Robustness analysis of traditional timber trusses

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Abstract In the present work, the safety of existing traditional timber trusses is evaluated, with particular emphasis on the structural robustness. Traditional Portuguese timber trusses are analyzed probabilistically, using the information provided in the JCSS model code, combined with action and resistance models provided in the Eurocodes.

Robustness is evaluated through introduction of a localized defect, simulating deterioration, construction error or damages. The comparison between the reliability index considering a defect and the corresponding index for an intact structure is defined as a measure of susceptibility to local damage.

The reliability index is computed using Monte-Carlo simulation combined with linear elastic finite elements for different examples.

Keywords traditional timber trusses, probabilistic analysis, Monte-Carlo simulation, defects and robustness.

1. INTRODUCTION

The deterioration process, human errors in design and construction, as well as, damage causes by humans or other sources, are extremely difficult to predict. Although significant effort has been placed on modeling some of these events, in particular, deterioration, more recently, a different approach has been suggested by different authors. In this approach, these defects are assumed unpredictable, and the consequences on safety are evaluated based on the definition of different damage scenarios. This approach allows the comparison between different structures to unexpected events of similar magnitude and the identification of critical events that should be monitored more closely.

Robustness was firstly defined as the ability of a structure to sustain localized damage without disproportionate consequences. An example of disproportionate consequences is the progressive collapse of a structure following damage in a single element. Robustness was first defined for events such as explosions and impacts following the partial collapse of the Ronan Point Building in 1968 (Baker et al., 2008), and received renewed attention as a measure to limit the effects of terrorist attacks.

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More recently, a set of accidents involving timber structures under loads below the design values increased the interest on its use in assessing the susceptibility of a structure to deterioration and design and construction errors.

However, existing codes have significant limitations in assessing robustness. On one hand, robustness can only be defined at the structural (global) level, rather than the element level, as under a significant defect, several elements are subject to stresses above the design values, and redistribution of stresses must be possible. On the other hand, robustness analysis requires the comparison between different levels of safety, which can only be achieved in a probabilistic framework, as semi-probabilistic methods, as used in current codes, provide only a pass/fail information.

As a result, although current codes, including the Eurocodes (CEN, 2001), define robustness as a desirable property, no method is indicated to assess it.

The analysis of structural safety of existing timber structures is relevant, not only from a robustness perspective, but also for allowing a more detailed characterization of material properties, actions, and models. In general, these properties are significantly different for an existing structure, in particular, a deteriorated structure, and the values proposed in codes should be used with significant reserves.

2. ROBUSTNESS ASSESSMENT

Firstly, it is fundamental to clarify the difference between robustness and resistance to accidental loads. A good design includes significant robustness, independently of the susceptibility of the structure to sustain accidental loads. In fact, robustness is a property of the structure to sustain unexpected loads, defined independently of any particular loading.

A structure can be considered robust when the fundamental elements to keep structural safety have the ability to sustain unexpected loading and defects, or when progressive collapse does not occur as a result of localized collapse (Sørensen and Christensen, 2006). If total or significant partial collapse results from the failure of an element, this element is referred to as a key element.

The partial safety method, used in current design codes, defines safety in terms of the safety of each element, disregarding the global behavior of the structure and the possibility of progressive collapse (Starossek and Wolff 2005). As a result, it is fundamental to get a better understanding of the structural behavior following a localized failure. In the past, these issues concerned only special structures, as long span bridges or building exposed to higher threats (e.g., embassies).

A simple method to analyze the robustness of the structure is to evaluate the structural safety assuming an element is removed. If the structure is safe to ultimate limit states, considering this notional removal, allowing repair works to take place, with a safety margin deemed acceptable, the structure can be considered robust. This type of safety assessment can be used, in design stages, to limit the consequences of unexpected events. However, it is not generalized to all structural systems.

In the last years, several authors presented different methods to quantify structural robustness. Most work focused on the application of probabilistic methods, although more consistent, introduces added complexity to the design process, usually unnecessary at a design stage. However, it is extremely useful in defining simplified rules, easier to apply in practice.

The first proposals towards a definition of structural robustness are focused on assessing redundancy, as these two properties were considered equivalent (Canisius *et al*. 2007). The first proposal presented (Frangopol and Curley, 1987) defines robustness in terms of the reliability index of the damaged and intact structure as:

$$\beta_r = \frac{\beta_i}{\beta_i - \beta_d} \tag{1}$$

where β_i is the reliability index of the intact structure and β_d is the reliability index of the damaged structure. The redundancy index β_r can vary from zero (unredundant system) to infinity (very redundant structural system).

Alternatively, (Lind, 1995) defines robustness in terms of a vulnerability index (V) indicating the increase in the probability of failure resulting from structural damage:

$$V = \frac{P(r_d, S)}{P(r_0, S)} \tag{2}$$

where $P(\ )$ represents the probability of failure, r_0 is the resistance of the intact structure, r_d is the resistance of the damages structure and S is the effect of actions.
More recently, a robustness index (I_R) was proposed by (Baker et al. 2008) in terms of the relation between direct and indirect risks associated with unforeseeable events as:

$$I_R = \frac{R_{Dir}}{R_{Dir} + R_{Ind}} \tag{3}$$

where R_{Dir} is the direct risk and R_{Ind} the indirect risk, both defined as the product of the probability of occurrence of a undesirable event and its cost. The direct consequences are associated with localized damage in structural elements, as indirect consequences are related to partial or global failure triggered by the localized damage. The robustness index (I_R) can assume values between zero and one, corresponding to no robustness and very high robustness, respectively.

The robustness index (I_R) is the most consistent and correct measure of assessing robustness, as it considers both the redundancy and the consequences of an unexpected event. However, its computation is extremely complex in real cases, as it requires the assessment of the probability of occurrence of complex events and the estimation of its costs.

The probability of a given consequence $P(C)$ can, from a robustness viewpoint, be defined as:

$$P(C) = \sum_i \sum_j P(C | E_i \cap D_j) \times P(D_j | E_i) \times P(E_i) \tag{4}$$

where $P(E_i)$ is the probability of exposure to the event i, $P(D_j|E_i)$ is the probability of occurrence of damage j given exposure i, and $P(C|E_i \cap D_j)$ is the probability of consequences given damage j.

Since the events considered in equation (4) are independent, the probability of structural collapse, $P(C)$, is given by the product of the probability of occurrence of the following events: exposure to a given occurrence (E_i), occurrence of damage of a given magnitude, considering that the exposure is of smaller magnitude ($D_j|E_i$) and occurrence of consequences given damage ($C|E_i \cap D_j$).

The probability of exposure, $P(E)$, depends on location, type of structure, nearby structures or infra-structures, among a large set of other properties that are, fundamentally, not influenced by design. As a consequence, two possible paths can be followed to improve robustness: improving the resistance to damage or limit the indirect consequences of damage. In this work, the indirect consequences of damage are evaluated, considering that damage can be modeled as the notional removal of one element (Kirkegaard and Sørensen, 2008), and the possible indirect consequence is the collapse of the structure.

This analysis can only be performed in a probabilistic framework, as traditional partial safety factor methods are incapable of quantifying safety.

3. PROBABILISTIC PROPERTIES OF TIMBER STRUCTURES

In order to analyze the robustness of timber structures it is fundamental to evaluate the probability of failure of both the intact and the damaged structure. As a natural material, the properties of timber present significant dispersion.

The probabilistic analysis of timber structures safety was executed considering the information included in the Joint Committee of Structural Safety (JCSS) model code (JCSS, 2006). The mechanical behavior of timber is defined using a relatively large set of parameters, as a result of the anisotropic non-linear behavior of timber. The different mechanical properties of timber are strongly related and, an empirical relation between the mean and standard deviation, as well as correlation factors, can be defined among these properties.

The JCSS model code (JCSS, 2006) defines three fundamental and independent properties: bending strength (f_m), Young modulus (E), and density (ρ). All other relevant mechanical properties are defined, in a probabilistic sense, based on these three properties, as presented in Table 1. The correlations between different properties are presented in Table 2.

Table 1 – Mechanical properties of timber (JCSS, 2006)

Property X	Distribution	E[X]	CoV[X]
Bending strength (f_m)	Lognormal	$E[f_m]$	0,25
Bending modulus of elasticity (E_m)	Lognormal	$E[E_m]$	0,13
Density (ρ_m)	Normal	$E[\rho_m]$	0,1
Tension strength parallel to the grain ($f_{t,0}$)	Lognormal	$0{,}6E[f_m]$	$1{,}2COV[f_m]$
Tension strength perpendicular to the grain, ($f_{t,90}$)	Weibull	$0{,}015E[\rho_m]$	$2{,}5COV[\rho_m]$
MOE - tension parallel to the grain, ($E_{t,0}$)	Lognormal	$E[E_m]$	$COV[E_m]$
MOE - tension perpendicular to the grain ($E_{t,90}$)	Lognormal	$E[E_m]/30$	$COV[E_m]$
Compression strength parallel to the grain, ($f_{c,0}$)	Lognormal	$5E[f_m]^{0,45}$	$0{,}8COV[f_m]$
Compression strength perpendicular to the grain ($f_{c,90}$)	Normal	$0{,}008E[\rho_m]$	$COV[\rho_m]$
Shear modulus (G_v)	Lognormal	$E[E_m]/16$	$COV[E_m]$
Shear strength (f_v)	Lognormal	$0{,}2E[f_m]^{0,8}$	$COV[f_m]$

Table 2 – Correlation between mechanical properties of timber (JCSS, 2006)

	f_m	E_m	ρ_m	$f_{t,0}$	$f_{t,90}$	$E_{t,0}$	$E_{t,90}$	$f_{c,0}$	$f_{c,90}$	G_v	f_v
f_m	1	0,8	0,6	0,8	0,4	0,6	0,6	0,8	0,6	0,4	0,4
E_m	-	1	0,6	0,6	0,4	0,8	0,4	0,6	0,4	0,6	0,4
ρ_m	-	-	1	0,4	0,4	0,6	0,6	0,8	0,8	0,6	0,6
$f_{t,0}$	-	-	-	1	0,2	0,8	0,2	0,5	0,4	0,4	0,6
$f_{t,90}$	-	-	-	-	1	0,4	0,4	0,2	0,4	0,4	0,6
$E_{t,0}$	-	-	-	-	-	1	0,4	0,4	0,4	0,6	0,4
$E_{t,90}$	-	-	-	-	-	-	1	0,6	0,2	0,6	0,6
$f_{c,0}$	-	-	-	-	-	-	-	1	0,6	0,4	0,4
$f_{c,90}$	-	-	-	-	-	-	-	-	1	0,4	0,4
G_v	-	-	-	-	-	-	-	-	-	1	0,6

4. EXAMPLE OF APPLICATION

In order to evaluate the robustness of traditional timber roof structures, a typical typology was selected. The structure analyzed is a queen post truss (Figure 1), used for medium spans in Portugal.

Different reliability methods can be employed to evaluate the probability of failure. Monte-Carlo simulation is a simple, although slow, methodology. In particular, this method is useful in computing the reliability of structural systems, with several relevant failure modes. The probability of failure can be computed through Monte-Carlo simulation by generating a large number of samples and evaluating the fraction of samples for which failure occurs.

The main disadvantage of this method is the large number of required samples and, consequently, structural analysis. Considering that a simple linear elastic model was used, Monte-Carlo simulation was relatively fast. The error in the probability of failure can be estimated with a 95% confidence interval, by (Haldar and Mehadevan, 2000):

$$erro(\%) = 200 \times \sqrt{\frac{1-P_f}{N \times P_f}} \tag{6}$$

where P_f is the estimated probability of failure and N is the number of samples.

Figure 1 – Queen post truss analyzed

For computing the probability of failure of a timber structure, it is fundamental to define probabilistic models for both actions and material properties, a structural model relating actions with stresses, and a resistance model, comparing acting stresses and resistance stresses. The resistance model was based on the models defined in Eurocode 5 (CEN 2004), replacing design values by samples. The properties of timber were defined considering *Pinus pinaster*, and the empirical relations defined in Table 1. The fundamental properties considered were based on Brites *et al.* (2008) (Table 3).

Table 3 – Properties of timber

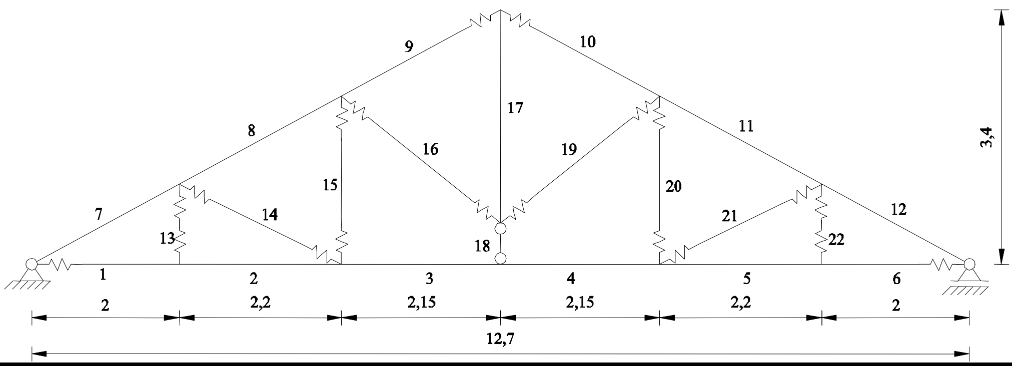
Property	Distribution	E[X]	CoV[X]
Bending Strength (f_m) [MPa]	Lognormal	18	0,25
Modulus of Elasticity in Bending (E_m) [GPa]	Lognormal	12	0,13
Density (ρ) [kg/m^3]	Deterministic	600	-

The loads considered were the dead load and the snow load. Both were simulated as concentrated forces applied in the joints. The permanent load was considered based on the weight of traditional roofs in Portugal. A normal distribution with a coefficient of variation of 10% was considered for the dead loads (JCSS, 2001a). The snow load was defined considering an altitude of 1000 meters, for a location in Portugal. The characteristic value of the snow weight at the ground was calculated based on the prescriptions of Eurocode (CEN, 2003), and a Gamma distribution (JCSS, 2001b) with a coefficient of variation of 40% (Toratti *et al.*, 2007).

The structure was modeled considering linear elastic behavior and the stiffness of connections, based on experimental results described in Branco (2008). The 10 damage scenarios considered are shown in Table 4.

Table 4 – Damage scenarios considered

Scenario	Connection	Element removed
AC1	Semi-rigid	-
AC2	Semi-rigid	13
AC3	Semi-rigid	14
AC4	Semi-rigid	15
AC5	Semi-rigid	16
AC6	Semi-rigid	17
AC7	Semi-rigid	19
AC8	Semi-rigid	20
AC9	Semi-rigid	21
AC10	Semi-rigid	22
AC11	Semi-rigid	18



The computed reliability indices are shown in Table 5. The second column represents the global reliability index, and the following columns represent the reliability index of each element. The reliability of the intact structure is 3.71, close to the target reliability defined in Eurocode. The different damage scenarios result in dramatically different levels of safety.

Table 5 – Reliability index

Scenario	**Global**	**Element**														
		1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**15**	**18**	**22**
AC1	3,71	4,36	3,88	4,15	4,21	4,15	4,39	-	5,15	-	-	5,28	-	-	-	-
AC2	3,77	4,29	3,98	4,23	4,18	4,16	4,36	-	4,96	-	-	5,06	-	-	-	-
AC3	1,47	1,82	1,60	4,42	2,96	3,20	4,29	2,77	2,14	4,75	4,22	3,91	-	-	-	-
AC4	2,23	4,61	2,44	2,53	3,62	4,61	-	4,16	4,14	-	-	-	-	-	-	-
AC5	< 0	4,22	0,75	0,22	0,23	1,61	-	-	2,49	1,87	3,80	4,04	-	-	-	-
AC6	< 0	3,56	2,64	< 0	< 0	1,65	3,53	4,31	< 0	< 0	< 0	0,12	-	4,26	4,42	*
AC7	< 0	4,13	0,70	0,23	0,23	1,68	-	-	2,41	2,28	3,30	3,95	-	-	-	-
AC8	2,78	4,61	3,09	3,12	3,35	4,12	-	-	4,53	4,75	4,16	3,21	3,80	-	-	-
AC9	2,44	4,24	4,17	4,75	3,58	2,61	2,81	-	-	-	-	3,30	3,90	-	-	-
AC10	3,66	4,39	3,75	3,98	4,66	4,60	4,98	-	4,87	5,25	-	4,80	5,25	-	-	-
AC11	3,72	4,33	3,92	4,22	4,14	4,13	4,37	-	5,09	-	-	4,87	-	-	-	-

In order to simplify the analysis, the redundancy factor (β_r) defined in Frangopol and Curley (1987) was applied to all damage scenarios. As shown in Table 6, the redundancy factor for AC2 is not shown, as removing element 13 leads to an increase in the safety factor. In fact, the removal of this element increases the bending moment in the tie-beam. However, this element is traditionally overdesigned. As a result, the increase in probability of failure in the tie-beam is small, and a small decrease in stresses in more critical elements results in an increase in safety. A similar result is observed for scenario AC11. The redundancy factors associated with scenarios AC5, AC6, and AC7 are negative, as the probability of failure considering removal of elements 16, 17, and 19 (king-post and struts) leads to a probability of failure above 50%, making global failure very likely. On the other hand, scenarios AC3, AC4, AC8, AC9 and AC10 lead to redundancy indices that are significant, meaning these elements are not critical to the overall safety.

Table 6 – Redundancy index (β_r) for the queen post truss analyzed

Scenario	AC2	AC3	AC4	AC5	AC6	AC7	AC8	AC9	AC10	AC11
β_r	*	1,657	2,501	-	-	-	3,995	2,914	74,12	*

* leads to an increase in the safety factor

It can be concluded that the structure is resistant to damage to the external posts, presenting acceptable performance under damage in external struts and interior posts. The inner struts and the main post are the key elements of this structure, and should be analyzed more carefully in an inspection or assessment.

5. CONCLUSIONS

The truss analyzed is structurally redundant, meaning that failure of one member will not, necessarily, lead to failure of the all structure. However, the results presented showed that, for this particular type of truss, the inner elements (post and struts) are fundamental to guarantee safety, and can be defined as key-elements. The defects or deterioration in the outer struts and inner posts result in a reduction in safety acceptable considering the magnitude of damage considered. The removal of the outer did not result in any reduction in safety. Moreover, it was observed the removal of the element connecting the tie-beam to the main post did not result in a significant change in safety.

It was also observed that, although the comparison between the required dimensions, according to a design based on Eurocode, and the dimensions used in traditional structure, shows that the posts and struts are overdesigned, this can be extremely beneficial from a robustness point of view, as these elements are fundamental in ensuring safety in case of damage.

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