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On The Optimal Resource Allocation in Projects Considering the Time Value of Money

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Abstract: *The optimal resource allocation in stochastic activity networks had been previously developed by applying three different approaches: Dynamic Programming (DP), an Electromagnetism Algorithm (EMA) and an Evolutionary Algorithm (EVA). This paper presents an extension to the initial problem considering the value of money over time. This extended problem was implemented using the Java programming language, an Object Oriented Language, following the approaches previously used (DP, EMA and EVA).*

Keywords: *Resource Allocation, Project Scheduling, Project Management, Stochastic Models, Value of Money Over Time.*

1 Introduction

This paper is concerned with the optimal resource allocation in stochastic activity networks, considering the time value of money.

Studies of resource allocation and activity scheduling derive from studies of the famous 'resource constrained project planning problem' (RCPSP) and its numerous offshoots, see the book by Demeulemeester and Herroelen (2002), chapters 6 to 8 for an excellent treatment of this topic under deterministic conditions. Research on the stochastic version of the problem is more scant. The particular model presented in this paper has its genes in previous treatments in which three approaches were used: Dynamic Programming (DP) (Tereso et al., 2004a; Tereso et al., 2006b), an Electromagnetism Algorithm (EMA) (Tereso et al., 2004b; Tereso et al., 2006a) and an Evolutionary Algorithm (EVA) (Tereso et al., 2007). This paper adds the consideration of the time value of money when evaluating the cost involved in a project.

There are at least two reasons for taking the time value of money into consideration. Firstly, long term projects that span several years (sometimes referred to as 'mega projects') should take account of the changing value of money. For instance, the 'Big Dig' project in Boston was conceived in the mid-80's and was completed in 2007 – some 20 years later[3]. On the issue of 'Cost and Schedule Procedures' the introduction of the report referenced in the footnote states:

> "*Since the project's Final EIS approval by FHWA in 1985, costs (in constant dollars) have grown to more than three times the original estimate .... and the duration has increased by 6 years .... Analysis of the project's performance presented to the committee by the CA/T project management team indicated that about half of the cost growth was caused by inflation*[footnote] *(the original estimates were in 1982 dollars, as required by FHWA) and that a portion of this could be attributed to the extended schedule.* "

The footnote referred to in the main text quoted above reads as follows:

> "*The 'absolute' cost growth for this project, without considering the change in the value of money over time, is approximately $12.0 billion (current project cost estimate of $14.6 billion minus original project cost estimate of $2.6 billion). The project management team asserts that about half (approximately $6.5 billion) of that $12.0 billion can be attributed to inflation from 1985, when expenditures on the project began. The estimate of the effect of inflation is derived from the Engineering News-Record's Building Cost Index (BCI) and Construction Cost Index (CCI) combined into a single index. An escalation because of inflation is calculated for each year of the project by applying the index to the actual or projected annual expenditures, thereby determining their value in 1982 dollars. These yearly escalations (actual or projected expenditures minus their 1982 value) were summed up to arrive at the total escalation of approximately $6.5 billion.*"

These quotes leave little doubt as to the need to take into account the time value of money in projects, especially if they span a long period of time.

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3 http://www.nap.edu/openbook.php?record_id=10629&page=R1

Secondly, discounting future commitments is another way of expressing uncertainty in the total effort required to accomplish the planned activities in the distant future. Normally, one acknowledges the uncertainty in future engineering estimates of resources and their costs by hypothesizing a probability distribution; as will be seen below, the model we construct embeds such uncertainty in the estimate of the activity's work content to which we attribute a probability distribution. However, an alternate way of expressing such uncertainty is via discounting: an estimate of 100 man-days' effort three years hence accounts for only 72.90 man-days now at a discount rate of 0.9, and 42.19 man-days now at a discount rate of 0.75. Depending on the project manager's confidence in the accuracy of the engineering estimates of the work content, he may reflect such incertitude in his choice of the discount rate. Indeed, a discount rate of 0.3 would reduce the 100 man-days to an insignificant 2.70 man-days.

Consideration of the time value of money has appeared predominantly in studies of unconstrained projects [4] under completely deterministic conditions, and in studies concerned with 'projects portfolio selection'. The focus of the former studies has been on when to initiate each activity so as to maximize the net present value of the project; hence the name 'the max-npv problem'; see Herroelen et al. (1995), Vanhoucke et al. (2003), Wiesemann et al. (2010) and Sobel et al. (2009)[5], and the references cited therein. The focus of the studies on portfolio selection, on the other hand, has been on the selection of the subset of projects from among a larger set that has the highest 'promise of success', where 'success' is also measured relative to the net present value; see Archer and Ghasemzadeh (1999), Ye and Tiong (2000), Better and Glover (2006), and the references cited therein.

Our treatment differs from the norm of the studies in the max npv-problem in that it addresses the issue of the allocation of resources – which may be of limited availability – to the activities while taking into consideration the time value of money under stochastic conditions. To the best of our knowledge this has not been treated before.

We devise three approaches to treat this problem: the discrete-time discounting version 1 and 2 and the continuous-time discounting.

In this paper, following the introduction and review of the literature, we first briefly describe the implications of considering the time value of money in this type of problems, explaining how the present worth value of the cost may be evaluated. Then we explain how the three approaches were integrated with the previously developed models. Finally, we present some experimental results and the conclusions.

2 Problem Definition and Review of Prior Work

2.1 Problem Definition

The problem under study is a particular case of the RCPSP (Brucker et al., 1999), where the goal is to optimize the allocation of a single resource to the activities of a project so as to minimize the total cost. The graphic representation of the project is denoted by G(N,A) in which N is the set of nodes and A is the set of arcs in the activity-on-arc (AoA) mode of representation. This cost is composed of the resource cost and the tardiness cost.

In our problem, each activity $i \in A$ has an associated work content (W_i), a random variable (r.v.), assumed to be exponentially distributed with parameter λ_i.

$$W_i \sim exp(\lambda_i). \quad (1)$$

Let x_i represent the amount of resource allocated to activity i, restricted to lie between a lower and upper bound, $x_i \in [l_i, u_i]$ with $l_i \geq 0$ and $u_i < +\infty$. We assume that the availabilities of the resources are unlimited.

We assume that the duration of an activity is related to its work content and the resource allocated to it by the relationship

$$Y_i = \frac{W_i}{x_i}, \quad (2)$$

whence Y_i is also a r.v. possessing the same distribution as W_i, modified by x_i.

As stated before, the total cost of the project is the sum of the resource cost, assumed quadratic in the allocation, for the duration of the activity, and the tardiness cost, assumed linear in the tardiness:

$$Total\ Cost = rc + tc, \quad (3)$$

where rc is the resource cost

$$rc = \sum_{i \in A} c_R\, x_i^2\, Y_i = \sum_{i \in A} c_R x_i\, W_i, \quad (4)$$

and tc is the tardiness cost

$$tc = c_L \times max\{0, \Upsilon_n - T\}, \quad (5)$$

where c_R represents the unit resource cost, normalized to 1, and c_L represents the marginal cost per period, Υ_n is the time of realization of the last node of the network (a r.v.) and T is the specified due date of the project.

[4] A project is 'unconstrained' when the resources are abundant, and therefore any number of activities can be executed at the same time.

[5] The study by Sobel et al. (2009) is distinguished from the others cited here in that it treats a stochastic environment. It assumes the activities to have exponentially distributed durations and constructs a continuous time Markov chain model of the process.

In addition to the above statement of our problem, we are also concerned with the time value of money. In particular, we are concerned with the present value (p.v.) of the project using a continuous discount rate, assuming disbursements and revenues at different epochs throughout the progress of the project.

2.2 Review of Literature

Consideration of the time value of money and its impact on the schedule of undertaking the various activities of the project has been the subject of three recent papers by Buss and Rosenblatt (1997), Creemers et al. (2008) and Sobel et al. (2009). All three deal with stochastic activity duration (not work content) which is assumed to be exponentially distributed, with positive (income) and negative (disbursement) cash flows and a penalty for tardiness in completing the project. The concern of all three contributions is with the manipulation of the start time of each activity, perhaps intentionally delaying some after being sequence-feasible, in order to maximize the net present value (NPV) of the project. Herein resides the main difference between these treatments and ours: we do not delay the start of any activity but assume that an activity shall be started as soon as it is precedence-feasible; our decision mechanism resides in varying the resource allocation within its permitted bounds to effect such maximization.

The problem of optimal resource allocation without any consideration of the time value of money and from the vantage point of the activity's work content to minimize the project cost as specified in (3) above was addressed by Tereso et al. (2004a), in Matlab, using dynamic programming (DP), then in a distributed platform using Java (Tereso et al., 2006b). The computational burden imposed by the DP model stimulated treatment by other techniques and led to implementation of the Electromagnetism Algorithm (EMA), first in Matlab; see Tereso et al. (2004b), then in Java; see Tereso et al. (2006a), followed by implementation of the Evolutionary Approach (EVA) in Java (Tereso et al., 2007). As expected, implementation of the EMA and EVA achieved better execution time results than DP, which was effective only in small networks.

3 On The Present Worth of Resource Cost

The resource allocation models presented in this section are: Dynamic Programming (DP), Electromagnetism Algorithm (EMA) and Evolutionary Algorithm (EVA). For each model we shall present two different approaches: Discrete-Time Discounting and Continuous-Time Discounting. In either approach the goal is to determine the resource allocation that optimizes the p.v. of the project. This section is devoted to a brief review of some basic concepts in "interest" and "discounting" which may not be familiar to all.

3.1 Discrete-Time Discounting

3.1.1 *Version 1*

In discrete-time discounting the duration of the activity is divided into discrete time intervals and discounting is applied to the receipts/disbursements in each interval.

Suppose the annual interest rate is given as i_a. Then the annual discount rate, denoted by β, is given by

$$\beta = \frac{1}{1+i_a} \tag{6}$$

If one wishes to use a different time interval from a year (referred to in the sequel as a "period") then one must evaluate the number of periods n_p in a year. The period interest rate, denoted by i_p, is given by the solution to the equation

$$(1+i_p)^{n_p} = 1 + i_a,$$
$$\text{which} \Rightarrow i_p = (1+i_a)^{1/n_p} - 1. \tag{7}$$

The period discount factor, α, is evaluated from an expression similar to (6) but with i_p instead of i_a. Alternatively, one may evaluate the period discount rate as the solution to the equation

$$\alpha^{n_p} = \beta. \tag{8}$$

where α is the discount rate per period; $0 < \alpha < 1$.

Assuming that the work content W is expended uniformly over the activity duration Y, then the work content in each period is W/Y, which, by the definition of Y, is equal to x. The p.v. of the work content at the start of the activity (denoted by VW) at discount rate α is given by

$$VW = \underbrace{x + \alpha x + \alpha^2 x + \cdots + \alpha^{Y-1} x}_{Y\ terms}$$
$$= x\frac{1-\alpha^Y}{1-\alpha}. \tag{9}$$

If the activity starts at time d then the p.v. of the activity work content, denoted by PVW, is given by

$$PVW = VW.\alpha^d. \tag{10}$$

Assuming the unit resource cost is c_R and the cost is quadratic in the allocation for the duration of the activity, then the p.v. of the resource cost is given by

$$rc = c_R x^2 . \frac{PVW}{x} = c_R \times x \times PVW \tag{11}$$

Without discounting we would have estimated the cost to be

$$rc = c_R \times x \times W, \tag{12}$$

3.1.2 *Version 2*

After some analysis to the Discrete-Time Discounting model with a daily time interval, we concluded that, because of the relative smallness of the period to the overall planning horizon, it is almost the same as the Continuous-Time Discounting. Therefore a second version of this model is relevant to our analysis.

In this second version, we assume that the cost of the work content of an activity is incurred at its completion. Thus we shall avoid the daily discounting, and the resulting model will be different with different results as well.

To calculate the p.v. of the work content (VW) at the start of the activity, when the cost of the activity is incurred at its completion, we need to know its duration (Y). This is evaluated by the expression used before ($Y = W/x$). Then, using the periodic discount rate α and the cost of the activity we get:

$$VW = W\alpha^{Y}. \tag{13}$$

And PVW is obtained as before (see expression (10)).

So, if the unit resource cost is c_r then, the p.v. of the resource cost of this activity would be given by expression (11), which would give a result slightly lower than the value obtained in the previous version due to the delay in the cost encumbrance.

3.1.3 *Continuous-Time Discounting*

An alternate approach is to consider time as a continuum and the effort is continuously applied to the activity.

The continuous discounting of \$1 spent at time t is given by $e^{-i_p t}$.

For the whole year we have the sum

$$rc = 1 + e^{-i_p} + e^{-2i_p} + \cdots + e^{-364 i_p} = \frac{1-\left(e^{-i_p}\right)^{365}}{1-e^{-i_p}}. \tag{14}$$

If the work content is continuously discounted each day, during n days, then the p.v. of the work content would be

$$VW = x + xe^{-i_p} + xe^{-2i_p} + \cdots + xe^{-(Y-1)i_p} = x \times \frac{1-\left(e^{-i_p}\right)^{n}}{1-e^{-i_p}}. \tag{15}$$

If the activity starts approximately d days from present time:

$$PVW = VW \times e^{-d\times i_p} \tag{16}$$

So, if the unit resource cost is c_R then, the p.v. of the resource cost of this activity would be given by expression (11).

4 The Dynamic Programming Model

The Dynamic Programming Model (DP) (Tereso et al., 2004a) (Tereso et al., 2006b) divides the activities into two groups: those with fixed resource allocations, denoted by the set F, and those with yet-to-be-decided resource allocations, the *decision variables*, denoted as the set D, with $F \cup D = A$, the set of all activities. The set D is the set of activities on the longest path in the network (the path containing the largest number of activities). The set F is its complementary set of activities in A. A stage is defined as an epoch of decision making. We define stage (k) as the decision epoch of the allocation x_a for each activity $a \in D$. In each stage only one decision variable is optimized since each uniformly directed cutset (u.d.c.) in the network contains exactly one activity in D; therefore there are as many stages as there are decision variables, which is equal to $|D|$, the cardinality of the set D. There is also the concept of state, which is defined as a vector of times of realization of the set of nodes that allows us to decide on x_a and evaluate the contribution of the stage, for $a \in D$. The stage corresponds to the project's evolution over time; the state specifies its condition (in particular, the time of realization of each "source" node in the u.d.c.), and the decision taken results in the "stage reward" (a cost, in our case) and moves the project to a new stage and a new state. Since we assume that the work content of each activity in the project is a random variable (r.v.), the realization of any stage or state shall also be a r.v., so is the cost incurred.

In DP, the numbering of stages is done backwards. The decision variable of stage k is identified as $x_{[k]}$, where k means the number of stages that are still missing for the conclusion of the project (stages "to go" to project completion). So, in stage $k = 1$, starting from the ending node n; the contribution of the stage is the sum of the resource cost ($= x_{[1]}W_{[1]}$) and the tardiness cost, if it exists ($= c_L \times \max\{0, \Upsilon_n - T\}$), in which Υ_n is the time of realization of node n, a r.v., and T is the target project completion time. Therefore, we obtain

$$f_1(s_1|F) = rcf + \min_{x_{[1]}\in D} \varepsilon\left\{c_r x_{[1]} W_{[1]} + c_L \times U\right\}, \tag{17}$$

where

$$U = max\{0, \Upsilon_n - T\} \tag{18}$$

In this stage we also add the resource cost of the fixed activities (rcf).

In all other stages, the contribution to the total cost is just the resource cost, a random variable equal to $x_{[k]}W_{[k]}$, applied until $s_k = t_1 = 0$, using:

$$f_k(s_k|F) = \min_{x_{[k]}\in D} \varepsilon\{c_{[k]}([x_{[k]}], s_k) + \varepsilon f_{k-1}(s_{k-1}|F)\}. \quad (19)$$

Using this method and a discrete time approach, we get for the first stage,

$$f_1(s_1|F) = PV_{rcf} + \min_{x_{[1]}\in D} \varepsilon\{c_r x_{[1]} PVW_{[1]} + PV(c_L \times \mathrm{U})\}, \quad (20)$$

with

$$\mathrm{PV_{rcf}} = \varepsilon \sum_{k\in F} \mathrm{c_r x_k PVW_k} = \sum_{k\in F} \mathrm{c_r x_k} \varepsilon(\mathrm{PVW_k}). \quad (21)$$

In version 1 we will have

$$PVW_k = x_k \frac{1-\alpha^Y}{1-\alpha} \times \alpha^d \quad (22)$$

and in version 2

$$PVW_k = W\alpha^Y \alpha^d \quad (23)$$

where Y represents the time of the activity duration (a r.v.) and d represents the time that the activity starts, and

$$PV(c_L \times \mathrm{U}) = c_L \times \mathrm{U} \times \alpha^{\Upsilon_k} \quad (24)$$

For the other stages:

$$f_k(s_k|F) = \min_{x_{[k]}\in D} \varepsilon\{PVW_{[k]}([x_{[k]}], s_k) + \varepsilon f_{k-1}(S_{k-1}|F)\} \quad (25)$$

If we use this method and a continuous time approach, we need to use this equation:

$$PVW_k = x_k \times \frac{1-(e^{-i_p})^Y}{1-(e^{-i_p})} \times e^{-d\times i_p} \quad (26)$$

where Y represents the activity duration, d the time that the activity starts and i_p the periodic interest rate, and

$$PV(c_L \times \mathrm{U}) = c_L \times \mathrm{U} \times e^{-\Upsilon_k \times i_p}. \quad (27)$$

5 The Electromagnetism Algorithm

The Electromagnetism Algorithm (EMA) is based on the principles of electromagnetism and it was developed by Birbil and Fang (2003). Those principles say that two particles experience forces of mutual attraction or repulsion depending on their charges.

This algorithm is divided in four phases that are: initialization of the algorithm, calculation of the vector of total force exerted on each particle, movement along the direction of the force, and application of neighborhood search to exploit the local minima (Birbil et al., 2004).

The initialization disperses randomly the *m* particles in the n-dimensional space (hyper-cube); each particle is a vector of dimension |A| with a fixed allocation of the resources to the activities. For each particle the value of the objective function is calculated and the best point is saved in x^{best}.

In the next step, the vector of total force exerted on each particle (x^i) is calculated. The charge of each particle determines the level of attraction or repulsion between any two particles of the population in the n-dimensional space. The charge is calculated as:

$$q^c = \exp\left[-\mathrm{n} \times \frac{f(x^c) - f(x^{best})}{\sum_{k=1}^{\mathrm{m}}[f(x^k) - f(x^{best})]}\right], \quad c = 1,2,\dots,m. \quad (28)$$

As the value of the objective function becomes better, the value of those charges increases.

The total force exerted on a particle, F^c, is determined by:

$$F^c = \sum_{b\neq c}^{m} (x^b - x^c) \frac{q^c q^b}{\|x^b - x^c\|^2}, \quad c = 1,2,\dots,m. \quad (29)$$

After determining $\mathrm{F^c}$, it is just necessary to move the particle according to:

$$x^{m'} = x^{\mathrm{m}} + \beta \frac{F^c}{\|F^c\|}(RNG), \quad (30)$$

where β is a random parameter that influence the movement length.

For our purpose, to obtain the minimum cost of the project, we have to evaluate the total cost after each iteration, keeping the best one stored for later use.

The total cost of the project is given by the sum of the p.v. of the resource cost (*RC*) and the tardiness cost (*TC*),

$$PVC = \sum_{a=1}^{n} PVRC_a + PVTC. \quad (31)$$

If we use this method and a discrete time approach we need to evaluate the p.v. of the resource cost and the p.v. of the corresponding tardiness cost.

The p.v. of the resource cost is given by

$$\mathrm{PVRC} = \sum_{a=1}^{n} c_R \times x_a \times \mathrm{PVW}_a \tag{32}$$

where c_R is the constant of proportionality.

In version 1 we will have

$$\mathrm{PVW}_a = x_a \frac{1-\alpha^{Y}}{1-\alpha} \times \alpha^{d}, \tag{33}$$

and in version 2

$$\mathrm{PVW}_a = W\alpha^{Y}\alpha^{d}, \tag{34}$$

in which Y represents the time of the activity duration and d represents the time that the activity starts.

The p.v. of tardiness cost is evaluated using:

$$\mathrm{PVTC} = c_L \times \max(0, \Upsilon_n - T) \times \alpha^{\Upsilon_n}. \tag{35}$$

If we use this method and a continuous time approach we need to evaluate the p.v. of the resource cost and the p.v. of the corresponding tardiness cost.

The p.v. of the resource cost is given by

$$PVRC = \sum_{a=1}^{n} c_R \times x_a \times PVW_a \tag{36}$$

where c_R is the constant of proportionality, and

$$\mathrm{PVW}_a = x_a \times \frac{1-\left(e^{-i_p}\right)^{Y}}{1-\left(e^{-i_p}\right)} \times e^{-d \times i_p}, \tag{37}$$

in which Y represents the activity duration, d represents the time that the activity starts and i_p the interest rate per period.

The p.v. of the expected tardiness cost is evaluated using:

$$\mathrm{PVTC} = c_L \times \max(0, \Upsilon_n - T) \times e^{-\Upsilon_n \times i_p} \tag{38}$$

6 The Evolutionary Algorithm

The Evolutionary Algorithm (EVA) is based on the natural evolution of the species, and it was developed by (Costa and Oliveira, 2001). It is usually used in optimization problems and it is based on the population evolution. In this kind of problem, it is very easy to be trapped in a local optimum, so it is crucial to use global optimization methods in order to achieve the best global solution.

Nowadays there are two important approaches to EVA: Evolutionary Strategies (EVA-ES) and Genetic Algorithms (EVA-GA). In our study we adopted the EVA-ES because several studies (Hoffmeister and Bäck, 1991) (Dianati et al., 2002) indicate that ES's are usually more efficient than GA in terms of the number of objective function evaluations, especially in continuous optimization problems.

The solution is obtained evaluating the fitness of the individuals and selecting the best ones to pass to the next generation. Thus, we start to generate an initial population (ancestors) of size μ that will create a new population (descendents) of size λ, after applying mutation and recombination operations. In each generation, λ descendents are generated from μ ancestors, and the best individuals are chosen to go to the next generation. All of these individuals are represented by vectors of real decision variables.

The mutation and recombination processes are used to preserve the genetic diversity between ancestors and descendents so that the algorithm will not be trapped in a local minimum.

The nomenclature often used for representing ES is based on the number of the ancestors μ, on the number of the descendents λ and on the type of selection chosen. If we adopt the $(\mu + \lambda)$ nomenclature, after the descendent population has been generated they are added to the ancestors population and then the μ best individuals are selected to go to the next generation.

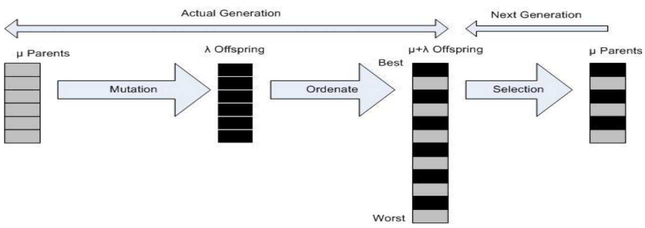

Figure 1 – (μ+λ) nomenclature.

Alternatively, if we adopt the (μ, λ) nomenclature, after the descendent population has been generated the μ best individuals are selected for the next generation.

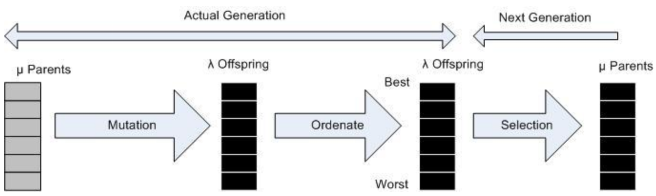

Figure 2 - (μ,λ) nomenclature

The total cost of the project when using this method is given by the same formulas used in EMA, applied to both versions of the Discrete Time Approach and to the Continuous Time Approach.

7 Results

7.1 Experiment layout

The program was tested on a set of fourteen projects (see main characteristics in table 1) that range in size from 3 to 49 activities[6]. The networks chosen enabled analysis of a spectrum of different network complexities. These networks were also used in prior studies (Tereso et al., 2006a; Tereso et al., 2007; Tereso et al., 2006b), allowing for comparison of performance and results. Each activity i has stochastic work content W_i, assumed to be exponentially distributed (as in prior studies).

Table 1 – Main Characteristics of the Networks Tested

Net	1	2	3	4	5	6	7	8	9	10	11	12	13	14
|A|	3	5	7	9	11	11	12	14	14	17	18	24	38	49
T	16	120	66	105	28	65	47	37	188	49	110	223	151	221
c_L	2	8	5	4	8	5	4	3	6	7	10	12	5	5

The due date T was selected to be slightly greater [1.04,1.09] than the length of the "critical path" in the CPM calculations, assuming the mean work content and the quantity of resource x_i equal to 1, thus the duration of each activity is fixed at $y = \bar{W}$, without considering the time value of money. The tardiness cost c_L was chosen to be 2 to 12 times the marginal resource cost c_R, which was normalized at 1.

Both in the EMA and EVA tests, we generated a set of work contents randomly (100) to represent the possible values for each activity and then we kept these values for all the experiments, for each network. The results presented were obtained by evaluation the mean of four runs.

7.2 Results

The results reported here were obtained using an Intel Core 2 6400 CPU at 2.13 GHz with 1GB of RAM under Microsoft Windows XP Professional SP3.

In our case, some values are equal in all runs (annual interest rate=8.6957%, unitary resource cost=$1 and number of periods in a year=365).

Appendix A presents an example of application of the three models and also a brief algorithm description. Appendix B explains the impact on the results of taking discounting into account. Appendix C presents the execution times obtained in all the experiments.

For the networks tested, using Dynamic Programming, the results are shown in tables 2 and C1. The work contents and realization times were discretized at 4 points. The range of the decision variables (fixed variables) was discretized at 5 (3) points. For the larger networks we could not get results due to excessive time to complete experiment. The program was aborted after 8 hours running.

Table 2 - Total Cost: Dynamic Programming

Net	No discounting	Discrete Version 1	Discrete Version 2	Continuous
1	$43.326	$43.240	$43.188	$43.240
2	$297.513	$294.629	$293.330	$294.629
3	$197.979	$197.070	$196.623	$197.070
4	$385.321	$382.813	$381.082	$382.813
5	$135.340	$134.974	$134.886	$134.974
6	$293.851	$292.599	$291.886	$292.599
7	$161.825	$161.352	$161.125	$161.352
8	$123.931	$123.671	$123.533	$123.671
9	(*)	(*)	(*)	(*)
10	(*)	(*)	(*)	(*)
11	(*)	(*)	(*)	(*)
12	(*)	(*)	(*)	(*)
13	(*)	(*)	(*)	(*)
14	(*)	(*)	(*)	(*)

(*) – Program aborted after 8 hours running.

For the Electromagnetism Algorithm, with the number of particles equal to 15, we got the results shown in tables 3 and C2. The total cost reported is the mean of the values obtained in the 4 runs.

Table 3 - Total Cost: Electromagnetism Algorithm

Net	No discounting	Discrete Version 1	Discrete Version 2	Continuous
1	$43.945	$43.537	$43.314	$43.728
2	$337.025	$339.608	$324.178	$326.310
3	$225.952	$218.901	$221.431	$220.266
4	$406.242	$387.221	$388.199	$389.353
5	$138.008	$133,808	$134.877	$135.987
6	$263.557	$253.173	$248.337	$251.678
7	$158.929	$156.139	$155.176	$156.772
8	$94.510	$94.442	$93.175	$93.681
9	$801.433	$750.093	$743.219	$746.081
10	$106.720	$105.945	$105.298	$105.508
11	$453.402	$443.930	$443.159	$444.894
12	$1,381.696	$1,167.805	$1,157.423	1.175,338
13	$811.971	$795.434	$776.087	$774.685
14	$532.055	$546.510	$511.184	$518.341

For the Evolutionary Algorithm, using 15 generations, a population of descendents and ancestors equal to 15, a recombination population equal to 10 and the type of selection (μ, λ), we got the results shown in tables 4 and C3. The total cost reported is, as above, the mean of the 4 runs.

[6] See full characteristics of the networks tested in www.dps.uminho.pt/pessoais/anabelat

Table 4 - Total Cost: Evolutionary Algorithm

Net	No discounting	TC – EMA Discrete Version 1	TC – EMA Discrete Version 2	TC – EMA Continuous
1	$44.499	$44.065	$44.097	$44.469
2	$343.077	$336.700	$338.077	$348.476
3	$238.832	$233.379	$229.840	$227.811
4	$413.791	$406.344	$402.645	$407.611
5	$148.573	$156.557	$143.630	$142.791
6	$266.330	$257.522	$255.039	$262.052
7	$166.982	$164.889	$160.329	$166.183
8	$106.403	$102.360	$102.608	$97.008
9	$814.795	$785.968	$787.030	$788.552
10	$116.157	$111.512	$113.428	$112.399
11	$489.945	$470.083	$475.137	$475.716
12	$1,518.377	$1,430.934	$1,437.272	$1,434.103
13	$903.669	$829.685	$830,003	$824.082
14	$569.911	$549.359	$551.289	$525.174

8 Conclusion

We started by comparing the results for the three algorithms without considering the time value of the money, and as expected the costs are higher (see tables 2, 3 and 4) than the costs obtained when using discounting; and the execution times are smaller (see tables C1, C2 and C3). This happens because when we consider the time value of money we discount the costs to the present time, turning them smaller and we need to do more evaluations, so it takes longer to achieve comparable results.

After doing an analysis to the Discrete Time Approach model with a daily time interval (discrete version 1), and comparing with the Continuous Time Approach model, we concluded that these two approaches are very similar. So, we decided to create a second version using the Discrete Time Approach model where we will only pay the work content of an activity when it is finished (discrete version 2). This avoids the daily discount and these two models will be different and have different results as well, as can be seen, in particular on the DP Model (table 2).

In table 2, which represents the DP results, we verify that the higher total cost considering discounting is given by the discrete-time approach (version 1), where we assume the work content is incurred at the start of the activity and by the continuous-time approach, which does the discount without establishing time intervals. There is no difference between these two approaches, because the number of periods per year used in the discrete-time approach is 365, staying very close to continuous time. The cost for the second version of discrete-time approach, where it is assumed that the work content is appointed to the end of the activity, is always smaller, but the different is not very high. This is because the time intervals used are not very large and the cost of the resource used is small.

When we analyze the total cost results presented on table 3 (EMA) and table 4 (EVA) we verify that the difference in the results is higher between the two approaches. This happens because these two algorithms have a random component that conditions the results. In the calculation of the resultant force (in the EMA) as well as in the formation of new generations (in the EVA) the random factor is always present. This random factor helps these algorithms not to be stuck in local minima.

For the smaller networks, DP achieved better results than EMA and EVA, but when networks increase their number of activities, DP results are worst than EMA and EVA, in terms of cost and also in terms of execution times. This is because the DP Model has to discretize the stochastic continuous variables during execution and the search space may not be well covered when the number of discretized points is small. The number of points used is a compromise between better results and slower execution time. The time needed to do the search also increases exponentially when the number of activities increases. This is because the number of nested cycles also increase, making the algorithm less efficient. In EMA and EVA we represent the stochastic variables using simulation and their search has a random component which allows exploring other regions of the search space, making these algorithms more efficient for larger networks.

Comparing the EMA and the EVA algorithm, we can conclude that EMA reached better results in terms of cost, but EVA was faster. Both of them are superior to the DP model, for larger networks, as concluded before.

We also conducted another experiment modifying network 1 in order to illustrate better the difference between taking discounting into account or not (see appendix B). Considering the results obtained, the following remarks are pertinent. Firstly, as expected, the optimal cost under discounting is less than that without discounting. However, the magnitude of the difference is rather surprisingly large, amounting to approximately 111% of its value, despite the closeness of the daily discount factor α to 1. Secondly, the optimal resource allocation without discounting is maximal for activity 1, almost ‘normal’ for activity 2 (very close to 1.0), and less than ‘normal’ for activity 3; which reflects the ‘anxiety’ at the start of the project relative to activity 1. This is quite different from the optimal resource allocation with discounting which is minimal for activities 1 and 3 and slightly above minimal for activity 2, which reflects the ‘steadiness’ in the decision brought about by discounting the future. Thirdly, the ‘PERT-based’ estimate of cost is closer to the undiscounted cost (difference = $122) than the discounted cost (difference = $1912), as expected, since in the PERT calculations we didn’t use discounting. If nothing else, this simple example forcefully illustrates the difference in decision as well as in value when analysis is conducted taking the time value of effort into account. This result reflects the inherent tendency of the PERT calculations to under-estimate the expected completion time of the project. In a sense, the `PERT-based' estimate of cost is based on a myopic view of the future, which is akin to what discounting does.

This paper presented the results for the resource allocation problem in stochastic activity networks as in previous papers of the same first author (Tereso, 2002; Tereso et al., 2003; Tereso et al., 2004a; Tereso et al., 2004b; Tereso et al., 2008; Tereso et al., 2006a; Tereso et al., 2007; Tereso et al., 2006b; Tereso et al., 2009), but introduced a new component on the models: the time value of money. This model may be better suited for representing real life situations, when this factor is important to be considered.

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Appendix A: Application Example

We will try to explain better how the models were applied using the simpler network tested (figure A1).

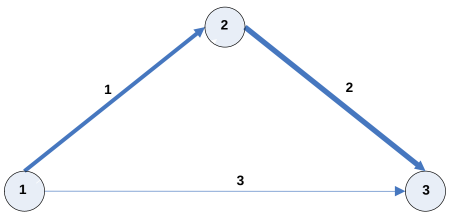

Figure A1 - Example network 1

The due date of this network T is 16 and the tardiness penalty c_L is 2 per unit time. The remaining parameters are represented in table A1. These parameters are the parameter (λ) of the exponential distribution that represents the Work Content of each activity, and the minimal and maximal amount of resource to allocate to each activity (min and max). The expected duration of activity 1 is 1/λ =1/0.2 = 5, and for activity 2 and 3, 10 and 14.29 respectively. In this way, the PERT expected duration for this network is 15. The due date of the project is selected to be a value above the PERT expected duration (approximately 5% more).

Table A1 - Parameters for network 1

Activity	1	2	3
λ	0.2	0.1	0.07
x_{min}	0.5	0.5	0.5
x_{max}	1.5	1.5	1.5

A1: The Dynamic Programming Model

First we determine the longest path in the network shown in heavy lines in figure A1. The activities along the longest path are the decision variables; set D = $\{x_1, x_2\}$. The set of fixed activities is the set F = $\{x_3\}$. For simplicity reasons we will illustrate the application of the models without considering the time value of money. The inclusion of this component is achieved through the use of the equations presented in sections 3, 4 and 5. The activities on the set F were discretized in 3 points {0.5, 1.0, 1.5} and the activities on the set D in 5 points {0.5, 0.75, 1.0, 1.25, 1.5}. The Work Contents were discretized in 4 values each with probability 0.25, with the same expected value as $\varepsilon(W_a)$ for instance, W2 ~ exp(0.1) was assumed to take only four values: {1.3695, 4.7675, 10.00, 23.8629}, all with equal probability. To be sure, the average of these four values is 10, which is the expected value of the r.v.. For each of the values of the fixed activities we evaluate the resource cost of the fixed activities, by the following expression. For example, considering $x_3 = 0.5$ we will have:

$$rcf = \sum_{i \in F} c_R x_i \, \varepsilon(W_i) = 1 * 0.5 * \frac{1}{0.07} = 7.143,$$

The DP iterations are initiated at stage 1 which is defined by the decision variable x_2 (the allocation to activity 2). The state may be defined by t_2, the time of realization of node 2. We have that

$$f_1(s_1|F) = rcf + \min_{x_2} \varepsilon\{c_r x_2 W_2 + c_L \times \text{U}\}$$

Where

$$U = \max(0, \Upsilon_3 - T)$$

And

$$\Upsilon_3 = \max\left\{t_2 + \frac{W_2}{x_2}; \frac{W_3}{x_3}\right\}$$

In this case we only have two stages. Stage 2 is the final stage.

$$f_2(s_2|F) = \min_{x_1} \varepsilon\{x_1 * W_1 + \varepsilon f_1(s_1|F)\}$$

t_2 is determine to be in the range between the minimal and maximal possible durations of activity 1. Then this range is also discretized. All the variables represented in upper case are random variables that were discretized for simplicity. In order to do evaluations with this kind of variables we need to keep also the associated probabilities and do the correct evaluation of the sum or maximum of two random variables, as needed.

After all the evaluations, the final result obtained for this network was \$43.326 with $x_1 = 1$ and $x_3 = 1$. The value of x_2 depends on the time of realization of node 2, a r.v..

A2: The Electromagnetism Algorithm

The Electromagnetic Algorithm works in a different way compared to the DP model. Instead of discretizing the random variables, namely the Work Content of each activity, their possible values are obtained though simulation. To start the algorithm, we generate randomly K (=100) vectors of work contents. These vectors were stored and used in all runs, for the same network, to keep the objective function stable. Then we generated *m* (=15) vectors of X (allocation quantities); *m* represents the size of the population of particles. For each vector of particles (X) and for each vector of work contents (W) the total cost is evaluated, using the equation (here without the discounting factors):

$$c = \sum_{a=1}^{n} c_R \times x_a \times W_a + c_L \times \max(0, \Upsilon_n - T)$$

The objective function value of each particle is the mean cost of all W's. Charges and forces are then evaluated. Points are moved to obtain a set of new *m* points. This process continues until the limit number of iterations is reached. In figure A2 we present the generic algorithm that describes the steps of this process.

1. Generate K vectors of $W = (w_1..w_n)$ randomly
2. Generate m vectors of $X = (x_1..x_n)$ to start with
3. For each vector X
4. For each vector W
5. $rc = c_R \times x_a \times W_a$
6. $tc = c_L \times max\{0, \Upsilon_n - T\}$
7. $c = rc + tc$
8. End for
9. $f = \sum \frac{c}{K}$
10. Evaluate charges
11. Evaluate forces
12. End for
13. Move the points
14. Go to step 3 until nº of iterations specified is reached

Figure A2 - The EMA algorithm

Suppose one of the work contents generated is equal to:

W1 = {6, 12, 15}

And one of the particles generated is:

X1 = {1.5, 0.5, 0.5}

For W1 and X1, we first evaluate the duration of the activities as being

$$y_a = \frac{W_a}{x_a} = \{4, 24, 30\}$$

Using CPM we evaluate $\Upsilon_n = 30$.

The resource (with c_R=1) and tardiness costs for this particle are:

$$rc = 1.5 \times 6 + 0.5 \times 12 + 0.5 \times 15 = 22.5$$

$$tc = 2 \times \max(0, 30 - 16) = 28$$

The total cost is then the sum of these two costs, 50.5.

The algorithm proceeds repeating this kind of evaluations for each work contents generated. The objective function value (cost) of each particle will be the mean of the cost values obtained for each vector of work contents.

Then the particles will attract and repel each other, originating movements that will produce other particles. At the end of this process convergence to the minimum is expected. In the case of network 1, the result obtained was a total cost equal to $43.945 with the following values for the allocation variables X^* = (0.500, 0.695, 0.914).

A3: The Evolutionary Algorithm

The evolutionary algorithm is basically applied in the same way as the electromagnetic algorithm. The Work Contents generated for the EMA are also used for the EVA. Then the initial population, of size $\lambda = 15$, of ancestors is generated, and through mutation and recombination operations, λ descendents are generated from $\mu = 10$ ancestors, and the best individuals are chosen to go to the next generation. The generic algorithm that describes the steps of this process can be seen in figure A3.

1. Generate K vectors of $W = (w_1..w_n)$ randomly
2. Generate m vectors of $X = (x_1..x_n)$ to start with
3. For each vector X
4. For each vector W
5. $rc = c_R \times x_a \times W_a$
6. $tc = c_L \times max\{0, \Upsilon_n - T\}$
7. $c = rc + tc$
8. End for
9. $f = \sum \frac{c}{K}$

12. End for

10. Apply mutation

11. Apply recombination

13. Generate the next population

14. Go to step 3 until stop criteria is reached.

Figure A3 - The EVA algorithm

In the case of network 1, the result obtained was a total cost equal to $44.499 with the following values for the allocation variables X^* = (1.296, 0.944, 0.989).

Appendix B: Impact of taking discounting into account

As a simple illustration of the impact of taking the time value of effort into account consider the miniscule project composed of three activities in figure A1, but with the following parameters: due date $T = 1600$, penalty for tardiness $c_L = 2$ and the new parameters of table B1 (where we included the expected duration at *x=1*).

Table B1 – New parameters for network 1

Activity	1	2	3
λ	0.002	0.001	0.0007
x_{min}	0.5	0.5	0.5
x_{max}	1.5	1.5	1.5
Expected duration at x=1	500	1000	1428.57

Based on the 'PERT-type' calculations[7], the 'Critical Path' is of expected duration 1500 days, with variance given by

$$\frac{1}{0.002^2} + \frac{1}{0.001^2} = 1{,}250{,}000\ days^2$$

and the expected tardiness is given by $L(z_0)$, in which $L(\cdot)$ is the standard loss function (under the standard normal distribution),

$$\begin{aligned} L(z_0) &= \int_{\tau=z_0}^{\infty} (\tau - z_0)\varphi(\tau)d\tau \\ &= \int_{\tau=z_0}^{\infty} \tau\varphi(\tau)d\tau - z_0[1-\Phi(z_0)] \\ &= \varphi(z_0) - z_0[1-\Phi(z_0)] \end{aligned}$$

in which $\varphi(\tau)$ is the standard normal density function, with the last equality a consequence of a well-known property of the standard normal distribution. With the due date given at 1600, we have $z_0 = (1600 - 1500)/\sqrt{1{,}250{,}000} = 0.089443$, which yields $L(z_0) = 0.355856$, which, in turn, translates into expected tardiness of ≈ 298 days. Hence the expected cost would be

$$C_{PERT}(1,1,1) = \$3524.29$$

Now we seek the optimal resource allocation without and with discounting at the annual interest rate of $i_a = 8.6957\%$, or daily discount rate of $\alpha = 0.9998$.[8]

The results, secured by the Electromagnetism Approach, are as follows:

	Opt. Resource Allocation *X*	Opt. Cost *C(X)*
Without discounting	(1.499, 0.951, 0.865)	3402.223
With discounting	(0.501, 0.635, 0.555)	1612.047

[7] Still assuming the 'normal' resource allocation x = 1 for all three activities.

[8] Secured from $\alpha = \left(\frac{1}{1+i_a}\right)^{1/365}$

Appendix C: Execution Times

Table C1 - Execution Time: Dynamic Programming

Net	No discounting	Discrete Version 1	Discrete Version 2	Continuous
1	0.000s	0.001s	0.001s	0.001s
2	0.032s	0.063s	0.063s	0.047s
3	0.062s	0.093s	0.094s	0.078s
4	2.546s	3.359s	3.312s	3.031s
5	8.266s	11.000s	11.187s	10.719s
6	1m 31.094s	1m 53.359s	1m 54.296s	1m 49.594s
7	10m 36.156s	11m 58.671s	11m 44.734s	11m 42.546s
8	52m 18.594s	1h 01m 25.859s	1h 00m 40.860s	56m 47.453s
9	(*)	(*)	(*)	(*)
10	(*)	(*)	(*)	(*)
11	(*)	(*)	(*)	(*)
12	(*)	(*)	(*)	(*)
13	(*)	(*)	(*)	(*)
14	(*)	(*)	(*)	(*)

(*) – Program aborted after 8 hours running

Table C2 - Execution Time: Electromagnetism Algorithm

Net	No discounting	Discrete Version 1	Discrete Version 2	Continuous
1	0.235s	0.485s	0.468s	0.453s
2	1.109s	1.781s	1.750s	1.718s
3	2.953s	4.906s	4.890s	4.719s
4	7.844s	10.984s	10.937s	10.782s
5	13.891s	19.422s	19.297s	18.922s
6	16.484s	22.140s	21.922s	21.500s
7	25.344s	31.188s	31.078s	30.328s
8	35.531s	43.172s	43.625s	42.781s
9	53.609s	1m 00.047s	59.328s	59.109s
10	1m 39.125s	1m 52.422s	1m 52.250s	1m 49.985s
11	2m 54.750s	3m 02.235s	3m 02.609s	2m 59.797s
12	27m 43.625s	9m 55.781s	10m 02.172s	9m 50.984s
13	55m 18.406s	56m 47.266s	55m 43.859s	54m 46.610s
14	5h 26m 15.860s	5h 28m 26.969s	5h 26m 15.860s	5h 25m 36.891s

Table C3 - Execution Time: Evolutionary Algorithm

Net	No discounting	Discrete Version 1	Discrete Version 2	Continuous
1	0.172s	0.390s	0.406s	0.375s
2	0.594s	1.046s	1.031s	1.016s
3	1.469s	2.265s	2.250s	2.157s
4	3.187s	4.406s	4.328s	4.297s
5	4.984s	6.516s	6.735s	6.500s
6	5.875s	7.500s	7.531s	7.437s
7	8.593s	9.906s	10.156s	9.844s
8	10.985s	12.828s	12.891s	12.875s
9	15.391s	17.735s	18.062s	17.328s
10	25.812s	28.578s	28.797s	27.984s
11	43.344s	45.266s	46.422s	44.766s
12	5m 29.094s	1m 58.485s	2m 00.656s	1m 55.766s
13	7m 29.469s	7m 27.453s	7m 59.016s	7m 05.031s
14	35m 59.500s	37m 08.063s	38m 41.781s	36m 34.500s