Uses and goals of mathematical tasks: an experiment with pre-service teachers

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Abstract
Mathematical tasks can be used in teacher education for several purposes: to introduce mathematical ideas, to enhance mathematical understanding or to explore issues related to the pedagogy of mathematics, among others.
In this paper we will describe a model to examine the use of mathematical tasks, in teacher education, proposed by Liljedahl, Chernoff and Zazkis (2007) and based on this model we will describe an experiment that went on with pre-service teachers. They were given a task to explore during their training sessions. They were then asked to adapt, try out and evaluate the task, with their students.
We will discuss the way teachers dealt with the task during the training sessions, how the task was used in the classroom with children and then the reflection teachers made about the experience.

Keywords: mathematical tasks; pre-service teachers; mathematical knowledge; pedagogical knowledge.

Introduction

No one doubts that "what students learn is largely defined by the tasks they are given" (Hiebert & Wearn, 1993). Not only should we use tasks that involve significant mathematics and stimulate students to make mathematical connections, we also need to be aware of our intentions, in using a certain mathematical task.
Tasks are used all the time and for a variety of goals. Even the same task can be used for different goals. The task-related activity, which takes place, is highly dependent on: how the task is prepared; how the task is presented; teacher-learner interactions during the activity; and how the work is debriefed.
When we use a mathematical task we, of course, use mathematical knowledge but since we work within a classroom we also use pedagogical knowledge. Our intentions in using the task also relate to the development of mathematical and pedagogical knowledge in our students.

The model

In order to untangle this complexity, Liljedahl, Chernoff and Zazkis (2007) propose a model that disaggregates mathematics from pedagogy and allows the examination of the use of mathematical tasks in teacher education, from a number of different perspectives.
In the following grid, each cell can be read as "*the use of x to promote understanding of y*".

		GOALS	
		Mathematics (M)	Pedagogy (P)
USAGE	mathematics (m)	**mM**	**mP**
	pedagogy (p)	**pM**	**pP**

(from Liljedahl, Chernoff and Zazkis, 2007)

Let us begin by describing in more detail the sense of each cell.

1. The use of mathematics to promote understanding of mathematics (**mM**)

When we use a mathematical task our intention is to address some mathematical idea. If we need to address a specific idea then the design and development of the task is much more challenging. This implies that, we need to have a "*deep and diverse understanding of the mathematics embedded within the task*" (Liljedahl et al., 2007, p.240).

2. The use of pedagogy to promote understanding of Mathematics (**pM**)

When we use a mathematical task in a class, even though it may be a good mathematical task, and we have a good knowledge of the task, this may not be enough to have a significant mathematical activity. Pedagogical task knowledge is also important in order to release the mathematics within the task. Therefore, we need to have a profound knowledge and understanding of what students know and what kind of procedures should we undertake in order to mobilize that knowledge. These are important conditions to effectively use a task and make the most of it.

3. The use of mathematics to promote understanding of Pedagogy (**mP**)

This aspect is more related with the context in which the tasks are developed.
"*Mathematical tasks provide a context within which to explore issues such as: the nature of mathematics, assessment, constructivism, social constructivism, group work, and so on.*" (Liljedahl et al., 2007, p.240). It is then important that one knows the affordances and limitations of a task in this regard. For instance, an exercise, with a procedural approach and a closed path is not a good task to explore problem solving and reasoning. It is also important to know how the task will progress within a classroom context. Of course this requires knowledge of the classroom and its dynamics and also be conscious of the intended goals.

4. The use of pedagogy to promote understanding of Pedagogy (**pP**)

Again, the knowledge of the limitations and affordances of a task may not be enough in order to release all the richness of a task. So, according to the authors, teachers "*must be able to orchestrate the emergence of the affordances from within the task".* (Liljedahl et al., 2007, p.241) Teachers need to know the embedded pedagogical

affordances of the task and also how to manage them within the classroom.

This model provides a good framework to analyse mathematical tasks, forcing teachers to reflect upon the intended uses and intended goals of a task and also the qualities of a good mathematical task.

The experiment

In this section we will relate an experiment that took place with pre-service teachers and is composed by two phases. In the first phase, we proposed a mathematical task to a class of pre-service teachers (future primary school teachers). The task was explored and then they were asked to adapt the task and implement it with their students, in primary school (second phase).

First Phase

The following task was proposed to a class of pre-service teachers:

Exploring polyminoes

Polyminoes are shapes created by joining, side by side, congruent squares along shared edges.
If we have only 1 square, we have 1 shape - Monomino.
If we have 2 squares, we have 1 shape - Domino.
If we join another square, we will have 2 different shapes – Triominoes.

1. Considering 4 (5 or 6) congruent squares, how many different shapes can you get?
2. From the shapes you got with 6 squares (hexaminoes), which ones are networks for the cube?

This kind of task is still not usual for the pre-service teachers. After discussing the meaning of polymino and what was meant by "different shapes", the pre-service teachers started their exploration in pairs.
With this particular task we intended to explore issues pertaining to the beliefs of pre-service teachers with respect to:

- the nature of mathematics;
- what it means to learn (and teach) mathematics and;
- the creative process of problem solving.

So, the principal goals can be classified, using the described model, as mp.
Of course, it was also our intention to explore some mathematical ideas like:

- symmetry;
- area and perimeter;
- spatial thinking;
- mathematical reasoning;
- …

These ideas emerged during the exploration carried on by the pre-service teachers and were addressed and discussed. So, we also used the task and the mathematics embedded inside it to promote understanding of mathematics.

Referring to the first question, we found out that the pre-service teachers' major concern was to find every polymino. There was not a great concern for organizing the answer or to try to justify whether they discovered all of the shapes. This made us reflect upon the importance of proposing tasks that challenge their beliefs about what it means to "do" mathematics and makes them understand the importance of organizing their reasoning and justifying their procedures. It should not be enough, for them, to just do things without knowing (or even questioning) why they do them.
As for the second question, they had some difficulties in "seeing" that some shapes were nets for a cube (they had to try with material) and again, they didn't try to justify why certain shapes aren't nets for cubes. This confirmed the significance of addressing some mathematical ideas, like spatial thinking, but also the importance of using the task to access topics related to the pedagogy of mathematics.

Second Phase

In the second phase of the study, pre-service teachers were asked to adapt the task, to present it to their students (children) and after the implementation, to write a small reflection upon what went on in the class.
We will relate two different adaptations, done by two groups of pre-service teachers.

In the first one, the group of pre-service teachers used only the first question of the task - exploring different shapes. This task was used with a 3rd grade class (8-9 years old).
The goals stated by the group of pre-service teachers for this particular task were:

- Work in groups (mP);
- Manipulate didactical material (mP);
- Discover new mathematical knowledge (mM);
- Reflect upon this new knowledge (mP);
- Represent/draw the conclusions (mP + mM);
- Discuss with colleagues (pP)

Looking at these goals, and using the model above, we can see that they relate essentially to mP (1, 2, 4, 5) and to mM (3, 5).

For the exploration of the task, children were organized in groups of 3 and each group was given a bunch of paper squares. Pre-service teachers started by explaining the meaning of polyminoes and let children explore.
After some attempts pre-service teachers felt the need to clarify that shapes could not be repeated.
At the end, a member of each group presented one of the shapes to the whole class.

In the reflection pre-service teachers wrote, after the implementation, they said that children were happy and motivated during the exploration of the task. That surprised the Pre-service teachers that did not expect such involvement. We must have in mind that this kind of task is not very used in primary schools. Teachers tend to use exercises and closed tasks and not problems or investigations.
Also surprising for the pre-service teachers was the fact that children were focused and not making much noise. This was interesting because often teachers avoid using manipulatives and working in groups because children make noise.

Pre-service teachers also referred that the sharing of ideas (results) was very interesting.
Unfortunately they didn't provide us data of the discussion part.
As a conclusion of their reflections, they said that this kind of tasks should be used more often.

The other group of pre-service teachers used only the second question of the task – discovering nets of a cube. The task was used with a 4rd grade class (9-10 years old).
Their goals, as they sated, were:

- Find criteria for discovering cube nets (mM + mP);
- Discover the different nets for the cube (mM).

These goals relate to mM (1, 2) and mP (1) of the model.

As in the other adaptation, children were organized in groups of 3 but in this case, they weren't given any material. Pre-service teachers asked children to draw, in squared paper, different shapes using 6 squares (the number of faces in a cube), Then they would select which of that shapes were nets for the cube.
After some difficulties Pre-service teachers felt the need to clarify some issues, namely:

- squares could not be joined by the vertices, only by the edges.
- reflected or rotated images count as one.

Pre-service teachers organized the work so that, as each group found 7 nets, a representative student went to the board and presented "their" shapes to the whole class.
Students were encouraged to search for the 11 nets. This suggests that pre-service teachers tend to emphasize the results (even though they referred that the most important was the comprehension of the process).
After the class, pre-service teachers wrote their reflections. They felt that the task was too abstract and that they should have given some support (manipulative). They also mentioned that the class should have been more dynamic. These comments mean that pre-service teachers don't have the necessary pedagogical task knowledge to properly explore the task.

They also thought that the concept of reflection and rotation should have been discussed in the beginning and that more mathematical contents should have been explored. In this case we notice that the mathematical aspects of the task were not sufficiently explored by pre-service teachers.

Final considerations

Mathematical tasks are used all the time and for a variety of goals. A task is not necessarily "good" or "bad", but even a potentially good task can become a very poor one. When using tasks, teachers need to have a deep knowledge of the mathematics inside the task and of the mathematics that can be reached with the task. Also fundamental is the pedagogical knowledge of the task that enables it to become fruitful.
Teachers should think about **why** they are using a certain mathematical task with their students and **how** they are going to use it.

As mathematics teacher trainers we must be very careful in selecting tasks and be conscious of our goals in using the tasks.

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