Purchasing Power Parity:
The rule and the exception

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Licença concedida aos utilizadores deste trabalho

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STATEMENT OF INTEGRITY

I hereby declare having conducted this academic work with integrity. I confirm that I have not used plagiarism or any form of undue use of information or falsification of results along the process leading to its elaboration.

I further declare that I have fully acknowledged the Code of Ethical Conduct of the University of Minho.

Paridade do Poder de Compra: A regra e a exceção

Resumo

A Paridade do Poder de Compra (PPC) tem sido debatida na literatura há várias décadas. Embora seja consentida a sua falta de capacidade em explicar flutuações de curto prazo, o seu relacionamento de longo prazo é ainda muito debatido.

Podemos dividir o debate em várias frentes. (1) Uma é sobre se, mesmo no longo prazo, a PPP é uma teoria útil. (2) Outra é sobre a própria definição de longo prazo, ou seja, quanto tempo é necessário para as taxas de câmbio convergirem para o seu valor de equilíbrio. (3) Há também um debate metodológico sobre a melhor abordagem econométrica para testar a teoria.

Neste trabalho pretendemos contribuir nessas três frentes. Em relação a esta última questão, estamos entre os primeiros a implementar um conjunto de ferramentas da Transformada de Wavelet Contínua (TWC) para testar a teoria. Através da decomposição tempo-frequência fornecida pela TWC, a resposta à segunda pergunta surge de forma muito natural. As taxas de câmbio flutuam em torno dos seus valores teóricos da PPC. Com o Wavelet Power Spectrum, mostramos que as flutuações mais críticas correspondem a ciclos de períodos de 8 e 16 anos. Neste estudo também contribuímos para a primeira questão: a teoria PPP verifica-se muito melhor quando o indicador de preço se baseia nos Índices de Preços de Produção (IPP) em vez de fundamentar-se no mais comum Índice de Preços ao Consumidor (IPC).

Palavras-chave: Paridade do Poder de Compra; Convergência das Taxas de Câmbio; Estimativa de tempo-frequência; A Transformada Wavelet Contínua.

Classificação JEL: F30, F44, C39

Purchasing Power Parity: The rule and the exception

Abstract

For several decades, researchers have debated the Purchasing Power Parity (PPP) theory. It is agreeable that PPP is not too helpful in explaining short-run fluctuations. However, there is a hot debate about the long run.

We can divide the debate among several fronts. (1) One is about whether, even in the long run, PPP is a helpful theory. (2) Another is about how long the long-run is, i.e., how long it takes for the exchange rates to converge to their equilibrium value. (3) There is also a methodological debate about the best econometric approach to test the theory.

We aim to make contributions on those three fronts. Regarding the latter matter, we show how the Continuous Wavelet Transform time-frequency decomposition answers several problems raised in the literature (non-linearities, instabilities, etc.). We rely on the Wavelet Power Spectrum to respond to the second issue: we show that the most critical exchange rate fluctuations around their PPP theoretical values correspond to cycles of 8 and 16 years. Finally, regarding the first question, we show that PPP theory holds much better when, as a price indicator, one relies on the Production Prices Indexes (PPI) instead of relying on the more common Consumer Price Index (CPI).

Keywords: Purchasing Power Parity; Exchange Rate Convergence; Time-Frequency Estimation; The Continuous Wavelet Transform.

JEL Classification: F30, F44, C39

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List Of Abbreviations

ADF	Augmented Dickey-Fuller
CAD	Canadian Dollar
CPI	Consumer Price Index
CWT	Continuous Wavelet Transform
DEC	Germany Currency
G7	Group of the World's Seven Largest Advanced Economies
GBP	British Pound Sterling
GDP	Gross Domestic Product
HBS	Harrod-Balassa–Samuelson
JPY	Japanese Yen
KPSS	Kwiatkowski–Phillips–Schmidt–Shin
OLS	Ordinary Least Squares
PP	Phillips-Perron
PPI	Producer Price Index
PPP	Purchasing Power Parity
USD	United States Dollar

1. Introduction

Over the last few decades, there has been a significant dispute over the validity of the Purchasing Power Parity (PPP) theory. Even though the theory has been around since the sixteenth century, it was only after World War I that academics took it seriously. In particular, there was an international debate among industrial countries about the appropriate level of exchange rates due to the large-scale increase in inflation rates in the years following the war. Since then, the exchange rates system moved from a fixed to a flexible regime. In this context, Cassel (1920), in a series of influential articles, proposed using the PPP concept to determine the respective value of exchange rates necessary to maintain the equilibrium of gold parities.

Consequently, this theory became an increasingly important topic in economic literature since it acts as an anchor for exchange rates. Several authors have tested the relationship between PPP and exchange rates, and most researchers found that the PPP does not hold in the short run. However, there is still no consensus about its existence in the long run or the convergence speed towards the PPP (Murray & Papell, 2002). The difficulty relies on three main issues.

First, despite the extreme short-term volatility of exchange rates, deviations from the PPP are persistent. It is hard to understand whether the exchange rates are persistently disconnected from the equilibrium value or whether they merely deviate from the PPP for extended periods and, if so, for how long (Obstfeld & Rogoff, 2000; Taylor & Taylor, 2004; Taylor et al., 2001).

Second, some previous studies have discarded the importance of the price indices choice to check the theory. Most of the time, prices were measured using CPI; however, other price indices, such as the PPI, that are differently constructed, might serve as a better proxy to test the PPP.

Finally, several previous studies have not addressed essential features of exchange rate data, namely structural instability, non-linearities, and non-stationary fluctuations (Jiang et al., 2015; Moon & Perron, 2005; Nagayasu, 2021).

We test the Purchasing Power Parity using data on exchange rates and prices from 1973 to 2021 for five G7 countries.[1] We aim to contribute to the literature in three main regards.

The first is methodological. In particular, we show how the Continuous Wavelet Transform (CWT) time-frequency decomposition answers several problems raised in the literature. We are the first to employ CWT tools with an approach involving a sequential analysis of wavelet coherency and phase differences

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[1] The United States, the United Kingdom, Germany, Canada, and Japan. We excluded France and Italy because, since 1999, they have had a common currency with Germany.

to study the evolution of the relationship between the nominal exchange and the theoretical exchange rates. With this technique, we estimate not only different relationships at various frequencies but also see how they evolve. Because of its local nature, with wavelet analysis, problems such as time-varying parameters or other instabilities, non-linearities, and non-stationary data do not pose a challenge.

The second contribution is regarding whether PPP is a helpful theory or not. In particular, we show that the PPP theory works very well in the long run, as long as one relies on the Production Prices Indexes (PPI) instead of the more common Consumer Price Index (CPI).

Finally, we estimate the Wavelet Power Spectrum (WPS) of the exchange rate deviations from PPP to determine how long the long-run is. We show that they have a cyclical nature and that the main fluctuations of nominal exchange rates around their theoretical expected values can be characterized by cycles of 8 and 16 years.

The remainder of the paper is organized as follows. Section 2 provides a review of the empirical literature. Section 3 introduces the model specification and describes the methodology. Section 4 describes the data. In Section 5, we present our results. In Section 6, we conclude and discuss the implications of this work and the steps for further developments.

2. Literature review

Until the late '80s, there had been some inertia around the rejection of the PPP theory- see e.g., Frenkel (1978), Rush & Husted (1985), or Patel (1990). Granted that, in the short run, everyone expected the failure of the theory, e.g., due to the exchange rate overshooting (Dornbusch et al., 1976), in the long run, this refusal was harder to accept, given the incongruency with the widely accepted neutral money theory, which states that the effect of money shocks in real variables should disappear as time passes.

Several researchers attributed the rejection of the PPP theory to the use of linear models, such as the ordinary least squares (OLS). In particular, applying these methods to study PPP has been the target of several criticisms among academics since it ignores possible time-variant cycles, i.e., the non-stationarity of the residuals – Granger & Newbold (1974); Taylor et al. (2001); Lo & Morley (2015). In fact, according to Banerjee et al. (1986) and Su & Roca (2014), even with stationary residuals, the statistical inference might still be invalid due to a potential bias in the estimated standard errors due to disturbances heteroscedasticity. According to Taylor (2001), Chen & Engel (2005), and Ahmad & Craighead (2011), some problems might also result from limiting conventional time series methodology, such as reenforcing temporal aggregation biases.

The lack of explanatory power of the models has been another of the most cited reasons in literature for the difficulty in rejecting the real exchange rates random walk[2] hypothesis (Frankel, 1985; Lothian & Taylor, 1996). To increase the statistical power, several researchers used different econometric approaches such as panel unit root tests and cointegration methods (Engle & Granger, 1987; Johansen, 1991; Zorzi & Rubaszek, 2020), more extended periods (Cheung & Lai, 1993a; Diebold et al., 1991; Lothian & Taylor, 1996), and more countries (Wu, 1996). Still, evidence from those studies is mixed. While some results favor the long-run PPP hypothesis (Cheung & Lai, 1993b; Corbae & Ouliaris, 1988; Frankel & Rose, 1995; Lothian, 2016), others still report the absence of a significant mean reversion of real exchange rates (Hakkio, 1984; Mark, 1990; Chang et al., 2010). And, of course, others get mixed results- e.g., Mahdavi & Zhou (1994) and Papell & Prodan (2020) find that the evidence of long-run PPP is much stronger for high inflation/high depreciation countries than for low inflation/low depreciation countries.

There are some reasons why using panel data, or long-sample studies might still be contentious in solving this debate. As far as panel-data studies are concerned, one possible drawback, as highlighted by Taylor&

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[2] The real exchange rate corresponds to the nominal exchange rate adjusted for relative national price differences, and so, the real exchange rate must be stationary for the PPP to hold.

Sarno (1998) and evidenced in the Monte Carlo simulation, is that the null hypothesis states that none of the real exchange rates is mean-reverting, which is a somewhat restrictive assumption.[3] As for the long-sample studies, some data may be inappropriate due to differences in exchange rate regimes and behavior across historical periods or unavailable for many currencies, thereby generating a "survivorship bias" in the tests (Baxter & Stockman, 1989; Hegwood & Papell, 1998). Thus, as argued by Adler & Lehmann (1983), one should take the rejection of the exchange rates convergence toward the PPP in these findings with a grain of salt.

Some authors also consider that the divergence in the results rests on the difficulty of finding a price index that precisely measures the inflation rate of the countries under study. For example, they defend that the choice between using the producer or consumer index might affect the theory's acceptance (Kim, 1990; MacDonald, 1993). For example, Thygesen (1978) and Taylor and Taylor (2004) found that the degree of correlation between exchange rate changes and relative inflation tends to be higher when using producer price indexes than consumer price indexes. This is a clue that we pursue in this work.

Finally, some researchers, such as Dropsy (1996) and Omay et al. (2018), found noise in turn of the PPP related to exchange rates' structural breaks. International markets have often been subject to unforeseen changes such as the oil shocks, the 2007 subprime mortgage crisis, and the recent covid crisis. And, although these structural breaks assume an essential source of bias when forming volatility estimation and forecasts for exchange rate return, they were rarely accommodated in the previous empirical work.

Nonetheless, the debate around the PPP theory is not just about its acceptance. Even among the studies that confirm the theory, there is disagreement concerning the length of time it takes for the nominal exchange rate to reflect the difference in countries' inflation rates.

Some authors, such as Frenkel (1981), Mussa (1986), Baxter & Stockman (1989), and Flood & Rose (1995), pointed out that the differences in the adjustment period rise from the difference between flexible and fixed exchange rate regimes. In particular, they showed that both real and nominal exchange rates tended to be more volatile under fluctuating exchange systems than in fixed regimes. Moreover, other authors like Wei & Parsley (1995), Lothian & Taylor (1996), and Macedoni (2021) have found dissimilarities in the PPP convergence across monetary systems and currencies. On top of that, recent studies have argued that estimations tend to be more questionable when results come from linear models

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[3] To remedy this shortcoming, some researchers have tried to design alternative tests to examine the existence of at least one non-mean reverting real exchange rate. However, according to Sarno & Taylor (1998), such tests are generally less powerful and might lead to ambiguous conclusions.

(Ali et al., 2021; Chen & Engel, 2005; Murray & Papell, 2005; Rabe & Waddle, 2020; She et al., 2021). Nevertheless, according to Rogoff (1996), the literature has an overall consensus about the time it takes for exchange rates to revert to their equilibrium level. In particular, the size of the half-life deviations from PPP at the aggregate level found in panel data and long-span studies generally range from 3 to 5 years.[4]

However, "Rogoff consensus," is not at all consensual. For instance, Vo and Vo (2020) show that the PPP tends to last at a horizon longer than five years by applying panel unit root and conventional univariate tests to the different multi-frequency settings of the real exchange rate data. Furthermore, Kilian & Zha (2002) provided minimal support for the half-life between 3 and 5 years. Taylor (2001) and Murray & Papell (2002) reject using autoregressive models to provide information on the size of the half-lives.

Rossi (2005), Imbs et al. (2005), and Murray & Papell (2005) also pointed out a significant heterogeneity in terms of confidence intervals and point estimates on some previous empirical studies sourced through sectors, countries, or types of tests applied.

Lothian & Taylor (2008) and Altavilla & Grauwe (2010) pointed out that most of the previous research-like Cheung et al. (2004)- did not consider the nonlinear evolution of exchange rates toward the equilibrium, i.e., the changes in the dynamic of the rate of convergence to PPP relative to initial deviations. If studies perform statistical tests for exchange rates convergence, not considering that equilibrium exchange rates may gradually move over time, estimates of convergence speed towards the mean are biased. These non-linearities can be caused by productivity differentials of tradable goods in different countries (Lee & Tang, 2007; Wu et al., 2018; Grisse & Scheidegger, 2021) or by the inclusion of countries with high inflation[5], which can exaggerate the extent of convergence to PPP[6]. Additionally, they might be associated with an abrupt switch of exchange rate regimes (Mussa, 1986; Habimana et al., 2018; Zeev, 2019; Macedoni, 2021). Not accounting for nonlinearities in real exchange rates might also explain why PPP was not confirmed in many studies (Obstfeld & Taylor, 1997).

Lastly, the heterogeneity of foreign exchange market participants in terms of investment purposes[7] and agents' expectations (Sarantis, 1999; Kilian & Taylor, 2003), and the arbitrage limitations because of market imperfections and frictions in trade (Sercu et al., 1995; Wei & Parsley, 1995; Michael et al., 1997; Ohanian & Stockman, 1997; Curran & Velic, 2019) may also explain the delay in adjustments.

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[4] For example, Abuaf & Jorion (1990) suggested an average of 3.3 years, Diebold et al. (1991) pointed to 2.8 years, and Frankel (1985) estimated an annual 14 percentage decay for real exchange rate deviations, implying an average of 4.6 years half-life deviations for PPP.

[5] High inflation is associated with a higher predominance of monetary shocks that usually bring additional uncertainty to the foreign exchange market.

[6] Given our sample of very developed countries, it is unlikely that we will encounter this problem.

[7] Different investor's objectives may arise from different geographical horizons or types of risk profiles and institutional investment constraints.

3. Methodology

We have seen that simple linear unit root and cointegration tests without incorporating non-linearities, non-stationarities, and cycle irregularities led to mixed results on the PPP theory's validity. Because of that, some authors included several time-varying features in their tests- Li et al., 2015; Yoon et al., 2019. More recently, alternative nonlinear unit-root tests based on the Fourier analysis have been considered (Mike & Kızılkaya, 2019; Doğanlar et al., 2021; She et al., 2021). Overall, the authors showed that these extensions added some support to the PPP theory, but the results were still mixed.

The problem with these approaches is that although they return the signal's frequency content, they offer no information on when the frequencies occur in time. We combine both approaches using the Continuous Wavelet Transform, which operates in the time-frequency space. On the one hand, we allow for time-varying relationships; on the other, relationships are also frequency-varying. We can, therefore, automatically allow for any time-varying features (irregular cycles, non-stationary variables, structural breaks, etc.) and distinguish between higher and lower frequencies. At the same time, due to its local nature, non-linearities pose no challenge.

Several types of wavelet analysis are possible to implement, and choosing one depends mainly on the goal of its application. Because information on cycles' phase and amplitude is required, a complex analytic wavelet is in this case a suitable choice– see Aguiar-Conraria & Soares (2014). In particular, this article applies the Morlet Wavelet since, in addition to easing the conversion from scales to frequencies, it also contains both an optimal time-frequency concentration and an excellent compromise between the accuracy in these two dimensions.

3.1. Continuous Wavelet Transform

A wavelet, ψ(t), for all practical uses, is a function that oscillates around the time-axis and behaves similarly to a small wave as it moves away from its center. The wavelet used in this paper, as denoted before, is a specific complex-valued wavelet ψ(t), also known as Morlet Wavelet, defined as:

$$\psi_{wo}(t) = Ke^{i\omega_0 t}e^{-\frac{t^2}{2}} \qquad (1)$$

This wavelet is famous mainly due to its four properties. First, it can act as an analytic wavelet and therefore be convenient for analyzing modulated signals[8]. Second, it is easier to convert from scales to frequencies because the peak, the energy, and the central instantaneous frequencies of the Morlet wavelet are all equal. To be precise, given our choice of the Morlet wavelet, we have that the usual Fourier frequency f (expressed in cycles per unit time) is given by $f(s) = 6/(2\pi s) \approx 1/s$. Therefore, we can use both terms interchangeably. This greatly facilitates the interpretation of the results by economists familiar with the Fourier Transform's frequency analysis. Third, this family of wavelets has an optimal joint time-frequency concentration and, consequently, there is a minimum trade-off between time and frequency precision. Lastly, because both the time and the frequency radius are equal, the Morlet wavelet is assumed to achieve an excellent accuracy compromise between these two dimensions.

To ensure that the admissibility condition was satisfied, the normalizing constant K in equation (1) was assumed to be equal to $\pi^{-\frac{1}{4}}$ so that ψ_{ω_0} (t) could have unit energy. Furthermore, since most economic papers consider some value of $\omega_0 \in [5, 6]$[9], this paper undertook a ω_0 of 6. Therefore, the previous equation can be rewritten as:

$$\psi(t) = \pi^{-\frac{1}{4}} e^{6it} e^{-\frac{t^2}{2}} \qquad (2)$$

Starting with this specific "mother wavelet," it is possible to obtain a family of Morlet "wavelet daughters" $\psi_{\tau,s}$, by simply scaling and translating ψ(t), that is:

$$\psi_{\tau,s}(t) := \frac{1}{\sqrt{|s|}} \psi\left(\frac{t-\tau}{s}\right), \quad s, \tau \in \mathbb{R}, \ s \neq 0 \qquad (3)$$

where the parameter s is responsible for adjusting the wavelet width[10] while the parameter τ fixes its position along the time-axis.[11]

For a given time series x(t), the CWT is a function of two variables, $W_{x,\psi}(s,\tau)$ expressed as:

$$W_{x,\psi}(s,\tau) = \int_{-\infty}^{\infty} x(t) \frac{1}{\sqrt{|s|}} \bar{\psi}\left(\frac{t-\tau}{s}\right) dt \qquad (4)$$

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[8] That is, signals with time-varying frequency and amplitude

[9] That is because, for ω0 ≥ 5, the Morlet wavelet respective Fourier transform values $\Psi_{\omega0}(\omega) = \sqrt{2}\pi^{\frac{1}{4}} e^{-\frac{1}{2}(\omega-\omega0)^2}$ for $\omega \leq 0$ are extremely small, and $\Psi_{\omega0}$ can be considered an analytic wavelet

[10] For s < 1, the CWT becomes narrower, therefore, pulling down the higher frequencies, and for s>1, it becomes larger hence capturing lower frequencies.

[11] The symbol := means "by definition".

where x(t) denotes the data time series, and the over-bar symbolizes complex conjugation.

3.2. Wavelet tools

The main wavelet tools we apply are the wavelet power spectrum, the wavelet coherence, and the wavelet phase and phase difference. This section briefly presents the mathematical theory behind these tools, starting with the most straightforward situation of univariate analysis and then progressing to bivariate analysis. Since all the wavelet measures introduced here are functions of τ and s, from now on, to simplify the notation, these variables, unless strictly necessary, will be omitted from future formulas. Furthermore, the notation $W_{x,\psi}$ will also be abbreviated and written as W_x.

3.2.1. Univariate Wavelet tools

The (local) wavelet power spectrum portrays the variance distribution of a particular time series at different frequencies and is defined by:

$$(\mathrm{WPS})_x = \mathrm{W}_x\overline{\mathrm{W}}_x = |\mathrm{W}_x|^2 \qquad (5)$$

Given that the wavelet ψ is complex-valued, the corresponding wavelet transform W_x is also complex-valued and so can be expressed in the polar form as:

$$W_x = |W_x|e^{i\phi_x},\ \phi_x \in (-\pi, \pi), \qquad (6)$$

where $|W_x|$ is the amplitude and ϕ_x denotes the argument, often referred to as the (wavelet) phase angle. The latter term is specifically crucial because it yields information on the position of the variable in the cycle as a function of its frequency[12].

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[12] The phase angle of the complex number Wx, can be obtained from the formula $\phi_x = \mathrm{Arctan}\left(\frac{\Im\{W_x\}}{\Re\{W_x\}}\right)$ where $\Im\{W_x\}$ corresponds to the imaginary part of the transform and $\Re\{W_x\}$ to its real part.

3.2.2. Bivariate wavelet tools

When dealing with time-frequency dependencies between two-time series, it is necessary to introduce the concepts of cross-wavelet coherency and phase difference. These tools help detect and quantify possible relationships between two-time series.

The cross-wavelet transform (W_{xy}) of two series x(t), and y(t) is defined as:

$$\mathrm{W}_{xy} = \mathrm{W}_x \overline{\mathrm{W}}_y \qquad (7)$$

where W_x and W_y are the wavelet transforms of x and y, respectively. From the absolute value of the cross-wavelet transform, it is possible to obtain the cross-wavelet power, which can be interpreted as the local covariance between two-time series at each moment and frequency. To further analyze the correlation coefficient between two series in the time-frequency space, it is necessary to resort to the wavelet coherence, R_{xy}[13] which can be written as:

$$R_{xy} = \left|\varrho_{xy}\right| = \frac{|S_{xy}|}{\sigma_x \sigma_y} \quad \forall\, 0 \leq R_{xy} \leq 1, \qquad (8)$$

where ϱ_{xy} is the complex wavelet coherence, S_{xy} denote a smoothing operator of the cross-wavelet transform[14], and σ_x and σ_y respectively represent the square root of the smoothed wavelet power of series x and y.

Finally, as in the case of the univariate complex-valued wavelet, the complex wavelet coherence can also be written in polar form as $\varrho_{xy} = \left|\varrho_{xy}\right| e^{i\phi_{xy}}$, where now the angle ϕ_{xy} is called the phase difference and can be approximated to $\phi_{xy} = \phi_x - \phi_y$. This measure provides information on the possible delays of the series' oscillations as a function of frequency and time. In particular if $\phi_{xy} = 0$, then both series move together at a given time and scale. If instead $\phi_{xy} \in \left(0, \frac{\pi}{2}\right)$ then both series are in phase with x leading y, and if $\phi_{xy} \in \left(-\frac{\pi}{2}, 0\right)$, the series are in phase, with x lagging. On the other hand, if $\phi_{xy} \in \left(-\pi, -\frac{\pi}{2}\right)$, series x leads y in an out of phase relation, and if $\phi_{xy} \in \left(\frac{\pi}{2}, \pi\right)$ the series x and y

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[13] This measure is analog to the correlation in a typical econometric regression.

[14] Smoothing is necessary, because, otherwise, coherency would be identically one at all scales and times, and the same happens with the Fourier coherency.

are out of phase, with y leading.

3.3. Model Specification

The absolute version of the PPP theory tells us that the exchange rate between two countries is equal to the ratio of prices: $\mathrm{E} = \frac{\mathrm{P}}{P^*}$. This version is, of course, rather stringent. A less restrictive version tells us that changes in the exchange rate are the difference between the inflation rates in both countries. That implies not that the exchange rate is equal to the price ratio but merely proportional to it: $\mathrm{E} = \mathrm{A}\frac{\mathrm{P}}{P^*}$.

To test if PPP holds, we first need a theoretical benchmark. For that purpose, we create a new variable, $PPP_t = \mathrm{A}\frac{P_t}{P_t^*}$, where A is a constant chosen such that the mean of the exchange rate (E_t) is equal to the mean of PPP_t. Taking logs, its empirical counterpart in the time-frequency space is then:

$$\ln\left(E_{t,f}\right) = \alpha_{t,f} + \beta_{t,f} \ln\left(PPP_{t,f}\right) + \varepsilon_{t,f}, \qquad (9)$$

where $\varepsilon_{t,f}$ represents the perturbation term. Because we have precisely two variables (the dependent and the independent variable), we can rely on the bivariate wavelet tools presented earlier. According to the theory, it is then expected that the parameter $\beta_{t,f}$ is statistically significant and with a positive sign. If the fit were perfect, then $\beta_{t,f} = 1$.

Nevertheless, as we have seen before, even if the PPP ends to be confirmed, there is still a debate about how much time it takes for the exchange rates to reflect the differences in countries' inflations correctly. For this purpose, in the second stage, we compute the local wavelet power spectrum for the difference between the market and the theoretical exchange rate $(\ln(E_t) - \ln(PPP_t))$, to estimate the length of time it takes for the nominal exchange rate to reflect the changes in countries' prices ratio properly. In other words, this process will allow us to study the strength of real exchange rates' reversion towards their equilibrium value in a time-frequency domain.

4. Data

We collected data for the period between 1973:1 and 2021:10 on the countries' price levels and nominal exchanges rate for five of the G7 currencies: the United States (USD), the United Kingdom (GBP), Germany (DEC), Canada (CAD) and Japan (JPY). We excluded France and Italy because, since 1999, these two countries, like Germany, have adopted the euro currency.

By selecting these countries, we reduce the data heterogeneity since all these countries are at the same stage of economic development and were under flexible exchange rate regimes in the chosen study period. Data is monthly and normalized using the average values of 2010. We extracted the data on exchange rates from the FXTOP historical rates page[15] , using the Spot Exchange Rate converted to a monthly basis.

We rely on the Consumer Price Indexes (CPI) and the Production Price Indexes (PPI) retrieved from the International Monetary Fund Database for countries' prices. These measures in this work leans on the literature debate about which one corresponds to the best price measure. Contrarily to CPI, which shows the general level of inflation, which comprises many goods and services, PPI is an indicator of changes in average output price charged by domestic producers, covering both capital equipment prices and consumer goods. Because of that, the determination of PPI does not consider excise and sales taxes, so it represents an immediate measure to deflate revenue streams and measure output growth and not just adjust income and expenditure as the CPI.

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[15] FXTOP Company's site, https://fxtop.com/en/historical-exchange-rates.php?MA=0&TR=1 (05-01-2022), complies with the European Union Council Regulation, 97/1103 to provide daily official exchange rates published by Central banks.

5. Results

We divide this section into two parts: in the first part, we present the results associated with the direct application of equation (9); in the second, we study the deviations of the observed exchanged rates from their theoretical counterparts in the time frequency-domain.

5.1. Does the PPP hold?

We schematize the figures in this section as follows: on top, we have the plots of both the observed and theoretical exchange rate; in the middle, we estimate the wavelet coherency; and, at the bottom, we have the phase differences. On the left, we test the theory using CPI and, on the right, using PPI.

As standard, we interpret the coherency as a local correlation measure in the time-frequency domain between the two variables under study. The color code for the wavelet coherency ranges from blue (low coherency) to red (high coherency). The black (grey) contours indicate the 5% (10%) significance level for the wavelet coherency, computed with 5000 Monte-Carlo simulations. The cone of influence, indicating the region affected by edge effects, is shown with a parabola-like black line.

The phase information at the bottom provides information on which variable leads and whether the correlation is positive or negative. If the phase difference is between 0 and $\pi/2$ ($-\pi/2$ and 0), the correlation is positive, with the nominal exchange rates leading (lagging). If it is between $\pi/2$ and π ($-\pi$ and $-\pi/2$), the correlation is negative, with the nominal exchange rates lagging (leading). We display the mean values of phase differences in two frequency bands: 1.5~8 years cycles (the typical business cycle frequencies) and 8~34 years (long-run fluctuations).

After inspecting the pictures, four results immediately emerge. First, just by eyeballing the plots, on top, we see that PPP does not hold continuously for any country: the nominal exchange and the theoretical exchange rate are recurrently disconnected. In other words, there are substantial deviations from PPP at the business cycle frequencies. These recurrent divergences might be related to the behavior of the series, i.e., computed exchange rates tend to be less volatile than those prevailing in the market, as the exchange rate overshooting hypothesis predicts. Furthermore, one can see that, at business cycle frequencies, changes in the theoretical exchange rate lag changes in the nominal exchange rates. We can verify this statement by checking that the phase difference is typically between 0 and $\pi/2$ for this frequency range whenever the coherency is statistically significant. If we think about it, these results align with both the neoclassical and the monetary neutrality theory since it suggests a causal relationship transmitted through nominal to real variables at business cycle frequencies and not the other way around due to the ease of adjustment each of the variables presents.

Second, the nominal and the theoretical exchange rates follow the same trend over the long run. This statement is supported by phase-difference diagrams, which are consistently located in the interval (-π/2, π/2), confirming that the market and theoretical exchange rate cycles are in-phase, i.e., their relationship is positive. Therefore, the relationship between price ratio and exchange rate changes is consistent with the PPP theory.

Third, the PPP theory holds better when PPI is used instead of CPI. Put another way; exchange rates have a closer relationship with producer price ratios than with consumer prices. We see that when the CPI, the PPP theory only seems to be a significant determinant of exchange rates for a limited number of cases. Nonetheless, the PPP proves to be a good model for determining exchange rates when using the PPI instead. We observe this either by looking at the plots or the estimated coherences, where we observe much larger red and statistically significant regions when one uses PPI. This appeal for the use of PPI is in line with Taylor & Taylor (2014) and Yoon et al. (2020).

Lastly, the correlations between the nominal and the theoretical exchange rates vary across countries. In fact, for some economies, exchange rates tend to show more significant deviations from their fundamental value, while for others, they follow the theoretical rate quite closely. Also, their relationship seems to be irregular over time; cycles' periodicity is time-dependent. One crucial feature retrieved from all analyses is that the adjustment periodicity of the economic cycles changes across time, i.e., there is a time-frequency dependence between exchange rate and prices. This dependency might explain why linear estimations are so controversial, supporting our methodological choice of working in the time-frequency domain. Besides, over the years, the deviation tendency between actual and hypothetical exchange rates seems to be decreasing, being their relation more stable in the past years, which is not surprising given the growing globalization movement witnessed in recent past decades. To sustain these conclusions, we focus on five situations: USD/DEM, USD/CAD, GBP/CAD, GBP/JPY, and DEM/JPY, and leave the other cases in the appendix.

We start by looking at the United States and Germany (USD/DEM Figure 1). When resorting to CPI as a price indicator, we see that even though the phase difference is consistently located in the interval (-π/2, π/2), the two series are, in general, weakly correlated, as we can see by the scarcity of red regions in the wavelet coherence. The coherency between the two variables becomes more significant when using the PPI than CPI. It is evident in the 8~12 years frequency band and at higher frequencies (2~4 years-cycles). We can also see from these diagrams that the phase differences are located between 0 and π/2 at business cycles, indicating that price differentials adjust after and in parallel with changes in the exchange rates, which is indeed consistent with the price stickiness hypothesis. In the long run, the two series tend to move again in the same direction, with a slight lead from nominal exchange rates.

The conclusions of the GBP/CAD case (Figure 2) are similar to the previous case, although more substantial. On the left, when using CPI, the wavelet coherence is low almost everywhere, and the observed phase-differences are sometimes inconsistent with the theory. In particular, we can see that the series are negatively correlated for high frequencies between 1988 and 2003 and low frequencies between 1978 and 1983, given the respective phase difference values located in the intervals (π/2, π) and (-π/2, -π). With PPI, on the right, red regions are more extensive and significant. This observation is accurate for high frequencies (until 2008) and low frequencies (16~24 years), especially from the 90s onwards. Also, note that the phase differences at lower frequencies, although negative, become close to zero, suggesting an almost simultaneous relation, with the theoretical exchange rate slightly leading.

The case of the United States and Canada (USD/CAD, Figure 3) has the most dramatic differences between using CPI and PPI. Based on the pictures on the left, with CPI, one would even question whether the purchasing power parity theory is of any use since there is a lack of regions of high coherency. Moreover, the estimated phase differences do not make much sense for a few regions at high and low frequencies, suggesting a negative correlation between 1993 and 2010 and from 2013 onwards. However, these results should not be given much primacy as they correspond to low coherence regions (business cycle frequencies) or are outside the cone of significance (long-term cycles). It is no surprise that papers relying on CPI (e.g., Kim, 1990, or Al-Zyoud, 2015) rejected the theory for these two countries, while the same did not happen with papers using PPI (e.g., see Moon & Perron, 2005, or Li et al., 2015). Using PPI, we can see that the data is perfectly compatible with the theory in the long run. The estimations at business cycle frequencies are also consistent with the theory, at least until 2010.

There are also examples for which the distinction between CPI and PPI is mostly irrelevant. It is the case in the United Kingdom and Japan (GBP/JPY, Figure 4). We see a very significant relationship between the nominal and theoretical exchange rates derived from the CPI and PPI at low frequencies. In this frequency range, the phase difference is stable and close to 0, indicating a simultaneous co-movement between the price ratio and exchange rate changes. For this currency pair, data strongly supports the theory, no matter which price indices — although it is still more robust when using PPI.

Lastly, considering the DEM/JPY case (Figure 5), it is also possible to see the same similarities of results found in the previous example. However, the exchange rate changes do not seem to be so well connected with the variations in countries' prices' differential, as denoted by the lower predominance of red regions in the wavelet coherency.

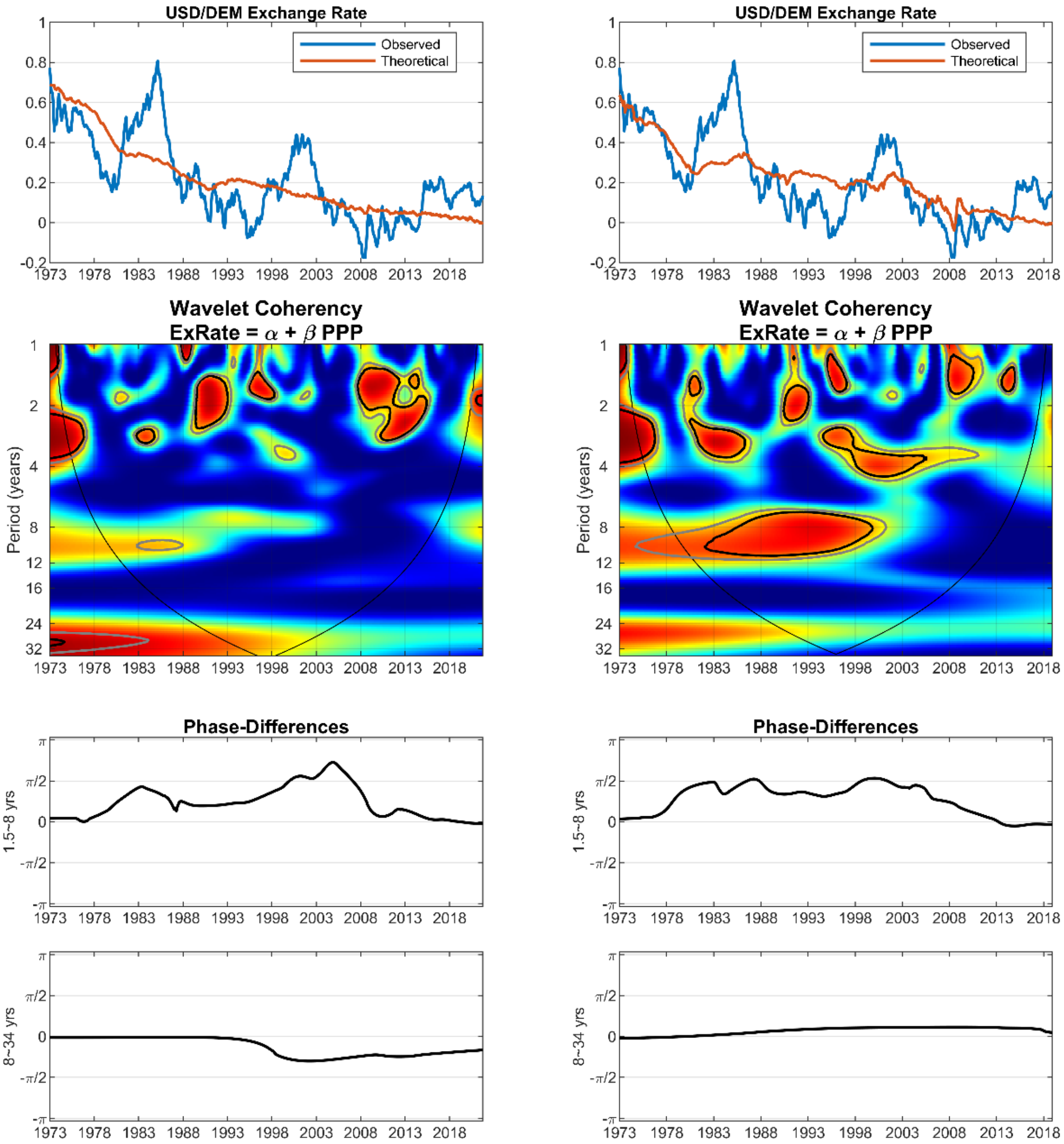

Figure 1: Top: Time-series plots of the USD/DEM market exchange rate and the ratio between Germany and United States price indexes. **Middle:** Coherency between the market and the expected PPP exchange rates. The color code for coherency ranges from blue (low coherency- close to zero) to red (high coherency- close to one). The black (grey) contours indicate the 5% (10%) significance level for the wavelet coherency, computed with 5000 Monte-Carlo simulations. The cone of influence, indicating the region affected by edge effects, is shown with a parabola-like black line. **Bottom:** Phase-differences between cyclical fluctuations of the theoretical and market exchange rates, for the two frequency bands. **Left:** PPP values obtained through consumer price index **Right:** PPP values obtained through producer price index.

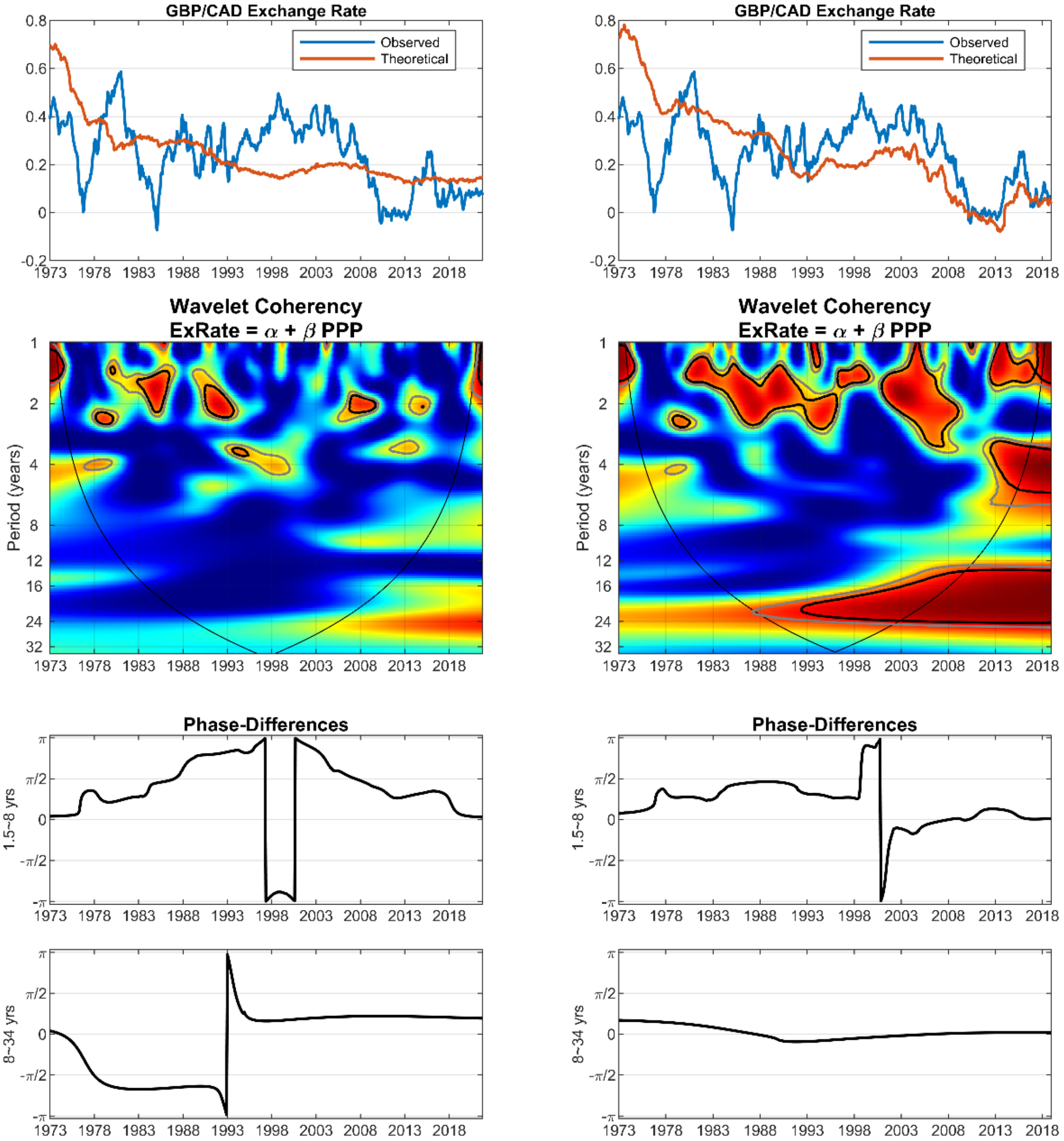

Figure 2: Top: Time-series plots of the GBP/CAD market exchange rate and the ratio between Canada and United Kingdom price indexes. **Middle:** Coherency between the market and the expected PPP exchange rates. The color code for coherency ranges from blue (low coherency- close to zero) to red- high coherency {close to one). The black (grey) contours indicate the 5% (10%) significance level for the wavelet coherency, computed with 5000 Monte-Carlo simulations. The cone of influence, indicating the region affected by edge effects, is shown with a parabola-like black line. **Bottom:** Phase-differences between cyclical fluctuations of the theoretical and market exchange rates, for the two frequency bands. **Left:** PPP values obtained through consumer price index **Right:** PPP values obtained through producer price index.

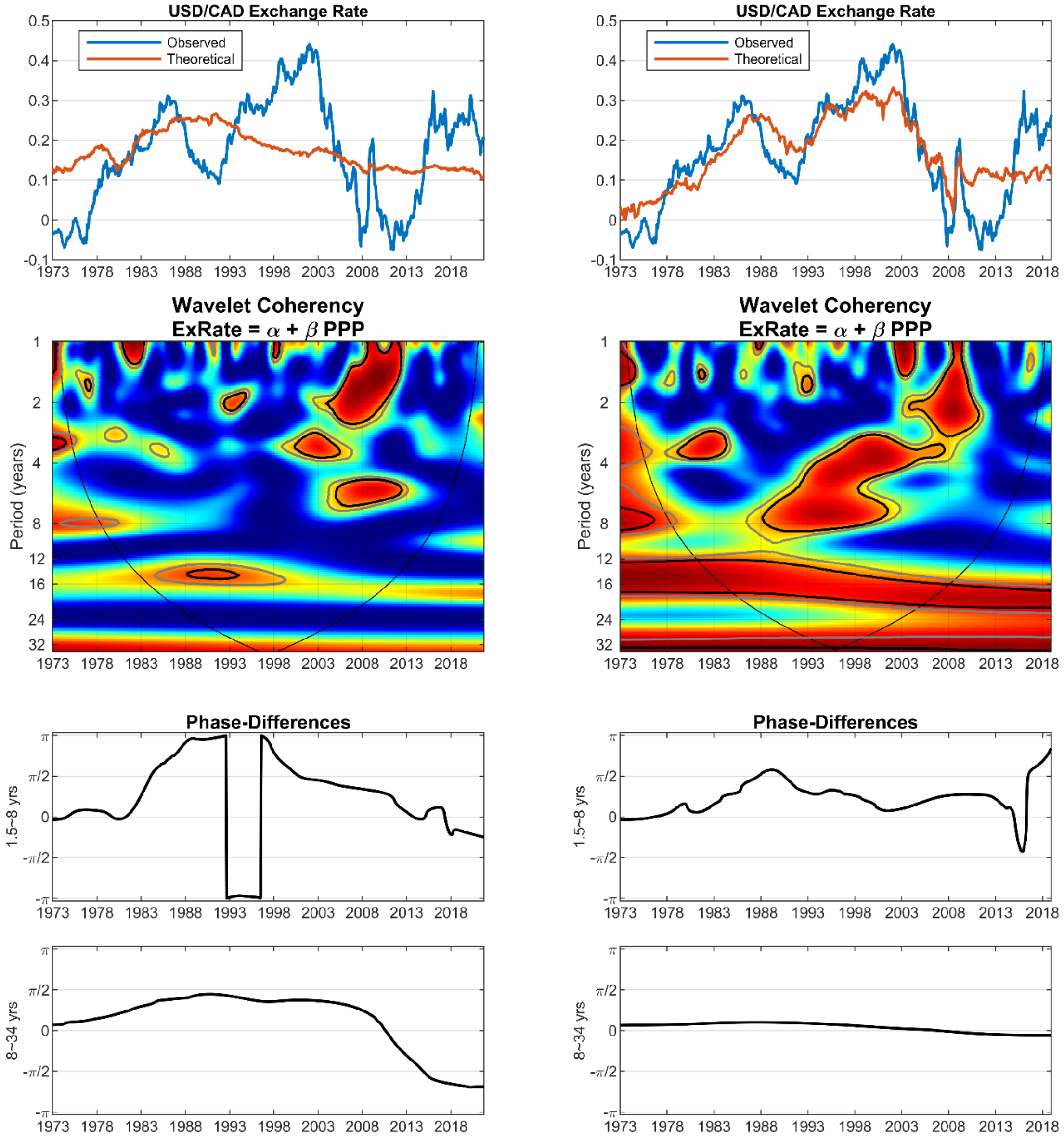

Figure 3: Top: Time-series plots of the USD/CAD market exchange rate and the ratio between Canada and United States price indexes. **Middle:** Coherency between the market and the expected PPP exchange rates. The color code for coherency ranges from blue (low coherency- close to zero) to red (high coherency- close to one). The black (grey) contours indicate the 5% (10%) significance level for the wavelet coherency, computed with 5000 Monte-Carlo simulations. The cone of influence, indicating the region affected by edge effects, is shown with a parabola-like black line. **Bottom:** Phase-differences between cyclical fluctuations of the theoretical and market exchange rates, for the two frequency bands. **Left:** PPP values obtained through consumer price index **Right:** PPP values obtained through producer price index.

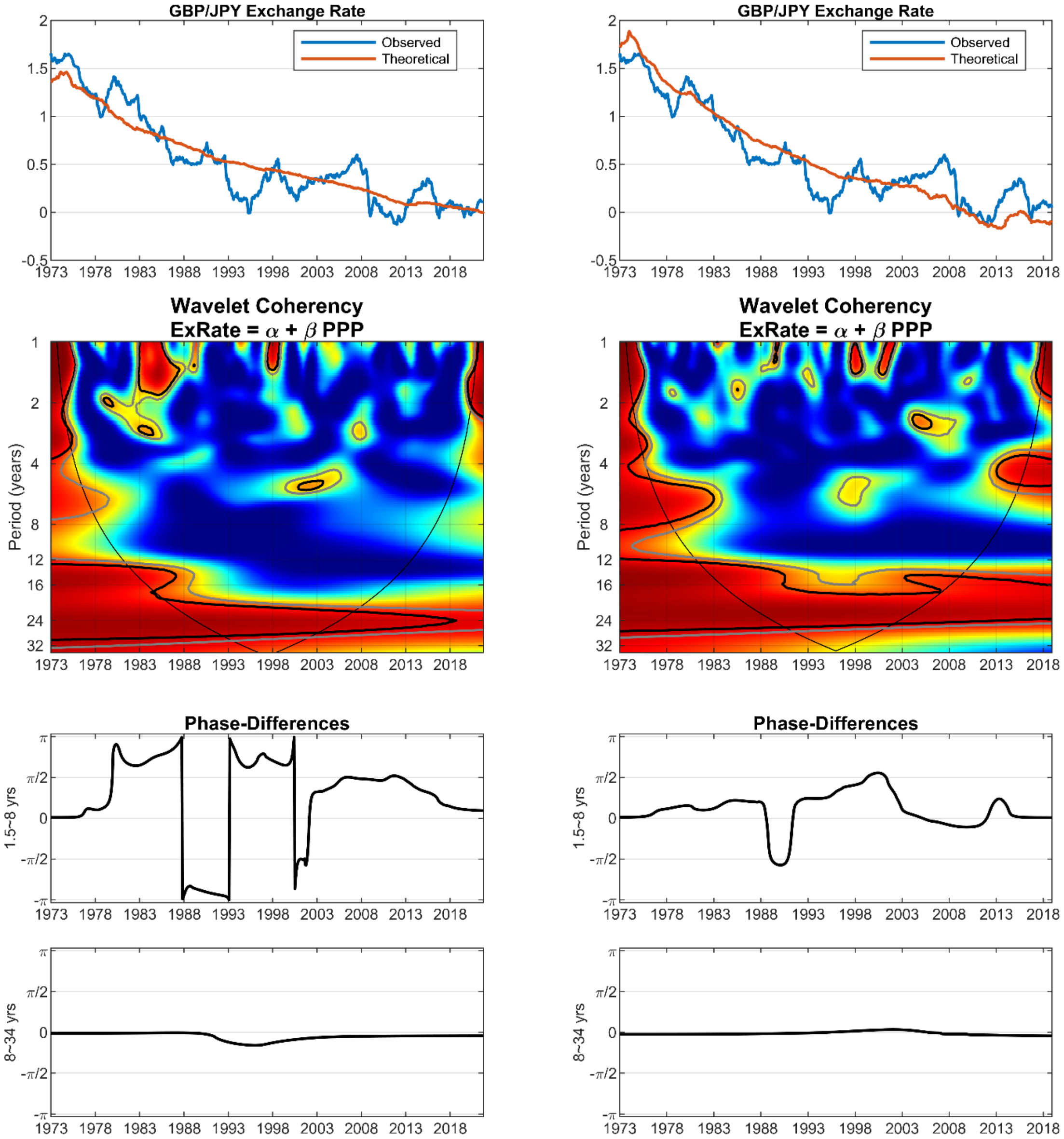

Figure 4: Top: Time-series plots of the GBP/JPY market exchange rate and the ratio between Japan and United Kingdom price indexes. **Middle:** Coherency between the market and the expected PPP exchange rates. The color code for coherency ranges from blue (low coherency- close to zero) to red (high coherency- close to one). The black (grey) contours indicate the 5% (10%) significance level for the wavelet coherency, computed with 5000 Monte-Carlo simulations. The cone of influence, indicating the region affected by edge effects, is shown with a parabola-like black line. **Bottom:** Phase-differences between cyclical fluctuations of the theoretical and market exchange rates, for the two frequency bands. **Left:** PPP values obtained through consumer price index **Right:** PPP values obtained through producer price index.

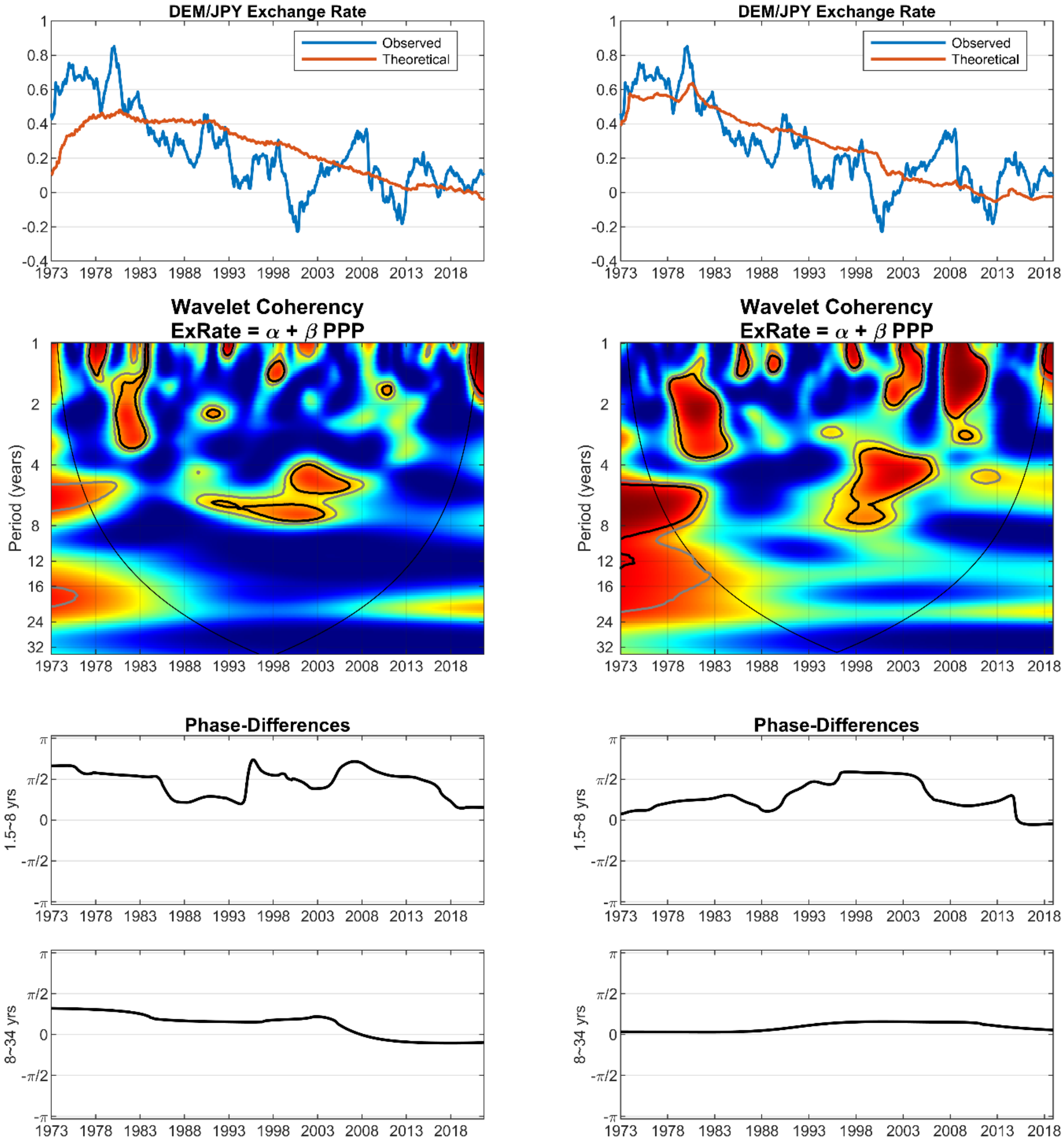

Figure 5: Top: Time-series plots of the DEM/JPY market exchange rate and the ratio between Japan and Germany price indexes. **Middle:** Coherency between the market and the expected PPP exchange rates. The color code for coherency ranges from blue (low coherency- close to zero) to red (high coherency- close to one). The black (grey) contours indicate the 5% (10%) significance level for the wavelet coherency, computed with 5000 Monte-Carlo simulations. The cone of influence, indicating the region affected by edge effects, is shown with a parabola-like black line. **Bottom:** Phase-differences between cyclical fluctuations of the theoretical and market exchange rates, for the two frequency bands. **Left:** PPP values obtained through consumer price index **Right:** PPP values obtained through producer price index.

5.2. How long do the exchange rates take to reach the PPP equilibrium?

Having shown that the PPP theory represents a good portrait of the reality experienced in international markets, now we determine the speed of convergence of the exchange rate towards the purchasing power parity theoretical equilibrium value. For this purpose, in this subsection we use the wavelet power spectrum to estimate which frequencies are more important to explain the overall variance of the deviations from theoretical values[16].

Our results are in Figure 6. We have the percentage deviations of the nominal exchange rate from its theoretical value on the right. On the left, we have its wavelet power spectrum. Red regions correspond to the regions that are more important to explaining the total variance of the series. The white lines show the local maxima of the wavelet power spectrum, providing an estimate of the most significant frequency. It is analogous to the peak Fourier spectrum and is, therefore, an estimate of the period of the most relevant cycles.

In the plots (Figure 6), we can identify major economic shocks. In particular, we can see substantial exchange rates equilibrium deviations during the energy crisis in 1973, 1979, and again in the 2000s; in the early 1980s recession; throughout the Japanese asset price bubble between 1986 and 1992; in the black Monday and Wednesday in 1987 and 1992, respectively; over the dot-com bubble in early 2000s; in the great recession occurring from 2007 until 2009 and more recently during the European debt crisis in early 2010.

Take the early 1980s recession as an example. This crisis was one of the most severe recessions since World War II. It started in 1980 and continued until early 1983 due to the sharp rise in oil prices due to the energy crisis. The increase in energy prices pushed the already high inflation rates in several major advanced countries to new double-digit highs, tightening monetary policies to control inflation. As we can see through the graphs, this caused the appreciation of the US dollar against the other currencies. According to Engel & Hamilton (1990), this change resulted from investors looking for a safe haven. Therefore, the dollar demand increased drastically, although the prices did not change that much.

This overreaction of market participants is possibly the reason for the slow adjustment towards the equilibrium value. In fact, according to the information provided in the power spectra (Figure 6), deviations from PPP are usually subject to reasonably long swings. In particular, one pattern that arises in several cases is the juxtaposition of peaks in the spectra that mainly occur at frequencies corresponding to periods

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[16] Given the results in our previous section, we use the PPI indexes to compute the equilibrium values.

close to 8- and 16- years. This is evident for example in the case of the United States and Canada. However, we can also see similar patterns in the case of the United States and Japan, the United States and Germany, the United Kingdom and Germany, and Canada and Germany. But this behavior is not a rule. There are also some exceptions.

Consider Germany and Japan (DEM/JPY). Three dominant cycles coexist. One is concentrated around the 6-year frequency, appearing in the mid-80s. There is also an 8-year cycle that occurred at the beginning of this century. Lastly, we can also see a cycle with a periodicity of 16 years, starting at the beginning of the sample and ending when the 8-year cycle begins, corresponding to when Germany joined the euro currency union. This faster convergence of nominal exchange rates towards their equilibrium value after the euro implementation is consistent with the results obtained by Koedijk et al. (2004) and Bergin et al. (2017) regarding the impacts of this event on the member countries' PPP speed of convergence.

The decrease in the exchange rate time of adjustment can also be seen in the case of the United Kingdom and Germany (GBP/DEM), but in this situation the change is more gradual and starts earlier, coinciding with the United Kingdom joining the European Economic Communities (EEC). In particular, while in 1973 there was an overlap of 10- and 16- years cycles, at the beginning of this century, the 10-year adjustment became reduced to 8 years. Once again, these results show how heroic the assumptions made by linear time-series models were.

The United Kingdom and Canada and the United States and the United Kingdom represent other peculiar cases. In particular, there seems to exist only a predominant cycle of 10-year cycles, mainly until the ’90s, as shown by the white line appearing in this frequency band.

The case of Canada and Japan is also interesting for the same reasons but this time the relationship is more stable throughout the sample, with a predominant cycle at an average of 8 years frequency.

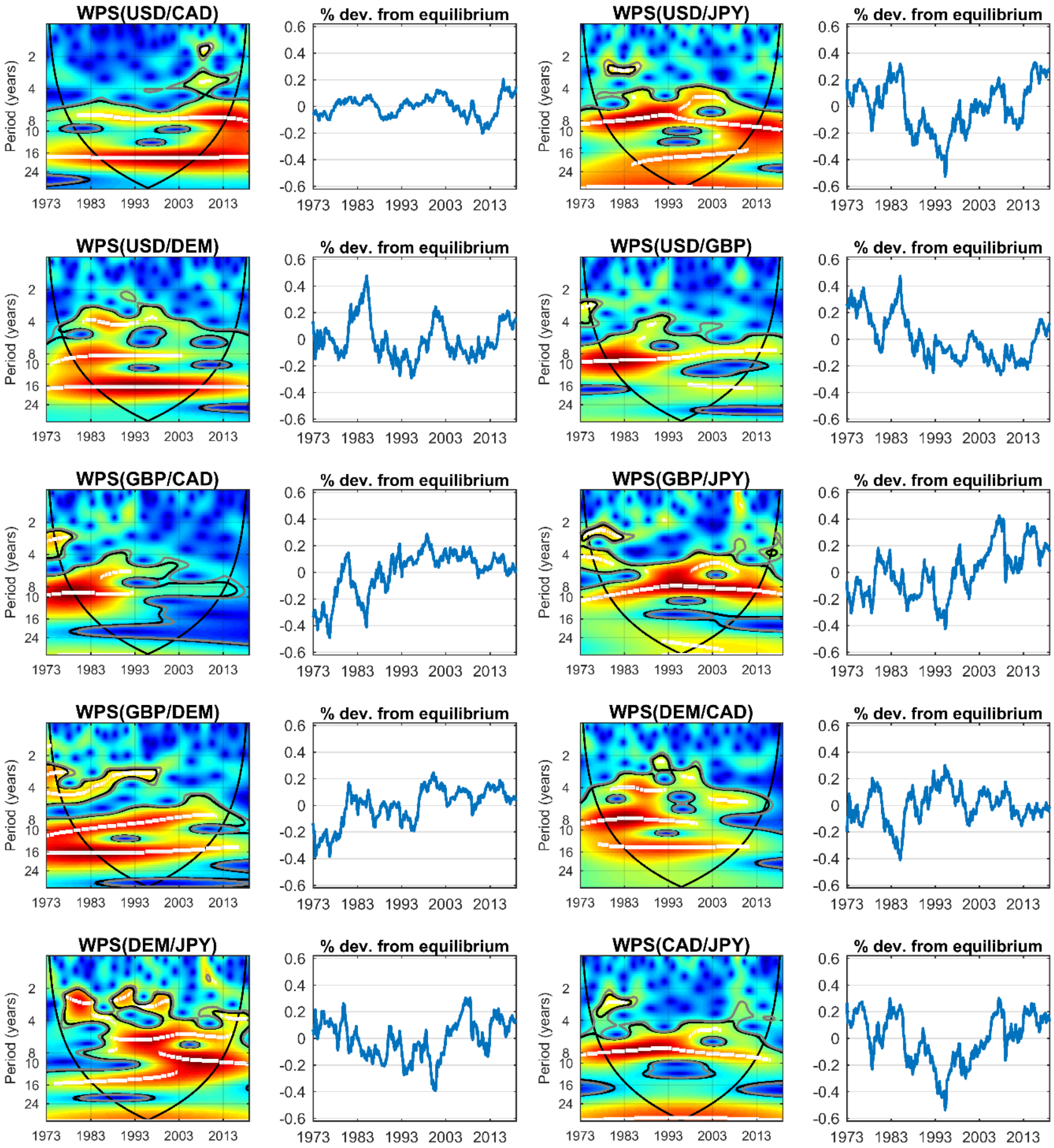

Figure 6: Even Columns: Time-series plots of the nominal exchange rates deviations from their equilibrium value for each pair of countries. **Odd columns:** Wavelet power spectrum, showing the variance of residuals at each time-frequency locus. The color code for power ranges from blue (low power) to red (high power). The white lines show the local maxima of the wavelet power spectrum. The black (grey) contours indicate the significance level of 5% (10%), computed from a theoretical distribution with white noise as null. The cone of influence, which indicates the region affected by edge effects, is shown with a parabola-like black line.

6. Conclusions and Next steps

We presented an overview of the extensive literature created over the last few decades around PPP. Although it is consensual that the theory does not work well in the short-run, long-run evidence is mixed. Much of the uncertainty on the PPP validity in the existing literature is related to estimation problems and econometric weaknesses. This study intended to undergo a more complex analysis, employing a set of continuous wavelet tools, both time and frequency-dependent. Because of the local nature of the CWT, non-linearities and non-stationarities do not represent a challenge. We uncover new stylized facts about the PPP theory that would be hard to detect in the pure time or frequency domains.

We relied on monthly data for the exchange rates and price indexes between the United States, the United Kingdom, Germany, Canada, and Japan from 1973 to 2021. Note that by considering developed low inflation countries, we choose a sample biased toward the rejection of PPP. Previous work has shown that high inflation/depreciation countries tended to confirm the theory.

Our findings confirm that exchange rates are often disconnected from inflation differentials at business cycle frequencies. However, in the long run, these two variables are highly correlated and moving closer to one another, i.e., we found the PPP to be a good description of the data and statistically significant for most of the time low frequencies.

Nevertheless, even at business cycle frequencies, we also found some periods of exception, in which exchange rate changes precede variations in price changes differentials. In other words, the relationship between these variables might still be substantial for business cycle frequencies at times. Notwithstanding, the causality runs from exchange rates to price changes in those situations. This result is in line with the ones obtained by Al-Zyoud (2015). This way of formulating the theory is also consistent with the neoclassic idea and the Fisher Effect in the sense that, due to the goods' sticky prices in the short run, it is reasonable to expect that exchange rates can adjust quickly and more profusely to market and policy changes- e.g., see Cheung & Lai, 2000 and Forbes et al., 2018.

Another significant result retrieved from this work was the fact that the theory works better when PPI is used as a price gauge rather than CPI. Eventually, and according to Shively (2001), Bache et al. (2013), and Rabe & Waddle (2020), this conclusion may be related to the fact that, in the PPI, the weight of tradable goods is more substantial.

We also found evidence that nominal exchange rate deviations from its theoretical values can be described as irregular cycles of periods between 8- and 16-years. Still, these cycles are not always verified simultaneously for all countries.

One of the problems emerging from this analysis is the paradox between the short-run volatility of exchange rates and the time adjustment of exchange rates towards their equilibrium value. Some researchers defend that the predominance of real shocks, such as technological and taste shocks, and the existence of market frictions and inefficiencies not predicted by the model can explain the long period of adjustment of exchange rates (Garratt et al., 2008; Garratt & Lee, 2010; Cheung et al., 2019). Recent studies have stated the importance of considering the impact of other variables on the study of the co-movement and causality between exchange rates and prices differentials, such as the geographical distance between countries, the strength of their trade links, countries' productivity structures, and economic conditions, and the magnitude of financial and market cycles and policies spillovers between different economies (Cho & Doblas-Madrid, 2014; Fornalo, 2015; Ariff & Zarei, 2016; Metiu, 2021; Aristidou et al., 2022).

Although this work does not test the veracity of these hypotheses, we will include other macroeconomic factors in the model in future research.

Besides, although we have used two different indicators to measure prices in this study, they both exclude asset prices. According to some authors, such as Eleftheriou & Müller-Plantenberg (2018) and Reitz & Umlandt (2021), this approach limits the exchange determination to global trade that only involves transactions of economic goods, which nowadays might not be a reasonable assumption given the voluminous weight of international financial assets transactions. Future work should also study the PPP relationship using a more diversified index that includes both tradable goods and assets.

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Appendix

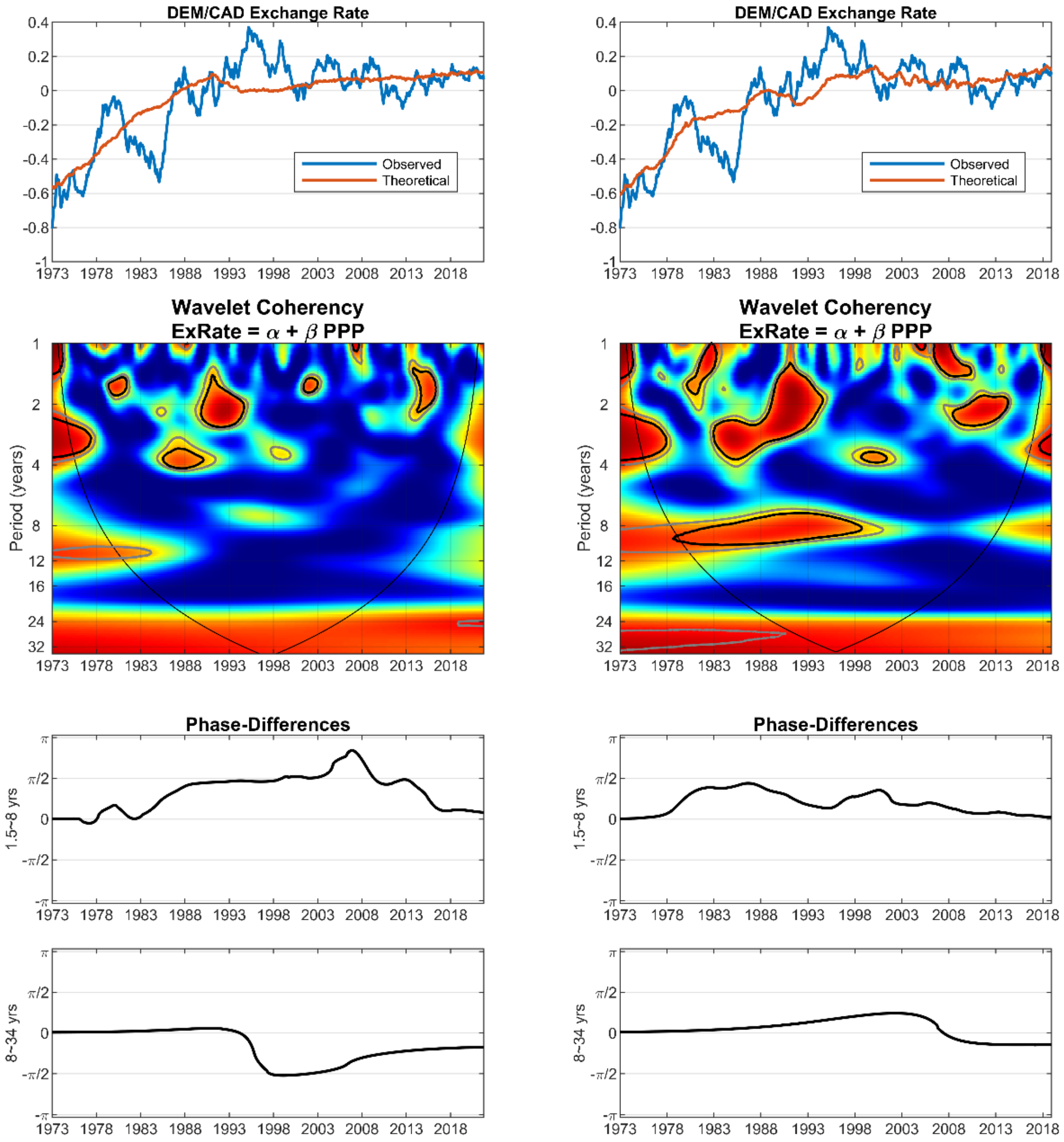

Figure A.1 – Top: Time-series plots of the DEM/CAD market exchange rate and the ratio between Canada and Germany price indexes. **Middle:** Coherency between the market and the expected PPP exchange rates. The color code for coherency ranges from blue (low coherency- close to zero) to red (high coherency- close to one). The black (grey) contours indicate the 5% (10%) significance level for the wavelet coherency, computed with 5000 Monte-Carlo simulations. The cone of influence, indicating the region affected by edge effects, is shown with a parabola-like black line. **Bottom:** Phase differences between cyclical fluctuations of the theoretical and market exchange rates, for the two frequency bands. **Left:** PPP values obtained through consumer price index **Right:** PPP values obtained through producer price index.

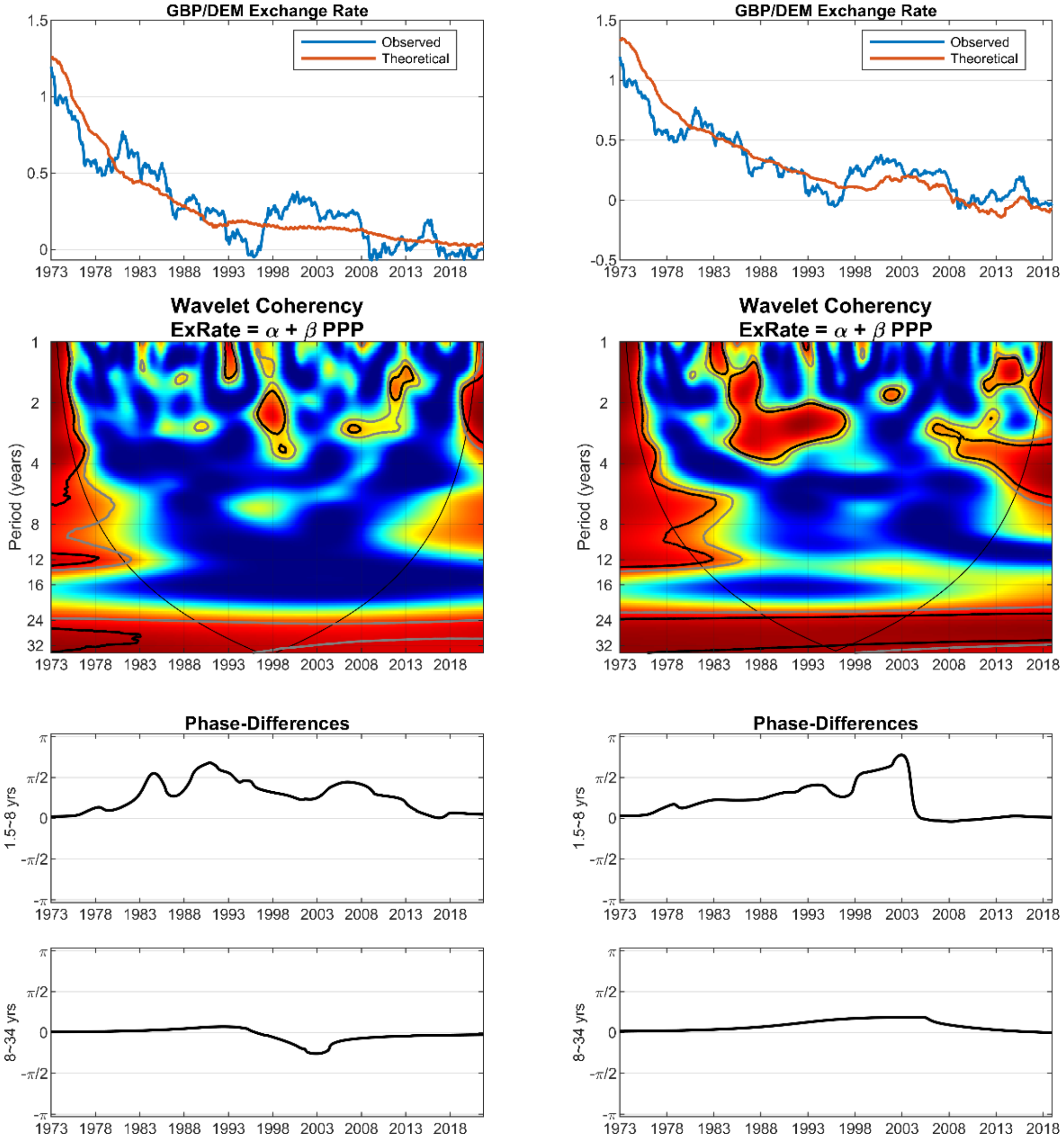

Figure A.2 – Top: Time-series plots of the GBP/DEM market exchange rate and the ratio between Germany and United Kingdom price indexes. **Middle:** Coherency between the market and the expected PPP exchange rates. The color code for coherency ranges from blue (low coherency- close to zero) to red (high coherency- close to one). The black (grey) contours indicate the 5% (10%) significance level for the wavelet coherency, computed with 5000 Monte-Carlo simulations. The cone of influence, indicating the region affected by edge effects, is shown with a parabola-like black line. **Bottom:** Phase differences between cyclical fluctuations of the theoretical and market exchange rates, for the two frequency bands. **Left:** PPP values obtained through consumer price index **Right:** PPP values obtained through producer price index.

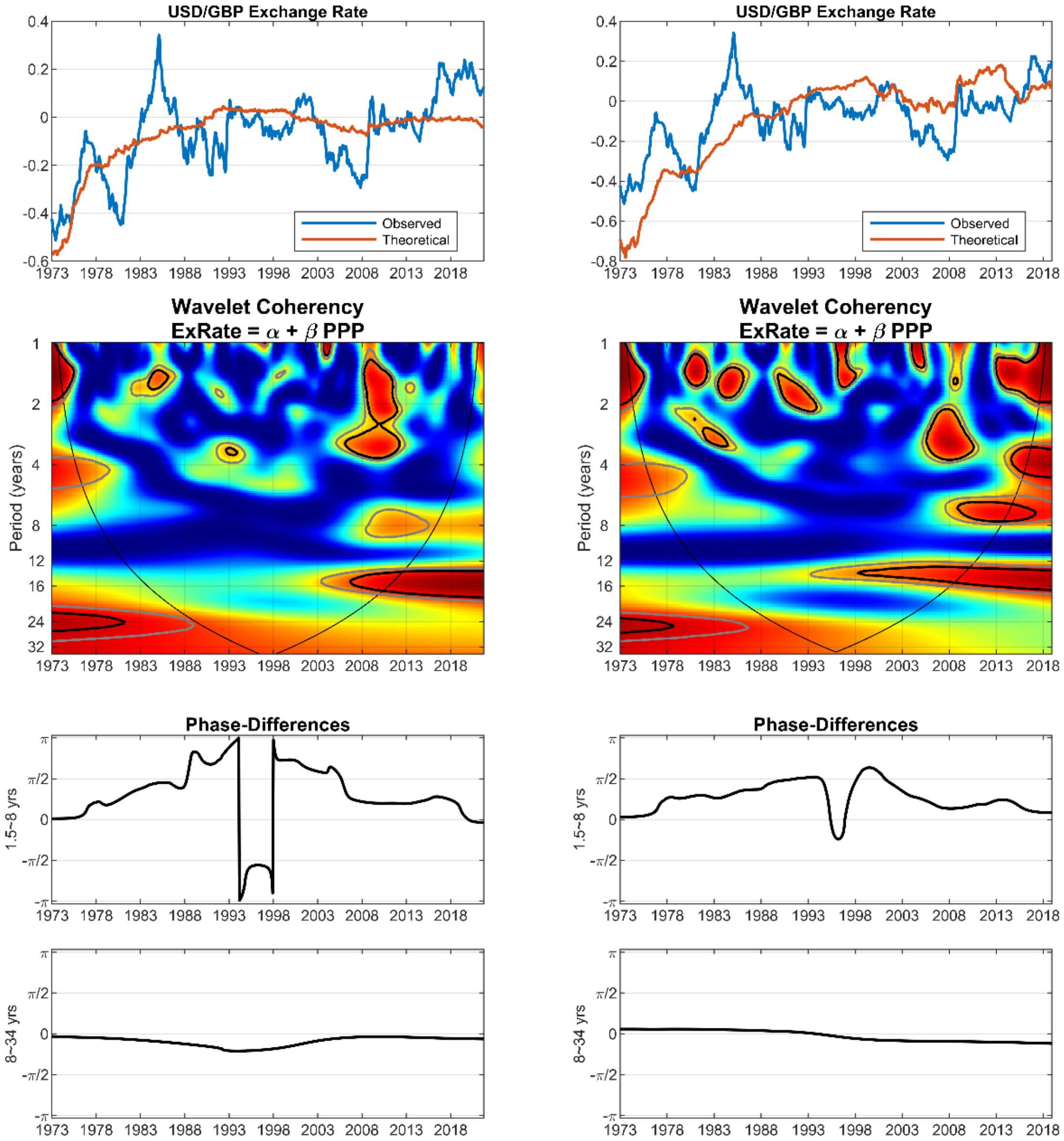

Figure A.3 – Top: Time-series plots of the USD/GBP market exchange rate and the ratio between Germany and United States price indexes. **Middle:** Coherency between the market and the expected PPP exchange rates. The color code for coherency ranges from blue (low coherency- close to zero) to red (high coherency- close to one). The black (grey) contours indicate the 5% (10%) significance level for the wavelet coherency, computed with 5000 Monte-Carlo simulations. The cone of influence, indicating the region affected by edge effects, is shown with a parabola-like black line. **Bottom:** Phase differences between cyclical fluctuations of the theoretical and market exchange rates, for the two frequency bands. **Left:** PPP values obtained through consumer price index **Right:** PPP values obtained through producer price index.

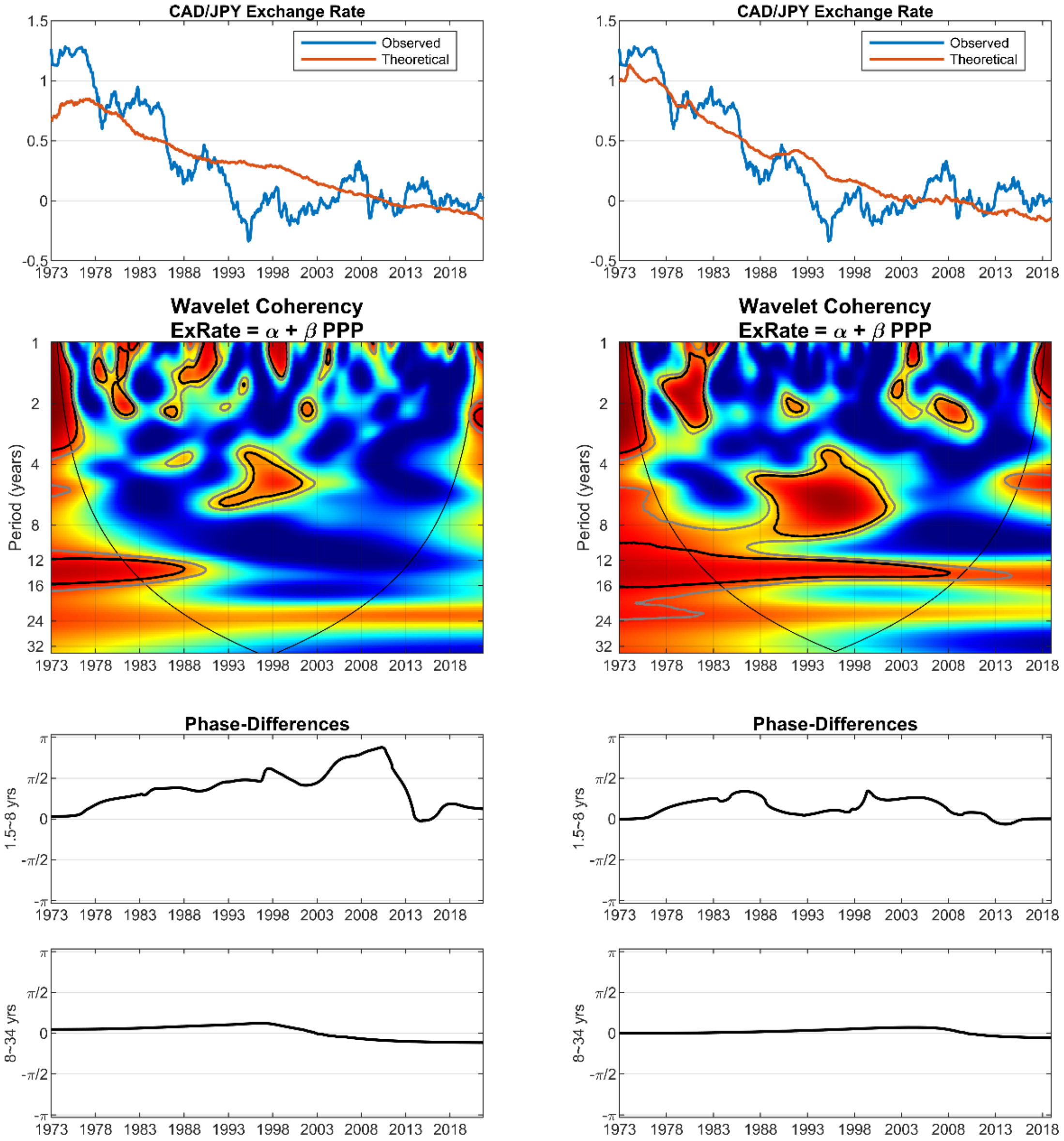

Figure A.4 – Top: Time-series plots of the CAD/JPY market exchange rate and the ratio between Japan and Canada price indexes. **Middle:** Coherency between the market and the expected PPP exchange rates. The color code for coherency ranges from blue (low coherency- close to zero) to red (high coherency- close to one). The black (grey) contours indicate the 5% (10%) significance level for the wavelet coherency, computed with 5000 Monte-Carlo simulations. The cone of influence, indicating the region affected by edge effects, is shown with a parabola-like black line. **Bottom:** Phase differences between cyclical fluctuations of the theoretical and market exchange rates, for the two frequency bands. **Left:** PPP values obtained through consumer price index **Right:** PPP values obtained through producer price index.

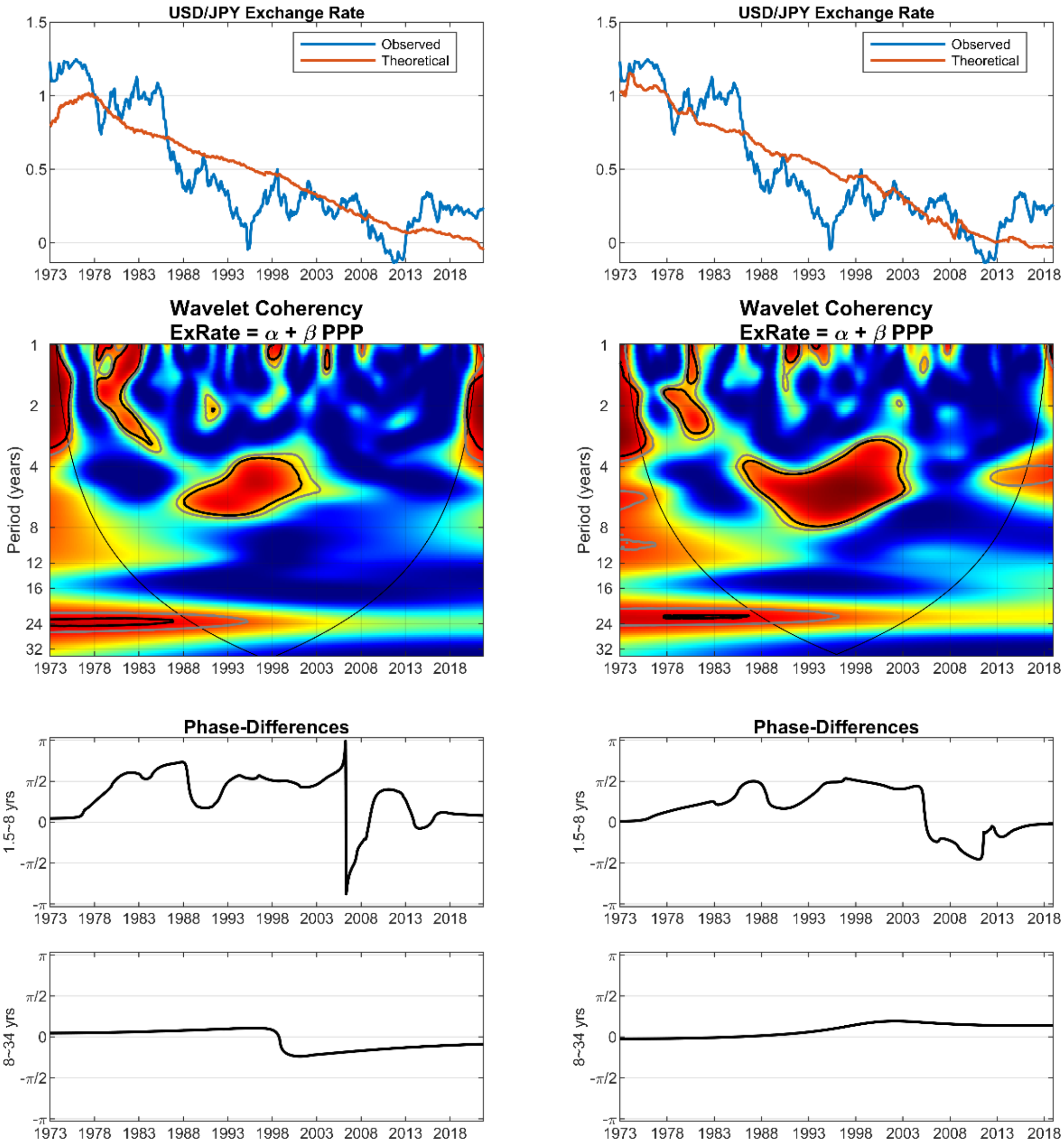

Figure A.5 – Top: Time-series plots of the USD/JPY market exchange rate and the ratio between Japan and United States price indexes. **Middle:** Coherency between the market and the expected PPP exchange rates. The color code for coherency ranges from blue (low coherency- close to zero) to red (high coherency- close to one). The black (grey) contours indicate the 5% (10%) significance level for the wavelet coherency, computed with 5000 Monte-Carlo simulations. The cone of influence, indicating the region affected by edge effects, is shown with a parabola-like black line. **Bottom:** Phase differences between cyclical fluctuations of the theoretical and market exchange rates, for the two frequency bands. **Left:** PPP values obtained through consumer price index **Right:** PPP values obtained through producer price index.

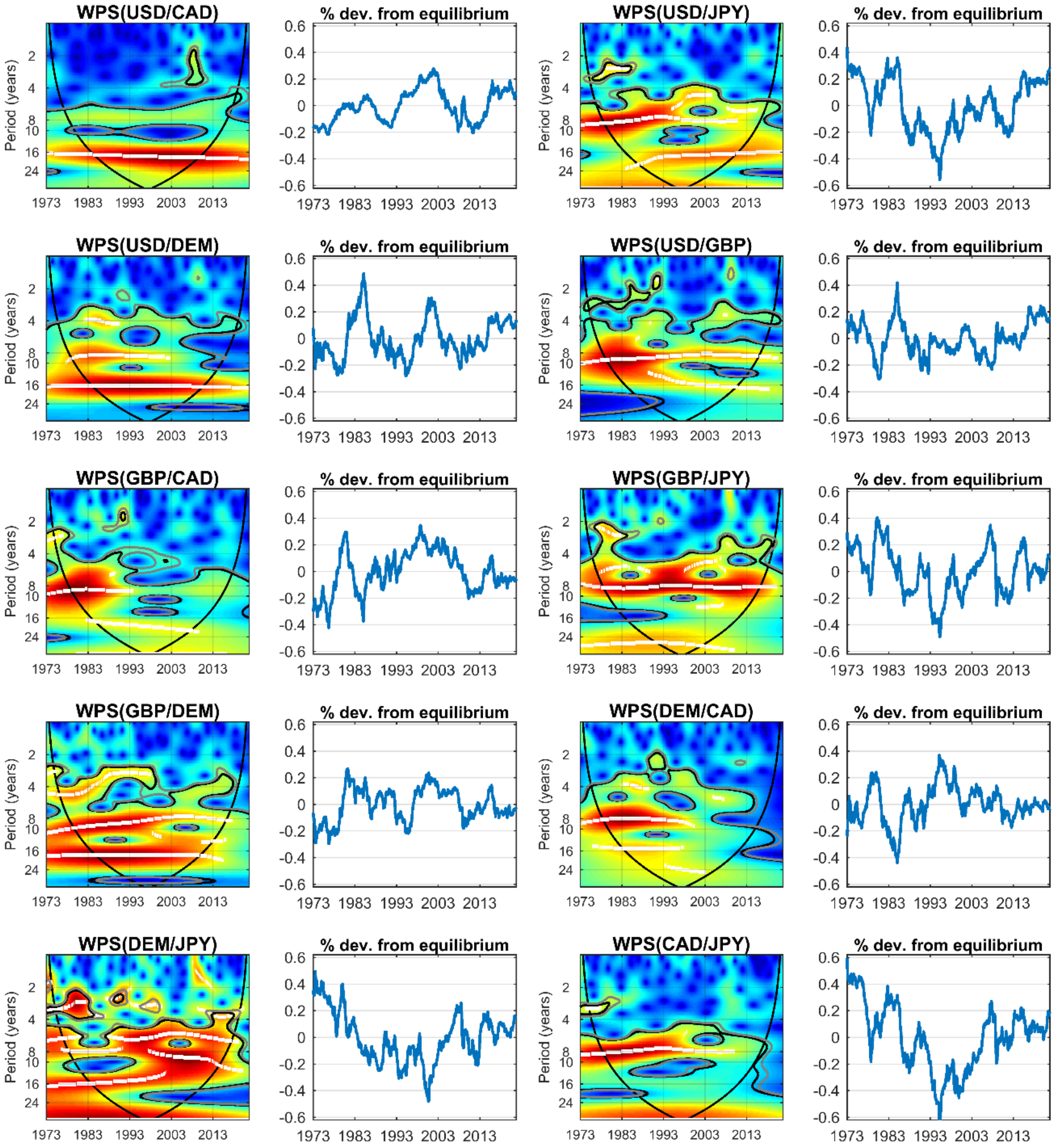

Figure A.6 – Even Columns: Time-series plots of the nominal exchange rates deviations from their equilibrium value for each pair of countries derived from the consumer price index. **Odd columns:** Wavelet power spectrum, showing the variance of residuals at each time-frequency locus. The color code for power ranges from blue (low power) to red (high power). The white lines show the local maxima of the wavelet power spectrum. The black (grey) contours indicate the significance level of 5% (10%), computed from a theoretical distribution with white noise as null. The cone of influence, which indicates the region affected by edge effects, is shown with a parabola-like black line.