MESH GENERATION AND REFINEMENT IN 2D MODELLING. A CASE STUDY

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ABSTRACT

Ria de Arosa is a bay located in the western region of Galicia-Spain, and is a well-known producer of commercially valuable shellfish, especially by raft culture of the edible mussel. Due to the ever increasing of sediments concentration in the water column, mussel productivity has been lowered in the past years. A research project on information systems for water quality assessment in that coastal zone is being developed at the University of Minho for decision making support.

Two numerical solvers of the shallow water equations based on the finite element method (FEM) were used for hydrodynamic simulations. This paper describes the general framework to generate meshes for simulations and presents the principal characteristics of a mesh generator that produces exact Delaunay triangulations, constrained Delaunay triangulations, and quality conforming Delaunay triangulations. A method for conditioned mesh refinement is also presented, where several area constraint criteria related to water depth, velocity gradients, and pollutant concentration gradients can be established.

KEYWORDS

Hydroinformatics, mathematical modelling, mesh generation, mesh refinement, Ria de Arosa, triangulations, water quality.

INTRODUCTION

The *Rias Bajas* are oceanic bays located on the Western coast of Galicia-Spain. These coastal zones, support high shellfish production by intensive raft culture of the edible mussel *Mytilus edulis* due to intermittent upwelling of nutrient-rich deep waters phenomena. Ria de Arosa, the largest of those *rias*, has a surface area of 230 km^2 and is 25 km long approximately. It is 67 m deep at the mouth and averages of 19 m depth. Mean tidal range is 2.7 m. Two rivers, Ulla and Umia, have discharge flows into the Ria de Arosa of about 70 to 120 m^3/s in winter and 5 to 75 m^3/s in summer.

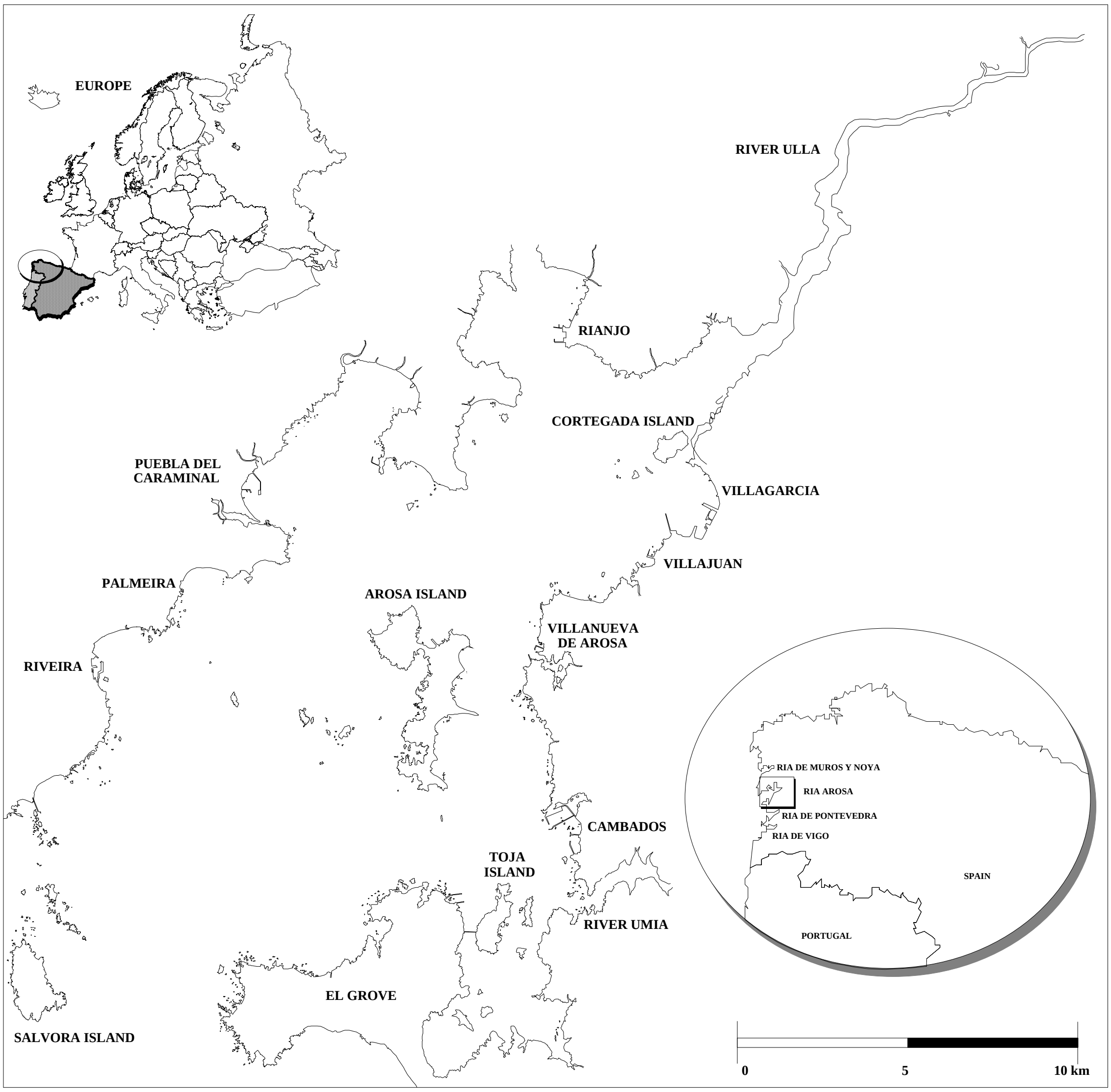


Figure 1: Location and general layout of the Ria de Arosa

Seasonal variations in salinity and temperature are negligible. Annual surface-water temperature ranges from 10 to 20 ºC and bottom temperature have small variability (11 to 13 ºC). Surface salinity ranges from 30 to 35 ‰, and bottom salinity remains constant at 35.6 ‰. The water column is stratified in summer and mixed or weakly stratified at other times (Gomez-Gallego, 1971 and 1975).

Special efforts have been done in the application of FEM models to solve both hydrodynamics and biogeochemical variables distribution in that bay. For successful application of this type of models pre-processing phase is of crucial importance. In this phase special care must be taken with the mesh generation process. The approximation error is affected by the following factors: the size of the largest element, the largest angle in the triangulation, the accuracy of the approximation to the geometry, and the ability of the function space to represent singularities and discontinuities in the solution function. Different mesh generation techniques have been used in the past, ranging from simple to complex. Many of the early mesh generation techniques required time consuming manual labour or they were strict application-specific.

With new computational capacities, general techniques and rules have emerged (Bern, M. and Eppstein, D., 1992).

Flexibility in fitting complicated domains, rapid grading from small to large elements and relatively easy refinement and derefinement are advantages of unstructured meshes, that make this kind of meshes well suitable for hydrodynamics and water quality models in coastal zones.

METHODS

Hydroinformatic environment

The Ria de Arosa hydrodynamics and water quality model was implemented using both Telemac2D (EDF, 1994) and RMA2-RMA4 (US WES-HL, 1996). Pre and post-processing work were carried out using several informatic tools, namely SMS (BOSS SMS, 1996), Geographical Information System (GIS), data bases with models results output, and field surveys data coupled as shown in Figure 2.

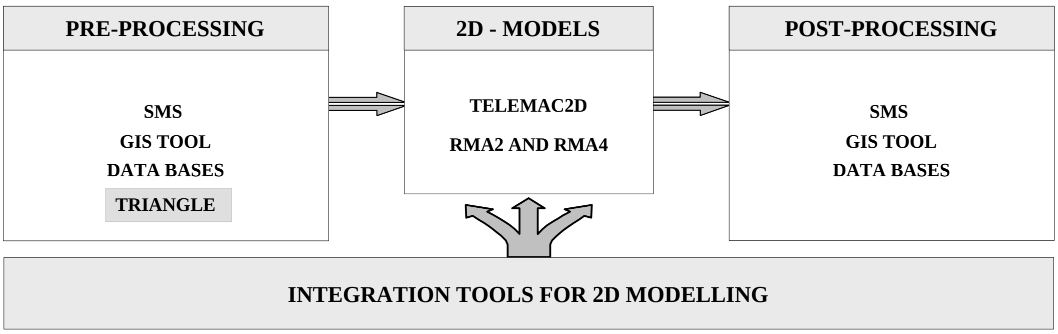


Figure 2: 2D hydrodynamic and water quality problems modelling tools

Linking and exchange information between programmes associated to different stages of the modelling process have been developed using several tools (Pinho, J.L.S. et al., 1998). In order to complement and improve the general capabilities of the above pre-processing programs the mesh generator TRIANGLE (Shewchuk, J. R., 1997) was added to the hydroinformatic environment.

Mesh generation

General considerations

The shapes of elements in a mesh have a pronounced effect on numerical methods. An usual measure of the shape of an element is its aspect ratio defined as the length/with ratio. Large aspect ratios lead to poorly conditioned matrices, worsening the speed and accuracy of the linear solver. A triangular mesh formed by elements with small interior angles can be noticeably slower than a triangular mesh with a minimum angle of 45 º. Moreover, even assuming that the solver gives an exact answer, large aspect ratios may give unacceptable interpolation error. The following rules can be established for mesh elements: restrict element shapes to undistorted triangles or rectangles; create elements with no small corner angles (greater than 20º); and limit the maximum element slope.

Among two-dimensional triangular mesh generators techniques, Delaunay triangulation is the most popular. The Delaunay triangulation *D* of a set of vertices *V*, introduced by Delaunay, is a graph defined as follows: in two dimensions, a triangulation of a set *V* of vertices is a set *T* of triangles whose vertices collectively form *V*, whose interiors do not intersect each other, and whose union completely fills the convex hull of *V*. Any circle in the plane is said to be empty if it contains no vertex of *V* in its interior (vertices allowed on the circle). Let *u* and *v* be any two vertices of *V*. The edge *uv* is in *D* if and only if there exists an empty circle that passes trough *u* and *v*. An edge satisfying this property is said to be Delaunay. The constrained Delaunay triangulation of a domain defined by his boundary (planar polygonal line) is similar to the Delaunay triangulation, but every input segment appears as an edge of the triangulation.

There are many Delaunay triangulation algorithms, some of which were surveyed and evaluated in the past. These evaluations indicate a rough parity in speed among the incremental insertion algorithm, the divide-and-conquer algorithm and the plane-sweep algorithm. Triangle is an implementation of the above methods and of Ruppert's Delaunay refinement algorithm (Ruppert, J. 1995). This algorithm has four main stages: the first stage is to find the Delaunay triangulation of input vertices belonging to the domain boundary; the second stage is to simply use a constrained Delaunay triangulation; the third stage is to remove triangles from islands or concavities of the domain (this stage is not considered in the Ruppert's algorithm); and the last stage, which is the core of the implementation, refines the mesh by inserting additional vertices into the mesh until all constraints on minimum angle and maximum triangle area are met.

Before designing the mesh, considerations should be given to the purpose of the study. Additional work is required in order to obtain good mesh properties. The resolution requirements for a hydrodynamic study must also consider the end use of the resolution. Resolution is not only a matter of distances within the study area, but also an issue of supporting post-hydrodynamic purposes.

Phases of mesh generation

Adopted methodology for 2D mesh generation in this case study includes four main phases as represented in Figure 3.

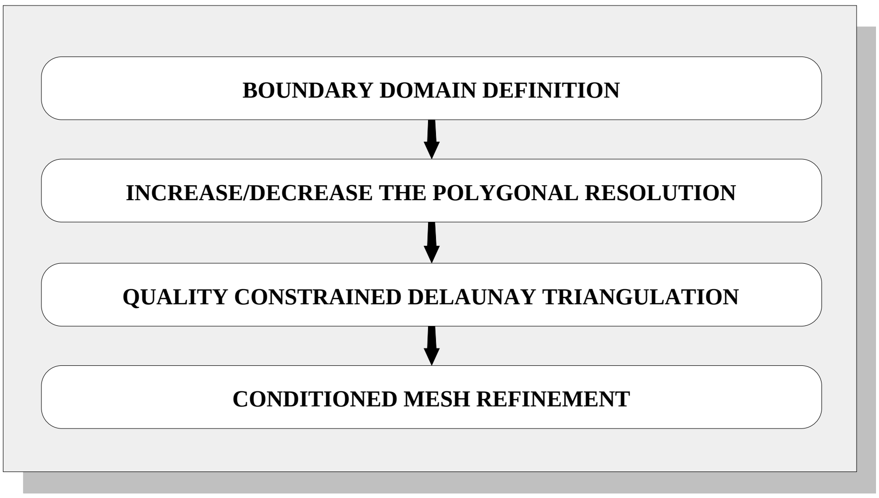


Figure 3: Main phases of the mesh generation process

Assessment on environmental problems in coastal zones recurring to the implementation of numerical models, based on FEM, begins with domain representation and simplification. Two main properties of the numerical model must be established at this phase: the number (or size) and the shape of elements in the mesh (according to the hardware/software capabilities and the desired space resolution. When a constrained Delaunay is used, these properties are related to the resolution (number of vertices) of the boundary polygonal line.

A high resolution of the boundary polygonal will result on an over refinement of the mesh near the boundary and high total number of elements; a low resolution will result in a coarse mesh. Thus, special care must be put on the resolution of the boundary polygonal line. For a regular mesh, composed of equilateral triangles we can estimate the average distance between vertices of the boundary polygonal line, assuming that wall the elements will be of the same size.

In order to manipulate the resolution of the boundary polygonal lines several tools were developed. Three meshes generated with constraints in the minimum interior angle (20º) and on the maximum element area (125000 m^2) are presented in Figure 4. These meshes are associated with different resolutions of the boundary polygonal line.

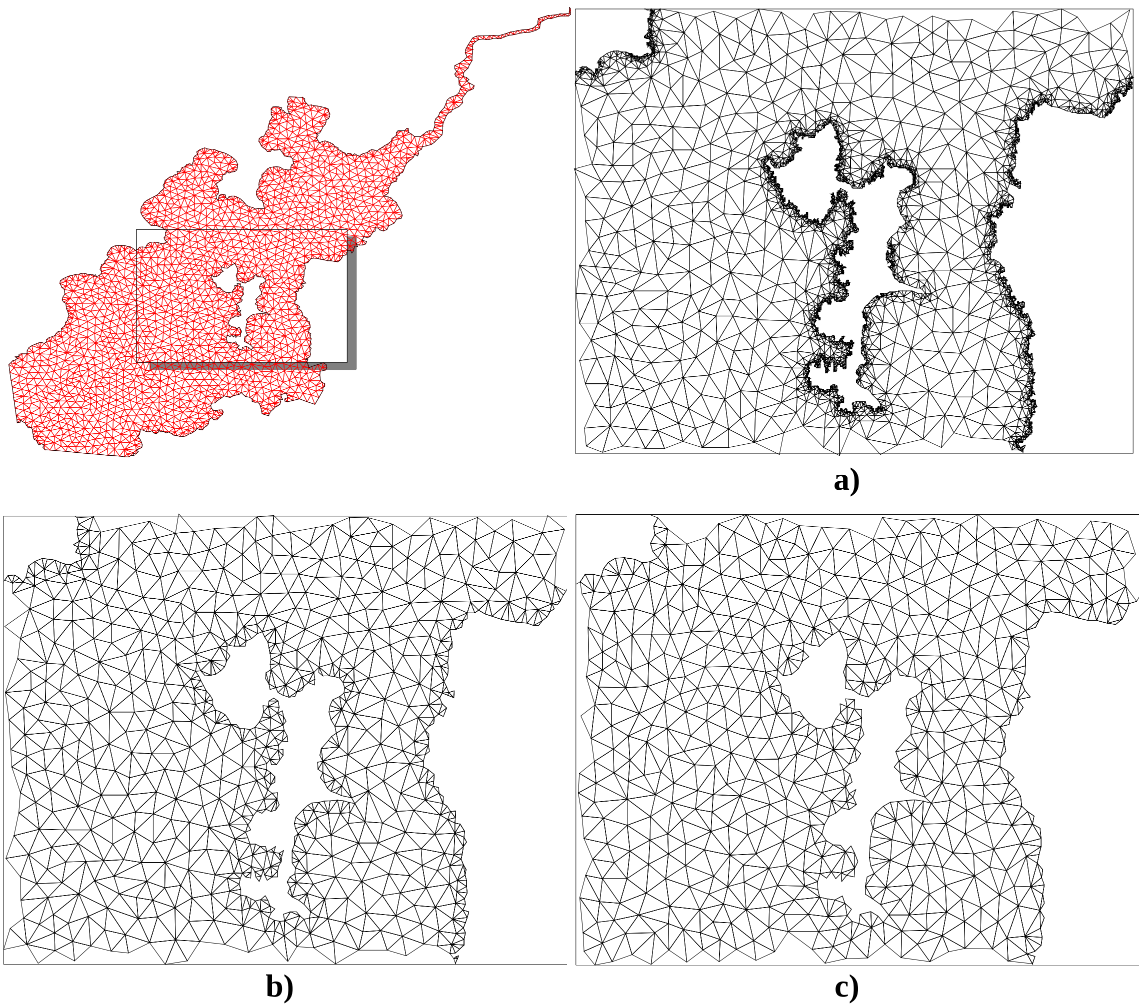


Figure 4: Meshes with different boundary polygonal line resolutions in the Ria de Arosa. Average distance between vertices: a) 14.4 m, b) 66.8 m, and c) 95.5 m.

Main mesh characteristics are summarised in Table 1. It is apparent an excessive refinement near the boundary in mesh a), concerning to an average distance between vertices of 14.4 m.

TABLE 1
MESHES CHARACTERISTICS DIFFERENT BOUNDARY POLYGONAL LINE RESOLUTIONS

Mesh	Average distance between vertices [m]	Total number of elements	Total number of points
a)	14.4	21680	14685
b)	66.8	4669	3008
c)	95.5	3723	2317

The general procedure described above, guaranties the generation of a mesh with limited interior minimum angles and elements with area size lower than a specified value. However, in most practical engineering problems, the initial mesh must be designed with spatial variations (local refinements) of the element size according to the local gradients of the modelled variables. The general purpose in the last phase is to define a governing function space that can be in some way related to the maximum area constraints within the planar domain. A *priori* or a *posteriori* error estimates became very complex and are strictly engaged with the particular technique of the FEM applied in the numerical model. Thus, this possibility to establish the governing function of the local element sizes was not used. A three steps simple methodology was established:

- **step 1**: definition of the governing function space based on: bottom depth, maximum velocity gradients or pollutant gradient concentrations; and new maximum number of elements;
- **step 2**: computation of the weighting area function and the maximum area constraints for each element of the initial mesh;
- **step 3**: mesh refinement according to the area constraints computed in step 2.

To illustrate this methodology an example of conditioned mesh refinement of a water flume with a continuous midpoint source of pollutant is presented in Figure 5.

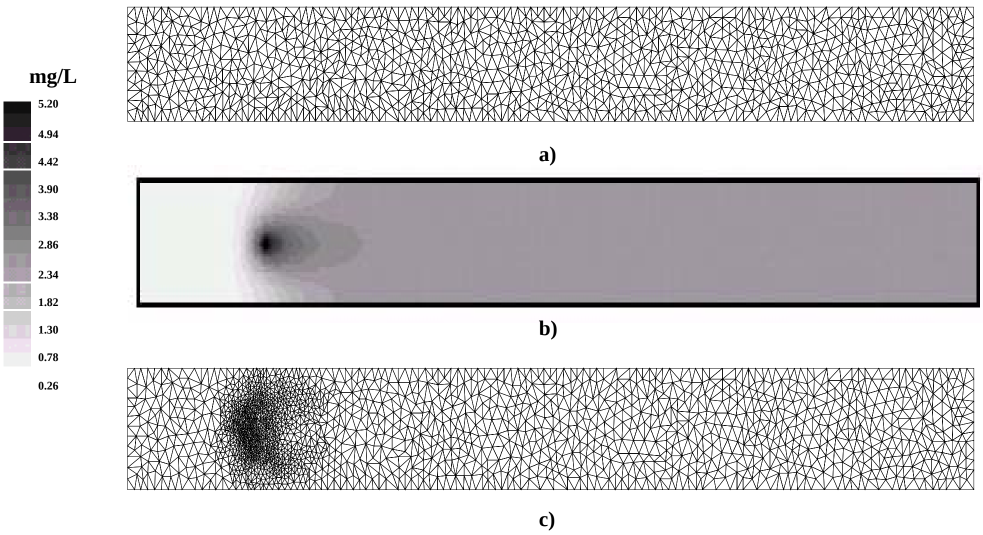
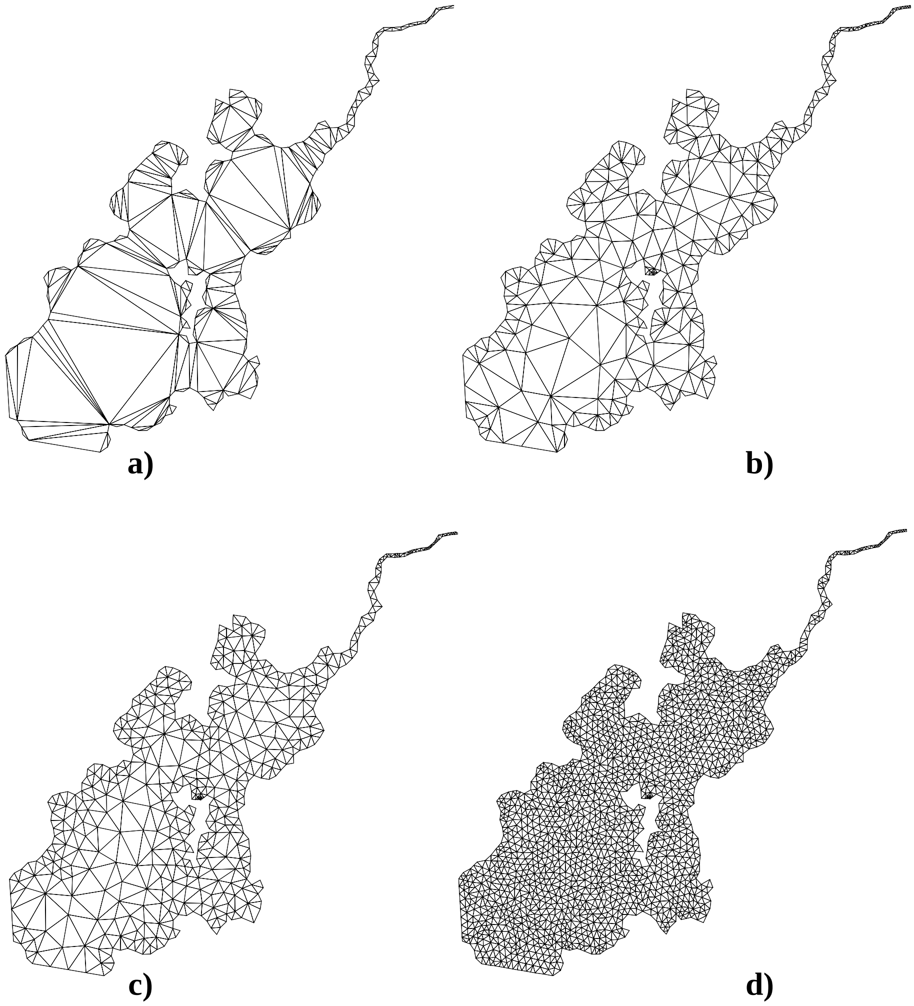


Figure 5: Conditioned mesh refinement - transport of a point source pollutant in a flume of constant flow: a) initial mesh; b) scalar field (concentration of the pollutant); c) refined mesh according to the gradient pollutant concentration.

RESULTS AND CONCLUSIONS

Generated meshes for the Ria de Arosa bay are presented in Figure 6. The total number of elements has tripled when the minimum interior angle constraint increased from 0º to 30º, as can be observed in meshes a) to c) and Table 2.

Mesh d) was generated with a minimum interior angle of 30º and a maximum area constraint of 125000 m^2 which results in a mesh with 3230 elements that has been used for currents prediction forced by tidal and wind action (details of hydrodynamic model are out of the scope of this paper). These simulations have shown reasonable approximation to measured values in stations located in the inner part of the *ria* and bad results in locations south of Arosa Island, near the boundary and in the Ulla river.



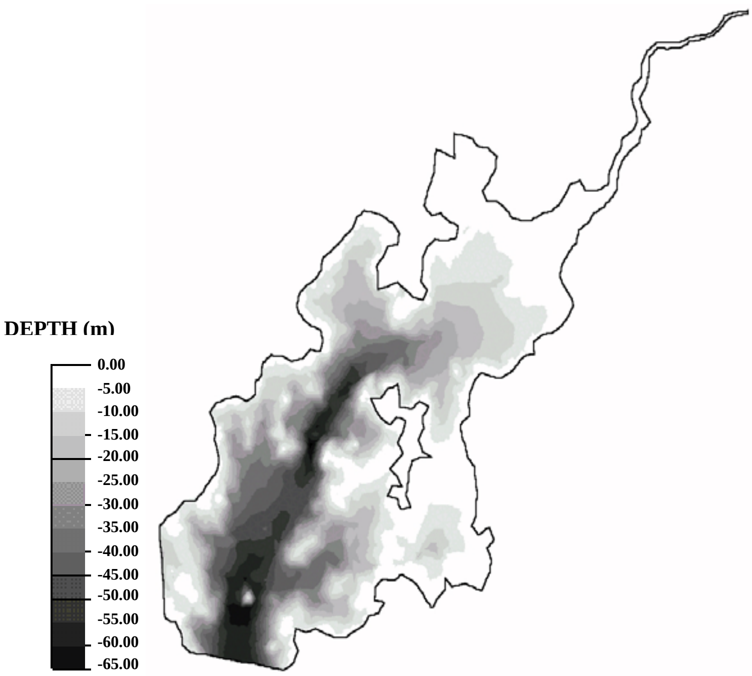
a) b) c) d)

Figure 6: Quality constrained meshes for Ria de Arosa bay: a) minimum interior angle: none, area constraint: none; b) minimum interior angle: 20º, area constraint: none; c) minimum interior angle: 30º, area constraint: none; d) minimum interior angle: 30º, area constraint: 125000 m^2.

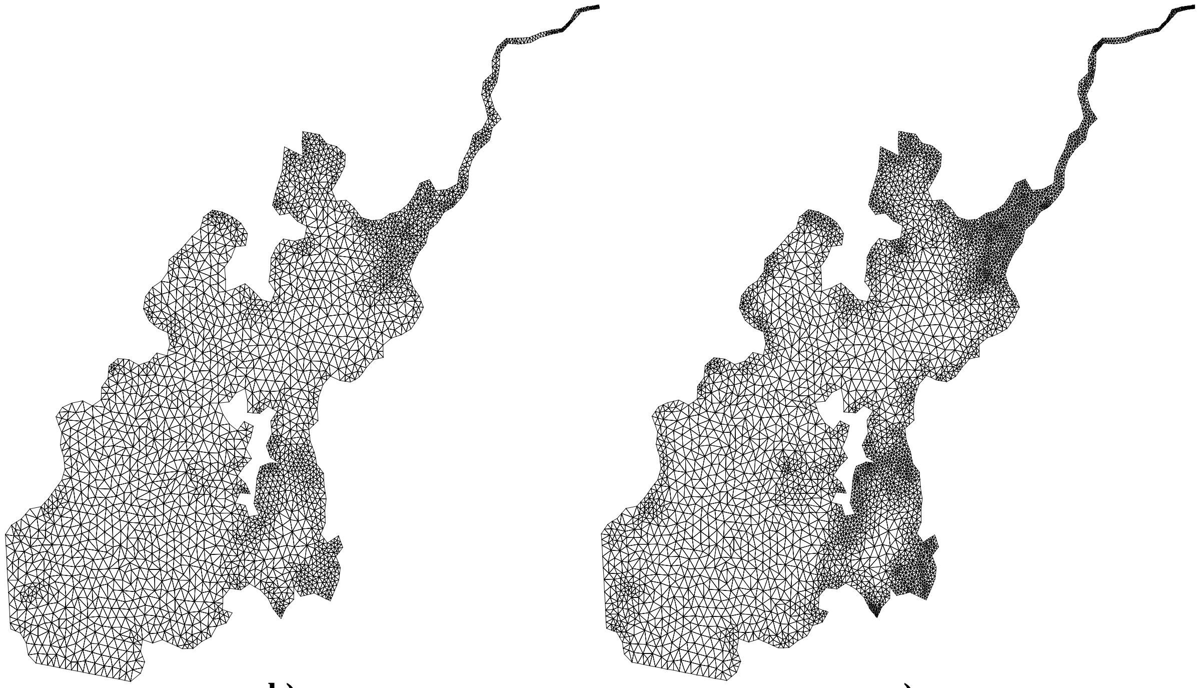
TABLE 2
MESHES CHARACTERISTICS - DIFFERENT CONSTRAINTS

Mesh	Average distance between vertices [m]	Total number of elements	Total number of points
a)	95.5	262	262
b)	95.5	494	395
c)	95.5	797	555
d)	95.5	3230	1814

To enhance the general performance of the model, a conditioned mesh refinement was carried out, taking the depth as the governing function for local mesh refinement as presented in Figure 7.



a)



b) **c)**

Figure 7: Mesh refinement conditioned to depth: a) bottom topography, b) allowing an increase of 3200 elements to 5300 ; c) allowing an increase of 3200 to 8500 elements.

The general framework for mesh generation and refinement applied in the Ria de Arosa research work appeared to be a sound procedure for 2D hydrodynamics and water quality modelling in coastal zones. Pollutant concentration gradients and water depth were used as the governing function space for mesh refinement. Other criteria for conditioned mesh generation can be established using the methodology presented in this work.

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