Instability in cointegration regressions: a brief review with an application to money demand in Portugal

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This study addresses some modelling questions related to the possibility of structural change in models with nonstationary variables. Focusing on cointegration issues, some methodological aspects are discussed, attempting to integrate coherently the several steps of the modelling strategy. These range from unit root to cointegration testing and to testing for instability in the cointegration vector. An empirical example with Portuguese data tries to illustrate the usefulness of this approach, where a simple money demand function is estimated using an error-correction model (ECM). If a break is explicitly allowed in the cointegration vector the forecasting performance of the ECM improves.

I. INTRODUCTION

In their search for simple and interpretable models, which describe the fundamentals of economic relations, economists are becoming more aware of the importance that structural changes can have on their analysis. In fact, models that implicitly assume a constant and stable economic environment, disregard an essential aspect of economic reality. Taking into account that the instability of the economic system may be reflected in the parameters of the models which try to describe it, their use for inference, policy simulation and forecasting may lead to very misleading results. The perception of these implications is notorious, as confirmed by the remarkable growth of theoretical and empirical research on the subject.

The emphasis of this paper is on linear regression models with cointegrated variables. In this context, several test procedures have recently appeared in the literature dealing with the problem of structural changes. A methodology is discussed that incorporates these new tests in a cointegration analysis, and illustrate how this may be useful for a more correct specification of an econometric model.

Sections II to IV briefly review some recent test procedures related to cointegration and structural changes in models with nonstationary variables. In Section V, methodological aspects are discussed, attempting to integrate coherently the several steps of the modelling strategy. Subsequently, in Section VI, an empirical example is presented, focusing on a simple model for Portuguese money demand, where the study tries to show the importance of considering the possibility of structural changes. The paper ends with some concluding remarks.

II. TESTING PARAMETER INSTABILITY WITH NONSTATIONARY VARIABLES

When a time series model is estimated, one frequently wants to know if the assumed relationship is temporally stable, that is, whether all model coefficients (or some of

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them) are the same for different sub-periods of the available sample. One of the basic hypotheses of the linear regression model is parameter constancy throughout the sample. Hence, testing for structural change is a form of testing the model specification, bearing at the same time important consequences in terms of economic analysis.

The most commonly used test has been, perhaps, the 'sample-split' Chow (1960) test. However, this procedure requires previous knowledge of the possible break point, that is, any event (changes in economic policy, institutional changes, etc.) which could lead to the suspicion that one is facing distinct economic conditions. The situation when there is no a priori information requires a particular type of analysis, one for which the adopted solution has been to endogenize the break-point selection in the testing problem, maintaining the inference valid. See Quandt (1960) and Davies (1977) for a first approach, and Nyblom (1989) and Andrews and Ploberger (1994), inter alia, for an extension to a class of optimal tests in terms of local power.

These tests were developed upon the assumption that the involved variables are stationary. Here, we are mainly interested in models with nonstationary and cointegrated variables.

Consider the following model:

$$\begin{aligned} y_t &= \beta' x_t + u_t, \qquad t = 1, \dots, T \\ \Delta x_t &= v_t \end{aligned} \tag{1}$$

where u_t and v_t are stationary disturbances and $x_t = (x_{1t}, \dots, x_{kt})'$ is a vector that may contain observations of deterministic components. Allowing β to depend on t, that is, considering $y_t = \beta_t' x_t + u_t$, then the null hypothesis for all tests is

$$H_0 : \beta_1 = \beta_2 = \cdots = \beta_T \tag{2}$$

meaning that the elements of the cointegration vector are temporally stable.

Using this framework, it is possible to specify two alternative hypothesis of coefficient instability, one corresponding to the usual single break of unknown timing, m,

$$\begin{aligned} H_1 : \beta_t &= \beta_1, t \leq m \\ \beta_t &= \beta_2, t > m \end{aligned}$$

the other treating β as a vector martingale process

$$H_1' : \beta_t = \beta_{t-1} + \varepsilon_t \qquad E(\varepsilon_t \varepsilon_t') = \gamma^2 Q$$

with zero mean, stationary and serially uncorrelated ε_t's. In this last case, the null hypothesis can be rewritten as a nullity restriction upon the variance of the martingale difference sequence formed by the ε_t's, i.e., $H_0 : \gamma^2 = 0$. This way, β_t is no longer a stochastic process, remaining constant along the period considered.

For the first alternative, H_1, Hansen (1992) proposed a test in the spirit of Quandt (1960), making use of the statistic

$$\sup -F = \sup_{\lambda \in J} F_{Tm} \tag{3}$$

that is, it consists in taking the largest of the statistics in a sequence of F_{Tm} tests, these corresponding to an LM formulation of the usual Chow tests. The above λ denotes the relative position of the break in the sample, so that $[T\lambda] = m$, with [.] denoting the integer operator. We take the usual trimming interval $J = (0.15, 0.85)$.

If the alternative hypothesis is that of time-varying parameters, H_1', then other procedures should be used. One of these is a test presented by Hansen (1992), also based on the F_{Tm} statistics, which simply requires the computation of the average value of the F_{Tm} sequence, that is, the *mean-F* test. A test that is similar to this one but which does not require the trimming of the sample is Hansen's (1992) nonstationary counterpart of the test discussed by Nyblom (1989), labelled L_c and of the form

$$L_c = T^{-1} \sum_{t=1}^{T} \hat{S}_t' \hat{V}^{-1} \hat{S}_t \tag{4}$$

where $\hat{S}_t$ represent the 'scores' of the FM-OLS estimation and $\hat{V}^{-1}$ is a weighting matrix based on an estimate of the covariance matrix $\gamma^2 Q$.

A different test, originally proposed by Andrews and Ploberger (1994) and extended to a nonstationary context by Hao (1996), is an exponential average of the type

$$Exp\text{-}F = \log \sum_{\lambda \in J} \exp\left(\frac{1}{2} F_{Tm}\right) \tag{5}$$

As demonstrated by Andrews and Ploberger (1994) and Hao (1996), these tests share the property of local asymptotic optimality. Monte Carlo simulations conducted by these authors show the relatively good performance of the procedures and tend to suggest that the *Exp-F* test is the most well-balanced of all, in terms of power and size distortion.

III. TESTING FOR COINTEGRATION

An important feature of the above tests is the possibility of being used as cointegration tests. In fact, when the alternative hypothesis is that of the intercept following a random walk, structural change testing becomes cointegration testing, the null hypothesis being that of cointegration. Writing model (1) as $y_t = \beta_1 + \beta_2' x_{2t} + u_t$, if y_t and x_{2t} are not cointegrated then the error term u_t is integrated of order one. Decomposing u_t such that $u_t = w_t + v_t$,

w_t denoting a random walk and v_t a stationary term, then the model can be written as

$$y_t = \beta_{1t} + \beta_2' x_{2t} + v_t \quad (6)$$

where $\beta_{1t} = \beta_1 + w_t$, that is, with the intercept 'absorbing' the random walk w_t when there is no cointegration. This is a special case of partial structural change, since only the constant term is allowed to follow a martingale, thus rendering impossible a cointegrating relationship.

Having this fact in consideration, Hansen (1992) suggested the use of the L_c statistic when testing the null of cointegration against the alternative of no cointegration. However, as this statistic was designed to test the stability of the whole cointegration vector, there are advantages in considering a version that only tests (partial) structural change in the intercept. Hao (1996) developed this version and arrived to a known statistic, already used by Kwiatkowski, Phillips, Schmidt and Shin (1992) (KPSS) to test for stationarity and by Shin (1994) to test for the null of cointegration. Furthermore, Hao (1996) obtained this type of version for the other tests discussed above, labelling them L_c^0, sup-F^0, mean-F^0 and Exp-F^0.

Nevertheless, as pointed out by Hansen (1992), a researcher should be cautious in interpreting the outcomes from these tests, since a rejection does not entangle the immediate acceptance of the alternative hypothesis for which they were designed. For instance, if the sup-F test rejects its null hypothesis, that should not be automatically interpreted as evidence that there are two cointegration regimes (as hypothesized in the alternative) and, at the same time, a rejection by the L_c test does not mean that there is no cointegration. Any of these tests has power against each of the alternatives considered here (H_1 and H_1'). Hence, the only plausible conclusion one can draw is that the traditional specification of a cointegration model such as Equation 1, assuming parameter stability, is not supported by the data. The same applies to partial structural change tests used as cointegration tests.

Having this in mind, Hao (1996) presented a robust test for cointegration, with the purpose of overcoming an eventual rejection of the null hypothesis due to a discrete break in the constant term. Given that the change point is assumed as unknown, the test consists of taking the smallest L_c^0 statistic computed for all possible break dates, that is, the test statistic is $\inf_{\lambda \in J} L_c^0$. The model is now written as

$$y_t = \mu_1 + \mu_2 D_t + \beta' x_{2t} + u_t \quad (7)$$

where D_t is a dummy variable equal to 0 if $t \leq [T\lambda]$ and to 1 if $t > [T\lambda]$. Again, there is no cointegration when the error term is $I(1)$ and it can be decomposed, as previously, into a random walk (which is absorbed by μ_1, producing μ_{1t}) and a stationary component.

IV. COINTEGRATION TESTS ALLOWING FOR REGIME SHIFTS

The power properties of the augmented Engle-Granger (AEG) test deteriorate substantially when there is a break in the cointegrating relationship. In such circumstances, the usual cointegration tests (just as the unit root ones) would hardly indicate a result of cointegration, since the existence of breaks is almost indistinguishable of nonstationary error terms. Gregory and Hansen (1996a,b) propose some modifications of the usual procedures in order to accommodate, under the alternative, the possibility of changes in the cointegration vector. In their most general formulation a regime and trend shift model (R/T) is considered,

$$y_t = \mu_1 + \mu_2 D_t + \alpha_1 t + \alpha_2 t D_t + \beta_1' x_t + \beta_2' x_t D_t + u_t \quad (8)$$

where x_t is an $I(1)$ vector of dimension k, u_t is stationary and D_t is a dummy variable as in Equation 7. Other formulations include a regime shift with trend model (R) when $\alpha_2 = 0$, a level shift with trend model (C/T) when $\alpha_2 = \beta_2 = 0$, and a level shift model (C) when $\alpha_1 = \alpha_2 = \beta_2 = 0$.

As is well known, the most popular cointegration tests, such as the AEG and the Phillips and Ouliaris (1988) Z_α and Z_t tests, are based on OLS residuals, evaluating when the error term, u_t, is $I(1)$ under the null (that is, no cointegration). In this framework, since the change point is unknown, the solution involves again the computation of the usual statistics for all possible break points and selecting the smallest value obtained: $\inf_{\lambda \in J} Z_\alpha$, $\inf_{\lambda \in J} Z_t$ and $\inf_{\lambda \in J} AEG$. A Monte Carlo simulation conducted by the authors showed that these tests have reasonable power, and that the inf Z_t test seems to be the most well-balanced of the three.

V. SOME METHODOLOGICAL ISSUES

From the previous review, it could be relevant to think of an integrated and coherent set of procedures to follow when the researcher wishes to model relationships between nonstationary variables. As previously mentioned, when testing for cointegration one must bear in mind that the traditional tests (AEG, Phillips-Ouliaris (PO), Johansen, etc.) are not the most adequate if there occurred breaks in the cointegration vector, since they fail to reject the null hypothesis of no cointegration less often than they should, inducing the researcher to conclude that a long run equilibrium relationship does not exist.

To overcome this difficulty, the tests of Gregory and Hansen should be used. However, these tests detect cointegration relationships subjected to regime shifts as well as invariant cointegration vectors. Therefore one should be cautious when interpreting a rejection of the null hypothesis of no cointegration. In such cases, Gregory and Hansen (1996a,b) advocate complementing the use of their procedures with those reviewed in Section II.

Another possibility to test for cointegration is to resort to the new cointegration tests (L_c^0, $Exp\text{-}F^0$, etc.) that detect situations where the intercept behaves like a random walk (non-cointegration). However, as these tests also capture single breaks in the constant term, false rejections of the null hypothesis of cointegration may emerge. To remedy this undesired property, one may apply the robust cointegration test $\inf L_c^0$ of Hao (1996).

Facing the results of the joint use of these procedures, three conclusions may be drawn. One is the inexistence of a long run relationship between the analysed series (at least of the form treated here), and it may be achieved if the tests are consensual, that is, non-rejection of the hypothesis of no cointegration by the usual tests and the Gregory-Hansen tests, and rejection of the null hypothesis of cointegration using the other new tests.

Another possible conclusion is the existence of 'Engle-Granger' cointegration with a stable cointegration vector. Even if the usual tests do not allow for this inference, the Gregory-Hansen and these recent tests may point for the existence of cointegration. The stability of the cointegration vector would be admissible if confirmed by the instability tests of Section II.

Last, the evidence suggested by the data may indicate a situation of cointegration with structural change, when the usual tests do not support the case for cointegration, but the Gregory-Hansen tests do and the instability is confirmed by the respective tests.

A different question is raised when evaluating the stability of an error-correction model (ECM), constructed in two steps from an initial estimate of the equilibrium errors. In this case, assuming cointegration, as all the variables are stationary the tests are the stationary counterparts of those discussed in Section II. If one applies the instability tests, then the focus is on the short run dynamics, since the long run relationship is estimated previously. Two situations may occur when interpreting the test results: one, in which the tests detect instability, and other in which stability is not rejected.

The first leads the researcher to reject the proposed specification and to look for a model with stable coefficients, probably implying a return to the initial stage of specification of the cointegration relationship. In the second case, even if the short run dynamics present no instability problems, it does not imply an invariant long run relationship, since the instability of the cointegration vector may appear mitigated in the ECM, that is, the tests may fail to reveal the instability of the coefficient of the error-correction term. Therefore, it is essential to test the stability of the cointegration vector in the first step. Otherwise one faces the risk of arriving at an apparently reasonable model, but with a misspecified long run relationship (with the negative consequences that this may produce, for instance, in terms of forecasting). To illustrate this, an empirical application is presented in the next section.

VI. MONEY DEMAND IN PORTUGAL: A SIMPLE MODEL

The money demand function has always been at the centre of the economic debate, both in theoretical and in empirical terms, mainly because of the implications that it may have for economic policy purposes. Here, we provide a simple illustration of the previous discussion, complementing the available empirical analysis for the Portuguese case. Two recent studies dealt with the subject, namely Sousa (1996) and Covas (1996), and the results obtained by the latter are taken as the departing point. Covas considered that real money demand is related to the level of real economic activity (measured by GDP), the inflation rate (obtained from the Consumer Price Index) and to a 6-months deposits interest rate. His long-run model is

$$(m_t - p_t) = \beta_1 + \beta_2 y_t + \beta_3 \Delta p_t + \beta_4 r_t + \beta_5 t + u_t \qquad (9)$$

where m is the log of the aggregate M_1, p is the log of the CPI, y is the log of GDP, Δp gives the inflation rate and r is the interest rate. The data is quarterly and covers the period between the first quarter of 1977 and the fourth quarter of 1996. The last year will be taken out of the estimation sample for the forecasting evaluation exercise. Johansen's procedure suggests the existence of a unique cointegration vector. Furthermore, as he also found evidence supporting the weak exogeneity of y_t, Δp_t and r_t (for the parameters of the cointegration vector), the conditional ECM for $(m_t - p_t)$ seems to provide a sound basis for conducting valid inferences.

However, Portugal suffered several institutional and policy changes in the last 20 years. The monetary system was highly nationalized until a few years ago, with controlled interest rates, and since the mid-1980s some innovations were introduced along with liberalization measures. It is very likely that these changes affected real money demand, as can be seen graphically in Fig. 1, and perhaps the long run relationship modelled through (9). This is what we will try to ascertain resorting to the tests discussed in the previous sections. Anyway, notice that this suspicion is reinforced by the poor forecasting ability of Covas's (1996) model.

Preliminary statistical analysis

Results for the 'classical' tests (ADF and Phillips-Perron Z_α test) and the KPSS stationarity test, support that all variables in the model are $I(1)$. Zivot and Andrews (1992)

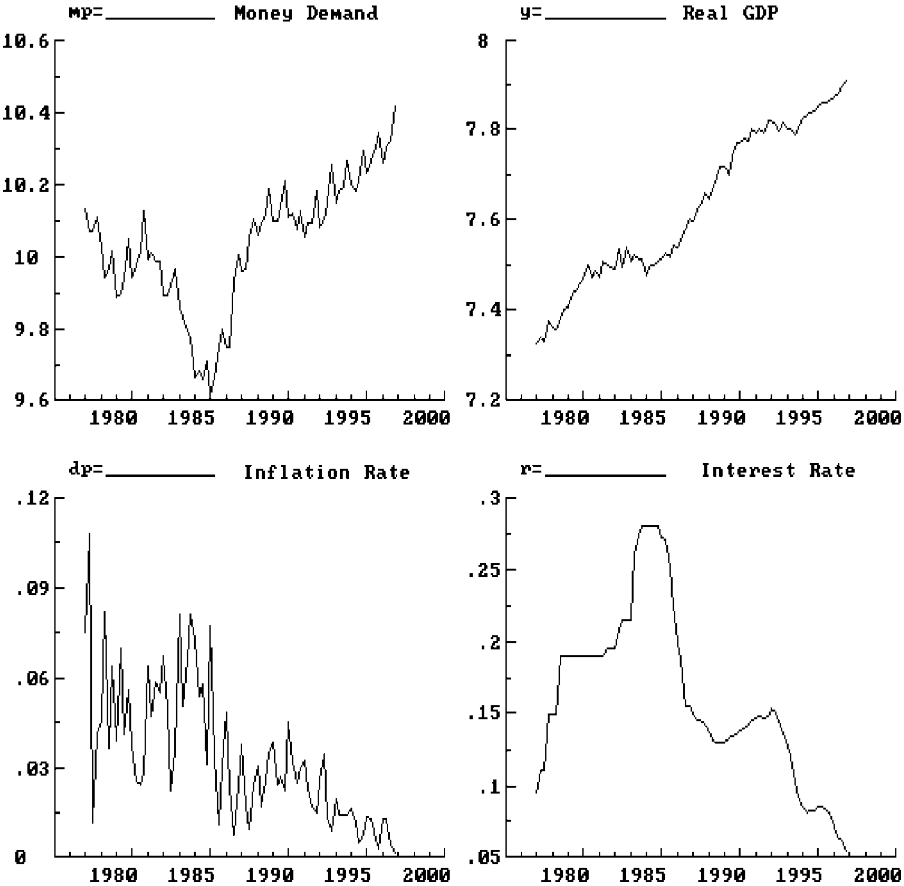

Figure 1. *Times series graphs of Portuguese data*

tests for a unit root allowing for a regime shift in the intercept and/or trend also confirm these results, thus justifying the cointegration analysis. These tests were also computed as in Nunes *et al.* (1997) with the null of a unit root with a break at an unknown time. Since the estimated break dates from the unit root tests were all different, these do not seem to carry much information for multivariate analysis. Detailed results are available from the authors upon request.

In what concerns evidence on cointegration, an expected contradiction arises when comparing the results from the different tests (Table 1). The conventional tests suggest that there is no cointegration, and this evidence is also supported by the L_c test, whereas the Shin-L_c^0 and $\inf L_c^0$ tests favour cointegration. Anyway, as previously pointed out, evidence against cointegration provided by the L_c test can arise in a situation where there is a break in the intercept. On the other hand, one should recall that the $\inf L_c^0$, test was built to be robust in this situation.

The Gregory-Hansen tests may be useful to further clarify this issue. The results presented in Table 2 generally provide evidence favouring cointegration. The estimated break dates range between the fourth quarter of 1983 and the first quarter of 1991. Notice that the $\inf Z_t$ statistic always rejects the null of no cointegration and that the

Table 1. *Cointegration tests*

H_0 : No cointegration		H_0 : Cointegration	
AEG (*k*)	−3.504(4)	*Shin*-L_c^0	0.069
PO-Z (*l*)	−43.151(4)	$\inf L_c^0$	0.032
		L_c (*p-value*)	5.781(< 0.01)

$\inf Z_\alpha$ and $\inf AEG$ statistics for models C and C/T also provide strong evidence for cointegration. It must be recalled that all the models involve a shift in the intercept and that, according to Gregory and Hansen's (1996a,b) simulations, the $\inf Z_t$ is the most powerful when this situation occurs.

Overall, evidence points to the existence of cointegration, possibly with at least one regime shift. Consequently, given the preceding methodological notes, it is indispensable to test for instability in the cointegration vector, in order to ascertain the existence of structural change.

Cointegration vector and ECM stability

For the estimation of the cointegration vector we resorted to the FM-OLS estimator of Phillips and Hansen (1990), mainly because the asymptotic theory for structural change

Table 2. *Gregory-Hansen tests*

Tests	C	C/T	R	R/T
inf Z_t ($\hat{m}$)	−6.858**(84:1)	−6.941**(84:3)	−7.041**(84:1)	−7.277**(90:1)
inf Z_α ($\hat{m}$)	−58.750*(84:1)	−61.451*(84:3)	−60.857(84:1)	−63.581(90:1)
inf *AEG* ($\hat{m}$)	−5.502*(83:4)	−6.114**(85:4)	−4.459(83:4)	−5.892(84:4)

Notes: *−5% significant statistic; **−1% significant statistic; $\hat{m}$-estimated break date.

Table 3. *Cointegration vector estimates*

Coefficients	OLS	FM-OLS
β_1	0.845 (0.591)	2.892 (1.932)
$\beta_2(y)$	1.300 (6.720)	1.027 (5.804)
$\beta(\Delta p)$	−0.466 (−1.181)	−0.286 (−4.392)
$\beta(r)$	−2.291 (−15.570)	−2.588 (−15.788)
$\beta(t)$	−0.0086 (−6.101)	−0.0067 (−4.589)

Note: *t*-statistics in parentheses.

Table 4. *Instability tests*

Tests	
sup-*F* (*p-value*)	738.269 (< 0.01)
mean-F (*p-value*)	45.190 (< 0.01)
Exp-F (*p-value*)	365.127 (< 0.01)
L_c (*p-value*)	2.206 (< 0.01)
Estimated break date	1990.2

Note: The *p-values* were computed using the GAUSS code kindly provided by Bruce Hansen.

testing is available for this method. Briefly, this method corrects for the finite sample bias in the OLS estimator of the static regression, providing median-unbiased and asymptotically normal estimates of the (long run) parameters of the cointegration vector.

For comparison purposes, Table 3 also presents the OLS estimates. As expected, the two methods propose somewhat different values for the parameters. Notice the FM-OLS estimate of β_2, which is very close to the homogeneity hypothesis, and that all the variables appear significant, according to the 'modified' *t*-statistics.

The results of the test statistics for evaluating the cointegration vector stability (see Table 4) all clearly reject the null hypothesis of constant parameters. Therefore, if there is cointegration, it possibly concerns one that has suffered structural change(s). The sup F test statistic suggests as break point the second quarter of 1990. This is a reasonable date, since a substantial set of monetary liberalization measures were taken at approximately that time.

Another problem may arise when this cointegration vector estimate is used in the estimation of an error-correction model, since it might be conjectured that the obtained relation presents instability problems, at least concerning the coefficient of the error-correction term (the remaining short run dynamics not revealing the presence of the problem). Starting from a generic ECM with lag order of 5, one arrives at the following, more parsimonious, model:

$$\Delta(m_t - p_t) = \underset{\substack{(6.819)\\ [0.14]}}{0.054} - \underset{\substack{(-5.022)\\ [0.15]}}{0.389}\,\hat{u}_{t-1} + \underset{\substack{(2.783)\\ [0.08]}}{0.396}\,\Delta y_t - \underset{\substack{(-4.188)\\ [0.08]}}{1.238}\,\Delta r_{t-1} - \underset{\substack{(-11.198)\\ [0.15]}}{0.123}\,D_{1t} - \underset{\substack{(-6.460)\\ [0.28]}}{0.063}\,D_{2t} - \underset{\substack{(-3.238)\\ [0.09]}}{0.035}\,D_{3t} + \hat{\varepsilon}_t \tag{10}$$

where $\hat{u}_t$ is the estimated equilibrium error and the D_{it} ($i = 1, 2, 3$) are seasonal dummies. The *t-ratio* (inside parentheses) for the coefficient of the error-correction term can be used for cointegration testing (t_{ECM}). In fact, the value of −5.022 comfortably rejects the null of no cointegration, even if the critical values from MacKinnon (1991) are used. This reinforces the evidence of cointegration, but says nothing about the possibility of structural change.

Since one is assuming the existence of cointegration, one can compute 'stationary' stability tests for each coefficient and for the whole regression. (These tests correspond to the Nyblom (1989) tests adapted by Bruce Hansen and computed by PCGIVE. The critical values are 0.47 for the individual tests and 2.54 for the global test, for 5% significance level tests.) The individual test statistics are presented inside square brackets, below the *t*-ratios, and the value for the global test statistic is 1.289. None of the tests rejects the null of coefficient stability, including the coefficient of the error-correction term which is associated with a cointegration vector whose instability seems to be clear. This probably means that the instability of the long run relationship is attenuated by the 'interaction' with the short run dynamics and, somehow, remains hidden when one just analyses the coefficients stability of the ECM.

The above results suggest the need to complement the evidence for cointegration, with an improved specification of the long run relationship, in order to obtain a stable cointegration vector or a relationship that explicitly models the eventual structural change(s).

Table 5. *FM-OLS estimates of the new cointegration vector*

Coefficients	1997.1–1990.2	1990.3–1995.4
β_1	−0.794 (−0.573)	5.025 (1.733)
$\beta_2(y)$	1.519 (8.123)	0.659 (1.983)
$\beta_3(\Delta p)$	−0.252 (−0.789)	−3.184 (−3.415)
$\beta_4(r)$	−2.177 (−15.014)	−0.019 (−0.041)
$\beta_5(T)$	−0.0104 (−7.704)	0.0054 (1.860)

Note: *t*-statistics in parentheses.

Reestimating the cointegration vector

To illustrate this last idea, the cointegration vector is re-estimated, assuming the existence of two 'regimes', separated at the estimated break date of 1990.2. The new estimates in Table 5 are quite different for each regime and from the ones on the second column of Table 3, a feature which is especially noticeable in the estimated GDP elasticities, now far from the homogeneity hypothesis. Also, the statistical significance of the regressors in the second period is very feeble. However, the FM-OLS method is efficient only asymptotically and its properties are very likely to deteriorate in small samples, as in this case.

Anyway, assuming that this specification is better than the previous one, it is natural that this improvement would be reflected in the ECM, namely in the magnitude and significance of the error-correction term coefficient, as well as in the forecasting ability of the model. Reconstructing the ECM, one obtains

$$\Delta(m_t - p_t) = \underset{\substack{(7.098)\\ [0.12]}}{0.050} - \underset{\substack{(-6.581)\\ [0.03]}}{0.601}\,\hat{v}_{t-1} + \underset{\substack{(3.052)\\ [0.08]}}{0.602}\,\Delta y_t - \underset{\substack{(-4.066)\\ [0.09]}}{1.109}\,\Delta r_t - \underset{\substack{(-10.339)\\ [0.06]}}{0.109}\,D_{1t} - \underset{\substack{(-7.264)\\ [0.37]}}{0.064}\,D_{2t} - \underset{\substack{(-3.810)\\ [0.07]}}{0.038}\,D_{3t} + \hat{\epsilon}_t \quad (11)$$

where $\hat{v}_t$ represents the estimated equilibrium error, now from the two sub-periods considered. Its structure is the same as the previous ECM and most of the estimated coefficients are close to those in Equation 10. However, the coefficients associated with Δy_t and $\hat{v}_{t-1}$ are now substantially larger in absolute value. We stress the *t*-ratio from the latter, −6.581, which again allows one to reject the null of no cointegration.

To confirm the superiority of this specification, a simple *ex-post* forecasting exercise was performed in an attempt to compare the forecasting ability of the two ECMs, Equations 10 and 11. The results are presented in Table 6. As expected, in terms of prediction error measures (root mean squared error, RMSE, and mean absolute error, MAE) the second ECM reveals some gains in incorporating the new equilibrium error estimate. These statistics are about 12.4% and 15.3% smaller for model 11, which is reasonable considering the total sample size and, particularly, the number of observations for the second regime. Apparently, the new ECM is more flexible in incorporating different 'structures' or 'regimes' and this flexibility is clear in terms of forecasts.

Table 6. *Prediction errors*

Periods	Model (10)	Model (11)
1996.1	−0.0208709	0.0086408
1996.2	0.0441698	0.0400536
1996.3	−0.0089913	−0.0113974
1996.4	0.0272226	0.0256318
$RMSE \times 100$	2.832	2.482
$MAE \times 100$	2.531	2.143

VII. CONCLUDING REMARKS

Despite the simplicity of the empirical analysis, merely illustrative, the paper tried to discuss and to show the usefulness of some statistical procedures that have recently appeared in the literature. Even though not allowing for definite answers, the new tests may provide information that otherwise would be concealed in the data. In particular, the paper tried to illustrate how structural change testing may be integrated in a methodology of cointegration analysis. Evidence was presented showing the time instability of a traditional long-run money demand function for Portugal, and the analysis also illustrated how the explicit incorporation of the information on structural change in the ECM may improve forecast accuracy.

Certainly, the results may be specific to the model under study. But this only reinforces the importance of considering additional misspecification tests in order to improve empirical modelling. This could be achieved, for example, by respecifying the theoretical relationship, or alternatively, by modelling explicitly eventual regime change(s) using, for instance, Markov-switching models, but this is beyond the scope of the analysis in this work. Nevertheless, the main message is clear: the traditional cointegration methodology might not be able to capture the complexity of economic problems when structural changes occur.

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