

# Comparison of Three Control Theories for Single-Phase Active Power Filters

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**Abstract:** Active Power Filters have been developed in last years, mostly for three-phase systems applications. The use of Shunt Active Power Filters on single-phase facilities brings many benefits for the electrical grid, since these installations have non linear loads and power factor problems, and in their total, they are responsible by a significant portion of the total electric energy consumption. Harmonics and reactive power consumed by single-phase installations cause additional power losses on the electrical grid. So, mitigate harmonics at the origin helps reducing these extra losses and other problems caused by the harmonics. The drawback of this solution is the necessity of a large number of Active Power Filters distributed by the generality of the single-phase facilities. So, it becomes necessary a simple and low cost Shunt Active Power Filter to install on single-phase installations. This paper presents three simple control theories to use on single-phase Shunt Active Power Filters. Simulation and experimental results comparing the three different control theories are presented and analyzed.

## I. INTRODUCTION

With the progress of power electronics, engineers started the development of electronically controlled devices that have non linear current consumption. These devices were primordially developed to increase the energy efficiency and the controllability of advanced production processes, but, since they produce harmonics, these devices are now responsible for extra energy losses and bad operation of the electrical distribution system and its components [1]. Many of harmonic sources are single-phase loads, such as computers, fluorescent compact lamps, copiers, printers and other home and office electronic equipments. To reduce the impact of harmonics produced by this type of loads, Gyugi and Strycula [2] introduced the concept of Active Power Filters. To avoid harmonic distortion on system voltages it is necessary to mitigate the harmonics at customer's side. This is the best way to avoid problems on neighbors' facilities and to keep a good electric power quality service. The harmonic currents on the electrical grid degrade the voltages waveform. The Shunt Active Power Filters are nowadays a good solution, since they can solve harmonic current problems, and also compensate the power factor. Shunt Active Power Filters have various advantages over Passive ones, since they don't need to be configured to a specific harmonic, but, all harmonics can be simultaneously compensated. To assure a good performance Passive Power Filters have to be carefully tuned for which particular case. Active Power Filters can be

easily installed without any commissioning, and instantaneously compensate the current harmonics and the power factor [3-4].

The aim of this paper is to contribute to the development of low cost Shunt Active Power Filters, by proposing three simple control theories, to single-phase Shunt Active Power Filters. The presented theories were developed to be simple, so inexpensive microcontrollers can operate with it. The proposed Active Power Filter topology uses only two switching devices (IGBTs) in a half-bridge arrangement with split capacitors at the DC side.

## II. CONTROL THEORIES PRESENTED

The Shunt Active Power Filters work as current sources, connected in parallel with the electric grid (Fig. 1), and they are capable of providing the harmonics and the reactive power required by the loads, exchanging the current between the electrical grid and the capacitors at DC side of the power inverter. The performance of the Active Power Filter is strongly dependent of the control theory. In the next topics three different approaches to the control of single-phase Shunt Active Power Filters will be presented:

### A. *p-q Theory*

The theory of the instantaneous reactive power also known as p-q theory, has been introduced by Akagi *et al.* [5] in 1983 for three-phase three-wire systems, and expanded to four-wire systems by Aredes *et al.* [6]. This theory allows the separation of the power components in mean and oscillating values, assuring the separation of the undesired components which

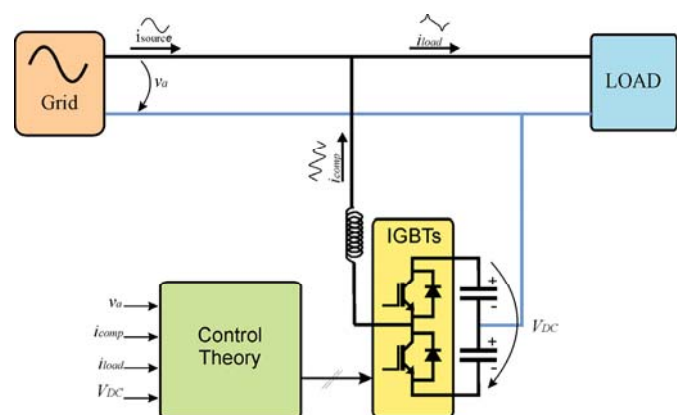


Fig. 1. Block Diagram of the Single-Phase Active Power Filter.

should be supplied by the Active Power Filter. The involved mathematical calculations to determinate the compensation currents can be completely executed with digital operations and implemented in low cost microcontrollers [7]. With some modifications, this theory can be used to control a single-phase Shunt Active Power Filter [8-9]. To apply this theory in single-phase systems, it is possible to consider a balanced three-phase system were the phase *a* corresponds to the single-phase system and the other phases (*b* and *c*) are used only with mathematical purposes [10]. The load current on a single-phase system can be mathematically represented by (1):

$$i_a = \sum_{i=0}^n \sqrt{2} I_i \cdot \sin(\omega_i + \theta_i) \quad (1)$$

Assuming that this current (1) is the phase *a* current, in a three-phase equilibrated system, then the corresponding *b* and *c* phase currents can be represented by (2) and (3):

$$i_b = \sum_{i=0}^n \sqrt{2} I_i \cdot \sin(\omega_i + \theta_i - 120^\circ) \quad (2)$$

$$i_c = \sum_{i=0}^n \sqrt{2} I_i \cdot \sin(\omega_i + \theta_i + 120^\circ) \quad (3)$$

The currents in the  $\alpha$ - $\beta$  reference frame are determined applying the Clarke transformation, to the three-phase currents (4) and (5):

$$i_\alpha = \sqrt{\frac{2}{3}} \sum_{i=0}^n \sqrt{2} I_i \cdot \sin(\omega_i + \theta_i) = \sqrt{\frac{2}{3}} \cdot i_a \quad (4)$$

$$\begin{aligned} i_\beta &= \sqrt{3} \sum_{i=0}^n \sqrt{2} I_i \cdot \cos(\omega_i + \theta_i) \\ &= \sqrt{3} \left[ \sum_{i=0}^n \frac{1}{\sqrt{3}} \sin(\omega_i + \theta_i) + \frac{2}{\sqrt{3}} \sin(\omega_i + \theta_i - 120^\circ) \right] \\ &= \sqrt{\frac{2}{3}} \sum_{i=0}^n A_i \sin(\omega_i - i \cdot 90^\circ) \end{aligned} \quad (5)$$

Assuming that system voltage is sinusoidal, the voltages are represented by

$$v_\alpha = \sqrt{\frac{2}{3}} \cdot v_a \quad (6)$$

$$v_\beta = v_\alpha e^{-j90^\circ} \quad (7)$$

The equation to calculate the active power *p* is given by (8). The instantaneous reactive power *q* is determined by (9):

$$p = v_\alpha i_\alpha + v_\beta i_\beta \quad (8)$$

$$q = v_\alpha i_\beta - v_\beta i_\alpha \quad (9)$$

The active power *p* can be separated in mean and oscillating, values (10):

$$p = \bar{p} + \tilde{p} \quad (10)$$

The  $\bar{p}$  value is obtained by a low pass filter on the *p* signal. After getting  $\tilde{p}$  and *q* it is possible to calculate the current  $i_{ca}$  (11) that is the compensation current in the  $\alpha$ - $\beta$

reference frame. The  $p_{reg}$  (regulation power) is the value of the power to control the voltage on the DC side of the power inverter.

$$i_{ca} = \left( \frac{v_\alpha \cdot (\tilde{p} - p_{reg}) - v_\beta \cdot q}{v_\alpha^2 + v_\beta^2} \right) \quad (11)$$

The compensation current in the a-b-c reference frame is determined by (12):

$$i_{comp} = \sqrt{\frac{2}{3}} \cdot i_{ca} \quad (12)$$

Fig. 2 shows the block diagram of the p-q theory. The value of  $p_{reg}$  is obtained with the use of a Proportional Integral (PI) controller. The  $v_\alpha$  and  $v_\beta$  are stored in a two arrays and is used a Phase Locked Loop (PLL) to keep the arrays index synchronized with the grid voltage. The use of this solution reduces the processing time.

### B. Control by the DC side voltage

This theory is based on the voltage value at the capacitors in the DC side of the power inverter. According to the DC side voltage, the control, estimates the ideal source current. This calculation uses a proportional integral controller (PI). The adjustment of the PI parameters is crucial on this theory, since the behavior of the Active Power Filter is hardly dependent of this controller. So the PI response must be fast, enabling the Active Power Filter to quickly respond to load changes, and also must be stable. Oscillations on the response of the PI controller will cause variations in the current amplitude. The value obtained by the PI controller is used to generate a sinusoidal signal that is equivalent to the ideal current at source, necessary to supply the load. To compensate the power factor, the generated sinusoid must be

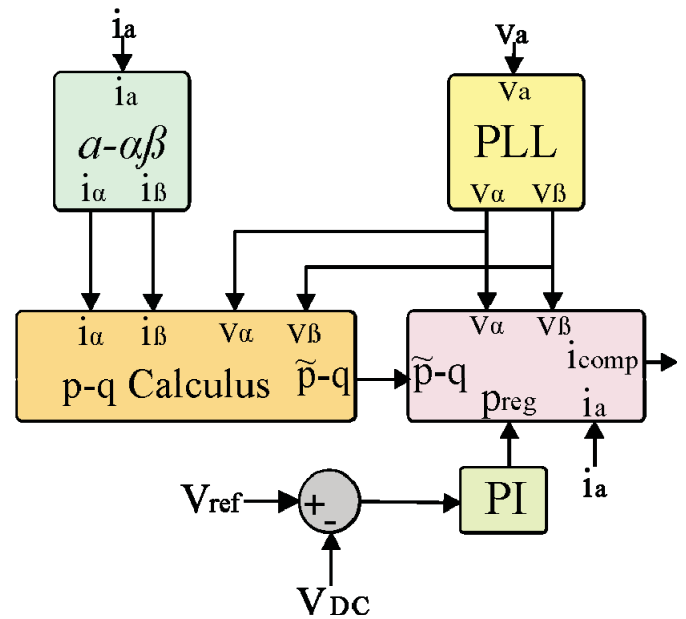


Fig 2 Function Block for the simplified p-q theory.

synchronized with the system voltage. This synchronization can be done through a PLL algorithm. After, the load current waveform will be subtracted to the obtained sinusoid to get the compensation current. Fig. 3 shows a block diagram that represents the algorithm used to control an Active Power Filter based by the regulation of the DC side voltage.

### C. Mean Power Theory

To apply this theory is necessary to calculate the average active power consumed by the load. Then the average value is divided by a constant value that represents the mean RMS voltage at the system, giving the amplitude of the ideal current on the source. The average power is calculated by the multiplication of the instantaneous value of the load current by the instantaneous value of the system voltage. Then a sliding window average is applied to obtain the mean power. The control of the DC voltage on the capacitors of the Active Power Filter is done by a PI controller that will return a value of power that may be exchanged with the electric grid in order to maintain the DC voltage regulated. This power value given by the PI control is added to the mean power value achieved by the sliding window average. With the resulting value from the sum of this two powers, it is generated a sinusoid which correspond to the ideal current that should be supplied by the source. This sinusoid is then subtracted to the load current waveform, obtaining the compensation current. To control the power factor is used the same strategy previously used in the control by the DC side voltage. Fig. 4 shows a block diagram that represents the algorithm used to control an Active Power Filter based on the mean power.

## III. SIMULATION RESULTS

The presented simulation results are achieved using an electromagnetic transient simulation tool the PSCAD/EMTDC version 4.2. The load used to simulate

the Shunt Active Power Filter is composed by: a 5 Ω resistor in series with a 50 mH inductance, a 10 Ω resistor and a single-phase bridge rectifier with a 20 Ω resistor and a 3300 μF capacitor in the DC side (fig 5). This load arrangement represents a typical house or small office current profile. The Fig. 6 shows the current and voltage waveforms and the Total Harmonic Distortion (THD) of the load current. The THD of the load is 43% and the power factor is 0.8. The  $I_{rms}$  26.6A and  $V_{rms}$  110V. At DC side of the Active Power Filter are used two 1410 μF capacitors.

### A. Simulation Results with p-q theory:

The results obtained with the p-q Theory were 2.4% of THD in the source current, after compensation. Fig. 7 shows the source current and the system voltage. As it can be seen, the source current becomes almost sinusoidal and in phase with the system voltage. The Active Power Filter mitigates most of the harmonics as it can be seen on the harmonic spectrum presented in Fig. 7. Fig. 8 presents the voltage at the capacitors at the DC side of the power inverter.

### B. Simulation Results with Control by the DC side:

With the control by the DC side, the source current THD becomes 2.4%, after compensation. This theory can also compensate the power factor. Fig. 9 presents the source

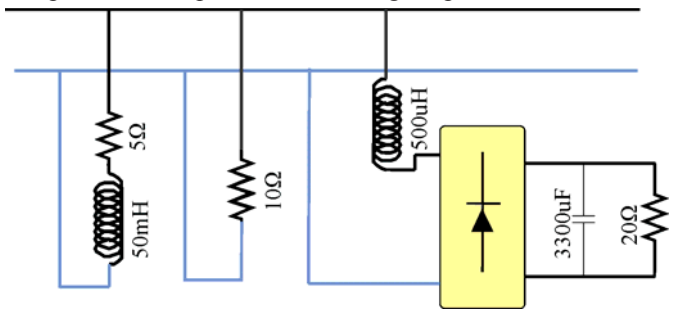


Fig. 5. Loads used in simulation

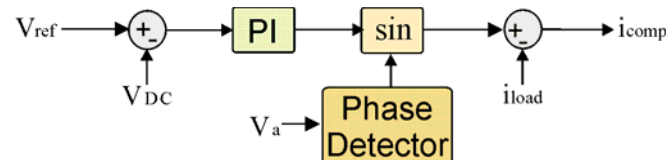


Fig 3 Block Diagram for DC side control algorithm

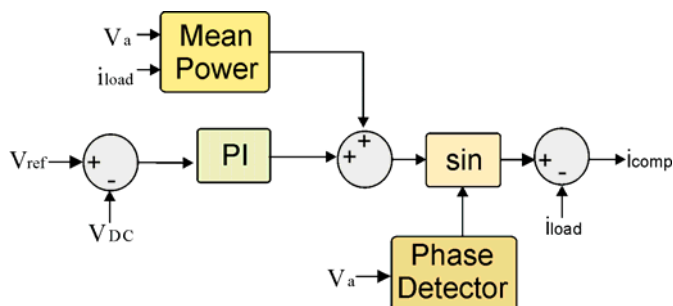


Fig 4 Block Diagram for Mean Power Control.

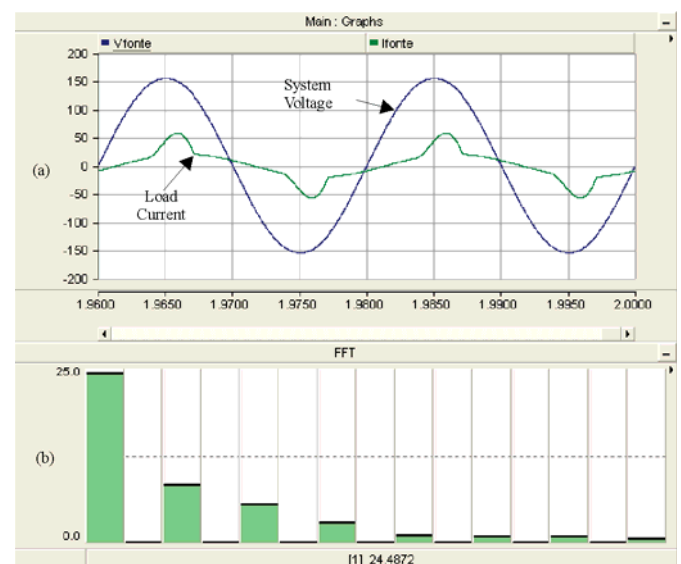


Fig. 6. Shunt Active Filter turned-off: (a) Source voltage and current waveforms (b) Current harmonics.

current and the system voltage. In this figure it is possible to see that the source current is almost sinusoidal in phase with the system voltage i.e. power factor near to 1.

On Fig. 10 it is presented the voltage in the DC side of the power inverter.

C. *Implementation Respects with the Mean Power Theory:*

The results obtained with the Mean Power Theory were a THD of 1.6% in the source current. Similar to the other ones this theory also compensates the power factor. In Fig. 11 it is possible to see that the load current is almost sinusoidal and in phase with the system voltage. The Active Power Filter with this theory mitigates the undesired harmonics as it can be seen on Fig. 11 in the harmonic spectrum. Fig. 12 represents the voltage on the DC side of the filter.

The results obtained with the three theories are very similar, all of the three presented theories, showed to be able to compensate harmonic currents and the power factor. The small difference between the theories is caused by the control used to regulate the capacitors voltage at the DC side.

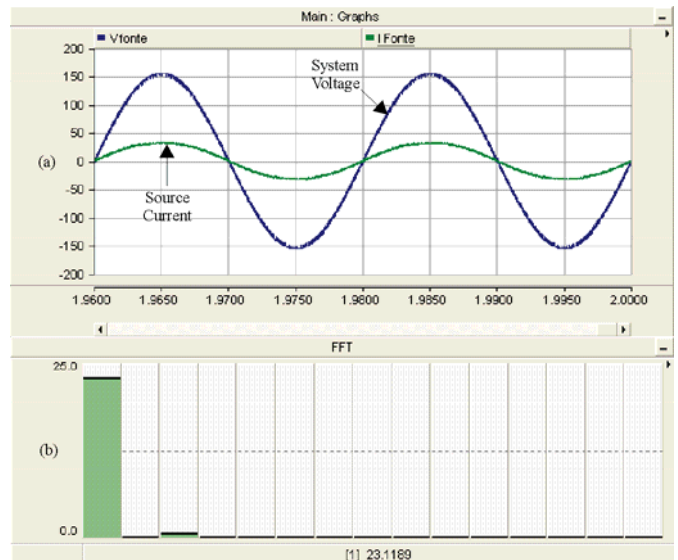


Fig. 9. Shunt Active Filter turned-on: (a) Source voltage and current waveforms (b) current harmonics for DC side control

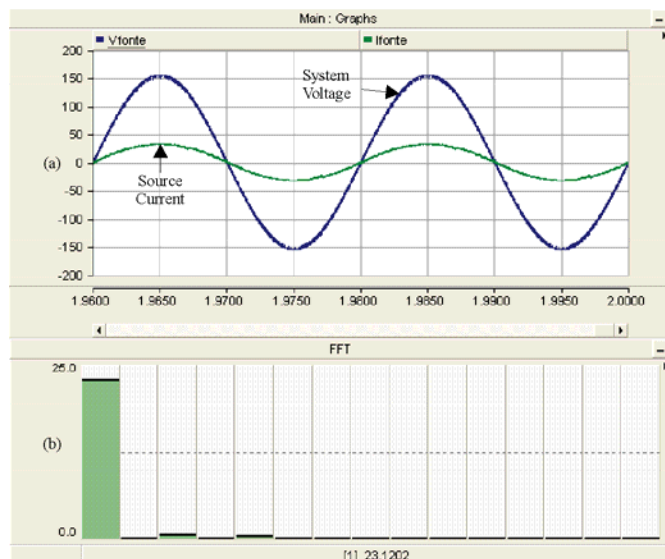


Fig. 7. Shunt Active Filter turned-on: (a) Source voltage and current waveforms (b) current harmonics for p-q theory

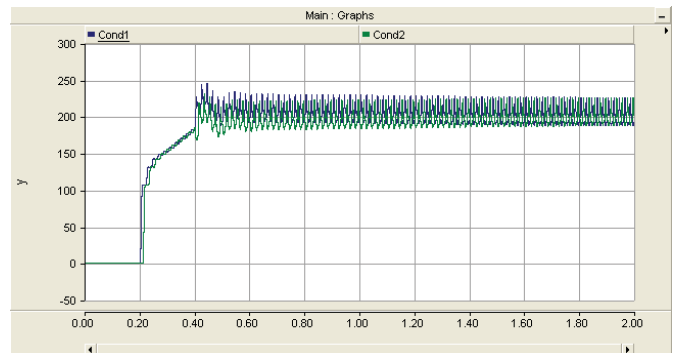


Fig. 10. Voltages on the Active Power Filter capacitors for DC side control

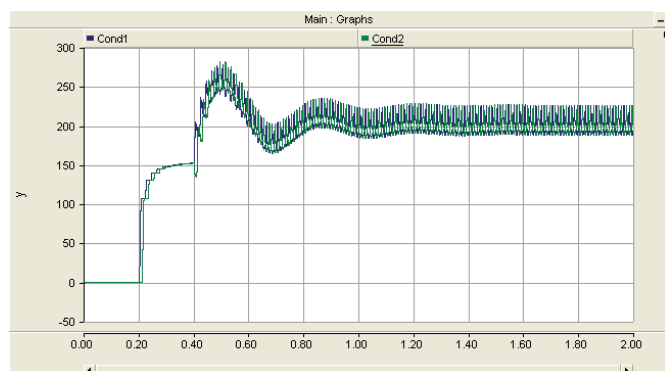


Fig. 8. Voltages on the Active Power Filter capacitors for p-q theory

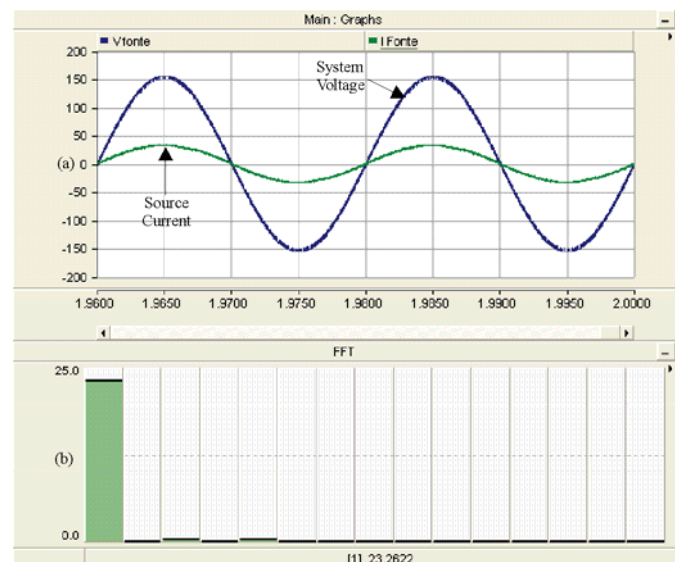


Fig. 11. Shunt Active Filter turned-on: (a) Source voltage and current waveforms (b) current harmonics for Mean Power Theory



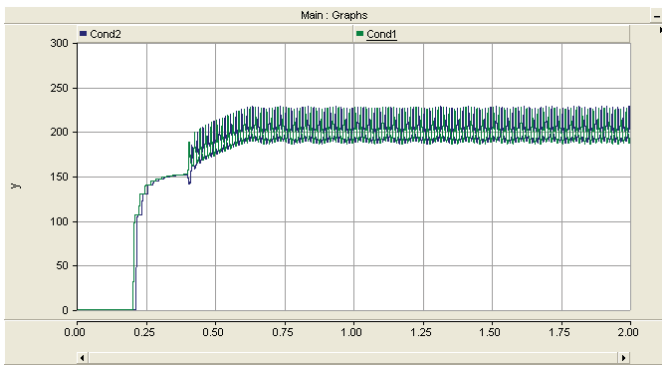


Fig. 12. Voltages on the Active Power Filter capacitors for Mean Power Theory

III. LABORATORY RESULTS

It was implemented a laboratory prototype to test the Mean Power Theory. The load used in the laboratory prototype consists in a group of two AC rectifiers, one with an inductive filter and the other one without it, and an inductive load, connected as shown in the Fig. 13. Fig. 14 shows the system voltage and load current waveforms. The RMS current of the load is 12.8 A and the THD 43.2%.

The voltage harmonics were also reduced after compensation. Before the compensation the voltage THD was 3.32%. After compensation this value becomes 2.9%. Note that the voltage waveform was already disturbed by the other

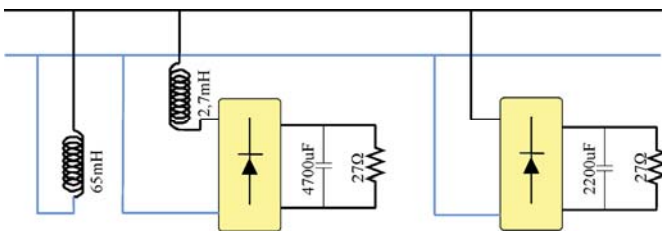


Fig. 13 Loads

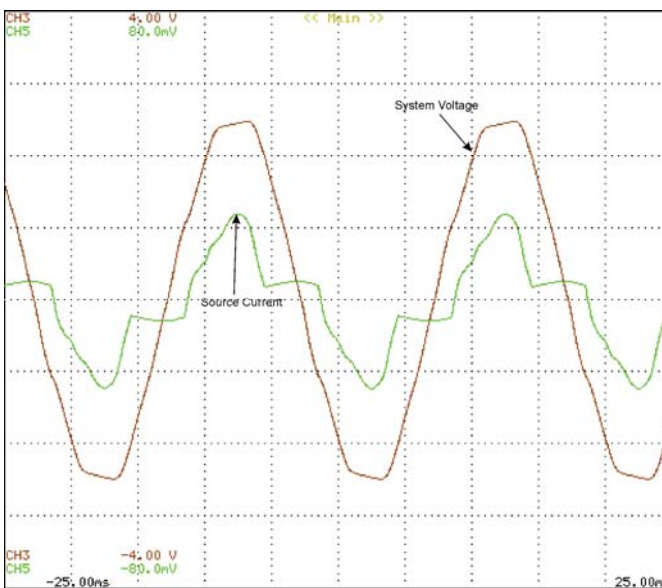


Fig. 14 Current and voltage waveforms with the Active Power Filter OFF

loads in the facility not compensated by the Active Power Filter. The power factor was also compensated.

Since the generated source current reference is sinusoidal, the current after compensation is independent of the voltage waveform. Harmonics in the voltage waveform don't affect the compensation current as it can see in the Fig. 16.

Fig. 15 shows the harmonics current spectrum, on the source before compensation. The THD was 43.2% and the transformer K factor is 3.4. In the Fig. 17 is represented the harmonics current spectrum on the source after compensation. It shows that the THD decreases from 43.2% to 2.8% and the K factor also decreases from 3.4 to 1.3. This improvement may represent a significant reduction on power losses in the electric grid. The Fig. 18 shows the voltage in the DC side capacitors of the power inverter. The ripple in the DC side voltage of the laboratory prototype was lower than the obtained on the simulations. This is caused, by the difference between the capacitors on the DC side that is 2350 μF in the

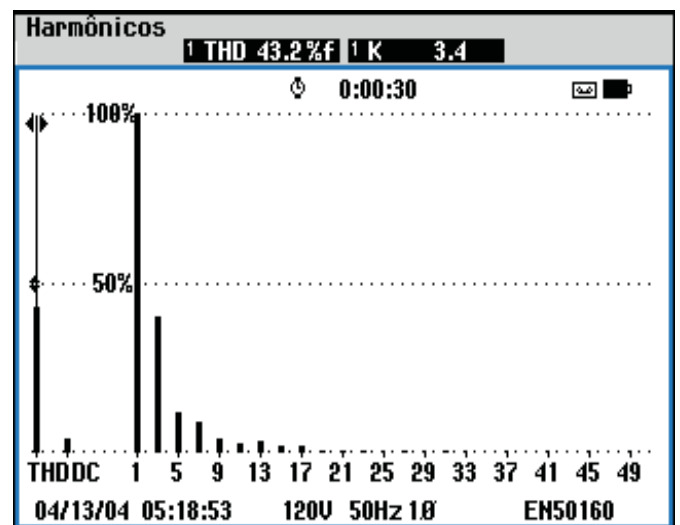


Fig. 15. Harmonic Spectrum of the Load current

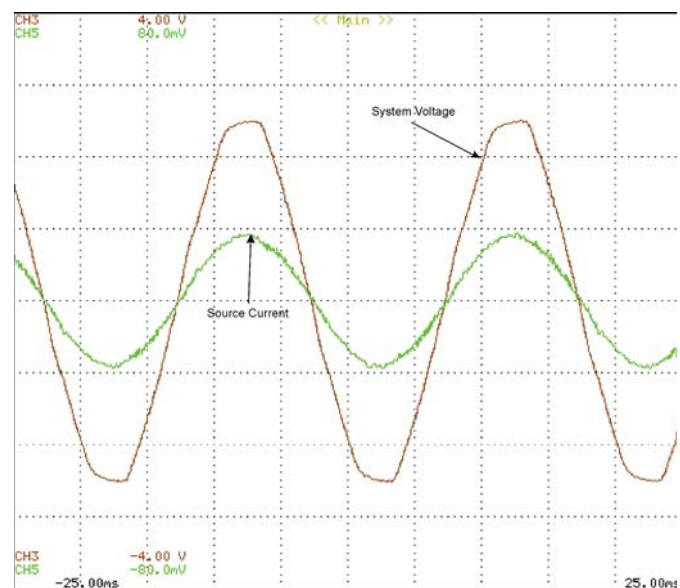


Fig. 16 Current and voltage waveforms with the Active Power Filter ON

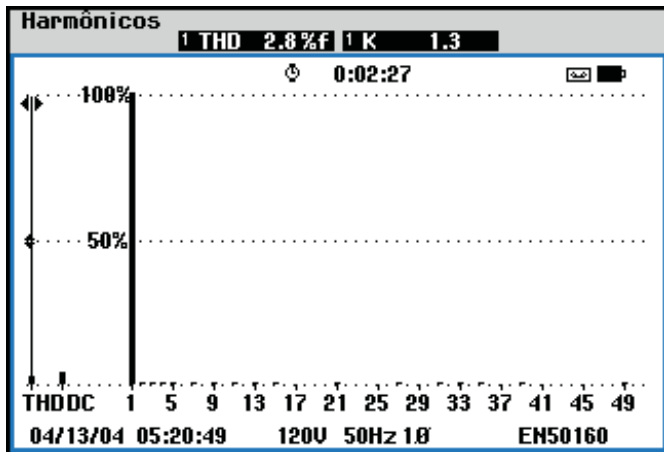


Fig 17. Harmonic Spectrum with the filter compensating

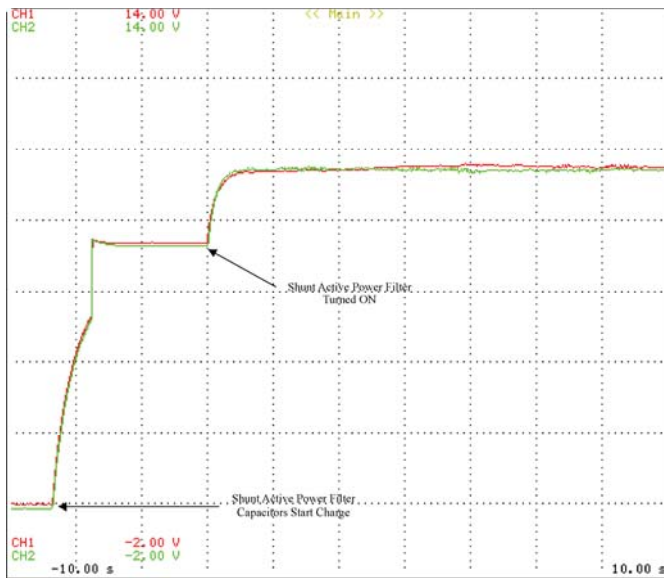


Fig 18 Capacitors voltage waveforms

prototype and 1410  $\mu\text{F}$  in the simulation. The load is also different, the RMS current in the simulation is 26.6 A and in the laboratory prototype is 12.81 A.

The switching technique used on the prototype, is the periodic sampling. This technique limits the maximum switching frequency to 16 kHz but not imposes a fixed frequency [9].

#### CONCLUSIONS

The designed and implemented single-phase Shunt Active Power Filter has demonstrated to be able to compensate the harmonic currents and the power factor produced by loads, making the current at the source side to become almost sinusoidal and in phase with the system voltage. This current compensation (if performed for all non linear loads of the

electrical system) can also prevent voltage harmonics on the power grid, caused by the harmonics currents on the line impedances. The experimental results presented in this paper shows that the developed Shunt Active Power Filter can be a good solution for residential, small offices and services customers, which are connected to the power grid by single-phase installations.

The results obtained through computer simulation for the three different control theories used in the single-phase Shunt Active Power Filter, proved to be very similar. All of the three presented theories showed to be able to compensate the harmonic currents and the power factor. The small difference obtained with these three theories is caused by the adjustment of the algorithm used to regulate the capacitors voltage at the DC side of the Active Power Filter.

#### ACKNOWLEDGMENTS

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