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Behavior-Based Price Discrimination with Non-Uniform Distribution of Consumer Preferences *

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Abstract

Firms commonly price discriminate across consumers based on purchase history, a practice known as behavior-based price discrimination (BBPD). Existing studies usually assume that consumer preferences follow uniform distribution, and find that BBPD benefits consumers at the cost of firms, prisoners' dilemma. In this paper, we consider a class of consumer preferences distribution and show that new profit and welfare results arise. In particular, when consumer preferences are sufficiently clustered at the center of the market (e.g. triangular distribution), BBPD boosts industry profits at the expense of consumers. This is opposite to the standard prisoners' dilemma results under uniform distribution. On the other hand, when consumer preferences are not clustered at the center of the market, the usual findings prevail. Our results highlight the important role of *the shape of preferences* and provide useful implications for relevant players including managers, regulators and consumer advocates.

1 Introduction

Today many companies collect and utilize consumer data for price discrimination purposes. In markets with repeat purchases, firms can use consumers' purchase history to price discriminate, a

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practice known as behavior-based price discrimination (BBPD). BBPD is widely observed in many markets. For example, Hannack et al (2014) find that "human shoppers got worse bargains on a number of websites" (including Home Depot and Wal-Mart), compared to an automated shopping browser that did not have any personal data trail associated with it. Similarly, a letter by the Consumer Education Foundation's Represent Consumers project, sent to the Federal Trade Commission (FTC) in June 2019, identifies that many companies create secret scores on their clients and charge some consumers more than others taking into account their purchase history, search behavior, location, mobile devices used, and so on.[1] The letter calls for an FTC investigation into who is targeted using the scores, and the scores' impacts on consumers and the marketplace.

Given that behavior-based price discrimination is a ubiquitous feature in today's business, it is important to understand under what conditions BBPD is a winning strategy for firms and/or consumers. Existing research shows that BBPD can intensify competition and hurt firm profitability at the benefit of consumers. In fact, a common finding in models with (i) fixed uniform distributed preferences across time, (ii) symmetric informed firms and (iii) fully informed consumers, is that equilibrium profits decrease with price discrimination (e.g. Fudenberg and Tirole, 2000). In spite of the fact that BBPD has received wide attention in the economics and marketing literature,[2] to our knowledge little is known about its implications in markets where the *distribution of consumer preferences is non-uniform.* In fact, we should stress that while Fudenberg and Tirole (2000) employ a general model, their welfare comparison between uniform pricing and BBPD is made for uniform distribution. This leaves open the question of how BBPD affects welfare under non-uniform distributions.

Although the uniformity assumption is convenient for deriving analytical results, it may be inaccurate in many markets. For example, a market may consist of many consumers with *average* preferences and few with *extreme* preferences. The main goal of this paper is to assess the impacts of behavior-based price discrimination in markets where the density of consumer preferences is non-uniform. By so doing, this paper provides useful implications for competition policy agencies and consumer advocates. It highlights that *the shape of preferences* plays an important role in understanding the welfare effects of BBPD.

We look at the following questions. What happens to prices and profits when consumer tastes change from uniform distribution to non-uniform distribution? Under what conditions should managers expect BBPD to enhance profitability? Under what conditions should competition agencies expect BBPD to hurt consumers, and thus take appropriate enforcement actions to restrict the use of consumer purchase history data for price discrimination?

With these questions in mind, we model a class of non-uniform distribution characterized by a

[1] These companies include Home Depot, Wal-Mart, ASOS, Cheaptickets, Orbitz, Expedia, Hotels.com, Travelocity, Priceline etc.

[2] Chen (2005) and Fudenberg and Villas-Boas (2006) present excellent surveys of the BBPD literature.

single parameter β.[3] When $\beta > 0$, consumers are clustered at the center of the market relative to the edges, and the reverse holds when $\beta < 0$. We recover the commonly assumed uniform density when $\beta = 0$. Our analysis reveals that the profit and consumer welfare effects of BBPD depend on the shape of consumer preferences. When consumer preferences are uniformly distributed, in comparison to non-discrimination, BBPD reduces industry profits and boosts consumer surplus. The same happens if consumer preferences are reverse triangular ($\beta < 0$), i.e., when more consumers are located at the edges of the market than at the center. However, BBPD raises industry profits at the expense of consumer surplus for a wide range of positive β. This suggests that managers should expect to employ BBPD in a profitable way in markets where consumer preferences are more likely to be concentrated at the center of the preference line than at the edges.

1.1 Literature review

There is an extensive literature on price discrimination, covering both monopolistic and oligopolistic price markets.[4] Our paper is related to the strand on oligopolistic price discrimination with best-response asymmetry (e.g., Thisse and Vives (1988), Shaffer and Zhang (1995), Liu and Serfes (2004) and Esteves (2009)). A common finding in this literature is that price discrimination leads to a prisoners' dilemma game, benefiting consumers at the cost of firms.[5]

Our paper is most closely related to the literature on behavior-based price discrimination. In this literature, firms can try to poach their rivals' customers with special "introductory" prices. Two approaches have been considered so far. In the switching costs approach (e.g. Chen, 1997 and Taylor, 2003), consumers initially view the two firms as perfect substitutes; but in the second period they face a switching cost if they change suppliers (ex-post heterogeneity). In the other approach, consumers have ex-ante heterogeneous brand preferences (e.g. Villas-Boas, 1999; Fudenberg and Tirole, 2000; Esteves, 2010). Models in both approaches exhibit best-response asymmetry (Corts, 1998): the strong market of one firm is the weak market of the competitor. Firms charge lower prices to customers in weak market segments (new/rival's customers) than to customers in strong segments (old customers). In comparison to uniform pricing, the ability to price discriminate across strong/weak markets reduces equilibrium profits, the classic prisoners' dilemma.[6] There are also important differences between the two approaches. In the brand preference approach, initial prices are high and then decrease. The reverse happens in the switching costs approach.

We build on the second approach of ex-ante heterogeneity, and assume that consumer preferences

[3] Details about the model can be found in Section 2.

[4] Armstrong (2006) and Stole (2007) provide excellent surveys of the literature.

[5] Corts (1998) shows that best-response asymmetry is necessary but not sufficient for price discrimination to lead to all-out competition. Liu and Shuai (2013) show that price discrimination can raise profit in a model with best-response asymmetry.

[6] Both Chen and Pearcy (2010) and Shin and Sudhir (2010) allow consumer preferences to change over time, and look at when a firm should reward its loyal customers and when it should offer discount to entice brand switching. They also identify conditions under which BBPD would raise firms' profits.

are *non-uniform.* A few papers (e.g., Chen and Zhang, 2009 and Esteves, 2010) have taken a similar approach, but consider discrete distributions and generally equilibrium in mixed strategies. In Esteves (2010), all consumers are loyal to either of the two firms, up to a degree $\gamma > 0$. Firms have an incentive to price far apart from each other (by more than γ) so all consumers (assumed to be naive) buy from only one firm in period 1. This avoids market sharing and learning, and prevents competition-intensifying BBPD in period 2. The problem is that each firm prefers to be the lower priced firm and thus both firms price aggressively in period 1. As a result BBPD leads to lower prices in both periods relative to uniform pricing. In contrast, Chen and Zhang (2009) find that BBPD raises firms' profits relative to uniform pricing, similar to our paper. But the driving force behind their result is quite different. In their model each firm has their respective loyal consumers and competes for switchers. Only the high price firm in period 1 will have the required information to engage in price discrimination in period 2. Thus having a lower price (and in turn serving more consumers) in period 1 actually puts the firm in a disadvantageous position subsequently, because it cannot distinguish loyals from switchers and price discriminate in period 2. Thus both firms have an incentive to raise prices in period 1. In contrast, for BBPD to raise profits in our model, density needs to be high around marginal consumers under uniform pricing ($x = \frac{1}{2}$) but low around marginal consumers under BBPD (x away from $\frac{1}{2}$). This puts downward pressure on profit under uniform pricing. In addition, forward-looking consumers become less price sensitive and in turn raise period 1 prices under BBPD. Similarly, when $\beta > 0$, forward looking firms also raise period 1 prices under BBPD, taking into account their impacts on period 2 profits.

Our analysis reveals that BBPD can boost industry profits at the expense of consumers when consumer preferences are sufficiently clustered around the center of the market. This result, which depends on β – the consumer distribution parameter, has a similar flavor as that in Chen and Pearcy (2010) whose result depends on α – the preference dependence parameter. Chen and Pearcy allow consumer preferences between the two periods to be imperfectly correlated, and use a copula to admit various degrees of positive dependence (α). When firms cannot commit to future prices, an increase in α always worsens the reduction of second period profits under BBPD, but initially raises and then reduces first period profits. Combined, BBPD raises firms' discounted overall profits when α is small. Analogous to their result, in our paper, BBPD has monotonic impacts on profits in each period (up in period 1, down in period 2), and raises discounted overall profits when β is large. Shin and Sudhir (2010) also allow consumer preferences to be imperfectly correlated between the two periods, and further introduce consumer heterogeneity (low/high demand). They find that profits can increase with price discrimination when preference dependence is low and consumer heterogeneity is high.

Other relevant papers have shown that profits can increase with price discrimination when a firm has superior information on its existing consumers. For example, Subramanian et. al. (2013) introduce consumer heterogeneity in terms of how much it costs to serve them, and each firm is able to learn the cost of its own consumers. They show that retaining high-cost-to-serve customers, even when they are unprofitable, can help increase the overall profitability of the firm. Similarly, Colombo

(2018) allows consumers to have different price sensitivity, and each firm is able to learn the price sensitivity of its own consumers. He shows that the access to this additional information may boost profits in comparison to uniform pricing provided that consumers are heterogeneous enough with respect to price sensitivity.

The rest of the paper is organized as follows. We present the model and analyze the benchmark case of uniform pricing in Section 2. Section 3 presents the equilibrium analysis of behavior-based price discrimination, for the two special cases of uniform distribution and triangular distribution respectively. In Section 4 we generalize the analysis to a class of non-uniform distributions, and illustrate how the insights from the two special cases carry over. We also consider the extension of piece-wise uniform distribution. Section 5 presents policy discussions and we conclude in Section 6. Proofs of lemmas and propositions are relegated to the Appendix.

2 The model

Two firms, A and B, produce differentiated nondurable goods at constant marginal cost which we normalize to zero. The firms are located at the opposite ends of the Hotelling line with firm A (B) at 0 (1). A unit mass of consumers are continuously distributed on the line. There are two periods of consumption and consumer preferences for the goods are constant over time.[7] In each period, consumer demands at most one unit of the good in each period. A consumer located at x receives instantaneous utility of $u_A = v - p_A - tx$ if she buys from firm A at price p_A. If she buys from firm B, her utility will be $u_B = v - p_B - t(1-x)$. We assume that v is sufficiently large so that all consumers buy in equilibrium (covered market). Like in Fudenberg and Tirole (2000), firms and consumers have a common discount factor $\delta \in (0, 1]$. Each firm maximizes its discounted sum of expected profit, and each consumer maximizes his/her discounted sum of expected utility.

Distribution of consumer preferences

A key departure of our paper from the literature is that we allow for non-uniform distribution of consumer preferences. Let $F(x)$ denote the cumulative distribution function with support $[0, 1]$. $f(x)$ is the corresponding density function which we assume to be smooth and symmetric around $x = \frac{1}{2}$. Specifically, we consider a class of distribution characterized by a single parameter β as follows,

$$f(x) = \begin{cases} 4\beta x + (1-\beta), & \text{if } x \in \left[0, \frac{1}{2}\right], \\ 4\beta(1-x) + (1-\beta), & \text{if } x \in \left(\frac{1}{2}, 1\right]. \end{cases}$$

The parameter β has a simple interpretation – it measures the density difference between the edges and the center of the market, i.e., $\beta = \frac{f(\frac{1}{2})-f(0)}{2} = \frac{f(\frac{1}{2})-f(1)}{2}$. This symmetric distribution has the advantage of allowing us to investigate BBPD under uniform and nonuniform distribution

[7] For an analysis of BBPD with imperfect correlated preferences across time see Chen and Pearcy (2010).

of consumer preferences. We restrict that $\beta \in \left[\underline{\beta}, 1\right]$ with $\underline{\beta} = \frac{\sqrt{5}-3}{2}$.[8] This class of distribution covers the commonly studied uniform distribution ($\beta = 0$) and pure triangular distribution ($\beta = 1$) as two special cases. When β increases from 0 to 1 more consumers are located around the center of the market than at the edges. When $\beta = 1$ consumer preferences follows a pure triangular distribution with no consumers at the edges. In contrast, when $\beta < 0$, the reverse happens and more consumers are located at the edges of the market than at the center (which we call reverse triangular distribution).

2.1 Benchmark case: Uniform pricing

Suppose that price discrimination is infeasible due to regulation, costs of changing prices, consumers' actions to behave anonymously, technological restrictions, etc. This benchmark case of uniform pricing will be used as a comparison later on when we allow firms to price discriminate across consumers.

Under uniform pricing, the two-period model reduces to two replications of the static equilibrium. Given the uniform prices p_A^U and p_B^U where the superscript U denotes uniform pricing, the marginal consumer who is indifferent between buying from the two firms is determined by

$$v - x_1 - p_A^U = v - p_B^U - (1 - x_1),$$

which yields,

$$x_1^U = \frac{1}{2} + \frac{p_B^U - p_A^U}{2}.$$

Firms' profits are given by,

$$\Pi_A^U = (1+\delta) p_A^U F(x_1^U),$$

$$\Pi_B^U = (1+\delta) p_B^U \left[1 - F(x_1^U)\right].$$

Each firm chooses its uniform price to maximize its profit. Solving firm A's FOC, for example, we can obtain

$$p_A^U = \frac{2F(x_1^U)}{f(x_1^U)}.$$

We look for a symmetric equilibrium, which is presented in the next proposition.

Proposition 1 *Under uniform pricing, each firm chooses the price*

$$p^U = \frac{t}{1+\beta} \tag{1}$$

[8] We cannot allow β to be too negative. Let us illustrate why, using the uniform price case. The equilibrium price is given by $p^U = \frac{1}{1+\beta}$. If β approaches -1, the equilibrium price, as well as the equilibrium profit, goes to infinity. This cannot be supported as an equilibrium, because both firms would have an incentive to lower price and take the whole market. Also, if we look at the second order condition, it is negative only when $\beta \geq \frac{\sqrt{5}-3}{2}$. We thus set the lower bound $\underline{\beta}$ at this value.

each period, and each firm's overall profit is

$$\pi^U = \frac{1+\delta}{2(1+\beta)}t. \tag{2}$$

Proof. See the Appendix. ■

Corollary 1 *When we depart from the uniform distribution of consumer preferences ($\beta = 0$) to a triangular distribution with $0 < \beta \leq 1$, prices and profits fall. The reverse happens when consumer preferences follow a reverse triangular distribution ($\underline{\beta} \leq \beta < 0$).*

Since $\frac{\partial p^U}{\partial \beta} < 0$ and $\frac{\partial \pi^U}{\partial \beta} < 0$, the proof of the corollary is trivial. The economic intuition is also relatively straightforward. Consider the uniform price $p^U = \frac{2F(\frac{1}{2})}{f(\frac{1}{2})}$. Since f is symmetric about $\frac{1}{2}$, $F(\frac{1}{2}) = \frac{1}{2}$ always holds. That is, what matters is the number of marginal consumers who are indifferent between firms, $f(\frac{1}{2})$. This determines how many more consumers a firm can gain by reducing its price slightly, which in turn determines firms' incentive to undercut each other's prices. Note that consumer density is the same everywhere ($f(x) = 1$) under uniform distribution, but varies with x under a non-uniform distribution. In particular, under triangular distribution ($\beta \in (0, 1)$), more consumers are located at the center of the market and $f(\frac{1}{2}) > 1$. In this case, the uniform price falls relative to the level when $\beta = 0$. In contrast, when $\beta < 0$, fewer consumers are located at the center and $f(\frac{1}{2}) < 1$, supporting a higher uniform price relative to when $\beta = 0$.

Next, we calculate social and consumer surplus under uniform pricing. In the symmetric equilibrium, all consumers buy from the closer firm which is efficient. Let TC denote the transport cost incurred by all consumers in each period, i.e.,

$$TC^U = 2\int_0^{\frac{1}{2}} t \cdot x \cdot f(x)dx = \frac{\beta t}{12} + \frac{t}{4}. \tag{3}$$

The discounted social surplus is

$$SS^U = (1+\delta)(v - TC) = (1+\delta)\left(v - \frac{\beta t}{12} - \frac{t}{4}\right). \tag{4}$$

Intuitively transport cost incurred by all consumers is higher when more consumers are located at the center of the market. Therefore, as β increases TC^U also increases and so welfare falls.

Finally, let CS^U denote consumer surplus with uniform pricing. It is straightforward to show that

$$CS^U = SS^U - 2\pi^U = (1+\delta)\left(v - \frac{\beta^2 + 4\beta + 15}{12(\beta+1)}t\right). \tag{5}$$

It can be easily verified that $\frac{\partial CS^U}{\partial \beta} = (1+\delta)\frac{t(2\sqrt{3}-1-\beta)(2\sqrt{3}+1+\beta)}{12(\beta+1)^2}$, which is always positive for our range of $\beta \in [\underline{\beta}, 1]$.

3 Behavior-based price discrimination with uniform and triangular distributions

In this section, we allow firms to price discriminate across consumers based on their purchasing history, focusing on two special cases. In the first case, consumers are uniformly distributed ($\beta = 0$). This has been analyzed in the literature before (e.g., Fudenberg and Tirole (2000), Chen (2005), Chen and Pearcy (2010)), and BBPD is found to reduce firm profits. In the second case, we consider the triangular distribution ($\beta = 1$). We show that under this specific non-uniform distribution, BBPD raises firm profits, in contrast to the case of uniform distribution. We lay out the intuitions, and illustrate that the contrast is driven by the density ratio at the middle relative to the sides for the consumer distribution.[9]

Under BBPD, in each period, firms choose their prices simultaneously and non-cooperatively. For $i = \{A, B\}$, a strategy of firm i specifies p_i^1 in period 1 and prices (p_i^o, p_i^n) in period 2 targeting its old and new consumers respectively. We impose that players' strategies are sequentially rational, in the sense that in equilibrium, these strategies must induce a Nash equilibrium at any second period subgame as well as a Nash equilibrium in period 1. Given that firms are initially symmetric, we only look for a *symmetric* pure strategy subgame perfect Nash equilibria.

3.1 Uniform distribution

BBPD with uniform distribution has been analyzed in many existing studies (see, for example, Chen (2005) and Chen and Pearcy (2010)). The results are presented in the next lemma.

Lemma 1 *(BBPD-Uniform distribution)*

When consumer preferences follow uniform distribution and firms can practice BBPD, in the equilibrium,

(i) Both firms choose $p_A^1 = p_B^1 = \frac{t}{3}(3 + \delta)$ *in the first period.*

(ii) Firms choose

$$p_A^o = p_B^o = \frac{2}{3}t$$

to their old customers and choose

$$p_A^n = p_B^n = \frac{1}{3}t$$

to poach rivals' old customers.

(iii) Each firm's period 2 profit is

$$\pi_A^2 = \pi_B^2 = \frac{5}{18}t,$$

[9] Extrapolating this finding to the whole class of non-uniform distribution, we conjecture that whether BBPD can raise overall profits depends on the aforementioned density ratio, emphasizing the importance of non-uniform distribution. This is verified in Section 4 where we analyze the case of general β.

and each firm's profit over the two periods is

$$\pi^{BBPD} = \frac{1}{2}t + \frac{4}{9}\delta t.$$

(iv) Discounted consumer surplus over the two periods is

$$CS^{BBPD} = (1+\delta)v - \frac{5}{4}t - \frac{43}{36}\delta t.$$

We refer the readers to previous studies (e.g., Chen (2005) and Chen and Pearcy (2010)) for more details.

3.2 Triangular distribution

Next, we consider triangular distribution ($\beta = 1$). Note that the density function is maximized at the middle, $f\left(x = \frac{1}{2}\right) = 2$, and minimized at the extremes, $f(x = 0) = f(x = 1) = 0$.

Second period:

We solve the game backwards, starting with period 2. Suppose that first period prices lead to marginal consumer $x_1 \in [0, 1]$ who is indifferent between buying from either firm in period 1.

Next, look at firm A's turf on $[0, x_1]$. These are firm A's old consumers and thus will observe the following second period prices: p_A^o and p_B^n. Given these prices, some consumers will buy again from A while others will be willing to switch to firm B. Let x_A denote the marginal consumer who is indifferent between staying with A paying p_A^o and switching to B paying p_B^n.

Then

$$p_A^o + x_A = p_B^n + (1 - x_A) \Rightarrow x_A = \frac{1}{2} + \frac{p_B^n - p_A^o}{2}. \tag{6}$$

Thus, conditional on purchasing from A in period 1 (i.e., $x \in [0, x_1]$), consumers in $[0, x_A]$ will buy from A again, while consumers in $(x_A, x_1]$ will switch to B. Similarly, conditional on purchasing from B in period 1 (i.e., $x \in (x_1, 1]$), consumers in (x_1, x_B) will switch to A while consumers in $(x_B, 1]$ will buy from B again, with

$$x_B = \frac{1}{2} + \frac{p_B^o - p_A^n}{2}. \tag{7}$$

In period 2, firm i chooses (p_i^o, p_i^n) to maximize $\pi_i^o(p_i^o, p_j^n)$ and $\pi_i^n(p_i^n, p_j^o)$ respectively, where

$$\pi_i^o(p_i^o, p_j^n) = p_i^o q_i^o, \quad \pi_i^n(p_i^n, p_j^o) = p_i^n q_i^n.$$

We can calculate firms' demand functions as follows,

$$\begin{aligned}
q_A^o &= F(x_A) = \frac{1}{2}(1+p_B^n-p_A^o)(1+p_B^n-\beta p_A^o), \\
q_B^n &= F(x_1)-F(x_A) = \frac{1}{2}(p_A^o-p_B^n+2x_1-1)(1+2\beta x_1+\beta p_B^n-\beta p_A^o), \\
q_A^n &= F(x_B)-F(x_1) = \frac{1}{2}\left[(p_B^o-p_A^n)(1+\beta+\beta p_A^n-\beta p_B^o)+(1-2x_1)(1+2\beta x_1)\right], \\
q_B^o &= 1-F(x_B) = \frac{1}{2}(p_A^n-p_B^o+1)(\beta p_A^n-\beta p_B^o+1). \qquad (8)
\end{aligned}$$

Note that the demand functions are presented for general β, even though we are considering triangular distribution here ($\beta = 1$). Substituting $\beta = 1$ and solving firms' first-order conditions, we can obtain equilibrium prices in the second period. The results are presented in the next Lemma.

Lemma 2 *(Triangular distribution - Period 2) When consumer preferences follow triangular distribution and firms can practice BBPD, in period 2,*

(i) Firms' prices, in firm A's and B's turfs respectively, are

$$p_A^o = \frac{t}{8}\left(1+\sqrt{1+16x_1^2}\right), \qquad p_B^n = \frac{t}{8}\left(3\sqrt{1+16x_1^2}-5\right),$$

$$p_A^n = \frac{t}{8}\left(3\sqrt{9-16x_1^2}-5\right), \qquad p_B^o = \frac{t}{8}\left(\sqrt{9-16x_1^2}+1\right).$$

(ii) Firms' profits are

$$\pi_A^2 = \frac{t}{256}\left((16x_1^2+1)^{\frac{3}{2}}-3(9-16x_1^2)^{\frac{3}{2}}-192\sqrt{9-16x_1^2}x_1^2+384x_1^2+3\sqrt{16x_1^2+1}+103\sqrt{9-16x_1^2}-160\right),$$

$$\pi_B^2 = \frac{t}{256}\left(192\sqrt{16x_1^2+1}x_1^2-3(16x_1^2+1)^{\frac{3}{2}}+(9-16x_1^2)^{\frac{3}{2}}-384x_1^2+7\sqrt{16x_1^2+1}+3\sqrt{9-16x_1^2}+32\right).$$

Proof. See the Appendix. ■

First period:

Next, we analyze the first period. If firms have no commitment power, then their market shares in the first period affect their second period pricing. Forward-looking firms take this interdependence into account when setting their first period prices. Similarly, consumers are forward-looking and anticipate that their choice in period 1 affect the prices they receive in period 2 as well. Let x_1 denote the marginal consumer in period 1, who must be indifferent between buying from A in the first period at price p_A^1, and buying from B in the next period at the poaching price p_B^n; or buying from B in the first period at price p_B^1, and switching to firm A in the second period at the poaching price p_A^n. Therefore,

$$v-x_1-p_A^1+\delta\left(v-p_B^n-(1-x_1)\right) = v-p_B^1-(1-x_1)+\delta\left(v-p_A^n-x_1\right)$$

This yields:

$$x_1 = \frac{1}{2} + \frac{p_B^1 - p_A^1}{2(1-\delta)} + \frac{\delta(p_A^n - p_B^n)}{2(1-\delta)} \tag{9}$$

Notice that equation (9) implicitly determines the marginal consumer x_1, because p_A^n and p_B^n both are functions of x_1, as laid out in Lemma 2.

Firms' overall profits are given by

$$\Pi_A = p_A^1 F(x_1(p_A^1, p_B^1)) + \delta(\pi_A^o + \pi_A^n), \tag{10}$$

$$\Pi_B = p_B^1 \left[1 - F(x_1(p_A^1, p_B^1))\right] + \delta(\pi_B^o + \pi_B^n), \tag{11}$$

where

$$\begin{aligned}
\pi_A^o &= p_A^o F(x_A(x_1(p_A^1, p_B^1))), \\
\pi_A^n &= p_A^n \left[F(x_B(x_1(p_A^1, p_B^1))) - F(x_1(p_A^1, p_B^1))\right], \\
\pi_B^o &= p_B^o \left[1 - F(x_B(x_1(p_A^1, p_B^1)))\right], \\
\pi_B^n &= p_B^n \left[F(x_1(p_A^1, p_B^1)) - F(x_A(x_1(p_A^1, p_B^1)))\right].
\end{aligned}$$

Note that second period prices are functions of x_1 which in turn is a function of first period prices. Then firms' overall profits are defined on first period prices as well, and each firm maximizes its overall profit with respect to its first period price. Consider firm A for example. Because firm A's second period prices are chosen to maximize its second period profit, we can use the envelop theorem to simplify the first-order condition, $\frac{\partial \Pi_A}{\partial p_A^1} = 0$, where Π_A is defined in equation (10). This yields:

$$F(x_1) + p_A^1 f(x_1)\frac{\partial x_1}{\partial p_A^1} + \delta\left[\frac{\pi_A^o}{\partial p_B^n}\frac{\partial p_B^n}{\partial x_1} + \frac{\pi_A^n}{\partial p_B^o}\frac{\partial p_B^o}{\partial x_1} + \frac{\pi_A^n}{\partial x_1}\right]\frac{\partial x_1}{\partial p_A^1} = 0.$$

After computing all these derivatives, we impose symmetry by evaluating the previous expression at $x_1 = \frac{1}{2}$. We are able to obtain closed form solution for equilibrium first period prices. Substituting $x_1 = \frac{1}{2}$ into second period equilibrium, we can obtain second period prices and profits. Combined, we can obtain overall profits and consumer surplus.

Next, we illustrate how BBPD affects first period prices under non-uniform distribution, via a consumer-side effect and a firm-side effect respectively. We do this without the need to substitute $\beta = 1$ (triangular distribution). The results are presented in the next Proposition.

Proposition 2 *(Decomposing first period impact) The impact of BBPD on first-period prices can be decomposed into a consumer-side effect and a separate firm-side effect, in the form of*

$$p_1^{BBPD} = (\textit{consumer-side effect}) \times p_1^U + (\textit{firm-side effect}).$$

Proof. See the Appendix. ■

The exact form of decomposition is shown in the proof. Note that in Fudenberg and Tirole (2000), there is no firm-side effect so consumer-side effect alone determines the increase in first period prices. In contrast, when consumers are naive (thus no consumer-side effect), the firm-side effect can still raise or lower first period prices in our model.[10] We will discuss this in more details later on in Section 3.3 when we explore the welfare impacts of BBPD.

Next, we come back to triangular distribution ($\beta = 1$). Equilibrium prices and profits under BBPD are presented in the next proposition.

Proposition 3 *(BBPD-Triangular distribution)*

When consumer preferences follow triangular distribution and firms can practice BBPD, in the equilibrium,

(i) Both firms choose $p_A^1 = p_B^1 = t\left(\frac{1}{2} + \frac{13}{20}\sqrt{5}\delta - \frac{5}{4}\delta\right)$ *in the first period.*

(ii) In the second period, firms choose

$$p_A^o = p_B^o = \frac{t}{8}\left(1 + \sqrt{5}\right)$$

to their old customers and choose

$$p_A^n = p_B^n = \frac{t}{8}\left(3\sqrt{5} - 5\right)$$

to poach rivals' old customers.

(iii) Each firm's period 2 profit is

$$\pi_A^2 = \pi_B^2 = \frac{t}{16}\left(3\sqrt{5} - 4\right),$$

and each firm's overall profit over the two periods is

$$\pi^{BBPD} = t\left(\frac{1}{4} + \frac{41\sqrt{5} - 70}{80}\delta\right).$$

(iv) Overall consumer surplus over the two periods is

$$CS^{BBPD} = (1+\delta)v - t\left(\frac{5}{6} + \frac{59\sqrt{5}}{60}\delta - \frac{31}{24}\delta\right).$$

Proof. See the Appendix. ■

[10] In our model, consumers and firms share the same discount factor $\delta > 0$, under both strategic and naive consumers. When making period 1 decisions, forward-looking (or strategic) consumers take into account their surplus from both periods. In contrast, naive consumers only take into account their immediate consumer surplus in period 1. The distinction between forward-looking and naive consumers lies in their level of sophistication, not in δ. In particular, we are not using $\delta = 0$ to capture naive consumers.

3.3 The welfare effects of BBPD

Using results under BBPD (in Lemma 1 and Proposition 3) and under uniform pricing (in Proposition 1 and Equations (4)-(5)), one can easily see how BBPD affects profits, consumer surplus and social surplus for the two special cases of uniform distribution and triangular distribution. This is summarized in the next Proposition.

Proposition 4 *(The welfare impacts of BBPD)*

If consumer preferences follow uniform distribution, BBPD harms industry profits and welfare but boosts consumer surplus, relative to uniform pricing. In contrast, if consumer preferences follow triangular distribution, compared to uniform pricing, BBPD harms consumer surplus and overall welfare but boosts industry profits.

Next, we explore the intuitions behind the above results. In particular, we decompose how BBPD affects prices and profits in the two periods respectively.

3.3.1 Effect on prices

Most of the existing academic literature on BBPD suggests that when the market exhibits best-response asymmetry – one firm's weak market is the other's strong market – the optimal choice for each firm is to offer a lower price to its rival's old consumers than to its own old consumers (e.g. Chen, 1997; Villas-Boas, 1999; Fudenberg and Tirole, 2000; Esteves, 2010).[11] In what follows we first check whether this happens in our setting, which applies to the second period. We then study how BBPD affects first period prices.

As aforementioned, in an oligopoly price competition model with covered market, what matters for equilibrium prices is the number of marginal consumers who are indifferent between firms. In a Hotelling model with uniform distribution, the number of marginal consumers is always the same no matter where they are. In contrast, under non-uniform distribution the density of consumers will vary with location. We look first at second period prices.

Second period prices:

Using the results presented earlier, in Table 1 we report second period prices under uniform pricing and BBPD, for $\beta = 0$ or $\beta = 1$ respectively. Due to symmetry, we only report firm A's prices. To facilitate comparison, we convert some values to decimal points when needed.

For both β values, relative to uniform pricing, prices are lower for all consumers in period 2, the same as in Fudenberg and Tirole (2000) and Chen and Pearcy (2010) when firms cannot commit

[11] An exception is Shin and Sudhir (2010) who show that firms can charge a low price to their strong customers when consumer preferences stochasticity across time and consumer heterogeneity are also high enough. Chen and Pearcy (2010) find similar results when firms are able to commit to future prices and there is weak preference dependence over time.

	Uniform pricing	BBPD	
When $\beta = 0$	$p^U = t$	$p_A^o = \frac{2t}{3}$	$p_A^n = \frac{t}{3}$
When $\beta = 1$	$p^U = \frac{t}{2}$	$p_A^o \approx 0.4045t$	$p_A^n \approx 0.2135t$

Table 1: Second period prices

to future prices. Since purchase history reveals consumers' brand preferences, each firm has an incentive to reduce the price to customers who bought from the rival, enticing them to switch. However, because both firms set lower prices to customers in their weak markets, they induce rivals to be more aggressive in their strong markets too. This intensifies competition and all prices fall in period 2. Our analysis confirms the usual findings in the literature, suggesting that even when the distribution of consumer preferences is non-uniform, firms also quote lower prices to their weak segment than to their strong segment of consumers.

However, it is important to stress that although second period equilibrium prices fall with price discrimination in our set-up, when the distribution of consumer preferences approaches a pure triangular distribution, we find that in comparison to uniform pricing the reduction in second period prices is smaller than the reduction under uniform distribution. In particular, if preferences are uniform, then

$$\begin{aligned} p^U(\beta = 0) - p_A^o(\beta = 0) &= t - \frac{2t}{3} \approx 0.3333t, \\ p^U(\beta = 0) - p_A^n(\beta = 0) &= t - \frac{t}{3} \approx 0.6667t, \end{aligned}$$

while if preferences follow a pure triangular distribution, then

$$\begin{aligned} p^U(\beta = 1) - p_A^o(\beta = 1) &\approx \frac{t}{2} - 0.4045t = 0.0955t, \\ p^U(\beta = 1) - p_A^n(\beta = 1) &\approx \frac{t}{2} - 0.2135t = 0.2865t. \end{aligned}$$

The intuition is as follows. Under the triangular distribution marginal price changes capture more additional consumers when the point of indifference is in the center of the market, than when the point of indifference is closer to one or the other firm. This reduces the aggression with which a firm seeks to drive the point of indifference into its rival's market segment in the second period, as the bulk of consumers are clustered near the margin of this segment. Put differently, when β increases from 0 to 1, the density of marginal consumers under uniform pricing ($x = \frac{1}{2}$) increases which hurts the prices and profits sharply. In contrast, under BBPD, marginal consumers are away from the center, so second period prices are not as hurt by the increase in β.

Therefore, our analysis reveals that relative to uniform pricing, when firms engage in BBPD, they compete more aggressively in period 2 if consumer preferences are uniformly distributed than if consumer preferences follow a triangular distribution.

First period prices:

Next, we move on to first period. The prices are reported in the next table. We can see that under both uniform and triangular distribution, BBPD raises period 1 prices relative to uniform pricing.

	Uniform pricing	BBPD
When $\beta = 0$	$p^U = t$	$p_A^1 \approx t + 0.3333\delta t$
When $\beta = 1$	$p^U = \frac{t}{2}$	$p_A^1 \approx \frac{t}{2} + 0.2034\delta t$

Table 2: First period prices

The intuition for higher first period prices is that consumers become less price sensitive in period 1 when they take into account prices in period 2. This same mechanism is also found in Fudenberg and Tirole (2000) and Chen and Pearcy (2010). But there are also subtle differences between our paper and theirs. Let us start with Fudenberg and Tirole (2000). When consumers are naive, BBPD has no impact on first period prices, relative to uniform pricing. Consider next our framework. Here the decreased elasticity of demand can also explain in part why BBPD raises first period prices, when consumers are forward-looking. The difference is that BBPD still raises first period prices even when consumers are assumed to be naive.

Our analysis suggests that apart from reduced elasticity of demand, there should be another driving force behind the higher first period prices. In general, BBPD generates two effects on first period prices: a consumer-side effect and a firm-side effect. The first effect occurs because under BBPD, forward-looking consumers correctly anticipate second period prices, become less price sensitive in period 1 and raise equilibrium first period prices (consumer-side effect). Consider next the firm-side effect. When firms are forward-looking, they also take into account that changes in the first period prices change the first period marginal consumer, and in turn change the nature of the second period competition. When consumer preferences are uniform and firms employ BBPD, a change in the first period price has no effect on second period profit because with uniform distribution a firm's marginal gain in one market is exactly offset by its loss in the other ($\frac{\partial \pi_2}{\partial p_1} = 0$). Thus, in Fudenberg and Tirole (2000), under symmetry ($x_1 = \frac{1}{2}$), the firm-side effect plays no role and the consumer-side effect fully determines the result that BBPD raises first period prices.

In the current model we find that if consumer preferences are non-uniform ($\beta = 1$), a change in the first period price does not cancel out in the neighborhood of $x_1 = \frac{1}{2}$. Specifically, we find that $\frac{\partial \pi_A^2}{\partial p_A^1} > 0$, suggesting that firm A's marginal gain in one second period market is higher than its loss in the other market. In other words, if firm A becomes more aggressive in the first period, its strong market becomes larger in the second period, especially if consumer preferences are clustered at the center of the market. This forces firm B to reduce its prices in period 2. Thus aggressive pricing in the first period will induce its competitor not only to be aggressive in the first period, but also in the second period. To avoid this, firms want to raise their first period prices above the uniform price when $\beta = 1$. Therefore, even if consumers are naive (i.e., no consumer-side effect), the firm-side effect can still lead to higher first period prices under BBPD.

In our paper, consumer preferences stay the same in the two periods. In contrast, Chen and Pearcy (2010) allow the preference dependence to be imperfect, captured by a preference dependence parameter α. Because of the imperfect preference dependence, BBPD may raise or lower first period prices depending on α, even though it always reduces second period prices. An increase in α always worsens the reduction of second period profit under BBPD, but first period profit first increases and then decreases with α. Combined, BBPD raises firms' discounted overall profits only when α is small. This has a flavor similar to our results with respect to β. That is, BBPD always worsens period 2 profit relative to uniform pricing, but tends to increase first period profit when β is large. Combined, BBPD raises discounted overall profit when β is sufficiently high. Next, we look at the profit effects of BBPD in details.

3.3.2 Profit effects and managerial implications

An important question for managers and marketing practitioners is when they should expect BBPD to improve profitability. Next, we look at how BBPD affects firms' profits in period 1 and 2 respectively, as well as their discounted overall profits. The results are presented in the next table, where the top (bottom) panel is for $\beta = 0$ ($\beta = 1$).

Uniform pricing	BBPD	Ratio
$\pi_A^1 = \frac{t}{2}$	$\pi_A^1 \approx 0.5t + 0.1667\delta t$	$1 + 0.3333\delta$
$\pi_A^2 = \frac{t}{2}$	$\pi_A^2 \approx 0.2778t$	0.5556
$\pi_A = 0.5t + 0.5\delta t$	$\pi_A \approx 0.5t + 0.4444\delta t$	< 1

($\beta = 0$)

Uniform pricing	BBPD	Ratio
$\pi_A^1 = \frac{t}{4}$	$\pi_A^1 \approx \frac{t}{4} + 0.1017\delta t$	$1 + 0.4068\delta$
$\pi_A^2 = \frac{t}{4}$	$\pi_A^2 \approx 0.1693t$	0.6772
$\pi_A = 0.25t + 0.25\delta t$	$\pi_A \approx 0.25t + 0.2710\delta t$	> 1

($\beta = 1$)

Table 3: Profits by periods and overall profits

In the table, π_A^t is each firm's profit in period $t = 1, 2$, while π_A is each firm's discounted overall profit. To facilitate comparison between BBPD and uniform pricing, we also calculate the BBPD to uniform pricing profit ratio (third column). From the table one can see that for both $\beta = 0$ or $\beta = 1$, BBPD raises profits in period 1 but lowers profits in period 2. This is in line with the price effects we discussed earlier. However, it is important to stress that the decrease in second period profits due to price discrimination is smaller when β increases. This suggests that the negative effect of BBPD on second period profits falls when we depart from a uniform distribution of consumer preferences to a triangular one. In fact, from the table we can see that the BBPD to Uniform pricing ratio is higher for both first and second period profits when $\beta = 1$, relative to $\beta = 0$. This allows BBPD to reduce overall profits when $\beta = 0$ but the opposite happens when $\beta = 1$. In the next section, we will consider general β and show that BBPD raises overall profits when β is sufficiently high (e.g., $\beta > 0.194$ for any $\delta \in (0, 1]$).

For managers and practitioners our analysis highlights that while BBPD with uniform preferences is bad for profits, it can be a winning strategy in markets where consumer preferences are clustered at the center of the market. When consumers are increasingly clustered at the center, BBPD makes companies compete less aggressively in both periods relative to uniform pricing, permitting overall profits to increase. Therefore, our analysis suggests that a good understanding of the distribution of consumer preferences is a necessary condition for managers to assess the profitability of this price discrimination based on consumers' purchase history. A simple proxy is to view consumers at the edges (center) as loyals (switchers). To determine whether BBPD is profitable, managers need to estimate their loyal-switcher ratios.

4 BBPD with general β

In the previous section we have explored the welfare implications of BBPD under the special cases of uniform distribution ($\beta = 0$) and pure triangular distribution ($\beta = 1$). In this section, we generalize the analysis to $\beta \in [\underline{\beta}, 1]$. Our goal is to show that the spirit of the results in the previous section carries through, in particular, when consumers are sufficiently clustered around the center relative to the sides, BBPD raise firms' overall profits.

We solve the game backwards, starting with period 2. Denote the marginal consumer by $x_1 \in [0, 1]$ who is indifferent between buying from either firm in period 1. Earlier when analyzing triangular distribution, we have laid out firms' demand functions for general β (equation (8)). Using these demand functions, after some (tedious) computation, we can solve the second period equilibrium and obtain closed form solutions for period 2 prices. The results are quite tedious and are relegated to the Appendix (see "**BBPD with general** β" toward the end of the Appendix). The steps to solve for equilibrium first period prices are similar to those in the case of triangular distribution, except without imposing $\beta = 1$. After computing all relevant derivatives, we impose symmetry, i.e., $x_1 = \frac{1}{2}$, and derive the equilibrium first period prices.

Having solved the subgame perfect Nash equilibrium, we can then explore the welfare effects of BBPD relative to uniform pricing (see Lemma 1) for general β. We look at this in details next.

4.1 Price and profit effects of BBPD

This section presents our main findings in terms of prices and profits, when we depart from a uniform distribution of consumer preferences to a class of non-uniform distributions. Since equilibrium prices and profits are all linear in the unit transport cost t, we normalize $t = 1$ in the rest of the paper. We first explore the role of δ in the prices and profits. The results are presented in the next Lemma.

Lemma 3 *(The role of δ) Under BBPD with general β, in the equilibrium,*

(i) Second period prices are independent of δ.

(ii) First-period price takes the form of

$$p_1^{BBPD} = p_1^U + g(\beta) \cdot \delta,$$

where $g(\beta)$ is a function of β only.

(iii) The impacts of BBPD on social surplus, profit and consumer surplus, measured by $SS^{BBPD} - SS^U$, $\pi^{BBPD} - \pi^U$, and $CS^{BBPD} - CS^U$ respectively, are all proportional to δ, i.e., linear in δ with zero intercept.

Proof. See the Appendix. ■

Our focus is on how BBPD affects prices and profits, and we are able to obtain closed form solutions under general β and δ. Based on results (ii) and (iii) above, we assume $\delta = 1$ without loss of generality. But even with $t = 1$ and $\delta = 1$, the expressions for prices and proits are too lengthy to report and compare.[12] Therefore, we present our main findings using figures plotted as a function of β, the single parameter which characterizes the class of distributions. Specifically, we plot prices and profits when price discrimination is not permitted (uniform pricing) and when BBPD is permitted. Using the closed form expressions, we can easily check how variables of interest vary with β.

4.1.1 Effect on prices

Second period prices:

Figure 1 plots second period prices under uniform pricing and BBPD respectively. Relative to uniform pricing, prices are lower for all consumers in period 2, the same as in the previous section (when $\beta = 0$ or $\beta = 1$). Our analysis confirms the usual findings in the literature, suggesting that price discrimination intensifies competition even under non-uniform distribution. Moreover, when β increases, in comparison to uniform pricing the reduction in second period prices becomes smaller. This is in similar spirit as what we have found previously when comparing $\beta = 1$ to $\beta = 0$.

First period prices:

Figure 2 left panel plots first period prices under uniform pricing and BBPD respectively. We can see that BBPD raises first period prices for the main range for β. Specifically, this happens as long as $\beta > -0.378$. As previously discussed, in our model, BBPD generates two effects on first period

[12] Equilibrium first period prices are given at the end of the Appendix. Equilibrium second period prices and profits can be found by substituting $x_1 = \frac{1}{2}$ into second period equilibrium.

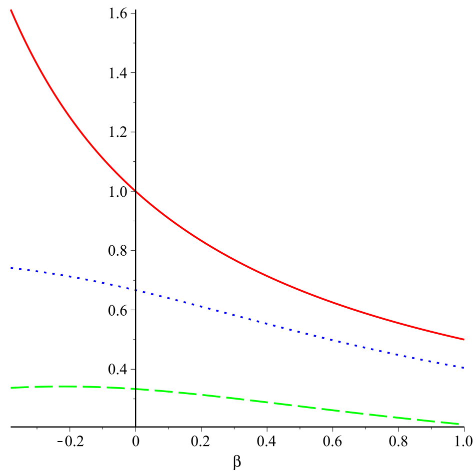

Figure 1: Second period prices

Red line: Uniform price; Blue dots: Price to old customers; Green dash line: Price to new customers

prices: a consumer-side effect and a firm-side effect, and their combination is reflected in Figure 2 left panel. The consumer-side effect always induces firms to charge higher first period prices. The firm side effect, while absent under uniform distribution, has been shown to exist under triangular distribution ($\beta = 1$) earlier. Here, for general β, it turns out that the direction of the firm-side effect depends on the sign of β. To see this, we shut down the consumer-side effect by considering naive consumers. Only the firm-side effect is present. The first period prices are plotted in Figure 2 right panel. We can see that BBPD leads to higher first period prices under naive consumers if and only if $\beta > 0$.

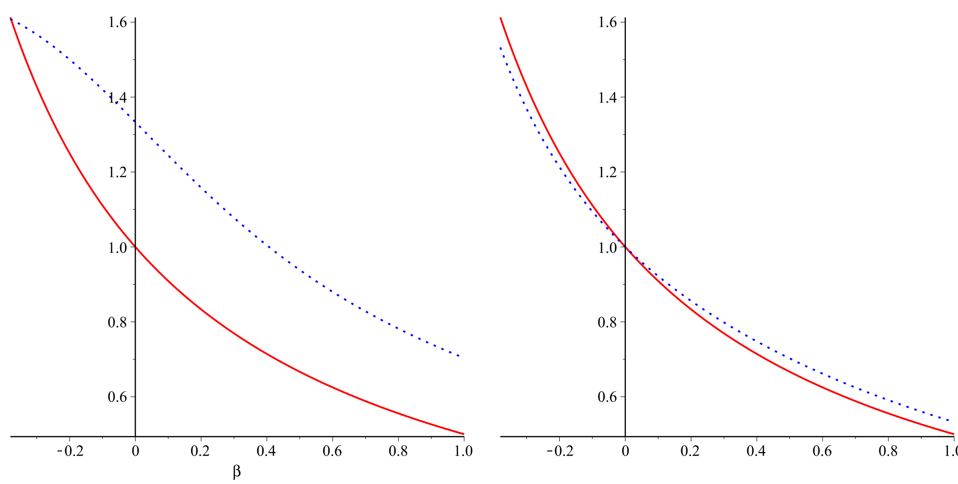

Figure 2: First period prices: Strategic consumers (left panel); Naive consumers (right panel)

Red line: Uniform pricing; Blue dots: BBPD

4.1.2 Effects on profits

We first look at how BBPD affects first period profit in our two-period model. The results are plotted in Figure 3 left panel. We can see that first period profits with BBPD are above the non-discrimination counterparts when $\beta > -0.378$. Note that the support is $\beta \in [\underline{\beta}, 1]$ where $\underline{\beta} = \frac{\sqrt{5}-3}{2} \approx -0.382$.

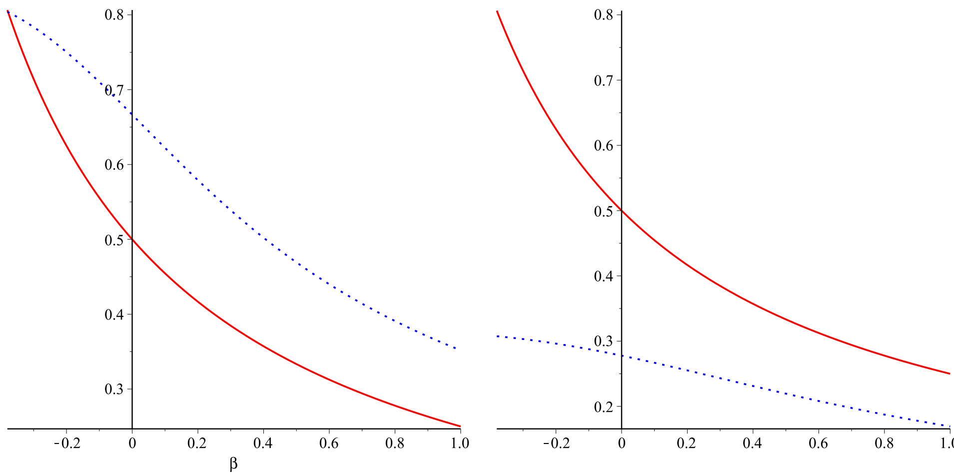

Figure 3: Left panel: first period profits ($\delta = 1$); Right panel: Second period profit

Red line: Uniform pricing; Blue dots: BBPD

While BBPD generally raises first-period profits, the opposite happens for second period profit (see Figure 3 right panel). It is important to stress that the decrease in second period profits due to price discrimination is smaller when β increases. This suggests that the negative effect of BBPD on second period profits falls when we depart from a uniform distribution of consumer preferences to a triangular one. The reverse happens when the distribution of preferences becomes a reverse triangle.

Next, we report the discounted overall profit over the two periods. From Figure 4, we can see that BBPD raises overall profits if β is sufficiently large. The exact threshold is presented in the next proposition. While this is for $\delta = 1$, based on Lemma 3 result (iii), the same threshold applies for all $\delta > 0$.

Proposition 5 *For any $\delta > 0$, BBPD boosts overall profits in comparison to uniform pricing as long as $\beta > 0.194$. Otherwise the reverse happens.*

4.2 Alternative distributions

Previously we have investigated a class of non-uniform distributions, captured by a single parameter β. This approach allows the density at the middle to differ from the density at the sides, and we are able to confirm the intuition laid out in Section 3 where we compare uniform distribution

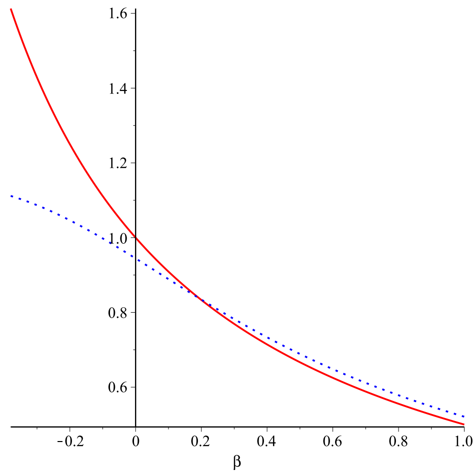

Figure 4: Overall profits ($\delta = 1$)

Red line: Uniform pricing; Blue dots: BBPD

and triangular distribution. That is, when consumer density is higher in the middle but lower on the sides, the density difference (captured by β) hurts uniform price but helps with discriminatory prices, everything else the same. When the density difference is sufficiently large, BBPD raises firms' overall profits relative to uniform pricing.

The downside of considering a class of non-uniform distribution is that the equilibrium prices and profits become extremely tedious. Next, we go back to uniform distribution, but allow piece-wise distribution so densities differ between the middle and the sides. Suppose that some consumers are uniformly distributed in the middle interval $[a, 1-a]$ with density f_{middle}, and the remaining consumers are uniformly distributed on the sides: $[0, a) \cup (1-a, 1]$ with density f_{side}. Let $r \equiv \frac{f_{middle}}{f_{side}}$ define the density ratio. If the insights from our main model carry through here, we would expect that BBPD raises overall profits when r is large, which we verify next.

Consumer mass is 1 so

$$f_{middle} \cdot (1 - 2a) + f_{side} \cdot 2a = 1.$$

Combined with $r = \frac{f_{middle}}{f_{side}}$, we can obtain,

$$f_{middle} = \frac{r}{r - 2ra + 2a}, \quad f_{side} = \frac{1}{r - 2ra + 2a}.$$

We look for symmetric equilibrium under uniform pricing and under BBPD.

Start with uniform pricing. The marginal consumer $x = \frac{1}{2}$ must be in the middle. The equilibrium candidate is characterized by

$$p^U = \frac{r - 2ra + 2a}{r}, \quad \pi^U = (1 + \delta) \cdot \frac{p^U}{2}.$$

Second-order conditions ensure that neither firm has an incentive to deviate when marginal consumer is still in the middle. However, a firm can change its price significantly and push marginal consumer to either side. Such an incentive is easier to curb if a is smaller. Later on under BBPD, we want marginal consumers to be on the sides and need to curb firms' incentive to deviate and push marginal consumers to the middle. Such an incentive is easier to curb if a is larger. Taking into account the opposite constraints on a under uniform pricing and BBPD respectively, we pick an intermediate value of $a = \frac{2}{5}$. We then derive the equilibrium candidate under BBPD and find that $\pi^{BBPD} - \pi^U$ is proportional to δ, similar to the case of non-uniform distribution with general β earlier. Thus we further normalize $\delta = 1$. With $a = \frac{2}{5}$ and $\delta = 1$, the equilibrium candidate under BBPD is characterized by

$$p_A^1 = p_B^1 = \frac{2}{5}r + \frac{14}{15},$$

$$p_A^o = p_B^o = \frac{3}{5} + \frac{1}{15}r, \quad p_A^n = p_B^n = \frac{1}{5} + \frac{2}{15}r,$$

$$\pi^{BBPD} = \frac{(23r^2 + 144r + 258)}{90(r+4)}.$$

It can be easily verified that $\pi^{BBPD} > \pi^U$ if and only if $r > r^* \approx 1.0561$. We pick $r = 1.06$ and verify that neither firm has an incentive to deviate, with or without changing the demand structure. Note that if $r = 1$, then we are back to traditional uniform distribution, and BBPD reduces overall profit.

5 Policy implications

In this section we discuss the main implications of our analysis for regulators and/or consumer advocates. Policy recommendations should be based on whether BBPD benefits consumers or not. A general presumption in the literature with uniformly distributed preferences fixed across time is that "price discrimination by purchase history ... is by and large unlikely to raise significant antitrust concerns. In fact, as the economics literature suggests, such pricing practices in oligopoly markets often intensify competition and potentially benefit consumers." (Chen, 2005, p. 123). Our goal in this section is to go beyond uniform distribution of consumer preferences and see how BBPD affects consumer surplus and welfare. Our results show that the distribution of consumer preferences within the product space plays an important role in determining these welfare implications.

In the absence of price discrimination, consumers always buy from the closer firm in both periods which is efficient. With BBPD the first period equilibrium outcome is also efficient. However, the second period switching lowers welfare. Next, we look at the question of *how much* switching should we expect to occur. In the symmetric equilibrium, the share of consumers who switch firms in the second period is

$$S = F(x_B) - F(x_A) = \frac{\left(3\beta + 3 - \sqrt{2\beta + 9\beta^2 + 9}\right)\left(5\beta + 5 + \sqrt{2\beta + 9\beta^2 + 9}\right)}{64\beta}.$$

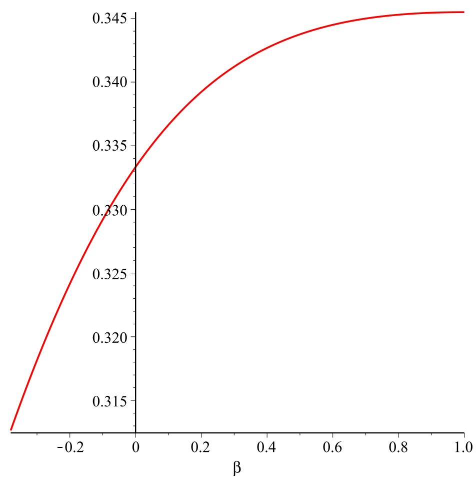

Figure 5: Share of switchers

Figure 5 plots the share of switchers under BBPD as a function of β. With triangular distribution, more consumers are in the middle, and consumers in the middle are also more likely to switch. Therefore, BBPD leads to more switching in markets where the density of preferences at the center is higher than when the density is the same everywhere ($\beta = 0$). The reverse happens when there are more consumers located at the edges ($\beta < 0$).

Under BBPD, social surplus in period 2 is given by

$$\begin{aligned} ss_2 &= v - 2\left[\int_0^{x_A} x f(x)dx - \int_{x_A}^{\frac{1}{2}} (1-x) f(x)dx\right] \\ &= v - \frac{48\beta + 240\beta^2 + 64\beta^3 + \left(2\beta + 9\beta^2 + 9\right)^{\frac{3}{2}} - 9\left(\beta+1\right)^2 \sqrt{2\beta + 9\beta^2 + 9}}{768\beta^2}, \end{aligned}$$

after substituting the equilibrium $x_A = \frac{1}{16\beta}\left(5\beta + \sqrt{9\beta^2 + 2\beta + 9} - 3\right)$.

Social surplus in period 1 is given by

$$ss_1 = v - 2\int_0^{\frac{1}{2}} x\left(4\beta x + (1-\beta)\right)dx = v - \frac{1}{12}\beta - \frac{1}{4}$$

Overall social surplus under BBPD is

$$\begin{aligned} SS^{BBPD} &= ss_1 + \delta \cdot ss_2 \\ &= v\left(1+\delta\right) - \frac{1}{12}\beta - \frac{1}{4} \\ &\quad -\delta\left(\frac{48\beta + 240\beta^2 + 64\beta^3 + \left(2\beta + 9\beta^2 + 9\right)^{\frac{3}{2}} - 9\left(\beta+1\right)^2 \sqrt{2\beta + 9\beta^2 + 9}}{768\beta^2}\right). \end{aligned}$$

Social surplus under uniform pricing is

$$SS^U = (1+\delta)\left(v - \frac{1}{12}\beta - \frac{1}{4}\right).$$

Figure 6 left panel plots the social surplus under uniform pricing and BBPD respectively, assuming $v = 2$ and $\delta = 1$. Because of inefficient switching, total transport cost is always higher and overall social surplus is always lower under BBPD, relative to uniform pricing.

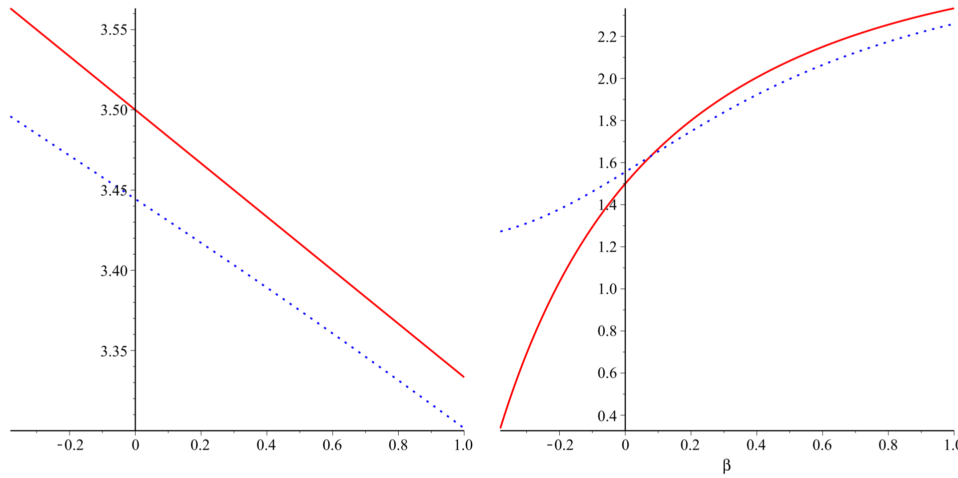

Figure 6: Social surplus (left) and consumer surplus (right)

Red line: Uniform pricing; Blue dots: BBPD

Look next at consumer surplus. The expressions are too lengthy to report, so we plot them in Figure 6 right panel for BBPD and uniform pricing respectively, assuming that $\delta = 1$ and $v = 2$. We can see that the BBPD raises consumer surplus only when β is below a threshold (about 0.077).

Combining the results for consumers surplus and social surplus, we have the next proposition.

Proposition 6 *(Consumer and social surplus) For any $\delta \in (0, 1]$, relative to uniform pricing, BBPD hurts consumers if $\beta > 0.077$. In contrast, if $\beta < 0.077$ consumers are better off with BBPD. BBPD always reduces social surplus.*

An important contribution of our analysis is to highlight that welfare effects of BBPD in a uniform distribution setting – higher consumer surplus at the expense of industry profits – do not prevail when preferences are better represented by triangular distribution. We show that when β is sufficiently high, BBPD can increase industry profits at the expense of consumer surplus and inefficient switching becomes more prominent.

To summarize the main literature findings and inform competition policy agencies, Table 1 shows the comparative static results obtained in a model of fixed uniform preferences across time (Fudenberg and Tirole, 2000), in a model of uniform correlated preferences across time (Chen and Pearcy, 2010) and in a model fixed non-uniform preferences across time (our analysis).

For overall welfare, because there is no role for price discrimination to increase aggregate output, variations in welfare are uniquely explained by the "disutility" incurred by consumers who buy

Models	Overall Profits	CS	W
Uniform distribution with fixed preferences Fudenberg and Tirole (2000)	Below U	Above U	Below U
Uniform distribution with correlated preferences Chen and Pearcy (2010)	Below U (Hα)/ Above U (Lα)	Below U (Lα)/ Above U (Hα)	Below U
Non-uniform distribution with fixed preferences This paper	Above U (Hβ)/ Below U (Lβ)	Below U (Hβ)/ Above U (Lβ)	Below U

U: uniform pricing; H(L)α:: high (low) α; H(L)β: high (low) β

Table 4: Comparative static results

inefficiently.[13] There is no such inefficient purchase under uniform pricing, thus BBPD always hurts overall welfare. Table 1 also shows that the profit and consumer welfare effects under fixed uniform preferences are the reverse of their counterparts under non-uniform preferences with high β. Chen and Pearcy (2010) shows that when consumer preferences' dependence across time (α) is high, BBPD benefits consumer and hurts profits (confirming the results in Fudenberg and Tirole, 2000). However, when consumer preferences' dependence is low (or independent), BBPD boosts industry profits at the expense of consumers. Their intuition is as follows. Unstable preferences across time introduces uncertainty, which acts to soften price competition under BBPD. This in turn enhances industry profits at the expense of consumers.

Summing up, the profit and welfare effects of BBPD depend on the way consumer preferences are modeled and on what is learned about consumers. Our results, thus, carry important policy implications. If competition authorities focus on consumer welfare when appraising price discrimination, our model suggests that they should scrutinize these pricing strategies with greater zeal in markets where consumer preferences seem to be clustered at the center of the market.[14]

6 Conclusion

Existing research shows that access to consumer purchase history for price discrimination goals can intensify competition and hurt firm profitability at the benefit of consumers. This is generally the case when firms are symmetrically informed and consumer preferences are uniformly distributed and fixed across time. Although the uniformity assumption is convenient for deriving analytical results, it may not be satisfactory in representing actual consumer preferences in many markets, frequently better represented by a non-uniform density. Therefore, the main goal of this paper is to

[13] For a model where BBPD can affect aggregate output see Esteves and Reggiani (2014).

[14] Policy intervention may depend on the legal standard or general mission of the competition law in a specific jurisdiction. According to the OECD (2018) report on price discrimination in digital markets, in 89% of the jurisdictions consumer welfare is the primary goal or one of the goals of competition law. See also Lyons (2002).

assess whether price discrimination based on purchasing history benefit/harm industry profits and consumers in markets where the density of consumer preferences is non-uniform. With this goal in mind, we depart from the base model of Fudenberg and Tirole (2000) by assuming that consumer preferences follow a non-uniform distribution.

By so doing, this paper provides useful implications for managers and marketing practitioners willing to use data on consumers' purchase history for price discrimination. It is also useful for competition agencies and consumer advocates. It highlights that *the shape of preferences* plays an important role to the understanding of the profit and welfare effects of BBPD. Specifically, we show that managers can use BBPD as a profitable strategy in markets where consumer preferences are clustered at the center of the market. In other words, if consumer preferences follow a pure triangular distribution, in contrast to the Fudenberg and Tirole conclusions, BBPD boosts industry profits at the expense of consumer welfare. On the other hand, if consumer preferences follow reverse triangular distribution, then the standard results under uniform distribution still prevail.

A Appendix

Proof of Proposition 1

Due to symmetry, assume that $x_1^U \leq \frac{1}{2}$. We calculate each firm's demand and profit, and then take first-order conditions to obtain:

$$\frac{\partial \pi_A}{\partial p_A} = \frac{1}{2} + \frac{1}{t^2}\left(\frac{3}{2}\beta p_A^2 - 2\beta p_A p_B - \beta p_A t - p_A t + \frac{1}{2}\beta p_B^2 + \frac{1}{2}\beta p_B t + \frac{1}{2}p_B t\right),$$

$$\frac{\partial \pi_B}{\partial p_B} = \frac{1}{2} + \frac{1}{t^2}\left(2\beta p_A p_B - \frac{3}{2}\beta p_B^2 - \beta p_B t - p_B t - \frac{1}{2}\beta p_A^2 + \frac{1}{2}\beta p_A t + \frac{1}{2}p_A t\right).$$

The second order conditions are:

$$\frac{\partial^2 \pi_A}{\partial (p_A)^2} = \frac{1}{t^2}\left(3\beta p_A - 2\beta p_B - \beta t - t\right),$$

$$\frac{\partial^2 \pi_B}{\partial (p_B)^2} = \frac{1}{t^2}\left(2\beta p_A - 3\beta p_B - \beta t - t\right).$$

We focus on the symmetric equilibrium, i.e., $p_A = p_B$. Solving the FOCs we can obtain

$$p_A = p_B = \frac{t}{1+\beta}.$$

The discounted overall profit is

$$\pi^U = \frac{1+\delta}{2(1+\beta)}t.$$

Note that when $\beta \neq 0$, density function $f(x)$ changes functional form at $x = \frac{1}{2}$. Thus demand has a kink at $x = \frac{1}{2}$. Nevertheless, we show that first-order condition continues to work for the symmetric equilibrium, because $\frac{\partial \pi_A}{\partial p_A^-} = \frac{\partial \pi_A}{\partial p_A^+}$ when $p_A = p_B$. Let $f_L(x)$ and $f_R(x)$ denote the density functions when $x \leq \frac{1}{2}$ (left) and $x \geq \frac{1}{2}$ (right) respectively. At symmetry ($p_A = p_B$), if firm A reduces or raises p_A slightly, we denote the marginal consumers by x_1^{U+} and x_1^{U-} respectively. Note that

$$\frac{\partial \pi_A}{\partial p_A^-} = \frac{1}{2} + p_A \cdot f_R(x_1^{U+}) \cdot \frac{\partial x_1^{U+}}{\partial p_A^-},$$
$$\frac{\partial \pi_A}{\partial p_A^+} = \frac{1}{2} + p_A \cdot f_L(x_1^{U-}) \cdot \frac{\partial x_1^{U-}}{\partial p_A^+}.$$

$\frac{\partial x_1^{U+}}{\partial p_A^-} = \frac{\partial x_1^{U-}}{\partial p_A^+}$ always holds, and $f_L(x_1^{U-}) = f_R(x_1^{U+})$ because the density function $f(x)$ is symmetric around $x = \frac{1}{2}$. Therefore, evaluated at symmetry $\frac{\partial \pi_A}{\partial p_A^-} = \frac{\partial \pi_A}{\partial p_A^+}$ holds, and $\frac{\partial \pi_A}{\partial p_A}$ is thus defined.[15]

The signs of SOCs are undetermined without substituting equilibrium prices. Evaluated at the equilibrium prices, we have

$$\frac{\partial^2 \pi_A}{\partial (p_A)^2} = -\frac{1 + \beta + \beta^2}{t(1+\beta)} < 0, \qquad \forall \beta \in [-1, 1].$$

Thus it is a local maximum for firm A. Similarly, for firm B,

$$\frac{\partial^2 \pi_B}{\partial (p_B)^2} = -\frac{1 + 3\beta + \beta^2}{t(1+\beta)}.$$

It is negative only if $\beta > \frac{\sqrt{5}-3}{2} \equiv \underline{\beta}$, which we designate as the lower bound of β.[16] Next, we show that when $\beta \geq \underline{\beta}$, neither firm has an incentive to deviate so our equilibrium candidate is indeed an equilibrium. This is needed because the SOCs above are evaluated at the equilibrium candidate only.

We want to show that neither firm has an incentive to raise or lower price. Due to symmetry, we only rule out firm A's incentive to raise p_A and firm B's incentive to lower p_B. Under both deviations, $x_1 \leq \frac{1}{2}$ holds so the assumed demand structure of the preceding analysis is satisfied. First fix p_B at the equilibrium level ($p_B = p^U$), and allow firm A to raise its price to p_A^{dev}. Denote firm A's deviation profit by $\pi_A(p_A^{dev}, p_B = p^U)$. We find that $\pi_A(p_A^{dev}, p_B = p^U)$ is lower than the equilibrium level for any $p_A^{dev} > p^U$. Thus firm A has no incentive to deviate and raise p_A. Next, fix $p_A = p^U$ and allow firm B to lower its price to p_B^{dev}. We find that its deviation profit $\pi_B(p_A = p^U, p_B^{dev})$ is lower than the equilibrium level for any $p_B^{dev} < p^U$. Thus firm B has no incentive to deviate and lower p_B.■

Proof of Lemma 2

[15] This is not true for second derivative, in particular, $\frac{\partial^2 \pi_A}{\partial (p_A^-)^2} \neq \frac{\partial^2 \pi_A}{\partial (p_A^+)^2}$ at symmetry ($x_1^U = \frac{1}{2}$). Later on we derive expressions for $\frac{\partial^2 \pi_A}{\partial p_A^2}$ and $\frac{\partial^2 \pi_B}{\partial p_B^2}$, which are valid only when $x_1^U < \frac{1}{2}$. When evaluated at $x_1^U = \frac{1}{2}$, they are only directional directives $\frac{\partial^2 \pi_A}{\partial (p_A^+)^2}$ and $\frac{\partial^2 \pi_B}{\partial (p_B^-)^2}$, to be consistent with $x_1 \leq \frac{1}{2}$. Moreover, $\frac{\partial^2 \pi_A}{\partial (p_A^+)^2} \neq \frac{\partial^2 \pi_B}{\partial (p_B^-)^2}$ at $x_1^U = \frac{1}{2}$.

[16] There may exist asymmetric equilibria when $\beta < \underline{\beta}$. But exploring the asymmetric equilibria is beyond the scope of this paper.

Let x_1 denote the marginal consumer in period 1. That is, consumers to the left of x_1 (firm A's turf) buy from firm A in period 1, while consumers to the right of x_1 (firm B's turf) buy from firm B. Without loss of generality, assume that $x_1 \leq \frac{1}{2}$.

In the second period, each firm charges different prices on these two turfs. Let x_A and x_B denote the marginal consumers on firm A's and B' turf respectively, characterized by,

$$x_A = \frac{1}{2} + \frac{p_B^n - p_A^o}{2t}, \qquad x_B = \frac{1}{2} + \frac{p_B^o - p_A^n}{2t}.$$

Since the density function $f(x)$ takes different functional form depending on whether x is above or below $\frac{1}{2}$, we need to determine whether x_A and x_B are above or below $\frac{1}{2}$. First note that $x_A < x_1 \leq \frac{1}{2}$. Next, if we assume $x_B \leq \frac{1}{2}$, then the "equilibrium" second period prices lead to $x_B > \frac{1}{2}$, a violation. Thus $x_B > \frac{1}{2}$ must hold.

Let p_i^j denote the price which firm $i = A, B$ charges to its $j = o, n$ (old, new) group of consumers. The corresponding demand q_i^j can be calculated as

$$q_A^o = \int_0^{x_A} f(x)dx, \quad q_B^n = F(x_1) - q_A^o, \quad q_A^n = 1 - F(x_1) - q_B^o, \quad q_B^o = \int_{x_B}^1 f(x)dx.$$

Calculating the profit $\pi_i^j = p_i^j q_i^j$ and solving the FOCs, we can obtain equilibrium prices in firm B's turf,

$$p_A^n = \frac{t}{8}\left(3\sqrt{9 - 16x_1^2} - 5\right), \qquad p_B^o = \frac{t}{8}\left(\sqrt{9 - 16x_1^2} + 1\right), \quad \forall x_1 \in \left[0, \frac{1}{2}\right].$$

In firm A's turf, however, equilibrium prices have different functional forms depending on whether x_1 is above a threshold level. In particular,

$$p_A^o = t(1 - 2x_1), \qquad p_B^n = 0, \quad \text{if } x_1 \leq \frac{1}{3},$$

$$p_A^o = \frac{t}{8}\left(1 + \sqrt{1 + 16x_1^2}\right), \qquad p_B^n = \frac{t}{8}\left(3\sqrt{1 + 16x_1^2} - 5\right), \quad \text{if } x_1 \in \left(\frac{1}{3}, \frac{1}{2}\right].$$

The case of $x_1 < \frac{1}{3}$ is needed when we check firms' incentive to deviate from the equilibrium. We focus on the case of $x_1 \in \left(\frac{1}{3}, \frac{1}{2}\right]$ since we look for the symmetric equilibrium ($x_1 = \frac{1}{2}$), and the corresponding period 2 profits are,

$$\pi_A^2 = \frac{t}{256}\left((16x_1^2 + 1)^{\frac{3}{2}} - 3(9 - 16x_1^2)^{\frac{3}{2}} - 192\sqrt{9 - 16x_1^2}x_1^2 + 384x_1^2 + 3\sqrt{16x_1^2 + 1} + 103\sqrt{9 - 16x_1^2} - 160\right),$$

$$\pi_B^2 = \frac{t}{256}\left(192\sqrt{16x_1^2 + 1}x_1^2 - 3(16x_1^2 + 1)^{\frac{3}{2}} + (9 - 16x_1^2)^{\frac{3}{2}} - 384x_1^2 + 7\sqrt{16x_1^2 + 1} + 3\sqrt{9 - 16x_1^2} + 32\right).$$

■

Proof of Proposition 2

Consider firm A's FOC when maximizing overall profit with respect to first-period price,

$$F(x_1) + p_A^1 f(x_1)\frac{\partial x_1}{\partial p_A^1} + \delta\left[\frac{\pi_A^o}{\partial p_B^n}\frac{\partial p_B^n}{\partial x_1} + \frac{\pi_A^n}{\partial p_B^o}\frac{\partial p_B^o}{\partial x_1} + \frac{\pi_A^n}{\partial x_1}\right]\frac{\partial x_1}{\partial p_A^1} = 0.$$

Under symmetry, $F(x_1) = \frac{1}{2}$. Then

$$\begin{aligned}
\frac{1}{2} + p_A^1 f(x_1)\frac{\partial x_1}{\partial p_A^1} + \delta\underbrace{\left[\frac{\pi_A^o}{\partial p_B^n}\frac{\partial p_B^n}{\partial x_1} + \frac{\pi_A^n}{\partial p_B^o}\frac{\partial p_B^o}{\partial x_1} + \frac{\pi_A^n}{\partial x_1}\right]}_{\Lambda}\frac{\partial x_1}{\partial p_A^1} &= 0 \\
\Rightarrow \quad \frac{1}{2} + \frac{\partial x_1}{\partial p_A^1}\left[p_A^1 f(x_1) + \delta\Lambda\right] &= 0 \\
\Rightarrow \quad \frac{1}{2} + \frac{\partial x_1}{\partial p_A^1}\left[p_1^{BBPD} f(x_1) + \delta\Lambda\right] &= 0,
\end{aligned}$$

where p_1^{BBPD} denotes the optimal p_A^1 under BBPD.

Note that under BBPD, a change in p_A^1 affects x_1 which in turn changes second period prices. Forward-looking consumers take this into account. This strategic thinking helps determine how x_1 responds to a change in p_A^1, and is reflected in the term $\frac{\partial x_1}{\partial p_A^1}$. We can define a new term $\widehat{\frac{\partial x_1}{\partial p_A^1}}$ which ignores this strategic thinking. $\widehat{\frac{\partial x_1}{\partial p_A^1}}$ can be calculated for uniform pricing and BBPD respectively, and they must be the same.

Under uniform pricing, with symmetry, we have

$$\frac{1}{2} + p_1^U \cdot f(x_1)\frac{\partial x_1}{\partial p_A^1} = 0,$$

where p_1^U denotes the optimal p_A^1 under uniform pricing.

Under uniform pricing, x_1 has no impact on second period prices so $\frac{\partial x_1}{\partial p_A^1} = \widehat{\frac{\partial x_1}{\partial p_A^1}}$. Then

$$\frac{1}{2} = -p_1^U f(x_1)\widehat{\frac{\partial x_1}{\partial p_A^1}}.$$

Combining the FOCs under uniform pricing and BBPD, we can obtain,

$$-p_1^U f(x_1)\widehat{\frac{\partial x_1}{\partial p_A^1}} + \frac{\partial x_1}{\partial p_A^1}\left[p_1^{BBPD} \cdot f(x_1) + \delta\Lambda\right] = 0$$

$$\Rightarrow p_1^{BBPD} f(x_1) + \delta\Lambda = \frac{p_1^U f(x_1)\widehat{\frac{\partial x_1}{\partial p_A^1}}}{\frac{\partial x_1}{\partial p_A^1}}$$

$$\Rightarrow p_1^{BBPD} = \underbrace{\frac{\widehat{\frac{\partial x_1}{\partial p_A^1}}}{\frac{\partial x_1}{\partial p_A^1}}}_{\text{consumer-side effect}} \times \; p_1^U \underbrace{- \delta\frac{\Lambda}{f(x_1)}}_{\text{firm-side effect}}.$$

From the above equation, the impact of BBPD on first period prices can be decomposed into a consumer-side effect and a separate firm-side effect.■

Proof of Proposition 3

Earlier we have solved the game in period 2. Next, we move on to period 1. Consider the marginal consumer, located at x_1. If she purchases from firm A, she will be poached by firm B in period 2, and enjoy an overall utility of

$$u_A(x_1) = u_{1A} + \delta \cdot u_{2B} = (v - p_A^1 - tx_1) + \delta(v - p_B^n - t(1 - x_1)).$$

In contrast, if she purchases from firm B in period 1, she will be poached by firm A in period 2 and enjoy an overall utility of

$$u_B(x_1) = u_{1B} + \delta \cdot u_{2A} = (v - p_B^1 - t(1 - x_1)) + \delta(v - p_A^n - tx_1).$$

Since consumer x_1 is indifferent between buying from either firm, it must be that

$$u_A(x_1) = u_B(x_1) \Rightarrow p_B^1 - p_A^1 + t(1 - 2x_1)(1 - \delta) - \frac{3}{8}\delta t \left(\sqrt{16x_1^2 + 1} - \sqrt{9 - 16x_1^2}\right) = 0.$$

Closed form solution for x_1 cannot be obtained. Instead, we use implicit function theorem to solve the FOCs. A change in p_A^1 affects discounted overall profit π_A through 3 channels: (i) directly through p_A^1 on π_A^1; (ii) indirectly through x_1 on π_A^1; (iii) indirectly through x_1 on π_A^2 (needs discounting). Firm A's FOC is given by

$$\frac{\partial \pi_A}{\partial p_A^1} = \frac{\partial \pi_A^1}{\partial p_A^1} + \frac{\partial \pi_A^1}{\partial x_1} \cdot \frac{dx_1}{dp_A^1} + \delta \frac{\partial \pi_A^2}{\partial x_1} \cdot \frac{dx_1}{dp_A^1} = 0,$$

We calculate the three terms in the FOC, and add them up to get firm A's FOC. Solve firm A's FOC and imposing symmetry ($p_A^1 = p_B^1$), we can obtain

$$p_A^1 = p_B^1 = t\left(\frac{1}{2} + \delta \frac{13\sqrt{5} - 25}{20}\right).$$

Due to symmetry ($p_A^1 = p_B^1$), $x_1 = \frac{1}{2}$. Substituting it into period 2 prices and profits, we have

$$p_A^o = p_B^o = \frac{t}{8}\left(1 + \sqrt{5}\right), \quad p_A^n = p_B^n = \frac{t}{8}\left(3\sqrt{5} - 5\right).$$

Firms' period 2 profits are

$$\pi_A^2 = \pi_B^2 = \frac{t}{16}\left(3\sqrt{5} - 4\right),$$

each firm's overall profit is

$$\pi^{BBPD} = p_A^1 \cdot \frac{1}{2} + \delta \pi_A^2 = t\left(\frac{1}{4} + \frac{41\sqrt{5} - 70}{80}\delta\right).$$

Using period 2 equilibrium prices, we can obtain

$$x_A = \frac{1+\sqrt{5}}{8}, \quad x_B = \frac{7-\sqrt{5}}{8}.$$

Overall consumer surplus over the two period can be calculated as,

$$\begin{aligned} CS^{BBPD} &= 2 \times \left(\int_0^{1/2} u_{1A} f(x)dx + \delta \int_0^{x_A} u_{2A} f(x)dx + \delta \int_{x_A}^{1/2} u_{2B} f(x)dx \right) \\ &= (1+\delta)v - t\left(\frac{5}{6} + \frac{59\sqrt{5}}{60}\delta - \frac{31}{24}\delta \right) \end{aligned}$$

The first integration is consumers' utility of buying from firm A in the first period, the second and third integration are these consumers' utility in period 2 when they buy from firm A and B respectively. Multiplying by 2 gives the surplus for all consumers.

Under triangular distribution, demand kinks at $x = \frac{1}{2}$. However, $\frac{\partial \pi_A}{\partial p_A^1}$ continues to be defined at $x_1 = \frac{1}{2}$ (i.e., $\frac{\partial \pi_A}{\partial p_A^{1-}} = \frac{\partial \pi_A}{\partial p_A^{1+}}$) and $\frac{\partial \pi_A}{\partial p_A^1} = \frac{\partial \pi_B}{\partial p_B^1}$, just as in the case of uniform pricing which we discussed in proof of Proposition 1. We also show that neither firm has an incentive to deviate from the equilibrium period 1 prices. To do so, we check how a firm's deviation profit varies with x_1 rather than its period 1 deviation price. With $x_1 \leq \frac{1}{2}$, firm A can raise its price without violating the assumed demand structure. We find that firm A's deviation profit increases with x_1 when $x_1 \leq \frac{1}{2}$. Thus firm A (and by symmetry firm B) has no incentive to raise price. Similarly, with $x_1 \leq \frac{1}{2}$, firm B can lower its price without violating the assumed demand structure. We find that firm B's deviation profit increases with x_1. Thus firm B (and by symmetry firm A) has no incentive to lower price.■

BBPD with general β

We solve the game backwards, starting with the second period.

<u>Second period:</u>

Recall that firms' period 1 prices are p_A^1 and p_B^1, and the marginal consumer is located at $x_1 \leq \frac{1}{2}$. Period 1 demand is given by

$$q_A^1 = F(x_1), \qquad q_B^1 = 1 - F(x_1).$$

In period 2, firm A (B) charges a price of p_A^o (p_B^o) to the consumers locating on firm A's (B's) turf; and a price of p_A^n (p_B^n) to the consumers on its competitor's turf. Let x_A (x_B) denote the marginal consumer on firm A's (B's) turf, which can be calculated as

$$x_A = \frac{1}{2} + \frac{p_B^n - p_A^o}{2}, \qquad x_B = \frac{1}{2} + \frac{p_B^o - p_A^n}{2}.$$

.

Substituting x_A and x_B, we have

$$q_A^o = F(x_A) = \frac{1}{2}(1 + p_B^n - p_A^o)(1 + p_B^n - p_A^o),$$

$$q_B^n = F(x_1) - F(x_A) = \frac{1}{2}(p_A^o - p_B^n + 2x_1 - 1)(1 + 2x_1 + p_B^n - p_A^o),$$

$$q_A^n = F(x_B) - F(x_1) = \frac{1}{2}\left[(p_B^o - p_A^n)(2 + p_A^n - p_B^o) + (1 - 2x_1)(1 + 2x_1)\right],$$

$$q_B^o = 1 - F(x_B) = \frac{1}{2}(p_A^n - p_B^o + 1)(p_A^n - p_B^o + 1).$$

Firm $i = A, B$'s problem is to choose (p_i^o, p_i^n) to maximize $\pi_i^o(p_i^o, p_j^n)$ and $\pi_i^n(p_i^n, p_j^o)$. The four FOCs are,

$$\frac{\partial \pi_A^o}{\partial p_A^o} = F(x_A) + p_A^o \cdot \frac{\partial F(x_A)}{\partial p_A^o} = 0,$$

$$\frac{\partial \pi_B^n}{\partial p_B^n} = F(x_1) - F(x_A) - p_B^n \cdot \frac{\partial (F(x_1) - F(x_A))}{\partial p_B^n} = 0,$$

$$\frac{\partial \pi_A^n}{\partial p_A^n} = F(x_B) - F(x_1) - p_A^n \cdot \frac{\partial (F(x_B) - F(x_1))}{\partial p_A^n} = 0,$$

$$\frac{\partial \pi_B^o}{\partial p_B^o} = 1 - F(x_B) - p_B^o \cdot \frac{\partial F(x_B)}{\partial p_B^o} = 0.$$

We solve $\frac{\partial \pi_i^j}{\partial p_i^j} = 0$ (where $i = A, B,\ j = o, n$) for p_i^j. There are two roots to each FOC, one of which leads to a violation. We pick the other root.[17]

We need to make sure that $p_A^n \geq 0$ and $p_B^n \geq 0$. With $x_1 \leq \frac{1}{2}$, $p_A^n \geq 0$ always holds in firm B's turf. In firm A's turf, however, $x_1 \geq \frac{2\beta - 1 + \sqrt{\beta^2 - \beta + 1}}{6\beta}$ is needed to have $p_B^n \geq 0$. Otherwise firm A's turf is too small that it will drive firm B out of the market completely (with $p_B^n = 0$ and $p_A^o = 1 - 2x_1$). Combining both cases of x_1, we have the following:

(i) Equilibrium second period prices in firm B's turf are given by,

$$p_A^n = \begin{cases} -\frac{1}{16\beta B}\left(40\beta^3 x_1^2 - 20\beta^3 x_1 + 40\beta^2 x_1^2 - 3\beta^3 - 17\beta^2 + 20\beta x_1 + E - 17\beta - 3\right), & \text{if } \beta \in (0, 1], \\ \frac{1}{48\beta B}[H(8\beta^2 x_1^2 - 4\beta^2 x_1 - \beta^2 + 4\beta x_1 - 2\beta - 1) - 112\beta^3 x_1^2 + 56\beta^3 x_1 - 112\beta^2 x_1^2 + 10\beta^3 \\ \quad + 46\beta^2 - 56\beta x_1 + 2D + 46\beta + 10], & \text{if } \beta \in [\underline{\beta}, 0). \end{cases}$$

$$p_B^o = \begin{cases} -\frac{1}{48\beta B}[24\beta^3 x_1 - 48\beta^3 x_1^2 - 48\beta^2 x_1^2 + 10\beta^3 + 14\beta^2 - 24\beta x_1 + 14\beta + 10 \\ \quad + (8\beta^2 x_1^2 - 4\beta^2 x_1 - \beta^2 + 4\beta x_1 - 2\beta - 1)I + 2E], & \text{if } \beta \in (0, 1], \\ \frac{1}{16B}\left(8\beta^3 x_1^2 - 4\beta^3 x_1 + 8\beta^2 x_1^2 - 3\beta^3 - \beta^2 + 4\beta x_1 - \beta - 3 + D\right), & \text{if } \beta \in [\underline{\beta}, 0). \end{cases}$$

[17] For example, when solving $\frac{\partial \pi_A^o}{\partial p_A^o} = 0$ for p_A^o, one of the root leads to $x_A < 0$. We do not report the intermediate results here as they are too lengthy. Maple programs are available upon request.

(ii) In firm A's turf, however, equilibrium prices have different functional forms depending on whether x_1 is above a threshold. In particular,

(ii-a) If $x_1 \in \left[0, \frac{2\beta-1+\sqrt{\beta^2-\beta+1}}{6\beta}\right]$, then

$$p_A^o = 1 - 2x_1, \qquad p_B^n = 0,$$

(ii-b) If $x_1 \in \left(\frac{2\beta-1+\sqrt{\beta^2-\beta+1}}{6\beta}, \frac{1}{2}\right]$, then

$$p_A^o = \begin{cases} -\frac{1}{48\beta A}[G(8\beta^2x_1^2 - 4\beta^2x_1 + \beta^2 + 4\beta x_1 - 2\beta + 1) + 24\beta^3x_1 - 48\beta^3x_1^2 - 48\beta^2x_1^2 - 10\beta^3 \\ \quad +10\beta^2 - 24\beta x_1 - 2C + 10\beta - 10), & \text{if } \beta \in (0,1], \\ \\ \frac{1}{16\beta A}\left(8\beta^3x_1^2 - 4\beta^3x_1 + 8\beta^2x_1^2 + 3\beta^3 - 3\beta^2 + 4\beta x_1 - 3\beta + 3 - F\right), & \text{if } \beta \in [\underline{\beta}, 0). \end{cases}$$

$$p_B^n = \begin{cases} \frac{1}{16\beta A}(20\beta^3x_1 - 40\beta^3x_1^2 - 40\beta^2x_1^2 - 3\beta^3 + 3\beta^2 - 20\beta x_1 + 3\beta - 3 + C), & \text{if } \beta \in (0,1], \\ \frac{1}{48\beta A}(J(8\beta^2x_1^2 - 4\beta^2x_1 + \beta^2 + 4\beta x_1 - 2\beta + 1) - 112\beta^3x_1^2 + 56\beta^3x_1 - 112\beta^2x_1^2 - 10\beta^3 \\ \\ \quad +10\beta^2 - 56\beta x_1 + 10\beta - 10 - 2F), & \text{if } \beta \in [\underline{\beta}, 0), \end{cases}$$

where

$$A = 8\beta^2x_1^2 - 4\beta^2x_1 + \beta^2 + 4\beta x_1 - 2\beta + 1,$$

$$B = 8\beta^2x_1^2 - 4\beta^2x_1 - \beta^2 + 4\beta x_1 - 2\beta - 1,$$

$$C = \sqrt{(64\beta^2x_1^2 - 32\beta^2x_1 + 9\beta^2 + 32\beta x_1 - 14\beta + 9)(24\beta^2x_1^2 - 12\beta^2x_1 + \beta^2 + 12\beta x_1 - 2\beta + 1)^2},$$

$$D = \sqrt{-(64\beta^2x_1^2 - 32\beta^2x_1 - 9\beta^2 + 32\beta x_1 - 18\beta - 9)(8\beta^2x_1^2 - 4\beta^2x_1 + \beta^2 + 4\beta x_1 - 6\beta + 1)^2},$$

$$E = \sqrt{-(64\beta^2x_1^2 - 32\beta^2x_1 - 9\beta^2 + 32\beta x_1 - 18\beta - 9)(24\beta^2x_1^2 - 12\beta^2x_1 - \beta^2 + 12\beta x_1 - 10\beta - 1)^2},$$

$$F = \sqrt{(64\beta^2x_1^2 - 32\beta^2x_1 + 9\beta^2 + 32\beta x_1 - 14\beta + 9)(8\beta^2x_1^2 - 4\beta^2x_1 - \beta^2 + 4\beta x_1 + 2\beta - 1)^2},$$

$$\begin{aligned} G &= (\frac{2}{A^2}(113 - 626\beta + 952\beta x_1 + 113\beta^6 + 18432\beta^6x_1^6 - 27648\beta^6x_1^5 + 24384\beta^6x_1^4 + 27648\beta^5x_1^5 \\ &- 12864\beta^6x_1^3 + 4544\beta^6x_1^2 - 952\beta^6x_1 - 626\beta^5 + 1487\beta^4 - 1948\beta^3 + 24384\beta^4x_1^4 - 46464\beta^5x_1^4 \\ &+ 36288\beta^5x_1^3 - 17024\beta^5x_1^2 - 36288\beta^4x_1^3 + 4472\beta^5x_1 + 24960\beta^4x_1^2 - 8656\beta^4x_1 + 12864\beta^3x_1^3 \\ &- 17024\beta^3x_1^2 + 8656\beta^3x_1 - 4472\beta^2x_1 + 4454\beta^2x_1^2 + 1487\beta^2 \\ &+ (5\beta^3 - 5\beta^2 - 5\beta + 5 - 12\beta^3x_1 + 24\beta^2x_1^2 + 12\beta x_1 + 24\beta^3x_1^2)C))^{1/2}, \end{aligned}$$

$$
\begin{aligned}
H &= (-\frac{2}{B^2}(-113 - 1030\beta + 1656\beta x_1 - 113\beta^6 + 100352\beta^6 x_1^6 - 150528\beta^6 x_1^5 + 43200\beta^6 x_1^4 \\
&+ 150528\beta^5 x_1^5 + 19520\beta^6 x_1^3 - 4704\beta^6 x_1^2 - 1656\beta^6 x_1 - 1030\beta^5 - 3423\beta^4 - 5012\beta^3 + 43200\beta^4 x_1^4 \\
&- 243328\beta^5 x_1^4 + 98368\beta^5 x_1^3 + 12736\beta^5 x_1^2 - 98368\beta^4 x_1^3 - 8296\beta^5 x_1 + 65600\beta^4 x_1^2 - 7664\beta^4 x_1 \\
&- 19520\beta^3 x_1^3 + 12736\beta^3 x_1^2 + 7664\beta^3 x_1 + 8296\beta^2 x_1 - 4704\beta^2 x_1^2 - 3423\beta^2) \\
&+ (56\beta^2 x_1^2 - 5\beta^3 - 23\beta^2 - 23\beta - 5 + 56\beta^3 x_1^2 - 28\beta^3 x_1 + 28\beta x_1)D)^{1/2},
\end{aligned}
$$

$$
\begin{aligned}
I &= (-\frac{2}{B^2}(-113 - 326\beta + 952\beta x_1 + 6592\beta^3 x_1^2 + 880\beta^3 x_1 + 808\beta^2 x_1 - 736\beta^2 x_1^2 - 607\beta^2 - 113\beta^6 \\
&+ 18432\beta^6 x_1^6 - 27648\beta^6 x_1^5 + 3264\beta^6 x_1^4 + 27648\beta^5 x_1^5 + 8256\beta^6 x_1^3 - 736\beta^6 x_1^2 - 952\beta^6 x_1 + 3264\beta^4 x_1^4 \\
&- 36480\beta^5 x_1^4 + 5184\beta^5 x_1^3 + 6592\beta^5 x_1^2 - 5184\beta^4 x_1^3 - 808\beta^5 x_1 + 4416\beta^4 x_1^2 - 880\beta^4 x_1 - 326\beta^5 \\
&- 607\beta^4 - 788\beta^3 - 8256\beta^3 x_1^3 + (24\beta^2 x_1^2 + 12\beta x_1 + 24\beta^3 x_1^2 - 7\beta - 5\beta^3 - 7\beta^2 - 5 - 12\beta^3 x_1)E))^{1/2},
\end{aligned}
$$

$$
\begin{aligned}
J &= (\frac{2}{A^2}(113 - 626\beta + 1656\beta x_1 + 11328\beta^2 x_1^2 - 7736\beta^2 x_1 - 150528\beta^6 x_1^5 + 100352\beta^6 x_1^6 + 150528\beta^5 x_1^5 \\
&+ 113\beta^6 - 44608\beta^6 x_1^3 + 11328\beta^6 x_1^2 - 1656\beta^6 x_1 + 7736\beta^5 x_1 + 107328\beta^6 x_1^4 - 626\beta^5 \\
&+ 1487\beta^4 - 1948\beta^3 - 208256\beta^5 x_1^4 + 127424\beta^5 x_1^3 - 42624\beta^5 x_1^2 - 127424\beta^4 x_1^3 + 62592\beta^4 x_1^2 \\
&+ 107328\beta^4 x_1^4 - 14928\beta^4 x_1 + 44608\beta^3 x_1^3 + 14928\beta^3 x_1 - 42624\beta^3 x_1^2 + 1487\beta^2 \\
&+ (5\beta^3 - 5\beta^2 - 5\beta + 5 + 56\beta^2 x_1^2 + 28\beta x_1 + 56\beta^3 x_1^2 - 28\beta^3 x_1)F)^{1/2}.
\end{aligned}
$$

We also verify that the above prices constitute an equilibrium, by showing that in each firm's turf, no firm has an incentive to deviate unilaterally.

First period:

Due to the complexity of the distribution function, closed form solution for period 1 marginal consumer x_1 cannot be obtained. Instead, we rely on implicity function theorem to solve the FOCs.

Firm $i = A, B$'s first period profit is,

$$\pi_i^1 = p_i^1 \cdot q_i^1,$$

and their overall discounted profit π_i is

$$\pi_i = \pi_i^1 + \delta\pi_i^2 = \pi_i^1 + \delta(\pi_i^o + \pi_i^n), \quad i = A, B.$$

A change in p_A^1 affects π_A through 3 channels: (i) directly through p_A^1 on π_A^1; (ii) indirectly through x_1 on π_A^1; (iii) indirectly through x_1 on π_A^2 (needs discounting). Firm A's FOC is given by

$$\frac{\partial \pi_A}{\partial p_A^1} = \frac{\partial \pi_A^1}{\partial p_A^1} + \frac{\partial \pi_A^1}{\partial x_1} \cdot \frac{dx_1}{dp_A^1} + \delta \frac{\partial \pi_A^2}{\partial x_1} \cdot \frac{dx_1}{dp_A^1} = 0.$$

We calculate the three terms in firm A's FOC, and then add them up. Solving the FOC and imposing symmetry ($p_A^1 = p_B^1$), we can obtain,

$$
\begin{aligned}
p_A^1 \;=\;& \frac{-1}{288KL\beta(\beta+1)(\beta^2+1)^4} \times ((8278\beta^{16}\delta + 42232\beta^{15}\delta - 92472\beta^{14}\delta + 265160\beta^{13}\delta - 432648\beta^{12}\delta \\
+\;& 614744\beta^{11}\delta - 833480\beta^{10}\delta + 842728\beta^{9}\delta - 994972\beta^{8}\delta + 842728\beta^{7}\delta - 833480\beta^{6}\delta + 614744\beta^{5}\delta \\
-\;& 432648\beta^{4}\delta + 265160\beta^{3}\delta - 92472\beta^{2}\delta + 42232\beta\delta - 32544\beta^{15} + 43200\beta^{14} - 241632\beta^{13} + 249984\beta^{12} \\
-\;& 752544\beta^{11} + 611136\beta^{10} - 1277280\beta^{9} + 808704\beta^{8} - 1277280\beta^{7} + 611136\beta^{6} - 752544\beta^{5} + 249984\beta^{4} \\
-\;& 241632\beta^{3} + 43200\beta^{2} - 32544\beta + 8278\delta)K - 41218\beta^{19}\delta - 354402\beta^{18}\delta - 681914\beta^{17}\delta - 900090\beta^{16}\delta \\
-\;& 3922656\beta^{15}\delta - 1195392\beta^{14}\delta - 9649104\beta^{13}\delta - 3195888\beta^{12}\delta - 12327988\beta^{11}\delta - 8539796\beta^{10}\delta \\
-\;& 8539796\beta^{9}\delta - 12327988\beta^{8}\delta - 3195888\beta^{7}\delta - 9649104\beta^{6}\delta - 1195392\beta^{5}\delta - 3922656\beta^{4}\delta - 900090\beta^{3}\delta \\
-\;& 681914\beta^{2}\delta - 354402\beta\delta - 12960\beta^{18} - 109152\beta^{17} - 345024\beta^{16} - 783936\beta^{15} - 1732032\beta^{14} \\
-\;& 2525760\beta^{13} - 4084416\beta^{12} - 4739904\beta^{11} - 5573376\beta^{10} - 5573376\beta^{9} - 4739904\beta^{8} - 4084416\beta^{7} \\
-\;& 2525760\beta^{6} - 1732032\beta^{5} - 783936\beta^{4} - 345024\beta^{3} - 109152\beta^{2} - 12960\beta - 41218\delta \\
+\;& ((201\beta^{15} + 807\beta^{14} + 157\beta^{13} + 4019\beta^{12} + 1329\beta^{11} + 7999\beta^{10} + 4861\beta^{9} + 8275\beta^{8} + 8275\beta^{7} \\
+\;& 4861\beta^{6} + 7999\beta^{5} + 1329\beta^{4} + 4019\beta^{3} + 157\beta^{2} + 807\beta + 201)K + 421\beta^{18} + 988\beta^{17} + 3113\beta^{16} \\
+\;& 21520\beta^{15} + 19896\beta^{14} + 64816\beta^{13} + 75696\beta^{12} + 102896\beta^{11} + 149706\beta^{10} + 117224\beta^{9} + 149706\beta^{8} \\
+\;& 102896\beta^{7} + 75696\beta^{6} + 64816\beta^{5} + 19896\beta^{4} + 21520\beta^{3} + 3113\beta^{2} + 988\beta + 421)\frac{\sqrt{2}\delta}{\beta^2+1}L^{1/2})
\end{aligned}
$$

if $\beta \in (0, 1]$,[18]

where

$$K = (\beta^2 + 4\beta + 1)\sqrt{9\beta^2 + 2\beta + 9},$$

$$L = 113\beta^6 - 150\beta^5 + 387\beta^4 - 268\beta^3 + 387\beta^2 - 150\beta + 113 + (5\beta^3 + \beta^2 + \beta + 5)K,$$

[18] Note that when $\beta \to 0$, the results converge to case of uniform distribution.

and

$$
\begin{aligned}
p_A^1 &= \frac{1}{288\beta MN(\beta+1)(\beta^2+1)^4} \times ((32544\beta^{15} + 58176\beta^{14} + 260064\beta^{13} + 358272\beta^{12} + 844704\beta^{11} \\
&+ 909504\beta^{10} + 1461600\beta^9 + 1218816\beta^8 + 1461600\beta^7 + 909504\beta^6 + 844704\beta^5 + 358272\beta^4 + 260064\beta^3 \\
&+ 58176\beta^2 + 32544\beta - 11802\beta^{16}\delta - 79656\beta^{15}\delta - 206136\beta^{14}\delta - 569112\beta^{13}\delta - 935240\beta^{12}\delta \\
&- 1749832\beta^{11}\delta - 2166728\beta^{10}\delta - 3014766\beta^9\delta - 2859836\beta^8\delta - 3014776\beta^7\delta - 2166728\beta^6\delta - 1749832\beta^5\delta \\
&- 935240\beta^4\delta - 569112\beta^3\delta - 206136\beta^2\delta - 79656\beta\delta - 11802\delta)M + 6734\beta^{19}\delta + 56222\beta^{18}\delta - 99450\beta^{17}\delta \\
&- 210090\beta^{16}\delta - 1098144\beta^{15}\delta - 1886784\beta^{14}\delta - 3962640\beta^{13}\delta - 5796528\beta^{12}\delta - 8210772\beta^{11}\delta \\
&- 9448916\beta^{10}\delta - 9448916\beta^9\delta - 8210772\beta^8\delta - 5796528\beta^7\delta - 3962640\beta^6\delta - 1886784\beta^5\delta - 1098144\beta^4\delta \\
&- 210090\beta^3\delta - 99450\beta^2\delta + 56222\beta\delta + 6734\delta + 12960\beta^{18} - 77472\beta^{17} + 116928\beta^{16} - 232128\beta^{15} \\
&+ 582336\beta^{14} + 90432\beta^{13} + 1497024\beta^{12} + 1198656\beta^{11} + 1972224\beta^{10} + 1972224\beta^9 \\
&+ 1198656\beta^8 + 1497024\beta^7 + 90432\beta^6 + 582336\beta^5 - 232128\beta^4 + 116928\beta^3 - 77472\beta^2 + 12960\beta \\
&+ (133\beta^{18} - 2700\beta^{17} + 585\beta^{16} + 10608\beta^{15} + 34296\beta^{14} + 77200\beta^{13} + 110960\beta^{12} + 142480\beta^{11} \\
&+ 160202\beta^{10} + 157176\beta^9 + 160202\beta^8 + 142480\beta^7 + 110960\beta^6 + 77200\beta^5 + 34296\beta^4 + 10608\beta^3 \\
&+ 585\beta^2 - 2700\beta + 133 + (553\beta^{15} + 4359\beta^{14} + 10557\beta^{13} + 28499\beta^{12} + 44561\beta^{11} \\
&+ 73247\beta^{10} + 88477\beta^9 + 103027\beta^8 + 103027\beta^7 + 88477\beta^6 + 73247\beta^5 + 44561\beta^4 + 28499\beta^3 + 10557\beta^2 \\
&+ 4359\beta + 553)M)\frac{\sqrt{2}\delta}{\beta^2+1}N^{1/2}),
\end{aligned}
$$

if $\beta \in (\underline{\beta}, 0)$, *where*

$$M = (\beta^2 - 4\beta + 1)\sqrt{9\beta^2 + 2\beta + 9},$$

$$N = 113\beta^6 + 202\beta^5 + 451\beta^4 + 436\beta^3 + 451\beta^2 + 202\beta + 113 + (5\beta^3 + 9\beta^2 + 9\beta + 5)M.$$

With symmetry, $p_B^1 = p_A^1$ and $x_1 = \frac{1}{2}$, which allows us to calculate first period profit. Substituting $x_1 = \frac{1}{2}$ into the second period price expressions, we can obtain the second period equilibrium prices and in turn profits.

Similar to the case of $\beta = 1$, we verify that no firm has an incentive to deviate unilaterally. Instead of checking how a deviating firm's profit varies with its price, we check how its deviation profit varies with x_1.[19] We first look at firm A, which can raise its price without violating the assumed demand structure $x_1 \leq \frac{1}{2}$. We find that firm A's profit increases with x_1 when $x_1 \leq \frac{1}{2}$. Thus firm A (and by symmetry firm B) has no incentive to raise price. Next, consider firm B, which can lower its price without violating the assumed demand structure $x_1 \leq \frac{1}{2}$. We find that firm B's profit increases with x_1 when $x_1 \leq \frac{1}{2}$. Thus firm B (and by symmetry firm A) has no incentive to lower its price.■

[19] The reason is that because we cannot get a closed form solution for x_1, the deviating firm's profit cannot be written explicitly as a function of its first period price.

Proof of Lemma 3

(i) is obvious. Given that $x_1 = \frac{1}{2}$, δ does not enter into second period pricing decisions.

(ii) While the p_1^{BBPD} expressions are lengthy, using software we can easily verify that $p_1^{BBPD} - p_1^U$ is proportional to δ and the coefficient is a function of β only, which we denote as $g(\beta)$.

(iii) We start with social surplus. Under inelastic demand, social surplus is determined by total transport cost. In particular, in period 2 under BBPD, consumers in $[x_A, 1/2]$ (and by symmetry those in $(1/2, x_B]$) purchase from the firm which is further away, leading to efficiency loss relative to uniform pricing. Therefore,

$$\begin{aligned} SS^{BBPD} - SS^U &= \delta \cdot \left[-2 \cdot \int_{x_A}^{1/2} [t(1-x) - tx] \cdot f(x) dx \right], \\ &= \delta \cdot h(x), \end{aligned}$$

where $f(x) = 4\beta x + (1-\beta)$ is the density function, and the marginal consumer x_A is linear in second period prices which depend on β but not on δ, after we normalize $t = 1$. There is a δ in $SS^{BBPD} - SS^U$ because the transport cost difference occurs in the second period only.

Next, we consider overall profits, which are given by

$$\pi^{BBPD} = \frac{p_1^{BBPD}}{2} + \delta \cdot \pi_2^{BBPD}, \quad \pi^U = \frac{p_1^U}{2}(1+\delta).$$

Then

$$\begin{aligned} \pi^{BBPD} - \pi^U &= \frac{1}{2}\left[p_1^U + g(\beta)\delta\right] + \delta \cdot \pi_2^{BBPD} - \frac{p_1^U}{2}(1+\delta) \\ &= \delta \left[\frac{g(\beta)}{2} + \pi_2^{BBPD} - \frac{p_1^U}{2} \right]. \end{aligned}$$

The term in the square bracket is a function of β only, because of each of its 3 components is a function of β only. Then BBPD raises (lowers) overall profits if and only if this term is positive (negative), independent of δ.

Next, we consider consumer surplus. Note that

$$\begin{aligned} CS^{BBPD} - CS^U &= \left(SS^{BBPD} - \pi^{BBPD}\right) - \left(SS^U - \pi^U\right) \\ &= \left(SS^{BBPD} - SS^U\right) - \left(\pi^{BBPD} - \pi^U\right). \end{aligned}$$

Since both $SS^{BBPD} - SS^U$ and $\pi^{BBPD} - \pi^U$ are proportional to δ, $CS^{BBPD} - CS^U$ must be proportional to δ as well.■

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