Study on the effect of uncertainty on Multi-objective Optimization Problems

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Abstract. In general, parameters in multi-objective optimization are assumed as deterministic with no uncertainty. However, uncertainty in the parameters can affect both variable and objective spaces. The corresponding Pareto optimal fronts, resulting from the perturbed problem, define a cloud of curves. In this work, the main objective is to study the resulting cloud of curves in order to identify regions of more robustness and, therefore, to assist the decision making process. Preliminary results, for a very limited set of problems, show that the resulting cloud of curves exhibits regions of less variation, which are, therefore, more robust to parameter uncertainty.

Keywords: parameter uncertainty, multi-objective optimization, evolutionary algorithms, numerical optimization
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INTRODUCTION

Many problems are modeled assuming a deterministic approach in their formulation. For instance, in demand supply problems, the demand, although stochastic, is formulated as deterministic since this leads to a problem formulation much easier to solve. Furthermore, in general, although there is uncertainty with respect to the objective functions parameters, these are assumed fixed and with no uncertainty. Yet, if the parameters are uncertain, this can have profound implication in the optimization. In this work, our objective is to study the implications of parameter uncertainty in multi-objective optimization. Parameter uncertainty will affect the optimization at the variable space but also at the objective space. Uncertainty will change the shape of the Pareto optimal front and, if this is the case, one important question is to assist the decision making process in the presence of uncertainty. Specifically, as the shape changes, is it possible to identify regions of greater robustness of the Pareto optimal front? How does this affect the decision making process? In this preliminary work, we investigate how parameter uncertainty in very simple problems affects the Pareto optimal front and we propose an approach to deal with uncertainty. Future work, will study more complex problems, with larger number of objectives.

MULTI-OBJECTIVE OPTIMIZATION

Mathematically, a multi-objective optimization problem with q objectives and n real decision variables can be formulated as, without loss of generality:

$$\begin{array}{lll} \underset{x\in\Omega}{\text{minimize}} & f_k(x) & k=1,\ldots,q \\ \text{subject to} & h_i(x)=0, & i=1,\ldots,m \\ & g_j(x)\leq 0, & j=1,\ldots,p \end{array} \tag{1}$$

where x is an n dimensional vector and $\Omega \subset \mathbb{R}^n$ ($\Omega = \{x \in \mathbb{R}^n : l \leq x \leq u\}$), $f_k(x)$ are the objective functions, $h(x)=0$ are the equality constraints and $g(x) \leq 0$ are the inequality constraints. The vectors $l, u \in \mathbb{R}^n$ define the lower and upper bounds on x, respectively.

For a multi-objective minimization problem, a solution $x \in \mathbb{R}^n$ dominates $y \in \mathbb{R}^n$, i.e., $x \prec y$ if and only if, $\forall_{k\in\{1,\ldots,q\}} : f_k(x) \leq f_k(y) \quad \wedge \quad \exists_{k\in\{1,\ldots,q\}} : f_k(x) < f_k(y)$. A solution $x \in \mathbb{R}^n$ is Pareto optimal if and only if, there is no solution $y \in \mathbb{R}^n$ which dominates x, i.e., $\nexists_{y\in\mathbb{R}^n} : y \prec x$.

The main goal of a multi-objective algorithm is to find a good and balanced approximation to the Pareto-optimal set. In order to produce a good approximation to the Pareto optimal front, evolutionary algorithms generate a population of points [5, 3, 4].

We apply the Multi-objective Elitist Genetic Algorithm (MEGA), described in [3]. This approach [6, 8], in contrast to other algorithms, does not require any differentiability or convexity conditions of the search space. Moreover, since it works with a population of points, it can find, in a single run, multiple approximations to the solutions of the Pareto optimal set without the need of fixing any weights and a well distributed representation of the Pareto optimal front induced by the use of diversity-preservation mechanisms. We now shortly describe some technical features and the parameters of the MEGA paradigm (see Algorithm 1).

Algorithm 1 Multi-objective Elitist Genetic Algorithm

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Require: e ≥ 1, s > 1, 0 < p_c < 1, η_c > 0, 0 < p_m < 1, η_m > 0, s_SP > s, σ_share > 0
k ← 0
for l = 1,...,s do
    Randomly generate the main population P ∈ Ω
end for{(Initialization of the population)}
while stopping criterion is not met do
    Fitness assignment FA(P, σ_share) for all points in main population P
    Update the secondary population SP with the non-dominated points in P
    Introduce in P the elite with e points selected at random from SP
    Select by tournaments s points from P
    Apply SBX crossover [7] to the s points, with probability p_c
    Apply mutation to the s points with probability p_m
    k ← k + 1
end while
Update the secondary population SP with the non-dominated points in P
return Non-dominated points from SP
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MEGA starts from a population of points P of size s. In our implementation, a real representation is used since we are leading with a continuous problem. Additionally, a secondary population SP that archives potential Pareto optimal solutions found so far during the search process is maintained. The elitist technique implemented is based on the secondary population with a fixed parameter e ($e \geq 1$) that controls the elitism level, i.e., e is the maximum number of non-dominated solutions of the secondary population that will be introduced in the main population. These non-dominated solutions will effectively participate in the search process that is performed using the points of the main population.

EXPERIMENTAL SETTING

In order to investigate how uncertainty affects the Pareto optimal front we have selected three very simple and well know problems. The Schaffer(1) problem [1] was modified by introducing uncertainty in a parameter in the second objective:

$$\begin{cases} \min f_1(x) & = x^2 \\ \min f_2(x) & = (x-z)^2 \text{ where } z \sim N(2, 0.1) \end{cases}$$

The Schaffer(2) problem [1] was modified by introducing uncertainty in a parameter in the second objective:

$$\begin{cases} \min f_1(x,y) & = x^2 + y^2 \\ \min f_2(x,y) & = (x-z)^2 + (y-5)^2 \text{ where } z \sim N(5, 1) \end{cases}$$

The Rendon(1) problem [1] was modified by introducing uncertainty in a parameter in the first objective:

$$\begin{cases} \min f_1(x,y) & = \dfrac{1}{(x^2 + y^2 + z)} \text{ where } z \sim N(1, 0.5) \\ \min f_2(x,y) & = (x^2 + y^2 + 1) \end{cases}$$

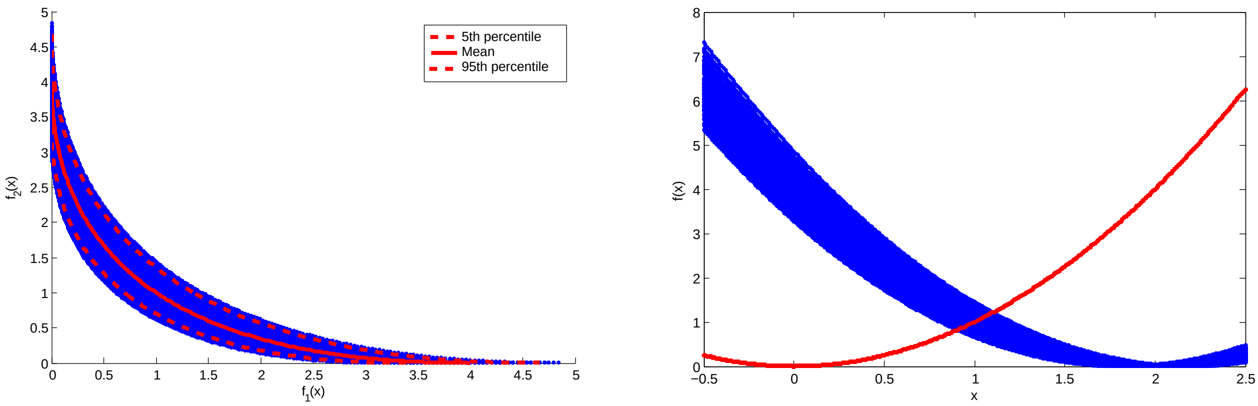

FIGURE 1. Effect on Pareto front and variable space for Schaffer(1).

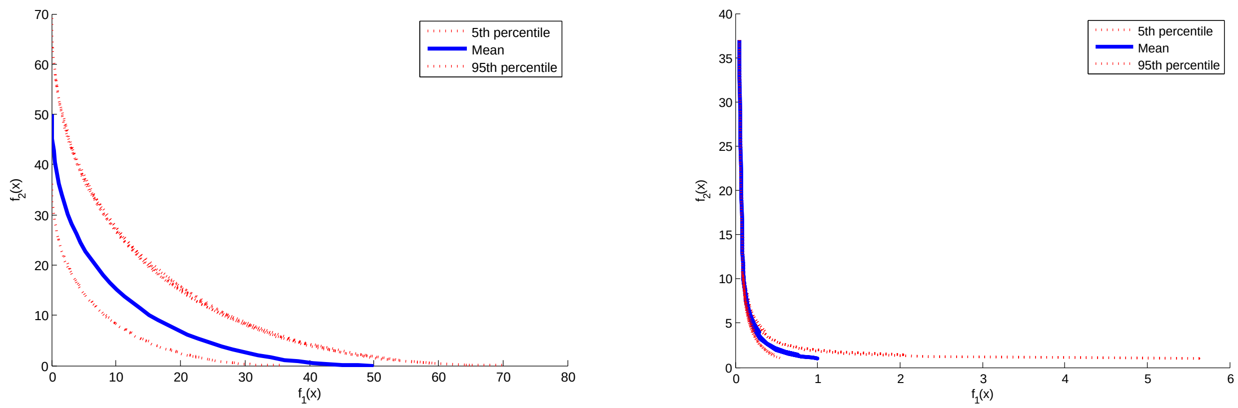

FIGURE 2. Effect on Pareto front for Schaffer(2) and Rendon(1).

RESULTS

In order to investigate how the Pareto optimal front changes with parameter uncertainty, we have assumed that the parameters were perturbed by a normal distribution with mean zero and a given variance. For Schaffer(1) problem, we have generated a random sample of size 100 of perturbed objective functions. Figure 1 (left hand side) presents the resulting Pareto optimal fronts. In order to facilitate its reading, we have superimposed the curves corresponding to the percentiles 5, 50 and 95 of the perturbed parameter. This facilitates the comprehension of the graph. Figure 1 (right hand side) shows how the second objective changes as a result of the perturbation.

For the remaining problems we present only the corresponding Pareto optimal fronts which result from the consideration of the above percentiles (Figures 2).

In all the figures, the resulting Pareto optimal fronts exhibit regions of larger variation and regions where the variation is much lower. Under the point of view of the decision maker, it is preferable to choose operating points that correspond to smaller variations in the Pareto optimal front.

CONCLUSIONS AND FUTURE WORK

In this preliminary work we have studied how perturbations in the objective function parameters affect the shape of the Pareto optimal fronts. In this very simple problems, it can be seen that the perturbations result in cloud of curves exhibiting different degrees of variation. In order to produce a more robust answer in the presence of parameter uncertainty, the decision maker should choose operating points which correspond to regions with less variation. In order to provide a more insightful answer to this problem, future work will focus in problems with a larger number of objectives as well of decision variables.

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