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Customer Poaching and Advertising

Rosa Branca Esteves*
University of Minho and NIPE

Abstract

This article is a first look at the dynamic effects of customer poaching in homogeneous product markets, where firms need to invest in advertising to generate awareness. When a firm can recognize customers with different past purchasing histories, it may send them targeted advertisements with different prices. It is shown that only the firm that advertises the highest price in the first period will engage in price discrimination, and that poaching clearly benefits the discriminating firm. This gives rise to "the race for discrimination effect", through which price discrimination may act to soften price competition rather than to intensify it. As a result of that, all firms might become better off, even when only one of them can engage in price discrimination. This article offers a first attempt to evaluate the effects of price discrimination on the efficiency properties of advertising. In markets with low or no advertising costs, allowing firms to price discriminate leads them to provide too little advertising, which is not good for consumers and overall welfare. Only in markets with high advertising costs, may firms overadvertise. Regarding the welfare effects, price discrimination is generally bad for welfare and consumer surplus, though good for firms.

1 Introduction

In many markets firms need to invest in advertising to inform potential consumers about the existence and price of their new products. The informative view of advertising claims that the primary role of advertising is to transmit information to, otherwise, uninformed consumers.[1] Notwithstanding the development of information technologies and the Internet that allowed consumers an easier access to relevant information about products, there is evidence that advertising still plays a very important informative task.[2] Firms are also increasingly able to recognize customers with different purchasing histories and to implement behavior-based price discrimination by means of behavior-based advertising. When a firm can distinguish its old customers and its rival's previous customers, it may want to send targeted advertising messages (henceforth,

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[1] In contrast, the persuasive view of advertising holds that the main role of advertising is to increase a consumer's willingness to pay for the advertised product. In this sense, firms may use advertising to alter consumers' tastes and to increase brand loyalty. For a review of models in the persuasive view, see Bagwell's (2005) survey on the "The Economic Analysis of Advertising."

[2] According to the Internet Advertising Revenue Report, conducted by the New Media Group of Pricewaterhouse Coopers, on behalf of the Internet Advertising Bureau, web advertising revenue totalled $4.06 billion in the second quarter of 2006. Additionally, the report found that consumer-related advertising was the top category in online advertising spending, accounting for 49% of total revenues for the second quarter in 2006 (being the top advertised categories nearly homogeneous). See www.iab.net/resources/adrevenue/pdf/IAB_PwC%202006Q2.pdf

ads) with differentiated content—like prices—to customers with different past behavior. Yet, the literature on behavior-based price discrimination has, hitherto, focused on the assumption that there is no role for advertising. This article departs from this hypothesis by assuming that, apart from its informative role, advertising may be used by firms as a price discrimination device.

Consider the following simple example. An outdoor recreation seller designs an online advertising campaign for new camping equipment. After receiving one of the seller's ads, a certain customer visits the seller's website and follows the usual registration process. Empowered with the customer's relevant information, as whether he bought the product or not and his email, the seller may send a tailored email next time. When the seller realizes that a certain reached customer was in its site, but did not buy, he may design a campaign to entice that customer back with a special email offer.[3,4]

Much attention has lately been given in the economics literature to the importance of understanding the competitive and welfare effects of firms capable of pricing differently towards customers with different past behavior.[5] The welfare properties of advertising have also been the subject of an ongoing debate that goes back to Kaldor's (1950) classical paper, later stimulated by Butters (1977). Yet, little theoretical attention has been dedicated to date to the interaction between advertising decisions and price discrimination. To fill that gap, this article proposes a simple two-period model, in which all information about price and product availability flows through advertising,[6] and where advertising may also be used as a way to implement behavior-based price discrimination in the second stage of the game. In doing so, we will revisit, with a new perspective, some of the following questions. Will the temptation of the firm to poach its rival's customers lead to higher profits? What are the effects on the firms' first-period pricing, if price discrimination is permitted? Being price discrimination permitted, is there too little or too much advertising? Do firms select a more efficient level of advertising under non-discrimination or under discrimination?

With that in mind, this article addresses a homogeneous product market, where, for example, two online firms advertise a new product (and its price) to otherwise uninformed consumers. In the initial period, each firm chooses what price to quote in its ads and the advertising intensity that will be used in that period, as well as in the next one. After firms have sent their ads independently, *ex-ante* uninformed identical consumers become *endogenously* differentiated on an informational basis. Some consumers receive ads from both firms, will be *selective* and will always buy from the lowest-priced known firm. Other consumers receive ads from only one firm, will be *captive* customers and will buy from that firm provided that the price does not exceed the reservation value. Finally, some consumers receive no ads, will remain uninformed and out of the market. In this setting, when a firm is in the market for more than one period, it may learn whether a previous contacted consumer bought its product or not. Therefore, it may be tempted to poach the rival's previous customers by sending ads with better deals to that group of customers.

[3] Before the new novel of Terry Pratchett's 'The Truth' was launched, Amazon was targeting its existing customers with a special direct email offer for that book, but offering it at a price higher than it was charging to first-time customers (see www.theregister.co.uk/2000/11/06/amazon_rips_off_regular_customers/).

[4] *CDnow*, an online vendor of music albums, emails certain buyers (e.g. first-time buyers) a special Web site address with lower prices. (Kotler (2000), p. 476).

[5] For a comprehensive survey on behavior-based price discrimination, see Fudenberg and Villas-Boas (2005).

[6] This assumption is in line with the stream of the informative advertising which considers that, in the absence of advertising, potential buyers do not purchase the advertised good. Implicitly, this means that consumers are passive, which may be due to high search costs in relation to the expected surplus generated by the good in question. For models in this vein, see for instance Butters (1977), Grossman and Shapiro (1984), Stegeman (1991), Stahl (1994) and Iyer, *et al* (2005).

In order to evaluate the competitive effects of behavior-based price discrimination, we first analyze the benchmark case, in which price discrimination is illegal. In this setting, it is shown that, as in other models with imperfectly informed consumers, the unique equilibrium is characterized by price dispersion,[7] and that firms fully advertise when advertising is costless (e.g. Butters (1977) and Stahl (1994)). Then, we analyze the case in which, whether feasible, price discrimination is permitted in period 2 (Section 5). Here, we find that the unique symmetric subgame perfect equilibrium entails pure strategies in advertising choices and mixed strategies in prices. We show that only the highest priced firm in period 1 has information to engage in behavior-based price discrimination in the subsequent period, and that it is always better off when it can price discriminate. As in the second period firms benefit from the asymmetry in the available information about customers, we identify the *"race for discrimination effect"*, through which the benefit of price discrimination gives firms an incentive to pursue price discrimination by setting high first-period prices. In this context, we show that, under discrimination, first-period prices are on average, above their non-discrimination counterparts and that, when advertising is significantly expensive, there is more advertising with discrimination; when advertising is cheap, the reverse happens. Further, unlike the no-discrimination benchmark, we show that the symmetric subgame perfect equilibrium entails incomplete market coverage for *both* firms, even when advertising is costless (Proposition 7).

We also analyze the profitability of price discrimination based on the consumer's purchase history, when consumers are imperfectly informed (Subsection 6.3). Our main finding here is that, at least when advertising costs are not too high, price discrimination increases the overall expected profits, regardless of the advertising technology considered (Corollary 5). This result is in contrast with the Prisoner's Dilemma result obtained in the existing literature (e.g. Thisse and Vives (1988), Villas-Boas (1999) and Fudenberg and Tirole (2000)).

Finally, we revisit, with a new perspective, the welfare issues associated with price discrimination based on customer recognition and informative advertising (Section 7). To our best knowledge, there is no analytical research that examines the effects of price discrimination on the efficient properties of informative advertising. In this regard, under no discrimination, we show that firms select the social optimal level of advertising (Proposition 10). Under price discrimination we show that for any advertising cost function, private and social incentives to advertise only coincide if the advertising cost is such that either the firms or the regulator choose an intensity of advertising equal to 50%. Otherwise firms under or over advertise (Proposition 11). By looking at the impact of discrimination on the advertising choices of firms, we can understand whether to encourage or ban price discrimination in the context of competitive homogeneous product markets, with no switching costs, where consumers are left out of the market without advertising. Our main finding here is that, regardless of the advertising technology considered, permitting price discrimination is generally bad for consumers and overall welfare.

Related literature In broad terms, the model presented in this article is related to the following strands of the literature. One is the extensive literature on informative advertising and price competition in homogeneous product markets. The other is the recent literature on behavior-based price discrimination in competitive markets.

Stimulated by Butters (1977), several papers investigated how sellers can influence the buyer's information about the product existence and price, by investing in advertising. In particular, Butters showed that, in a monopolistically competitive model with a homogeneous product, where advertising is the sole source of information to otherwise uninformed customers, the equilibrium in prices is characterized by price dispersion, and the market equilibrium level

[7] See, for instance, Butters (1970), Varian (1980) and Narasimhan (1988) to name a few.

of advertising is socially optimal.[8] This latter puzzling result was confirmed by Stahl (1994), who extended the Butters model to oligopolistic markets with more general demand curves and advertising technologies. Variations on the Butters model such as the introduction of product differentiation (Grossman and Shapiro (1984)), or the heterogeneity among buyers (Stegeman (1991)), were shown to easily offset this result. They also helped to establish the idea that increased competition stimulated additional advertising—the *business stealing effect*—, while the incapability of the firm to appropriate the social surplus it generates acts as a deterrent to advertising —the *nonappropriability of social surplus effect* (Tirole (1988)).

Our model is also related to those studies that look at the strategic effects of advertising in sequential games, where firms first invest simultaneously in advertising and, then, compete simultaneously in prices (e.g. Ireland (1993) and Roy (2000)[9]). In order to investigate the strategic interaction between advertising choices and price discrimination, we concentrate on a sequential game, where firms compete simultaneously in advertising and prices in the initial period and, if permitted, engage in behavior-based price discrimination in the next stage of the game. When firms are forward-looking, by incorporating price discrimination into a model where advertising is used as a long-run variable, we are able to understand how the advertising decisions of today affect the pricing and profits of tomorrow and also how price discrimination affects the advertising decisions of today.

Finally, this article engages the recent literature on behavior-based price discrimination in competitive markets. Broadly speaking, two approaches have been considered in this literature so far. In the switching costs approach, consumers initially view the two firms as perfect substitutes; but in the second period they face a switching cost, if they change supplier. Thus, purchase history is useful in disclosing information about exogenous switching costs (e.g. Chen (1997) and Taylor (2003)). In the brand preferences approach (e.g. Villas-Boas (1999), Fudenberg and Tirole (2000), Chen and Zhang (2004), Esteves (2005)), purchase history discloses information about a consumer's exogenous brand preference for a firm. Being price discrimination permitted firms will be tempted to price low in order to poach the rival's previous customers. Although the framework of competition differs in these two approaches, their predictions have some common features. First, if price discrimination is allowed firms offer better deals to the competitor's consumers than to its previous customers. Second, because both firms can discriminate and each firm regards its previous clientele as its strong market and the rival's clientele as its weak market (i.e. there is best-response asymmetry[10]), firms find themselves in the classic Prisoner's Dilemma. Third, there is socially excessive switching between firms. Nonetheless, important differences arise in both approaches when we take into account the effects of poaching on initial prices. While in the brand preferences approach when behavior-based price discrimination is permitted initial prices are high and then decrease (Fudenberg and Tirole (2000)), in the switching costs approach the reverse happens (Chen (1997)).

Our model is also related to Chen and Zhang (2004), who revisit the profitability issue of behavior-based price discrimination using a discrete version of the Fudenberg-Tirole (2000) model. They assume that each firm has an exogenous "loyal" segment of the market, and that they compete for the remaining consumers, who are price-sensitive, with a reservation value lower than the loyal consumers' one. In this context, they show that customer recognition only occurs if the first-period price of a firm is high enough, so that it is not accepted by all consumers, and that price discrimination might benefit firms.

This article contributes to the literature in this field by analyzing the dynamic effects of this

[8] For a complete survey on customer search and price dispersion see, for instance, Baye, *et al* (2006).

[9] An interesting contribution of Roy (2000) is to assume that firms can target consumers on the basis of their address. For other models of targeted advertising, see, for instance, Esteban, *et al* (2001) and Iyer, *et al* (2005).

[10] See Corts (1998).

kind of price discrimination using a different form of market competition. In many markets, advertising and pricing decisions are taken together. Especially in new product markets, advertising may be the consumers' sole source of information, and thus, after advertising decisions have been taken, *ex-ante* identical consumers become imperfectly informed. Unlike Chen and Zhang (2004), in which the market segmentation is exogenous, we allow firms to endogenously segment the market into captive and selective. Moreover, unlike the switching costs approach, some consumers are locked in with a certain firm, not due to switching costs, but because they ignore the other firm. Finally, the existing models on behavior-based price discrimination have assumed that firms can perfectly reach, and with no cost, all the potential customers in the market. While this assumption may fit well in some cases, it may be inappropriate in others. Then, it is relevant to evaluate the competitive effects of behavior-based price discrimination in markets where advertising plays an important informative and strategic role.

The rest of this article is organized as follows. Section 2 sets out the model. Section 3 analyzes the benchmark case, where price discrimination is illegal. Section 4 looks at the equilibrium advertising and pricing strategies, when firms are permitted to engage in behavior-based price discrimination. The competitive effects of customer poaching are discussed in Section 5. Issues as the efficient properties of advertising and the welfare effects of price discrimination are addressed in Section 6. Conclusions are provided in Section 7.

2 The model

Consider a duopolistic market where two firms, A and B, produce a new homogeneous good at a constant marginal cost, which, without loss of generality, is assumed zero. There are two periods, 1 and 2, and the firms act to maximize the expected discounted value of their profits, using a common discount factor $\delta \in [0,1]$. On the demand side, there is a large number of consumers, with mass normalized to one, who desire to buy at most one unit of the good in each period. Consumers are *naive* in the sense that they do not take into account that second-period prices may be affected by their initial buying decisions. This assumption is especially relevant in a new product market, because, in this context, consumers have not yet learned the firm's pricing behavior. Consumers have a common reservation price v, have identical preferences and are initially unaware of the existence of the firms' products and prices. Consumers will be actual buyers if two conditions hold: (i) if they become aware of a firms' product and price by being exposed to its advertising and (ii) if the price is not higher than v. Accordingly, a potential consumer cannot be an actual buyer unless firms invest in advertising.[11] This assumption fits well in those markets where firms are introducing new products, in which awareness is clearly the first stage in creating demand for a product.

Following the informative view, advertising is a necessary activity for transmitting relevant information to otherwise uninformed consumers. As in Butters (1977), each firm sends advertising messages, which contain truthful[12] and complete information about the existence of its products and price. Each consumer's behavior is quite simple. After being exposed to the firms' advertising, each consumer acquires one unit of the product from the firm that advertises the lowest price, whenever that price is below v. When a consumer is indifferent between the two firms, he assigns an equal chance to any of the firms.

After firms have sent their ads independently (i.e., advertising reach is independent for each firm), there are, in principle, four different mutually exclusive and exhaustive market segments. Some consumers are *captive* to a given firm, because they are only aware of that firm. Each

[11] One alternative interpretation is to assume that for new products search costs are prohibitively high.

[12] This is guaranteed by the FTC regulation that prohibits advertisers from making false and deceptive statements about their products (see www.ftc.gov/bcp/conline/pubs/buspubs/ad-faqs.htm).

firm enjoys a monopolistic power within its captive group. Other consumers receive ads from both firms and are *selective*. In this latter group, consumers have complete information and will buy from the firm that offers them the highest surplus. Finally, the remaining consumers receive no ad from either firm, are *uninformed* and excluded from the market. Thus, by investing in advertising, firms endogenously determine the partition of *ex-ante* identical consumers into captive, selective and uninformed consumers. Although we let consumers to be identical in the beginning of the game, after advertising choices have been made, consumers will be differentiated in terms of the information they have.

Advertising in this model is used as a variable with a long-run nature. After advertising decisions have been made in period one, we assume that, in a repeated interaction, each firm is able to address the same group of consumers it was able to reach in period one. Further, we assume that firms reach the same customers with no additional cost, which means that advertising costs sank in period 1. Several examples justify this assumption. First, in online markets, for instance, after firms have sent their ads, a reached consumer will be directed to the firm website. If he is interested in the firm's product, he may be asked to fill a registration form, in which the email may be one of the requirements. The next time, empowered with the consumers' email accounts, the firm may send its ads by email with almost no cost. Second, in the internet markets, firms can easily identify consumers who had clicked on one of their ads, and, next time, identify on its website those that bought and those that decided not to buy. Finally, firms have access to syndicated vendors of information about potential consumers; thus, they may decide to buy external databases with certain market coverage for advertising purposes. In this case, the cost of advertising is entirely borne in the beginning of the game, since thereafter firms can contact the same consumers with no additional cost.

The game proceeds as follows. In the first-period firms choose advertising intensities and prices simultaneously and non-cooperatively. Notice that in period one all consumers are anonymous and so each firm puts the same price in each of its period 1 ads. However, in a repeated interaction between sellers and customers, sellers might be able to learn whether a previous contacted is an old customer or rather a non-purchaser that previously bought from the rival. When a firm achieves that type of learning, it may have incentives to poach the selective customers that previously had bought from the rival, by offering them a better second period deal. Here, the fact that an informed consumer did not buy from the firm in the past reveals to that firm that he is a selective one—i.e., he also received an ad from the rival with a lower price. Likewise, this allows the firm to infer that all consumers that bought its product must be captive. Conversely, when the firm sells its product to all consumers that received one of its ads, it learns nothing; thereby, price discrimination is unfeasible in the next period.

In the second-period, firms observe each other's choice of first-period advertising intensity and price. If price discrimination is permitted and feasible, they select a different price to customers with different past behavior. In other words, firms may implement behavior-based price discrimination by means of behavior-based advertising—i.e., consumers with different past behavior will receive targeted ads with different prices. Note that while the introductory prices tend to be publicly quoted (e.g. through the use of banner ads), we can think of second-period prices being quoted via private offers (e.g. direct mail, email, targeted websites,[13] and so on).

Finally, the no-discrimination benchmark considers that somehow public policies prohibit price discrimination, and thus, in the second period, firms are forced to set the same prices. This means that, once prices are publicly announced through advertising in period one, they must remain for the entire duration of the game.

Before proceeding, we will look at the different advertising cost functions that have been

[13] In the Internet some retailers operate a few different websites, which frequently have different prices for the same prducts.

used in the literature and that will be used throughout the analysis.

Advertising technologies Advertising is a costly activity for firms. The cost of reaching a fraction ϕ of consumers is given by the function $A(\phi) = \alpha\eta(\phi)$. Following the extant literature on informative advertising (e.g. Grossman and Shapiro (1984)), it is assumed that the cost of reaching consumers increases at an increasing rate, which formally can be written by $\frac{\partial A(\phi)}{\partial \phi} = A_\phi > 0$ and $\frac{\partial^2 A(\phi)}{\partial \phi^2} = A_{\phi\phi} \geq 0$. The latter condition means that it is increasingly more expensive to inform an additional customer or likewise, to reach a higher proportion of customers. Several justifications support this assumption. One justification has to do with the advertising technology itself. In other words, if ads are sent randomly at a fixed cost per ad, then the probability of reaching a consumer not yet informed decreases with the amount already advertised, e.g. the Butters' urn technology and the Constant Reach Independent Readership (CRIR) technology proposed by Grossman and Shapiro (1984). Other justifications are the existence of different predispositions to view ads on the part of the target population and the possibility of media saturation. It is also assumed that $A(0) = 0$ or, in other words, there are no fixed costs in advertising. These assumptions hold for all specific advertising cost functions that have been used in the literature, including, for instance, the CRIR technology, the Butters' technology and the Quadratic Advertising cost function proposed in Tirole (1988). Finally, in order to make advertising viable, it is assumed that $A_\phi(0) < v$. In a two-period game, this latter assumption translates to $A_\phi(0) < v(1+\delta)$.

Different types of online advertising—display ads, sponsorship advertising, keyword-related search advertising, and so on—may be reasonably well represented by the Butters, or even by the quadratic technology. Although this latter technology is not based upon an underlying technology of message production, as happens with the Butters' technology, it has the advantage of being extremely simple to manipulate algebraically. It is given by $A(\phi) = \alpha\eta(\phi)$, where $\eta(\phi) = \phi^2$. When the classic urn technology proposed by Butters is the most adequate the advertising cost function is $A(\phi) = \alpha\eta(\phi)$ where $\eta(\phi) = \ln\left(\frac{1}{1-\phi}\right)$. Butters (1977) notes that messages are sent out in a purely random fashion at a fixed cost per unit. If L messages are sent to M buyers and both L and M are large, then the fraction of buyers who do not receive any ad is $1-\phi = (1-\frac{1}{M})^L \simeq e^{-\frac{L}{M}}$. If each ad has a fixed cost of λ, then the total advertising cost is λL, or in terms of ϕ, $\lambda M \ln\left(\frac{1}{1-\phi}\right)$. If we let α equal λM, we find that the cost of informing ϕ consumers is $A(\phi) = \alpha \ln\left(\frac{1}{1-\phi}\right)$. Because in our model there is a large number of buyers, which we normalized to one, α can be identified with the cost per ad. In what follows, whenever a functional form is needed, we will make use of one of these technologies.

3 No discrimination benchmark

Let us establish the benchmark case in which, even if feasible in period 2, price discrimination is generally illegal. As mentioned, each firm is forced to quote the same price in period 1 and 2 (i.e., $p_1 = p_2$, where p_t is the price in period $t = 1, 2$.). In other words, when a firm advertises its product and price, it publicly announces that the introductory price will remain equal for the entire game. (Note that in this benchmark case, there is neither inter-temporal price discrimination nor behavior-based price discrimination.) Throughout the article, we will use this benchmark to evaluate the competitive and welfare effects of price discrimination enabled by informative advertising. To solve for the equilibrium without discrimination, we will start looking at the behavior of firms in a static game.

There are two components to a firm's strategy: Firm i must choose its advertising level (denoted by ϕ_i), as well as its price (denoted by p_i). After firms have sent their ads independently, a proportion ϕ_i and ϕ_j of customers is reached, respectively, by firm i and j advertising. Therefore, the potential demand of firm i is made of a group of captive (locked-in) customers, namely $\phi_i\left(1-\phi_j\right)$, and a group of selective customers, namely $\phi_i\phi_j$. Since the product is homogeneous, buyers care only about price. When consumers receive ads from both firms (i.e., when they are selective), they are indifferent between firms, if these quote the same price; otherwise, they purchase at the lowest advertised price—provided it does not exceed v. When consumers receive ads only from one of the firms, they only purchase from that firm, as long as the advertised price is below the reservation price v. The size of each group of consumers is jointly determined by the advertising intensity of both firms, and it is not under the control of any particular firm. In the price setting game, each firm faces a trade-off between quoting a low price and compete for the segment of selective customers; or setting a high price and extract surplus from its captive segment.

Proposition 1. *There is no symmetric Nash equilibrium in pure strategies in prices.*[14]

Proof. See the Appendix.

However, there is a symmetric Nash equilibrium in prices involving mixed strategies.[15] Suppose that firm i selects a price randomly from the distribution function $F_i(p)$ with support $[p_{\min}, v]$. Since the marginal cost of production is assumed null, no ads will be sent specifying prices below the marginal cost of advertising or above v. Given the price and advertising strategies of firm B, the expected profit of firm A is equal to

$$E\pi_A = p\phi_A\left(1-\phi_B\right) + p\phi_A\phi_B\left[1-F_B(p)\right] - A(\phi_A). \tag{1}$$

In equilibrium firm A should be indifferent between quoting any price that belongs to the equilibrium support. When it chooses price v, it knows that this price is only accepted by a consumer who is not aware of any other price. In this case its expected profit is equal to $v\phi_A\left(1-\phi_B\right)$. This implies that in equilibrium we must observe,

$$p\left(1-\phi_B\right) + p\phi_B\left[1-F_B(p)\right] = v\left(1-\phi_B\right).$$

Solving for $F_B(p)$, we get

$$F_B(p) = 1 - \frac{(v-p)\left(1-\phi_B\right)}{p\phi_B}. \tag{2}$$

From $F_B(p_{\max}) = 1$ and $F_B(p_{\min}) = 0$ it follows that $p_{\max} = v$ and $p_{\min} = v\left(1-\phi_B\right)$.

The equilibrium level of advertising maximizes (1) with respect to ϕ_A. Plugging the expression that defines $F_B(p)$, the first-order condition is equal to $v\left(1-\phi_B\right) = A_\phi$.[16] Under symmetry we obtain that[17]

$$v\left(1-\phi\right) = A_\phi \text{ or, equivalently, } p_{\min} = A_\phi. \tag{3}$$

[14] More precisely, a price equilibrium in pure strategies fails to exist unless firms' advertising gives rise to perfect information, that is to say, unless $\phi_i = \phi_j = 1$.

[15] For a similar derivation of the mixed strategy equilibrium in prices, but with exogenous captive and selective consumers, see for example Shilony (1977), Varian (1980) and Narasimhan (1988).

[16] Second-order conditions also hold since $-A_{\phi\phi} < 0$.

[17] It is worth noting that the condition that defines the equilibrium level of advertising is simply the same we would obtain by solving the Butters static model with only two firms.

Basically, the equilibrium condition for advertising established above stipulates that, in equilibrium, firms should advertise up to the point where the cost of the last ad sent equals the expected revenue of a sale at the highest price to an uninformed consumer.

Turning now to the two-period game without discrimination, as we assume that in the second-period firms are constrained to quote the same prices, the overall expected profit for firm A is

$$p\phi_A (1-\phi_B) + p\phi_A\phi_B [1 - F_B(p)] - A(\phi_A) + \delta v \phi_A (1-\phi_B). \tag{4}$$

Once more, in equilibrium, firm A should be indifferent between quoting any price that belongs to the equilibrium support. When it chooses price v, its expected profit is now equal to $v(1+\delta)\phi_A(1-\phi_B) - A(\phi_A)$. In equilibrium, we must observe

$$p\phi_A (1-\phi_B) + p\phi_A\phi_B [1 - F_B(p)] - A(\phi_A) + \delta v \phi_A (1-\phi_B) = v(1+\delta)\phi_A(1-\phi_B) - A(\phi_A)$$

from which it follows that

$$F_B(p) = 1 - \frac{(v-p)(1-\phi_B)}{p\phi_B}, \tag{5}$$

which is the same expression obtained in the static game. The equilibrium level of advertising maximizes (4) with respect to ϕ_A. From the first-order condition, it follows that the equilibrium level of advertising is implicitly given by,[18]

$$v(1+\delta)(1-\phi_B) = A_\phi. \tag{6}$$

Equation (6) gives firm A's best response function for advertising intensity, when price discrimination is not permitted. Let the superscript *nd* identify the no-discrimination case.

Proposition 2. *In the benchmark case without discrimination,*

(i) there is a symmetric mixed strategy subgame perfect Nash equilibrium in prices described as follows. Each firm chooses a price randomly from the distribution

$$F^{nd}(p) = \left\{ \begin{array}{ccc} 0 & if & p < v\left(1-\phi^{nd}\right) \\ 1 - \frac{(v-p)\left(1-\phi^{nd}\right)}{p\phi^{nd}} & if & v\left(1-\phi^{nd}\right) \leq p \leq v \\ 1 & if & p > v \end{array} \right\} \tag{7}$$

with minimum equilibrium price equal to

$$p^{nd}_{\min} = v\left(1-\phi^{nd}\right). \tag{8}$$

(ii) There is a symmetric Nash equilibrium in pure strategies, in which each firm chooses an advertising reach $\phi^{nd} \in (0,1)$ defined by

$$v(1+\delta)\left(1-\phi^{nd}\right) = A_\phi\left(\phi^{nd}\right) \tag{9}$$

where $A_\phi(0) \leq v(1+\delta)$.

(iii) Each firm earns overall expected equilibrium profits equal to

$$E\Pi^{nd} = v(1+\delta)\phi^{nd}\left(1-\phi^{nd}\right) - A\left(\phi^{nd}\right) = \phi^{nd}A_\phi - A\left(\phi^{nd}\right) \tag{10}$$

[18] Second-order conditions also hold since $-A_{\phi\phi} < 0$.

The equilibrium identified above entails pure strategies in advertising choices and mixed strategies in prices. Regarding the price-setting solution, it shares features present in Butters (1977), Varian (1980), Narasimhan (1988) and Baye and Morgan (2001), where firms use a price strategy that prevents their opponents from predicting their price setting behavior. This suggests that, even in homogeneous product markets, where consumers care only about prices, price dispersion may remain persistent and substantial. Bynjolfsson and Smith (2000), for instance, argue that the observed price dispersion in online markets is largely explained by awareness, branding and trust—factors clearly affected by promotion activities of online sellers, such as advertising. Obviously, our results give support to the explanation of price dispersion based on awareness.

The next corollary summarizes the effects of changes in advertising costs on the equilibrium outcomes, when a legal restriction forbids sellers from engaging in price discrimination.

Corollary 1. (Effects of advertising costs)
(i) As advertising becomes costless, i.e., as α approaches 0, $\phi^{nd} \to 1$, $p_{\min}^{nd}$ and expected profits approach zero and F^{nd} converges to unit mass at zero;
(ii) As $A_\phi(0)$ approaches $v(1+\delta)$, ϕ^{nd} and expected profits approach zero, $p_{\min}^{nd}$ approaches v and F^{nd} converges to unit mass at v.

It is straightforward to observe that, when advertising is costless, both firms fully advertise in equilibrium (i.e., $\phi^{nd} = 1$). The competitive implications of complete market coverage are well known: in a homogeneous product market, as all consumers become perfectly informed, the equilibrium approaches the classic Bertrand outcome; firms set the marginal cost price, profits go to zero and consumers are clearly better off than under incomplete coverage. Conversely, as advertising costs rise, advertising decreases and higher prices are more likely.

4 Equilibrium analysis of poaching with advertising

In the first period all consumers are anonymous, meaning that price discrimination is unfeasible. However, after consumers have made their buying decisions in period one, a seller may be able to separate those customers that previously bought its product from non-purchasers (i.e., those customers that received one of its ads, but decided not to buy its product in period 1). When a firm achieves that type of learning, it may have an incentive to deliver targeted ads with better deals to the latter group of consumers, in an effort to poach them from the rival firm. Note that in period 2, while a seller can identify those consumers that were reached by its advertising campaign in period one, he cannot identify those customers that are only aware of the rival firm (i.e., the opponent's captive segment).

Considering that firms are forward-looking $(\delta > 0)$, the next step is to solve the game working back from the second period.

4.1 Second-period pricing game

Depending on first period pricing and advertising decisions, two scenarios are possible in period 2 for each firm. In one scenario, firm i $(i = A, B)$ is the lowest-priced firm in period 1, thereby it sells to its captive group of customers, as well as to the entire group of selective customers. In this situation, first-period demand for the highest-priced firm—in this case, firm j $(i \neq j)$—comes exclusively from its captive segment. Since the lowest-priced firm sells to all consumers that received one of its first period ads, it learns nothing and, consequently, it cannot engage in price discrimination in period 2. In opposition, the highest-priced firm in period 1 learns that some consumers that received one of its ads did not buy its product. Likewise, it also infers that those customers that bought its product did not receive any of the rival's ads and, for that reason they must be captive. Based on this reasoning, the highest-priced firm in period 1 is able to sort out customers into different segments: its own captive customers and selective customers that bought from the rival. As this firm can, in the second period, reach again the same customers, it will tailor ads with different content—i.e., prices—to its own previous customers and to selective customers that bought from the competitor before. Summing up, the highest-priced firm in period 1 will be the discriminating firm in period 2.

For a given price p_i chosen by firm i in period 1, firm i is the lowest-priced firm in period 1 (or the non-discriminating firm in period 2) with a probability equal to $\Pr(p_i \leq p_j) = 1 - F_j(p_i)$; and it is the highest-priced firm in period 1 (or the discriminating firm in period 2) with a probability equal to $\Pr(p_i > p_j) = F_j(p_i)$.

4.1.1 Subgame 1: Firm i is the discriminating firm

Firm i is the discriminating firm in period 2, if it is the highest priced firm in period 1. While the first-period demand of firm i embraces uniquely its captive customers, namely $\phi_i\left(1 - \phi_j\right)$, the demand of firm j comprises its captive customers, as well as the selective group of customers, that is $\phi_j(1 - \phi_i) + \phi_i\phi_j$. In this case, in the second stage of the game, firm i is able to recognize old customers and selective customers that bought from j before, and to entice the latter customers to switch, by offering them a lower price. In this way, firm i will advertise a different price to its old captive customers and to the group of customers that bought the rival's product before.[19] Let us denote by p_i^o the price of firm i to its old customers and by p_i^r the price of firm i to the rival's customers.

Corollary 2. *The discriminating firm will charge its old captive customers the highest possible price, namely $p_i^o = v$, regardless of the price it charges to the rival's customers.*

The proof of this result is quite simple. The ability of firm i to fully separate its captive customers from the selective customers that bought from the rival before, together with the incapability of firm j to reach any of firm i's captive customers, allows firm i to charge its captive customers their reservation price, without fearing any poaching attempt by its rival. Then, firm i's profit from its old captive customers, denoted by π_i^o, is equal to

$$\pi_i^o = v\phi_i\left(1 - \phi_j\right). \tag{11}$$

Since consumers remain anonymous to firm j, it is forced to advertise the same price to all consumers. We denote by $\widehat{p}_j$ the non-discrimination price of firm j.

Proposition 3. *There is no pure strategy equilibrium in the price setting game for the group of the non-discriminating firm's previous customers.*

Proof. See the Appendix.

The intuition behind the above result runs as follows. Firm i is able to increase its second period market share, by offering a better price to the group of selective customers that bought from firm j before, without damaging the profit it can extract from its captive segment. Even

[19] Especially in internet markets, where firms can easily track the behavior of customers online; to identify them, firms may easily send targeted offers to first-time and old customers by means of email or by creating special websites.

though firm j can always guarantee itself a profit equal to $v\phi_j(1-\phi_i)$, the presence of a positive fraction of selective consumers creates a tension between the incentives of the firm to price low, in order to attract the selective customers, and to price high, in order to extract rents from its locked in customers. This tension results in an equilibrium displaying price dispersion.

Let $G_i^r(\widehat{p}_j) = \Pr(p_i^r \leqq \widehat{p}_j)$ be the probability of firm i to quote a price no higher than $\widehat{p}_j$. Additionally, let $p_{i\min}^r$ and $p_{i\max}^r$ be, respectively, the lowest and the highest advertised price by firm i with positive density in equilibrium. Then, the equilibrium support of prices, in which $G_i^r(\widehat{p}_j)$ is defined, is $[p_{i\min}^r, p_{i\max}^r]$. Similarly let $\widehat{G}_j(p_i^r) = \Pr(\widehat{p}_j \leqq p_i^r)$, with support $[\widehat{p}_{j\min}, v]$. As mentioned, firm j can guarantee itself a profit equal to $v\phi_j(1-\phi_i)$, by focusing only on its captive group. Thus, in equilibrium, it must be indifferent for firm j to quote price v (and sell only to its captive customers) or to quote a lower price with the aim of serving again captive and selective consumers. Formally, the minimum price that the non-discriminating firm, in this case firm j, will be willing to advertise in equilibrium must satisfy the following condition:

$$\widehat{p}_{j\min}\left(\phi_j(1-\phi_i)+\phi_i\phi_j\right) = v\phi_j(1-\phi_i),$$

from which we get that:

$$\widehat{p}_{j\min} = v(1-\phi_i). \tag{12}$$

What is then the minimum price firm i can charge in order to grab the entire group of selective consumers? It is a dominated strategy for firm i to set a price below $\widehat{p}_{j\min}$. Quoting $p_i^r = \widehat{p}_{j\min}$ allows firm i to poach the entire group of selective consumers and makes firm j indifferent. Therefore, the minimum price of firm i to its rival's previous customers is equal to $\widehat{p}_{j\min}$ and its expected profit is also equal to $\widehat{p}_{j\min}\phi_i\phi_j = v(1-\phi_i)\phi_i\phi_j$. Thus, in equilibrium, for the non-discriminating firm, we must observe:

$$\widehat{p}_j\phi_j(1-\phi_i)+\widehat{p}_j\phi_i\phi_j\left[1-G_i^r(\widehat{p}_j)\right] = v\phi_j(1-\phi_i),$$

from which we obtain:

$$G_i^r(\widehat{p}_j) = 1-\frac{(v-\widehat{p}_j)(1-\phi_i)}{\widehat{p}_j\phi_i}, \tag{13}$$

with $G_i^r(\widehat{p}_{j\min}) = 0$ and $G_i^r(v) = 1$. For the discriminating firm i, we must observe in equilibrium:

$$p_i^r\phi_i\phi_j\left[1-\widehat{G}_j(p_i^r)\right] = \widehat{p}_{j\min}\phi_i\phi_j$$

which yields,

$$\widehat{G}_j(p_i^r) = 1-\frac{\widehat{p}_{j\min}}{p_i^r} = 1-\frac{v(1-\phi_i)}{p_i^r}, \tag{14}$$

with $\widehat{G}_j(p_i^r = \widehat{p}_{j\min}) = 0$ and $\widehat{G}_j(v) = 1-(1-\phi_i) < 1$. In consequence, firm j has a mass point at v.

Proposition 4. *When one firm can engage in price discrimination, whilst the other cannot, the competition within the group of selective consumers previously buying from the non-discriminating firm, gives rise to an asymmetric mixed strategy equilibrium in prices.*

(i) The non-discriminating firm chooses a price randomly from the distribution

$$\widehat{G}_j(p_i^r) = \left\{\begin{array}{ccc} 0 & for & p_i^r < \widehat{p}_{j\min} \\ 1-\frac{v(1-\phi_i)}{p_i^r} & for & \widehat{p}_{j\min} \leqq p_i^r \leqq v \\ 1 & for & p_i^r > v \end{array}\right\} \tag{15}$$

with support $[\widehat{p}_{j\min}, v]$ *, and the discriminating firm chooses a price randomly from the distribution*

$$G_i^r(\widehat{p}_j) = \left\{ \begin{array}{ccc} 0 & for & \widehat{p}_j < \widehat{p}_{j\min} \\ 1 - \frac{(v-\widehat{p}_j)(1-\phi_i)}{\widehat{p}_j\phi_i} & for & \widehat{p}_{j\min} \leqq \widehat{p}_j \leqq v \\ 1 & for & \widehat{p}_j \geqq v \end{array} \right\} \tag{16}$$

with support $[\widehat{p}_{j\min}, v)$ *, where*

$$\widehat{p}_{j\min} = v\left(1 - \phi_i\right). \tag{17}$$

The non-discriminating firm has a mass point at v *equal to*

$$m_j = \left(1 - \phi_i\right). \tag{18}$$

(ii) The expected profit for the discriminating firm, coming from poached customers, equals

$$\pi_i^r = v\left(1 - \phi_i\right)\phi_i\phi_j, \tag{19}$$

and the expected profit for the non-discriminating firm equals

$$\widehat{\pi}_j = v\phi_j\left(1 - \phi_i\right). \tag{20}$$

Therefore, the total second-period expected profit for the discriminating firm equals

$$\pi_i = \pi_i^o + \pi_i^r = v\phi_i\left(1 - \phi_j\right) + v\left(1 - \phi_i\right)\phi_i\phi_j. \tag{21}$$

In this equilibrium, the non-discriminating firm uses a "Hi-Lo" pricing strategy. To squeeze more surplus from its locked in customers, it charges the highest price v, with probability m_j, which increases with the size of that segment. However, in order to avoid being poached, it quotes occasionally a low price.

Corollary 3. *From the equilibrium distribution functions defined by (15) and (16) it follows that:*
(i) $\widehat{G}_j(p) < G_i^r(p)$*, that is,* $\widehat{G}_j(p)$ *first-order stochastically dominates* $G_i^r(p)$*;*
(ii) $E(\widehat{p}) > E(p^r)$ *; and*
(iii) the mass point m_j *is decreasing in* ϕ_i*.*

Proof. See the Appendix.

In other words, part (i) states that $\widehat{p}_j$ is stochastically larger than p_i^r, because it assumes large values with higher probability. Consequently, regarding price competition between firms for the group of selective consumers, (ii) says that on average the non-discriminating firm charges higher prices than its competitor. Indeed, the discriminating firm has an advantage over its rival. It sets lower prices on average, because it is able to entice some of the rival's customers to switch, by offering them a low price, without damaging the profit that comes from its locked in segment. Conversely, when the non-discriminating firm decreases its price, in order to avoid poaching, it damages the profit that comes from its captive segment. As the discriminating seller has less to lose, he is more aggressive and, therefore, he charges, on average, lower prices. Obviously, this means that, with some likelihood, there is customer poaching in equilibrium.

Not surprisingly, part (iii) states that the greater is the size of the non-discriminating firm's captive group, the higher is the probability of this firm charging the reservation price v. Note that there is a negative relation between the captive segment size of a firm and the rival's advertising intensity. Clearly, when, for example, firm i increases its advertising reach, the size of firm j's group of captive customers falls down, as more consumers will be aware of both firms. By decreasing its advertising effort in period 1, firm i expects firm j to play less aggressively in period 2. This strategic reasoning will be important to understand the advertising choices of firms in the first stage of the game.

Corollary 4. *With symmetric first period advertising intensities, the firm that can engage in price discrimination earns higher profits than the firm that cannot.*

From equations (20) and (21), it is trivial to see that under symmetry (i.e., $\phi_i = \phi_j$) it follows that $\pi_i > \widehat{\pi}_j$.

4.1.2 Subgame 2: Firm i is the non-discriminating firm

In this subgame, firm i is the firm offering the better deal in period 1 and, therefore, it captures all informed consumers, except those that are locked in with firm j. It immediately follows that, in comparison to the previous subgame, firms interchange positions with each other. Using our previous computations, we have that second-period profit for the non-discriminating firm, now firm i, is equal to $\widehat{\pi}_i = v\phi_i\left(1-\phi_j\right)$. Looking at the difference between the second-period profit of a firm, with and without discrimination, we find that $\pi_i - \widehat{\pi}_i = v\left(1-\phi_i\right)\phi_i\phi_j > 0$. In this way, we can estimate the benefit of engaging in price discrimination. This benefit will give a firm an incentive to pursue price discrimination, when choosing its pricing and advertising behavior in the first-stage of the game. Remarkably, the benefit of engaging in price discrimination motivates a firm to be the highest priced firm in period 1. In fact, we will see below that the race between firms to become the discriminating firm may result in higher first-period prices in comparison to the non-discrimination case.

We may now compute the expected profit for a representative firm in the second-stage of the game. As seen before, firm i is the discriminating firm in period 2, if it advertises the highest price in period 1, which occurs with a probability equal to $\Pr\left(p_i \geq p_j\right) = F_j\left(p_i\right)$. With the remaining probability, firm i advertises the lowest price, whereby it is the non-discriminating firm in period 2. This occurs with a probability equal to $\left[1-F_j\left(p_i\right)\right]$. Hence, the second period expected profit of firm i, denoted by π_i^2, is equal to

$$\pi_i^2 = v\phi_i\left(1-\phi_j\right) + F_j\left(p_i\right)v\left(1-\phi_i\right)\phi_i\phi_j. \tag{22}$$

4.2 First-period pricing and advertising

In the initial period, firms make their advertising and pricing decisions, rationally anticipating how such decisions will affect their profits in the subsequent period. We have seen that the benefit of engaging in price discrimination in period 2 may give firms an incentive to price high in period 1. Before proceeding, it is appropriate to investigate whether or not the monopoly price v is a pure strategy equilibrium of this game.

Proposition 5 *There is no subgame perfect Nash equilibrium in which both firms set the monopoly price. Moreover, there is no subgame perfect Nash equilibrium in pure strategies.*

Proof. See the Appendix.

If there were such a pure strategy equilibrium in period 1, both firms would quote the same first-period price and, therefore, they would share equally the group of selective consumers. In period 2, within the group of old customers, no firm would be able to distinguish the fraction of selective and captive customers. However, each firm would be able to recognize those selective customers that bought from the rival before. As a result, in the second period, both firms could engage in price discrimination, as they could advertise a different price to their old clients and their rival's previous (selective) customers. As the reader can see in the appendix provided, when both firms advertise price v in period 1, it is always profitable for a given firm to decrease slightly its price to $v-\varepsilon$ and capture the remaining selective customers. The increase in its first-period profit that comes from a deviation is higher than the decrease in its second-period profit, as a result of becoming the non-discriminating firm. For that reason, the monopoly price cannot be a pure strategy equilibrium of this game. Nonetheless, there is a mixed strategy equilibrium. The existence of such equilibrium is proved by construction.

The overall expected profit for firm i, when it charges first-period price p_i, uses a discount factor equal to δ, and its competitor charges a first-period price equal to p_j according to $F_j(p_i)$, is equal to:

$$E\Pi_i = p_i\phi_i\left(1-\phi_j\right)+p_i\phi_i\phi_j\left[1-F_j(p_i)\right]-A(\phi_i)+\delta\left[v\phi_i\left(1-\phi_j\right)+F_j(p_i)\,v\left(1-\phi_i\right)\phi_i\phi_j\right].$$

Again, in equilibrium, firm i must be indifferent between quoting any price that belongs to its equilibrium support, where $p_i\in[p_{i\min},v]$. Thus, we must observe

$$\begin{aligned} & p_i\phi_i\left(1-\phi_j\right)+p_i\phi_i\phi_j\left[1-F_j(p_i)\right]-A(\phi_i)+\delta\left[v\phi_i\left(1-\phi_j\right)+F_j(p_i)\,v\left(1-\phi_i\right)\phi_i\phi_j\right] \\ = \; & v\phi_i\left(1-\phi_j\right)(1+\delta)-A(\phi_i)+\delta v\left(1-\phi_i\right)\phi_i\phi_j, \end{aligned}$$

Solving for $F_j(p_i)$ we have

$$F_j(p_i)=1-\frac{(v-p_i)\left(1-\phi_j\right)}{\phi_j\left(p_i-\delta v\left(1-\phi_i\right)\right)}. \tag{23}$$

Note that if the discount factor is zero, we get the same distribution function as in the static case (see equation (2)). From the conditions which establish that $F_j(p_{i\min})=0$ and $F_j(p_{i\max})=1$, it follows that:

$$p_{i\min}=v\left(1-\phi_j\right)+\delta v\phi_j\left(1-\phi_i\right) \quad \text{and} \quad p_{i\max}=v. \tag{24}$$

Now consider the equilibrium choice of advertising intensity. Plugging $F_j(p_i)$ into the overall expected profit of firm i, we have:

$$\begin{aligned} E\Pi_i \; = \; & \left[p_i\phi_i\left(1-\phi_j\right)+p_i\phi_i\phi_j\left(\frac{(v-p_i)\left(1-\phi_j\right)}{\phi_j\left(p_i-\delta v\left(1-\phi_i\right)\right)}\right)\right]-A(\phi_i) \\ & +\delta\left[v\phi_i\left(1-\phi_j\right)+\left(1-\frac{(v-p_i)\left(1-\phi_j\right)}{\phi_j\left(p_i-\delta v\left(1-\phi_i\right)\right)}\right)v\left(1-\phi_i\right)\phi_i\phi_j\right]. \end{aligned}$$

The first-order condition with respect to ϕ_i is given by[20]

$$v\left(1-\phi_j\right)+v\delta\left(1-2\phi_j\phi_i\right)=A_\phi. \tag{25}$$

The equation above gives firm i's best response function for advertising intensity. It shows that advertising choices are strategic substitutes. It also shows that, when there are no costs to advertising, the best response of firm i to firm j choosing an advertising intensity of 1 (i.e.,

[20] Second order condition also holds, since $\frac{\partial^2 E\Pi_i}{\partial\phi_i^2}=-2v\delta\phi_j-A_{\phi\phi}<0$.

complete market coverage), is to choose an advertising intensity of 0.5. The intuition is that, in doing so, firm i is able to strategically reduce the size of the selective group of consumers, thereby avoiding the head-to-head price competition that would otherwise arise under full market coverage. Based on these two facts we will see below that in the symmetric subgame perfect equilibrium with price discrimination, both firms select incomplete market coverage, even when advertising is costless.

Proposition 6. *There is a symmetric mixed strategy subgame perfect Nash equilibrium in which,*

(i) each firm advertises a first-period price randomly chosen from the distribution

$$F^*(p) = \left\{ \begin{array}{ccc} 0 & for & p \leqq p^*_{\min} \\ 1 - \frac{(v-p)(1-\phi^*)}{\phi^*(p-\delta v(1-\phi^*))} & for & p^*_{\min} \leqq p \leqq v \\ 1 & for & p \geqq v \end{array} \right\} \tag{26}$$

with minimum equilibrium price equal to

$$p^*_{\min} = v(1-\phi^*) + \delta v \phi^* (1-\phi^*); \tag{27}$$

(ii) each firm selects an advertising reach, denoted ϕ^*, implicitly defined by

$$v(1-\phi^*) + v\delta \left(1 - 2(\phi^*)^2\right) = A_\phi(\phi^*) \tag{28}$$

where $A_\phi(0) \leq v(1+\delta)$.

(iii) Each firm earns expected overall equilibrium profits equal to

$$E\Pi^* = v\phi^*(1-\phi^*)(1+\delta(1+\phi^*)) - A(\phi^*) = \phi^* A_\phi - A(\phi^*) + \delta v (\phi^*)^3. \tag{29}$$

The next proposition characterizes the advertising behavior of firms when advertising is costless.

Proposition 7. *When advertising is costless, forward-looking firms do not select full market coverage. To be exact, in this case, the symmetric equilibrium level of advertising is given by* $\phi^* = \frac{1}{4\delta}\left(\sqrt{1+8\delta+8\delta^2} - 1\right) < 1$.

The proof of the above result is trivial. Given any general advertising technology, with form $A(\phi) = \alpha\eta(\phi)$, when α goes to zero, A_ϕ goes to zero as well. In that way, the right-hand side of (28) equals zero; thus, solving for ϕ, we obtain the result.

Unlike the behavior of firms in the no-discrimination benchmark (see corollary 1), we find that, being price discrimination permitted, firms do not fully advertise, even when advertising is costless. Considering, for instance, that $\delta = 1$ and that advertising costs are null, our findings show that there is a *symmetric* equilibrium in pure strategies, where each firm provides information to 78% of the market.[21]

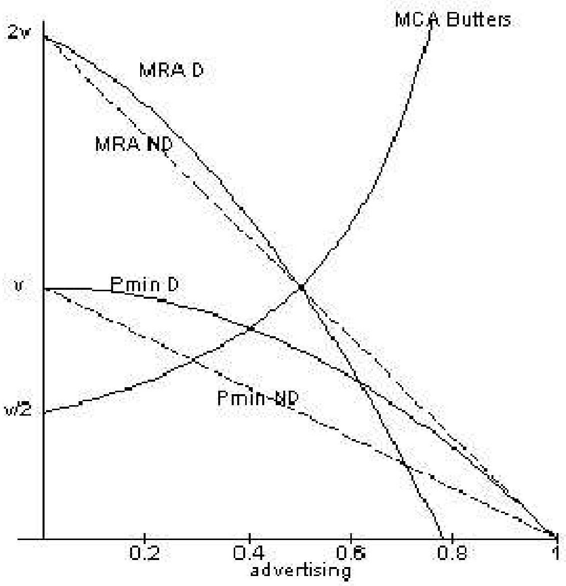
[21] It is important to stress that full market coverage for both firms, even when advertising is costless, does not arise in a simpler model, where advertising choices are committed to in period one. Firms, then, choose prices in a second period, after observing the opponent's advertising levels. To be precise, when advertising is costless and firms compete in prices after advertising decisions have been made, the subgame perfect equilibrium is asymmetric; while one firm fully advertises, the other covers 50% of the market. (See, for instance, Ireland (1993)).

Our findings shed some light on the dynamic effects of price discrimination on firms' advertising decisions. As we allow firms to compete in prices in period two and to price discriminate, we need to take these two facts into account when explaining the above result. First, when advertising is costless, each firm realizes that, by reducing its market coverage in period one, it strategically reduces the size of the selective group of consumers, thereby avoiding the head-to-head price competition that would otherwise arise under full market coverage. (This would be the only strategic effect in a model with no discrimination, but where firms could set different prices in period two.) Second, when price discrimination is permitted in the second stage, apart from the previous reasoning, each firm acknowledges that the benefit of price discrimination will be higher, when its advertising reach is not too high, as this induces the rival to play less aggressively in period 2. Altogether, each firm can commit to play less aggressively in the next stage of the game, by not selecting very high market coverage in period 1.

5 Competitive implications of price discrimination

In this section, we analyze how the permission of price discrimination affects the equilibrium outcomes—i.e., advertising intensity, prices and profits—in comparison to the situation in which price discrimination is banned. Figure 1 illustrates the solutions to equations (9) and (28) , and to equations (8) and (27), for the special case where $\delta = 1$ and $\alpha = \frac{v}{2}$, and using the technology proposed by Butters without loss of generality.

Figure 1: Equilibrium solutions with and without discrimination



The downward-slopping curves "MRA D" and "MRA ND" are, respectively, the marginal revenue of advertising with discrimination and with no discrimination. The upward-slopping curve "MCA" is the marginal cost of advertising for the Butters' technology. The intersection between "MRA D" and "MCA" provides the equilibrium level of advertising with price discrimination and, similarly, the intersection between "MRA ND" and "MCA" provides the equilibrium level of advertising with non discrimination. Additionally, the curves "Pmin D" and "Pmin ND" are, respectively, the minimum price selected by firms in period 1, when discrimination is permitted and when it is banned. In this manner, given the advertising equilibrium level in each case, we may determine the support of equilibrium prices, with and without discrimination. This figure is helpful to see mainly: (i) the effects of moving from non-discrimination to discrimination, (ii) the effects of advertising intensity on the equilibrium outcomes and (iii) the effects of changes in the exogenous parameters, namely α, v and δ.

5.1 Advertising decisions

Figure 1 shows that the left-hand side of equation (28)—i.e., the marginal revenue of advertising—is strictly decreasing, and that shifts in that curve arise when there is a change in v or in δ. Concretely, the effect of an increase in v is the upward shift of the curve, whilst the effect of a decrease in δ, from one to zero, is the downward shift of the curve until overlapping the "Pmin ND", which is exactly equal to the MRA in the static game. (Recall that in the static case, the equilibrium level of advertising is given by $v(1-\phi) = A_\phi$.) The right-hand side of equations (9) and (28)—i.e., the marginal cost of advertising—is the same with and without discrimination, and whatever the advertising technology considered. Shifts in the latter curve are only due to changes in α. Thus, we can see that an increase (decrease) in marginal advertising costs makes the curve move upwards (downwards) and gives rise to less (more) advertising in equilibrium. To be precise, because $A_{\phi\alpha} > 0$ and $A_{\phi\phi} \geq 0$, the static comparative analysis shows that $\frac{\partial \phi^*}{\partial \alpha} = -\frac{A_{\phi\alpha}}{v+4\delta v\phi + A_{\phi\phi}} < 0$ and $\frac{\partial \phi^{nd}}{\partial \alpha} = -\frac{A_{\phi\alpha}}{v+\delta v + A_{\phi\phi}} < 0$.

Let us turn now to the comparison between the equilibrium advertising decisions with and without discrimination. We can observe in Figure 1 that firms advertise less under discrimination for marginal advertising costs not too high, whilst they advertise more for high advertising costs. Clearly, price discrimination has no effect on advertising decisions when $\phi^* = \phi^{nd}$. From Figure 1, it is easy to see that both advertising intensities coincide when the marginal revenues of advertising with and without discrimination cross. Indeed, using (9) and (28), the reader can easily check that for $\delta > 0$, regardless of v and δ, it follows that the latter condition is satisfied whenever $\phi^* = \phi^{nd} = 0.5$ (which obviously occurs for a specific α). Otherwise, when α is such that $\phi^* > 0.5$, it follows that $\phi^* < \phi^{nd}$. Likewise, when α is such that $\phi^* < 0.5$, it ensues that $\phi^* > \phi^{nd}$.[22] Thus, the comparison is ambiguous. When the advertising costs are high, there is more advertising with discrimination; when advertising is cheap, the reverse happens.

5.2 Prices

This subsection examines the impact of behavior-based price discrimination on second and first-period prices.

Second-period prices Being price discrimination permitted, the discriminating firm increases (or at least does not reduce) the price to its captive group of consumers. Thus, those consumers that bought from the highest priced firm in period 1—the discriminating firm in period 2—are expected to pay higher first and second-period prices. From a comparison between F^* and G^r, we observe that $F^* < G^r$, which means that the first-period price is stochastically larger than p^r, as it assumes large values with higher probability.[23] From the latter result, it follows that, if poaching occurs, the group of selective consumers will pay, on average, a lower price in period 2. Finally, regarding the group of captive consumers that bought from the lowest priced firm before, the conclusion is not so clear-cut. We have seen that the non-discriminating

[22] For the Butters' technology, we find that $\phi^* \geq \phi^{nd}$, when $\alpha \geq \frac{1}{4}v(1+\delta)$ and $\phi^* < \phi^{nd}$ otherwise. Similarly, if $\eta(\phi) = \phi^2$, then $\phi^* \geq \phi^{nd}$ when $\alpha > \frac{1}{2}v(1+\delta)$; otherwise $\phi^* < \phi^{nd}$.

[23] Because ϕ is the same in period one and two, F^* first-order stochastically dominates G^r, if $1 - \frac{(v-p)(1-\phi)}{\phi(p-\delta v(1-\phi))} \leq 1 - \frac{(v-p)(1-\phi)}{p\phi}$; i.e., if $-\delta v(1-\phi) \leq 0$, which is always tue.

firm uses a "Hi-Lo" pricing strategy in the second period. With a probability equal to m_j, its locked-in customers will pay a higher second-period price, namely v. Otherwise, because it is not possible to establish a general stochastic order between F^* and $\widehat{G}$, this set of consumers may end up paying a higher or lower second-period price.

First-period prices The benefit of price discrimination gives rise to what we designate as the "*race for discrimination effect*". This effect is related to the idea that the benefit of price discrimination motivates a firm to advertise high first-period prices, in order to secure the discriminating position in the second-stage. As the future becomes more important, the higher is that benefit and so the stronger the "*race for discrimination effect*" becomes. Conversely, as δ declines, the benefit of price discrimination shrinks, weakening the incentives to set high first-period prices (note that $\frac{\partial p_{\min}}{\partial \delta} > 0$). Furthermore, because prices are strategic complements, when one firm raises its price, it induces the rival firm to do the same. In other words, the "*race for discrimination effect*" acts to soften first-period price competition, suggesting that, price discrimination being permitted, firms tend to charge higher first-period prices than if discrimination were banned. A similar result is obtained by Chen and Zhang (2004). In their model, the market is exogenously divided into captive customers and switchers, and firms can perfectly reach all of them. In particular, they assume that switchers have a lower reservation price than captive customers and that a firm is only able to distinguish a switcher from a captive customer, if it sells exclusively to its captive customers in period 1, that is if its price is high enough. They show that, in period 2, it may happen that only one firm discriminates or both of them do. Analogously, they show that the benefit of price discrimination in the second period motivates a firm to increase first-period prices, in order to obtain the required information for price discrimination.

In the present model, apart from the "*race for discrimination effect*", there is also the effect of advertising on first-period prices. This is the "*advertising effect*" on first-period prices. If price discrimination had no effect on advertising (i.e., if $\phi^* = \phi^{nd}$), this latter effect would play no role; in which case, F^* would dominate stochastically F^{nd}, and first-period prices would move upwards (exclusively due to the former effect).[24] Because there is a negative relation between the advertising reach and prices,[25] this effect is expected to reinforce the "*race for discrimination effect*", when price discrimination gives rise to less advertising and, so, to higher prices. This conclusion is not so clear cut when price discrimination gives rise to more advertising (this is particularly the case when α is too high). In this situation, the "*advertising effect*" would act in favor of lower first-period prices due to more advertising. That is, when $\phi^* > \phi^{nd}$, although firms have an incentive to raise prices, they also invest in more advertising—which, in turn, has a negative effect on prices.

Even though it is not possible to establish a general stochastic ordering between $F^*(p)$ and $F^{nd}(p)$, we can see in Figure 1 that, when marginal advertising costs are not too high, price discrimination leads firms to select a lower advertising reach and higher first-period prices than under non-discrimination case. To confirm this, we carried out a numerical analysis for the Butters' technology, as well as for the quadratic technology, and for different values of α. For both technologies considered, we found that first-period prices with discrimination are, on average, above their non-discrimination counterparts, even when α is extremely high. Thus, our numerical investigation suggests that, even when advertising costs are high, first-period prices

[24] In this case we would have $\phi^* = \phi^{nd}$. Then F^* first-order stochastically dominates F^{nd} if $1 - \frac{(v-p)(1-\phi)}{\phi(p-\delta v(1-\phi))} \leq 1 - \frac{(v-p)(1-\phi)}{p\phi}$, i.e., if $-\delta v(1-\phi) \leq 0$, which is always tue.

[25] Note that $\frac{\partial p_{\min}}{\partial \phi^*} < 0$ and $\frac{\partial F^*}{\partial \phi^*} > 0$.

tend to be, on average, above their non-discrimination counterparts.[26]

In sum, we can say that, for marginal advertising costs such that $\phi^* < \phi^{nd}$, first-period prices are, on average, above their non-discrimination counterparts. Whether first-period prices are above or below their non-discrimination counterparts, when α is such that $\phi^* > \phi^{nd}$, strongly depends on which effect dominates. As a general rule, it can be said that, when the "*race for discrimination effect*" dominates, we expect first-period prices to be on average above the non-discrimination levels; otherwise, the reverse might happen.

At this point, it is appropriate to compare our predictions with those derived in the extant literature on behavior-based price discrimination. As mentioned, our model proposes a new ground to evaluate the dynamic effects of this price discrimination form. We have seen in the Introduction that two different approaches have been considered so far. While in the brand preferences approach purchase history discloses important information about a consumer's exogenous brand preference that is present from the beginning, in the switching costs approach, past behavior reveals information about exogenous switching costs. Here, as in the switching costs approach, consumers are ex-ante identical regarding their preferences for the firms. However, after advertising decisions have been made, some consumers are *endogenously* locked in with a certain firm, not due to the existence of switching costs, but rather because they ignore the other firm. Thus, in our setting, each consumer's first-period choice discloses information about his awareness.

Regarding second-period prices, the Fudenberg-Tirole and Chen models share the feature that second-period prices are *all* lower than if behavior-based discrimination were not permitted. In the present analysis, in contrast, some consumers are expected to pay higher second-period prices. More precisely, the firm's existing customers are expected to pay higher prices than first-time customers do.

Additionally, unlike the switching costs approach, where firms price below the non-discrimination levels in period 1, our findings show that, at least for advertising costs not too high, first-period prices are, on average, above their non-discrimination counterparts. Similar results were obtained, within the brand preference approach, by Fudenberg and Tirole (2000). However, the intuition behind their result is different from ours. In their model, firms are able to quote higher first-period prices than if price discrimination were banned, because consumers anticipate poaching and become less sensitive to prices in period 1. Here, first-period prices are, on average, above their non-discrimination counterparts, due to a strategic behavior of competing firms, i.e., due to the "race for discrimination effect", which tends to be stronger when price discrimination leads firms to underinvest in advertising.

Finally, the next proposition summarizes some of the effects of changes in α on first-period prices. Advertising costs affect the distribution of prices through ϕ^*. Since increases in α make ϕ^* fall, the price distribution shifts towards higher prices.[27]

Proposition 8. *Regardless of the advertising technology considered, as advertising becomes cheaper, the minimum price in the equilibrium support is smaller and lower prices are more likely. However, as advertising becomes costless,*

$$p^*_{\min} \to v\left[1 - \frac{1}{4\delta}\left(\sqrt{1+8\delta+8\delta^2} - 1\right)\right]\left(\frac{3}{4} + \frac{1}{4}\sqrt{1+8\delta+8\delta^2}\right) > 0.$$

[26] Details are available from author on request.

[27] More precisely, it is easy to see that $\frac{\partial F^*}{\partial \alpha} = \frac{\partial F^*}{\partial \phi^*}\frac{\partial \phi^*}{\partial \alpha} < 0$ and $\frac{\partial p_{\min}}{\partial \alpha} = \frac{\partial p_{\min}}{\partial \phi^*}\frac{\partial \phi^*}{\partial \alpha} > 0$.

We have seen in proposition 7 that, when marginal advertising costs go to zero, firms do not fully advertise. Therefore, as long as $\delta > 0$, the classic Bertrand equilibrium is not a solution of the price discrimination game.

Note also that price dispersion, measured by the range of prices, tends to increase with decreases in advertising costs. Proposition 7 predicts that we should observe higher levels of price dispersion in markets characterized by low costs of information provision. A similar finding is obtained by Baye and Morgan (2001), who show that price dispersion is greater, as it becomes less costly for firms to list prices at price-comparison sites. As it becomes less costly to provide information to customers, more consumers will be informed. Thus, as more and more people have access to more information, prices tend to fall. This reduces the lower bound of the distribution of prices, which gives rise to a wider range in advertised prices. Hence, this result challenges the view that price dispersion should decrease as more consumers become informed.

5.3 Profitability

The profitability issue of price discrimination is one of the major concerns when it comes to assess the competitive effects of this practice. We revisit this issue in what ensues. Regarding the firm that can price discriminate in period 2, we have already seen that it is always better off under discrimination. This is not, of course, a surprising result. It is well known that price discrimination is privately profitable for each firm. Given the rival's strategy, each firm is better off, if it is free to adjust its prices to different groups of customers (e.g. Thisse and Vives (1988)).

What is more interesting is the fact that, depending on the level of market coverage selected by firms in the first-stage of the game, the non-discriminating firm might also become better off in period 2. The non-discriminating firm would face no effect on its second period profit, if each firm's advertising decision were not affected by discrimination. However, as said, this is not usually the case. Specifically, under symmetry, and denoting by ϕ^* each firm's first-period advertising decision when discrimination is permitted, from the comparison between $v\phi^{nd}\left(1-\phi^{nd}\right)$ and $v\phi^*\left(1-\phi^*\right)$ (see equation (20)), it immediately follows that the non-discriminating firm might as well gain when its rival discriminates. On the one hand, when $\phi^{nd} < \phi^* < 0.5$, the size of the captive segment is greater than the size of the selective segment, and it is even higher under poaching in period 2. With discrimination, the non-discriminating firm faces a greater captive segment in period 2 and, consequently, its second-period expected profit moves upwards. On the other hand, the non-discriminating firm might also become better off when $0.5 < \phi^* < \phi^{nd}$. In this case, although the size of the captive segment is smaller than the size of the selective segment, moving from no discrimination to discrimination increases the size of the former segment at the expense of the selective group's size. Clearly, this benefits the non-discriminating firm. A key finding is that whenever price discrimination increases the proportion of captive customers, the non-discriminating firm will also become better off when its rival discriminates. Since the two conditions hold in equilibrium for any α, it immediately follows that the non-discriminating firm might also gain, when its competitor discriminates.

A common feature of the extant models on customer poaching is that price discrimination in period 2 tends to intensify competition between firms and to decrease equilibrium profits. In contrast, in our model, *only* one firm has information to engage in behavior-based price discrimination in period two, as its opponent learns nothing. This kind of asymmetry in the available information for price discrimination purposes acts to soften price competition in period two, permitting second-period profit to move upwards. Esteves (2005) and Armstrong (2006) point out that, in the context of the Fudenberg-Tirole model, where there is best response asymmetry, equal market shares in the first-period give rise to the most informative outcome, which tends to destroy second period profits.[28]

Let us now look at the effect of price discrimination on overall expected profit. Intuitively, as first-period prices tend to be above their non-discrimination counterparts, we may expect first-period profits to be also above the non-discrimination levels. Obviously, this seems to suggest that overall expected profits would move upwards, when price discrimination was introduced. It is worth recalling that from (10) when price discrimination is banned, the overall expected profit is equal to

$$E\Pi^{nd} = v\left(1+\delta\right)\phi^{nd}\left(1-\phi^{nd}\right) - A\left(\phi^{nd}\right), \tag{30}$$

whereas when price discrimination is allowed, the overall expected profit is equal to

$$E\Pi^{*} = v\left(1+\delta\right)\phi^{*}\left(1-\phi^{*}\right) + \delta v\phi^{*}\left(1-\phi^{*}\right) - A\left(\phi^{*}\right). \tag{31}$$

It follows immediately that if $\phi^* = \phi^{nd}$, the overall expected profit under discrimination is always above the overall expected profit under non-discrimination. Additionally, the reader can easily verify that the first term in (30) and (31) has an inverted-U relationship with the advertising intensity, reaching its maximum value at $\phi^{nd} = 0.5$ and $\phi^* = 0.5$. From our previous discussion, it has become clear that, in equilibrium, depending on α, we may obtain: (i) $\phi^* = \phi^{nd} = 0.5$, (ii) $0.5 < \phi^* < \phi^{nd}$ or (iii) $\phi^{nd} < \phi^* < 0.5$. The case in (i) was already discussed. Let us now compare overall equilibrium profits when $0.5 < \phi^* < \phi^{nd}$. In this case, it follows that the first term in (31) is greater than the first term in (30). On the other hand, because $\phi^* < \phi^{nd}$, it immediately follows that $A\left(\phi^*\right) < A\left(\phi^{nd}\right)$. If we further take into account that the second term in (31) is positive, it ensues that $E\Pi^*$ is strictly higher than $E\Pi^{nd}$. Thus, in this case price discrimination increases overall profits. In sum, when α is such that $\phi^* \leq \phi^{nd}$, overall equilibrium profits under discrimination are always above their non-discrimination counterparts, regardless of the advertising technology considered.

As previously mentioned, the more complex situation occurs when price discrimination leads firms to increased advertising. This happens when α is such that $\phi^{nd} < \phi^* < 0.5$. Again, it is easy to confirm that, for this range, the first term in (31) is larger than the first term in (30). We already know that the second term in (31) is also positive. However, since $\phi^* > \phi^{nd}$, it follows that $A\left(\phi^*\right) > A\left(\phi^{nd}\right)$. This means that overall expected profits under discrimination will be above their non-discrimination counterparts, as long as the increase in advertising costs does not offset the increase in sales revenue. In order to verify whether profits increase with discrimination in the latter case, we have carried out a numerical analysis using the quadratic and Butters technologies for different values of α. Our numerical simulations confirmed our previous intuition that the increase in advertising costs is not enough to offset the increase in sales revenue that results from departing from non-discrimination to discrimination.[29]

Corollary 5. *Regardless the advertising technology considered, the overall equilibrium profit with discrimination is above its non-discrimination counterpart, at least for advertising costs not too high (i.e., when α is such that $\phi^* \leq \phi^{nd}$).*

The previous result is in contrast with findings in other models on behavior-based price discrimination, in which firms are clearly worse off, if price discrimination is permitted (Chen

[28] Esteves (2005) proposes a variant of the Fudenberg-Tirole model, by assuming that consumers' preferences follow a binary distribution: 50% prefer firm A by a fixed amount and the remaining consumers prefer firm B by the same fixed amount. In this setting, if firms quote extremely different first-period prices, the market is entirely served by the same firm, and nothing is learned from period one. When this happens, price discrimination is not feasible and, so, the second period profit is high. On the other hand, when firms share the initial market equally, both have the same information for price discrimination, and the second period profit decreases.

[29] Details are available from author on request.

(1997), Villas-Boas (1999), Fudenberg and Tirole (2000) and Esteves (2005)). However, it is closely related to the Chen and Zhang's (2004) result, that price discrimination may benefit competing firms.

Two important common features that arise in those models where firms are negatively affected by price discrimination are: (i) *both* firms have information that is obtained by observing the customers' past behavior, meaning that *both* firms engage in price discrimination, and (ii) the market displays *best-response asymmetry* (i.e. the weak market of one firm is the rival's strong market). In this set of models, as each firm tries to poach the rival's previous customers, price discrimination intensifies competition, and profits fall down. In contrast, in our model, only one firm has information for price discrimination purposes and, in the second period, the weak market of the discriminating firm is the selective group of consumers that previously bought from the rival, which is the weak market of the non-discriminating firm as well. Since only one firm discriminates and the non-discriminating firm uses a "Hi-Lo" pricing strategy, price competition for the group of selective consumers is weaker than if both firms were competing exclusively for that group. Note that we have seen that firms competing less aggressively in the second period when the non-discriminating firm has a greater captive segment (as in this case the non-discriminating firm is more likely to charge the highest price, v). Finally, in our analysis there is another important effect at work—namely, the "*race for discrimination effect*"—through which price discrimination acts to *soften* price competition in period 1, permitting profits to rise.

Next, we investigate the impact of the exogenous parameters on profits. As expected, profits are increasing in the consumers' willingness to pay, v. More interesting is the response of firms' profits to variations in advertising costs.

Proposition 9. (Advertising costs and profits)
For the quadratic technology, it follows that, when $\delta = 1$:

(i) under non-discrimination, sellers benefit from advertising costs increases if advertising costs are low;

(ii) under discrimination, sellers are always better off with decreases in advertising costs.

Proof. See the Appendix.

As expected, part (i) confirms a well known result in the extant literature on informative advertising: profits of a firm may increase with advertising costs (e.g. Grossman and Shapiro (1984), Stahl (1994), to name a few). In general, whilst an increase in advertising costs has a negative *direct effect* on profits, there is, as well, a *strategic effect*: as advertising costs increase, firms respond with less advertising, permitting prices to rise. When the strategic effect dominates, profits may increase with advertising costs. In particular, in the appendix provided it is shown that, for the quadratic technology, firms benefit from advertising costs increases, if advertising costs are relatively low (i.e., if $\alpha < \frac{v}{2}(1+\delta)$). If we depart from a situation where ϕ^{nd} is low (α is high), we observe that additional advertising is more likely to increase the fraction of captive customers than the fraction of selective customers. It turns out that the probability of reaching an uninformed buyer is high and, then, firms have more incentives to focus on the group of captive consumers, thereby quoting high prices. However, as α becomes increasingly smaller and advertising becomes increasingly higher, more buyers will be aware of both firms' prices. As the size of selective segment increases, the danger of losing those consumers to the other firm is greater and, therefore, low prices are more likely. An increase in α induces less advertising, relaxes price competition and allows firms to increase prices by enough to offset the direct effect on profits.

Interestingly, proposition 9 claims that (at least for the quadratic technology) when price discrimination is allowed, profits and advertising costs move in opposite directions. That is, an increase in advertising costs is always bad for profits. Following the theory, this suggests that the direct effect is stronger than the strategic effect. When α is high (ϕ is low), the arguments presented above are valid to explain why profits decrease with the increases in α. However, when α is low (ϕ is high) and we move from non-discrimination to discrimination, we find that the relationship between profits and advertising costs is reversed—i.e., profits no longer increase with the increases in advertising costs. The reason is that under no discrimination, lower and lower advertising costs tend to push firms to the Bertrand outcome, while with price discrimination the Bertrand result is far from being reached. Expressed differently, under non-discrimination, an increase in α, when α is low, has the strategic effect of avoiding a more aggressive behavior, thereby increasing the firms' profits; in contrast, under price discrimination, the strategic effect plays a very small role, as full market coverage is never provided in equilibrium. The direct effect dominates, and profits are clearly negatively affected by increases in advertising costs.

6 Welfare issues

This section evaluates the welfare effects of price discrimination, enabled by informative advertising. First, we examine the conventional question of whether there is too little or too much informative advertising. With that in mind, we draw a comparison between the level of advertising that would be selected by an advertising regulator, whose aim would be to maximize total welfare, and compare that level with the market equilibrium. Second, we discuss how the transition from non-discrimination to discrimination affects aggregate welfare, as well as the sellers' and consumers' surplus. Although prices play no welfare role here—due to the unit demand assumption, no dropping out of consumers and no loyalty costs—price discrimination being permitted would affect advertising decisions and thereby, the efficiency properties of the equilibrium advertising level. In this way, given that the regulator would select the same level of advertising, regardless of price discrimination, we also evaluate whether the introduction of price discrimination enhances or not the efficiency properties of sellers' advertising decisions. To simplify the analysis, throughout this section, it is assumed that $\delta = 1$.

Since production costs are assumed to be zero, total welfare is equal to the value of the good for all buyers that enter the market in both periods minus total advertising costs, that is

$$W = 2v\left[1 - (1-\phi)^2\right] - 2A(\phi). \tag{32}$$

6.1 Do firms under or over advertise?

In a homogeneous product market with competition, there are mainly two effects behind the divergences between the equilibrium and the social optimal level of advertising. Firstly, an ad that reaches any consumer that would be otherwise uninformed generates a sale whatever the price. From a social point of view, this action is desirable, because more consumers can enter the market, thereby increasing total welfare. Nonetheless, due to price competition, firms cannot extract all the surplus generated by advertising, because they are not able to charge the highest price. As a result, due to the *"nonappropriability of social surplus effect"*, firms tend to underadvertise. Secondly, when a firm decides to invest in more advertising, it does not take into account the profit reduction of the rival firm, as it increases its own advertising level and poaches some of the rival's customers. When this latter effect—the "*business stealing effect*"—is the dominant one, firms tend to overadvertise. In this case, more advertising may be socially undesirable, if it merely leads to a swap of consumers between firms.

In what follows, we assume that the regulator has access to the same advertising technology as firms and that maximizes W, given by (32), with respect to ϕ. From the first-order condition, the social optimal level of advertising, denoted by ϕ^w, is implicitly given by

$$2v\left(1-\phi^{w}\right)=A_{\phi}. \tag{33}$$

Recall that, in the overall game with price discrimination, the equilibrium level of advertising, ϕ^*, is implicitly given by

$$v\left(1-\phi^{*}\right)+v\left(1-2\phi^{*2}\right)=A_{\phi}; \tag{34}$$

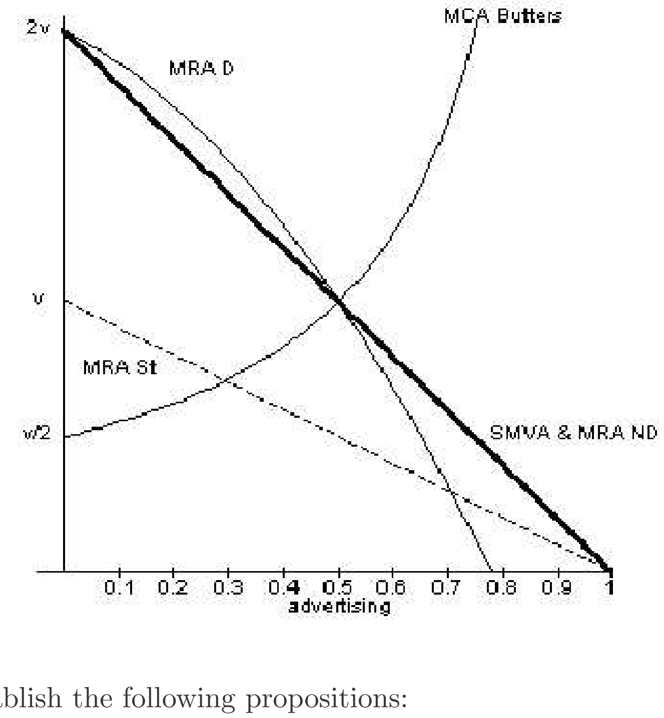
whilst, without discrimination, the equilibrium level of advertising, ϕ^{nd}, is implicitly given by

$$2v\left(1-\phi^{nd}\right)=A_{\phi}. \tag{35}$$

It immediately follows that the expression that defines the social optimal level of advertising is equal to the expression that defines the equilibrium level of advertising under non-discrimination.

Figure 2 is based on Figure 1 except that now the left-hand side of equation (33) is introduced, which is the social marginal value of advertising net of advertising costs, namely "SMVA", and the equation that defines the minimum equilibrium price is removed. In addition, the MRA with and without discrimination, for $\delta=0$, given by "MRA St", is also represented. As in Figure 1, without loss of generality, it is assumed that $\alpha=\frac{v}{2}$, and the advertising technology considered is that proposed by Butters. We can see below that as δ decreases, the marginal revenue of advertising with and without discrimination, as well as the social marginal value of advertising, move downward until reaching the marginal revenue in the static (or myopic) case.

Figure 2: Efficiency of advertising decisions for the Butters's technology



We may now establish the following propositions:

Proposition 10. (Efficiency properties with no-discrimination) *Suppose there is a ban on price discrimination. Then, firms select the social optimal level of advertising.*

Proposition 11. (Efficiency properties with discrimination) *Regardless the advertising technology considered, when price discrimination is permitted and* $\delta = 1$, *firms select the social optimal level of advertising if* α *is such that* $\phi^* = \phi^w = 0.5$; *underadvertise if* α is such that $0.5 < \phi^* < \phi^w$; *and overadvertise if* α is such that $\phi^w < \phi^* < 0.5$.

Proof. See the Appendix.

We illustrate these results using of the Butters's technology. However, both propositions are quite robust for any advertising technology, because we only need to look at MRA and SMVA. Proposition 10 can be simply proved using (33) and (35). Since the left-hand side of both equations coincides, one gets the result. A similar result was obtained for the first-time in Butters' seminal paper. He showed that in a static monopolistically competitive setting, the market equilibrium level of advertising is socially optimal. The same always happens in a static version of our model, as well as in our no discrimination benchmark, but with only two-firms and for any advertising technology. Although Butters was unable to offer an intuition for his welfare finding, the intuition for our result is partially based on Bagwell (2005). It runs as follows. Since a consumer that receives an ad today will enter the market in both periods, welfare only increases if an additional ad is received by an uninformed consumer. That is, the social benefit of an additional ad is equal to $v(1+\delta)$ times the probability of that consumer receiving no other ad. In a mixed strategy equilibrium in prices, the overall private benefit to a firm of sending an ad at any price $p \in [p_{\min}, v]$ is constant and equal to $v(1+\delta)$ times the probability of the consumer receiving no other ad. It is, therefore, evident that in the benchmark game without discrimination, private and social benefits coincide.

New theoretical findings arise when the model is extended to allow price discrimination using advertising in period 2. As far as our knowledge is concerned, this is the first attempt to evaluate the advertising behavior of firms in a market where price discrimination may occur at the firm level. Interestingly, when $\delta = 1$, we get that from the social point of view, firms may supply too little, too much or even the efficient level of advertising. To understand this outcome let us observe separately the effect of advertising on firms and consumers surplus. From the sellers overall expected profit, given by (29), we obtain that when $\delta = 1$,

$$\frac{\partial E\Pi^*}{\partial \phi} = 2v(1-\phi) - 3v\phi^2 - A_\phi.$$

Evaluating at the non-cooperative equilibrium level of advertising, where $A_\phi = v(1-\phi) + v\left(1-2\phi^2\right)$, it follows that at $\phi = \phi^*$

$$\frac{\partial E\Pi^*}{\partial \phi} = -\phi^* v(1+\phi^*) < 0.$$

Hence, from the firms' point of view, there is too much advertising. When a firm decides to invest in more advertising, it does not take into account the profit reduction at the rival firm, as it increases its own advertising level and poaches some of the rival's customers. Due to this "*business stealing effect*", firms tend to overadvertise.

Conversely, from the consumers' point of view, we will see below that there is underadvertising. Expected consumer surplus, denoted by ECS, is given by total welfare minus industry profits. In the price discrimination game, the expected consumer surplus, when $\delta = 1$, is equal to

$$ECS^* = W^* - 2E\Pi^* = 2v\left(\phi^*\right)^3.$$

Thus, at $\phi = \phi^*$

$$\frac{\partial ECS^*}{\partial \phi} = 6v(\phi^*)^2 > 0.$$

It is obvious that, from the consumers' point of view, for $\phi = \phi^* < 1$, there is always too little advertising . When firms are not able to appropriate the entire surplus generated by advertising, they tend to underadvertise.

Given that consumers desire more advertising and firms desire less advertising, the net social effect is not immediately obvious. Adding the two previous effects we find that, evaluated at $\phi = \phi^*$,

$$\frac{\partial ECS^*}{\partial \phi} + 2\frac{\partial E\Pi^*}{\partial \phi} = 2\phi^* v \left(2\phi^* - 1\right).$$

Therefore, the net social effect is equal to zero if and only if $\phi^* = 0.5$. This advertising intensity is efficient from the social point of view. However, the net effect is positive, when α is such that $\phi^* > 0.5$, which means that firms advertise too little. (Indeed, Figure 2 shows that, when α is such that $\phi^* > 0.5$, it follows that $\phi^* < \phi^w$.) The reason is that, in this case, the *"nonappropriability of social surplus effect"* dominates. From our previous discussion, recall that firms charge, on average, lower prices, when ϕ is high (α is low). Finally, when $\phi^* < 0.5$, it follows that $\phi^* > \phi^w$ (see Figure 2), so firms overadvertise.[30] To summarize, when price discrimination is allowed, firms may provide too little, too much or even the social optimal level of advertising.

6.2 Welfare effects of price discrimination

A recurrent policy question in the price discrimination literature is whether to restrict or encourage the practice of setting discriminatory prices. We have already noted that in this model, there is a unique source of social inefficiency, the advertising intensity selected by firms, which endogenously determines the fraction of consumers that will enter the market. In this context, it immediately follows that a key ingredient to understand the welfare effects of behavior-based price discrimination is the impact of price discrimination on the firms' advertising choices. When firms can engage in price discrimination, welfare is equal to

$$W^* = 2v\left[1 - \left(1 - \phi^*\right)^2\right] - 2A\left(\phi^*\right),$$

whilst when there is a ban on price discrimination it is equal to

$$W^{nd} = 2v\left[1 - \left(1 - \phi^{nd}\right)^2\right] - 2A\left(\phi^{nd}\right).$$

Proposition 12. *Regardless the advertising technology considered, when* α is such that $\phi^* \neq 0.5$, *welfare falls with price discrimination.*

Proof. See the Appendix.

The intuition for this result is straightforward if we take into account our previous findings. We have seen that a ban on price discrimination leads firms to *always* select the social optimal level of advertising. Therefore, apart from the case in which α is such that $\phi^* = 0.5$, the price discrimination permission introduces a distortion into the level of advertising provided by firms, thereby reducing total welfare. Thus, proposition 12 provides a simple criterion for predicting whether price discrimination will be desirable or not in a market where advertising is the consumers' sole source of information. The main conclusion is that, apart from the special case in which $\phi^* = 0.5$, from the social point of view, price discrimination should be banned.

[30] Considering, for instance, the Butters' technology, we can see in Figure 2 that, when $\delta = 1$ and $\alpha = \frac{v}{2}$, $\phi^* = \phi^w = 0.5$. For higher values of α, firms overadvertise; whilst for smaller values, they underadvertise.

Because advertising choices are ultimately dependent on advertising costs, any policy drawn in favor or against discrimination should also take into account the costs' structure of each advertising-product market. As a whole we observe that, as advertising costs rise, welfare falls $\left(\frac{\partial W}{\partial \alpha} < 0\right)$. Figure 2 illustrates this result. If price discrimination is permitted, when advertising is cheap firms advertise too little, and thereby, from the social point of view, too many consumers are left out of the market. On the contrary, when advertising costs are high, firms provide too much advertising from the social point of view, and less advertising would be welfare enhancing. Only in the special case where the advertising cost is such that $\phi^* = 0.5$, will price discrimination introduce no welfare distortion.

We have seen that, at least in markets where advertising costs are not too high, profits rise when firms may add price discrimination to their pricing strategies in period 2. Together, the profit and the welfare effects of price discrimination, suggest that the consumer surplus falls when price discrimination is introduced.

The present model highlights the importance of taking into account different forms of market competition when public policies try to evaluate the welfare effects of behavior-based price discrimination. In broad terms, in the two approaches considered in the literature so far, behavior-based price discrimination has been proved to be welfare reducing, due to excessive inefficient switching (e.g. Chen (1997), Fudenberg and Tirole (2000)).[31] Here, in contrast, price discrimination based on purchase history causes welfare to fall, not due to excessive switching, but because firms do not advertise in a socially efficient manner. Furthermore, in contrast with Fudenberg and Tirole, our analysis shows that price discrimination does not benefit consumers, as at least some of them are expected to pay higher prices.

7 Conclusions

This article has provided a first look at the dynamic effects of customer poaching in homogeneous product markets, where advertising plays two major roles: it is used by firms as a way to transmit relevant information to otherwise uninformed consumers, and it is used as a price discrimination device. When a firm can recognize customers with different past purchasing histories, it may send them targeted ads with different prices. In particular, it was shown that only the firm that advertises the highest price in the first-period has information to engage in price discrimination in the second period, and that poaching clearly benefits the discriminating firm. When a firm foresees that benefit, "*the race for discrimination effect*" is identified, through which each firm has an incentive to price high in the initial period. Mainly on the basis of this strategic effect, it was proved that price discrimination may act to *soften* price competition rather than to intensify it. As a result of that, it was shown that, regardless the advertising technology considered, *all* firms might become better off, even when only one of them can engage in price discrimination.

By proposing a framework in which advertising is the consumers' sole source of information, our analysis has allowed us to investigate the interaction between advertising decisions and behavior-based price discrimination. For the special case where advertising is costless and discrimination is not permitted, it was found that the market provides full market coverage and firms realize no economic profit. Conversely, departing from non-discrimination to discrimination has proved to change that result—i.e., both firms choose incomplete market coverage—, which in turn translates into positive profits. Further, it was shown that when advertising is costly, there is more advertising with discrimination than with no discrimination; when advertising is cheap, the reverse happens.

[31] By considering a discrete framework for consumer types, Esteves (2005) shows that the static and the first-period equilibrium of the two period price discrimination game are in mixed strategies. Thus, under certain conditions, she shows that behavior-based price discrimination may increase efficiency.

The welfare effects of informative advertising and price discrimination were also analyzed with a new perspective. In the no-discrimination benchmark the market provides the social optimal level of advertising. In contrast, when firms can engage in price discrimination, it was found that firms might provide too little, too much or even the social optimal level of advertising. Obviously, the effect of price discrimination on the efficiency properties of advertising is strongly related to the costs structure of each advertising-product market. In this way, this article has predicted that, in markets with low advertising costs, allowing firms to price discriminate, leads them to provide too little advertising. Only in markets with very high marginal advertising costs may firms overadvertise from a social point of view. Based on the effects of discrimination on the efficiency properties of informative advertising, it was possible to establish a simple criterion for predicting whether price discrimination would be desirable or not. In this regard, it was found that behavior-based price discrimination is generally good for firms (at least when advertising costs are not too high), but bad for overall welfare and consumer surplus.

In light of the above, this article has tried to contribute to the ongoing debate on the implications of new forms of price discrimination, only made possible in the context of new media markets. It is obvious that the specificity of each market plays an important role on the conclusions derived. A special limitation of the stylized model addressed in this article is the assumption that advertising is the consumers' sole source of information. Although this assumption may at first sight seem odd in the context of online markets, it helped us to isolate the effects of price discrimination on the advertising decisions of firms. Evidently, while in new product markets this assumption might not be very restrictive, in other markets it might be inadequate. Allowing consumers to obtain information through advertising and costly search could be a natural extension, bringing new insights to the analysis. Another worthwhile extension would be to depart from the unit demand assumption. Extending the present analysis, as well as the existing models of customer poaching, to elastic demands would bring new interesting insights, improving the economic understanding of behavior-based price discrimination in competitive markets.

APPENDIX

This appendix collects the proofs that were omitted from the text.

Proof of Proposition 1.

Suppose (p_A^*, p_B^*) is an equilibrium in pure strategies. Then, by definition, there is no such p_i, $i = A, B$, such that $\pi_i\left(p_i, p_j^*\right) > \pi_i\left(p_i^*, p_j^*\right)$. The proof proceeds by contradiction.

(i) If $p_A^* = p_B^*$,

$$\pi_A\left(p_A^*, p_B^*\right) = \phi_A\left(1-\phi_B\right)p_A^* + \frac{1}{2}\phi_A\phi_B p_A^*. \tag{36}$$

If firm A deviates and charges $p_A = p_A^* - \varepsilon$, with $\varepsilon > 0$, then, its profit from deviation is

$$\pi_A\left(p_A, p_B^*\right) = \phi_A\left(1-\phi_B\right)\left(p_A^* - \varepsilon\right) + \phi_A\phi_B\left(p_A^* - \varepsilon\right). \tag{37}$$

From (36) and (37), $\pi_A\left(p_A, p_B^*\right) > \pi_A\left(p_A^*, p_B^*\right)$ if $\varepsilon < \frac{1}{2}\phi_B p_A^*$. Since $\phi_B > 0$ such an ε always exists. A contradiction. *Q.E.D.*

(ii) Without loss of generality, let $p_A^* < p_B^*$. Then $\pi_A\left(p_A^*, p_B^*\right) = \phi_A\left(1-\phi_B\right)p_A^* + \phi_A\phi_B p_A^*$. Let $p_A = p_A^* + \varepsilon < p_B^*$, then, firm A's profit from deviation is $\pi_A\left(p_A, p_B^*\right) = \phi_A\left(1-\phi_B\right)\left(p_A^* + \varepsilon\right) + \phi_A\phi_B\left(p_A^* + \varepsilon\right)$, from which it is easy to see that there is an ε such that the deviation is profitable. A contradiction. *Q.E.D.*

(iii) If $p_A^* > p_B^*$, then $p_B^* < p_A^*$ and, as firm A does in (ii), there is a profitable deviation for firm B. A contradiction. *Q.E.D.*

Proof of Proposition 3.

Suppose $\left(p_i^{r*}, \widehat{p}_j^*\right)$ is an equilibrium in pure strategies. Then, by definition, there is no such p_i^r, $i = A, B$, such that $\pi_i^r\left(p_i^r, \widehat{p}_j^*\right) > \pi_i^r\left(p_i^{r*}, \widehat{p}_j^*\right)$. The proof proceeds by contradiction.

(i) If $p_i^{r*} = \widehat{p}_j^*$, then

$$\pi_i^{r*} = \frac{1}{2}\phi_i\phi_j p_i^{r*}. \tag{38}$$

If firm i deviates and quotes $p_i^r = p_i^{r*} - \varepsilon$, with $\varepsilon > 0$, its profit from deviation is $\pi_i^r = \phi_i\phi_j\left(p_i^{r*} - \varepsilon\right)$. It is then trivial to see that there exists such an ε that makes the deviation profitable. A contradiction. *Q.E.D.*

(ii) Let $p_i^{r*} < \widehat{p}_j^*$ then

$$\pi_i^{r*} = \phi_i\phi_j p_i^{r*}. \tag{39}$$

Let $p_i^r = p_i^{r*} + \varepsilon < \widehat{p}_j^*$, then, firm i's profit from deviation is $\pi_i^r = \phi_i\phi_j\left(p_i^{r*} + \varepsilon\right)$, from which it is straightforward to see that the deviation is profitable. A contradiction. *Q.E.D.*

Proof of Corollary 3.

To prove part (i) take as given the level of advertising selected by firms in period 1. Thus, $G^r(p) - \widehat{G}(p) = (1-\phi)\frac{v\phi - v + p}{p\phi} > 0$. This condition is satisfied if and only if $p \geq v(1-\phi)$. Since $v(1-\phi) = \widehat{p}_{\min}$ we get that $G^r(p) - \widehat{G}(p) > 0$, as long as $p \geq \widehat{p}_{\min}$, which is true for the equilibrium support of prices. When (i) holds, result (ii) follows. *Q.E.D.*

Proof of Proposition 5.

We prove this result by contradiction. Suppose that $(p_A, p_B) = (v, v)$ is a subgame perfect equilibrium of this game. Because both firms have the same price in period 1, they share equally the selective group of consumers. In period 2, within the group of old customers, each firm is not able to distinguish the fraction of selective and captive customers. However, each firm is able to recognize those selective customers that bought from the rival before. As a result, being price discrimination permitted, both firms will offer a different price to their old clients and their rival's previous customers. As before, p_i^o is the price charged by firm i to its old customers whilst p_i^r is the price charged by firm i to the rival's customers. Again, there is no pure strategy equilibrium and, thus, we need to prove the existence of a mixed strategy equilibrium. Each firm's prices are selected according to the distribution functions $G_i^r\left(p_j^o\right)$ and $G_i^o\left(p_j^r\right)$.

When, for example, firm A decides what price to charge to the rival's previous customers it takes into account that firm B is not willing to charge a price yielding an expected profit below $v\phi_B(1-\phi_A)$ (even in the case where it is able to sell to both groups of old customers). Therefore, it easy to see that, the minimum price firm B is willing to quote, $p_{B\min}^o$, is $p_{B\min}^o\left(\phi_B(1-\phi_A) + \frac{1}{2}\phi_A\phi_B\right) = v\phi_B(1-\phi_A)$ or $p_{B\min}^o = \frac{2v(1-\phi_A)}{2-\phi_A}$. It is also easy to see that, it is a dominated strategy for firm A to set a price below $p_{B\min}^o$. Quoting $p_A^r = p_{B\min}^o$, allows firm A to grab the entire group of selective customers that bought from B before, whilst makes firm B indifferent. Therefore, firm A's minimum price to the rival's customers is $p_{B\min}^o$, and its expected profit, from that group, is equal to $p_{B\min}^o\left(\frac{1}{2}\phi_A\phi_B\right)$. The derivation of the mixed strategy equilibrium follows the same steps as before. In equilibrium we must observe $p_A^r\left(\frac{1}{2}\phi_A\phi_B\right)\left[1 - G_B^o\left(p_A^r\right)\right] = 2v\left(\frac{1-\phi_A}{2-\phi_A}\right)\left(\frac{1}{2}\phi_A\phi_B\right)$ which leads to

$$G_B^o\left(p_A^r\right) = 1 - \frac{2v(1-\phi_A)}{p_A^r(2-\phi_A)} \tag{40}$$

with $G_B^o(p_A^r = p_{B\min}^o) = 0$ and $G_B^o(p_A^r = v) = 1 - \frac{2v(1-\phi_A)}{v(2-\phi_A)}$. Thus, firm B has a mass point at v within the group of its old customers.

Looking now at firm A's pricing strategy for its old customers, we know that firm A's expected profit from its old customers group is equal to $v\phi_A(1-\phi_B)$. In equilibrium we must observe $p_A^o\phi_A(1-\phi_B) + p_A^o\left(\frac{1}{2}\phi_A\phi_B\right)[1 - G_B^r(p_A^o)] = v\phi_A(1-\phi_B)$ which yields,

$$G_B^r(p_A^o) = 1 - \frac{2(v - p_A^o)(1-\phi_B)}{p_A^o\phi_B}. \tag{41}$$

From $G_B^r(p_A^o) = 0$ we obtain $p_{A\min}^o = \frac{2v(1-\phi_B)}{2-\phi_B}$ and from $G_B^r(p_A^o) = 1$ we obtain $p_{A\max}^o = v$. Second-period expected profit for firm A is, therefore,

$$\begin{aligned}\pi_A^2 &= \pi_A^o + \pi_A^r = v\phi_A(1-\phi_B) + 2v\left(\frac{1-\phi_A}{2-\phi_A}\right)\left(\frac{1}{2}\phi_A\phi_B\right) \\ &= v\phi_A\left(\frac{2-\phi_A-\phi_B}{2-\phi_A}\right).\end{aligned}$$

In sum, overall profit when both firms play (v, v) in period 1 is equal to

$$\Pi = v\left(\phi_A(1-\phi_B) + \frac{1}{2}\phi_A\phi_B\right) + \delta v\phi_A\left(\frac{2-\phi_A-\phi_B}{2-\phi_A}\right) - A(\phi_A).$$

Under symmetric decisions in advertising, we obtain:

$$\Pi = v\left(\phi(1-\phi) + \frac{1}{2}\phi^2\right) + \delta 2v\phi\left(\frac{1-\phi}{2-\phi}\right) - A(\phi).$$

Now we need to prove that no firm has an incentive to deviate from the equilibrium proposed. Taking as given its rival pricing and advertising strategy in period 1, if, say, firm A decides to undercut its rival, by quoting a price slightly below v, it is able to sell to the entire group of selective consumers in that period, but it is not allowed to discriminate in period 2. In this situation, the first-period profit of firm A from deviation is $(v-\varepsilon)(\phi_A(1-\phi_B) + \phi_A\phi_B) \approx v(\phi_A(1-\phi_B) + \phi_A\phi_B)$, for ε infinitely small. In period 2, firm A is the non-discriminating firm and its expected profit is, then, $v\phi_A(1-\phi_B)$. Summing up, firm A's profit from deviation is equal to

$$\Pi^d = v(\phi_A(1-\phi_B) + \phi_A\phi_B) + \delta v\phi_A(1-\phi_B) - A(\phi_A).$$

Under symmetry we have:

$$\Pi^d = v\left(\phi(1-\phi) + \phi^2\right) + \delta v\phi(1-\phi) - A(\phi).$$

This deviation is profitable as long as $\Pi^d - \Pi > 0$. Since,

$$\Pi^d - \Pi = \underbrace{v\phi^2 - \frac{1}{2}v\phi^2}_{\text{Increase in first-period profit due to deviation}} + \underbrace{\delta v\phi(1-\phi)\left(1 - \frac{2}{2-\phi}\right)}_{\text{Decrease in second-period profit due to deviation}},$$

the deviation is profitable as long as the increase in first-period profit overcomes the decrease in second-period profits. The above condition simplifies to

$$\Pi^d - \Pi = \frac{1}{2}v\phi^2\frac{2 - 2\delta + 2\delta\phi - \phi}{2-\phi}.$$

It is then easy to see that, since $\phi \in (0,1)$, $\Pi^d - \Pi > 0$ as long as $2 - 2\delta + 2\delta\phi - \phi > 0$. For $\delta \in (0,1)$ the previous condition is always positive. Consequently, firm A can deviate and increase its overall profit by slightly reducing its price below v. A contradiction. *Q.E.D.*

Proof of Proposition 9.

To prove part (i), due to algebraically reasons, we use the quadratic technology. From (10), we have $E\Pi^{nd} = \phi^{nd} A_{\phi} - A\left(\phi^{nd}\right)$, from which it follows that the derivative of overall profit, with respect to α, is

$$\frac{\partial E\Pi^{nd}}{\partial \alpha} = \phi^{nd} A_{\phi\phi} \phi_{\alpha}^{nd} + \phi^{nd} A_{\phi\alpha} - A_{\alpha}$$

where

$$\phi_{\alpha}^{nd} = \frac{\partial \phi^{nd}}{\partial \alpha} = -\frac{A_{\phi\alpha}}{v(1+\delta) + A_{\phi\phi}} < 0.$$

When the advertising technology is the quadratic one, $A(\phi) = \alpha\phi^2$, we obtain $\frac{\partial \phi^{nd}}{\partial \alpha} = \left(-\frac{2\phi}{v(1+\delta)+2\alpha}\right)$ and $\phi^{nd} = \frac{v}{v+2\alpha}$ and, thus,

$$\begin{aligned}
\frac{\partial E\Pi^{nd}}{\partial \alpha} &= \phi^{nd} A_{\phi\phi} \phi_{\alpha}^{nd} + \phi^{nd} A_{\phi\alpha} - A_{\alpha} \\
& \left(2\alpha\phi^{nd}\right) \phi_{\alpha}^{nd} + \left(2\phi^{nd}\right)^2 - \left(\phi^{nd}\right)^2 \\
&= \phi^2 \frac{(-2\alpha + v(1+\delta))}{v + v\delta + 2\alpha}.
\end{aligned}$$

Therefore $\frac{\partial E\Pi^{nd}}{\partial \alpha} > 0$ if and only if $\alpha < \frac{v}{2}(1+\delta)$. Otherwise, $\frac{\partial E\Pi^{nd}}{\partial \alpha} < 0$. *Q.E.D.*

To prove part (ii), from (29), we have $E\Pi^{*} = \phi^{*} A_{\phi} - A(\phi^{*}) + \delta v (\phi^{*})^3$. Therefore, the derivative of overall profit, with respect to α, is

$$\frac{\partial E\Pi^{*}}{\partial \alpha} = \phi^{*} A_{\phi\phi} \phi_{\alpha}^{*} + \phi^{*} A_{\phi\alpha} - A_{\alpha} + 3\delta v (\phi^{*})^2 \phi_{\alpha}^{*}$$

where

$$\phi_{\alpha}^{*} = \frac{\partial \phi^{*}}{\partial \alpha} = -\frac{A_{\phi\alpha}}{v(1+4\delta\phi) + A_{\phi\phi}} < 0.$$

When $A(\phi) = \alpha\phi^2$, it follows that,

$$\begin{aligned}
E\Pi_{\alpha}^{*} &= \phi^2 + \left(2\alpha\phi + 3\delta v\phi^2\right)\left(-\frac{2\phi}{v(1+4\delta\phi) + 2\alpha}\right) \\
&= -\phi\left(\frac{v\phi - 2\delta v\phi^2 - 2\alpha\phi}{v + 4\delta v\phi + 2\alpha}\right).
\end{aligned}$$

Thus, $\frac{\partial E\Pi^{*}}{\partial \alpha} < 0$ if $v\phi - 2\delta v\phi^2 - 2\alpha\phi > 0$. Using (28) and the quadratic technology, it follows that $v(1-\phi) + v\delta\left(1 - 2\phi^2\right) = 2\alpha\phi$ and, therefore, $\frac{\partial E\Pi^{*}}{\partial \alpha} < 0$ if $v(1+\delta-2\phi) > 0$, which, for $\delta = 1$, is always true. *Q.E.D.*

Proof of Proposition 11.

Evaluating the advertising regulator's first order condition, given by (33), at $\phi = \phi^{*}$, it follows that

$$\frac{\partial W}{\partial \phi} = 2v(1-\phi^{*}) - A_{\phi}(\phi^{*}).$$

Using the fact that when $\delta = 1$, $A_{\phi}(\phi^{*}) = v(1-\phi^{*}) + v\left(1 - 2\phi^{*2}\right)$, and plugging this expression into the above condition, we obtain that

$$\frac{\partial W}{\partial \phi} = v\phi^{*}(2\phi^{*} - 1). \tag{42}$$

In this case, it immediately follows that ϕ^* maximizes welfare, when $\phi^* = 0.5$. Otherwise, when $\phi^* > 0.5$, it follows that $\frac{\partial W}{\partial \phi} > 0$ and, so, welfare increases, if ϕ^* becomes increasingly higher. This implies that $\phi^* < \phi^w$. Similarly, when $\phi^w < \phi^* < 0.5$, welfare increases if ϕ^* becomes smaller. This completes the proof. *Q.E.D.*

Proof of Proposition 12.

From Proposition 10, it follows that welfare is maximized at $\phi = \phi^{nd.}$. In contrast, from Proposition 11, it follows that, when $\delta = 1$, $\frac{\partial W}{\partial \phi} = 0$ at $\phi = \phi^* = 0.5$; while $\frac{\partial W}{\partial \phi} \gtreqless 0$ when $\phi^* \gtreqless 0.5$. Thus, apart from the case in which $\phi^* = 0.5$, welfare always falls when moving from no discrimination to discrimination. *Q.E.D.*

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