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The duration of business cycle expansions and contractions: Are there change-points in duration dependence?

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Abstract

The issue of whether the likelihood of an expansion or contraction ending is dependent on its age, i.e whether they are duration dependent, is widely addressed in the business cycles literature and evidence of positive duration dependence is found in several studies. However, there is an important issue that has not been explored in this literature yet: the presence of change-points in duration dependence. All the studies in this field depart from the assumption that the magnitude of duration dependence is the same over time. However, we conjecture that the degree of likeliness of an expansion or contraction ending as it gets older might change after a specific duration. To test for that possibility, this paper will allow for the presence of a change-point in the analysis of the duration of expansions and contractions for a group of 13 European and Non-European industrial countries over the period 1948-2009.

The evidence provided by the estimation of a continuous-time Weibull duration model shows strong support for the presence of positive duration dependence, which is stronger for contractions than for expansions. Results also show that contractions have become longer over time and that their length is negatively affected by the length of the previous expansion. Most importantly, this paper provides quite interesting evidence for the presence of a change-point in duration dependence for expansions, but not for contractions. Results show that the magnitude of the duration dependence parameter decreases significantly when an expansion surpasses 10 years of duration. In particular, evidence of positive duration dependence is no longer found when an expansion surpasses that threshold.

Keywords: *business cycles; expansions; contractions; duration dependence.*

JEL Classification: *C41, E32.*

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1 Introduction

A widespread idea in the business cycles literature is that the older an expansion or contraction is, the more likely it is to end. Technically, this concept is known as positive duration dependence. The issue of whether business cycles are duration dependent - i.e. whether the likelihood of an expansion or contraction ending is dependent on its age - has gained special interest in the last two decades, due to an increase in the average duration of expansions and a decrease in the duration of contractions after World War II (WWII). Some studies have been successful in finding evidence of positive duration dependence for expansions or contractions (or both), in particular, for the United States (Sichel (1991), Diebold et al. (1993), Durland and McCurdy (1994), Kim and Nelson (1998), Zuehlke (2003), Lam(2004), Davig (2007), and Layton and Smith (2007) and Zhou and Rigdon (2008)), France, Germany and United Kingdom (Diebold et al., 1990), Australia (DiVenuto and Layton, 2005), Japan (Iiboshi, 2007), Switzerland (Perruchoud, 2008) and a panel of thirteen industrial countries (Castro, 2010).

Nevertheless, two other issues are not so well explored in this literature. The first is related to the very own duration of expansions and contractions. We know that expansions, after WWII, tend to last longer than contractions, therefore, we would expect that the magnitude of positive duration dependence might be superior for contractions than for expansions. However, little empirical analysis is done to explore this aspect. On the other hand, it is not sure that the magnitude of duration dependence is the same over the entire duration of an event, as it is assumed by all the studies in this field. In fact, the degree of likeliness of an expansion or contraction ending as it gets older may change after a certain duration. For example, the likelihood of an expansion ending as it gets older may increase over time (positive duration dependence) in a first moment, but become constant (no duration dependence) after a certain duration. While the first idea is somehow discussed by Lam (2004) and Castro (2010), this second important issue, to our knowledge, has not been addressed in the literature yet. This is an important aspect that we intend to explore in this paper.

Parametric and non-parametric duration models - and even Markov-switching models - have been the models mainly used to test for the presence of positive duration dependence, especially in the US business cycle phases. This has been the case because their turning-point dates have been well documented by the National Bureau of Economic Research (NBER) for a long time. As the Economic Cycle Research Institute (ECRI) has recently built similar chronologies for other countries, a new branch of research is open to be explored using duration analysis. The main aim of this paper is to explore that data applying a parametric continuous-time duration model to the analysis of the duration of expansions and contractions in a panel of 13 European and Non-European industrial countries over the post-WWII period. In particular, this study intends: (1) to confirm the evidence of positive duration dependence in expansions and contractions in that group of countries; (2) to show evidence of significant differences in the magnitude of positive duration dependence between expansions and contractions; (3) to find whether there are differences in the magnitude of positive duration dependence between European and Non-European countries and whether there are significant differences in the respective average duration of expansions and contractions; (4) to analyse whether the duration of the previous phase affects the duration of the next and whether their durations have become significantly longer or shorter over time; (5) and, most importantly, to check whether the magnitude of duration dependence may change over time, i.e. to test whether there is evidence for the presence of change-points in duration dependence.

Regarding these five points, the evidence provided in this paper is quite fruitful since we find strong support for the presence of positive duration dependence, which is stronger for contractions than for expansions but not significantly different for European and Non-European countries; however, expansions and contractions seem to last longer in the European than in the Non-European countries; additionally, results show that contractions (and expansions) have become longer over time and that their length is negatively affected by the length of the previous expansion; and, last but not least, we find quite interesting evidence for the presence of a change-point in duration dependence for expansions, but not for contractions. Results show that the magnitude of the duration dependence parameter decreases significantly when expansions surpass 10 years of duration. In particular, evidence of positive duration dependence is no longer found when an expansion surpasses 10 years of duration. This represents a remarkable new finding in this field of research and an important contribution to the literature.

The rest of the paper is organized as follows. Section 2 reviews the existing literature on the duration of expansions and contractions. Section 3 presents the empirical model and estimation methodologies. Section 4 describes the data and the hypotheses to test. Section 5 discusses the empirical results. Finally, Section 6 concludes emphasizing the main findings of this paper.

2 Review of the Literature

According to Sichel (1991), the literature on the duration of business cycles has focused in finding an answer to the question: "Are periods of expansion or contraction in economic activity more likely to end as they become older? More technically, do business cycles exhibit positive duration dependence?" (Sichel, 1991, p. 254). Several authors have tried to answer this question using either (parametric and non-parametric) duration models or Markov-switching models. Traditionally, far more interest has been given to the United States (US) business cycle because their turning-point dates are well documented by the NBER. Nevertheless, other industrial countries – like, for example, Australia, France, Germany, Japan and the United Kingdom – have also been under the scope of some of those studies. On the other hand, parametric duration models and Markov-switching models have proved to be more reliable in detecting the presence of positive duration dependence for expansions or contractions than non-parametric duration models.[1] For this reason, we will focus our brief review of literature on the first two kinds of models or approaches to the analysis of the duration of expansions and contractions.[2]

Using a continuous-time Weibull duration model and the NBER monthly chronology for the US from 1854 to 1990, Sichel (1991) finds significant evidence of positive duration dependence for pre-WWII expansions and post-WWII contractions, but not for the other phases. Diebold et al. (1990) also use a Weibull model to test for duration dependence in France, Germany and United Kingdom in the pre-WWII period and reach the same conclusion as Sichel (1991) for that period.

Other more flexible continuous-time parametric methods are applied in other studies that test for duration dependence. Diebold et al. (1993) employ an exponential-quadratic hazard model to business cycle data, but they are only able to reproduce the results obtained by Sichel's (1991). Using a generalized Weibull model, that nests the simple Weibull model, Zuehlke (2003) finds some additional evidence of duration dependence in pre-WWII US contractions, but his model does not improve upon Sichel's (1991) Weibull specification in any of the other cases. Moreover, he only finds evidence of positive duration dependence in post-WWII expansions and contractions once the sample is extended through 2001.[3] Davig (2007) criticizes the fact that Sichel (1991) and Zuehlke

[1] Sichel (1991) notices that two important advantages of the parametric approach are: the fact that parametric techniques may have higher power for detecting duration dependence than non-parametric methods; and the fact that it makes it possible to compute estimates of the magnitude of duration dependence. Another advantage is that parametric approaches permit testing of additional hypothesis by extending the basic model.

[2] Diebold and Rudebusch (1990) and Ohn et al. (2004) represent important references of non-parametric duration analysis applied to the US business cycle. For further references on non-parametric duration approaches to the analysis of the duration of expansions and contractions, see Castro (2010).

[3] Using a different approach that relies on a Poisson process, called modulated power law process, Zhou and Rigdon

(2003) have exogenously and arbitrarily split the sample at WWII. Hence, they develop a model that can endogenously detect a structural shift in a time series of durations. His modified Weibull model detects a shift in the US business cycle phases at around WWII and confirms Sichel (1991) and Zuehlke's (2003) findings: expansions (contractions) only exhibit positive duration dependence before (after) the WWII change-point. When some control variables are added to the model, the change-points for expansions and contractions occur earlier than WWII and contractions no longer exhibit positive duration dependence following the estimated threshold.

Abderrezak (1998) also uses parametric continuous-time hazard models to analyse the issue of duration dependence in a group of eleven industrial countries. However, instead of considering the classical business cycles, this author uses growth cycles.[4] Results from individual-country and pooled regressions show evidence of positive duration dependence in both the whole growth cycles and growth phases (upswings and downswings).

Zellner (1990), Sichel (1991), Abderrezak (1998) and Davig (2007) also analyse whether the duration of the previous business cycle phase may affect the length of the current phase. Neither Sichel (1991) nor Abderrezak (1998) were able to find evidence for this link. However, Zellner (1990) and Davig (2007) provide some evidence that shorter contractions tend to follow longer expansions in the US. In this study, we hope to provide further evidence to clarify this issue. Sichel (1991) and Davig (2007) also test whether the duration of expansions and contractions has become gradually longer or shorter over time. Their results show that US expansions have become longer, but US contractions have become shorter over time. This is another aspect to be considered in the analysis provided in this paper.

Extending the analysis to a panel of industrial countries, for which the ECRI provides business cycle turning points, and using essentially a discrete-time duration model, Castro (2010) provides significant evidence of positive duration dependence for both expansions and contractions for the post-WWII period. Moreover, he also notices that the probability of a contraction ending increases more quickly with its age than an expansion and that shorter contractions are preceded by longer expansions.[5] This same evidence is expected to be found in this paper, but, most importantly, we

(2008) also provide evidence of positive duration dependence in the US business cycles.

[4] Contrary to the classical business cycles, growth cycles are simply identified by increases and decreases in GDP growth rates.

[5] The aim of that paper is not only to find evidence of positive duration dependence but to look at other factors that may affect the duration of an expansion or contraction. To do so, the author has to use a discrete-time duration model since the additional variables to be included in the analysis are time-varying (leading indicators, investment, price of oil, etc.). On the contrary, in this paper we are using a traditional continuous-time duration model and our most important aim is to test whether there are change-points in business cycles duration dependence.

also expect to find new and significant evidence that the magnitude of positive duration dependence may change after a certain duration, i.e. we expect to find a change-point in duration dependence. In that sense, this paper represents an important improvement upon Castro's (2010).

Other authors have modelled the business cycle as the outcome of a Markov process that switches between the states of expansion and recession. Contrary to the approaches described above, this method regards the business cycle as an unobserved stochastic process, so that the reference cycle turning-point dates identified by the NBER are not necessary. Hamilton (1989) was the first to implement this kind of analysis. His model assumes that the likelihood of a country switching from an expansion to a recession (or vice-versa) is not affected by its own duration. Some later studies relaxed this assumption allowing for state transition probabilities to be duration dependent. Durland and McCurdy (1994) apply such a refinement to the US real GNP growth rate series and provide evidence of duration dependence for contractions but not for expansions after WWII.[6] A similar result is obtained by Kim and Nelson (1998) applying a Bayesian approach. Also using a Bayesian approach, Iiboshi (2007) finds evidence of positive duration dependence for Japanese expansions and contractions. Extending Durland and McCurdy's (1994) model by allowing for duration dependence not only in transition probabilities but also in mean growth rates and heteroscedasticity in the noise component, Lam (2004) shows that the probability of an expansion ending decreases *gradually* as it gets older, while the probability of a contraction ending increases *rapidly* a its age increases.

A slightly different approach to the study of business cycle dynamics was implemented by Di Venuto and Layton (2005) and Layton and Smith (2007). They develop a multinomial regime-switching logit model to examine the issue of duration dependence in the Australian and US business cycles, respectively. Contrary to the Markov-switching approach, this discrete-time approach assumes the ex-post observation of business cycle phases (as in the duration models). As this model allows for the use of time-varying covariates, they also include in the equation some leading indices as explanatory variables. Their findings provide evidence of positive duration dependence for both expansions and contractions and their indicators show some power in predicting the termination of either phase.

Despite all these developments, there is an important aspect that has not been addressed in this literature yet: the presence of change-points in duration dependence. All these studies depart

[6] Perruchoud (2008) reaches the same conclusion employing a Markov-switching approach over the Swiss business cycle: he only finds evidence of positive duration dependence for Swiss contractions.

from the assumption that the magnitude of duration dependence parameter is the same over the entire duration of expansions and contractions.[7] However, the degree of likeliness of an expansion or contraction ending as it gets older may indeed change after a certain time. Therefore, to test for that possibility, this paper will allow for the presence of a change-point in the analysis of the duration of expansions and contractions for a group of 13 industrial countries over the period 1948-2009. But before doing that empirical study, it is useful to make a brief description of the model used in this duration analysis. That is the aim of the next Section.

3 Empirical Methodology

The duration analysis has its genesis in studies developed in engineering and medical fields, but its use quickly spread to other sciences. In economics, it started to be used in labour economics to study the duration of periods of unemployment.[8] Due to its properties, this kind of analysis is also suitable for studying the duration of expansions and contractions. In the previous Section, we presented some of the most important references in this field. In this Section, we intend to clarify the methodological steps behind the duration analysis, making the bridge to the study of the duration of expansions and contractions. Next we will allow for the presence of a change-point in the structure of the model used in this analysis. This represents an important novelty of this paper.

3.1 Duration analysis

The duration variable is defined as the number of periods – months in this study – that a country is in a state of expansion or contraction, depending on which phase is being analysed. If T is defined as the discrete random variable that measures the time span between the beginning of an expansion (contraction) and its transition to the other state, the series of data at our disposal $(t_1, t_2, \ldots, t_n)$ will represent the observed duration of each expansion (contraction). The probability distribution of the duration variable T can be specified by the cumulative distribution function:

$$F(t) = Pr(T < t) \tag{1}$$

[7] Only Davig (2007) allows for the presence of a change-point or, more precisely, a structural break in a time series of durations, i.e. over the chronological time, but the approach that we develop in this paper is different since we control directly for the presence of a change-point in the duration dependence parameter and we test whether it changes or not over the duration of the business cycle phases.

[8] See Allison (1982) and Kiefer (1988) for a review of the literature on duration analysis.

This function measures the probability of the random variable T being smaller than a certain value t. The corresponding density function is then $f(t) = dF(t)/dt$. An alternative function to specify the distribution of T is the survivor function, which is $S(t) = Pr(T \geq t) = 1 - F(t)$. This function measures the probability of the duration of an expansion (contraction) being greater than or equal to t. A particularly useful function for duration analysis is the hazard function:

$$h(t) = f(t)/S(t) \tag{2}$$

which measures the rate at which expansion (contraction) spells will be completed at duration t, given that they last until that moment. In other words, it measures the probability of exiting from a state in moment t conditional on the length of time in that state. From the hazard function we can derive the integrated hazard function:

$$H(t) = \int_0^t h(u)du \tag{3}$$

and then compute the survivor function as follows:

$$S(t) = \exp[-H(t)] \tag{4}$$

The hazard function is useful to characterize the dependence path of duration. If $dh(t)/dt > 0$ when $t = t^*$, then there is positive duration dependence in t^*. This means that the probability of an expansion (contraction) ending in moment t, given that it has reached t, increases with its age. Thus, the longer is the expansion (contraction), the higher will be the conditional probability of it ending or reaching a peak (trough). An opposite conclusion is reached if the derivative is negative. If the derivative is zero there is no duration dependence. Several parametric countinuous-time models are proposed to measure the magnitude of duration dependence and the impact of other variables on the likelihood of an expansion or recession ending.[9] The functional form that has been used to characterize and parameterize the hazard function is the so-called proportional hazards model:[10]

$$h(t, \mathbf{x}) = h_0(t) \exp(\boldsymbol{\beta}'\mathbf{x}) \tag{5}$$

[9] See Diebold et al. (1990), Sichel (1991), Diebold et al. (1993), Abderrezak (1998), Zuehlke (2003), Davig (2007) and Castro (2010).

[10] This means that the ratio of the hazard rates for any two observations is constant over time.

where $h_0(t)$ is the baseline hazard function that captures the dependency of the data to duration, $\boldsymbol{\beta}$ is a $K \times 1$ vector of parameters to be estimated and $\mathbf{x}$ is a vector of covariates. The baseline hazard also represents an unknown parameter to be estimated. This model can be estimated without imposing any specific functional form to the baseline hazard function, which results in the so-called Cox Model. However, this procedure is not adequate when we are studying duration dependence. An alternative estimation imposes one specific parametric form for the function $h_0(t)$. The most popular model in the study of the duration of expansions and contractions is the Weibull model, which is described next.

3.2 Basic Weibull model

The Weibull model is characterized by the following (baseline) hazard function:

$$h_0(t) = \gamma p t^{p-1} \tag{6}$$

with $\gamma > 0$ and $p > 0$. In this hazard function, γ is essentially a constant term and p parameterizes the duration dependence. If $p > 1$, the conditional probability of a turning point occurring increases as the phase gets older, i.e. there is positive duration dependence; if $p < 1$ there is negative duration dependence; finally, there is no duration dependence if $p = 1$. In this last case, the Weibull model is equal to an Exponential model. Therefore, by estimating p, we can test for duration dependence in expansions or contractions.

Including this Weibull specification for the baseline hazard function in the proportional hazard function given above in equation 5, we have:

$$h(t, \mathbf{x}) = \gamma p t^{p-1} \exp(\boldsymbol{\beta}'\mathbf{x}) \tag{7}$$

Hence, the respective survival function can be written as follows:

$$S(t, \mathbf{x}) = \exp\left[-H(t, \mathbf{x})\right] = \exp\left[-\gamma t^p \exp(\boldsymbol{\beta}'\mathbf{x})\right] \tag{8}$$

This model can be estimated by Maximum Likelihood. The likelihood function for a sample of $i = 1, \ldots, n$ spells (expansions or contractions) is given by:

$$L(\cdot) = \prod_{i=1}^{n} f(t_i, \mathbf{x}_i) = \prod_{i=1}^{n} h(t_i, \mathbf{x}_i)^{c_i} S(t_i, \mathbf{x}_i) \tag{9}$$

where c_i indicates when observations are censored. They are censored ($c_i = 0$) if the sample period under analysis ends before the turning point has been observed; when the turning points are observed in the sample period, they are not censored ($c_i = 1$).

The corresponding log-likelihood function can be written as follows:[11]

$$\ln L(\cdot) = \sum_{i=1}^{n} \left[c_i \ln h(t_i, \mathbf{x}_i) + \ln S(t_i, \mathbf{x}_i)\right] \tag{10}$$

or, making use of the respective Weibull hazard and survival functions:

$$\ln L(\cdot) = \sum_{i=1}^{n} \left[c_i \left(\ln \gamma + \ln p + (p-1)\ln t_i + \boldsymbol{\beta}'\mathbf{x}_i\right) - \gamma t_i^p \exp(\boldsymbol{\beta}'\mathbf{x}_i)\right] \tag{11}$$

This is the basic structure of the log-likelihood function for the Weibull model that we will estimate in this study to analyse the presence of duration dependence in expansions and contractions in a group of industrial countries. However, this study intends to go a step further relatively to the previous studies in this field, which assume that the magnitude of duration dependence is the same over time. However, that may not always be the case. The degree of likeliness of an expansion or contraction ending as it gets older might change after a certain duration. Thus, we consider that the survival time for expansions and contractions may not follow a basic Weibull distribution, but a Weibull distribution where the parameters characterizing the baseline distribution (or the baseline hazard function) may vary over time for different intervals but remain constant within each interval. The points where the parameters change are called change-points. In particular, we will allow for the possibility of a structural break in the Weibull model used to study the duration of expansions and contractions. We conjecture that the parameters of the baseline hazard function (γ and p) may change at a certain point in time. In particular, we expect that the degree of duration dependence (p) may change after the event has last more than a certain time. Therefore, we do not only expect that the likelihood of an expansion or contraction ending increase over time, but we also expect that if they have last more than a certain time, the likelihood of ending may change significantly after that point, i.e. the magnitude of duration dependence (p) may decrease or increase from that point onwards.

[11] See Allison (1982) and Kiefer (1988) for details.

3.3 Weibull model with change-points

For commodity, lets re-write the Weibull baseline hazard function on a slightly different way:[12]

$$h_0(t) = \gamma p t^{p-1} = \lambda p (\lambda t)^{p-1} \tag{12}$$

where $\gamma = \lambda^p$. Hence, the survival function can be written as:

$$S(t, \mathbf{x}) = \exp\left[-H(t)\exp(\boldsymbol{\beta}'\mathbf{x})\right] \tag{13}$$

where $H(t) = (\lambda t)^p$ is the baseline integrated hazard function. Denoting $g(t) = \ln H(t)$ and considering a change point τ_c and two intervals, $t_0 < t \leq \tau_c$ and $\tau_c < t \leq t_T$, $g(t)$ becomes:

$$g(t) = \ln(\lambda_j t)^{p_j} \tag{14}$$

with $j = 1, 2$, regarding that we have two intervals. Due to the fact that the continuity of $g(t)$ has to be verified in the change-point τ_c, we must impose that:

$$\ln(\lambda_1 \tau_c)^{p_1} = \ln(\lambda_2 \tau_c)^{p_2} \tag{15}$$

solving this in order to p_2, we get:

$$p_2 = p_1 \frac{\ln(\lambda_1 \tau_c)}{\ln(\lambda_2 \tau_c)} \tag{16}$$

Thus, for the survival time ending at the first interval, we have:

$$g(t) = p_1 \ln(\lambda_1 t) \tag{17}$$

and for the survival time ending at the second interval, we have:

$$g(t) = p_1 \ln(\lambda_2 t) \frac{\ln(\lambda_1 \tau_c)}{\ln(\lambda_2 \tau_c)} \tag{18}$$

Considering the i-th spell (or individual), we get:

[12] The Weibull model with change-points developed in this study follows the general model proposed by Lara-Porras et al. (2005) for cases where the Weibull distribution, or the respective parameters characterizing the baseline hazard function, may vary over time for different intervals but remain constant within each interval.

$$g(t_i) = d_i p_1 \ln(\lambda_1 t_i) + (1-d_i) p_1 \ln(\lambda_2 t_i) \frac{\ln(\lambda_1 \tau_c)}{\ln(\lambda_2 \tau_c)} \tag{19}$$

where $d_i = 1$ if $t_0 < t \leq \tau_c$ and $d_i = 0$ if $\tau_c < t \leq t_T$ and $i = 1, 2, ..., n$ (number of spells).

Let $H(t_i, \mathbf{x}_i) = \exp[g(t_i) + \boldsymbol{\beta}'\mathbf{x}_i]$. Hence, the hazard function is given by:

$$h(t_i, \mathbf{x}_i) = H'(t_i, \mathbf{x}_i) = g'(t_i) H(t_i, \mathbf{x}_i) = \left[d_i \frac{p_1}{t_i} + (1-d_i) \frac{p_1}{t_i} \frac{\ln(\lambda_1 \tau_c)}{\ln(\lambda_2 \tau_c)} \right] H(t_i, \mathbf{x}_i) \tag{20}$$

and the respective survival function can be expressed as:

$$S(t_i, \mathbf{x}_i) = \exp\left[-H(t_i, \mathbf{x}_i)\right] \tag{21}$$

This means that the log-likelihood function $\ln L(\cdot) = \sum_{i=1}^{n} \left[c_i \ln h(t_i, \mathbf{x}_i) + \ln S(t_i, \mathbf{x}_i)\right]$ can be written as follows:

$$\ln L(\cdot) = \sum_{i=1}^{n} \left\{ c_i \left[\ln g'(t_i) + g(t_i) + \boldsymbol{\beta}'\mathbf{x}_i \right] - \exp\left[g(t_i) + \boldsymbol{\beta}'\mathbf{x}_i \right] \right\} \tag{22}$$

where $g'(t_i) = d_i \frac{p_1}{t_i} + (1-d_i) \frac{p_1}{t_i} \frac{\ln(\lambda_1 \tau_c)}{\ln(\lambda_2 \tau_c)}$. This model is estimated by Maximum Likelihood, given a particular change-point τ_c. The relevance of that change-point will be evaluated by testing whether there is a significant statistical difference between p_1 and p_2, i.e. whether there is a substantial difference in the parameter of most interest - the duration dependence parameter - between the two sub-periods.

4 Data and descriptive statistics

In duration analysis the data are organised in spells. In this study, a spell represents the number of periods in which a country is in either an expansion or a contraction. An expansionary spell ends when a business cycle peak is reached; on the other hand, a contractionary spell ends when the business cycle reaches a trough. These sequence of peaks and troughs in economic activity must be identified to generate the spells of expansions and contractions and the respective durations. In this study, we use the monthly business cycle phase chronology elaborated by the NBER Business Cycle Dating Committee for the US economy and a similar chronology recently elaborated by the ECRI for a group of 21 market-oriented economies for the period 1948-2009.[13] From that group, we

[13] There are other ways of identifying turning points in economic activity like: the already mentioned Markov-switching approach; the Bry and Boschan (1971) algorithm; and GDP growth rules. However, by the reasons

select all the European Countries (EC) for which this Institute reports data on the business cycle turning points: Austria, France, Germany, Italy, Spain, Sweden, Switzerland, and the United Kingdom. Additionally, data were also collected for other industrial Non-European Countries (NEC): Australia, Canada, Japan, New Zealand, and the US. Thus, a panel of 13 industrial countries will be analysed in this study over the period 1948-2009.

The methodology used by the ECRI to establish the business cycle chronology for these countries is the same that has been used by the NBER for the US business cycle. Those chronologies represent a set of reference dates (peaks and troughs) which are agreed upon by a group of experts at either the NBER or the ECRI and based on a system of monthly economic indicators.[14] A list of the business cycle chronologies for the countries used in this study is presented in Table A.1 in Appendix.

Our preference for studying the duration of business cycle phases using NBER and ECRI chronologies is justified by three reasons. First, we believe that a chronology determined by a committee of experts, using a large range of macroeconomic indicators and employing a consistent methodology, is likely to be superior to a method that regards the business cycle as an unobserved stochastic process and that uses a single variable such as GDP, GNP or industrial production to infer the state of the business cycle at a particular moment in time (Markov-switching approach).[15]

Second, using a Markov-switching model over a unique series is more like studying growth cycles than classical business cycles. Thus, the results from such models seem to be better suited to forecast output growth rates than to detect effective business cycles or to evaluate the causes of an expansion or contraction ending.[16]

Third, authors employing Markov-switching models usually measure the ability and quality of their approach in predicting business cycles turning points (and duration dependence) by comparing their results with NBER's chronology. Making this comparison they are giving credibility to the work of the NBER Committee.

Therefore, this study assumes that the dates provided by the NBER for the US and by the ECRI - employing the same methodology as the NBER for other countries - are credible and reliable.

mentioned in the text, we prefer to rely on the chronology elaborated by the NBER and ECRI.

[14] The most important indicators are: real personal income, employment, industrial production, sales and monthly estimates of real GDP. For further details on the indicators used, methodologies and chronologies see http://www.nber.org/cycles/main.html and http://www.businesscycle.com/resources/cycles/.

[15] In the words of Di Venuto and Layton (2005, p. 292), "... adopting a single measure of the business cycle fails to capture the many activities that constitute the complex phenomena that is the business cycle."

[16] Besides Markov-switching models, other methodologies could be used to identify the business cycle chronology, like the rule that considers a recession when the growth of real GDP is negative for two consecutive quarters or more, or the chronology resulting from the application of Bry and Boschan (1971) algorithm to this series. However, these methodologies also rely exclusively on the analysis of a single economic indicator, which may not be enough to provide all the necessary information to capture effective business cycles but only swings in economic growth.

Moreover, the aim of this study is not to fit a model for predicting turning points, but to test for the presence of duration dependence in expansions and contractions and to evaluate the presence of change-points in the distribution used in the duration analysis. Hence, these chronologies are a useful source of data to proceed with those tasks.

Table A.2 in Appendix provides a complete description of the business cycle variables that can be extracted from those chronologies. The variables $Peak$ (for the analysis of the duration of expansions) and $Trough$ (for the analysis of the duration of contractions) are fundamental to define whether an expansion or contraction has ended (1) or whether it has been censored (0). These variables correspond to c_i in the Weibull model presented above. The dummy variables $BCExpan$ and $BCContr$ are useful to distinguish expansions from contractions in the dataset. The duration of each spell is another important variable to be used in the model: $DurExpan$ measures the duration of an expansion, in months; and $DurContr$ measures the duration of a contraction, in months. These correspond to the variable t_i in the model described above. These variables are what we need to check for the presence of duration dependence in expansions and contractions.

However, other factors may affect the duration of an expansion or contraction. Zellner (1990), Sichel (1991) and Abderrezak (1998) and Davig (2007) suggest that the duration of the previous business cycle phase ($DurPrev$) may affect the length of the current phase. Moreover, Zellner (1990) theorizes that the solid fundamentals resulting from longer expansions may affect the duration of the following contraction. This author effectively provides some evidence that shorter contractions tend to follow longer expansions in the US, but only for the pre-WWII period. Davig (2007) confirms this finding for the entire period. However, Sichel (1991) and Abderrezak (1998) were not able to find any significant evidence of this link. With the panel analysis provided in this paper, we hope to provide further evidence to clarify this issue.

Like Sichel (1991) and Davig (2007), we also test whether the duration of expansions and contractions has become gradually longer or shorter over time. This is done by including a kind of a trend variable ($Event$) that reports the order or observation number of each event over time for each country. This variable equals one for the first event (i.e. for each expansion or contraction), two for the second, and so on. If the coefficient on this variable is significantly less than (greater than) zero, then phase durations get longer (shorter) over time.

Finally, besides estimating separate regressions for European and Non-European countries, we also test whether there is a significant statistical difference in the parameter of duration dependence

(p) between these two groups of countries and whether the difference in the average duration of expansions and contractions - if it exists - is statistical significant. That is done by including the dummy D_EC in the model. This dummy takes value 1 for European countries and 0 for Non-European countries.[17]

Before proceeding with the econometric analysis, some descriptive statistics for the duration of expansions and contractions are presented in Table 1. This table shows the number of spells of expansions and contractions (Obs.) for each country, for EC and NEC, and for all countries. In total, we have 82 expansions and 80 contractions over the period 1948-2009. Table 1 also shows the mean duration, standard deviation, minimum and maximum durations for each spell. On average expansions last about 4 times longer than contractions. Another interesting evidence is that the duration of expansions and contractions is, on average, slightly higher in the group of European countries than in the group of Non-European countries. Whether this difference is statistically significant is an issue that we will try to answer later with the estimation of the continuous-time Weibull model. Notice also that the shorter expansion (contraction) has last 5 (6) months and the longer has last about 23 (5) years.

Next we analyse graphically the survival functions for the group of all countries, European countries and Non-European countries. Figure 1 reports the respective survival functions for expansions and contractions. Each survival function measures the probability of an expansion or contraction surviving after duration t_i. Hence, this figure is showing the proportion of expansions and contractions surviving over time, as well as, the respective 95% confidence intervals. The evidence reported in Figure 1 shows a substantial decrease in the probability of expansions and contractions surviving, as they become older, for all groups of countries considered in the analysis. As this probability decreases quite rapidly, we may conjecture that positive duration dependence can be present in these events. However, to conclude whether the evidence of positive duration dependence for expansions and contractions is statistically significant (or not), we need to test whether the parameter p in the Weibull model is significantly higher than 1 (or not). With the parametric duration analysis, we can also check whether the magnitude of duration dependence is superior or inferior for expansions than for contractions. Moreover, we can also test whether there are significant differences in the duration dependence parameter between the European and

[17] Given that we will estimate a simple continuous-time duration model, it is not proper to include economic variables that vary over time in this model. Therefore, the list of possible covariates is quite limited. Nevertheless, discrete-time varying covariates are usually used in discrete-time duration models. For further details, see Castro (2010).

Table 1: Descriptive statistics by country (1948-2009)

Country	Duration of Expansions					Duration of Contractions				
	Obs.	**Mean**	**St.Dev.**	**Min.**	**Max.**	**Obs.**	**Mean**	**St.Dev.**	**Min.**	**Max.**
Austria	6	79.0	45.8	23	152	6	17.0	9.9	10	35
France	8	71.9	54.8	10	183	7	15.6	8.0	9	32
Germany	6	97.3	61.9	54	219	6	26.7	9.3	14	39
Italy	6	88.0	46.4	32	166	6	20.0	10.2	10	36
Spain	3	129.7	37.3	90	164	3	34.3	13.7	25	50
Sweden	5	70.8	64.2	22	177	5	27.6	11.3	13	40
Switzerland	6	84.2	70.7	15	220	6	23.8	9.6	14	42
United Kingdom	4	153.3	96.1	46	265	5	19.0	4.7	11	23
E.C.	44	91.4	61.0	10	265	44	22.0	10.3	9	50
Australia	7	96.6	65.7	39	216	6	11.3	4.3	7	18
Canada	6	109	104.8	5	278	5	18.0	4.1	13	24
Japan	6	91.7	98.5	9	227	6	22.3	7.3	13	32
New Zealand	8	51.6	35.5	6	113	8	20.4	16.0	7	57
United States	11	58.4	35.0	12	120	11	11.6	5.2	6	24
N.E.C.	38	77.2	67.7	5	278	36	16.2	9.6	6	57
All countries	82	84.8	64.1	5	278	80	19.4	10.3	6	57

Notes: See Table A.1 in Appendix. The duration of expansions and contractions is measured in months.
E.C. = European Countries; N.E.C. = Non-European Countries.
Sources: NBER website at http://www.nber.org/cycles/main.html, updated in January 2010;
ECRI website at http://www.businesscycle.com/resources/cycles, updated in January 2010.

Non-European countries. This is something that is not clear by looking only at this figure.

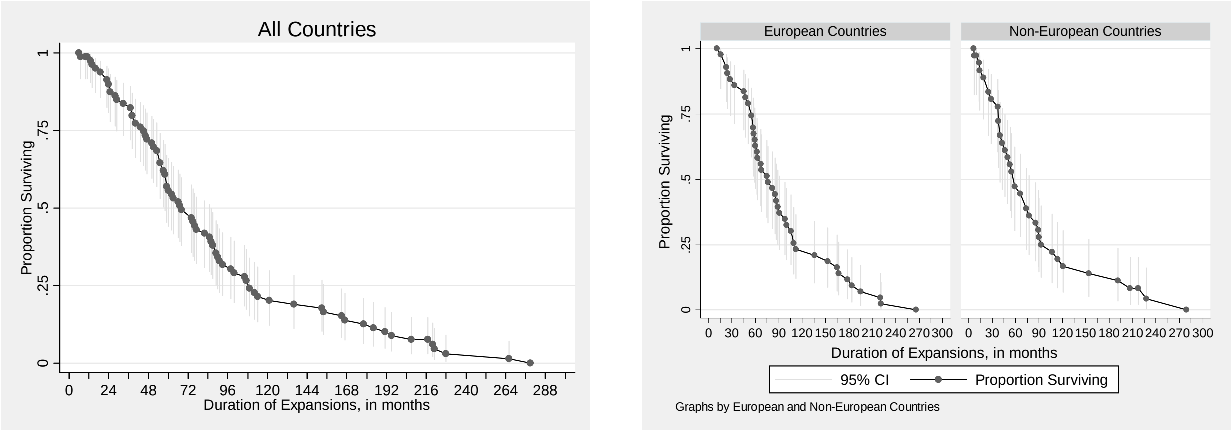
Finally, there is a very interesting aspect to analyse in Figure 1. Looking at the survival function for expansions, in particular, we see that it decreases very quickly until $t_i = 120$ but at a slower pace from there onwards. This is also evident for both the EC and NEC. Similar evidence is not so clear for contractions. Thus, this makes us wonder whether there is a break or change-point in duration dependence for expansions. In other words, we suspect that the magnitude of duration dependence can be inferior when expansions surpass the 120 months (or 10 years) of duration. This is a striking and very important issue not yet considered in the literature but that deserves to be analysed with special attention. That is precisely one of the aims of the empirical analysis developed in the next Section.

5 Empirical results

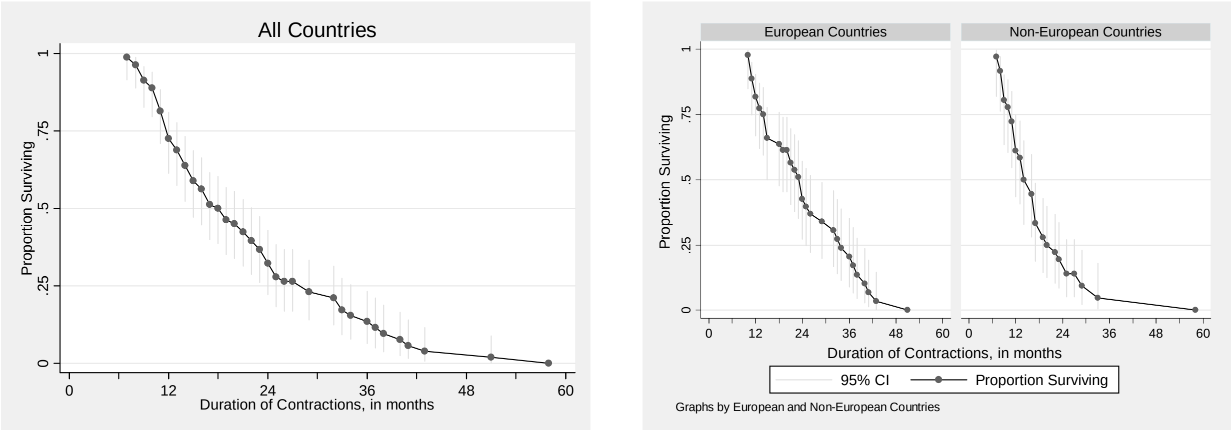
The empirical results from the estimation of a parametric continuous-time Weibull model over a panel of thirteen industrial countries for the period 1948-2009 are presented in this Section. We start by showing the results obtained with the estimation of a basic Weibull model and, then, we proceed to the analysis of the results from the estimation of a Weibull model with a change-point.

Figure 1: Survivor functions

Expansions



Contractions



Sources: NBER website at http://www.nber.org/cycles/main.html, updated in January 2010;
ECRI website at http://www.businesscycle.com/resources/cycles, updated in January 2010.

5.1 Basic results

Considering the basic Weibull model presented in sub-Section 3.2 and using the business cycles turning points data for thirteen European and Non-European countries over the period 1948-2009, we obtained the results that are show in Tables 2 and 3. In Table 2, we present results not only for the panel of thirteen countries but also for the sub-panels of European and Non-European countries.[18] This is a first step to check whether there are significant differences in duration

[18] As the sample sizes for individual countries are very small in most cases, individual estimates are likely to be unreliable. A way of circumvent that small sample problem is pooling all the countries in a single equation. This increases the power of the tests and provides more consistent estimates for duration dependence. Nevertheless, some regressions were run to test for the presence of positive duration dependence in each individual country, but the small sample size made difficult to detect its presence given that the standard errors were very high in some cases. Evidence of positive duration dependence for expansions was found only for Germany and the United States; for contractions, it was found for France, Germany, Switzerland, the United Kingdom, Australia, Canada, Japan, and the United States (results are available upon request).

dependence between this two groups.

The estimate of p measures the magnitude of the duration dependence and γ is the estimate for the constant term. Robust standard errors are reported for all the estimated coefficients. A one-sided test is used to detect the presence of positive duration dependence (i.e. whether $p > 1$). The sign '+' indicates when it is present at a significance level of 5%. Considering first the basic model, the results provide strong evidence of positive duration dependence for expansions and contractions in the three groups of countries considered. Nevertheless, there some differences that must be commented. First, the magnitude of duration dependence is higher in the European than in the Non-European countries, indicating that the likelihood of an expansion or contraction ending in moment t, given that it has last until t, increases at a faster pace in the first group than in the second. Whether this difference is statistically significant is something that we will check later.

On the other hand, the estimated parameter p is higher for contractions than for expansions. This shows that the probability of expansions and contractions ending evolves at different rates as their age increases. Regarding the model used, it is not possible to check whether this difference is statistically significant, but it is possible to show whether this difference is relevant. The analysis of the second derivative of the (baseline) hazard function ($h_0''(t) = \gamma p(p-1)(p-2)t^{p-3}$) shows that in presence of positive duration dependence ($p > 1$) the hazard function increases at a decreasing, constant or increasing rate if $p < 2$, $p = 2$ or $p > 2$, respectively. Therefore, the presence of decreasing, constant or increasing positive duration dependence can be detected by testing if p is lower, equal or higher than 2. We start by testing for the presence of constant positive duration dependence using a 10% two-sided test. The symbol c next to the estimated parameter indicates when this hypothesis is not rejected. Otherwise, a 5% one-sided test is performed to detect the presence of decreasing (d) or increasing (i) positive duration dependence. Results provide evidence of a relevant difference in the rates at which the probability of expansions and contractions ending evolves. As expansions become older the probability of ending increases at a decreasing rate, but for contractions it increases at a constant rate. This result is in line with the findings provided by Lam (2004) and Castro (2010). Nevertheless, a similar relevant difference is not found when European countries are compared with Non-European countries: evidence of decreasing (constant) positive duration dependence is found for expansions (contractions) in both groups.

Table 2: Duration dependence - Basic Weibull model estimations

	Basic			Frailty			Country dummies			Truncation		
	All	E.C.	N.E.C.	All	E.C.	N.E.C.	All	E.C.	N.E.C.	All	E.C.	N.E.C.
Expansions												
γ	0.0013	0.0004	0.0033	0.0003	0.0001	0.0005	0.0010	0.0003	0.0020	0.0012	0.0004	0.0027
	[0.0006]	[0.0003]	[0.0020]	[0.0004]	[0.0002]	[0.0008]	[0.0006]	[0.0002]	[0.0016]	[0.0008]	[0.0004]	[0.0026]
p	$1.454^{+,d}$	$1.666^{+,d}$	$1.268^{+,d}$	$1.890^{+,c}$	$2.133^{+,c}$	$1.853^{+,c}$	$1.653^{+,d}$	$1.788^{+,c}$	$1.503^{+,d}$	$1.614^{+,d}$	$1.705^{+,c}$	$1.436^{+,d}$
	[0.099]	[0.147]	[0.127]	[0.377]	[0.645]	[0.505]	[0.118]	[0.155]	[0.172]	[0.130]	[0.181]	[0.200]
θ				0.510	0.472	0.770						
				[0.360]	[0.549]	[0.550]						
Log-L	−93.43	−44.47	−47.58	−92.32	−43.99	−46.50	−84.79	−41.50	−42.81	−411.3	−226.9	−183.2
LR test				0.068	0.165	0.071	0.139	0.548	0.049	0.165	0.587	0.067
Wald test							0.009	0.316	0.029	0.009	0.311	0.037
SBIC	195.7	96.51	102.4	197.9	99.34	103.9	231.3	117.1	107.5	884.2	487.9	388.3
Obs.	82	44	38	82	44	38	82	44	38	82	44	38
Censored	4	1	3	4	1	3	4	1	3	4	1	3
Contractions												
γ	0.0022	0.0006	0.0050	0.0001	0.0006	0.0001	0.0015	0.0001	0.0023	0.0038	0.0010	0.0097
	[0.0010]	[0.0004]	[0.0034]	[0.0001]	0.0004	[0.0001]	[0.0008]	[0.0001]	[0.0016]	[0.0026]	[0.0013]	[0.0094]
p	$1.949^{+,c}$	$2.275^{+,c}$	$1.804^{+,c}$	$4.154^{+,c}$	$2.274^{+,c}$	$5.074^{+,i}$	$2.480^{+,i}$	$2.648^{+,i}$	$2.324^{+,c}$	$2.197^{+,c}$	$2.114^{+,c}$	$1.876^{+,c}$
	[0.137]	[0.204]	[0.258]	[2.946]	[0.204]	[2.166]	[0.182]	[0.228]	[0.275]	[0.224]	[0.363]	[0.350]
θ				2.098	0.0001	2.447						
				[2.831]	[0.005]	[1.920]						
Log-L	−70.98	−33.62	−33.07	−67.65	−33.62	−27.40	−53.29	−26.94	−26.09	−248.7	−133.5	−112.0
LR test				0.005	1.000	0.000	0.000	0.064	0.007	0.002	0.107	0.028
Wald test							0.000	0.017	0.011	0.000	0.033	0.017
SBIC	150.7	74.82	73.30	148.4	78.60	65.54	167.9	87.93	73.68	558.7	301.0	245.5
Obs.	80	44	36	80	44	36	80	44	36	80	44	36
Censored	9	7	2	9	7	2	9	7	2	9	7	2

Notes: The presence of any pattern of heteroscedasticity and autocorrelation is controlled for by using robust standard errors (reported in square brackets).
$^{+}$ indicates that p is significantly higher than 1 using a 5% one-sided test with robust standard errors.
d, c, and i indicate, respectively, decreasing, constant and increasing positive duration dependence at a 5% significance level.
The LR test for unobserved heterogeneity (frailty) gives the *p-value* from the likelihood ratio test that tests whether the estimated variance (θ) is different from zero (see further details in the text). On the other cases, the LR test gives the *p-value* from the likelihood ratio test for the significance of country-specific dummies (test for pooling): $LR = -2(logL_r - logL_u)$, where r and u state, respectively, for the restricted and unrestricted models; the *p-value* from the Wald test for the same set of linear restrictions is also reported (using robust standard errors). Truncation is made at the minimum observable durations for expansions and contractions: $d_0 = \min(d_i) - 1$, where $\min(d_i)$ is the shortest expansion/contraction observed in the sample. The Schwartz Bayesian Information Criterion is computed as follows: $SBIC = 2(-logL + (k/2)logN)$, where k is the number of regressors and N is the number of observations (Obs.); Censored represents the number of censored observations.
Sources: See Tables A.1 and A.2 in Appendix.

In this first estimations we assume that the population of individual-spells is homogeneous, i.e. that each expansion or contraction is under the same risk of ending, but that may not be the case. Therefore, in the next group of regressions, we allow for the presence of unobserved heterogeneity or frailty. In statistical terms, a frailty model is a kind of a random-effects model for duration analysis, which represents an unobserved random proportionality factor that modifies the hazard function of an individual spell. In practical terms, frailty is a statistical modelling concept which aims to account for heterogeneity caused by unmeasured covariates or measurement errors.[19]

To include the frailty in the Weibull model we have to modify the hazard function given in equation 7, as follows:

$$h(t, \mathbf{x}|v) = vh(t, \mathbf{x}) \tag{23}$$

where v is an unobserved individual-spell effect that scales the no-frailty component. The random variable v is assumed to be positive with unit mean, finite variance (θ) and distributed independently of t and $\mathbf{x}$. On the other hand, the survival function becomes:

$$S(t, \mathbf{x}|v) = [S(t, \mathbf{x})]^v \tag{24}$$

We cannot estimate the values of v themselves since they are unobserved. However, supposing, as is usual in this literature,[20] that v follows a Gamma distribution with unit mean and variance θ, the frailty survival function can be written in the following way:

$$S(t, \mathbf{x}|\boldsymbol{\beta}, \theta) = [1 - \theta \ln S(t, \mathbf{x})]^{-(1/\theta)} \tag{25}$$

and the frailty hazard function will be:

$$h(t, \mathbf{x}|\boldsymbol{\beta}, \theta) = h(t, \mathbf{x}) \left[S(t, \mathbf{x}|\boldsymbol{\beta}, \theta)\right]^{\theta} \tag{26}$$

Hence, the respective log-likelihood function can be written as follows:

$$\ln L(\cdot) = \sum_{i=1}^{n} \left\{ c_i \left[\ln \gamma + \ln p + (p-1)\ln t_i + \boldsymbol{\beta}'\mathbf{x}_i\right] - \left(c_i + \frac{1}{\theta}\right) \ln \left[1 + \gamma t_i^p \exp(\boldsymbol{\beta}'\mathbf{x}_i)\right] \right\} \tag{27}$$

[19] If unobserved heterogeneity is ignored when it is really present in the data, the magnitude of positive duration dependence will be under-estimated.

[20] See Lancaster (1990), for further details.

The variance-parameter (θ) is an additional parameter to be estimated, which measures the presence (or absence) of unobserved heterogeneity. As θ is something that is always greater than zero, the limiting distribution of the maximum-likelihood estimate of θ is a normal distribution that is halved or chopped-off at the boundary (zero in this case). As a result, the likelihood ratio (LR) test used to detect its presence is a 'boundary' test that takes account of the fact that the null distribution is not the usual chi-squared with one degree of freedom, but rather a fifty-fifty mixture of a chi-squared with no degrees of freedom (which is a point mass at zero) and a chi-squared with one degree of freedom.[21] The *p-value* of the LR tests reported for the frailty models estimated for the duration of expansions and contractions show no evidence of unobserved heterogeneity effects in the full sample and in the EC and NEC sub-samples. Hence, as frailty is not a problem, we can proceed the analysis with the basic Weibull model.

Even though the presence of frailty (or "random effects") has not been detected, individual-country effects may be present given that the sample is composed by thirteen countries which may have specific individual characteristics. Hence, country dummies are included in the third group of estimations presented in Table 2. An LR test and a Wald test are used to test for pooling, i.e. to test whether the model controlling for country-specific effects is preferred to simple pooling.[22] The results from these tests provide evidence that supports the presence of individual-country effects, which is particularly strong for contractions. These effects are not significant only for expansions in the group of European countries. Hence, this is the only case where we do not need to control for those effects.[23] An interesting finding that deserves to be to emphasized here is the fact that the magnitude of the estimated coefficient for duration dependence increases for both expansions and contractions when the presence of individual country effects is taken into account in the model. For contractions, we even detect evidence of increasing positive duration dependence, i.e. now the likelihood of a contraction ending increases at an increasing (and not constant) rate with its age. This means that when individual effects are present, but not controlled for, the magnitude of positive duration dependence is under-estimated. Therefore, in the next regressions those effects will be controlled for - when present - including individual-country dummies in the model.

[21] For further details, see Gutierrez et al. (2001). They also notice that the *p-value* of the LR test will be set to one if it is determined that the estimate is close enough to zero to be, in effect, zero for purposes of significance. Otherwise, the *p-value* is set to one-half of the probability that a chi-square with 1 degree of freedom is greater than the calculated LR test statistic.

[22] As robust standard errors are not allowed in the LR test, the *p-value* from a Wald test is also reported because this test permits the use of robust standard errors.

[23] In all the following regressions, if at least one of the tests indicates the presence of those effects, they are controlled for including country dummies in the model. Otherwise, no country dummies are used.

We have assumed that expansions and contractions may have a length from one month to the maximum observable in our sample. However, the NBER and ECRI do not consider a phase of expansion (contraction) with less than five (six) months. According to Sichel (1991, p. 255), "NBER durations are truncated because even though the economy is considered always to be in either the state of expansion or contraction, transitions to these states are observed only if the new state survives for some minimum number of periods". Therefore, in our duration analyse will be reasonable to truncate expansions and contractions to a minimum duration and check whether that affects our results or not. This means that the hazard rate must be identically zero for the first months and some non-zero value thereafter. Truncation is made at the minimum observable durations for expansions and contractions: $d_0 = \min(d_i) - 1$, where $\min(d_i)$ is the shortest expansion/contraction observed in the sample for each group of countries. This means that the survival function is now:

$$S(t_i, \mathbf{x}_i) = \exp\left[-\gamma(t_i^p - d_0^p)\exp(\boldsymbol{\beta}'\mathbf{x}_i)\right] \tag{28}$$

Truncation is allowed for in the last group of regressions presented in Table 2, but the main results are not substantially affected.[24] Only contractions are slightly affected, but evidence of decreasing positive duration dependence remains for expansions as well as the magnitude of the respective coefficient. However, later we will see that when some additional relevant variables are included in the model contractions are not affect by truncation either. In general, results in this kind of studies have not shown sensitive to the choice of this minimum observable duration and the qualitative conclusions tend to be identical in any case.[25]

A question that we have not answered yet is whether the difference in the magnitude of duration dependence between European and Non-European countries is statistically significant or not. To answer this question, we decided to estimate directly the difference in the duration dependence parameter replacing the parameter p by $p + \Delta p D\_EC$ in the model, where Δp will measure that difference and D_EC is a dummy variable that takes value one for European countries and zero otherwise. Results are presented in the first column of Table 3 and show that the estimated duration dependence parameter is higher for the group of European countries in both expansions and contractions, as we have already noticed in the separate regressions estimated above. However,

[24] Individual-country dummies are included in all regressions except for EC expansions given that the LR and Wald test do not detect the presence of those effects in this group of countries.

[25] Sichel (1991) and Layton and Smith (2007), for example, reach the same conclusion.

the estimated coefficient Δp is not statistically significant, which means that the differences in the duration dependence parameter - i.e. in likelihood of an expansion or contraction ending as it gets older - between European and Non-European countries are not significant. These results are not affected even if truncation is allowed for (see column 2). Thus, a unique duration dependence parameter can be estimated for the group of all countries.

Nevertheless, given the differences detected in the mean duration of expansions and contractions between these groups of countries (see Table 1), we decided, in alternative, to include the dummy variable D_EC in the list of the additional covariates ($\mathbf{x}_i$) to control for (the statistical significance of) those differences. That variable is included in the regression presented in column 3. The coefficient on this dummy is highly significant for both expansions and contractions and indicates that the likelihood of an expansion or contraction ending is lower in the group of European countries than in the group of Non-European countries. This result provides statistical support to the evidence reported in Table 1 that, on average, both expansions and contractions last longer in the European than in the Non-European countries. Hence, we conclude that the difference in the average duration of expansions and contractions between these two groups of countries is statistically significant, even though the same duration dependence parameter should be considered for both groups of countries.

Next we test whether the duration of the previous phase ($DurPrev$) affects the length of the current phase. The results provide some evidence that shorter contractions tend to be preceded by longer expansions, which is a result that complements Zellner (1990) and Davig's (2007) findings, now for the post-war period and over a panel of industrial countries.[26] This means that the solid fundamentals that characterize longer expansions tend to exert significant effects on subsequent contractions, making their durations shorter. On the other hand, no solid evidence is found in the opposite direction.

Like Sichel (1991) and Davig (2007), we also control for the possibility of the duration of expansions and contractions has been affected over time. In this task, a kind of a trend variable ($Event$) is included in the model (see column 5 in Table 3). This variable measures the order or observation number of each event over time for each country. Results show that contractions have become significantly longer over time, a result that is contrary to the one found by Sichel

[26] The interpretation of the coefficients is not easy in this kind of model. A way of interpreting them is using a factor-change analysis. For example, when the duration of the previous contraction increases by one month the hazard rate of an expansion ending increases by a factor of approximately $\exp(0.0037) = 1.0037$, i.e. by about $0,4\%$, ceteris paribus.

Table 3: Duration dependence: Weibull model estimations with control variables

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Expansions							
γ	0.0020	0.0027	0.0010	0.0006	0.0015	0.0012	0.0014
	[0.0016]	[0.0025]	[0.0006]	[0.0005]	[0.0010]	[0.0010]	[0.0012]
p	$1.503^{+,d}$	$1.438^{+,d}$	$1.653^{+,d}$	$1.789^{+,c}$	$1.661^{+,d}$	$1.822^{+,c}$	$1.789^{+,c}$
	[0.171]	[0.199]	[0.119]	[0.166]	[0.120]	[0.172]	[0.185]
$\triangle p$	0.285	0.330					
	[0.230]	[0.257]					
D_EC			-1.5846^{***}			-2.0783^{***}	-2.0485^{***}
			[0.4884]			[0.6831]	[0.6803]
DurPrev				-0.0168		-0.0157	-0.0156
				[0.0124]		[0.0124]	[0.0123]
Event					-0.0584	-0.1053^{*}	-0.1043^{*}
					[0.0505]	[0.0601]	[0.0600]
Log-L	−411.3	−410.7	−84.80	−69.29	−84.27	−68.11	−346.8
LR test	0.110	0.125	0.107	0.037	0.133	0.016	0.021
Wald test	0.018	0.018	0.045	0.015	0.003	0.004	0.005
SBIC	888.7	887.5	231.7	202.5	234.6	204.4	761.9
Obs.	82	82	82	71	82	71	71
Censored	4	4	4	4	4	4	4
Contractions							
γ	0.0023	0.0097	0.0015	0.0011	0.0041	0.0030	0.0063
	[0.0016]	[0.0093]	[0.0008]	[0.0006]	[0.0023]	[0.0018]	[0.0045]
p	$2.324^{+,c}$	$1.876^{+,c}$	$2.479^{+,i}$	$2.510^{+,i}$	$2.712^{+,i}$	$2.800^{+,i}$	$2.534^{+,i}$
	[0.273]	[0.347]	[0.183]	[0.193]	[0.208]	[0.224]	[0.265]
$\triangle p$	0.324	0.614					
	[0.355]	[0.442]					
D_EC			-1.1230^{**}			-2.3409^{***}	-2.2703^{***}
			[0.5478]			[0.4076]	[0.4067]
DurPrev				0.0037^{*}		0.0045^{**}	0.0041^{**}
				[0.0021]		[0.0019]	[0.0018]
Event					-0.2494^{***}	-0.2688^{***}	-0.2562^{***}
					[0.0665]	[0.0706]	[0.0699]
Log-L	−252.0	−248.0	−53.28	−51.90	−45.92	−43.71	−233.6
LR test	0.002	0.005	0.003	0.000	0.000	0.000	0.000
Wald test	0.000	0.000	0.002	0.000	0.000	0.000	0.000
SBIC	569.7	561.7	167.9	169.2	157.6	157.1	536.8
Obs.	80	80	80	78	80	78	78
Censored	9	9	9	9	9	9	9

Notes: See Table 2. Robust standard errors are reported in square brackets. $\triangle p$ is the estimated difference in the duration dependence parameter between European and Non-European countries. Truncation is used in regressions 2 and 7. Individual-country effects are controlled for in all estimations, given that at least one of the tests (LR or Wald) indicates the presence of those effects.

$^{+}$ indicates that p is significantly higher than 1 using a 5% one-sided test with robust standard errors.

d, c, and i indicate, respectively, decreasing, constant and increasing positive duration dependence at a 5% significance level.

* statistically significant at 10% level; ** at 5% level; *** at 1% level.

Sources: See Tables A.1 and A.2 in Appendix.

(1991) and Davig (2007) for the US. This new evidence can be partially explained by the longer than average duration of the contractions that followed the recent financial crises and by the fact that we are using information not only for the US but for a panel of industrial countries. There is also some evidence that expansions have become longer over time, but this evidence is only marginally significant when D_EC and $DurPrev$ are included in the model (see columns 6 and 7). Nevertheless, despite weak, this evidence is in line with Sichel (1991) and Davig's (2007) findings for the US.

So far, we have included the three additional regressors (D_EC, $DurPrev$ and $Event$) individually in the model, but now they are all simultaneously included in the regressions presented in columns 6 and 7 of Table 3. The results and conclusions obtained from these variables remain unchanged when compared with the regressions where they are included individually. In sum, we have evidence that: (i) expansions and contractions last longer in the European than in the Non-European countries; (ii) longer expansions are followed by shorter contractions; (iii) and contractions and expansions have become longer over time. Moreover, after controlling for the significant effects of these covariates, we find quite strong evidence of constant positive duration dependence for expansions and increasing positive duration dependence for contractions. This means that the likelihood of expansions ending increases over time at a constant rate, but for contractions it increases at an increasing rate. This result confirms Lam (2004) and Castro's (2010) findings and the observed tendency for much shorter contractions than expansions over the last half century.

In the regression presented in column 7 of Table 3, we account for truncation but none of the previous conclusions is affected and the magnitude of the estimated duration dependence coefficient does not change much. Hence, and once again, we find no practical advantage in complicating further the analysis with such a small truncation. However, this duration analysis deserves to be further explored by allowing for the presence of a change-point in duration dependence for expansions and contractions. That is the aim of the analysis provided in the next sub-Section.

5.2 Change-points in duration dependence

The results presented so far (in Tables 2 and 3) rely on the assumption that the magnitude of duration dependence is the same over time. However, as we noticed above, the degree of likeliness of an expansion or contraction ending as it gets older may change after a certain duration. Looking at Figure 1, we can see that the survival function for expansions (all countries, EC and NEC) decreases very quickly until 120 months, but proceeds decreasing at a slower pace from there onwards. This make us to suspect that the magnitude of duration dependence can be inferior when expansions surpass the 120 months of duration. To check whether there is a significant difference in the duration dependence coefficient, beyond the threshold $\tau_c = 120$, we allow for this change-point in the Weibull distribution. Considering the Weibull model with a change-point developed in sub-Section 3.3, it is possible to test for the presence of differences in the duration dependence parameter. Estimating two parameters, one for the first period (p_1) and other for the second (p_2),[27] we can evaluate the statistical significance of the difference between them ($p_2 - p_1$).[28] If this difference is statistically significant, we have evidence of a change-point and the magnitude of the estimated duration dependence coefficient has indeed changed after that threshold. Otherwise, we do not have a change-point and may rely on the results of a basic Weibull model.

The results from the estimation of the Weibull model for expansions with a change-point at 120 months are presented in the first part of Table 4. We start by estimating a simple equation without either covariates or country-dummies (see column 1). Then, the presence of individual-country effects is controlled for in regression 2. In regression 3, individual-country dummies and the three additional covariates are all included in the model. Results are quite interesting and provide remarkable new evidence that the magnitude of the estimated duration dependence coefficient for expansions changes over time. In particular, results show that the magnitude of positive duration dependence is significantly lower when expansions surpass the 120 months (or 10 years) of duration.[29] The evidence of constant increasing duration dependence is still present for expansions that last less than 120 months, but when their duration surpasses this threshold they are no longer duration dependent, i.e. the likelihood of ending is no longer dependent on their age.

[27] The estimates for the two different constant terms are obtained as follows: $\gamma_1 = \lambda_1^{p_1}$ and $\gamma_2 = \lambda_2^{p_2}$.

[28] The deltha method is used to compute the respective standard-errors.

[29] Notice that the coefficient on the difference ($p_2 - p_1$) is negative and statistically significant.

Table 4: Duration dependence: Weibull model estimations with change-points

	Expansions					Contractions				
	All Countries			E.C.	N.E.C	All Countries			E.C.	N.E.C
	(1)	(2)	(3)	(4)	(5)	(1)	(2)	(3)	(4)	(5)
γ_1	0.0005	0.0012	0.0006	0.0005	0.0017	0.0017	0.0061	0.0061	0.0095	0.0002
	[0.0004]	[0.0004]	[0.0004]	[0.004]	[0.0011]	[0.0008]	[0.0024]	[0.0062]	[0.0023]	[0.0004]
γ_2	0.0121	0.0100	0.0074	0.0122	0.0084	0.0052	0.0062	0.0097	0.0020	0.0044
	[0.0109]	[0.0175]	[0.0173]	[0.0381]	[0.0258]	[0.0071]	[0.0138]	[0.0307]	[0.0030]	[0.0194]
p_1	$1.690^{+,d}$	$1.775^{+,c}$	$1.937^{+,c}$	$2.188^{+,c}$	$1.623^{+,d}$	$2.037^{+,c}$	$2.481^{+,i}$	$2.830^{+,i}$	$2.472^{+,c}$	$3.203^{+,i}$
	[0.168]	[0.169]	[0.218]	[0.335]	[0.261]	[0.156]	[0.212]	[0.248]	[0.314]	[0.411]
p_2	1.019	1.333	1.438	1.040	1.283	$1.709^{+,c}$	$2.472^{+,c}$	$2.692^{+,c}$	$2.938^{+,c}$	$2.337^{+,c}$
	[0.163]	[0.250]	[0.367]	[0.260]	[0.540]	[0.366]	[0.565]	[0.585]	[0.730]	[0.907]
$p_2 - p_1$	-0.671^{**}	-0.442^{**}	-0.499^{**}	-1.147^{**}	−0.339	−0.329	−0.009	−0.137	0.466	−0.866
	[0.266]	[0.224]	[0.252]	[0.507]	[0.615]	[0.434]	[0.644]	[0.650]	[0.426]	[0.671]
D_EC			-1.9206^{***}					-2.3523^{***}		
			[0.6945]					[0.4052]		
$DurPrev$			−0.0162	-0.0292^{*}	−0.0053			0.0045^{**}	0.0042	0.0041^{**}
			[0.0126]	[0.0176]	[0.0123]			[0.0019]	[0.0034]	[0.0021]
$Event$			-0.1135^{*}	0.0789	-0.1362^{*}			-0.2694^{***}	-0.2936^{**}	-0.2637^{***}
			[0.0624]	[0.1094]	[0.0734]			[0.0705]	[0.1534]	[0.0842]
Log-L	−418.2	−411.1	−346.6	−186.3	−162.8	−269.7	−252.3	−236.5	−129.5	−104.4
LR test		0.287	0.056	0.154	0.044		0.001	0.000	0.006	0.001
Wald test		0.034	0.057	0.113	0.060		0.000	0.000	0.005	0.000
SBIC	849.7	892.8	769.9	394.3	360.8	552.6	574.6	551.4	307.9	244.3
Obs.	82	82	71	37	34	80	80	78	43	35
Censored	4	4	4	1	3	9	9	9	7	2

Notes: See Table 2. Robust standard errors are reported in square brackets. $p_2 - p_1$ is the estimated difference in the duration dependence parameters. The change-point is located at duration equal to 120 months for expansions and 30 months for contractions.
$^+$ indicates that p is significantly higher than 1 using a 5% one-sided test with robust standard errors.
d, c, and i indicate, respectively, decreasing, constant and increasing positive duration dependence at a 5% significance level.
* statistically significant at 10% level; ** at 5% level; *** at 1% level.
Sources: See Tables A.1 and A.2 in Appendix.

Considering now separate estimations for the two sub-groups of countries (EC and NEC - see columns 4 and 5), results show that the presence of this change-point is quite significant in the group of European countries but statistically insignificant in the group of Non-European countries. Even though the estimated difference in the duration dependence parameter ($p_2 - p_1$) is not significant in this second group, evidence of positive duration dependence is only found for expansions shorter than 120 months. Hence, combining this finding with the one reported for the EC estimation, it is perceptible why evidence of positive duration dependence is found for expansions shorter than 120 months but no evidence of duration dependence is noticed for "older" expansions in the panel of all countries. All this statistical evidence supports the one that was obtained by simple visual analysis of the survival function for expansions in Figure 1.

However, the picture is not so clear regarding the presence of a change-point in the survival function for contractions. In any case, looking at Figure 1, we decided to test for its presence considering a change-point at 30 months of duration (i.e. $\tau_c = 30$). This seems to be a nice threshold to begin with since the survival function becomes substantially flatter after that point. An analysis similar to the one implemented above for expansions is now provided for contractions in the second part of Table 4. In the first three columns all countries are included in the estimations; the separate estimations for the two sub-groups of countries (EC and NEC) are presented in columns 4 and 5. Results confirm our expectation of no significant differences in the estimated duration dependence parameter between contractions shorter than 30 months and "older" ones, for the groups of all countries, EC and NEC. This means that $\tau_c = 30$ is not a change-point in duration dependence for contractions, which increases the doubts on its existence for these events.

Other candidates to change-points can be considered either for expansions or contractions. As the thresholds considered above were not estimated directly in the model but selected by simply observing the survival function, we proceed with a sensitivity analysis where different thresholds are allowed for to check whether the choices made above are reasonable. The results from this sensitivity analysis are presented in Table 5.

For expansions we consider four different thresholds: 60 months or 5 years; 85 months, which corresponds to the average duration of expansions in this group of thirteen industrial countries; 150 months; and 180 months. These are some values around the change-point identified above for expansions ($\tau_c = 120$). Results show that the magnitude of positive duration dependence is not significantly different below or above those thresholds, which implies that none of these can be considered a change-point for expansions. Moreover, evidence of constant increasing duration dependence is found in any of the sub-periods. Other thresholds were tried, but results were quite similar. Only when the threshold is 120 months (or a value very close to it), we have found significant differences in the magnitude of the duration dependence coefficient. Hence, we can conclude that for expansions a change-point is present at a duration of 120 months, meaning that the likelihood of an expansion ending increases (at a constant rate) as it gets older, but when it surpasses 10 years of duration it is no longer dependent on its age.

On the other hand, the conclusion is not the same for contractions. We have shown above that $\tau_c = 30$ is not a change-point in duration dependence for contractions, but we also noticed that the choice of this threshold was quite arbitrary since that, looking at Figure 1, we do not find a clear break (or a kind of a kink) as we have found for expansions. Thus, we also tried to test the presence of a change-point considering different thresholds.

Table 5: Sensitivity analysis: Weibull model estimations with change-points

	Expansions				**Contractions**		
months	**60**	*mean*=**85**	**150**	**180**	*mean*=**19**	**24**	**36**
γ_1	0.0095	0.0505	0.0010	0.0005	0.0536	0.0005	0.0004
	[0.0064]	[0.0099]	[0.0006]	[0.0003]	[0.0206]	[0.0003]	[0.0052]
γ_2	0.0283	0.1174	0.0001	0.0001	0.2623	0.0107	0.0049
	[0.0689]	[0.2264]	[0.0003]	[0.6217]	[0.6502]	[0.0199]	[0.0505]
p_1	$1.992^{+,c}$	$1.905^{+,c}$	$1.754^{+,c}$	$1.751^{+,c}$	$2.974^{+,i}$	$3.147^{+,i}$	$2.478^{+,i}$
	[0.351]	[0.266]	[0.183]	[0.172]	[0.267]	[0.277]	[0.194]
p_2	$1.727^{+,c}$	$1.715^{+,c}$	$2.245^{+,c}$	$2.801^{+,c}$	$2.434^{+,c}$	$2.164^{+,c}$	$1.786^{+,d}$
	[0.225]	[0.262]	[0.611]	[0.883]	[0.581]	[0.480]	[0.506]
$p_2 - p_1$	−0.265	−0.190	0.491	1.050	−0.539	−0.983	−0.692
	[0.437]	[0.397]	[0.658]	[0.901]	[0.711]	[0.657]	[0.625]
D_EC	−2.0097***	−2.0139***	−2.1951***	−2.2537***	−2.5051***	−2.4533***	−2.5749***
	[0.6839]	[0.6899]	[0.7542]	[0.7764]	[0.5725]	[0.4229]	[0.5997]
DurPrev	−0.0158	−0.0161	−0.0155	0.0154	−0.0034**	−0.0040**	0.0041**
	[0.0122]	[0.0126]	[0.0123]	[0.0121]	[0.0018]	[0.0020]	[0.0018]
Event	−0.1056*	0.1070*	−0.0996*	−0.0968	0.2131***	−0.2745***	−0.2206***
	[0.0595]	[0.0604]	[0.0608]	[0.0614]	[0.0806]	[0.0707]	[0.0814]
Log-L	−346.9	−347.0	−346.8	−346.4	−243.0	−235.1	−244.9
LR test	0.029	0.038	0.011	0.010	0.022	0.000	0.014
Wald test	0.019	0.017	0.025	0.014	0.002	0.000	0.000
SBIC	770.6	770.8	770.3	769.5	560.1	548.7	563.9
Obs.	71	71	71	71	78	78	78
Censored	4	4	4	4	9	9	9

Notes: See Tables 2.and 4. Robust standard errors are reported in square brackets. $p_2 - p_1$ is the estimated difference in the duration dependence parameters. For each regression, the change-points are located at the values indicated in the second row (values in months). Individual-country effects are controlled for in all estimations, given that at least one of the tests (LR or Wald) indicates the presence of those effects.

$^+$ indicates that p is significantly higher than 1 using a 5% one-sided test with robust standard errors.

d, c, and i indicate, respectively, decreasing, constant and increasing positive duration dependence at a 5% level.

* statistically significant at 10% level; ** at 5% level; *** at 1% level.

Sources: See Tables A.1 and A.2 in Appendix.

Like for expansions, the average duration of contractions is also used as a threshold in one of the estimations presented in Table 5. Then two additional values around 30 months were considered: 24 and 36 months. Even though some evidence of increasing positive duration dependence is found in the first periods and constant positive duration dependence is found in the second ones, results are quite clear in indicating that no significant differences (in statistical terms) are present in the duration dependence parameter in any of the cases.[30] Therefore, we may argue that there is no change-point in duration dependence for contractions, meaning that a basic Weibull model can be used in the duration analysis of these events.

Thus, we can summarize the main finding of this paper saying that a change-point is present in duration dependence for expansions but not for contractions. In particular, evidence of positive duration dependence is no longer observed when an expansion surpasses 10 years of duration, which is a remarkable new finding in this field of research.

6 Conclusions

The issue of whether business cycles are duration dependent, i.e. whether the likelihood of an expansion or contraction ending is dependent on its age, has been the focus of a substantial part of the business cycles literature. Duration analysis and Markov-switching models have been the most used approaches in this literature to test for the presence of duration dependence and several studies have been successful in finding evidence of positive duration dependence for expansions and contractions in some countries. This is an issue that is also tested in this paper over a panel of thirteen industrial countries for the period 1948-2009. Results are very clear in confirming the evidence of positive duration dependence for both expansions and contractions. Hence, we can conclude that the likelihood of those events ending increases over time.

However, two other issues are not so well explored in this field of research. The first is related to the very own duration of expansions and contractions. As expansions tend to last longer than contractions, we would expect that the magnitude of positive duration dependence might be superior for contractions than for expansions. As the literature gives little attention to this issue, we decided to evaluate the presence of such differences. In general, our results show that as expansions become older the probability of ending increases at a constant rate, but for contractions it increases at an increasing rate. This is an interesting finding that confirms the observed tendency for much shorter contractions than expansions over the last half century.

[30] Similar results (not reported here) were obtained considering other values for the threshold.

The second and most important issue is related to the magnitude of duration dependence itself, which is usually assumed to be the constant over time. We conjecture that this may not always be the case. In fact, the degree of likeliness of an expansion or contraction ending as it gets older may change after a certain duration. This is an important issue that, to our knowledge, has never been tested in business cycles literature but that we explore in this paper. To proceed with such task, we extend the basic Weibull model used in duration analysis allowing for the presence of a change-point in the duration dependence parameter. Results from the estimation of this model are quite interesting. They indicate the presence of a change-point in duration dependence for expansions but not for contractions. They show that the magnitude of the duration dependence parameter decreases significantly when an expansion surpasses 10 years of duration. More precisely, evidence of (constant) positive duration dependence is found for expansions that last less than 10 years, but when their duration surpasses this threshold they are no longer duration dependent, i.e. the likelihood of an "older" expansion ending is no longer dependent on its age. This is a remarkable new finding that shows that the likelihood of an expansion ending increases at a constant rate with its age, but when it is running for more than 10 years, the likelihood of it ending will not depend anymore on its actual duration or age, but perhaps on other random factors.

Concerning all these findings, we can conclude that the likelihood of contractions and expansions ending increases with their age, at a higher rate for contractions than expansions, but evidence of positive duration dependence is no longer found when expansions surpass 10 years of duration. This evidence represents a striking new finding in this field of research and the most important result of this paper.

Given the specificities of the countries involved in this study, we also decide to compare the behaviour of two sub-groups of countries: European and Non-European countries. Results show no significant differences in the magnitude of the duration dependence parameter between these two groups, but reveal that expansions and contractions last longer in the European than in the Non-European countries. Additionally, this study also controls for the impact of the duration of the previous phase on the length of the current phase and whether the duration of expansions and contractions is affected over time. Results provide strong evidence that the length of contractions is negatively affected by the length of the previous expansion and that contractions and expansions have become longer over time.

All the empirical research provided by this paper can be extended in several directions. For example, as the ECRI also provides data for growth cycles for the panel of countries analysed in this study, a straightforward extension would be to test whether the conclusions obtained in this study for the classical business cycles can also be obtained using growth cycles. Additionally, we could implement an algorithm to identify the business cycle turning points using, for example, GDP, GNP or industrial production series. Such procedure will allow us to study the behaviour of the business cycle phases in other industrial countries for which the ECRI does not provide data on the business or growth cycle turning points. A drawback of this procedure is the fact that as we have to rely on a single series to identify the turning points, we may not be doing an effective analysis of the duration of classical expansions and contractions.

Another possible extension is related to the selection of the change-point(s). In this paper, that selection is exogenously determined by a sensible graphical analysis of the survival function. An interesting extension would be to make the selection of the change-point endogenous, maybe incorporating in the standard Weibull model a discrete latent variable that follows a Markov-chain. This represents a challenging and promising approach to be considered in future research.

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Appendix

Table A.1: Business cycle chronologies (1948-2009)

European Countries			Non-European Countries		
	Peak	Trough		Peak	Trough
Austria	August 1974	June 1975	Australia	June 1951	September 1952
(1962-2009)	February 1980	January 1983	(1948-2009)	December 1955	August 1956
	April 1992	June 1993		December 1960	September 1961
	May 1995	March 1996		June 1974	January 1975
	January 2001	December 2001		June 1982	May 1983
	February 2008	-		June 1990	December 1991
France	November 1957	April 1959	Canada	May 1953	June 1954
(1953-2009)	July 1974	June 1975	(1948-2009)	October 1956	February 1958
	August 1979	June 1980		April 1981	November 1982
	April 1982	December 1984		March 1990	March 1992
	February 1992	August 1993		January 2008	July 2009
	August 2002	May 2003	Japan	-	December 1954
	February 2008	February 2009	(1953-2009)	November 1973	February 1975
Germany	March 1966	May 1967		April 1992	February 1994
(1948-2009)	August 1973	July 1975		March 1997	July 1999
	January 1980	October 1982		August 2000	April 2003
	January 1991	March 1994		February 2008	March 2009
	January 2001	August 2003	New Zealand	June 1966	March 1968
	April 2008	-		April 1974	March 1975
Italy	January 1964	March 1965	(1962-2009)	March 1977	March 1978
(1956-2009)	October 1970	August 1971		April 1982	May 1983
	April 1974	April 1975		November 1984	March 1986
	May 1980	May 1983		September 1986	June 1991
	February 1992	October 1993		October 1997	May 1998
	August 2007	-		November 2007	-
Spain	March 1980	May 1984	United States	November 1948	October 1949
(1969-2009)	November 1991	December 1993		July 1953	May 1954
	February 2008	-	(1948-2009)	August 1957	April 1958
Sweden	October 1970	November 1971		April 1960	February 1961
(1969-2009)	July 1975	November 1977		December 1969	November 1970
	February 1980	June 1983		November 1973	March 1975
	June 1990	July 1993		January 1980	July 1980
	April 2008	-		July 1981	November 1982
Switzerland	April 1974	March 1976		July 1990	March 1991
(1956-2009)	September 1981	November 1982		March 2001	November 2001
	March 1990	September 1993		December 2007	-
	December 1994	September 1996			
	March 2001	March 2003			
	May 2008	-			
United Kingdom	-	August 1952			
(1951-2009)	September 1974	August 1975			
	June 1979	May 1981			
	May 1990	March 1992			
	May 2008	-			

Notes: The time period considered by the ECRI for each country is in parenthesis.
Sources: NBER website at http://www.nber.org/cycles/main.html, updated in January 2010;
ECRI website at http://www.businesscycle.com/resources/cycles, updated in January 2010.

Table A.2: Description of the variables

Business Cycle Variables	
Peak	Dummy variable that takes value 1 in the month in which a business cycle peak is reached, and 0 otherwise.
Trough	Dummy variable that takes value 1 in the month in which a business cycle trough is reached, and 0 otherwise.
BCExpan	Dummy variable that takes value 1 when a country is in expansion, and 0 otherwise.
BCContr	Dummy variable that takes value 1 when a country is in contraction, and 0 otherwise.
DurExpan	Variable that measures the duration of an expansion, in months.
DurContr	Variable that measures the duration of a contraction, in months.
DurPrev	Variable that measures the duration of the previous phase, in months.
Event	Observation number of the event (expansion or contraction) for each country.
D_EC	Dummy variable that takes value 1 for European Countries, and 0 otherwise.

Sources: NBER website at http://www.nber.org/cycles/main.html, updated in January 2010;
ECRI website at http://www.businesscycle.com/resources/cycles, updated in January 2010.

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