation parameter determines which member of the family of Weibull failure distributions best fits to data.

From table 5.1, when a=1 the failure time follow an exponential distribution otherwise, the process has not Markov property and we cannot modeled by Markov chain. To choose the right stochastic process in this maintenance application is very important, because this will give the accurate information about the transition probabilities according to our data. The shape parameters of Weibull distribution is equal to slope of the best straight line fit to the plotted data. From figure 5.26, we see that the shape parameters *a* has value 1.2408, which result different from 1, as results of testing Markov property based on graphical methods we may conclude that the process has not Markov property, for this reason we will model the transition stated used the delay time concept.

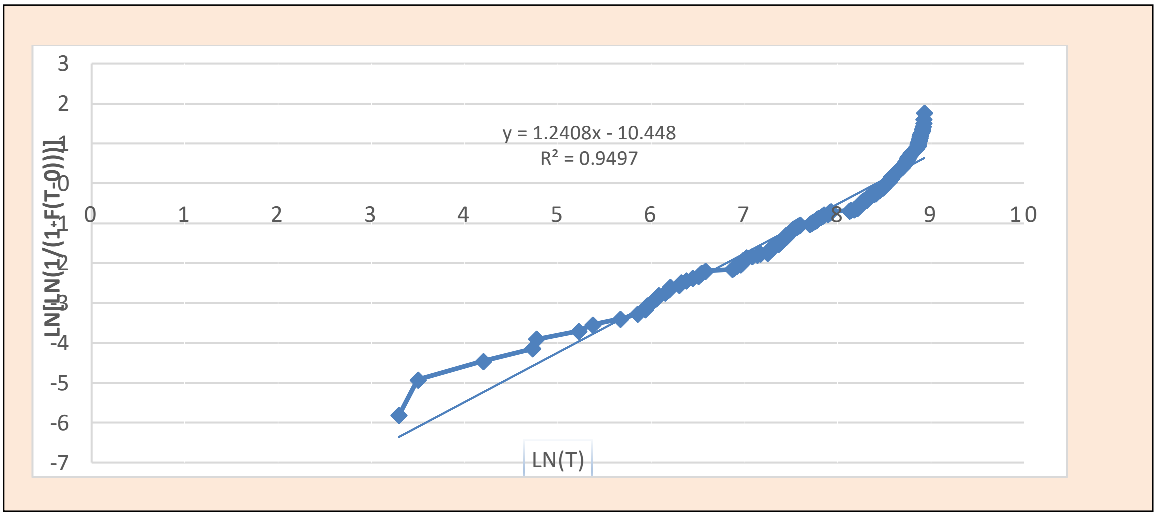


Figure 5.26- Probability plot of complete data.

5.7 Optimal Multivariate Bayesian control chart for real data

As result output fit distribution command in Matlab software, the probability distribution fit with observed data monitoring: temperature (C°), vibration(mm/s), pression (mbar), power of motor (KW/h) approximate to Weibull distribution which describes our data well with three parameters, these parameters include a, b, c. Besides, fit distribution in Matlab found that the empirical distribution describes well our data about size particle data. Indeed, Maximum likelihood method is often used for parameter estimation, it is considered as the most popular method according to their accuracy and precision. We can find widely applicable approximations for a number of useful probability distributions. In this context, the parameters were estimated by Maximum likelihood estimation which already implanted in Matlab. The goal is to estimate the following parametric vectors a, b, c for $p(y_i^k / x_i)$, where k is number of observed variable. The value of parameters vectors are presented in Table 5.8.

Table 5.8. Weibull distribution parameters.

		a	b	c
x_i =1	$k = 1$ (C°)	47.5325	3.8438	73.7108
	$k = 2$(mm/s)	2.8319	7.3618	0.8764
	$k = 3$ (mbar)	3842+e0.3	20.8422	-1739e+0.3
	$k = 4$ (KW/h)	36.0530	19.0481	14.8017
x_i =2	$k = 1$ (C°)	60.6202	2.2548	73.5773
	$k = 2$(mm/s)	6.9494	7.8453	-3.5999
	$k = 3$ (mbar)	4.1499e+0.3	8.1779	-2.3876+0.3
	$k = 4$ (KW/h)	86.4524	10.4453	-39.9999

Considering limited space, only Weibull probability distribution for temperature and pression were presented in Figure 5.27, other plot we can see them in annex2.

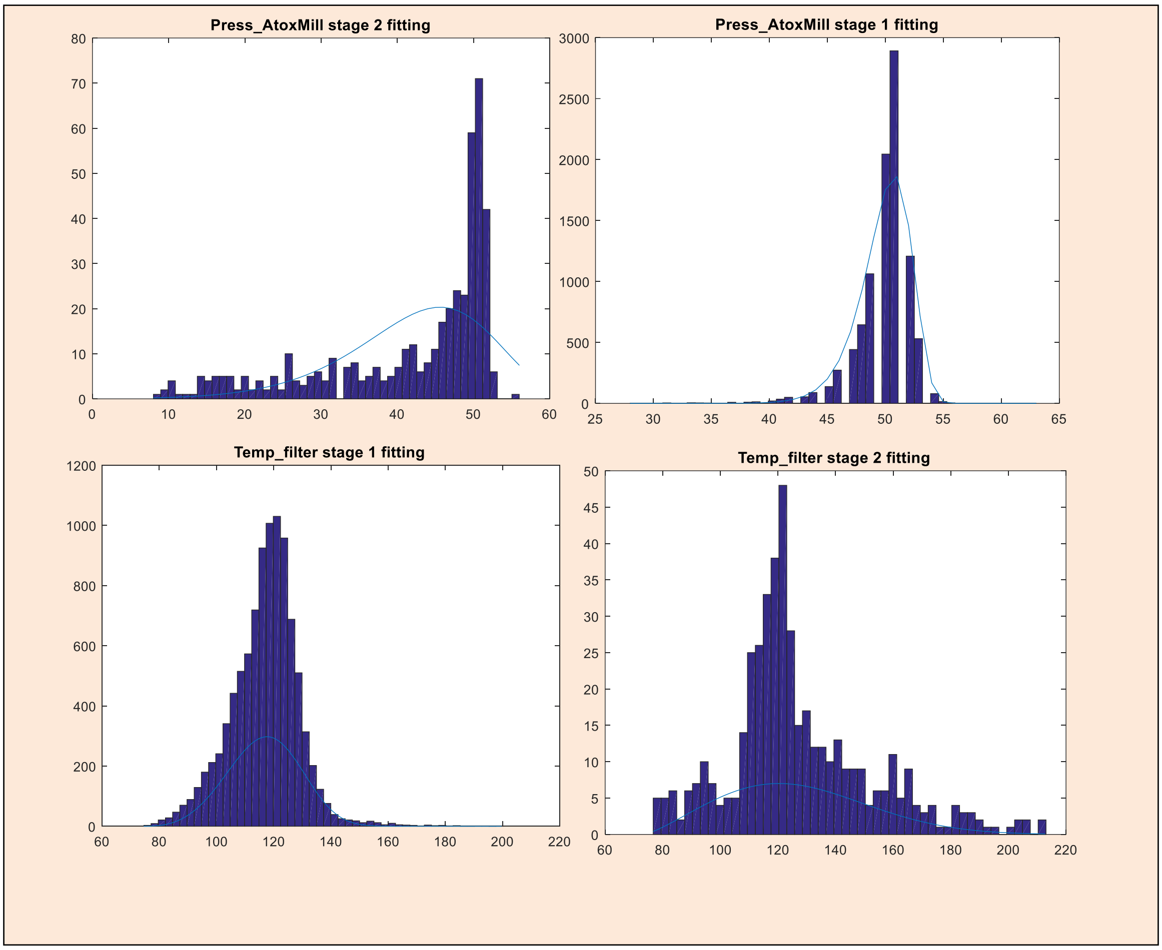


Figure 5.27- Weibull distribution plot of observed data (temperature and pression) in tow failure stages at sample epoch =30 min.

As said before, posterior probability expression contains transition probabilities, for that it is mandatory to compute those probabilities. Weibull probability density function of two failure stages fit well with distribution of time in the first stage and the second stage. The parameters of Weibull distribution of both stage l_1 and l_2 are presented in table below:

Table 5.9. Delay time concept parameters.

	a	b	c
Stage 1(L_1): 1→2	20.2151	1.0275	0.4974
Stage 2(L_2): 2→3	0.09239	0.2305	0.4999

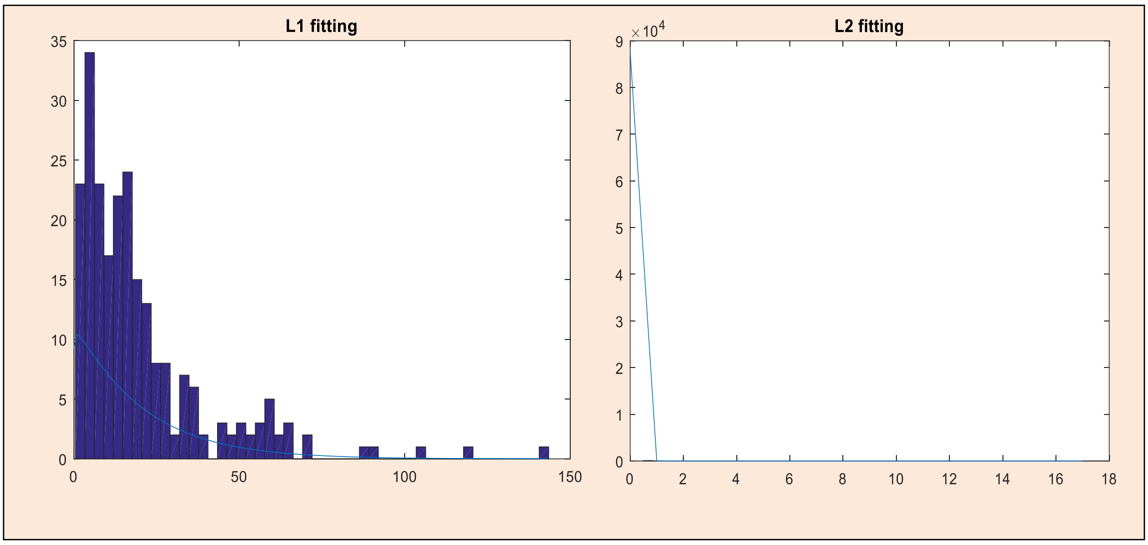


Figure 5.28- Weibull plot distribution of the historical data for stage 1 and stage 2.

Numerical results have been presented to illustrate the effectiveness of integrated model based simultaneously data quality control measurement and condition equipment Atox Mill data set. In this section we consider two control limit policy: the first case when only condition parameter Atox Mill were taking into account for estimating conditional probability of system being on Stage 1 and Stage 2 (without quality control measurement), the second case, we consider the quality control measurement in estimating conditional probability of system being on Stage 1 and Stage 2. We use the same observed data about condition monitoring of Atox Mill in the previous case, additional the quality control measurement, to illustrate the two control limit policy proposed, therefore, we evaluate the objective function and obtain the corresponding optimal $h*$ and $p*$ for both situation as shown in Table 5.9, Table 5.10. We Run PSwarm solver implementing in Matlab for finding optimal $h*$ and $p*$ using the following convergence criteria:We assume that the ranges for interval monitoring *(h)*=[0.05, 10], The ranges for upper control ($p*$)=[0.01, 0.99]. For both cases, we consider the same convergence criteria. Solving the expected average cost it difficult analytically (Eq 4.19) for this reason Pswarm and Monte Carlo simulation can be used to iteratively determine the objective function and the decision variables, we assume also that C_{monit}=1000, C_{inspe}=100, $C_{prevmain}$ =10000, C_{failed}= 6000000. It is important to note that PSO can find the optimum for both cases.

Considering the first case without quality measurement, the number of simulation equal to 3000 renewal cycle and as suggested by Hu and Eberhart (2001) by the best number of population size in PSO is about 40. In this case, the simulation consume about Elapsed time is about 86.84 (1.45 minutes)

The following table summarize the PSO results (maintenance cost, interval monitoring and upper control limit.

Table 5.10. Summarize of PSO results in case 1

(Global, size population=40, 3000 iteration=54).

Case 1	
Optimal h*	5.449
Optimal p*	0.67028
Optimal Maintenance cost	272.4543

Figure 5.29. Shows that for a given monitoring interval, the posterior probability in stage 1 decrease, in the following approximate time unit: t_i=4 time unit, t_i=16 time unit, But at time around t_i=25 time unitthe sytem shift state to state 2 where $P(x_i = 1/Y) = 0$and the posterior probability of being in stage 2 given observed data decrease at the same times and egual to 1 $(P(x_i = 1/Y) = 1)$.

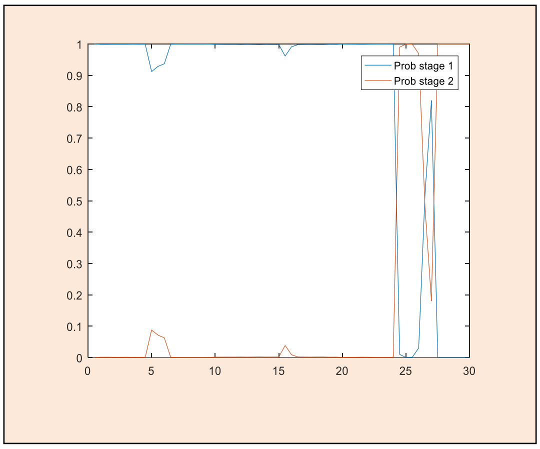


Figure 5.29- Posterior probability for stage 1 and stage 2 (case1).

In order to display observed data given monitored history up to time $t_i(Y_i^k)$ in a meaningful way, and to be plotted all these variable in the same plot requires divided all these variables by scale, we assume that temperature/100, vibration72, power/100, pression/4. Quality/1.

Figure 5.30. Showsthat for a given monitoring interval observed data about variables (temp, vibrat, power,press). It can see clearly from figure that $y_i^1, y_i^2, y_i^3, y_i^4$ from 25 time unit. However,

at 5 time unit and around 15 time unit we can see clear that only pression and vibration deacresing. This mean that those two variables affect strongly by the shift of stage form 1 to 2.

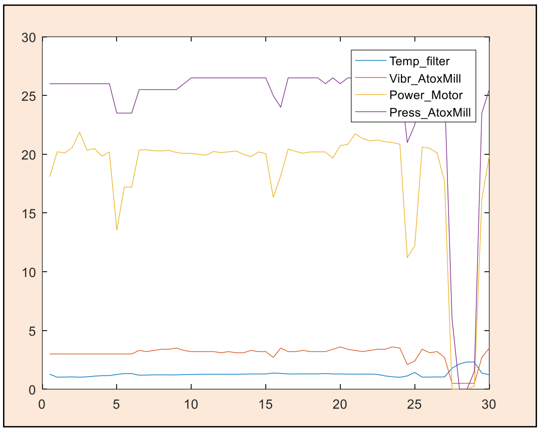


Figure 5.30- Observed data for a given interval monitoring (case1).

Now we simulate the same objective function for the case 2 where the quality control measurement is considered. The same cost and convergence criteria, the results become as follow:

Table 5.11. Summarize of PSO results in case 2
(Global, size population=40, 3000 iteration=76).

Case 1	
Optimal h*	5.3862
Optimal p*	0.03127
Optimal Maintenance cost	268.2472

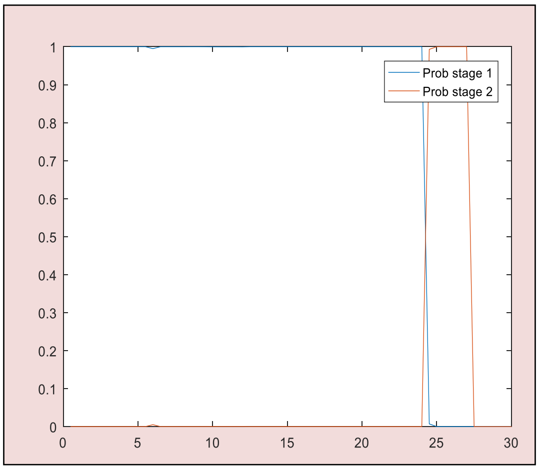


Figure 5.31-Posterior probability for stage 1 and stage 2 (case2).

Figure 5.31. illustrate that for a given monitoring interval, the posterior probability in stage 1 decrease, around t_i=25 time unit the system shift state 1 to state 2 where $P(x_i = 1/Y) = 0$ and the posterior probability of being in stage 2 given observed data decrease around t_i=25 time unit and equal to 1 ($P(x_i = P(x_i = 2/Y) = 1$)

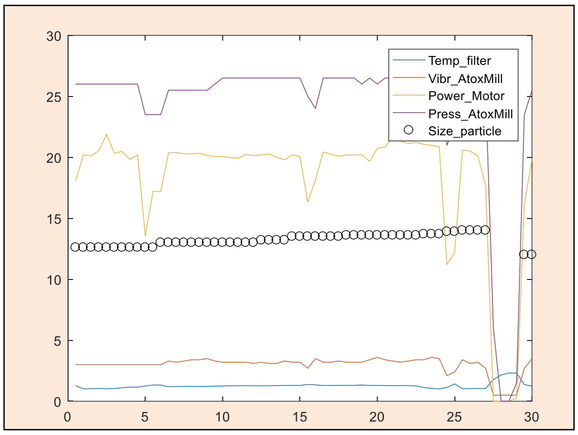


Figure 5.32- Observed data for a given interval monitoring (case2).

Figure 5.32-Shows that for a given monitoring interval observed data about variables (temp, vibrat, power, press) with presence of quality control measurement. It can see clearly from figure that around 25 time unit, the following variables decreasing: power motor, pression, vibration, we can see clear that we have poor quality refer to size particle deceasing and pression decreasing suddenly. However, the temperature increasing. This mean that those variables affect strongly by the shift of stage form 1 to 2.

From table 5.10 and table 5.11, we can see that the maintenance policy in the second case 2 where control quality measurement control taking into account yield the lower expected cost. In addition, the interval monitoring in the case 2 yield lower value than the first second case. The cost and interval monitoring have to be considered the two quantities in comparing whether the maintenance policy is effectiveness. However, sometimes there is tradeoff between these two quantities. As results, this study found that multivariate Bayesian when considering quality control measurement perform better than the previous Multivariate Bayesian control chart, and the maintenance cost was quite equal but the interval monitoring is higher for the proposed model. However, due to insufficient research in multivariate Bayesian control chart needs to improve the proposed model with another practical application.

Chapter 6

Conclusion and future research

This chapter presents the conclusions about the work performed. It presents guidelines for future work and research in order to expand and solidify knowledge about Condition based maintenance model.

Contents

6.1 Focus of the work and original contribution

In maintenance management, decision-makers are facing though challenges to conduct an appropriate and an accurate decision. A proper and well-performed CBM models are beneficial for maintenance decision making. In order to respond to this challenge, Condition based maintenance need to be integrated with Bayesian control chart. This integrated models have been considered as an intelligent model and a suitable strategy for forecasting items failures for non Markovian deterioration model, optimal asset management, providing an effectiveness maintenance cost, provides lower inventory costs for spare parts, etc. However, CBM models need new aspects and integrate a new type of information in maintenance modeling that can improve the results and reduce uncertainty. Thus, to be successful in a global competitive market. Mathematical approach have considered in this study instead engineering approach. In this case, my study was focused on developing optimal multivariable Bayesian control chart with two decision variables: sampling interval and upper control limit. The thesis emphasis to develop a new methodology based on Bayesian control chart for predicting failures of item incorporating simultaneously two types of data: key quality control and measured machine condition indicators. These observation can be fused by using Bayes theory to give a posterior probability estimate of the warning state which is unobservable (upper control limit), in this case the process is monitoring by plotting the posterior probability in control chart in which can be compared with a control limit to assess whether a full inspection is need or not. In this study, we review modelling approaches for system deterioration. A Markov chain property verifying in this thesis as result the process are not Markovian, in this case we used delay time concept for state transition, analyzing and estimate the transition probabilities in 3-State Models. State 1 and 2 are unobservable, represent normal and warning state, respectively. Only the failure state 3 is assumed to be observable. Although, failure modeling is important for designing such Bayesian control chart. Combining the delay time concept and Bayes theory to establish posterior probability the Atox Mill equipment is in warning state given data observed. The objective is to find a stopping rule under partial observations, minimizing the long-run expected average cost per unit time for a given sample size and sampling interval. AMonte Carlo Simulation and PSO algorithm developed to find the optimal control limit and the minimum average cost.

6.1.1 Summarize of study case

This empirical research was illustrated using real data obtained from condition monitoring technique collected at regular time epochs from Atox Mill equipment used in cement industry (Secil Portugal). We have benefit to access in real world maintenance problem with complete record data which is not common especially in the world with hard competiveness between companies.

The problems was investigated about impending the failure of Atox Mill equipment by optimizing maintenance in order to maximize the operation of equipment for the long time. It can be triggered by observing the parameters that affect the operation of Atox Mill and the quality control measurement of size particle (output of equipment), and it follow be fused through Bayesian theory to give a posterior probabilistic estimate the warning state of equipment which is not directly observable. This empirical research examine the alternative solutions, and propose the most effective solution as well as the limitation of proposed model.The study case was focused on this work is about Atox Mill equipment which considered the most important in cement industry (Complex system, Maintenance requirement are higher, Higher overall cost), its servers as equipment for crushing and grinding raw material, uses pressure and shear generated between the rollers and the rotating table to crush and grind raw materials.

In this practical application, observed monitoring information history come from Atox Mill including five variables: temperature, vibration, pression, power of motor, and size of particle. The sample of each variable was recorded each 30 minute for the period about one years (10 months), the size of data are quite big equal to 15160 samples. Data classifies as confidential any of the following data variables: temperature, pression, vibration, power motor, size particle when can appear in the thesis.

6.1.2 Summarize of scientific results

A new Multivariate Bayesian control chart for CBM has been found, the optimal control chart parameters, which are the interval monitoring and the upper control limit were determined. The Weibull analysis was adopted to verify the Markov property, the result found the process are not Monrovian, the time to failure not follow exponential distribution which means the process has not memory property (Markov property), this analysis will tell us about is it ok to move from Markov chain to delay time concept or not, as results the delay time concept was adopted in deterioration modelling where the history past was considered.

Empirical research should be comparative research, this is why it is necessary comparing the proposed model (case 2) with multivariate Bayesian control chart without considering the quality control measurement (case 1), it concluded that the propose model can reach economic efficiency where the cost of maintenance in the first case is quite equal to the second case and the interval monitoring in the first case (10 h) is higher than the second one (8 h). To conclude, this study found that multivariate Bayesian when considering quality control measurement perform better than the previous Multivariate Bayesian control chart, and the maintenance cost was quite equal but the interval monitoring is higher for the proposed model. However, due to insufficient research in multivariate Bayesian control chart needs to improvethe proposed model with another practical application.

6.2 Suggestion of further research

It is expected will submit 4 publications from this research related to the results of the analysis the proposed model, sensitivity analysis (validation and evaluation). Besides, the finding of the proposed model give rise to several issues, it is well recognized that uncertainty influence the results and the finding, in particular the one related to quality of data due to error, missing, frameworks and method collected data, it would be important to select the strong tools and appropriate techniques for these kind of situation. Even this research work showing interested effectiveness when comparing with other model but still this area needs more research to be performed. To cope with above situation, further research resulting from this work:

- Since CBM model based on observed data, further the application data mining in this area could be challenging steps,
- Further incorporation artificial intelligence (unsupervised and supervised) to be interesting research.
- Since the data itself is weak to reveal the reality… as well as there is not perfect model, it could be necessary to add a new information related to the machine and its environment for reducing risk related to uncertainty.
- Further research in Bayesian control chart for CBM with considering more than three state
- Consoling the efficiency of propose model by testing to another practical application.
- It could be interesting to consider further Statistical analysis for such CBM problems.
- Taking into account the concept big data and its requirement.
- Testing the performance of algorithm using in solving optimization problems.
- Development integrated model based on Bayesian control chart and Logic fuzzy.
- Development integrated model based on Bayesian control chart and Analytic hierarchy process (AHP).
- Prior probability affect the results of posterior probability.
- Validation the model with two kind of data: test data and trial data.
- Incorporating classification model such as: Receiver Operating Characteristic (ROC).
- Development framework for automatization CBM.

In addition, since my research work is novel in Department of Production and System (Engineering school), it would be useful to develop a CBM group in SEOR and Research group from statistic department in which it can many discipline will integrated in order to develop intelligent and optimal model.

References

Abdel-Hameed, M., 1975. A gamma wear process. IEEE Transactions on Reliability, Vol.24, No.2, 152-153.

Abdel-Hameed, M., 2010. Degradation process: An Overview. Springer science and business media.17-25.

Affonso, L. O. A., 2006. Machinery failure analysis handbook. Houston, TX: Gulf Publishing Company. American Institute of Chemical Engineers.

Aiwina, H., Sheng, Z., Andy, C.C.T., Joseph, M., 2009. Rotating machinery prognostics: State of the art, challenges and opportunities. Mechanical Systems and Signal Processing, Vol. 3, 724-739.

Allocco, M., Raheja, D.J., 2005. Assurance Technologies Principles and Practices: A Product, Process, and System Safety Perspective, Journal of Loss Prevention in the Process Industries, Vol. 25, No.3, 643-649.

Al-Najjar, B., 2007. Cost-effective and continuous improvement of production process and company's business when using total quality maintenance. International conference on maintenance engineering. China (Cheng Du): Proceedings of ICME, October 15-18.

Al-Najjar, B., 2007. The lack of maintenance and not maintenance which cost: A model to describe and quantify the impact of vibration-based maintenance on company's business. International Journal and Production Economics. No. 107, 260-273.

Amari, V., McLaughlin, L., Pham, H., 2006 Cost-Effective Condition-Based Maintenance Using Markov Decision .Processes Relex Software Corporation. Annual Reliability and Maintainability Symposium, 464-469.

Ahmad, R., 2012., Kamaruddin, S . A review of condition-based maintenance decision-making. European Journal of Industrial Engineering, Vol. 6, No. 5, 519-541.

Anderson, T.W., 2003. An introduction to multivariate statistical analysis. 3 rd edition, John Wiley and Sons, Inc: United States of American.

Archambeau, C., Opper, M., 2010. Approximate inference for continuous-time Markov processes. Inference and Learning in Dynamic Models. Cambridge University Press.

Ascher, H., Feingold, H., 1984. Repairable system reliability: modeling, inference, misconception and their cause. Marcel Deeker: New York.

Ashrae, 1999. Operation and maintenance management, ASHRAE handbook applications, Tullie Circle, N.E., Atlanta,

Aven. T, Castro, I.T., 2009. A delay-time model with safety constraint. Reliability Engineering System and Safety, Vol. 94, No. 2, 261–267.

Baker, R.D., 1993. A non-parametric estimator of renewal function. Computers and Operations research, Vol 20, 167-178.

Baker, R. D., Wang W. 1992. Estimating the delay-time distribution of faults in repairable machinery from failure data. IMA Journal of Mathematics Applied in Business & Industry.

Baker, R. D., Christer, A. H. 1994. Review of delay-time OR modelling of engineering aspects of maintenance. European Journal of Operational Research, Vol. 73, 407-422.

Baker, R. D., Wang W., 1992. Estimating the delay-time distribution of faults in repairable machinery from failure data. IMA Journal of Mathematics Applied in Business & Industry, No. 3, 259-281.

Baker, R.,Wang, W., 1993. Developing and testing the delay time model. Journal of Operational Research Society, Vol. 44, No. 4, 361-374.

Baker, R.D., and Wang, W., 1991. Determining the delay time distribution of faults in repairable machinery from failure data. IMA Journal of Mathematics Applied in Business and Industry, Vol. 3, 259-282.

Banerjee, P. and Rahim, M., 1993. A generalized model for the economic design of X-bar control charts for production systems with increasing failure rate and early replacement. Naval Research Logistics, Vol. 40, No. 6, 787-809.

Banjevic, D., Jardine, A.K.S., Makis, V., and Ennis, M., 2001. A control-limit policy and software for CBM optimization. INFOR, Vol. 39, No. 1, 32-50.

Bangston, M., 2007. A Condition-Based Maintenance and its implementation in industrial settings. Thesis. Departement of innovation, design and product development, Malardaren University. Sweden.

Barbera, F., Schneider, H., and Watson, E., 1999. A condition based maintenance model for a two-unit series system. European Journal of Operational Research, 281-290.

Barlow, R. E., Marshall, A. W., and Proschan, F., 1963. Properties of probability distributions and monotone hazard rate. Annals of Mathematical Statistics, Vol. 34, 375-389.

Barlow, R.E, and Hunter, L.C., 1960. Optimum preventive maintenance policies. Journal of Operations Research. Vol. 8, No. 1, 90-100.

Barlow, R.E, and Hunter, L.C., 1960. Optimum preventive maintenance policies. Journal of Operations Research. Vol. 8, No. 1, 90-100.

Barlow,R.E, Hunter, L.C., and Proschan, F., 1963. Optimum checking procedures. Journal of the Society for Industrial and Applied Mathematics, Vol. 11, No. 4, 1078-1095.

Barlow, R.E., and Proschan, F., 1965. Mathematical Theory of Reliability (Classics in Applied Mathematics). Society for industrial and applied mathematics: New York.

Barlow, R.E., and Proschan, F., 1975. Statistical theory of reliability and life testing probability models. Reliability Engineering and System Safety. Vol. 102, 16-26.

Barone, G. and Frangopol, D.M., 2014. Life-cycle maintenance of deteriorating structures by multi-objective optimization involving reliability, risk, availability, hazard and cost. Structural Safety, Vol. 48, 40-50.

Bartlett, M.S., 1978. An Introduction to Stochastic Processes, with Special Reference to Methods and applications: Cambridge University Press.

Basharin, G.P., Langville, A.N., Naumov, V.A., 2004. The life and work of Andrei Andreevich. Markov. Linear Algebra and its Applications, Vol. 386, 3-26.

Bather, J. A., 1963. Control charts and minimization of costs, Journal of the Royal Statistical Society, Series B, Vol. 25, 49-80.

Bauer, H. H., Falk, T., Hammer schmidt, M., 2006. A transaction process-based approach for capturing service quality in online shopping. Journal of Business Research, No. 59, 866-875.

Bauer, J. E., Duffy, G. L., Westcott, R., 2006. The quality improvement handbook (2^{nd}ed.). Milwaukee, WI: ASQ Quality Press.

Bazovsky. I., 1961. Reliability Theory and Practice by Dover Publications, INC. Mineola, New York.

Ben-daya, M. and Rahim, M.A., 2000. Effect of maintenance on the economic design of $\bar{x}$-control chart. European Journal of Operational Research, Vol. 120, No. 1, 131-134.

Ben-Daya, M., 1999. Integrated production maintenance and quality model for imperfect processes. IIE Transactions, Vol. 31, 491-501.

Ben-Daya, M., and Duffuaa, S.O., 1995. Maintenance and quality: the missing link. Journal of Quality in Maintenance Engineering, Vol. 1, No. 1, 20-26.

Ben-daya, M., and Rahim, M.A., 1998. A generalized economic model for joint determination of production run, inspection schedule and control chart design. International Journal of Production Research, Vol. 36, No. 1, 277-289.

Bengtsson, M., Fundin, A., Deleryd, M., Salonen, A., Olsson, E., Funk, P., Andersson, C. and Qureshi, H., 2010. Integrating quality and maintenance development-opportunities and implications. International conference on condition monitoring and diagnostic engineering management: Japan, 821-828.

Berg, M., and Cleroux R., 1982. The block replacement problem with minimal repair and random repair costs. Journal of Statistical Computation and Simulation, Vol. 15, No.1, 1-7

Bernardo, J.M., Smith, A.F.M., 2000.Bayesian Theory, Measurement Science and Technology, Vol. 12, No. 2, John Wiley & Sons, Inc., Hoboken, New Jersey. Canada.

Biroloni, A., 2007. Reliability engineering: Theory and Practice. Springer.

Biswas, A. and Sarkar, J., 2000. Availability of a system maintained through several imperfect repairs before a replacement or a perfect repair. Statistics & Probability Letters, Vol. 50, No. 2, 105-114.

Blache, K.M., Shrivastava, A.B., 1993. Reliability and Maintainability of Machinery and Equipment for Effective Maintenance. International Congress & Exposition: SAE Technical Paper,

Blache, K.M., Shrivastava, A.B., 1994. Defining failure of manufacturing machinery and equipment. Annual Reliability and Maintainability Symposium (RAMS), Anaheim, CA, USA, 69-75.

Blanchard, B.S., Lowery. E.E., 1969. Maintainability: principles and practices, Mc Graw Hill, New York.

Breuer, L., 2007. Introduction to Stochastic Processes.

Bruns, P., 2002. Optimal maintenance strategies for systems with partial repair options and without assuming bounded costs. European Journal of Operational Research, Vol. 139, No. 1, 146-165.

Bryman, A., 2006. Mixed methods. London: SAGE Benchmarks in social research methods.

Butcher, S.W., 2000. Assessment of Condition-based maintenance. Department of Defense: Mclean, Virginia.

Calabrese, J.M., 1995. Bayesian process control for attributes. Management Science, Vol. 41, 637-645.

Campbell, D.T. & Stanley, J.C. (1963). Experimental and Quasi-Experimental Designs for Research. Boston: Houghton Mifflin Company.

Cassady, C. R., and Pohl, E.A., 2003. Introduction to repairable systems modeling. Department of Industrial Engineering University of Arkansas, USA.

Cassady, C. R., Bowden, R.O., Liew, L., Pohl, E.A., 2000. Combining preventive maintenance and statistical process control: a preliminary investigation. IIE Transactions, Vol. 32, No.6, 471-478.

Chan, L.Y., Wu, S., 2009. Optimal design for inspection and maintenance policy based on the Control chart. Computers and Industrial Engineering, Vol. 57, No. 3, 667-676.

Charles, A.S., Floru I.R, Azzaro-Pantel, C., Pibouleau, L., and Domenech, S., 2003. Optimization of preventive maintenance strategies in a multipurpose batch plant application to semiconductor manufacturing. Computers and Chemical Engineering, Vol. 27, No. 4, 449-467.

Charongrattanasakul, P., and Pongpullponsak, A., 2011. Minimizing the cost of integrated systems approach to process control and maintenance model by EWMA control chart using genetic algorithm. Expert System with Application, Vol. 35, No. 8, 5178-5186.

Chaudhry, M.L., Templeton, J.G.C., 1983. A First Course in Bulk Queues, John Wiley & Sons, New York.

Chen, D., and Trivedi, K.S., 2002. Closed-form analytical results for CBM. Reliability Engineering and System Safety, Vol. 76, No. 1, 43-51.

Chen, D., Trivedi, K.S., 2004. Optimization for CBM with Semi-Markov Decision Process. Reliability Engineering and System Safety. Vol. 90, No. 1, 25-29.

Chen, W.S., Yu, F.J., Guh, R.S, and Lin, Y.H., 2011. Economic design of x-bar control charts under preventive maintenance and Taguchi loss functions. Journal of Applied Operational Research, Vol. 3, No. 2, 103-109.

Cho, D., and Parlar.M., 1991. A survey of maintenance models for multi-unit systems. European Journal of Operational Research, Vol. 51, 1-23.

Chiang, J.H., Jiang, J .2001. Optimal maintenance policy for Markovian system under periodic inspection. Reliability engineering and system safety, Vol. 71, 165-172.

Christer A. H. A., 2002. Review of Delay time analysis for modelling plant maintenance. Stochastic Models in Reliability and Maintenance, Berlin Heidelberg: Springer.

Christer A. H., Waller W. M., 1984. An operational research approach to planned maintenance: modelling P.M. for a vehicle fleet. Journal of the Operational Research Society, Vol. 35, No. 11, 967-984.

Christer, A. H. A., 2002. Review of Delay Time Analysis for Modelling Plant Maintenance. Stochastic Models in Reliability and Maintenance, Springer: Berlin Heidelberg, Germany.

Christer, A. H., 1982. Modelling inspection policies for building maintenance. Journal of the Operational Research Society, Vol. 33, 723-732.

Christer, A. H., 1987. Delay-time model of reliability of equipment subject to inspection monitoring. Journal of the Operational Research Society, Vol. 38, No. 4, 329-334,

Christer, A. H., 1992. Modelling condition monitoring inspection using the delay-time concept. Department of Mathematics and Computer Science: the University of Salford, U. K.

Christer, A. H., 1999. Developments in delay time analysis for modelling plant maintenance. Journal of the Operational Research Society, Vol. 50, 1120-1137.

Christer, A. H., Redmond, D. F. A., 1990. A recent mathematical development in maintenance theory. IMA Journal of Mathematics Applied in Business and Industry, No. 2, 97-108.

Christer, A. H., Redmond, D. F., 1992. Revising models of maintenance and inspection. International Journal of Production Economics, Vol. 24, 227-234.

Christer, A. H., Scarf, P. A. A., 1994. A robust replacement model with applications to medical equipment. Journal of the Operational Research Society, Vol. 45, No. 3, 261-275.

Christer, A. H., Scarf, P. A., 1994. A robust replacement model with applications to medical equipment. Journal of the Operational Research Society, Vol. 45, No. 3, 261-275,

Christer, A. H., Waller W. M., 1984. Delay Time Models of Industrial Inspection Maintenance Problems. Journal of the Operational Research Society, Vol. 35, No. 5, 401-406.

Christer, A. H., Waller, W. M. A., 1987. Descriptive model of capital plant replacement, Journal of the Operational Research Society, Vol. 38, No. 6, 473-477.

Christer, A. H., Waller, W. M., 1984. Reducing production downtime using delay-time analysis. Journal of the Operational Research Society, Vol. 35, No. 6, 499-512.

Christer, A. H., Whitelaw J., 1983. An operational research approach to breakdown maintenance: problem recognition. Journal of the Operational Research Society, Vol. 34, No. 11, 1041-1052,

Christer, A., Wang, W., and Sharp, J. M., 1997. A state space condition monitoring model for furnace erosion prediction and replacement. European Journal of Operational Research, Vol. 101, 1-14.

Christer, A.H., Waller W.M., 1984. Delay time models of industrial inspection maintenance problems. Journal of Operational Research Society, Vol. 35, No. 5, 401-406.

Christer, A.H., Wang, W., Choi, K., Van der Duyn Schouten F. A., 2001. The robustness of the semi-Markov and delay time single-component inspection models to the Markov assumption. IMA Journal of Management Mathematics, No. 12, 75-88.

Christer, A.H., Wang, W., Choi, K., Van der Duyn Schouten, F. A., 2001. The robustness of the semi-Markov and delay time single-component inspection models to the Markov assumption. IMA Journal of Management Mathematics, Vol. 12, 75-88.

Christopher, P., Fox, J., 2014. Comparing the comprehensiveness of three expert inspection methodologies for detecting errors in interactive systems, Safety Science, Vol. 62, 286-294.

Chu, C., Proth, J.M., and Wolff, P., 1998. Predictive maintenance: The one-unit replacement model. International Journal of Production Economics. Vol. 54, 285-295.

Cinlar, E., 2013. Introduction to stochastic processes. Englewood Cliffs , NJ : Prentice-Hall: New York.

Cook Fras, A., 2000. Success and failure in Newton’s lunar theory. Blackwell Publishing.

Cooke, R. M. 1996. The design of reliability data bases. Part I and 11. Reliability Engineering and System Safety, Vol. 51, 137-146 and 209-223.

Cox, D.R, Miller, H.D., 2001. The theory of stochastic processes. Chapman and Hall, CRC: United States of American.

Creswell, J.W., 2002. Research Design: Qualitative, Quantitative, and Mixed Methods Approaches. SAGE Publications.

Creswell, J.W., 2003. Research Design: Qualitative, Quantitative, and Mixed Methods Approaches. Thousand Oaks: Sage Publications.

Damien, P., Dellaportas, P., Polson, N.G., Stephens, D. A., 2013 Stephens, Bayesian Theory and Applications

Dekker, R. and Scarf, P., 1998. On the impact of optimization models in maintenance decision making: state of the art. Reliability Engineering and System Safety. Vol. 60, 111-119.

Dekker, R., 1996. Applications of maintenance optimization models: a review and analysis. Reliability Engineering and System Safety. Vol. 52, 229-240.

Frate, L., Franssen, M., Vermaas, P. E., 2011. Towards a trans-disciplinary concept of failure for integrated product development. International Journal of Product Development, 14(1–4), 72–95.

Desa, M. I., Christer, A. H., 2001. Modelling in the absence of data: a case study of fleet maintenance in a developing country. Journal of the Operational Research Society, No. 52, 247-260,

Dhillon, B.S., 2002. Engineering maintenance, a modern approach, CRC PRESS Boca Raton London New York Washington, D.C.

Dileo, M., Manker, C., and Cadick, J., 1999. Condition-based Maintenance. Cadick Corporation. Vol. 60, 111-119.

Dohi, T., Kaio, N., Osaki, h., 2002. Renewal processes and their computational aspects.Stochastic Models in Reliability and Maintenance. 1-30.

Dubrova, E., 2013. Fault-TolerantDesign. Springer-Verlag: New York.

Duncan, A., 1956. The economic design of x-bar charts to maintain current control of a process. Journal of the American Statistical Association 51, 222-242.

Eckles, J.E., 1968. Optimum maintenance with incomplete information. Operational Research, Vol. 16, 1058-1067.

EN 13306: 2001. Maintenance terminology, CEN.

Escudero, L.A.G., Perez, O.D., Sotelo, D.M., Alonso, M.P., 2011. Robust multivariate control charts for early detection of broken rotor bars in an induction motors fed by a voltage source inverter. Journal Expert Systems with Applications. Vol. 38, No. 3, 2653-2660.

Feller, W. 1968. An Introduction to Probability Theory and Its Applications, Vol. 1. Wiley, New York.

Flage, R., Coit, D.W., Luxhoj, J.T., and Aven, T., 2012. Safety constraints applied to an adaptive Bayesian maintenance optimization model. Reliability Engineering and System Safety, Vol. 102, 16-26.

Frate, L.D., 2012. Failure of Engineering Artifacts: A Life Cycle Approach. Springer science.

Frawley, D. J., 2002. ISO 9001 QMS. Policies, procedures and forms. St. Louis, Mo.: Bizmanualz.com, Inc109.

Frawley, G., 2002. International Directory of Military Aircraft. Motorbooks International; 4th edition edition.

Frees, E., 1986. Warranty analysis and renewal function estimation. Naval Research Logistics, Vol. 33, 361-372.

Gallasch, G.E. and Francis, B., 2013. Examining the Interaction between Condition Based Maintenance and the Logistics Supply Chain, Proc. 8th DSTO International Conference on Health and Usage Monitoring, Melbourne, Australia, 25-28.

Gallager, R.G., 2013. Stochastic processes: theory for applications. TJ International Ltd. Padstow cornwall: UK.

Garg, A. and Deshmukh, S.G., 2006. Maintenance management: literature review and directions. Journal of Quality in Maintenance Engineering, Vol. 12, 205- 238.

Gertsbakh, I. B. 1977. Models of preventive maintenance. North-Holland Pub. Co; New York

Gelman, A., Carlin, J.B., Stern, H.S., Rubin, D.B., 2014. Bayesian data analysis. Journal of the American Statistical Association, Vol. 109 Journal of the American Statistical Association.

George, T., Yiannis, N., 2001. Comparing the effectiveness of various Bayesian x-bar control chart. Operations research, Vol. 50, No. 5, 878-888.

Ghahramani, Z.,2001. An Introduction to Hidden Markov Models and Bayesian Networks. International. Journal of Pattern Recognition and Artificial Intelligence, Vol. 15, No. 1, 9-42.

Gheorghe, A.V., 1990. Decision Processes in Dynamic Probabilistic Systems. Springer.

Ghasemi, A., Yacout S., and Ouali M.S., 2010. Parameter estimation methods for condition based maintenance with indirect observations. IEEE Transactions on Reliability. Vol. 59, No. 2, 426-39.

Girshick, M.A., Rubin, H., 1952. A Bayes' approach to a quality control model. Annals of Mathematical Statistics, Vol. 23, 114-125.

Gosavi, A., 2013. A Tutorial for Reinforcement Learning. Department of Engineering Management and Systems, 3-12.

Guida, M., Pulcini. G., 2013. The inverse Gamma process: a family of continuous stochastic models for describing state dependent deterioration phenomena. Reliability Engineering and System Safety, Vol. 120, 72-79.

Gulati, R., Smith, R., 2009, Maintenance and Reliability Best Practices, Industrial Press.

Hammer, W., 1972. Handbook of System and Product Safety. Prentice-Hall, Englewood Cliffs.

Heng, A.S.Y., 2009. Intelligent prognostics of machinery health utilizing suspended condition monitoring data. PhD thesis, Queensland University of Technology.

Heng, A.S.Y., 2009. Intelligent prognostics of machinery health utilizing suspended condition monitoring data. PhD thesis, Queensland University of Technology, Australia.

Heyman, D. P, Sobel, M. J., 1982. Stochastic Models in Operations Research, McCraw-Hill, New York.

Ho, C., and Case, K.E., 1994. Economic Design of Control Charts: A literature review for 1981-1991. Journal of Quality Technology, Vol. 26, 39-53.

Hoel, P.G., Sidney, C.P., Stone, C.H., 1986. Introduction to Stochastic Process

Hosseini,M.M., Kerr, R.M. Randall, R.B., 2000 An inspection model with minimal and major maintenance for a system with deterioration and Poisson failures. IEEE, Trans on Reliability. Vol. 49, No. 1, 88-98.

Hu, X., Eberhart, R., 2001.Solving Constrained Nonlinear Optimization Problems with Particle Swarm Optimization.

Idhammar, C., 2014, Lean Maintenance, IDCON.

Ilangkumaran, M. and Kumanan, S., 2012. Application of Hybrid VIKOR Model in Selection of Maintenance Strategy. International Journal of Information Systems and Supply Chain Management. Vol. 5, No. 2, 59-81.

Ilangkumaran, M., and Kumanan, S., 2009. Selection of maintenance policy for textile industry using hybrid multi-criteria decision making approach. Journal of Manufacturing Technology Management, Vol. 20, No. 7, 1009-1022.

Isermann, R., 2001. Fault-Diagnosis Applications. Springer: Verlag Berlin Heidelberg, Germany.

Jamali, M.A., Ait-Kadi, D., Cléroux, R., and Artiba, A. 2005. Joint optimal periodic and conditional maintenance strategy. Journal of Quality in Maintenance Engineering, Vol. 11, No. 2, 107-114.

Jardine, A.K.S., Banjevic, D., and Joseph, T. 1999. Optimizing condition based maintenance decisions for equipment subject to vibration monitoring. Journal of Quality in Maintenance Engineering, Vol. 5, No. 3, 192-202.

Jardine, A.K.S., Banjevic, D., Khan, K., Wiseman, M., and Lin, D. 2003. An optimized policy for the interpretation of inspection data from a Condition Based Maintenance program at a nuclear reactor station. COMADEM: Vaxjo University, Sweden, 27-29.

Jardine, A.K.S., Banjevic, D., Wiseman, M., Buck, S., and Joseph, T., 2001. Optimizing a mine haul truck wheel motors condition monitoring program Use of proportional hazards modeling. Journal of Quality in Maintenance Engineering, Vol. 7, No. 4, 286-302.

Jardine, A.K.S., Lin, D., and Banjevic, D. 2006. A review on machinery diagnostics and prognostics implementing CBM. Mechanical Systems and Signal Processing. Vol. 20, No. 7, 1483-1510.

Jennings A.D., Drake P.R.,1998. Machine tool condition monitoring using statistical quality control charts. International Journal of Machine Tools and manufacture. Vol. 37, No. 9, 1243-1249.

Jiang, R., 2013. Relationship between delay time and Gamma process models. Chemical Engineering Transactions, Vol. 33, 19-24.

Jiang, R., and Makis, V., 2009. ARL criterion in Bayesian process control using hidden Markov model. Industrial Engineering and Engineering Management, 228- 232.

Jiang, R., Jong Kim, M., and Makis, V., 2011. A Bayesian model and numerical algorithm for CBM availability maximization. Annals of Operations Research Vol. 196, 333-348.

Jiang, R., Makis, V., 2009. ARL Criterion in Bayesian Process Control using Hidden Markov Model. Journal of Industrial Engineering and Engineering Management. 228-232.

Jiang, R., Yu, J., and Makis, V., 2012. Optimal Bayesian estimation and control scheme for gear shaft fault detection. Computer and Industrial Engineering, Vol. 63, No. 63, No. 4, 754-762

Jones, B., Jenkinson, I., Wang, J., 2009. Methodology of using delay-time analysis for a manufacturing industry. Journal of Reliability Engineering System and Safety, Vol. 94, No. 1, 111-124.

Jong Kim, M., Jiang, R., Makis, V., Lee, Chi-Guhn., 2011. Optimal Bayesian fault prediction scheme for a partially observable system subject to random failure. European Journal of Operational Research, Vol. 214, No. 2, 331–339.

Joseph, D., Pattron, Jr., 1983. Preventive maintenance. Instrument Society of America, Creative Service Inc. New York. USA.

Kallen, M.J., Noortwijka, J.M., 2006. Optimal maintenance decisions under imperfect inspection Reliability Engineering and System Safety. Vol. 90, No. 2-3, 177-185.

Kallen, M.J., Noortwijka, J.M.V., 2006. Optimal periodic inspection of a deterioration process with sequential condition states. International Journal of Pressure Vessels and Piping, Vol. 83, No. 4, 249-255.

Kancev, D., Zerovnik, G., Cepin, M., 2012. Uncertainty analysis in the nuclear industry: Analytical unavailability modelling incorporating ageing of safety components Journal of Loss Prevention in the Process Industries, Vol. 25, No. 3, 643-649 .

Karlin, S., Taylor, H.M., 1975. A first Introduction to Stochastic Processes. Elsevier.

Karlin, S. & Taylor, H.M. (1981). A Second Course in Stochastic Processes, London: Academic Press.

Keats, J., Del Castillo, E., von Collani, E. and Saniga, E., 1997. Economic modeling and statistical process control. Journal of Quality Technology, Vol. 29, No .2, 144-147.

Kelly, A., 1984. Maintenance planning and control. London.

Kim, M.G., Makis, V., 2009. Optimal maintenance policy for a multi-state deteriorating system with two types of failures under general repair. Journal Computers and Industrial Engineering, Vol. 57, No. 1, 298-303.

Kirwan, B., Ainsworth, L.K., 1992. A Guide to Task Analysis. CRC press.

Kniele, R., Stephens, G. and Vasudeva, K., 1989. Performance assessment and statistical process control. An approach to operation and maintenance cost reduction. Proceeding of the IFAC/ IFORS/IAEE symposium on energy, systems, management and economics, 215-221.

Kobacci, K.A.B., Murthy, D.N.P., 2008. Complex System Maintenance Handbook. Springer: England.

Kothamasu, R., Huang, S., 2007. Adaptive Mamdani fuzzy for CBM. Journal Fuzzy Sets and Systems. Vol. 158, No. 24, 2715-2733.

Kletz, T.A., 1992. HAZOP and HAZAN: Identifying and Assessing Process Industry Hazards. Institution of Chemical Engineers, Rugby, UK.

Lampreia, S.S., Requeijo, J.G., Dias, J.M., and Vairinhos, V., 2012. T^2 charts applied to mechanical equipment condition control. Intelligent Engineering Systems, 441-446.

Lee, B. H. and Rahim, M. A., 2001. An integrated economic design model for quality control, replacement, and maintenance. Quality Engineering, Vol. 13, No. 4. 581-593.

Lesage, A., Dehombreux, P., 2012. Maintenance and quality control: A first methodological approach for maintenance policy optimization. Symposium on Information Control Problems in Manufacturing, Vol. 45 , No. 6 , 1041-1046.

Linderman, K., McKone-Sweet, K.E., and Anderson, J.C., 2005. An integrated systems approach to process control and maintenance. European Journal of Operational research, Vol. 164, 324-340.

Liu, X., Li,J., and Al-Khalifa, K., Hamouda, A., Coit, D., and Elsayed, E., 2012. A framework for CBM scheduling. IEEE, Vol. 978, No. 1, 4577-1851.

Lin, D, Makis, V., 2004. On-line parameter estimation for a failure-prone system subject to condition monitoring, Vol. 41, No. 1, 211-220.

Lloyd, W., Condra. 1993. Reliability improvement with design of experiments: New York: Marcel Dekker.

Lochner, R., 1987. Improved Statistical process control using reliability concepts. Quality Congress Transactions, 648-653.

Louit, D., Pascual, R., Banjevic, D., and Jardine, A.K.S., 2011. Condition-based spares ordering for critical components. Mechanical Systems and Signal Processing, Vol. 25, No. 5, 1837-1848.

Lowhorn, G.L., 2007. Qualitative and Quantitative Research: How to Choose the Best Design.

Lu, C. J. and Meeker, W. Q., 1993. Using degradation measures to estimate a time-to-failure distribution. Technimetrics, Vol. 35, 161-174.

Luce, S., 1999. Choice criteria in conditional preventive maintenance. Mechanical Systems and Signal Processing, Vol. 13, No.1, 163-168.

Mahmoodian, M and Alani, A., 2013. Multi-failure mode assessment of buried concrete pipes subjected to time-dependent deterioration using system reliability analysis. Journal of Failure Analysis And Prevention, Vol. 13, No. 5, 634-642.

Makis, V., 2007. Multivariate Bayesian Control Chart. Operations Research, Vol. 56, No. 2,487-496.

Makis, V., and Jardine, A. K. S., 1992. Optimal replacement in the proportional hazards model. INFOR, Vol. 30, 172-183.

Makis, V., Jiang, R., 2003. Optimal replacement under partial observations. Mathematics of Operations Research, Vol. 28 No. 2, 382-394.

Makis, V., Jiang, X., 2003. Optimal replacement under partial observations. Mathematical Operational Research, Vol 28, 382-394.

Marguez, A., Heguedas, A., 2002. Models for maintenance optimization: As study for repairable system and finite time period. Reliability engineering and system safety, Vol. 75, 376-376.

Mariun, N., Mehrjou, M.R., 2011. An Experimental Study of Induction Motor Current Signature Analysis Techniques for Incipient Broken Rotor Bar Detection, International Conference on Power Engineering, Energy and Electrical Drives, Spain.

Markovich N.M., Krieger, U.R., 2006.Nonparametric estimation of the renewal function by empirical data, Stochastic Models, Vol. 22, 175-199.

Marseguerra, M., Zio, E., and Podofillini, L., 2002. CBM optimization by means of genetic algorithms and Monte Carlo simulation. Reliability Engineering and System Safety, Vol. 77, 151-166.

Martin, K.F.,1994. A review by discussion of condition monitoring and fault diagnosis in machine tools. International Journal of Machine Tools Manufacturing, Vol. 34, No. 4, 527-551.

McCall, J.J., 1965. Maintenance Policies for Stochastically Failing Equipment: A Survey. Journal of Management Science, Vol. 11, No. 5, 493-524.

McLachlan, G., Krishnan, T., 2008. The Expectation maximization algorithm and extensions. 2nd Edition. John Wiley and Son.Inc: United States of American.

McMillan, D., and Ault, G.W., 2008. Condition monitoring for onshore wind turbines: sensitivity to operational parameters. Institution of Engineering and Technology Renewable Power Generation. Vol. 2, No. 1, 60-72.

Meeker, W.Q., Escober, L.A., 1998. Statistical methods for reliability data. John Wiley and Son.Inc: United States of American.

Meel, A. & Seider, W.D., 2006. Plant-specific dynamic failure assessment using Bayesian theory, Chemical Engineering Science 61, 7036–7056.

Meel, A.,Seider ,W.D., 2006. Plant-specific dynamic failure assessment using Bayesian theory. Elsevier Chemical Engineering Science, Vol. 61, No. 21, 7036–7056.

Mehrafrooz, Z., and Nourossema, R., 2011. An integrated model based on statistical process control and maintenance. Computer and Industrial Engineering, Vol. 61, No. 4, 1245-1255.

Medhi, J. 1982. Stochastic process. First Edition, American Mathematical Monthly.

Mishra, R.C., and Pathak, K., 2002. Maintenance engineering and management. Prentice: India.

Mitchell, J.S., 1998. Five to ten year vision for CBM. United Stated American (Georgia): Advanced technology program Fall 98 National Meeting, 17-18.

Montgomery, D.C., 1980. Economic design of control charts: A review of literature survey. Journal of Quality Technology, Vol. 12, 40-43.

Moubray, J., 1997. Reliability centered-maintenance. 2 nd edition, Industrial Press Inc: United States of American.

Muller, A., Marquez, A.C. and Iung.B, 2008. On the concept of e-maintenance: Review and current research. Reliability Engineering and System Safety, Vol. 93 1165-1187.

Myrefelt, S., 2004. The reliability and availability of heating, ventilation and air conditioning systems, Energy and Buildings, Vol. 36, 1035-1048.

Nagai, H., Dohi, T., Osaki, S., 2000. The non-parametric estimator the renewal function applying the radial basis function neural network. Transaction of Japan Society for Industrial and applied mathematics.

Nguyen, D., Brammer, C., and Bagajewicz, M. 2008. New tool for the evaluation of the scheduling of preventive maintenance for chemical process plants. Industrial and Engineering Chemistry Research. Vol. 47, No. 6, 1910-1924.

Noortwijk, J.M.V., 2009. A survey of the application of gamma processes in maintenance. Reliability Engineering and System Safety. Vol. 94, 2-21.

Noortwijka, J.M.V., 2006. A survey of the application of gamma processes in maintenance. Reliability Engineering & System Safety, Vol. 94, No. 1, 2-21.

Octavio, P.L., Affonso, A., 2006. Machinery Failure Analysis Handbook: Sustain Your Operations and Maximize Uptime gulf publishing compagny, Houston, Texas., USA.

Okumura, S., 1997. An inspection policy for deteriorating processes using delay time concept. International, Vol. 4, No. 5/6, 365-375.

Osaki, S., 2002. Stochastic Models in Reliability and Maintenance. Springer-Verlag Berlin Heidelberg GmbH, Germany.

Panagiotidou, S., and Nenes, G., 2009. An economically designed, integrated quality and maintenance model using an adaptive Shewhart chart. Reliability Engineering and System Safety, Vol. 94, No. 3, 732-741.

Panagiotidou, S., and Tagaras, G., 2014. Statistical Process Control and CBM: A meaningful relationship through data sharing. Journal of Production and Operations Management, Vol. 19, No. 2, 156-171.

Pandey, D., Kulkarni, M.S., and Vrat, P., 2012. A methodology for simultaneous optimization of design parameters for the preventive maintenance and quality policy incorporating Taguchi loss function. International Journal of Production Research, Vol. 50, No. 7, 2030-2045.

Paroissin, C., Salami, A. 2014. Failure time of non-homogeneous gamma processes. Communications in Statistics - Theory and Methods, Vol. 43, No. 15, 3148-3161.

Parzen, E., 1999. Stochastic processes. Classic in Applied Mathematics.

Pedregal, D. J., and Carmen, C.M., 2009. Vibration analysis diagnostics by continuous-time models: A case study. Reliability Engineering and System Safety, Vol. 94, No.2, 244-253.

Peng, Y., Dong, M., and Zuo. M., 2010. Current status of machine prognostics in CBM: a Review. International Journal Advanced Manufacturing Technology, Vol. 50, 297-313.

Pierskalla, W.P., and Voelker, J.A., 1976. A survey of maintenance models: the control and surveillance of deteriorating systems. Naval Research Logistics Quarterly, Vol. 23, 353-388.

Porteus, E.L., Angelus, A., 1997. Opportunities for improved statistical process control. Management Science, Vol. 43, 1214-1229.

Power, C., Fox, J., 2014. Comparing the comprehensiveness of three expert inspection methodologies for detecting errors in interactive systems. Safety science. Vol. 62, 286-294.

Prasad, D., McDermid, J., Wand, I. (1996). Dependability terminology: Similarities and differences. Aerospace and Electronic Systems Magazine, IEEE, Vol. 11, No. 1, 14-21.

Pukite, J., Pukite, P., 1998. Modeling for reliability analysis. Markov Modeling for reliability, Maintainability, Safety, and supportability analysis of computer system. IEEE Press series on complex: United States of American.

Rabbani, M., Manavizadeh, N. and Balali, S. 2007. Stochastic model for indirect condition monitoring using proportional covariate model. International Journal of Engineering, Vol. 21, No. 1, 45-56.

Rabiner, L. R., 1991. Hidden Markov Models for Speech Recognition, Technometrics, Vol. 33, No. 3, 251-272.

Rabiner, L. R., Juang, B.H., 1986. An Introduction to Hidden Markov Models. IEEE ASSP Magazine.

Rahim, A., and Shakil, M., 2011. A tabu search algorithm for determining the economic design parameters of an integrated production planning, quality control and preventive maintenance policy. International Journal of Industrial and Systems, Vol. 7, No. 4, 477-497.

Rahim, M.A., 1994. Joint determination of production quantity, inspection schedule, and control chart design. IIE Transactions, Vol. 26, No. 6, 2-11.

Rahim, M.A., and Muhammad, S., 2011. A tabu search algorithm for determining the economic design parameters of an integrated production planning, quality control and preventive maintenance policy. International Journal of Industrial and Systems, Vol. 7, No. 4, 477-497.

Rahim, M.A., Banerjee, P.K., 1993. A generalized model for economic design of x-bar control charts for production systems with increasing failure rate and early replacement. Naval Research Logistics, Vol. 40, 787-809.

Rausand, M., Øien, K., 1996. The basic concepts of failure analysis. Reliability Engineering and System Safety, Vol. 53, No. 1, 73-83.

Rosenthal, J.S., 2005. Struck by Lightning: the Curious World of Probabilities. Harper Collings.

Rosmaini, A., Shahrul, K., 2011. A review of Condition based maintenance decision-making. European Journal of Industrial Engineering, Vol. 6, No. 5. 519-541.

Ross, S.M., 1971. Quality control under Markovian deterioration. Management Science, Vol. 17, 587-596.

Ross, S.M., 1989. Estimating the mean number of renewal by simulation. Probability in Engineering and Informational sciences, Vol. 3, 319-321.

Ross, S.M., 1996. Stochastic Processes. New York: John Wiley & Sons.

Ross, S.M., 2000. Introduction to probability and statistic for engineers and scientists. Willey Series in Probability and Mathematical Statistics.

Rubin. H.J., 1983. Applied social research, Longman Higher Education.

Saniga, E.M., 1989. Economic statistical control chart designs with an application to x-bar and R Control Charts. Technometrics, Vol. 31, 313-320.

Saranga, H., and Knezevic, J., 2001. Reliability prediction for condition-based maintained systems. Reliability Engineering and System Safety, Vol. 71, No. 2, 219-224.

Sarkar, J. and Chaudhuri, G., 1999. Availability of a system with gamma life and exponential repair time under a perfect repair policy. Statistics and Probability Letters, Vol. 43, No.2, 189-196.

Sarkar, J., Sarkar, S., 2000. Availability of a periodically inspected system under perfect repair, Journal of Statistical Planning and Inference, Vol. 91, No. 1, 77-90.

Salata, F., Vollaro, A.L., Vollaro, R.L., Davoli, F., 2014. Plant Reliability in hospital facilities. Conference of the Italian Thermal Machines Engineering Association Vol.45, 1195-1204.

Saunders, M.N.K., Lewis, P., Thornhill, A., 2006. Research Methods for Business Students**.**Prentice Hall.

Scarf, H., Gilford, M., Shelly, M., 1963. Multistage inventory models and techniques. Stanford University, California.

Schneider, H., Lin, B.S., O'Cinneide, C., 1990. Comparison of non-parametric estimators for the renewal function. Journal of Royal Statistical Society, Vol. 39, 56-61.

Shamshad, A., Bawadi, M.A., Wan Hussin, W.M.A., Majid, T., A., Sanusi, S.A.M.,2005. First and second order Markov chain models for synthetic generation of wind speed time m series. Energy, Vol. 30, 693-708.

Singpurwalla, N.D., 1995. Survival in Dynamic Environments. Journal of Statistical science, Vol. 10, No.1, 86-103.

Sorensen, D., Gianola, D., 2002. Likelihood, Bayesian and Monte Carlo Markov Chain Methods in quantitative genetics. Statistics for Biology and Health: United States of American.

Steven, E., R., Asit, P. B., 2000. Statistical methods for the reliability of repairablesystems. Johny Wiley and Sons, Inc: United States of American.

Stone, J.V., 2013. Bayes' Rule: A Tutorial Introduction to Bayesian Analysis. Applying Social Research: Exercises to Accompany Applied Social Research. C.E. Merrill Pub.

Tagaras, G., 1988. An Integrated cost model for the joint optimization of process control and maintenance. Journal of the Operational Research Society, Vol. 39, No. 8, 757-766.

Tagaras, G., Nikolaidis, Y., 2002. Comparing the effectiveness of various Bayesian x-bar control charts. Operations Research, Vol. 50, 878-888.

Tagaras, G.A., 1994. A dynamic programming approach to the economic design of x-bar charts. IIE Transaction, Vol.26, 48-56.

Tagaras, G.A., 1996. Dynamic control charts for finite production runs. European Journal of Operational Research, Vol. 91, 38-55.

Tam, A. S., Gordon, I. (2009). Clarification of failure terminology by examining a generic failure development process. International Journal of Engineering Business Management, Vol. 1, No. 1, 33-36.

Tapiero, C. S., 1986. Continuous quality production and machine maintenance. Naval Research Logistics, Vol. 33, 489-499.

Taylor, H.M., 1965. Markovian sequential replacement processes. Annals of Mathematical Statistics, Vol. 36, 1677-1694.

Taylor, H.M., 1967. Statistical control of a gaussian process. Technometrics, Vol. 9, 29-41.

Taylor, R. W., 1996. A linear programming model to manage the maintenance backlog Omega. The International Journal of Management Science Vol. 24, No. 2, pages 217-227.

Thomas, L.C., 1986. A survey of maintenance and replacement models for maintainability and reliability of multi-item systems. Reliability Engineering, Vol. 16, No. 4, 297-209.

Tijms, H.C., 2002. A First Course in Stochastic Models. Wiley and Sons, Ltd.

Tsang, A. C., 1995. Condition based maintenance: Tools and decision making. Journal of Quality in Maintenance Engineering, Vol. 1, No. 3, 1-17.

Vaishnavi, V., Kuechler, W.J., 2008. Design science research method and patterns: innovating information and communication technology, CRC: Taylor and Francis group.

Valdez-Flores, C., and Feldman, R.M., 1989. A survey of preventive maintenance models for stochastically deteriorating single-unit systems. Naval Research Logistics, Vol. 36, 419-446.

Valdma, 2007. A general classification of information and systems. Department of Electrical power engineering. Tallinn University of Technology: Estonia.

Vaughan, T. S., 1993. Variable sampling interval np process control chart. Community Statistical Theory Methods. Vol. 22, 147-167.

Vlok, P.J., Coetzee, J.L., Banjevic, D., Jardine, A.K.S, Makis, V., 2002. Optimal component replacement decisions using vibration monitoring and the proportional-hazards model. Journal Operational Research SocialVol. 53, No. 2, 193-202.

Waeyenbergh, G. and Pintelon, L., 2002. A framework for maintenance concept development. International Journal of Production Economics, Vol. 77, 299-313.

Wallace, R., Blischke D. N., Prabhakar, M., 2000. Case Studies in Reliability and Maintenance. John Wiley & Sons, Inc., Hoboken, New Jersey. Canada.

Wallance. W.L., 1971. The logic of science in sociology. Aldine Altherton, Chicago. USA.

Wang, H.Z., 2002. A survey of maintenance policies of deteriorating systems. European Journal of Operational Research, Vol. 139, No. 3, 469-489.

Wang, C.H., 2005. An optimal production and maintenance policy for imperfect production systems. Naval Research Logistic. Vol. 53, No. 2, 151-156.

Wang, W., Zhang, W., 2008. Early defect identification: application of statistical process control methods. Journal of Quality in Maintenance Engineering, Vol. 14, No. 3, 225-236.

Wang, L., Hu H, Wang. Y., 2011. The availability model and parameters estimation method for the delay time model with imperfect maintenance at inspection. Journal Applied Mathematic Model, Vol. 35, No. 6, 2855-2863.

Wang, W., 1997. Subjective estimation of the delay time distribution in maintenance modeling. European Journal of Operational Research, Vol. 99, 516-529.

Wang, W., 2003. Modeling condition monitoring intervals: A hybrid of simulation and analytical approaches, Journal of the Operational Research Society, Vol. 54, No. 3, 273-282.

Wang, W., 2007. A two-stage prognosis model in Condition-Based Maintenance. European Journal Operational Research, Vol. 182, No. 3, 1177-1187.

Wang, W., 2012. A simulation-based multivariate bayesian control chart for real time CBM of complex systems. European Journal of Operational Research, Vol. 218, No. 3, 726-734.

Wang, W., 2012. An overview of the recent advances in delay-time-based maintenance modeling. Reliability Engineering and System Safety. Vol. 106, 165-178.

Wang, W., Banjevic, D., Pecht., M., 2010. A multi-component and multi-failure mode inspection model based on the delay time concept. Journal of Reliability Engineering System and Safety Vol. 95, No. 8, 912-920.

Wang, W., Christer A. H., 1997. A modelling procedure to optimize component safety inspection over a finite time horizon. International Quality and Reliability Engineering.

Wang, W., Hussin, B., Tim, J., 2012. A case study of Condition Based Maintenance modeling based upon the oil analysis data of marine diesel engines using stochastic filtering. International Journal of Production Economics, Vol.136, 84-92.

Wang, W., Jia, X., 2007. An empirical Bayesian approach in delay time maintenance model parameters estimation using both subjective and objective data. Quality Maintenance and reliability International, Vol. 23, 95-105.

Webster, J.G., Eren, H., 2014. Measurement, Instrumentation, and Sensors Handbook. Second Edition: Set CRC PRESS, Taylor and Francis Group.

Wee, H.M:, Widyadana, G.A., 2013. A production model for deteriorating items with stochastic preventive maintenance time and rework process with FIFO rule. Omega, Vol.41, No. 6, 941-954.

Weide, J.A.M., Noortwijk, J.M., 2007. Renewal theory with exponential and hyperbolic discounting. Probability in the Engineering and Informational Sicences, Vol. 22, 53-74.

Werbińska, S., Paweł Zając, W., 2015. Use of delay-time concept in modelling process of technical and logistics systems maintenance performance. Case study, Maintenance and Reliability, Vol.17, No. 2.

White, C.C., 1977. A Markov quality control process subject to partial observation. Management Science, Vol. 23, 843-852.

Wireman, T., 1990. World class maintenance management. Industrial Press Inc: USA.

Wu, J. and Makis, V., 2007. Economic and economic-statistical design of a chi-square chart for Condition Based Maintenance. European Journal of Operational Research, Vol. 188, No. 2, 516-529.

Wu, S., and Wang, W., 2005. Optimal inspection policy for three-state systems monitored by control charts. Applied Mathematics Computation, Vol. 217, No. 23, 9810-9819.

Xiaoyue, J., Kan, C., and Makis, V., 1998. On the optimality of repair-cost-limit policies. Journal of Applied Probability, Vol. 35, No. 4, 936-949.

Yam, R. C. M., Tse, P. W., Li, L. and Tu, P., 2001. Intelligent predictive decision support system for CBM. International Journal Advanced Manufacturing Technology, Vol. 17, 383-391.

Yang, S.K.A., 2003. Condition-based failure prediction and processing-scheme for preventive maintenance. Reliability, IEEE Transactions on Reliability,Vol. 53, No. 3, 373-383.

Yang, Y., Klutke, G.A., 2000. Lifetime-characteristics and inspection-schemes for Levy degradation process. IEEE Trans Reliability, Vol. 49, No. 4, 377-382.

Yeung, T., Cassady, C. R., and Schneider, K., 2008. Simultaneous optimization of X-bar control chart and age-based preventive maintenance policies under an economic objective. Journal IIE Transactions , Vol. 40, No. 2, 147-159.

Yin, Z. and Makis, V., 2011. Economic and economic-statistical design of a multivariate Bayesian control chart for condition based maintenance. Journal of Management and Mathematics. Vol. 22, No. 1, 47-63.

Yin, G., 2008. Bayesian transformation cure frailty models with multivariate failure time data. Computational Statistic and Data Analysis, Vol. 54, 1921-1929.

Yu, J.B., 2011. Bearing performance degradation assessment using locality preserving projections. Expert system with application, Vol. 38, No. 6, 7440-7450.

Zhan, Y., Makis, V., and Jardine, A.K.S., 2006. Adaptive state detection of gearboxes under varying load conditions based on parametric modeling. Mechanical Systems Signal, Vol. 20, 188–221.

Zhang, G., andBerardi, V., 1996. Economic statistical design of x-bar control charts for systems with Weibull in-control times. Computers and Industrial Engineering, Vol. 32, No. 3, 575-586.

Zhang, G., Berardi, V., 1997. Economic statistical design of x-bar control charts. Journal of Computers and Industrial Engineering, Vol. 32, N. 3, 575-586.

Zhao, Z., Wang, F.L., Jia, M.X., and Wang, S., 2010. Predictive maintenance policy based on process data. Chemo metrics and Intelligent Laboratory Systems, Vol. 103, 137-143.

Zhijian, Y., 2008. Multivariate Bayesian process control. PhD thesis, University of Toronto.

Zhou, P., and Liu, D., 2011. Research on marine diesel's fault prognostic and health management based on oil monitoring. Prognostics & System Health Management Conference: China, 1-4.

Zhou, W. H., and Zhu, G. L., 2008. Economic design of integrated model of control chart and maintenance. Mathematical and Computer Modeling, Vol. 47, 1389-1395.

Zikmund, W.G. 2001. Exploring Marketing Research. (7th ed.). Fort Worth: The Dryden Press.

Zikmund, W.G., 2010. Business research methods. Mason, OH: South-Western Cengage Learning.

Annex A Descriptive analysis

Table A.1. Case Processing Summary

	Cases					
	Valid		Missing		Total	
	N	Percent	N	Percent	N	Percent
Temp_filter	15159	100.0%	0	0.0%	15159	100.0%
Vibr_Atox Mill	15159	100.0%	0	0.0%	15159	100.0%
Power_Motor	15159	100.0%	0	0.0%	15159	100.0%
Press_Atox Mill	15159	100.0%	0	0.0%	15159	100.0%
Size_particule	10047	66.3%	5112	33.7%	15159	100.0%

Table A.2. Descriptives

			Statistic	Std. Error
Temp_filter	Mean		124.7978	.39147
	95% Confidence Interval for Mean	Lower Bound	124.0304	
		Upper Bound	125.5651	
	5% Trimmed Mean		124.7208	
	Median		119.6400	
	Variance		2323.095	
	Std. Deviation		48.19849	
	Minimum		.00	
	Maximum		304.68	
	Range		304.68	
	Interquartile Range		24.32	
	Skewness		.218	.020
	Kurtosis		.735	.040
Vibr_Atox Mill	Mean		2.5078	.01195
	95% Confidence Interval for Mean	Lower Bound	2.4843	
		Upper Bound	2.5312	
	5% Trimmed Mean		2.5223	
	Median		3.3000	
	Variance		2.163	
	Std. Deviation		1.47083	
	Minimum		.40	
	Maximum		5.30	
	Range		4.90	
	Interquartile Range		3.20	
	Skewness		-.524	.020

	Kurtosis		-1.532	.040
Power_Motor	Mean		1323.5150	7.81822
	95% Confidence Interval for Mean	Lower Bound	1308.1904	
		Upper Bound	1338.8397	
	5% Trimmed Mean		1339.1212	
	Median		1951.5600	
	Variance		926586.415	
	Std. Deviation		962.59359	
	Minimum		.00	
	Maximum		2490.79	
	Range		2490.79	
	Interquartile Range		2032.95	
	Skewness		-.574	.020
	Kurtosis		-1.555	.040
Press_Atox Mill	Mean		33.29	.189
	95% Confidence Interval for Mean	Lower Bound	32.92	
		Upper Bound	33.66	
	5% Trimmed Mean		34.04	
	Median		49.00	
	Variance		541.671	
	Std. Deviation		23.274	
	Minimum		0	
	Maximum		63	
	Range		63	
	Interquartile Range		51	
	Skewness		-.679	.020
	Kurtosis		-1.492	.040
Size_particule	Mean		12.7287	.00757
	95% Confidence Interval for Mean	Lower Bound	12.7139	
		Upper Bound	12.7436	
	5% Trimmed Mean		12.6544	
	Median		13.0000	
	Variance		.576	
	Std. Deviation		.75920	
	Minimum		11.00	
	Maximum		19.00	
	Range		8.00	
	Interquartile Range		1.00	
	Skewness		2.089	.024
	Kurtosis		10.556	.049

Table A.3. Extreme Values

			Case Number	Value
Temp_filter	Highest	1	14415	304.68
		2	3526	299.68
		3	3525	293.60
		4	14226	289.32
		5	14225	288.75
	Lowest	1	6577	.00
		2	6576	.00
		3	6575	.00
		4	6574	.00
		5	6573	.00[a]
Vibr_Atox Mill	Highest	1	2175	5.30
		2	2398	5.30
		3	2176	5.00
		4	12343	4.90
		5	351	4.80[b]
	Lowest	1	6310	.40
		2	6309	.40
		3	6308	.40
		4	6307	.40
		5	6306	.40[c]
Power_Motor	Highest	1	12641	2490.79
		2	14123	2460.65
		3	13341	2452.70
		4	13278	2450.51
		5	14122	2440.61
	Lowest	1	15101	.00
		2	14784	.00
		3	14783	.00
		4	14782	.00
		5	14780	.00[a]
Press_Atox Mill	Highest	1	4333	63
		2	4332	57
		3	4442	56
		4	4443	56
		5	7687	56[d]
	Lowest	1	15102	0
		2	15101	0
		3	15100	0
		4	15061	0

		5	15060	0[a]
Size_particule	Highest	1	1340	19.00
		2	3034	19.00
		3	7653	19.00
		4	8693	19.00
		5	8958	19.00[e]
	Lowest	1	9368	11.00
		2	9367	11.00
		3	9366	11.00
		4	9365	11.00
		5	9364	11.00[f]

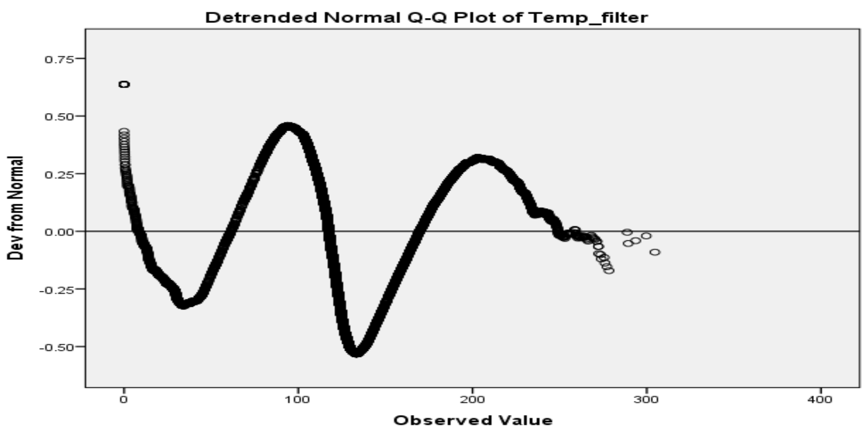


Figure A.1. Detrended Normal Q-Q plot of Temperature_filter.

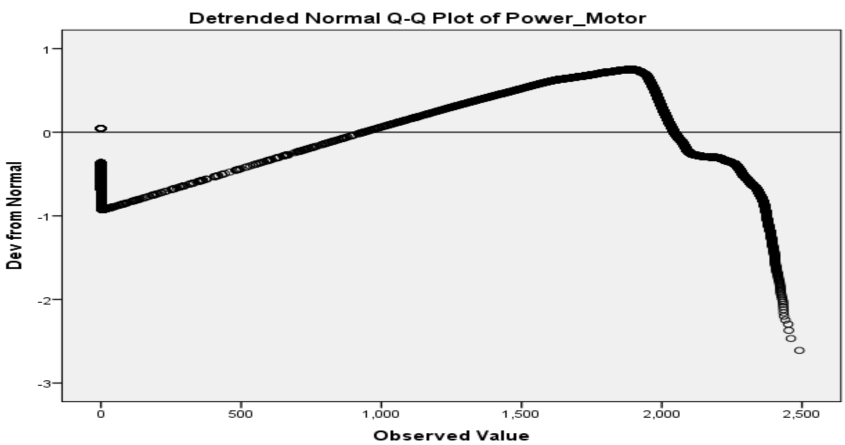


Figure A.2. Detrended Normal Q-Q plot of Power_filter.

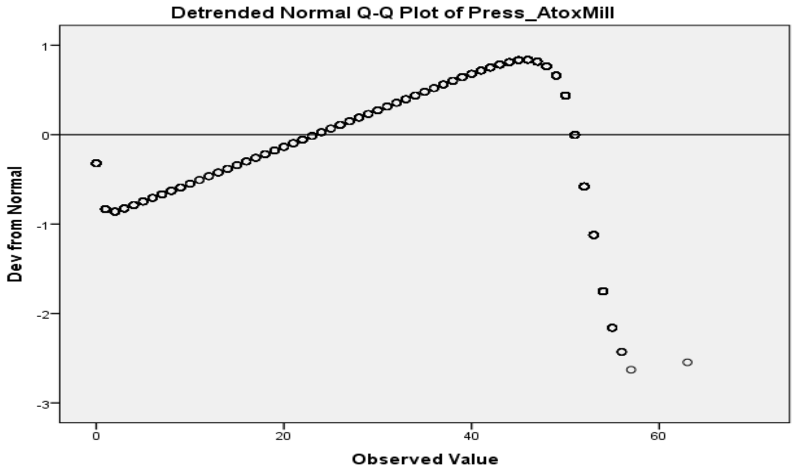


Figure A.3. Detrended Normal Q-Q plot of Power_filter.

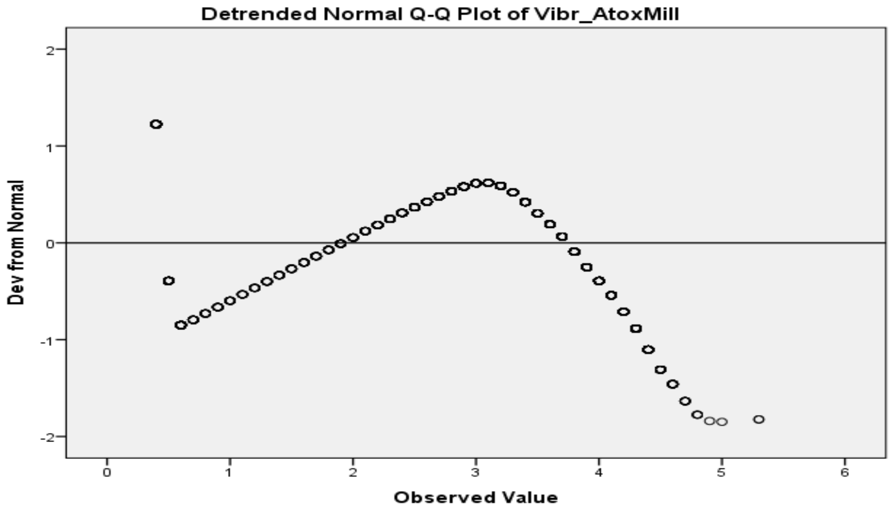


Figure A.4. Detrended Normal Q-Q plot of Power_filter.

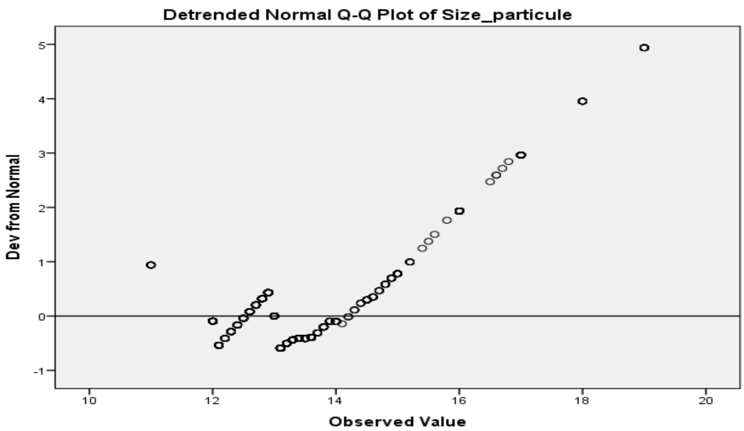
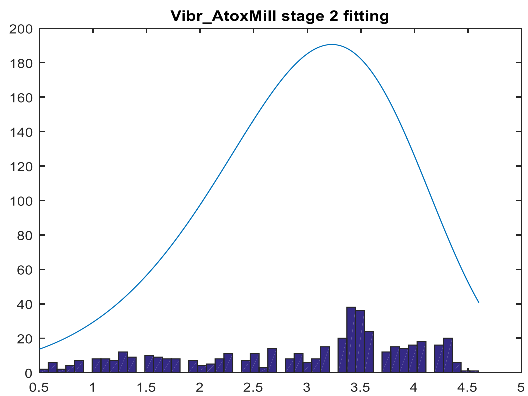


Figure A.5. Detrended Normal Q-Q plot of size_particle.

AnnexBFitting Weibull distribution for observed data monitoring



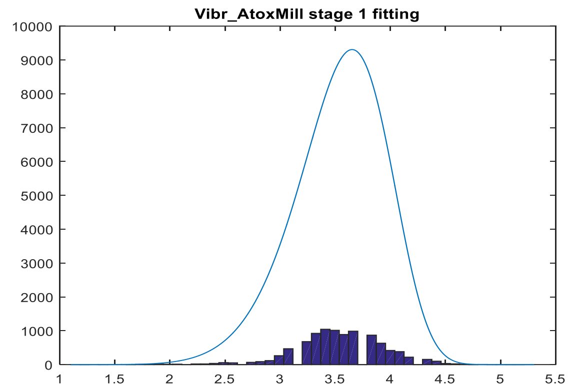
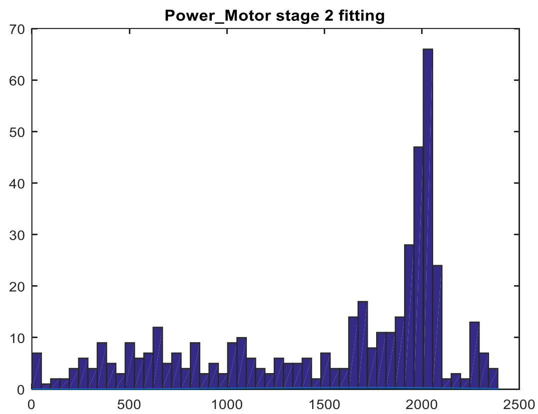


Figure B.1- Weibull distribution plot of vibration in stag e 2.

Figure B.2- Weibull distribution plot of vibration Data in stage 1



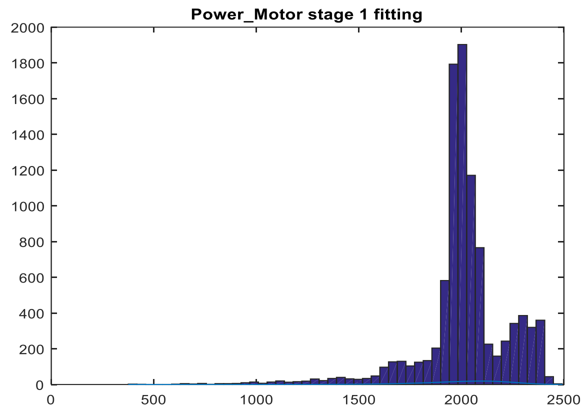


Figure B.3- Weibull distribution plot of power'motor Data in stage 2.

Figure B.3- Weibull distribution plot of power'motordata in stage 1.

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