Promoting the power of Mathematics – Children learning fractions with understanding

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Abstract. This paper focuses on a teaching experiment that helps second grade students to discover the power of mathematics by building on their informal knowledge of fractions with understanding. It describes a study on children's understanding of fractions when the quotient interpretation of fractions is used to introduce them this concept, after a contact with partitioning and equal sharing activities. The study tries to address two questions: How do children understand ordering of fractions when introduced to this concept using the quotient interpretation? How do children understand the equivalence of fractions in this interpretation? An intervention program comprising 7 sessions was conducted in a 2nd-grade classroom, with 7-years-old children, from a public primary school, in Portugal. The findings of this study suggest that to solve ordering problems of fractions using the quotient interpretation can help children to easily understand the inverse relation between the divisor and the quotient, when the dividend is the same; the equivalence problems presented in quotient interpretation can give the children an opportunity to promote their proportional reasoning. Issues on their learning process are analyzed and discussed here.

Rationale of the study

Fractions is one of the most complex concept that children have to learn during the elementary school, but also a necessary one. Literature already provided information about students' difficulties (see Behr, Wachsmuth, Post & Lesh, 1984; Hart, 1981; Kerslake, 1986) with fractions. More recently, literature has been discussing the issues related to the effects of the interpretations for fractions on children's understanding of this concept (see Mamede, Nunes & Bryant, 2006; Mamede & Nunes, 2008; Nunes, Bryant, Pretzlik, Evans, Wade & Bell, 2004) and on the children's schemes of action (Nunes, 2008; Nunes & Bryant, 2008).

Distinct interpretations of fractions seem to affect differently children's understanding of the ideas of fraction. According to the Portuguese curriculum, at the primary school, children are supposed to understand at least fractions in quotient, parte-whole and operator interpretations. But in these interpretations the meaning of the numerator and denominator differ, and because of this, the type of interpretation used to work with fraction in the school interferes with students understanding of fractions (Mamede, Nunes & Bryant, 2006; Mamede, 2008; Nunes et al., 2004).

Research has been giving evidence that quotient situations are more suitable for children to build on their informal knowledge for fractions (Mack, 1990; Mamede, 2008; Nunes et al., 2004). The informal ideas about fractional quantities appear much earlier than the formal learning of fractions in school. Research developed with younger children shows that in a division situation, there are some children as young as 6-year-olds who can

understand the inverse relation between the divisor and the quotient, when the dividend is the same (Correa, Nunes & Bryant, 1998) when discrete quantities are involved, and when continuous quantities are involved (Empson, 1999; Kornilaki & Nunes, 2005). The understanding of this inverse relation between the divisor and the quotient can be seen as a precursor of understanding of the logic of fractions: the greater the divisor (which would be represented by the denominator in a quotient interpretation), the smaller the quantity. Streefland (1991, 1997) recommends the use of quotient situations to introduce fractions to children because these situations rely on the idea of fair sharing, which can provide the model for fractions and the part-whole concept related to equivalence and operational relations. The author not only recommends but also provides evidence of success in the use of the quotient interpretation to introduce fractions to children, describing a theory for teaching fractions based on the realistic approach that uses this type of interpretation to introduce fractions to children (see Streefland, 1991). Starting from problems using situations taken from daily life focused on division situations, Streefland (1997) produced good improvements on children's understanding of fractions, helping them to perceive the meaning of numerator and denominator as connected to each other, forming a correct mental object for the concept of fraction.

Traditionally, in many European countries, including Portugal (see DEB, 1998; Kerslake, 1986) and the United States of America (see Berh, Harel, Post & Lesh, 1992; Berh, Lesh, Post & Silver, 1983) children are introduced to fractions at school using the part-whole interpretation and then this work with fractions is extended to include operator situations. In Portugal, in the primary school levels (1st to 4th-grades) students are introduced to fractions representation using the part-whole interpretation, and in some cases students have their first contact with fractions on the 5th grade. Portugal is now experiencing a new curriculum for the elementary school levels. This new curriculum refers that fractions should be introduced to children in an informal way, in the second grade, relying in partitioning and equal sharing; and explored in the third and fourth grades in the quotient, part-whole, operator and measure interpretations. Nevertheless that document gives no other indication for teachers to introduce and explore fractions in the classroom. However, for many Portuguese primary school teachers the concept of fraction only makes sense when part-whole interpretation is involved. Knowing that quotient interpretation of fractions can help children to build on their informal knowledge with understanding, how can teachers explore this interpretation in the classroom? And how can they connect the quotient interpretation of fractions to the part-whole interpretation in order to facilitate children's understanding of fraction ideas?

The study

The study reported here focuses on children's understanding of fractions when they are introduced to this concept using the quotient interpretation, after a contact with partitioning and equal shared activities. It tries to address two questions: (1) How do children understand ordering of fractions when introduced to this concept using the quotient interpretation? (2) How do children understand the equivalence of fraction in this interpretation?

Methods

An intervention study was conducted using qualitative methods to describe children's performances and characterize the processes involved in their learning to represent and compare fractions, when problems are presented to them using situations taken from

daily life. Children's answers, as well as their arguments and solving strategies were analyzed to reach an insight on their ideas of fraction.

The participants were a class of 8 students from a public primary school from Fafe, in the north of Portugal. The children were all 7 years-old. The teacher of the class is one of the researchers. These children had received no instruction about fractions.

The intervention comprised 7 sessions, of approximately 90 minutes each, in which children were introduce to fractions using quotient situations. In the first two sessions children were challenged to solve problems involving equal sharing; they were also introduced to the symbolic representation of fractions, in which the quotient situation or interpretation was used. The remaining sessions were designed to explore ordering and equivalence of fractions in quotient situations.

There were 6 task of ordering of fractions and 4 of equivalence of fractions. The fractions used in these tasks were all less than 1. In the ordering tasks children were asked to solve a problem such as: "Two girls are going to share fairly a chocolate bar, and there is nothing left; four boys are going to share fairly a chocolate bar and there is nothing left. These chocolate bars are equal. Do you think that each girl is going to eat more chocolate than each boy, each boy is going to eat more chocolate than each girl, each girl and each boy are eating the same amount of chocolate? Can you write the number that represents the amount of chocolate that each child eats?". They were also asked to compare fraction given only symbolically. Analogous tasks were presented to them involving equivalence of fractions, in a problem such as: "Two girls are going to share fairly a chocolate bar, and there is nothing left; four boys are going to share fairly two chocolate bars and there is nothing left. These chocolate bars are equal. Do you think that each girl is going to eat more chocolate than each boy, each boy is going to eat more chocolate than each girl, each girl and each boy are eating the same amount of chocolate? Can you write the number that represents the amount of chocolate that each child eats?". In some sessions, the ordering and equivalence problems were presented with no pictorial support.

Results

The results of this study allow us to establish some remarks. First, this experience gives evidence that children can understand fractions when introduced to them with the quotient interpretation, in agreement with Streefland (1991, 1997), Mamede, Nunes and Bryant (2006) and Mamede and Nunes (2008) who previously studied these issues. In the sessions of this study, the ordering problems seemed to help the children to easily understand the inverse relation between the divisor and the quotient, when the dividend is the same. This relation is essential to understand the meaning of fractions.

Second, these children learned easily fractions labels. They were introduced to the representation of fractions in the beginning of the intervention, and soon they mastered the symbolic representation of fractions, when quotient interpretation was used. In this type of interpretation, the magnitudes involved in the fractions refer to two variables of different nature (Nunes & Bryant, 2008; Nunes et al., 2004), - numerator refers to the number of items to share, denominator refers to the number of recipients - and this may facilitate children's learning of fractions labels.

Third, because in quotient interpretation the numerator and the denominator relate to variables that are different in nature (Nunes & Bryant, 2008), children easily relied on the use of correspondence to solve many of the tasks. This finding was also documented previously by Mamede, Nunes and Bryant (2006) when observing 6-7-year-olds

children's strategies solving ordering and equivalence problems, when interviewed individually. Nunes (2008) argues that in a division situation, there are two types of action schemes: partitioning, which involves dividing the whole into equal parts; and correspondence which involves two quantities (a quantity to be shared and a number of recipients of the shares). The development of these action schemes defers. Children of 5 to 6-year-olds can establish correspondence to produce equal shares (see Kornilaki & Nunes, 2005; Mamede, Nunes & Bryant, 2006; Nunes, 2008), but they find more difficult to accomplish partitioning of continuous quantities. These schemes of action (Nunes, 2008) are fundamental for the learning of the mathematical concepts.

Fourth, the equivalence problems presented in quotient interpretation gave the children an opportunity to promote their proportional reasoning. When solving the equivalence problems many children establish a proportional relation between the numbers of items to share and the number of recipients in order to reach the solution; some of them could express that relation in a written way, others by drawings. Proportional reasoning was also a strategy identified by Mamede (2007) and Nunes et al. (2004) when analysing students' strategies solving equivalence problems presented to them in quotient interpretation of fractions.

Final remarks

To conclude, this short intervention program allowed the teacher to understand children's possibilities of success with fractions when they are introduced to the children using quotient situations. We hope that this experiment can contribute to promote a change in the classroom practices, following the Portuguese official curriculum, giving the primary teachers an example of a well succeeded experience.

More research is needed in order to explore other ways of introducing fractions to children in the classroom, using quotient interpretation of fractions. Nevertheless, this study suggests that it is possible for children to learn fractions with understanding, building on their informal knowledge, linking school mathematics to their out-of-school experiences, giving meaning to their mathematical ideas, empowering their mathematics ideas.

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