SELECTION OF THE INVENTORY POLICY IN NON-REPAIRABLE SPARE PARTS MANAGEMENT USING ACTIVITY BASED LIFE CYCLE COSTING AND TRIANGULAR NUMBERS

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ABSTRACT

This paper presents the development of a model for costing the spare parts during a life cycle of a physical asset. The main aim of the model is to allow decision makers to select the most appropriate inventory policy for a given and spare part, anticipate budgets, and determining supply chain parameters. In addition, and since the model is designed to operate with triangular numbers, it can handle the uncertainty usually present in life cycle analysis, where the remarkable characteristic is the long term of the planning horizons.

INTRODUCTION

Life cycle costing (LCC) is a tool that allows the assessment of all the costs incurred during the lifetime of a project, a product or a service. Thus, LCC allows managers to predict scenarios and make decisions whose effects can be observed in the long-term (Rodríguez Rivero and Emblemsvåg, 2007). From the generation of diverse scenarios, it is possible to make comparisons and opt for a series of strategies, policies and models that will allow operating a given system at lower costs and with greater efficiency. In this paper, activity-based costing is used to implement multidimensional LCC models were both time and operation conditions can be combined. Thus, an activity-based life cycle costing model includes these dimensions and is particular relevant for the context of spare parts management.

However, since these processes involve long term decision making, they often deal with high degrees of uncertainty. This uncertainty prevents managers from anticipate precisely the exact values of important variables, such as the future discount rate, the unitary costs values and the demand pattern. To deal with this uncertainty, several authors suggest the use of simulation models. However, these models lose precision and validity when sufficient data is not available to define the appropriate and representative probability distributions. An alternative is the use of the so-called fuzzy reasoning. Through this technique, the decision-making processes can be carried out using incomplete, subjective and diffuse information. The approach based on the use of triangular numbers represents a useful method to model and operate in scenarios when only diffuse information is available to make long term assessments. The use of triangular numbers allows to implement, in a satisfactory way, models that lead to quantitative (*crisp*) values.

This paper proposes the incorporation of triangular numbers in an activity-based life cycle costing model for non-repairable spare parts management. The main objective of this model is to provide a method to make comparisons among different inventory policies and models. The comparison of costs is made through the net present value of the discounted cash flows (Net Present Value - NPV).

PROPOSED METHOD

In methodological terms, a generic logistic process for spare parts management is modeled using the activity-based costing approach. A series of resources, with their respective costs, are considered for the execution of the logistics activities. Relevant cost drivers are selected for each activity and for each resource group. To include uncertainty handling into the model, the main variables are represented as triangular numbers. These values are processed, in combination with crisp values. To estimate the future demand of the spare parts, a model based on the Weibull distribution which allows the estimation of the failure rates and corresponding spare parts demand. Through this model it is possible to measure the effect of the cost variation in the use of different inventory policies and compare the effects they generate on the long-term.

Table 1. Relationships between Activities and Resources

	Resource 1	Resource 2	Resource j	Resource n
Activity1	√			
Activity 2	√	√		√
Activity i			√	
Activity m		√		

Table 2. Relationships between Activities and Spare parts.

	Activity 1	Activity 2	Activity i	Activity m
Spare Part 1		√		
Spare Part 2	√	√		
Spare Part k			√	√
Spare Part K		√		

THE MODEL

The net present value of the global cost of critical non-repairable spare parts inventories along the entire life cycle, can be modelled as follows:

$$GC_k = DC_k + HC_k + LC_k \quad (1)$$

Where

DC_k = Net present value of the Direct Costs of item k
HC_k= Net present value of the Holding Costs of item k
LC_k= Net present value of the Logistics Costs of item k

If we detail the first two elements using traditional cost equations, we can express equation 1 as:

$$GC_k = c_k \cdot \lambda_k + \frac{h_k \cdot Q_k}{2} + LC_k \quad (2)$$

Where:
$c_k = unitary\ cost\ of\ item\ k$
$\lambda_k = demand\ for\ item\ k$
$Q_k = Replenishment\ lot\ size\ of\ item\ k$
$h_k = capital\ costs\ of\ item\ k$

Regarding the LC_k costs, we propose here an ABC-based methodology to compute those costs. In the next paragraphs we explain how to implement the logistics costs assessment.

Afonso and Paisana (2009) proposed a matrix-based representation of the ABC methodology. In the reported approach, information about resources, activities, cost objects and costs drivers is represented using matrix and vectors. The relationship between resources and activities and between activities and costs objects can be represented as shown in Table 1 and 2. There, the √ symbols represent a relationship between a resource and an activity or between an activity and a cost object.

As it is well known, the ABC methodology is a twofold (i.e. allocation phases) procedure. In the first allocation phase, the cost per activity is computed using the resource-activity matrix, composed of r_{ij} terms which represent the proportion of the resource driver *j* that is related to activity *i*, and the resource matrix whose terms r_j represent the total amount of the resource *j* expended during the period under analysis. That multiplication results in the so-called Activitiy Matrix, where the term a_i represents the amount of cost allocated to activity *i*.

$$[r_{ij}][r_j] = [a_i] \quad (3)$$

Note that in the resource-activity matrix, its elements represent the portion of the resource driver *j* that is consumed by the activity *i*. That is obtained as the ratio between the resource driver *j* related to activity *i* and the total amount of the resource driver *j*. Then, the cost allocated to each activity will be obtained by:

$$a_i = \sum_{j=1}^{n} r_{ij} * r_j \quad (4)$$

In the second allocation phase, the cost calculation per cost object will be carried out by multiplying the activity matrix (a_{ki}) by the vector column of the activity costs (a_i) obtaining the so-called cost object matrix (sp_k):

$$[a_{ki}]\ [a_i] = [sp_k] \quad (5)$$

Considering the Activity-Product matrix, each element a_{ki} is the proportion of the activity driver related to spare part *k*. That proportion is obtained by computing the ratio between the activity-driver *i*, related to product *k* (a_{ki}) and the total amount of the activity-driver *i* (a_i). Then, the cost imputed to each cost object (e.g. product) will be obtained as follows:

$$sp_k = \sum_{i=1}^{m} a_{ki} * a_i \quad (6)$$

In summary, the cost object matrix can be obtained in just one step as shown below:

$$[a_{kj}]\ [r_{ij}][r_j] = [sp_k] \quad (7)$$

One of the most important problems in modelling the costs elements in life cycle costing is that the planning horizons are significantly extended, and the available information is restricted. Besides, the subjectivity arises to represent some of the involved values and related drivers. To overcome such limitations and uncertainty, a fuzzy approach can be added to the ABC methodology.

Therefore, we propose to include, within the costing procedure, the use of triangular numbers.

A fuzzy number reveals the meaning of expressions such as '*about x*'. That meaning is represented using a membership function (Zadeh, 1965). Such a function could be defined using a triangular number, (a_1, a_2, a_3). The values of the triangular number are used to represent the output of the fuzzy membership function as shown in equation 8 and Figure 1.

$$\mu(x) = \begin{cases} 1 & x = a_2 \\ \frac{x-a_1}{a_2-a_1} & a_1 \leq x < a_2 \\ \frac{a_3-x}{a_3-a_2} & a_2 < x \leq a_3 \\ 0 & otherwise \end{cases} \quad (8)$$

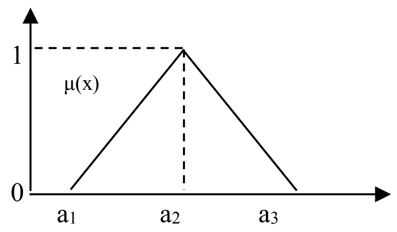

Figure 1: Membership function of a triangular number

To operate with triangular numbers, a set of operations is needed. Next, a brief description about addition, multiplication and division of triangular numbers is given. Basic mathematical operations over triangular fuzzy numbers were defined by Chang (1996) and are briefly explained in the following. Let A and B be two triangular fuzzy numbers, with their parameters as follows:

$$\tilde{A} = (a_1, a_2, a_3)$$
$$\tilde{B} = (b_1, b_2, b_3)$$

Then, the product of a pair of fuzzy numbers is defined by:

$$\tilde{A}\tilde{B} = (a_1 b_1, a_2 b_2, a_3 b_3) \quad (9)$$

At the other hand, fuzzy numbers division is defined as follows:

$$\tilde{A}/_{\tilde{B}} = (a_1/b_1, a_2/b_2, a_3/b_3) \quad (10)$$

Whilst the reciprocal value of a triangular fuzzy number (a, b, c) is given by (1/a, 1/b, 1/c). The power of a triangular fuzzy number is given by

$$\widetilde{A^n} = (a_1, a_2, a_3)^n = (a_1^n, a_2^n, a_3^n) \quad (11)$$

For converting the triangular number into a crisp one (*defuzzyfication*), we suggest the procedure indicated in the following equation:

$$A = \frac{a_1 + 2 \cdot a_2 + a_3}{4} \quad (12)$$

Where $\tilde{A}$= (a_1, a_2, a_3) is a triangular number and A represents the representative ordinal (crisp) of a triangular number.

Returning to the proposed model, we adapted the model created by Afonso and Paisana (2009) to accept three new characteristics:

- The life cycle concept (multiperiod).
- The fuzziness of certain variables (triangular numbers)
- The Weibull based reliability of the components (reliability model).

First of all, we use the fuzzy version of matrix representation of the ABC model. That is, the elements of the matrixes and vectors in the ABC matrix-based model are represented now by a triangular number. Therefore, we have rewritten the equation 5 to represent its elements as fuzzy ones:

$$\begin{bmatrix} \widetilde{a_{11}} & \widetilde{a_{12}} \cdots & \widetilde{a_{k1}} \\ \widetilde{a_{21}} & & \\ \vdots & \widetilde{a_{22}} \ddots & \vdots \\ \widetilde{a_{1\iota}} & \widetilde{a_{2\iota}} \cdots & \widetilde{a_{k\iota}} \end{bmatrix} \cdot \begin{bmatrix} \widetilde{r_{11}} & \widetilde{r_{12}} \cdots & \widetilde{r_{1J}} \\ \widetilde{r_{21}} & & \\ \vdots & \ddots & \vdots \\ \widetilde{r_{\iota 1}} & \cdots & \widetilde{r_{\iota J}} \end{bmatrix} \cdot \begin{bmatrix} \widetilde{r_1} \\ \widetilde{r_2} \\ \vdots \\ \widetilde{r_\iota} \end{bmatrix} = \begin{bmatrix} \widetilde{p_1} \\ \widetilde{p_2} \\ \vdots \\ \widetilde{p_k} \end{bmatrix} \quad (11)$$

And secondly, we incorporate the sub index *y* to represent the periods of the life cycle.
From that, we can express the equation 7 to represent the total life cycle costs of the spare parts management according to the following equation (Chiuh and Park, 1994):

$$CGG_{k\,cv} = \sum_{y=1}^{cv} \frac{1}{(1+\tilde{\iota}_y)^y} \left[\sum_{k=1}^{t} \tilde{c}_{ky} \cdot \lambda_{ky} + \frac{(\tilde{c}_{ky} \cdot \tilde{\iota}_y \cdot Q_{ky})}{2} + \sum_{k=1}^{t} \sum_{i=1}^{m} \sum_{j=1}^{n} \tilde{a}_{ki\,y} \cdot \tilde{r}_{ij\,y} \cdot \tilde{r}_{j\,y} \right]$$

The parameters using the symbol ~ correspond to a triangular number.
Last but not the least, the demand is modelled using the Weibull distribution. The Weibull statistical distribution is one of the most used function to represent the behavior of the reliability throughout the life of a physical asset. The reliability of a component can be represented by the following equation (Murthy et al., 2004):

$$R(t) = e^{-(\frac{t-t_0}{\eta})^\beta} \quad (15)$$

Where β is the shape parameter and characterizes the failure pattern. The higher value of β, the greater the failure probability in a given period of time. The time t_o

is the location parameter and provides an estimate of the earliest time at which a failure may be observed. Also, it can represent the beginning of the deterioration process of the equipment. Finally, η is the scale parameter and represents the characteristic life of the equipment. This parameter corresponds to the time in which 63.2% of the failures are expected to occur. Therefore, knowing both Weibull parameters, it is possible to estimate the failure rate for a given component in a time *t*. The equation (16) shows the relationship between β, η and λ:

$$\lambda(\mathrm{t}) = \frac{\beta}{\eta}\left(\frac{t}{\eta}\right)^{\beta-1} \qquad (16)$$

One of the traditional ways to represent the phases over time is the bathtub curve (Figure 2). That curve shows the behavior of the failure rate over time. The bathtub curve consists of three phases, a phase called infant mortality period, where the failure rate (λ) decreases over time. Next is a phase called the useful life, where λ remains constant. After that there is the end of life phase, also called wear out phase. In such a phase λ begins to increase in time to the point where it is decided to discard the equipment or decommissioning the installation.

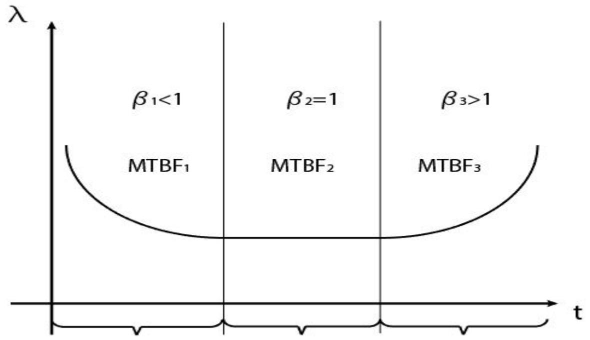

Figure 2: Bathtub curve of a physical asset

As already mentioned, these different phases of the life cycle have different characteristics in terms of reliability and failure rate. Table 3 summarizes these different characteristics.
As it is expected, when the aforementioned Weibull parameters of a component change over the life cycle, the failure rate will be affected, and the demand of a new non-repairable spare part will also vary. Therefore, future spare parts demand could be modelled using this distribution, and lambda values ($\lambda(y)$) for each year *y*, along the life cycle can be estimated using equation (16).

Table 3. Behavior of the Weibull parameters according to the stage of the life cycle

Value	**Characteristic**
$\beta < 1$	λ decreases with age, without reaching zero.
$\beta = 1$	λ remains constant, with random occurrences.
$1 < \beta$	λ increases with age on an ongoing basis.

To allow the selection of different inventory policies to be implemented in a set of critical spare parts, this aspect has to be incorporated into the cost model. After an analysis we realized that the inventory policy affects mainly the inventory average levels and the number of the activity executions during each period. Therefore, the model does not need to be reconfigured for assessing different inventory policies and its respective parameters. To illustrate the utilization and validate the proposed model, we develop a series of simulation examples. Based on the use of the proposed model, simulations of the use of two types of inventory policies were carried out: continuous review (*S, Q*) and periodic review (*R, s*) policies. Along with that, it is possible to assess the impact of the variation of some of the variables of the model, such as: the discount rate, some unitary costs (such as direct costs, which depend on the projected prices of the spare parts, and the holding costs, among others) and the main parameters of inventory policies (such as lot sizes, replacement periods and lead times). Those examples are shown and discussed in the next section.

ILLUSTRATIVE EXAMPLES

A two critical spare parts' logistic process is executed through two activities (*ACT1, ACT2*). Those activities consume two different resources (*R1, R2*). In the following examples the cost objects are represented as *SP1* and *SP2*. We considered that for *ACT1* the costs are not incurred to handle an individual unit but rather to handle a batch of the same units (Batch level activity). On the other hand, regarding *ACT2*, the costs are incurred to handle individual units (Unit level activity). We considered a life cycle of 9 periods. After obtaining the necessary demand information, we constructed a set of 9 fuzzy-ABC-based sub-models, one for each year of the life cycle. After obtaining the total logistics costs for each one of the spare parts in each of the nine years, the net present value of the sum of those costs is obtained as a crisp number. The present value can be used to make comparisons and to perform some sensibility analysis.
With the proposed model it is possible to perform several risk analyses representing as triangular numbers the following model's parameters:

- Activity executions volume
- Resources expenses
- Spare parts prices
- Inflation
- Discount taxes

Trying to reflect the most important sources of uncertainty, we represented as triangular numbers the activities´ execution volumes for each period. We consider that those executions volumes are highly correlated to the demand volumes, and, therefore are correlated with the projected failure rate. In a second case, we represented as triangular numbers the amounts of the expenses related to both resources. With that we tried to represent the uncertainty that may occur during the years with certain resources prices and costs (consider, for instance, energy prices, fuel, taxes, etc.). The third case is aiming at observing the behavior of the logistics total costs, summing up the cost of each one of the spare parts. Finally, we develop a simple, but useful analysis of the dispersion of the estimation of the logistic costs along the life cycle.
In the following subsections we present with more detail the analysis which can be performed in order to assist the spare parts managers in defining inventory policies and their parameters. It is worth noting that almost all of those analyses are usually pointed to the minimization of the present value of the life cycle spare parts costs. That is made in order to impact, at the lowest degree, the costs of ownership of a given asset or system of assets.

Decision 1

For *SP1* and *SP2*, are expected to have a constant failure rate along the entire life cycle. Through these assumptions, it is possible forecast both spare parts demand along the planning horizon. The demand values are not represented as fuzzy numbers, because they proceed from a Weibull-based risk analysis method, therefore, the uncertainty is already well considered in such computation. We develop a series of 5 experiments to compare both inventory policies. For carrying out the experiments we decided to equalize the inventories average levels according to the following values: 3, 5, 10, 15 and 20 units of each spare part.
Figure 3 shows a comparison of the behavior of the global costs considering both spare parts costs and both inventory policies.

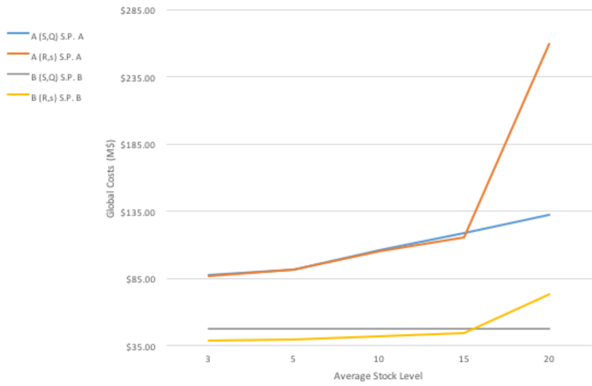

Figure 3: Analysis 1

It is observed as the average levels of stock increases, the global cost values tend also to increase. It is clearly observed that, when increasing the unitary prices of the spare parts, the global costs are also higher. On the other hand, if there are low levels of average stocks, the costs of both policies do not show important differences. However, from a given level up, the costs of carrying periodic review policies tend to generate significantly higher costs than those of the policy of continuous review. Notice that this is connected to the service level that is intended to have throughout the life cycle in this warehouse.

Decision 2

Figure 4 shows a comparison of the behavior of the global costs considering that the demand of both spare parts is increasing equally for both parts. It is observed as the average levels of stock tend to increase, the values of the global cost show also a tendency to increase. Clearly it is observed that, when increasing the unit prices of the spare parts, the global costs are also greater. The exception is presented for the spare part B with policy of continuous review, where the costs tend to fall as the values of average stock increase. In the case of the spare A (of greater unitary cost) it shows a reverse behavior to that shown in the case of a constant demand (and failure rate). Now, from a certain level, the costs of applying continuous review policies tend to generate significantly higher costs, than those obtained by using the periodic review policy. This leads us to think that the hypothesis that the predicted failure patterns and the other logistics parameters generate direct influence on the present value of the global costs of spare part management is valid.

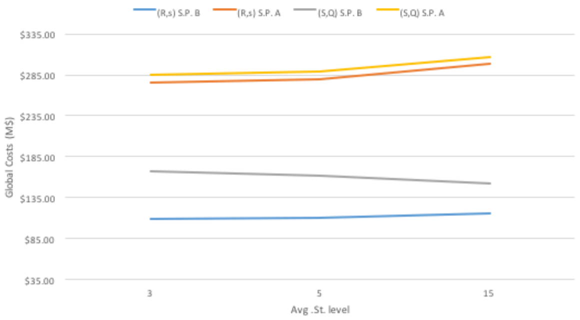

Figure 4: Analysis 2

Decision 3

We simulated a third scenario to evaluate the behavior of the sum of the costs of individual spare parts. To do that, we calculated the total costs considering different average stock levels. Through that analysis, the decision maker can evaluate the optimum average stock level to guarantee the lowest total costs of spare parts. In Figure 5, such analysis, considering just the continuous review policy, is shown. As it can be observed, the minimum cost is obtained when the average stock level is located at 10 units of both types of spare parts. This type of analysis helps managers to establish appropriate budgets in order to satisfy a planned service level.

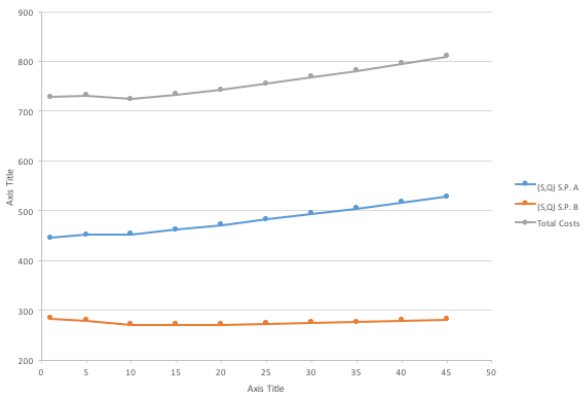

Figure 5: Analysis 3

Decision 4

Another possible analysis which is very useful for risk assessments is to verify how uncertainty propagates along the entire life cycle. In Figure 6 it is observed that the range of net present values of the continuous review policy is increasing as the age of the asset is also augmenting. Note that this result was obtained using the same range size for each one of the triangular numbers according to the following variation: -20%, 100%, 120%.

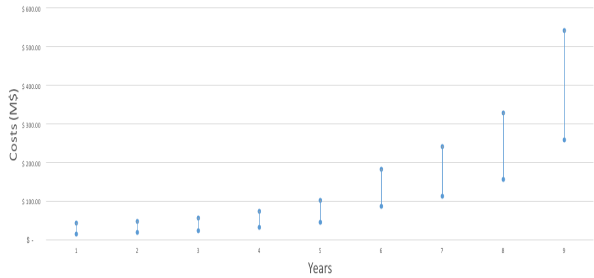

Figure 6: Analysis 4

CONCLUSIONS

In this paper a model for computing the life cycle costs of critical spare parts is proposed. In order to represent a valuable tool for decision making, the presented model consider three novel and important characteristics: it is supported on activity-based costing, it includes uncertainty as it uses triangular (fuzzy) numbers and, last but not the least, it integrates the use of a reliability prediction function into the model which allows a better demand estimation.

We believe that, through the use of this model, that can be extended with adding a higher number of parameters, such as activities, drivers, resource expenses and, a number of additional spare parts, the decision maker could perform a series of useful analysis namely, to determine the best inventory policy and the recommended patterns' values incorporating value ranges to represent the uncertainty in the assessments.

Future deployments are pointed to the application of other inventory policies, experiments with other types of fuzzy numbers, and applying the proposed model to a real case. Also, it seems to be possible to easily transform the proposed model into an optimization model, incorporating some restrictions such as a given budget, capacities or specific lot sizes or replenishments frequencies.

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