An Integer Programming Approach for Sensor Location in a Forest Fire Monitoring System

Beatriz Flamia Azevedo[1,2][0000−0002−8527−7409],
Filipe Alvelos[2][0000−0002−1851−4339],
Ana Maria A. C. Rocha[2][0000−0001−8679−2886],
Thadeu Brito[1,3,4][0000−0002−5962−0517],
José Lima[1,4][0000−0001−7902−1207], and
Ana I. Pereira[1,2][0000−0003−3803−2043]

[1] Research Centre in Digitalization and Intelligent Robotics (CeDRI), Instituto Politécnico de Bragança, Bragança - 5300-253, Portugal
[2] ALGORITMI Centre, University of Minho, Campus Azurém, Guimarães - 4800-058, Portugal
[3] Faculty of Engineering of University of Porto, Porto - 4200 - 465, Portugal
[4] INESC TEC - INESC Technology and Science, Porto - 4200 - 465, Portugal
beatrizflamia@ipb.pt, falvelos@dps.uminho.pt, arocha@dps.uminho.pt, brito@ipb.pt, jllima@ipb.pt, apereira@ipb.pt

Abstract. Forests worldwide have been devastated by fires. Forest fires cause incalculable damage to fauna and flora. In addition, a forest fire can lead to the death of people and financial damage in general, among other problems. To avoid wildfire catastrophes is fundamental to detect fire ignitions in the early stages, which can be achieved by monitoring ignitions through sensors. This work presents an integer programming approach to decide where to locate such sensors to maximize the coverage provided by them, taking into account different types of sensors, fire hazards, and technological and budget constraints. We tested the proposed approach in a real-world forest with around 7500 locations to be covered and about 1500 potential locations for sensors, showing that it allows obtaining optimal solutions in less than 20 minutes.

Keywords: forest fires · sensors location · integer programming · Geographic Information System (GIS) · technological and budget limitations.

1 Introduction

Forest fires are a severe threat to both natural ecosystems and human beings since forests have an essential role in the global environmental and recreational system, such as atmospheric carbon absorption, soil erosion reduction, moderation of the temperature, and regulation of rainfall [1]. So, perturbation into this ecosystem provokes enormous impacts on fauna and flora, causing economic losses and people's deaths, among other problems.

Globally, it is estimated that humans are responsible for around 75% of all forest fires and much of the increase in fire incidents during 2020 can be directly linked to human actions [17]. With the advent of COVID-19 pandemic, some governments diverted resources to the front line fight against the virus, and the forest patrols and enforcement has been scaled back or stopped altogether [17]. Consequently, the forests were even more unprotected, increasing the number of fire alerts across the globe. From the Amazon to the Arctic, in April 2020, the number of fire alerts across the world was up by 13% compared to last year – which was already a record year for fire [17].

In Europe, the Mediterranean region is the most affected by forest fire catastrophes [15]. Portugal, part of this region, is the country with the highest incidence of wildfires. According to [16], in 2017, Portugal registered 19105 rural fires, resulting in a burnt area of 537143 ha, causing more than 100 human deaths. The following year, 2018, was registered 11450 rural wildfires, with a burnt area of approximately 44078 ha. In 2019, were registered 10841 occurrences of rural fires, corresponding to 41622 ha of the burn area [16]. Although these numbers are decreasing, the number of fires and the burnt area annually recorded are still very high, which generates a high economic, environmental, and humanitarian impact on the affected regions.

There is an urgent need to develop strategies that can serve to map the occurrences and damages caused by forest fires in this context. One possible approach is through the development of forest monitoring systems via remote sensing [7,6]. This type of system consists of spreading a set of sensors in the forest environment to collect data. The data can be used both to monitor the environment under normal conditions (without fire occurrence) through temperature, humidity, and CO_2 level and quickly detect anomalies, such as fires, through flame or smoke sensors.

Forest monitoring systems provide authorities support in management, planning, resource, location, pre-fire planning, and emergency decision support. In a forest fire situation, a system like this can be determinant since, according to [8,26], the maximum time interval, from ignition to firefighters' alert response, should not exceed 6 minutes. Otherwise, the fire will be out of control due to the fire's fast propagation speed.

This paper focuses on defining the location of sensors inside the forest environment to constitute a forest fire monitoring system. For this, an integer programming model was developed, in which some sensors with different technical specifications (cost, range distance) are considered, and the forest characteristic (forest density and forest fire hazard) is taken into account to define the optimum position for each sensor.

Location problems have been addressed by optimization, in particular, by integer programming, since the 1960s. For an overview of the topic, we direct the reader to the comprehensive book Laporte et al. [19]. Although some work has been done on locating fire-related resources (e.g. vehicles [11] and stations and trucks in [21]), decisions related to where to locate sensors in an actual landscape, to the best of our knowledge, have not been addressed before.

The remainder of this paper is organized as follows: after the introduction, Sect. 2 presents the state of art of the mixed-integer programming models in the context of forest fire monitoring systems. After that, the parameters and the region used to build the mathematical model are presented in Sect. 3, and the methodology adopted is described in Sect. 4. Section 5 presents the results and discussion of the approach developed. Section 6 concludes the paper and identifies future paths for further research on this subject.

2 State of Art

Forest fires are considered complex events in causes, intensity, behavior, variability, control, size, and severity. Thus, their early detection is crucial since the time of response will determine the level of damage [9,24]. For this purpose, at least since the 1960s, as surveyed in [22], research has been done to support decision-making to prevent, detect, monitor, and control forest fires with integer programming.

In [4] a mixed-integer linear program is used to model the spatial fire behavior interacting with suppression placement. The authors defined the study area as cells, and the fire behavior and suppression placement decision are modeled using nodes associated with the cell centers from raster landscapes. The proposed model evaluates fire arrival times and fire lines intensities based on the direction that a fire spreads into a cell as a response to spatially explicit suppression placement. The information of fire spread rates is defined based on Rothermel's equation, and the maximum fire line intensity is based on Byram's fire line intensity [12]. The model presented also considers "control locations areas" that represent the fire suppression resources, modeled as decision variables that alter fire spread paths. Thus if a control area is located at a flammable node, it is assumed that fire will not spread into that node.

A similar approach can be seen in [27]. The authors introduced an algorithm based on the Delaunay triangulation, shortest path algorithms, and mesh refinement to evaluate surface wildfire propagation through a complex heterogeneous landscape. Geographic Information System (GIS) [23] was used to create a spatial model of the region, which includes data about fuel, topography, and weather conditions. To compute the dynamics of fire perimeter extension, a fire perimeter growth construction method to locate the fire perimeter location is utilized. A fire spread model is also used to evaluate the maximum rate of fire spread and other fire parameters. In this approach, each cell's wind and moisture conditions are defined as constant for some time to ensure continuous fire environments for the polygons used in the methods to delimit the area.

A stochastic model of frame spreading prediction is present on [14]. Although wind speed is considered an unpredictable phenomenon, the proposed model can deal with the unpredictable changes in wind speed. The authors considered this problem a stochastic shortest path problem. The landscape is represented as a graph network and the fire propagation time is associated with probabilities for the wildfire arrival time at a point of interest (residences, firemen camp, etc). To solve the proposed model, the Monte-Carlo simulation is used, and a network size reduction methodology is introduced to optimize the network, removing the redundant edges to speed up the simulation time.

Another interesting work is found in [25], which proposes an integer linear programming model aiming to select the optimal resources to be applied during a planning period for forest fire extinction. In this case, historical data is used to obtain the parameters of cost and resource performance.

In [2] four models integrating fire spread in mixed integer programming are presented in order to solve the location of the optimal resources for the fire forest problem. The first one is for protecting areas, the second for minimizing burned areas, and two others considering fire containment problems.

Resource location in large areas such as forests is hard due to the numerous possibilities to locate them. Thus, developing strategies based on mathematical models to define the optimal position that maximizes forest protection is an important area of study to support fighting the fire combats. Considering the state of the art presented, the mixed-integer linear program is a meaningful approach to deal with the sensor location problem; thus it will be used in this work. Integer programming, since its beginnings in the late 1950s [18] has been used in location analysis [20].

3 Case Study: Experimental Forest Region

The methodology to be developed in this work, will be applied in the region named "Serra da Nogueira", located in the municipality of Bragança, Portugal, as shown in Fig. 1.

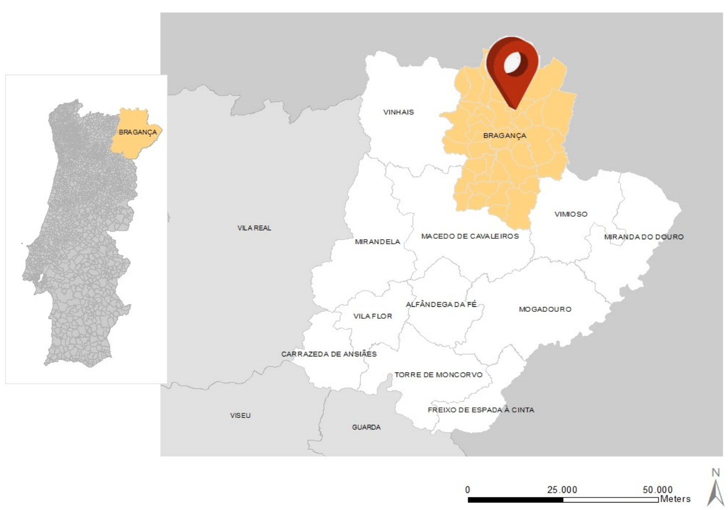


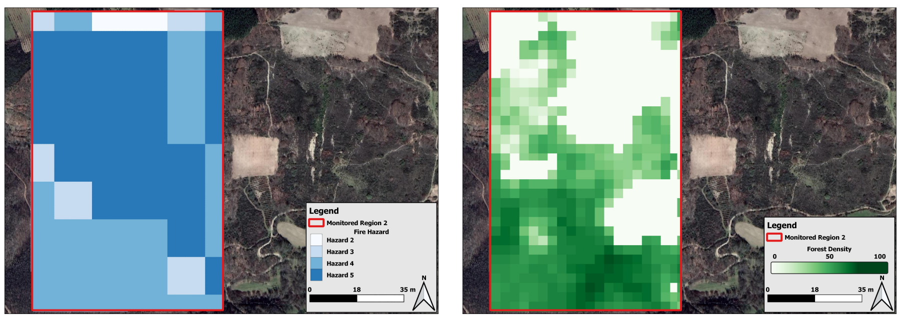
Fig. 1: Serra da Nogueira location

The "Serra da Nogueira" is composed of approximately 13 km^2, which represents a large territorial extension and a complex problem for the implementation of a forest monitoring system. For this reason, an experimental region was defined to carry out the methodology presented in this work. Thus, an area of nearly $246,875$ m^2 is defined as a forest experimental region, which is presented in Fig. 2. This region was strategically defined due to its heterogeneity, such as different fire hazards, density levels and presence of agriculture or non-agricultural areas.

By the QGIS software [23], it was possible to map this region, considering the fire hazard and forest density levels. These data are provided by ICNF [16] and Copernicus [10], respectively, according to the coordinate system ETRS89/PT-TM06 (EPSG:3763) UTM Zone 29N standard with Mercator Transverse Universal projection.

Fig. 2: Forest experimental region.

The fire hazard can be described as the probability of the event occurrence associated with the terrain conditions. As presented in [28], the fire hazard encompasses two dimensions, space and time, which are intrinsically related to probability and susceptibility. The probability assessment can be based on the historical data of the event for the region that can be considered an uncertain indicator of the fire occurrence [29]. On the other hand, the susceptibility is addressed to aspects of the terrain considered [29]. A territorial unit will be more or less susceptible as affected or potentiates the phenomenon's occurrence and development. In the case of forest fires, a given area will be more susceptible the better it allows the deflagration and/or the progression of the fire spreading [30]. According to [29], it is possible to estimate a fire hazard scale from 0 to 5, according to the previous information presented. In this way, a level 0 indicates a low fire hazard, and level 5 indicates a high hazard fire. Another parameter used is the forest density, which describes the size of the vegetation coverage inside the region demarcated. In this work, the forest density varies from 0 to 100, where 0 indicates no presence of vegetation and 100 indicates a high concentration of vegetation in $40\,\mathrm{m}^2$. More details about both parameters can be verified in [3]. Figure 3a shows the forest fire hazard map, whereas Fig. 3b presents the forest density map of the region considered. Note that fire hazard level is not strictly dependent on the forest density since it is a variable related to many other parameters; as previously presented, thence the regions with higher fire hazards are not the same areas with the highest concentration of vegetation.



(a) Forest fire hazard map.

(b) Forest density map.

Fig. 3: Forest Experimental Regions.

4 Problem definition and mathematical model

The problem herein described aims to decide where to locate a set of sensors of different types to cover the maximize the coverage (weighted by a hazard index). The region to be monitored was demarcated by cells of $5m^2$, with, consequently, 5 meters of distance between the centers of adjacent cells. This measure was defined by technical experiments, to evaluate the behavior of the sensors over different distances and constraints of the problem [5,7].

The cell's central point is represented by a node, thus considers, a given sensor k can be placed on a cell j, for $j = 1, ..., n$, with a given coverage that depends on the forest density parameter. When a sensor k, for $k = 1, ..., l$, is assigned to a cell j, it is necessary to identify which cells i are covered by this sensor. It is important to mention that it is considered that each sensor is capable of covering points in any direction, i.e., in 360 degrees. To define if a sensor covers a node, firstly, the Euclidean Distance, d_{ji}, between the cells' nodes c, inside the map, is evaluated by (1).

$$d_{ji} = ||c_j - c_i||_2, \quad j = 1, ..., n; \quad i = 1, ..., n. \tag{1}$$

However, the coverage sensor distance function, $v^k(c_j, c_i)$, is the k sensor view and it depends on the forest density of the cell where the sensor is located f_j^d, the forest density of the cell that can be covered f_i^d, and also the sensor maximum covered distance d_{max}^k. Thereby, the coverage sensor distance is given by Equation (2).

$$v^k(c_j, c_i) = d_{max}^k \times \left(1 - \frac{f_j^d + f_i^d}{2f_{max}^d}\right) \quad (2)$$

If the distance (d_{ji}) between the sensor located on c_j and the cell c_i is smaller than the coverage sensor distance $v^k(c_j, c_i)$, the cell c_i is covered by the sensor k placed on c_j cell, that is Equation (3),

$$d_{ji} \quad \leq v^k(c_j, c_i), \quad (3)$$

In our model, this information is represented by setting the parameter $a_{ij}^k = 1$, if sensor k located in j covers cell i; $a_{ij}^k = 0$ otherwise. Besides this, the following notation is introduced:

- n number of nodes (cells);
- l number of sensors that can be assigned to cells;
- c_k unit cost of sensor k, $k = 1, ..., l$;
- b available budget for sensors;
- h_i hazard of cell i, $i = 1, ..., n$.

Decision variables:

- $y_j^k = 1$, if sensor k ($k = 1, ..., l$) is located in cell j ($j = 1, ..., n$); 0 otherwise;
- $x_i = 1$, if a cell i is covered; 0 otherwise;

Objective function:

$$Max \quad z = \sum_{i=1}^{n} h_i x_i \quad (4)$$

Subject to:

$$\sum_{j=1}^{n} y_j^k \leq 1, \quad k = 1, ..., l \quad (5)$$

$$\sum_{k=1}^{l} y_j^k \leq 1, \quad j = 1, ..., n \quad (6)$$

$$\sum_{k=1}^{l} c_k \sum_{j=1}^{n} y_j^k \leq b \quad (7)$$

$$x_i \leq \sum_{j=1}^{n} \sum_{k=1}^{l} a_{ij}^k y_j^k, \quad i = 1, ..., n \tag{8}$$

$$x_i \in \{0, 1\}, \quad i = 1, ..., n \tag{9}$$

$$y_j^k \in \{0, 1\}, \quad j = 1, ..., n; \quad k = 1, ..., l \tag{10}$$

Objective function (4) maximizes the forest fire hazard covered. Constraints (5) and (6) state that a sensor is not used or is located in a single cell and each cell can accommodate at most one sensor. Constraint (7) is a budget constraint. Constraint (8) is the covering constraint, stating that a covered location must be within the distance of at least one sensor. Equation (9) and (10) are integrality constraints.

5 Results

The sensors will be fixed on the tree trunks, so at least one tree is required on the cell indicated by the solution. In this sense, only points with forest density over or equal to 80 were considered candidates to receive a sensor. This value also ensures appropriate trees in the region to fix the sensors. Moreover, only the cell with a forest fire hazard equal to 5 can receive a sensor in this approach. Thence, after a filter in the cell of the original map, 1499 remains cells on the map to locate the sensors, being 7495 the sum of forest fire hazard considering all points available. Five types of sensors were considered, in different quantities, cost, and maximum coverage range, as presented in Table 1.

Table 1: Available sensor types

Sensor Type	Quantity Available	Unit Cost (€)	Coverage Radio (m)
A	10	45	15
B	5	80	30
C	7	180	100
D	5	350	200
E	3	1000	500

It is important to mention that the coverage distance varies according to the forest density, as mentioned in Sect. 4. Thence, the values presented in Table 1 correspond to the maximum value that the sensor can reach when there is no forest density interference. In practice, the reached values are defined by Equation (2).

A maximum budget was stipulated to be spent on purchasing and installing sensors. In this way, a solution defines the optimal number and sensor types according to the budget for each experiment.

Gurobi interface for Python was used to define the model and the general purpose mixed integer programming solver, the Gurobi 9.5 [13], was used for the optimization in an Intel(R) i5(R) CPU @1.60 GHz with 8 GB of RAM machine. Python was used to manipulate the data structures involved.

5.1 Results of Experiment 1

On the first experiment the budget considered was 2000 euros. Figure 4 presents the results of the experimental region, having 5 sensors located: sensor 1 - type C, sensor 2 - type B, sensor 3 and 4 - type D, sensor 5 - type E.

The cells (or areas) demarcated by green are regions covered by at least one sensor, while any located sensor does not cover regions in red. It is essential to clarify that the white areas correspond to cells that are not eligible to receive sensors due to the forest density or the forest fire hazard constraints already mentioned.

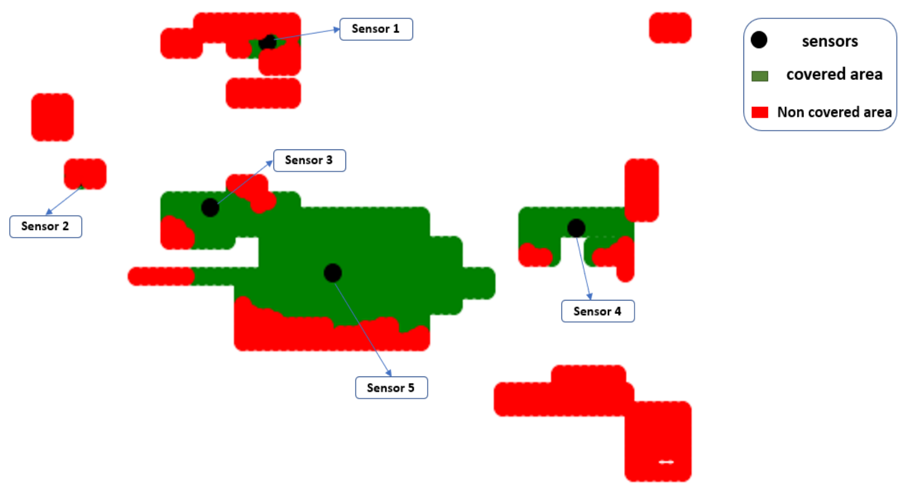


Fig. 4: Results considering a budget equal to 2000 euros.

Through the proposed layout, it can be highlighted that the optimal solution is to locate the more extended range sensors in the areas where there were more points to be covered to maximize the forest fire hazard covered (e.g. sensor 5, which has greater coverage capacity, was positioned in the central region where there are more points to be covered, while sensor 1 and 2, which have less coverage, are in areas with fewer points blue. Another point that can be observed is that although sensor 1 is in a region of a high concentration of points to be covered, the budget constraint did not support the allocation of a higher value sensor and, consequently, greater coverage capacity.

The cost of this layout is 1960 euros providing coverage of 932 cells, which corresponds to a fire hazard of 4660 units. In this way, it is possible to reduce 62.17 % in the region's total fire hazard. It is important to mention that the exact sensor's location can be found confronting the cell location with the map coordinates, considering the grid pre-established. The optimal solution for the default relative optimality gap of $1e-4$ was obtained in 98 seconds (GPU time), using an 8-core processor and 2316 nodes were explored in 46358 simplex iterations.

5.2 Results of Experiment 2

On the second experiment, the budget considered is 4000 euros. By this way 10 sensors was located as presents in Fig. 5: sensor 1 - type A, sensor 2 - type D, sensor 3, 4 and 5 - type C, sensor 6 and 7 - type D, sensor 8 - type E, sensor 9 - type D, sensor 10 - type E.

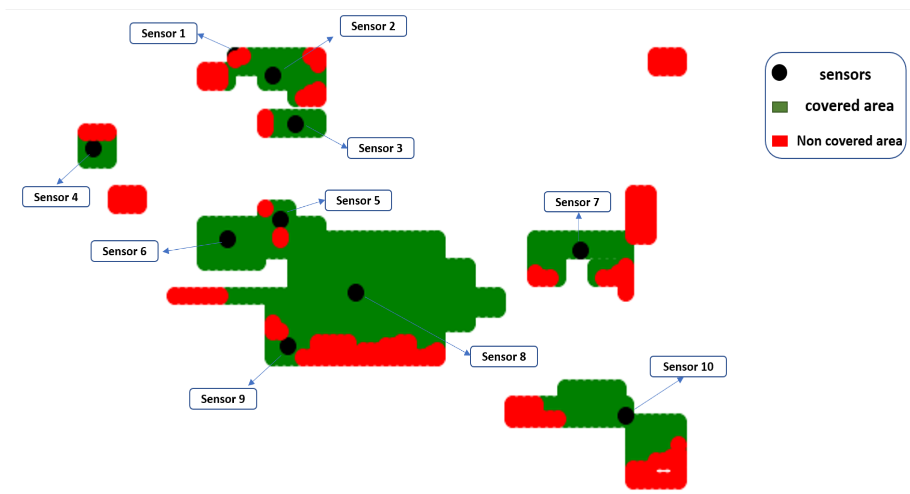


Fig. 5: Results considering a budget equal to 4000 euros.

In the second experiment, we have the same region used in the first, but this time we have a higher budget. However, the same behavior observed in the first experiment can be observed in the second one; that is, the longer-range sensors are allocated in regions with the highest concentration of points to maximize the sum of forest fire hazards over surveillance.

The cost of this layout is 3985 euros, providing coverage of 1253 cells, which corresponds to a forest fire hazard of 6280 units. In this way, it is possible to reduce 83.79% in the region forest fire hazard. In this experiment, the optimal solution, for the default relative optimality gap of $1e-4$, was obtained in 65 seconds (GPU time), using an 8-core processor and 1373 nodes were explored in 75411 simplex iterations.

6 Conclusion

Forest fire causes environmental disasters and physical and financial damage to the entire ecosystem. For this reason, studying the topic and developing solutions are of great relevance. Currently, multiple techniques and strategies are being searched, proposed, and implemented to solve the emergence problem of wildfires. However, finding an efficient solution to deal with forest fires and replicating them in different regions is not a simple task due to the forest environment's complex dynamics.

This work was conducted to solve the problem of locating wireless sensors in a forest to detect fire ignitions. Being an established approach for location problems, an integer programming model was developed and tested. The results demonstrate that the methodology under development has great potential to assist a decision support system of forest fire detection, in terms of optimal resource location, in this case, sensors.

The proposed integer programming model obtained optimal solutions to a case study in the region of Bragança in less than 20 minutes, which is acceptable given the time horizon of the decision-making process. The model can be extended to deal with several variants, including i) using different weights for cells with a different scale from the one used (e.g., increasing the relative importance of low-risk cells) and ii) minimizing the cost with constraints stating which cells must be covered.

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