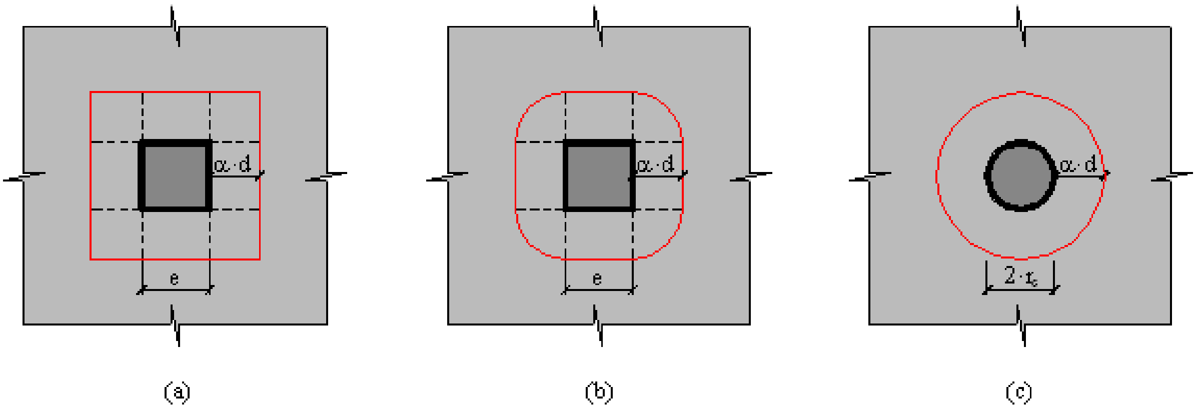

Figure 1. Punching failure perimeter adopted for the evaluation of the punching resistance when columns have: (a) and (b) square cross section, (c) circular cross section.

A MODEL TO SIMULATE THE CONTRIBUTION OF FIBRE REINFORCEMENT FOR THE PUNCHING RESISTANCE OF RC SLABS

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Abstract

In this paper analytical formulations are developed for the prediction of the punching resistance of flat slabs of steel fibre reinforced concrete (SFRC) flexurally reinforced with steel bars. By performing statistical analysis with a database that collects experimental results on the characterization of the post-cracking behaviour of SFRC, equations are determined for the evaluation of the residual flexural tensile strength parameters (f_{Ri}) from fundamental data that characterize steel fibres. The f_{Ri} strength parameters proposed by CEB-FIP 2010 were used for the definition of the stress-crack width law (σ-w) that simulates the fibre reinforcement mechanisms in cement based materials. In the second part of the paper is described an analytical formulation based on the concepts proposed by Muttoni and Ruiz, where the σ-w law is conveniently integrated for the simulation of the contribution of steel fibres for the punching resistance of SFRC slabs. By using a database composed of 154 punching tests with SFRC slabs, the good predictive performance of the developed proposal is demonstrated. The good performance of this model is also evidenced by comparing its predictions to those from other models.

Keywords: Reinforced concrete, Flat slab, Punching, Steel fibre reinforced concrete, Analytical models.

INTRODUCTION

Recent experimental programs have shown the possibility of building structural systems based on flat slabs of steel fibre reinforced concrete (SFRC) supported on reinforced concrete (RC) columns (Espion 2004; Mandl 2008; Destrée *et al.* 2009; Destrée and Mandl 2008; Barros *et al.* 2012). This type of slabs is generally designated by Elevated Steel Fibre Reinforced Concrete Slabs (ESFRC), and it includes a minimum continuity rebars, also referred as anti-progressive collapse rebars, placed in the bottom of the slab in the alignment of the columns (Sasani and Sagiroglu 2008). The results obtained in these experimental tests have demonstrated that this construction system fulfills the structural exigencies required for residential buildings, and is a competitive alternative to the availabe conventional construction methods. However, a reliable acceptance of this innovative construction system also requires the existence of design guidelines that can predict its structural behaviour with high accuracy, namely the punching resistance, since punching failure is quite brittle and, in general, conducts to the global collapse of a building (Gardner *et al.* 2002).

In terms of punching resistance of conventionally reinforced concrete flat slabs, in general, the actual design standards, such is the case of ACI 318 (2008), BS 8110 (1985), EC2 (2004), and CEB-FIP Model Code 1990, adopt the approach that the ultimate punching resistance, $V_{R,d}$, is obtained adding the parcel due to the concrete resistance, $V_{R,cd}$, to the term that simulates the contribution of shear reinforcement, $V_{R,sd}$, e.g., $V_{R,d}=V_{R,cd}+V_{R,sd}$. For slabs without shear reinforcement, $V_{R,d}=V_{R,cd}$, and the design procedure for punching is based on the verification that the nominal shear stress, $v_{N,d}$, in two or more critical sections around the column does not exceed the nominal shear strength, $v_{R,d}$, e.g., $v_{R,d}\geq v_{N,d}$.

Recently, the CEB-FIP Model Code 2010 proposed recommendations for the evaluation of the flexural and shear resistance of members made by fibre reinforced concrete (FRC). These recommendations are supported on residual flexural tensile strength parameters, f_{Ri}, that characterize the post-cracking behaviour of FRC, and are determined from three point bending tests with notched FRC beams. The full definition and the strategy to obtain f_{Ri} from experimental tests, as well as the aforementioned recommendations, are described in the following sections.The CEB-FIP Model Code 2010 has also proposed a very simple approach to simulate the contribution of fibre reinforcement for the punching resistance of FRC flat slabs, $V_{R,fd}$. Several studies have been done with the purpose of developing a design approach for the prediction of the contribution of fibre reinforcement for the punching resistance of SFRC slabs (Narayanan and Darwish 1987; Harajli *et al.* 1995; Muttoni and Ruiz 2010; Michels *et al.* 2012), but the predictive performance of these models is generally limited to a relatively small number of punching tests, and the contribution of fibre reinforcement is not based on the most recent knowledge about modelling the post-cracking behaviour of this composite.

In the present paper a design formulation is proposed for the evaluation of the punching resistance of SFRC slabs. This model is based on the principles proposed by Muttoni and Ruiz (2010), where a stress-crack width relationship is used to simulate the contribution of the fibre reinforcement mechanisms for the punching resistance of SFRC flat slabs, $V_{R,f}$. The $V_{R,f}$ is determined from the f_{Ri} parameters proposed by CEB-FIP Model Code 2010.

DESIGN FORMULATIONS PROPOSED BY STANDARD CODES

Design formulation for flat slabs only reinforced with conventional steel bars

ACI 318 (2008)

According to ACI 318, the punching resistance of the type of slabs analysed in the present paper is obtained from:

$$\phi \cdot V_{R,d} \geq V_{S,d} \tag{1}$$

where,

$$V_{R,d} = v_{R,d} \cdot b_0 \cdot d \tag{2}$$

with

$$v_{R,d} = min\left[\frac{\sqrt{f_c}}{6} \cdot \left(1 + \frac{2}{\beta_c}\right); \frac{\sqrt{f_c}}{6} \cdot \left(\frac{6 \cdot \alpha_s \cdot d}{b_0} + 1\right); \frac{\sqrt{f_c}}{3}\right] \qquad \text{[MPa, mm]} \tag{3}$$

In Eq. (2) b_0 is the perimeter corresponding to the formation of the punching failure surface, assumed localized at a distance α=0.5 from the external face of the column, and with the geometric configuration represented in Figure 1a.In Eq. (3) β_c is the ratio between the larger and the smaller edge of the column's cross section, α_s=3.32 for columns located in the interior of the building (assumed centrically loaded), such is the case treated in the present work, f_c is the average concrete compressive strength evaluated using cylinder specimens, and d is the internal arm of the longitudinal tensile reinforcement of the slab's cross section.

CEB-FIP MODEL CODE 1990

The CEB-FIP Model Code 1990 recommends that the punching resistance of a RC slab without shear reinforcement should be determined from the following equation:

$$V_{R,d} = v_{R,d} \cdot u_1 \cdot d \qquad \text{[MPa, mm]} \tag{4}$$

with

$$v_{R,d} = 0.12 \cdot \xi \cdot \left(100 \cdot \rho \cdot f_c\right)^{1/3} \qquad \text{[MPa, mm]} \tag{5}$$

where

$$\xi = 1 + \sqrt{\frac{200}{d}} \quad \text{[mm]}; \tag{6}$$

$$\rho = \sqrt{\rho_x \cdot \rho_y} \tag{7}$$

The ξ parameter in Eq. (6) aims to simulate the size effect. The reinforcement ratio of the tensile flexural reinforcement, ρ, is calculated from Eq. (7) by considering the reinforcement ratio ρ_x and ρ_y in the two main orthogonal directions (x and y). For the evaluation of ρ_x and ρ_y, a width of the slab cross section equal to

$e+6\cdot d$ (for the columns of square cross section), or equal to $2\cdot r_c+6\cdot d$ (for the columns of circular cross section), is considered, Figure 1.

EC2 (2004)

In the Eurocode 2, EC2, it is assumed that the punching resistance of RC slabs without shear reinforcement can be estimated by the following equation:

$$V_{R,d} = v_{R,d} \cdot u_1 \cdot d \qquad (8)$$

with

$$v_{R,d} = max\left[C_{Rd,c} \cdot k \cdot \left(100 \cdot \rho \cdot f_c\right)^{1/3} ; 0.035 \cdot k^{3/2} \cdot f_c^{1/2}\right] \qquad \text{[MPa, mm]} \qquad (9)$$

where k is defined as ξ (Eq. (6)), but a maximum limit of 2.0 is imposed to k, while ρ is obtained from Eq. (7) with an upper limit of 0.02. In Eq. (8) the critical perimeter, u_1, is localized at $2d$ from the external surface of the column (α=2), and can assume the configurations represented in Figures 1b and 1c. In Eq. (9) $C_{Rd,c}=0.18/\gamma_c$.

CEB-FIP Model Code 2010

According to the CEB-FIP Model Code 2010, the punching resistance of RC slabs without shear reinforcement, $V_{R,d}=V_{R,cd}$, is determined from the following equation:

$$V_{R,d} = V_{R,cd} = k_{\psi} \cdot \frac{\sqrt{f_c}}{\gamma_c} \cdot b_0 \cdot d \qquad \text{[MPa, mm]} \qquad (10)$$

where b_0 is the critical punching perimeter at a distance α=0.5 from the external surface of the column, as represented in Figures 1b and 1c. The k_{ψ} parameter depends of the rotation of the slab, and is determined from the following equation:

$$k_{\psi} = \frac{1}{1.5 + 0.9 \cdot \psi \cdot d \cdot k_{dg}} \leq 0.6 \qquad \text{[mm]} \qquad (11)$$

where k_{dg} parameter simulates the aggregate interlock:

$$k_{dg} = \frac{32}{16 + d_g} \geq 0.75 \quad \text{[mm]} \tag{12}$$

being d_g the maximum diameter of the aggregates.

The rotation of the slab, ψ, (Figure 2b) required to determine the k_ψ parameter, is evaluated according to the approach II indicated in CEB-FIP Model Code 2010, by applying the following equation:

$$\psi = 1.5 \cdot \frac{r_s}{d} \cdot \frac{f_{yd}}{E_s} \cdot \left(\frac{m_{sd}}{m_{Rd}} \right)^{1.5} \tag{13}$$

where r_s indicates the position, in relation to the axis of the column, at which the radial bending moment, m_r, is null (Figure 2a). The value of r_s can be considered equal to $0.22 \cdot L_s$ (L_s is replaced by $L_{s,x}$ for the analysis in x direction, and L_s is replaced by $L_{s,y}$ for the analysis in y direction, Figure 2c) in slabs where the $L_{s,x}/L_{s,y}$ ratio pertains to the interval [0.5 – 2.0]. In Eq. (13) the $m_{sd}=V_{S,d}/8$ (Johansen 1962) and m_{Rd} represents the design value of the actuating and resisting bending moment, respectively. Both m_{sd} and m_{Rd} are evaluated for a slab strip of a width of $b_s=1.5 \cdot (r_{s,x} \cdot r_{s,y})^{0.5} \leq L_{s,min}$, where $r_{s,x}$ and $r_{s,y}$ is the r_s in x and y direction, respectively, and $L_{s,min}$ is the minimum value between $L_{s,x}$ and $L_{s,y}$, Figue 2c. The strategy to evaluate m_{Rd} will be discussed in a posterior section.

Design formulation for FRC flat slabs flexurally reinforced with conventional steel bars - CEB-FIP Model Code 2010

For SFRC slabs, the CEB-FIP Model Code 2010 recommends the following equation for the evaluation of the punching resistance:

$$V_{R,d} = V_{R,cd} + V_{R,fd} \tag{14}$$

where $V_{R,cd}$ is calculated according to Eq. (10), and the parcel corresponding to the contribution of fibre reinforcement, $V_{R,fd}$, is evaluated from:

$$V_{R,fd} = \frac{f_{Ftuk}}{\gamma_F} \cdot b_0 \cdot d \tag{15}$$

being,

$$f_{Ftu} = f_{Fts} - \frac{w_u}{2.5} \cdot \left(f_{Fts} - 0.5 \cdot f_{R3} + 0.2 \cdot f_{R1} \right) \geq 0 ; \qquad f_{Fts} = 0.45 \cdot f_{R1} \tag{16}$$

In Eq. (15) f_{Ftuk} represents the characteristic value of the ultimate residual flexural tensile strength, which is calculated for an ultimate crack width (w_u) of 1.5 mm. When the SFRC slab also includes conventional flexural reinforcement, the CEB-FIP Model Code 2010 suggests the use of $w_u=\psi\ d/6$, where ψ is calculated from Eq. (13). The f_{Fts} in Eq. (16) represents the residual flexural tensile strength for the verifications of serviceability limit states. The f_{Ri} (i=1 and 3) parameters indicated in Eq. (16) represent the residual flexural tensile strength parameters of FRC, and are determined from the load versus Crack Mouth Opening Displacement (*CMOD*) registered in three point notched beam bending tests, by applying the following equation:

$$f_{Ri} = \frac{3 \cdot F_{Ri} \cdot L}{2 \cdot b \cdot {h_{sp}}^2} \tag{17}$$

where F_{Ri} is the force corresponding to $CMOD_i$, and L (500 mm), b (150 mm) and h_{sp} (125 mm) are the free span, the width of the cross section and the distance from the tip of the notch to the top surface of the beam, respectively (CEB-FIP Model Code 2010).

ASSESSMENT OF THE PREDICTIVE PERFORMANCE OF DESIGN FORMULATIONS PROPOSED BY STANDARD CODES

Introduction

The predictive performance of the the formulations proposed by standard codes, described in a previous Section, is evaluated in terms of the $\lambda=V_{exp}/V_{the}$ parameter, where V_{exp} is the punching failure load registered experimentally, and $V_{the}=V_R=V_{R,c}$ is the punching failure load estimated according to the analytical formulations of the considered standard codes. The purpose of this section is to determine the formulation that best predicts the punching failure load of RC flat slabs ($V_R=V_{R,c}$), in order to be adopted with the model proposed in the present work to estimate the failure load of SFRC flat slabs that are also flexurally reinforced with steel bars, e.g. $V_{the}=V_R=V_{R,c}+V_{R,f}$. This assessment was executed by considering a database of punching tests with flat slabs described in the following section.

Database (DB)

A database (DB) composed by 154 slabs submitted to punching test configuration was built, 137 of them were reinforced with longitudinal steel bars/grids in order to avoid the occurrence of flexural failure mode. None of these slabs has conventional shear/punching reinforcement. However, 105 slabs composing the DB were made by SFRC. In terms of concrete compressive strength, f_{cm}, the DB is composed of slabs with f_{cm} in the range of 14 to 93 MPa, so a quite high interval exists for a parameter that has a relevant impact on the punching resistance of concrete slabs. For the slabs that were flexurally reinforced with steel bars, the internal arm of this reinforcement (d, Figure 2) has varied from 13 mm to 180 mm, while the reinforcement ratio (ρ) is in the interval 0.4 to 2.75%. In the SFRC slabs, "*hooked*", "*twisted*", "*crimped*", "*corrugated*", "*paddle*" and other types of fibres were used, with an aspect-ratio that varied from 20 to 100, and in a volume percentage ≤2%. In some of the SFRC slabs (6 specimens), the SFRC was only applied in a region around the loaded area (that represents the position of the column), considered the region where punching failure could occur. In terms of loading conditions, all the slabs of the DB were submitted to a load distributed in a certain area of the slab without transferring any bending moments from the loading device to the slab. In the tests of the DB, the columns were simulated by a RC element monolithically connected to the slab or applying steel plates, or even introducing a semi-spherical device in between the piston of the actuator and the tested slab. The cross section of the columns and steel plates was square or circular. To avoid results that can compromise the reliability of this statistical analysis, the slabs with a thickness lower than 80 mm were discarded, since an eventual influence of size effect can have a detrimental consequence on this study. Furthermore, the slabs where the concrete compressive strength has decreased more than 15% in consequence of the addition of fibres were also neglected, since this decrease reveals that the SFRC mix composition was not properly designed. Further details about the DB can be found elsewhere (Moraes Neto 2013).

In this section, slabs only reinforced with steel bars are considered for the assessment of the predictive performance of the formulations proposed by standard codes, described in the previous chapter.

General statistical analysis procedures

The performance of the formulations proposed by the considered standard codes for the prediction of the punching resistance of RC slabs is appraised using the collected data registered in the DB. For each proposal, the obtained values of V_{the} are compared with V_{exp} and a λ factor corresponding to the V_{exp}/V_{the} ratio is evaluated. The values of λ were classified according to the modified version of the of the *Demerit Points Classification* (*DPC*) proposed by Collins (2001), where a penalty (PEN) is assigned to each range of λ parameter according to Table 1, and the total of penalties (Total PEN) determines the performance of the proposal. The penalty is a weighting factor determined from statistical analysis that takes into account safety, accuracy and economic aspects (Collins 2001). According to this strategy, the proposal with the minimum total of penalties is the best one under this framework.

In next section the models in analysis are designated as *MODi* (i=1 up to 4), with the corresponding formulation assigned in the footnote of Table 2.

In the analysis performed, unit value was assumed for all the safety factors (such is the case of γ_c and γ_F in equations (10) and (15), respectively) considered by the formulations, and average values were adopted for the properties of the materials (such as: f_c, f_{ct}, f_{R1}, f_{R3}, f_{Fts}, f_{Ftu}), since the analytical predictions will be compared to the experimental results. Furthermore, in the evaluation of Eq. (10) of the model proposed by CEB-FIP Model Code 2010, the Eqs. (11) and (12) that define the punching failure criterion are replaced by the following ones:

$$k_{\psi} = \frac{3/4}{1+\psi \cdot d \cdot k_{dg}} \quad \text{[rad, mm]} \tag{18}$$

$$k_{dg} = \frac{15}{16+d_g} \quad \text{[mm]} \tag{19}$$

in order to take into account that average values are now considered (Muttoni 2008).

Results

The results presented in the present section assess the performance of the formulations described in Section “Design formulation for flat slabs only reinforced with conventional steel bars” for the prediction of the punching failure load of flat slabs only reinforced with longitudinal bars, e.g. $V_{the}=V_R=V_{R,c}$. The results are presented in Table 2, and from the analysis it can be concluded that MOD4, corresponding to the one proposed by CEB-FIP Model Code 2010, has best predicted the punching failure load registered experimentally, with the lowest COV. Furthermore, it is the one with the largest number of predictions in the appropriate safety interval according to the DPC (Table 1), e.g. 21 samples with $\lambda \in [0.85\text{-}1.15[$. Therefore, it will be selected for the evaluation of the $V_{R,c}$ in the context of the formulation to be developed for the prediction of the punching failure model of SFRC flat slabs flexurally reinforced with steel bars, Eq. (14).

PRACTICAL PROPOSAL FOR THE ESTIMATION OF THE RESIDUAL FLEXURAL TENSILE STRENGTH PARAMETERS, f_{Ri}

Introduction

The predictive performance of the model proposed in the present work for the evaluation of the punching failure load of SFRC flat slabs will be assessed by comparing the estimated results with those available in the DB described in Section "Database (CB)". As already indicated, the contribution of fibre reinforcement for the punching resistance is simulated by using the concept f_{Ri}, however, these values are not available in the majority of the works composing the DB. Therefore, to apply the proposed model to the tests composing the DB, another database was built by collecting results (f_{Ri}) of the characterization of the post-cracking flexural behaviour of SFRC according to the recommendations of CEB-FIP Model Code 2010. Since the fibre volume percentage, V_f, and fibre aspect ratio, (l_f/d_f), (quotient between fibre length, l_f, and fibre diameter, d_f) are practically the unique common information available in the works forming the DB of the punching tests, the statistical analysis performed with the collected data for the characterization of the post-cracking behaviour of SFRC was governed by the criterion of deriving equations for the f_{Ri} dependent on the V_f and l_f/d_f. The authors are aware that this is a quite simple approach to simulate the fibre reinforcement mechanisms, since other variables like the fibre-matrix bond strength, fibre inclination and fibre embedment length influence the values of f_{Ri} (Cunha *et al.* 2010), but this information is not available in those works. Therefore, a relatively large scatter of results is naturally expected for the relationships f_{Ri}-(V_f, l_f/d_f), but actually this is the unique possibility of considering the fibre reinforcement mechanisms according to the CEB-FIP Model Code 2010 for the prediction of the punching failure load of the slabs collected in the DB by applying the proposed model. Preliminary statistical analysis by considering also the bond strength was also carried out (Moraes Neto 2013), but the obtained results have revealed that by also adopting these parameters, the dispersion of the results increase significantly, since a large scatter of bond strength values exists in the bibliography. Taking this into account, the statistical analysis was carried out in order to derive equations that conduct to safe predictions.

Database (DB) and procedures for the analysis

A database composed of 69 results from three point notched SFRC beam bending tests was collected (Moraes Neto 2013). The analysis of the DB has indicated that fibre reinforcement index, $V_f{\cdot}l_f/d_f$, is the most influential parameter on the f_{Ri} values. Taking into account the geometric characteristics and volume percentage of the steel fibres most used in the available experimental programs of punching SFRC slabs, this database is restricted to the tests with concrete reinforced with hooked ends steel fibres, in a volume percentage ranging from 0.13% to 1.25% and with fibre aspect-ratio in the interval 50 to 80.

In the first step of the analysis of the information available in the DB, relationships between f_{Ri} and $V_f{\cdot}l_f/d_f$ were established (Moraes Neto 2013). The predictive performance of the equations were then evaluated in terms of the $\lambda_i=f_{Ri,exp}/f_{Ri,the}$ parameter, where $f_{Ri,exp}$ and $f_{Ri,the}$ is, respectively, the residual flexural strength parameter recorded experimentally (available in the DB) and estimated according to the obtained f_{Ri}-$V_f{\cdot}l_f/d_f$ relationships. The predictive performance of these relationships was also appraised by using a modified version of the *DPC*, where a penalty is assigned to each range of λ_i parameter according to Table 1, and the total of penalties determines the performance of the f_{Ri}-$V_f{\cdot}l_f/d_f$ relationship. To assure stable predictions, the statistical analysis was executed in order to provide average values for $\lambda_1=f_{R1,exp}/f_{R1,the}$ and $\lambda_{2,the}$ in the lower bound of the interval considered as "conservative" (Table 1), which assures safe predictions in terms of design philosophy.

Assessement of the predictive performance of the f_{Ri}-$V_f{\cdot}l_f/d_f$ relationship

Analyzing the results of the DB it was realized that the f_{Ri}-$V_f{\cdot}l_f/d_f$ function that best fit the results is of the type (Moraes Neto 2013), $f_{Ri}=k_j\cdot\left(V_f\cdot l_f/d_f\right)^{c_j}$. To derive the k_j and the c_j values (j=1 and 2), a parametric analysis was executed (Moraes Neto 2013) in order to obtain the best compromise in terms of the lowest R-squared values (R^2) of the fitting process and the lowest total penalties according to the modified *DPC*, having resulted the following equations:

$$f_{R1} = k_1 \cdot \left(V_f \cdot \frac{l_f}{d_f} \right)^{c1} = 7.5 \cdot \left(V_f \cdot \frac{l_f}{d_f} \right)^{0.8} \qquad \text{[MPa]} \qquad (20)$$

$$f_{R3} = k_2 \cdot \left(V_f \cdot \frac{l_f}{d_f} \right)^{c2} = 6.0 \cdot \left(V_f \cdot \frac{l_f}{d_f} \right)^{0.7} \qquad \text{[MPa]} \qquad (21)$$

$$f_{R3} = k_3 \cdot f_{R1} = 0.85 \cdot f_{R1} \qquad (22)$$

Eq. (22) shows a tendency for a linear relationship between f_{R1} and f_{R3}, which was already pointed out in a previous work (Barros *et al.* 2005).

The predictive performance of Eqs. (20) and (21) was assessed by taking the results estimated for the λ_i parameter, and considering the dispersion of the results and total penalties according to the modified *DPC*. The obtained results are presented in Figure 3. A "box and whiskers" plot of the λ ratio for the f_{R1} and f_{R3} is represented in Figure 3b. The box plot diagram graphically depicts the statistical five-number summary, consisting of the minimum and maximum values, and the lower (Q1), median (Q2) and upper (Q3) quartiles. Table 3 resumes the obtained results. As expected, a relatively high dispersion was obtained for the predictions of both parameters, which is intrinsically dependent of the dispersion of results in the DB for the f_{R1} and f_{R3}, since the values of these parameters are also affected by the properties of the surrounding cement matrix, but not considered in the present approach due to the reasons already pointed out. The authors are doing an effort for increasing the database on the characterization of the post-cracking behaviour of SFRC, in order to derive more reliable equations for the determination of f_{Ri}.

In a design context of a SFRC slab, three point notched SFRC beam bending tests should be executed according to the recommendations of CEB-FIP Model Code 2010 in order to obtain the f_{Ri} of the SFRC, and these values are directly used in the proposed model for the evaluation of the punching failure load of a SFRC slab supported on columns.

THE CONTRIBUTION OF FIBRE REINFORCEMENT FOR THE PUNCHING RESISTANCE OF SFRC FLAT SLABS

The contribution of fibre reinforcement mechanisms for the punching resistance of SFRC flat slabs, $V_{R,f}$, has been investigated by several researchers (Narayanan and Darwish 1987; Harajli *et al.* 1995; Muttoni and

Ruiz 2010; Choi *et al.* 2007; Higashiyama *et al.* 2011), but none of them has acquired generalized conclusions to be considered as design guideline criteria.

With the aim of contributing for the development of a formulation that is sufficiently simple to be adopted in the design practice, and with scientific rigour capable of simulating with enough accuracy a phenomena that has a brittle character and huge impact if a collapse occurs (Gardner *et al.* 2002), a new approach to determine $V_{R,f}$ is described in this section by combining the most comprehensible knowledge available. This formulation is based on the principles proposed by Muttoni and Ruiz (2010), being the contribution of fibre reinforcement mechanisms simulated by a stress vs crack width law, $\sigma_f(w)$, recommended by the CEB-FIP Model Code 2010, but considering Eqs. (20) and (21) to determine $\sigma_f(w)$.

According to Muttoni and Ruiz (2010), it is acceptable to consider that a slab with axisymmetric structural conditions, when submitted to a load level corresponding to the failure state, can be regarded as a group of radial segments that rotate as rigid bodies, Figure 4.

The reinforcement mechanisms offered by the fibres crossing the critical punching surface are simulated by the stress-crack width relationship (Figure 5a), and after convenient integration of $\sigma_f(w)$ at the fracture surface, the fiber reinforcement contribution for the punching resistence $V_{R,f}$ can be obtained.

According to the Critical Shear Crack Theory (CSCT) (Muttoni 2008; Muttoni and Schwartz 1991), the crack opening of the punching failure crack, w, is proportional to the rotation of the slab, ψ, and the distance from the bottom surface of the slab, z (Figure 5b):

$$w(\psi, z) = \mu \cdot \psi \cdot z \tag{23}$$

where μ is the coefficient relating the rotation ψ with the crack opening w. The μ parameter was obtained by using the rotation values at punching failure load, ψ_u, of the slabs composing the database introduced in Section “Database (DB)”. For each slab its ψ_u was evaluated from available experimental data, and assuming that the ultimate crack width, w_u, should be in the interval 1.5 to 3.0 mm, for each w_u in this interval the corresponding value of the μ parameter is determined from Eq. (23). The w_u was determined at the level of the tensile flexural reinforcement, considering $z=d\text{-}x$ (Figure 5), where the position of the neutral axis, x, is determined by applying the approach recommended by CEB-FIP Model Code 2010 (Figure 6), where f_{Ftu} is obtained from Eq. (16) with w_u=2.5 mm, as suggested by this standard. Therefore, for the μ parameter that respect Eq. (23) for the considered w_u, it is obtained the contribution of fibre reinforcement for the punching resistance, $V_{R,f}$, and applying Eq. (14) the punching failure load is estimated and compared to the value registered experimentally. This algorithm was executed for the adopted interval of w_u, and the pair of w_u and μ parameters that have best predicted the punching failure load of the experimental programs collected in the database was μ=2.5 and w_u=2.5mm (Moraes Neto 2013).

According to Moraes Neto (2013) ψ_u is calculated from the following equation:

$$\psi_u = 0.35 \cdot \left(\frac{m_R}{E \cdot I_1} - \chi_{ts} \right) \cdot r_s \tag{24}$$

where,

$$E \cdot I_1 = \rho \cdot \beta \cdot E_s \cdot d^3 \cdot \left(1 - \frac{x}{d}\right) \cdot \left(1 - \frac{x}{3 \cdot d}\right) \tag{25}$$

and

$$\chi_{ts} = \frac{f_{ct}}{\rho \cdot \beta \cdot E_s} \cdot \frac{1}{6 \cdot h} \cong 0.5 \cdot \frac{m_{cr}}{E \cdot I_1} \tag{26}$$

In equation (25), $E \cdot I_1$ represents the flexural stiffness of SFRC cracked cross section, obtained according to the procedures adopted for RC members (Moraes Neto 2013), and assuming a stabilized cracking stage. The contribution of fibre reinforcement for the $E \cdot I_1$ is only indirectly taken in the evaluation of the neutral axis, x, Figure 6. In Eq. (25) β is a factor to take into account the real arrangement of the reinforcement, since the CSCT is supported on the principle of axisymmetric structural conditions, but the majority of the built and tested RC flat slabs have orthogonal distribution of the reinforcement (Guandalini 2005). According to Muttoni (2008), β=0.6 yields to satisfactory results. The evaluation of the position of the neutral axis, x, was made according to the recommendations of CEB-FIP Model Code 2010, see Figure 6.

The χ_{ts} factor in Eq. (26) simulates the post-cracking tensile strength of cracked concrete (tension stiffening effect), where f_{ct} is the concrete tensile strength, E_s is the elasticity modulus of the steel reinforcement, ρ is the reinforcement ratio of the tensile flexural reinforcement, h is the slab thickness, and $m_{cr}= f_{ct}\, h^2/6$ is the cracking moment.

To evaluate $V_{R,f}$ it is assumed that the post-cracking stress law, $\sigma_f(w)=\sigma_f(\psi,z)$, can be represented by the following linear constitutive σ-w approach recommended by CEB-FIP Model Code 2010:

$$\sigma_f(w) = f_{Ftu}(w) = f_{Fts} - \frac{w}{2.5}\cdot\left(f_{Fts} - 0.5\cdot f_{R3} + 0.2\cdot f_{R1}\right) \geq 0 \tag{27}$$

Replacing Eq. (23) into Eq. (27) yields:

$$\sigma_f(\psi, z) = f_{Fts} - \frac{\mu\cdot\psi\cdot z}{2.5}\cdot\left(f_{Fts} - 0.5\cdot f_{R3} + 0.2\cdot f_{R1}\right) \geq 0 \tag{28}$$

The $V_{R,f}$ is obtained by integrating $\sigma_f(\psi,z)$ on the area A_0, where A_0, see Figure 5a, represents the horizontal projection of the punching failure surface (Moraes Neto 2013):

$$V_{R,f} = \int_{A_0} \sigma_f(\psi, z)\cdot dA_0 \tag{29}$$

$$V_{R,f}(\psi) = \pi\cdot d\cdot(1-2\cdot k)\cdot\left(2\cdot\Omega 1\cdot\Omega 3 + d\cdot\left\{\begin{array}{l}\Omega 1\cdot\Omega 4 - \Omega 2\cdot\Omega 3\cdot\psi - \\ 2\cdot d\cdot\Omega 2(\psi)\cdot\Omega 4\cdot\left[\dfrac{(1-k)\cdot(1-2\cdot k+k^2)-k^3}{3\cdot(1-2\cdot k)}\right]\end{array}\right\}\right) \tag{30}$$

where,

$$\Omega 1 = f_{Fts}; \quad \Omega 2 = \frac{\mu\cdot\left(f_{Fts} - 0.5\cdot f_{R3} + 0.2\cdot f_{R1}\right)}{2.5}; \quad \Omega 3 = r_c; \quad \Omega 4 = \frac{1}{2\cdot(1-k)} \tag{31}$$

$$\psi = \psi_u = 0.35\cdot\left(\frac{m_R}{E\cdot I_1} - \chi_{ts}\right)\cdot r_s; \quad k = \frac{x}{d} \tag{32}$$

To evaluate the $V_{R,f}$ an assumption was assumed by considering $\sigma_f(\psi,z)=\sigma_f(w)=\sigma_f(w_u)$ (see Eq. (23)) and adopting for w_u the value 1.5 mm recommended by CEB-FIP Model Code 2010 (clause 7.7.3.5.3). Therefore:

$$V_{R,f} = \int_{A_0} \sigma_f(\psi, z)\cdot dA_0 = \int_{A_0} \sigma_f(w_u)\cdot dA_0 = \sigma_f(w_u)\cdot A_0 \tag{33}$$

where $\sigma_f(w_u)$ is obtained from Eq. (16):

$$\sigma_f(w_u) = f_{Ftu}(w_u) = f_{Fts} - \frac{w_u}{2.5}\cdot\left(f_{Fts} - 0.5\cdot f_{R3} + 0.2\cdot f_{R1}\right) \geq 0 \tag{34}$$

resulting:

$$V_{R,f} = \left[f_{Fts} - \frac{1.5}{2.5}\cdot\left(f_{Fts} - 0.5\cdot f_{R3} + 0.2\cdot f_{R1}\right)\right]\cdot A_0 \tag{35}$$

ASSESSMENT OF THE PREDICTIVE PERFORMANCE OF FORMULATIONS FOR SFRC FLAT SLABS

Since the formulation of CEB-FIP Model Code 2010 has best fitted the $V_{R,c}$ of the collected DB, it was selected to be coupled with the proposed model that predicts the contribution of fibre reinforcement for the punching resistance of SFRC flat slabs, $V_{R,f}$, resulting a model capable of estimating the the punching failure load of SFRC slabs reinforced with longitudinal steel bars: $V_{the}=V_R=V_{R,c}+V_{R,f}$. In the previous Section two equations were proposed to estimate $V_{R,f}$: i) Eq. (30) that requires performing a full integration of the crack opening–fibre stress law, thereby is herein designated as *The-refin*; 2) Eq. (35) that is more simple to obtain, thereby is herein designated as *The-simpl*, where *refin* and *simpl* means refined and simplified, respectively. In Table 4 the predictive performance of these two models is compared to the one resulting from the application of the formulation proposed by CEB-FIP Model Code 2010 that was already described. It can be concluded that both proposed formulations evidence excellent predictive performance with a relatively small COV. Both formulations present average value of λ much closer to the unit value than the CEB-FIP model, and lower COV. Furthermore, these models have also the largest number of predictions in the appropriate safety interval according to the DPC (Table 1). Based on the results and considering its easy attainement, it is recommended to use *The-simpl* approach to predict $V_{R,f}$, based on Eq. (35).

The CEB-FIP Model Code 2010 provides a large number of predictions against safety, e.g. a λ value in the interval [0.50-0.85[, considered "Dangerous" according to the DPC (see Table 1), was obtained for 39 slabs. In the CEB-FIP Model Code 2010 a constant post-cracking residual strength is assumed distributed in the punching fracture surface for the simulation of the fibre reinforcement contribution for the punching resistance. This is in fact the same strategy adopted in the simplified approach herein proposed (Eq. (35)), but the relatively high number of unsafe predictions demonstrates that the punching failure surface assumed in this standard ($b_0.d$) seems not realistic, or not compatible with the assumption of a constant residual strength distribution in the punching failure surface. In this model the $V_{R,f}$ is calculated from f_{Ftu} determined by Eq. (16). To evaluate f_{Ftu}, instead of adopting $w_u=\psi\cdot d/6$ as recommended by this standard, it was assumed w_u=1.5 mm, since the former approach lead to more unsafe predictions. If a proper safety criterion is considered as the one that 85% of the samples remains in the interval λ=[0.85-1.15[, a $w_u\approx$4.0 mm should be adopted in the model proposed by CEB-FIP Model Code 2010.

COMPARISON OF THE PREDICTIVE PERFORMANCE OF THE DEVELOPED MODELS

In this section the predictive performance of the refined and simplified models (*The-refin*, *The-simpl*) developed in the present work for the evaluation of the punching failure load of SFRC flat slabs is compared to the one of the following models found in the literature: Narayanan and Darwish (1987), Shaaban and Gesund (1994), Harajli *et al.* (1995), Holanda (2002), Choi *et al.* (2007), Muttoni and Ruiz (2010) and Higashiyama *et al.* (2011). The formulation of these models is presented in Moraes Neto (2013) and Moraes-Neto *et al.* (2012). Like in a previous Section, the predictive performance of the models was based on the evaluation of the $\lambda=V_{exp}/V_{the}$ parameter and on the analysis of λ according to the modified version of the DPC, where the V_{exp} is the punching failure load of the slabs collected in the database described in Section "Database (DB)". The models in analysis are designated as *MODi* (*i*=1 up to 9), with the corresponding formulation assigned in the footnote of Table 5 and in the caption of Figure 7. The box plot diagram in Figure 7 graphically depicts the statistical five-number summary, consisting of the minimum and maximum values, and the lower, median and upper quartiles of λ for each model. From the analysis of Figure 7 and the values included in Table 5 it can be concluded that the proposed models, together with the model of Muttoni and Ruiz (2010), are those that assure values of λ closer to the unity with the lowest COV. However, the models proposed in the present work provided the smallest total penalties, with the largest number of predictions in the appropriate safety interval.

CONCLUSIONS

In the present paper a model was proposed to predict the punching failure load of steel fibre reinforced concrete (SFRC) slabs centrically loaded (V_R). This model is supported on the assumption that $V_R=V_{R,c}+V_{R,f}$, where $V_{R,c}$ and $V_{R,f}$ is the contribution of concrete and fibre reinforcement for the punching resistance, respectively. To determine the best available formulation for the prediction of $V_{R,c}$, the predictive performance of models proposed by ACI 318, CEB-FIP Model Code 1990, EC2 and CEB-FIP Model Code 2010 was assessed by estimating the punching tests results collected in a database (DB) built for this purpose. From this study it was concluded that CEB-FIP Model Code 2010 evaluates more accurately the concrete contribution for the punching resistance of RC slabs, and consequently it was selected to be combined with the formulation developed for the prediction of the punching resistance of SFRC slabs. This formulation is supported in the critical shear crack theory, and integrates a stress-crack width relationship ($\sigma_f(w)$) for modelling the contribution of fibre reinforcement mechanisms. The $\sigma_f(w)$ was determined according to the recommendations of CEB-FIP Model Code 2010 for the characterisation of the post-cracking flexural tensile behaviour of FRC. The proposed model has two levels of sophistication, one of more laborious calculus, and the other with a simpler way of obtaining the $V_{R,f}$.

The predictive performance of these two versions of the developed model was appraised by simulating the punching tests composing the DB. Both versions of the model have predicted with high accuracy the failure load of the punching tests of the DB, and assured better and safer predictions than the ones obtained with available models for the evaluation of the punching failure load of SFRC slabs.

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NOTATION

A_0	Horizontal projection of failure surface
A_s	Area of tension reinforcement
A'_s	Area of compression reinforcement
b	Width of a isolated slab element
b_0	Critical perimeter for punching shear (ACI 318 and CEB-FIP Model Code 2010)
b_s	Strip of slab to avaluet the bending moment
d	Internal arm of the slab
d_f	Diameter of fibre
d_g	Maximum diameter of the
E	Modulus of elasticity of concrete
E_s	Modulus of elasticity of reinforcement
f_c	Average compressive strength of concrete in cylinder specimens
f_{ct}	Average tensile strength of concrete (Brazilian test)
f_{Fts}	Post-cracking strength for serviceability crack opening
f_{Ftu}	Post-cracking strength for ultimate crack opening
f_{Ri}	Residual flexural tensile strength of fibre reinforced concrete corresponding to $CMOD_i$
f_{yd}	Design yield strength of reinforcement
F_s	Internal compressive force of tensile reinforcement
F'_s	Internal compressive force of compressive reinforcement
h	Slab thickness
I_1	Second moment of area of cracked concrete cross-section
k, ξ	Size effect parameter
k_{dg}	Aggregate interlock parameter
k_ψ	Rotation of the slab parameter
l_f	Length of fibre
L	Span of slab
m_R	Resisting bending moment (plastic bending moment)
m_{sd}	Actuating bending moment
r_0	Radius of the critical shear crack
r_c	Radius of a circular column
r_q	Radius of the load introduction at the perimeter
r_s	Radius of circular isolated slab element
u_1	Critical perimeter for punching shear (EC2 and CEB-FIP Model Code 1990)
V	Shear force
V_f	Fibre volume percentage
$V_{R,cd}$	Design concrete contribution to punching shear strength
$V_{R,d}$	Design punching shear strength
$V_{R,fd}$	Design fibre contribution to punching shear strength
$V_{R,sd}$	Design shear reinforcement contribution to punching shear strength
$V_{s,d}$	Actuating shear force
w	Shear crack opening
w_u	Maximum acceptable crack width imposed by design conditions
x	Neutral axis of slab

α_s Parameter for columns located in the interior of the building

β Efficiency factor of the bending reinforcement for stiffness calculation

β_c Ratio between the larger and the smaller edge of the column’s cross section

χ_{ts} Tension stiffening parameter

ε_c Concrete strain

ε_{cu} Ultimate strain of concrete in compression zone

ε_{fu} Ultimate strain of fibre in tensile zone

ε_s Strain of steel reinforcement in tensile zone

ε_{su} Ultimate strain of steel reinforcement in tensile zone

ε'_s Compressive steel reinforcement strain

$\varepsilon_{t,bot}$ Concrete tensile strain at the bottom surface of the slab

$v_{N,d}$ Design nominal shear stress

$v_{R,d}$ Design shear strength

τ_f Average interracial bond strength of fibre matrix

ρ Tensile reinforcement ratio

σ Stress

ψ Rotation of slab

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Table 1. Modified version of the *Demerit Points Classification* (DPC).

$\lambda = V_{exp}/V_{the}$	Classification	Penalty (PEN)
< 0.50	Extremely Dangerous	10
[0.50-0.85[	Dangerous	5
[0.85-1.15[	Appropriate Safety	0
[1.15-2.0[	Conservative	1
≥ 2.0	Extremely Conservative	2

Table 2. Predictive performance of the design models according to the modified version of the DPC.

$\lambda = V_{exp}/V_{the}$		< 0.50	[0.50-0.85[	[0.85-1.15[	[1.15-2.00[	≥ 2.00	Total PEN	AVG	STD	COV (%)
MOD1	N° samples	0	3	3	18	0	24	1.28	0.32	25.28
	PEN	0	15	0	18	0	33			
MOD2	N° samples	0	7	14	3	0	24	0.95	0.20	20.91
	PEN	0	35	0	3	0	38			
MOD3	N° samples	0	1	15	8	0	24	1.16	0.25	21.30
	PEN	0	5	0	8	0	13			
MOD4	N° samples	0	2	21	1	0	24	1.01	0.09	9.34
	PEN	0	10	0	1	0	11			

MOD1=ACI 318, MOD2 =CEB-FIP Model Code 1990, MOD3=EC2, MOD4=CEB-FIP Model Code 2010.

Table 3. Predictive performance of Eqs. (20) and (21) in the context of the modified version of the DPC.

f_{Ri}	f_{R1}		f_{R3}	
$\lambda_i=f_{Ri,exp}/f_{Ri,the}$	N° samples	PEN	N° samples	PEN
< 0.50	0	0	1	10
[0.50-0.85[	4	20	7	35
[0.85-1.15[	17	0	18	0
[1.15-2.00[	43	43	38	38
≥ 2.00	5	10	5	10
Total PEN	69	73	69	93
Statistical resume				
f_{Ri}	f_{R1}		f_{R3}	
Average (AVG)	1.37		1.32	
STD	0.38		0.48	
COV (%)	27.88		36.08	

Table 4. Performance of models for the prediction of the punching failure load of SFRC flat slabs according to the modified version of the DPC.

Models	*The-refin*		*The-simpl*		CEB-FIP Model Code 2010	
$\lambda = V_{exp} / V_{the}$	N° samples	PEN	N° samples	PEN	N° samples	PEN
< 0.50	0	0	0	0	2	20
[0.50-0.85[	6	30	5	25	39	195
[0.85-1.15[	43	0	42	0	9	0
[1.15-2.00[	1	1	3	3	0	0
≥ 2.00	0	0	0	0	0	0
Total PEN	50	31	50	28	50	215
Statistical resume						
Models	*The-refin*		*The-simpl*		CEB-FIP Model Code2010	
Average (AVG)	0.97		0.98		0.73	
STD	0.11		0.11		0.13	
COV (%)	11.38		11.17		17.48	

Table 5. Performance of several models to predict V_{exp}: classification of the models according to the modified version of the DPC

$\lambda = V_{exp} / V_{the}$		< 0.50	[0.50-0.85[	[0.85-1.15[	[1.15-2.00[	≥ 2.00	Total PEN	AVG	STD	COV (%)
MOD1	N° samples	0	21	21	8	0	50	0.92	0.23	25.29
	PEN	0	105	0	8	0	113			
MOD2	N° samples	0	2	18	29	1	50	1.24	0.26	20.89
	PEN	0	10	0	29	2	41			
MOD3	N° samples	0	5	18	20	7	50	1.42	0.62	43.38
	PEN	0	25	0	20	14	59			
MOD4	N° samples	0	0	8	42	0	50	1.32	0.20	15.47
	PEN	0	0	0	42	0	42			
MOD5	N° samples	0	6	17	27	0	50	1.20	0.29	24.03
	PEN	0	30	0	27	0	57			
MOD6	N° samples	0	6	37	7	0	50	0.99	0.13	13.26
	PEN	0	30	0	7	0	37			
MOD7	N° samples	0	20	24	6	0	50	0.92	0.18	19.45
	PEN	0	100	0	6	0	106			
MOD8	N° samples	0	6	43	1	0	50	0.97	0.11	11.38
	PEN	0	30	0	1	0	31			
MOD9	N° samples	0	5	42	3	0	50	0.98	0.11	11.17
	PEN	0	25	0	3	0	28			

MOD1= Narayanan and Darwish (1987); MOD2= Shaaban and Gesund (1994); MOD3= Harajli *et al.* (1995); MOD4= Holanda (2002); MOD5= Choi *et al.* (2007); MOD6= Muttoni and Ruiz (2010); MOD7= Higashiyama *et al.* (2011); MOD8=*The-refin*; MOD9=*The-simpl*

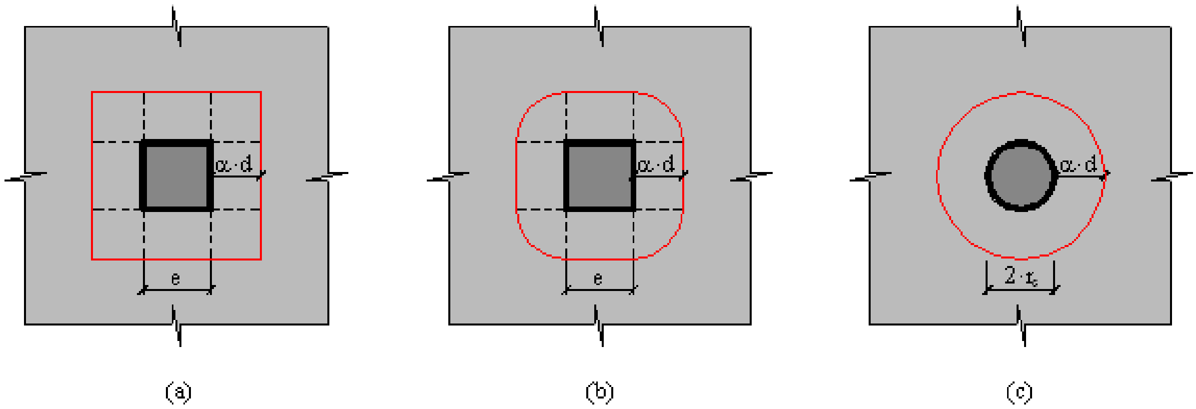

Figure 1. Punching failure perimeter adopted for the evaluation of the punching resistance when columns have: (a) and (b) square cross section, (c) circular cross section.

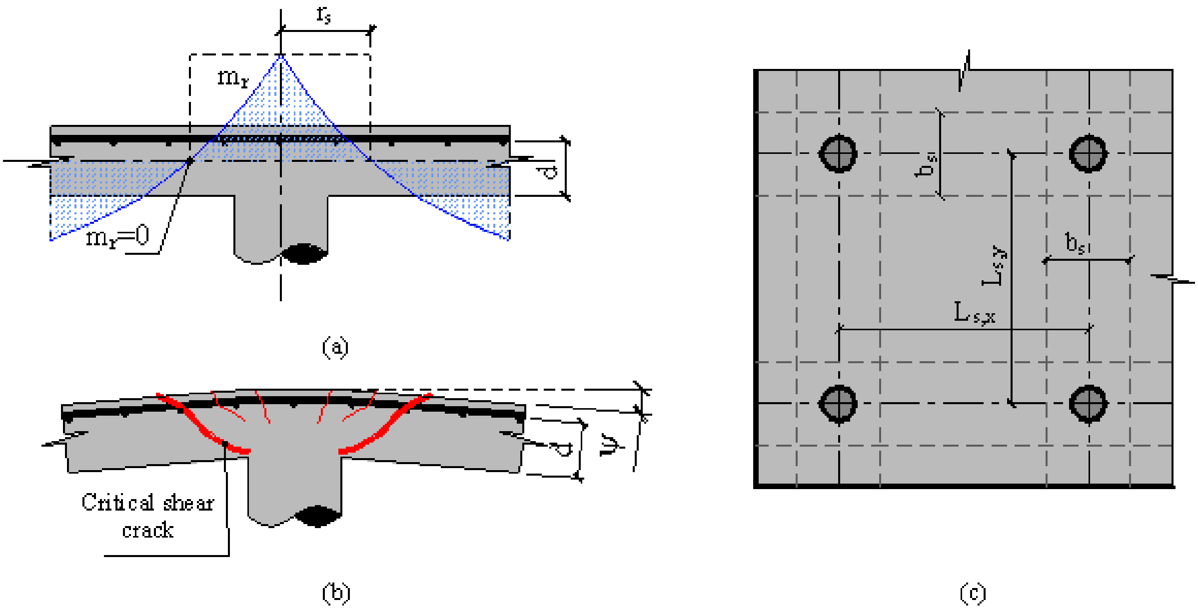

Figure 2. Characteristics of the slab: (a) Distribution of the radial bending moment, (b) Slab's rotation at failure (c) Representation of the slab's spans and width of slab's strips.

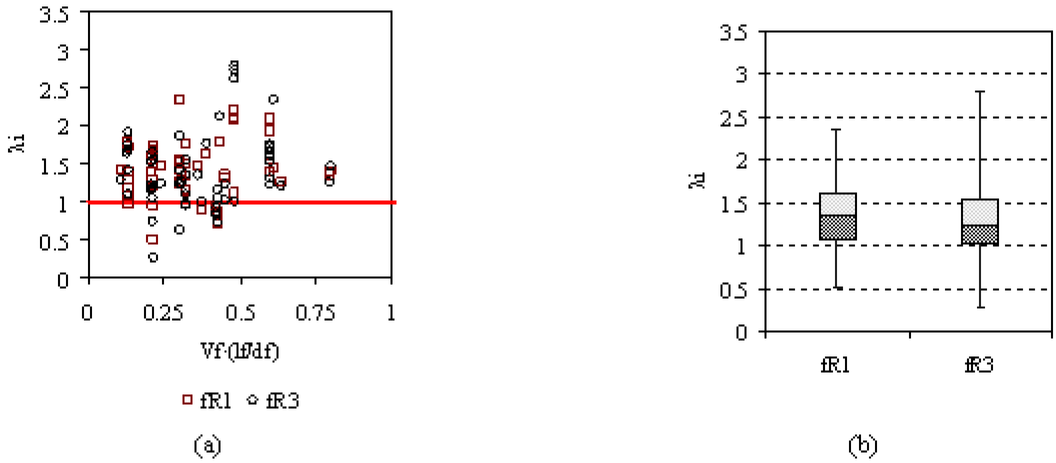

Figure 3. Predictive performance of Eqs. (20) and (21).

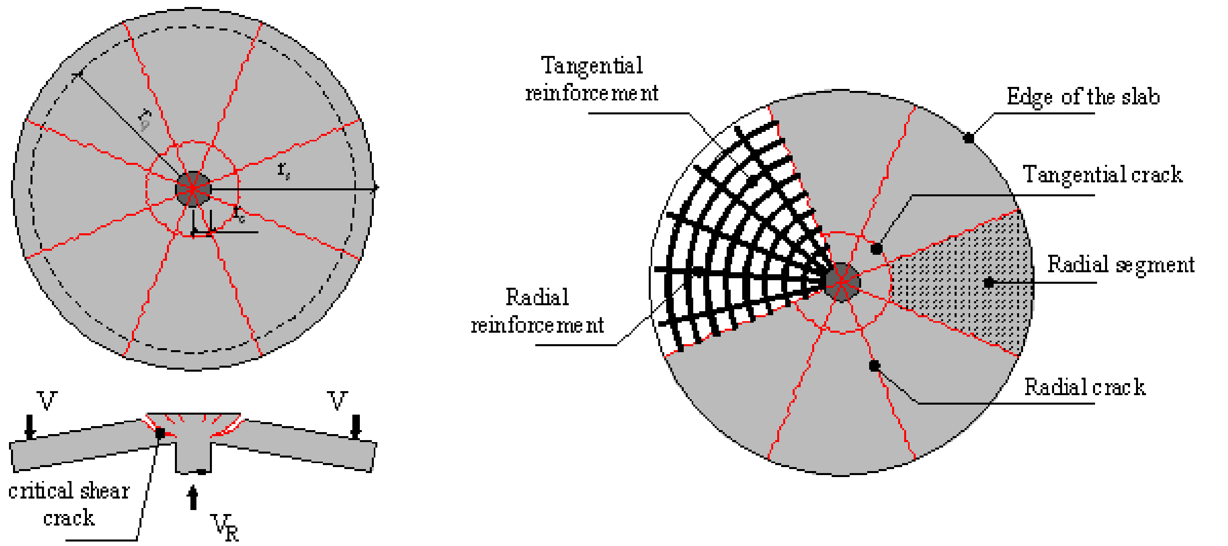

Figure 4. RC slab with axisymmetric structural conditions, crack pattern at ultimate limit state and reinforcement arrangement.

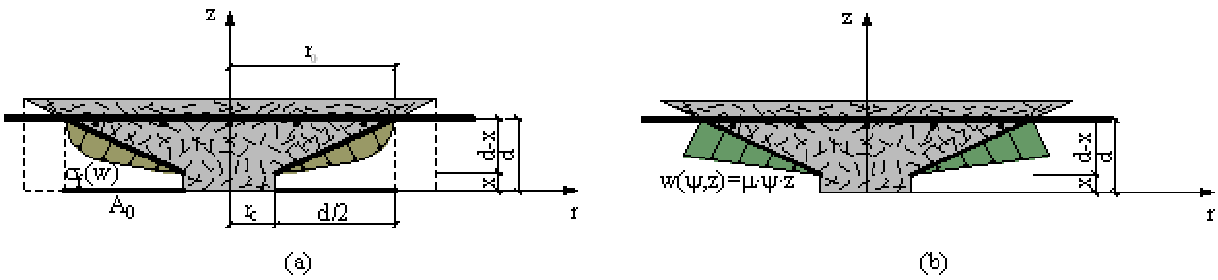

Figure 5. Contribution of fibre reinforcement mechanisms for the punching resistance: (a) stress distribution and (b) crack opening idealization.

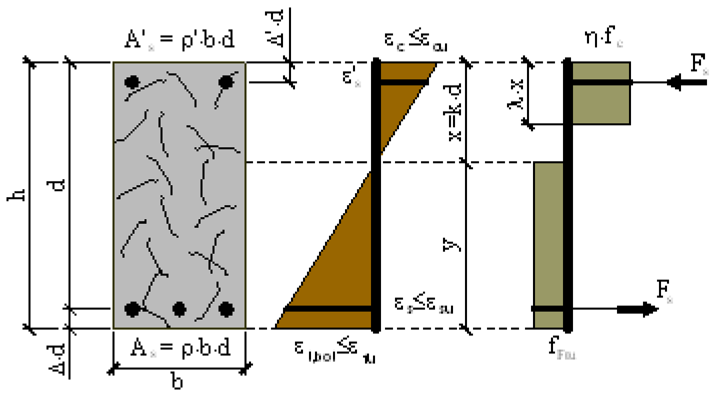

Figure 6. Adopted approach to evaluate x and m_R (adapted from the CEB-FIP Model Code 2010 [8]).

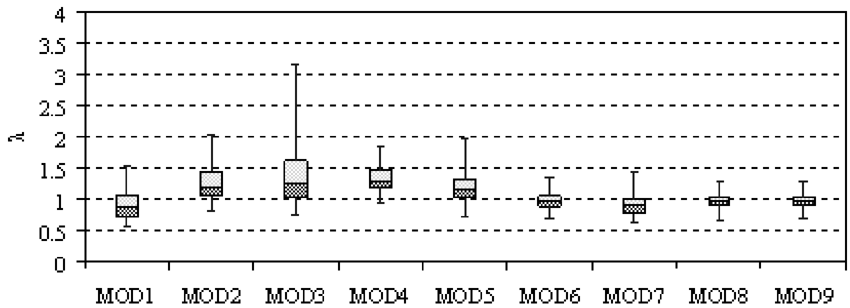

Figure 7. Comparison of the predictive performance of available and proposed models.

MOD1= Narayanan and Darwish [13]; MOD2= Shaaban and Gesund [27]; MOD3= Harajli *et al.* [14]; MOD4= Holanda [28]; MOD5= Choi *et al.* [23]; MOD6= Muttoni and Ruiz [15]; MOD7= Higashiyama *et al.* [24]; MOD8= *The-refin*; MOD9= *The-simpl*