



ADVANCED MASTERS IN STRUCTURAL ANALYSIS OF MONUMENTS AND HISTORICAL CONSTRUCTIONS

Master's Thesis

Anna Maria Milczarek

Robustness-Based Assessment of Railway Masonry Arch Bridges



Universidade do Minho

UNIVERSITAT POLITÈCNICA DE CATALUNYA



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This Masters Course has been funded with support from the European Commission. This publication reflects the views only of the author, and the Commission cannot be held responsible for any use which may be made of the information contained therein.

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ACKNOWLEDGEMENTS

I would like to express great thanks to talented, hard-working prof. Matos and prof Oliveira, who took time from their busy schedules to offer advice and guidance in regards to the thesis.

I would like to express my deep gratitude to Vicente Moreira, for his shared knowledge, help, and the patient hours he spent enlightening me with engineering concepts that I, as an architect, was not as familiar with.

I would like to thank the lecturers of SAHC that passed down their knowledge, making this thesis possible.

Thank you, to all of my fellow students that I have encountered during this program. I have learned and grown amongst you all, benefiting from the specific types of knowledge that you all had by coming from incredible backgrounds.

Most of all, I would like to thank my family—both sets of grandparents Babunia i Dziadzius Milczarek, and Babunia i Dziadzius Piszczako, my parents; Mama and Tata, and all of my siblings, Elzunia, Basia, Piotrus, Gabrysia, Stas and Wojtus. Your constant support, your belief in my success, and your love helped me greatly in the completion of the SAHC program.

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ABSTRACT

Railway masonry arch bridges (MAB) were first created to allow human beings to travel through bodies of water. The durable masonry allowed the bridges to withstand heavy subjection to loads, exposure to climate, and the passing of time. To this day, many older railway MAB remain standing—while never undergoing any renovations—supporting more loads than the original intent; standing true to their robustness. The concept of robustness gained interest following the collapse of the Roman Point building in the second half of the 20th century. The collapse of the multi-storey building—that resulted from a single oven gas explosion—lead to the development of a concept of robustness as proportionality between inflicted damage and consequence. A few decades later, at the beginning of the 21st century, the collapse of the Twin Towers increased the interest in developing efficient methods to measure robustness.

While various methods have been proposed to assess robustness, to this day there exists no consensual decision in which of those methods should be enforced. The paper assesses and compares the robustness of the bridge of Vila Mea through three applications; i) probabilistic based redundancy; ii) Frangopol and Curley approach; iii) Cavaco method. To compute the robustness of the bridge, the reliability indexes of the damaged and undamaged structure are defined. To determine the reliability indexes of the damaged and undamaged structure, the bridge of Vila Mea is modeled using the software Limitstate:RING. The software computes the resistance of the bridge by applying a modified version of the Kinematic theorem, initially formulated by Heyman. The reliability index of the undamaged structure is computed through the probabilistic analysis. Next, 5 typical damage scenarios are introduced into the bridge individually, and the reliability index for each of the 5 damaged structures is defined. By obtaining the reliability indexes of the damaged and undamaged structure, robustness of the bridge is assessed through the application of the 3 methods mentioned above. Results from each damage, as well from each method, are compared. The bridge of Vila Mea proved to be robust and the methods proved to be difficult to be compared amongst eachother.

KEY WORDS: masonry arch bridge, robustness, reliability index, probabilistic approach

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RESUMO

As pontes em arco de alvenaria de pedra (PAAP) foram construídas para permitir que os seres humanos transpusessem cursos de água. A alvenaria, material durável e resistente, permitiu que as pontes suportassem as fortes cargas, à exposição ao clima e à passagem do tempo às quais estão submetidas. Até ao presente, várias PAAP encontram-se em utilização – e não foram intervencionadas – suportando cargas superiores às consideradas no seu dimensionamento; reforçando a sua robustez. O conceito de robustez despoletou interesse após o colapso do edifício Ronan Point na segunda metade do século XX. O colapso do edifício de vários andares – que resultou de uma única explosão de gás – originou o desenvolvimento do conceito de robustez como medida de proporcionalidade entre o dano inicial e respetivas consequências. Poucas décadas mais tarde, no início do século XXI, o colapso das Torres Gêmeas aumentou o interesse e investigação no desenvolvimento de métodos eficientes para avaliação da robustez.

Embora vários métodos tenham sido propostos para avaliar a robustez, até hoje não existe uma decisão consensual sobre quais dos métodos desenvolvidos devem ser aplicados. A dissertação avalia e compara a robustez da ponte de Vila Meã através da aplicação de três métodos: i) redundância baseada em análises probabilísticas; ii) método de Frangopol e Curley; iii) método de Cavaco. Para calcular a robustez da ponte, os índices de fiabilidade da estrutura danificada e não danificada são determinados. Para determinar os índices de fiabilidade da estrutura danificada e não danificada, a ponte de Vila Meã é modelada através do software Limitstate: RING. O software calcula a capacidade de carga última da ponte aplicando uma versão modificada do teorema cinemático, inicialmente formulado pela Heyman. O índice de fiabilidade da estrutura intacta é calculado através métodos probabilísticos. Seguidamente, 5 cenários de dano típicos são considerados individualmente, e o índice de fiabilidade para cada uma das 5 estruturas danificadas é obtido. Ao obter os índices de fiabilidade da estrutura danificada e não danificada, a robustez da ponte é avaliada através da aplicação dos 3 métodos mencionados anteriormente. Os resultados de cada cenário de dano, bem como de cada método, são comparados. A ponte de Vila Meã apresentou ser robusta perante os cenários de dano estudados, enquanto que os métodos para avaliação da robustez estrutural revelaram-se difíceis de serem comparados entre si.

PALAVRAS-CHAVE: ponte em arco de alvenaria de pedra, robustez, índice de fiabilidade, análise probabilística

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KONSPEKT

Kamienne kolejowe wiadukty o łukowym sklepieniu (KWLS) zostały pierwotnie skonstruowane, żeby umożliwić ludziom swobodne przemieszczanie się przez rzeki, i wszelkie inne wodne zbiorniki. Konstrukcja mostu jest przykładem wytrzymałości; odporna jest na wszelkie czynniki klimatyczne a także upływ czasu. Zawalenie się budynku Roman Point w drugiej połowie dwudziestego wieku spowodowało zainteresowanie do koncepcji wytrzymałości. Wtedy zaczęła dojrzewać nowa koncepcja traktująca wytrzymałość konstrukcji z uwzględnieniem skutków ewentualnej katastrofy. Tragiczne w skutkach konsekwencje zawalenia się Twin Towers w Nowym Jorku, zwłaszcza liczba ofiar i stopień zniszczeń, spowodowały zainteresowanie w obliczeniu wytrzymałości konstrukcji.

Podczas gdy wiele różnych metod zostało zaproponowanych w celu zmierzenia wytrzymałości, do dziś dnia nie ma concensusu co do wyboru które z tych metod są najwłaściwsze. Papierkowe wyliczenia i porównania wytrzymałości mostu Vila Mea dokonano używając trzech kryteriów: a) koncepcja probabilistycznej niezawodności b) koncepcja Frangopola i Curleya c) metoda Cavaco. Do obliczenia wytrzymałości mostu, porównanie zniszczonej konstrukcji do niezniszczonej konstrukcji musi być precyzyjnie określone. Aby określić stosunek niezniszczonej części mostu do zniszczonej, do modelu mostu Vila Mea użyto oprogramowania Limistate:RING. Program wylicza wytrzymałość mostu przez zastosowanie zmodyfikowanej teorii kinematycznej pierwotnie sformułowanej przez Heymana. Wskaźnik niezawodności jest wyliczany poprzez analizę prawdopodobieństwa. Następnie most zostaje testowany przez pięć podstawowych rodzajów katastrof i wskaźnik wytrzymałości jest wyraźnie określony przez wskaźnik niezawodności zniszczonych do niezniszczonych części mostu. Wytrzymałość jest określona poprzez zastosowanie trzech metod wspomnianych powyżej. Rezultaty każdego rodzaju zniszczeń a także wyliczenia pochodzące z trzech metod zostają porównane. Most Vila Mea potwierdził swoją wytrzymałość a metody udowodniły swoją nieporównywalność.

KEY WORDS: kolejowe wiadukty o łukowym sklepieniu, wytrzymałość, wskaźnik niezawodności, koncepcja probabilistyczna

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1. INTRODUCTION

1.1 Problem Stated

Robustness allows existing structure to support unforeseen structural damages (natural or man-made), without being triggered global collapse [1]. Masonry arch bridges (MAB) are often subdued to larger load capacity than their original intent. Oftentimes, historical structure needs to undergo a strengthening intervention, however MABs are rarely retrofitted. The excellent performance in terms of structural capacity of MAB is an excellent example of the concept of robustness. Determining the robustness of MABs would allow to determine the amount of intervention needed to stabilize the structure once the structure undergoes damage. By understanding the amount of intervention necessary, an unnecessary over-modification of the structure would be avoided.

1.2 Objective

The main objective of the paper is to perform a robustness assessment of the bridge of Vila Mea. To perform the robustness assessment, other tasks must be accomplished. Some of the tasks performed include:

- i) Geometrical and material assessment of the bridge of Vila Mea;
- ii) Construction of a numerical model of the bridge of Vila Mea;
- iii) Performing a probabilistic analysis on the undamaged structure of the bridge;
- iv) Performing a probabilistic analysis on the damaged structure of the bridge;
- v) Compute the reliability indexes for both the undamaged and damaged numerical models of the bridge.

By performing the tasks above, the reliability indexes needed to perform the robustness assessment are obtained. Robustness of the bridge of Vila Mea is computed through three proposed methods, namely: i) Probabilistic based redundancy [2]; ii) Frangopol and Curley approach [18]; and iii) Cavaco method [18], to gather a better understanding on the methods available for robustness assessments.

1.3 Outline

The paper is created through literature review as well as computer simulations performed on numerical models. The information on the properties of the bridge of Vila Mea is based on literature, while the performance assessment of the bridge is performed through the modeling

of the bridge. The numerical models, constructed in Limitstate:RING [3], allows for the assessment of the reliability indexes of the undamaged and damaged structure, which in turn allow for the robustness assessment of the bridge of Vila Mea. The paper is divided into 6 chapters:

- i) Chapter 1 – An introductory part of the paper which summarizes the addressed problem, the objectives as well as the outline of the paper;
- ii) Chapter 2 – A state of art containing the description of MABs, the typical damages experienced by MAB, as well as the concept and means of measure of robustness;
- iii) Chapter 3 – A description of the case study of the bridge of Vila Mea addressing the history, and the modelling of the bridge;
- iv) Chapter 4 – The computation of the sensitivity analysis and reliability analysis (for both undamaged and damaged structure) of the bridge;
- v) Chapter 5 – The robustness assessment of the bridge of Vila Mea, as it undergoes 5 different types of damages: i) transversal cracking, ii) longitudinal cracking due to spandrel wall detachment, iii) longitudinal cracking due to bi-block sleepers, iv) mortar loss, v) delamination.;
- vi) Chapter 6 – A conclusion to the paper stating what was achieved through the assessment and the future considerations.

2. STATE OF THE ART

2.1 Masonry arch Bridges

2.1.1 Introduction

Innovation has always been, and still is, embedded in the minds of human beings. Humans first started building structures to protect themselves from the environment. At first, human beings used earth and branches as material for the structures. With time, the materials used were replaced with more durable ones, such as timber, then mud bricks, and finally masonry. The purpose of structures also evolved, from shelters, to walls to bridges. In fact, more than 4000 years ago, masonry bridges allowed for humans to cross barriers, such as bodies of water, instead of going around such obstacles. This allowed for a more effective travel of greater distances. Means of transportations evolved with time as well, leading to the use of train for traveling and transportation. To allow human beings to travel through bodies of water, various structures—such as railway masonry arch bridges (MAB) – were constructed. The masonry allowed the bridges to be durable. The arch shapes reduced the amount of material needed in the making of the bridge. Newer materials and building techniques lead to the creation of other types of bridges. However, a lot of railway MAB stand to this day. Some of those bridges underwent intervention to adapt to modern traffic, others were left unaltered, allowing trains to travel through bodies of water. Others are now left unused. Many older railway MAB are still standing—while never undergoing any renovations—supporting more loads than the original intent; standing true to their robustness.

2.1.2 Components

Structure design is dictated by function of the structure. A specific function imposes a structure with specific load cases and applications. The components of the structure act together as a system to resist those applied loads. In the case of MAB, the structure must distribute its dead weight, as well as the imposed loads due to traffic, through the structure into the its foundations, from the foundations into the ground.

The structural behaviour of MABs depends highly on the geometry of the structural elements and material properties. MAB are usually composed with three main materials; masonry, mortar as well as fill material [4]. Masonry is known as a heterogenic, anisotropic material, with a moderate compressive strength and very low or null tensile strength [4]. The

soil which comprises the fill material is made up of various materials of various sized materials placed over the masonry vaults [5].

The loads which are applied on a MAB are distributed in both the transversal and longitudinal direction [4]. In the transversal direction, the concentrated vertical live loads from the train are distributed in a Boussinesq load scheme manner. In the longitudinal direction, the most important load bearing element consists of the arch barrel. The arch barrel bears the loads from the fill as well as the live loads, and distributes the loads towards the piers, which in turn direct the loads into the foundation, abutments and the ground [4]. All the MAB components are demonstrated in **Figure 1**.

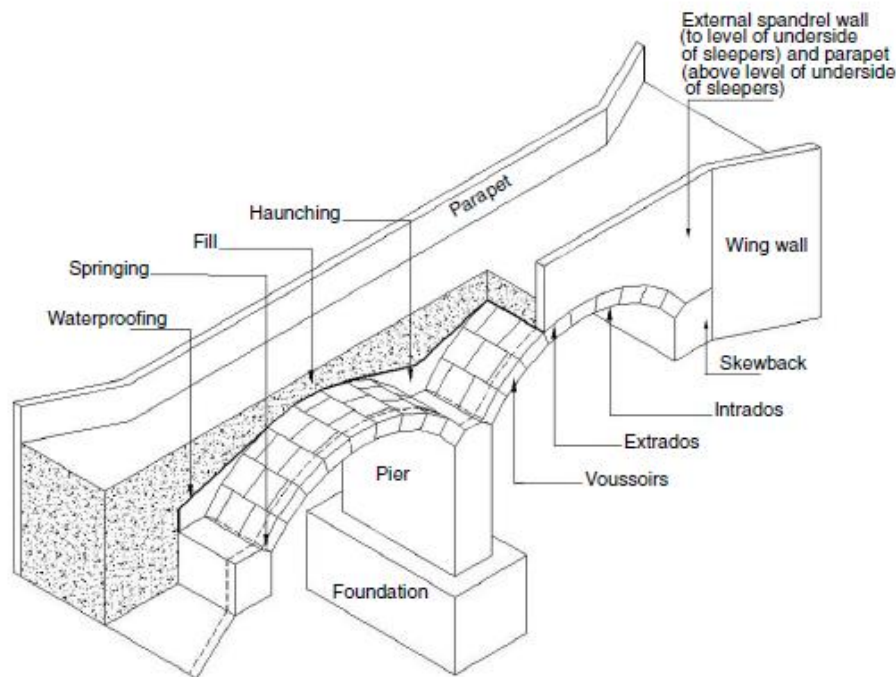


Figure 1 – Elements of a masonry arch bridge [6].

By acting as the main distributor of the longitudinal loads, the arch barrel is the primary element of an arch bridge [6]. The element can be made up of stone or brick. The overall shape of arch barrels is dictated by the clearance requirements and economic situation of the time of construction [6]. Therefore, bridges located on the same route, or built in the same period, display identical or very similar geometries [6]. The horizontal and vertical resistance of the spread of the arch is provided by abutments [6].

The pier elements are situated between the foundation of the bridge and the springing of the arches. Supporting each arch, the slenderness of the piers dictates the behavior of each arch

[6]. A pier which is slenderer will render the arch less robust. A stockier pier will render the arch more robust.

The spandrel exists over and between the arches. The spandrel comprises, amongst others, the fill, the spandrel walls and the backing and haunching. The fill distributes the live load over a larger area into the arches, and provides stability to the arches by keeping the arches under compression [6]. The spandrel walls contain the fill over the arches [6]. Backing and haunching constitute of additional masonry elements placed on the arch barrel or over the abutment or pier [6]. The additional masonry increases the load capacity of the bridge by allowing substantial distribution of the thrust before it reaches the next elements [6]. When the additional masonry has a horizontal upper surface, the element is known as backing. When the additional masonry has a sloped horizontal upper surface, it is known as haunching.

2.2 Failure of Masonry Arch Bridges

Failure of MAB occurs when the stresses applied onto the structure exceed the resistance capacity of the bridge. It is important that failure mechanism of MAB is dependent on the slenderness of the piers [4]. In fact, a bridge can fail either globally or locally. Local damage occurs when a bridge displays stocky piers, while global damage occurs when a bridge is built with slender piers [7]. When the stresses applied onto the bridge exceed the resistance capacity of the bridge, then the structure fails either due to snap through, ring separation or internal releases.

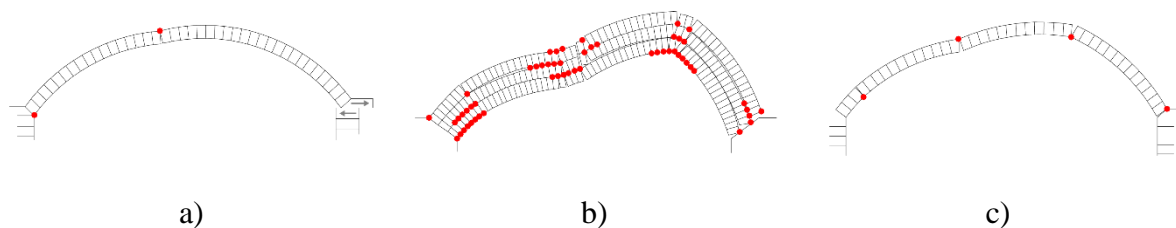


Figure 2 – Failure mode, a) snap through, b) ring separation, c) internal releases, based on [4] [8].

Snap through failure occurs prior to the full formation of arches [4]. It is a rare occurrence, usually seen in bridges that have very shallow arches. Ring separation occurs in multi ring MAB [4]. It affects brick masonry bridges, since the arch barrel is composed of a few layers of bricks which are usually not thoroughly bonded. The formation of internal releases—such as plastic hinges or bridges—is the most likely failure mode to occur in MAB.

2.3 Damage patterns

With time passing, damage appearing on a structure is an inevitable occurrence. Masonry is deemed to be one of the more durable materials [9]. However, just like with any structures, masonry structures further away from their pristine condition throughout their years of standing. The deterioration of the structure depends on the load application as well as the environment of the structure. MAB experience different types of damages in different sections of the bridges. Arch barrels commonly display longitudinal cracking, spandrel wall detachment, diagonal cracking in the arch barrel, transversal cracking, spalled masonry arch voussoirs, masonry deterioration and fatigue. Piers present vertical and stepped cracks. Overall, masonry of the whole structure suffers from fatigue and environmental deteriorations as years pass.

2.3.1 Arch Barrel

2.3.1.1 TRANSVERSAL CRACKING

Transversal cracking (**Figure 3**) can be caused by a few occurrences; detachment of spandrel walls, settlement of the supports, or displacement of masonry voussoir (caused by mortar loss in masonry voussoir). As the arches work in compression, transversal cracking does not directly impact the structural integrity of the structure. However, transversal cracks become openings in the arch which allow deterioration of the fill material [10] [11].

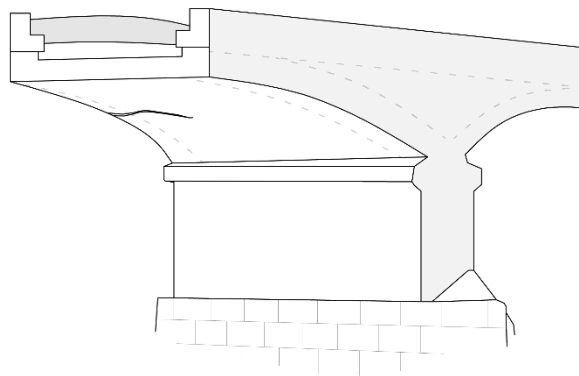


Figure 3 - Transversal Cracking

2.3.1.2 LONGITUDINAL CRACKING: TRANSVERSAL BENDING

Longitudinal cracking (**Figure 4**) may appear in any section of the arch barrel. When the crack appears in the centre of the barrel, it can be caused by relative settlement of the centre of the pier compared to the edges [12]. A centre crack can also be caused by the transverse

bending and axial tension forces present in the arch barrel [12]. Transverse bending and axial tension forces occur when the position of the train track over the barrel is non-symmetrical [12]. The crack prevents proper distribution of the applied load throughout the arch into the piers and abutments [10] [11]. In fact, the loads can no longer be distributed correctly in the transversal direction.

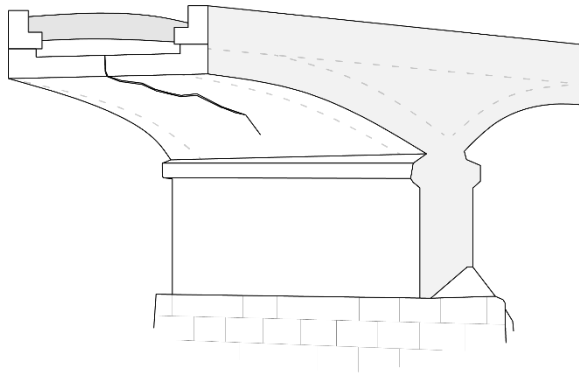
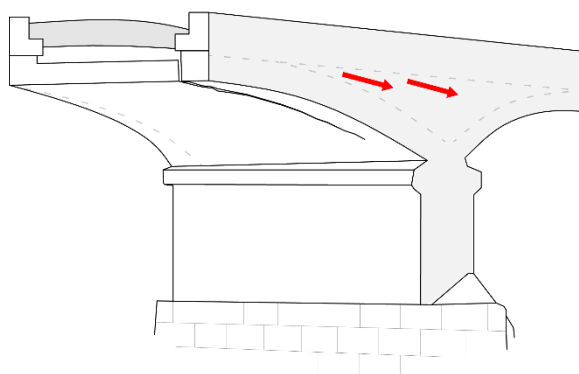


Figure 4 - Longitudinal crack diagram

2.3.1.3 LONGITUDINAL CRACKING: DETACHMENT OF SPANDREL WALLS

Spandrel wall detachment (**Figure 5**) occurs when the spandrel walls of the arch move outwards. The resulting crack is a type of longitudinal cracking. However, the crack continues down from the spandrel wall. Spandrel wall detachment does not only lead to cracking in the arch, but also isolates the spandrel wall from the rest of the arch. Consequently, the provided stiffness by the spandrel walls to the arch is reduced, particularly at the crown zone [10] [11].



-Figure 5 - Spandrel wall detachment

2.3.1.4 LONGITUDINAL CRACKING: BI-BLOCK SLEEPER LOAD CONCENTRATION

Another type of longitudinal cracking in the centre of the arch is caused by a concentration of loads in longitudinal lines on the load [12]. This cracking is further emphasized when the fill does not distribute the loads effectively [12]. This is often caused by the presence of bi-block sleepers: which are used instead of mono-block sleepers to increase resistance [13]. However, the concentration of longitudinal loads on the arch barrel caused by the bi-block method creates stresses and cracking at the centre of the arch. Those cracking patterns are characterized by 3 to 4 longitudinal cracks at the crown of the arch (**Figure 6**) [12].

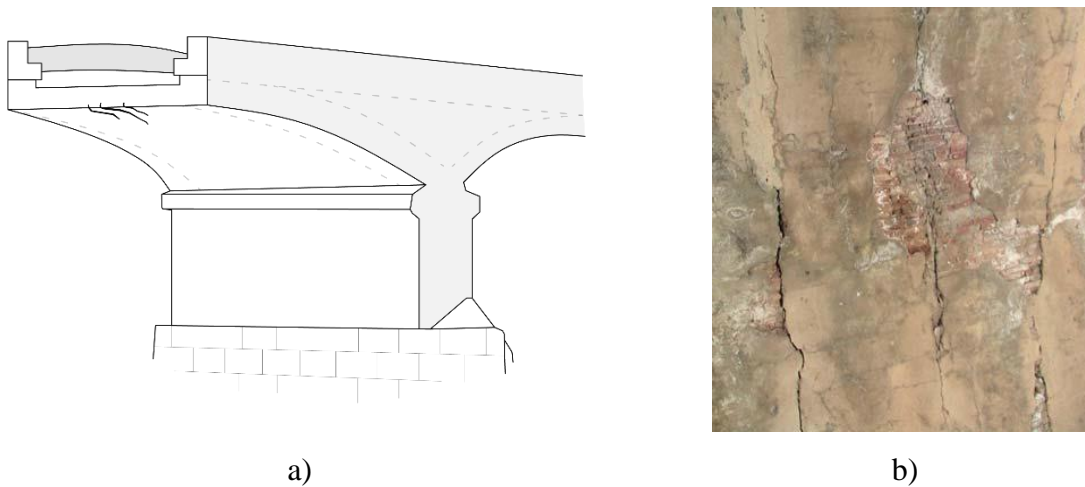


Figure 6 – a) crown cracking caused by bi-block sleepers load concentration, b) typical cracking pattern at crown of the arch due to bi-block sleepers load concentration [10]

2.3.1.5 DIAGONAL CRACKING

Diagonal cracking (**Figure 7**) in the arch barrel can be either caused by an inappropriate bond or by settlement and pier rotation [10]. With a differential rotation of the piers, torsional forces are applied onto the arch barrel [10]. The crack will appear from the edge of one of the rotating piers, running to the opposite edge of the second rotating pier. The crack reduces the load capacity of the arch barrel greatly, by preventing proper load distribution [10]. The diagonal crack also narrows the effective width of the springing, which results in an increase and concentration of stresses in the springing [10].

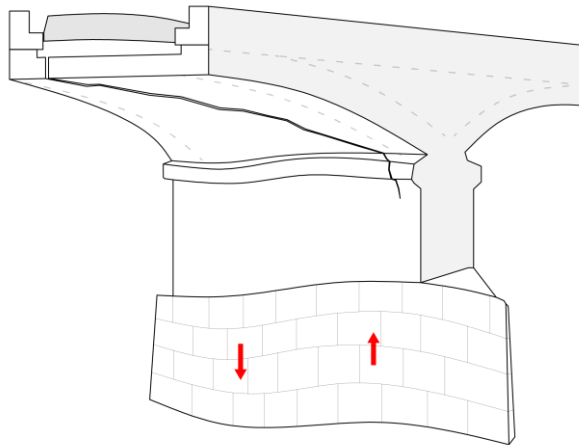


Figure 7 - Diagonal cracking on arch barrel, based on [10]

2.3.2 Piers

2.3.2.1 VERTICAL AND STEPPED CRACKING

Vertical cracking (**Figure 7**) on piers is a very severe deterioration and require immediate action. In fact, vertical cracks on piers act as a warning of an immediate collapse due to material failure [10]. However, this type of damage is rare since the piers are rarely subjected to stresses higher than the strength of the construction material [10]. Vertical pier cracking can also be caused by differential settlement of the bridge foundation [10].

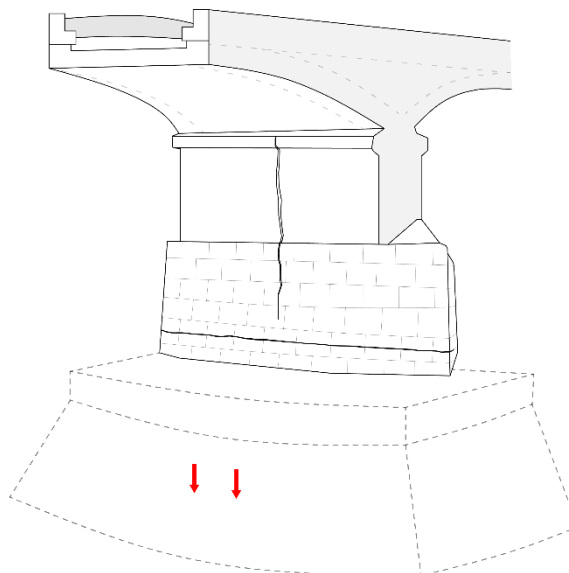


Figure 8 - Vertical cracking due to differential settlement, based on [10]

Stepped cracks occurring on the piers of the MAB are associated with the rotation of a footing on the horizontal and longitudinal axis [10]. It is usually a result of a local failure of the foundation [10].

2.3.3 Other Damage Types

2.3.3.1 MORTAR-LOSS

Mortar loss can be caused by movement in the structure as well as exposure to elements. Mortar is a weaker, softer material compared to masonry that the mortar binds [9]. The mortar is an extremely important material that ensures good adhesion between the masonry blocks, and adds flexibility to the structure. Flexibility in the structure is an important element as it adds some tensile strength to—an otherwise—non-tensile type of assemblage. Mortar also acts as an elastic component of a masonry bridge, allowing to deform under loading and unloading of stresses on the bridge, preventing stress propagation into the masonry units. Therefore, mortar reduces the possible cracking in the masonry units. With mortar loss, water can be easily trapped between the masonry units through capillary suction, inducing more mortar loss. The loss of mortar lowers the resistance of the masonry bridges by no longer limiting stress propagation. With a lot of mortar loss, the masonry may start to fall away from the wall.

2.3.3.2 DELAMINATION

Delamination of masonry occurs in most older masonry structures. In fact, delaminated masonry in arch voussoir is a damage that is almost always present in old MAB. Delamination is similar to spalling; both consist of the splitting of the stone's outer surface into thin layers or laminae [9]. However, delamination is a naturally occurring phenomenon in stone—especially sandstone and stone with high clay content—due to a stone's layered composition [9]. On the other hand, spalling is caused by pressure buildup due to trapped soluble salts or moisture particles under the surface of the stone or brick [9]. While delamination in a MAB does not affect the structural integrity of the bridge directly, the delamination can lead to mortar loss or mortar wash-out. A large amount of mortar loss may affect the arch thickness enough to negatively impact the ultimate load carrying capacity of the bridge [10].

2.3.3.3 FLOOD AND MASONRY SATURATION

Masonry is a porous material, allowing it to absorb water. In the case of MAB, the masonry of the bridges is greatly exposed to water. The water can reduce the adhesiveness of the bonding between the masonry units and the mortar. Furthermore, saturated masonry units suffer a reduction in their mechanical properties. Many tests concluded that the strength of the masonry units is negatively affected when the masonry is saturated. Therefore, water considerably lowers the resistance of masonry, particularly its compressive strength [14].

2.3.3.4 FILL WASHOUT

Fill washout is usually a result of water penetrating the bridge and reaching the fill. Transversal cracks allow easier access to the fill, resulting in its deterioration. Fill washout reduces the mass of the fill and eliminates the fine contents of the fill. Reduction in mass of the fill reduces the compression exerted on the arches. The stability of an arch is dependant on its compressive quasi static state. Without enough compressive strength acting upon the arch, the arch will become unstable, which can be the case with fill washout [6]. With a loss of finer particles, the fill suffers a reduction in density as well as a reduction in the friction angle. This impacts the stability of the fill.

2.3.3.5 FATIGUE

Masonry fatigue and deterioration occurs with time, and is dependant on the environment in which the MAB is situated as well as cyclic loading. In fact, masonry loses its strength when it experiences freeze-thaw cycles and sulfate attacks. Furthermore, masonry consists of a porous material, which can be susceptible to penetration of chemical substances. The freeze thaw cycles, sulfate attacks as well as chemical penetration result in reduction of the mechanical properties—especially compressive strength—of the masonry [10].

2.3.3.6 SPALLED BLOCKS

Similarly, other types of arch material loss can occur – such as dropped stones (**Figure 9**). The loss can either occur due to load application or deterioration. In most cases, the loss occurs due to movements of the supports at the springing of the arch barrel [10].

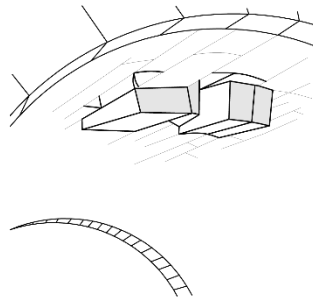


Figure 9 - Dropped stones based on [10]

2.4 Modelling of Masonry Arch Bridges

With numerical modeling of any structures, certain general steps must be taken. First a geometrical survey is performed. In terms of MAB, in addition to the general geometry of the structure, the specific geometry survey should include backing height h_h , piers thickness t_p , arch thickness t_a , ballast height h_b and, fill material at depth of crown h_f .

Furthermore, material characterization must be completed. When it comes to MABs, one of the most relevant property which needs to be determined is the compressive strength normal the bed joints [4]. Other relevant properties include: compressive strength normal to the load f_c , masonry density γ_m , masonry friction coefficient μ , fill material density γ_f , fill material friction angle ϕ , fill material cohesion c , ballast density γ_b and, track load T_l [4]. The properties are determined through various tests. However, tests can be sometimes too costly or not available. In that case, material data must be based on literature [4].

With the collected data, a numerical model is constructed. To compute the ultimate load-carrying capacity, several methods are available, namely: i) Thrust line analysis method ii) Limit analysis; iii) Finite element method (FEM); and iv) Discrete element method (DEM) [4].

While the thrust line analysis allows to determine if a structure is stable, it offers little information regarding the model other than its stability.

The limit analysis method is the most widely used, mainly since its complexity can be controlled with the amount of input parameter. The more amount of data is imputed, the more information will be gathered from the analysis. Limit analysis is based on the two theorems of the Plasticity theory: i) Lower bound or Static theorem; ii) Upper bound or Kinematic theorem. The Kinematic theorem, formulated by Heyman [15], assumes that masonry has infinite capacity in compression, zero capacity in tension, that masonry units are infinitely rigid, and that no

sliding occurs between the stone blocs [15]. The Kinematic theorem states that the arch will fail when a kinematically admissible mechanism can be found for which the work developed by the external forces is greater or equal to zero [15]. Simply stated, the thrust line resulting from the applied and resisting loads of the structure must stay within the arch (**Figure 10**).

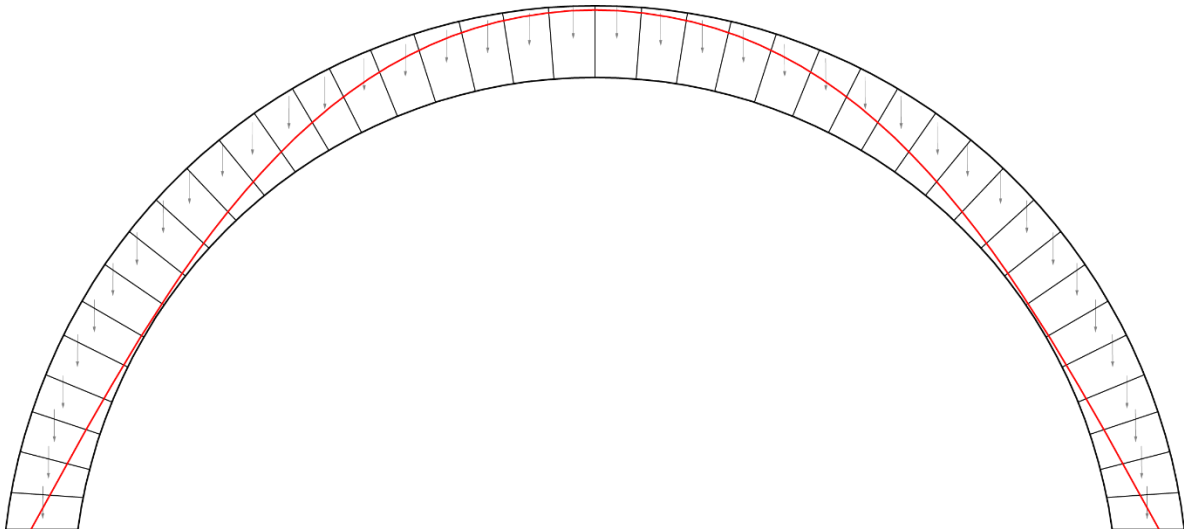


Figure 10 - Thrust line fitting in an arch [6].

When the thrust line travels from the extrados to the intrados (or vice versa) of the arch, a plastic hinge is formed. The kinematic theorem states that a local collapse will occur when 4 plastic hinges are developed in a single span MAB, and a global collapse will occur when 7 plastic hinges are developed in a multiple span MAB [4]. When the thrust line fits inside of the arch, the arch is deemed stable. A live load—such as a train axle—will disturb the thrust line [6]. When the thrust line deviates greatly from the centre of the arch, cracking may occur. The formation of new cracks can lead to arch deteriorations [6]. Due to the control on the complexity of computation of the limit analysis method, the former method is preferred.

FEM offers a lot of information, but also requires a lot of computation effort, rendering the analysis quite complex. FEM is a numerical method which allows to perform a structural assessment in a 3-dimensional, non-linear manner, without any simplifications to turn the model into a 2 dimensional one [4]. However, with a 3-dimensional analysis, a greater amount of data is required to conduct the FEM. The method consists of a division of a structure into smaller parts, called finite elements. A greater number of finite elements allows for greater accuracy in the solution [4]. However, a greater number of finite elements require high computational efforts.

DEM is used when a dynamic situation is being assessed [16]. A dynamic situation includes configuration exists when a collection of bodies is under constant variation under the action of some external forces [16]. Different methods exist under DEM, each referring to specific ways the methods deal with specific variations of the models—such as, amongst many, contact detection algorithm, treatment of contacts or deformability [16]. As with FEM, the main drawback of this method consists of the high computational efforts required with larger systems and accurate results.

2.4.1 Limit Analysis Method

Limit Analysis Method allows for a structural assessment of a structure using a limited amount of information. The method is based on Heyman’s Theorem which states that the masonry in the arch has no tensile strength, the masonry in the arch is incompressible, and that sliding between masonry units cannot occur [17]. Limit analysis method simplifies the problem into a resistance of the structure towards the applied load through a limit state (LS) approach. To understand the LS approach, the term “failure” must be understood [18]. The term “failure” has an ambiguous meaning. It can be stated that “failure” occurs when a structure or element loses its intended function. However, the function of the structure or element is not always clear [18]. The notion of LS helps define the term “failure” through structural reliability analysis [18]. A LS is the boundary between desired and undesired structural performance [18]. Once a structure’s performance decreases until it crosses the limit state, the boundary, its performance becomes undesired. While various representations of LS exist, for the context of the thesis, LS is represented through the difference between the resistance and the applied loads (1).

$$R - S \leq 0 = LS \quad (1)$$

where R consists of Resistance, S consists of applied loads and LS consists of limit state. Through that representation, the resistance must be larger than the applied loads for the structure to not fail. Resistance is typically a function of material strength, geometry and dimensions. [18] While LS allows for structural assessment, it is a simple method which is based in a 2-dimensional approach of a structure. In the case of 3-dimensional structures, such as bridges, some assumptions and simplifications must be made. The LS analysis of numerical models can be undertaken through a deterministic or probabilistic approach.

2.5 Deterministic Approach

In design of structures, the LS analysis is typically done through a deterministic approach [18]. The deterministic approach accounts for the uncertainties of the solution through employment of calibrated partial safety factors, which are prescribed in the current codes and standards [18]. The method addresses the uncertainties in an indirect way. Uncertainties to the solution can occur in material properties, in the fabrication of elements of the structure and in the analysis due to approximate methods [18]. A partial safety factor, usually already set and calibrated by code, considers those uncertainties. Through the use of a partial safety factor, the deterministic approach involves one input of information which produces one output of solution. The input consists of the design value which is calculated by expression (2):

$$X_d = \frac{X_{ck}}{\gamma} \quad (2)$$

where X_d consists of the design value, X_{ck} consists of the characteristic value, γ consists of the partial safety factor mentioned above. Since the partial safety factor is set by code, it is a good solution in design of new structures, but not the best option when assessing existing structures that were not built by code. The uncertainties of those existing structures create variations in the parameters which cannot always be represented by the partial safety factor. In that case, the probabilistic approach is more suitable.

2.6 Probabilistic Approach

The probabilistic approach accounts for the uncertainties of the solution by including randomness in selected model parameters, inputting and outputting probabilistic distribution function (PDF). In that manner, this method addresses the uncertainties of the solution in an explicit way. Regarding the LS equation, see expression (1), both resistance (R), and applied loads (S) consist of statistical distributions.

A PDF is defined as the first derivative of the cumulative distribution function (CDF) [18]. A CDF is defined for both discrete and continuous random variables. It is the total sum of all probability functions corresponding to the values less than or equal to x (the considered variable) (equ (3)) [18]:

$$F_x(x) = P(X \leq x) \quad (3)$$

A PDF for both the load function, S , and the resistance function, R , can lead to the computation of the probability of failure of a structure. The region of interest of both the S , and

R distributions lays in the tails of the functions. In fact, for the resistance distribution, the importance lays in the opening tail, to account for the possible lower values of resistance. For the applied loads distribution, the importance lays in the end tail, to account for the possible higher values of applied loads [18].

When both the resistance and applied load distributions are plotted on the same frequency and value graph, the area under which the two distributions overlap consists of the failure probability (p_f). The value of p_f is very low, to have an easier value to work with and in order to avoid possible numerical handling issues [4], p_f is expressed through reliability index (β). The transposition is done by expression (4) [4]:

$$\beta = -\phi(p_f)^{-1} \quad (4)$$

The probability of failure can be represented in a 3-dimensional space, through a user defined marginal distributions of resistance and applied loads. Both marginal distributions form the joint distribution in the 3-dimensional space. The joint distribution is sliced through with the LS plane, where on of the resulting volumes consists of the probability of failure. Diverse methods exist to determine the value of the failure probability.

The convolution integral consists of an expression which allows to calculate the area located under the overlapping load and resistance PDFs [19] (5).

$$p_f = \iint_{R \leq S} f_{R,S}(r,s) dr ds \quad (5)$$

The computation of the convolution integral may be quite complex, depending on the analytical expression. Therefore, its applicability and resolution is limited to only particular situations even though it is an analytical method which offers an exact solution. In fact, to solve the integral, special numerical techniques must be applied, which may reduce the accuracy in the solution [18]. Therefore, probability of failure, p_f , is usually calculated using other, simpler methods.

Basler Cornel method defines the reliability index as the inverse of the coefficient of variation (CV). In other words, the method defines the reliability index as the shortest distance from the origin of reduced variables of the load and resistance distributions to the Limit State function [18]. The distance, as reliability index, is computed with the following expression (6):

$$\beta = \frac{\mu_z}{\sigma_z} = \frac{\mu_R - \mu_S}{\sqrt{\sigma_R^2 + \sigma_S^2}} \quad (6)$$

Where β consists of the reliability index, μ_z consists of the mean of the safety margin, σ_z consists of the standard deviation of the safety margin, μ_R and μ_S consists of the mean of resistance and applied load respectively, σ_R and σ_S consists of the standard deviation of resistance and applied load respectively. While this method provides an exact solution, the method is only applicable when the variables are normal and the expression itself is linear.

Approximative methods exist to determine the failure probability of the structure; two of which consist of the First Order Reliability Method (FORM) and Second Order Reliability Method (SORM). Both methods attempt to distinguish the shortest distance between the Limit State plane and the center of the joint distribution function, just like the Basler Cornell method [18]. Contrarily to Basler Cornell method, the line is not calculated, it is approximated. To perform the approximation, a linear (FORM method) or parabolic (SORM method) plane is user defined to match as closely as possible the LS plane. The approximation is repeated until the shortest distance is achieved between the approximated curve and the origin of the joint distribution function (**Figure 11**).

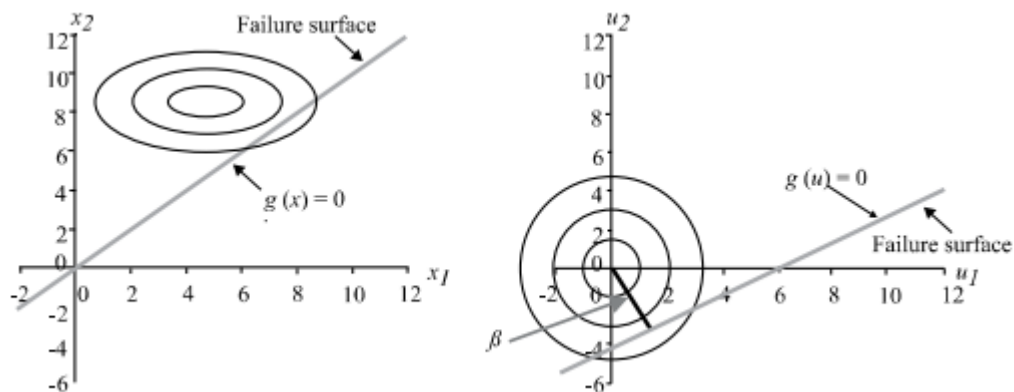


Figure 11 – Two-dimensional case of a linear limit state function and standardised Normal distributed variables U [20]

The probability of failure of a structure can also be achieved through simulation methods. Simulation methods allow to determine the probability of failure of a certain structure without the need of performing physical tests. One of such methods consists of the Monte Carlo Sampling (MCS), also known as crude Monte Carlo Simulation method [18]. MCS involves the simulation of a situation, to obtain a distribution which would encompass all possible solutions. In theory, an infinite number of samples would offer an exact solution. While an infinite sampling is not possible, a larger number of samples increases accuracy, offering an approximative solution. MCS is usually used for three different purposes: i) it is used to solve complex problems such as nonlinear finite element models; ii) to solve problems without the

need of making assumptions; iii) in order to compare results with another solution [18]. Due to the large number of samples, this method requires a large amounts of computation effort. A lot of the computation effort will be redundant since the most frequent samples will be situated around the mean of the distribution. The region of interest when performing LS Analysis consists of the tails of the distribution, not the mean [21].

To reduce the number of computations needed to perform the simulation, Latin Hypercube Sampling (LHS) should be performed [18]. This method decreases the amount of unnecessary overlap of sampling near the mean of the distribution by evenly dispersing the obtained samples. A sample scheme is applied to reduce the number of samples [18]. LHS divides both sampling curves from the resistance distribution and applied loads distribution into equal areas (Figure 12) [22]. From each equal areas of the curves, a grid is derived, and the samples are distributed evenly: one sample per row and column of the grid. This method works best with normal distributions, since symmetry allows for an easier sample scheme grid layout. The robustness of the bridge of Vila Mea will be assessed with this method.

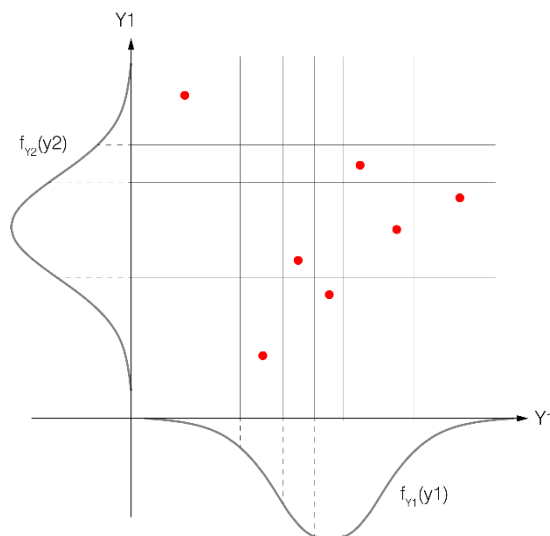


Figure 12 – An example of LHS sampling scheme, where each dot represents one simulation, based on [22]

2.7 Robustness

2.7.1 Importance of Robustness in Masonry Arch Bridges

Retrofitted historical structures are often subdued to larger load capacity than the original intent. In some cases, the historical structure needs to undergo a strengthening intervention, while in other cases the original design suffices to resist the increase in load application. MAB

were designed with the intent to withstand much lower loads than what modern day bridges are designed to resist. Moreover, MABs were rarely retrofitted during their life of service. However, with their original design and years of service, MABs still manage to resist the application of the increased modern loads. Therefore, MAB's visibly display excellent in-service performance [10], Robustness allows existing structure to withstand a new increase in loads without significant damage [1].

2.7.2 Interest in Robustness

The start of interest in the concept of robustness can be visibly noted following the collapse of the Roman Point building in 1968 [1]. The collapse of Roman Point sparked an interest in the relation of proportionality between damage inflicted on a building and consequences resulting from the damage. In fact, the multi-storey building partially collapsed after an oven gas explosion incident destroyed a load bearing wall, which led to a collapse of one side of the overall building [1]. Following that incident, the adopted concept of a robust structure consisted of a structure which experiences proportional consequences as a response to the inflicted damage. In 2001, the collapse of the Twin Towers renewed and increased the interest in correct measures of robustness [23]. This time, the focus was placed on developing more efficient methods to measure robustness.

2.7.3 Definition of Robustness

According to the current code, a structure is considered robust when proportionality exists between damage scenario and consequence [24]. However, robustness is a more complex notion than the proportionality between damage scenario and consequence. Robustness can be defined as the prevention of disproportional damage spreading and propagation into a structure—by eliminating failure consequences following a local damage—rendering a structure collapse resistant. The possible outcomes following a damage occurrence are represented by the event tree (**Figure 13**).

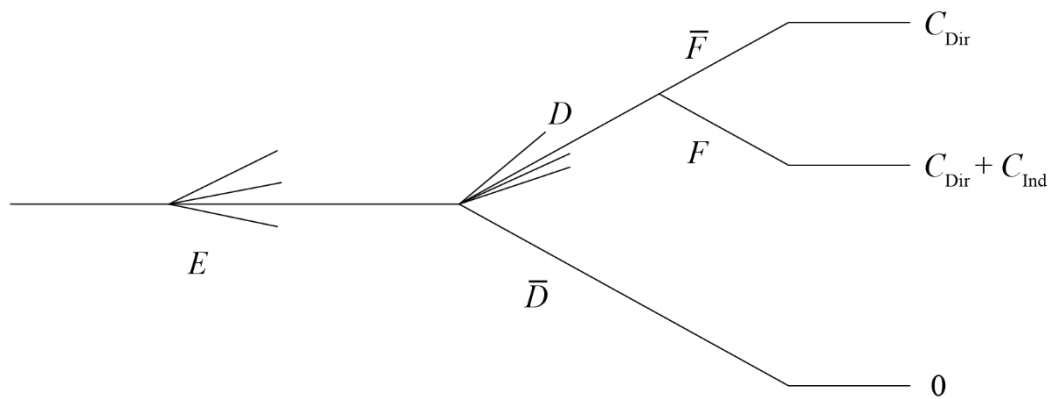


Figure 13 - Event tree [2]

when the exposure (E), occurs, either some damage (D) or no damage occurs (\bar{D}). When no damage occurs, no consequences occur. When damage occurs, either failure (F) or no failure will occur (\bar{F}). When failure occurs, the structure experiences direct (C_{Dir}) and indirect consequences (C_{Ind}). When no failure occurs, the structure experiences direct consequences.

When the total damage resulting from an action is much greater than the initial damage caused by the action, the failure of the structure is classified as disproportionate collapse/damage [1]. Disproportionate damage can be classified as progressive damage or immediate damage [25]. Methods can be used to avoid disproportionate damage; prevention of abnormal events, prevention of occurrence of initial damage which would occur in consequence of abnormal events, and prevention of disproportionate spreading of failure if an initial damage occurs [25].

It is important to note that local damage is acceptable, as long as it does not endanger stability of the whole structure [1]. By analyzing the event tree in **Figure 13**, robustness is defined as the prevention of disproportional damage spreading [25]. However, it must be noted that robustness is much more than proportionality between damage and consequence. It is a component which is related to *Redundancy*, *Vulnerability* and *Progressive Collapse*.

2.7.3.1 REDUNDANCY

Through redundancy, robustness can be understood as the prevention of damage propagation. Some authors interchange redundancy with robustness, defining redundancy as the ability to resist collapse following the failure of any single member of the structure [26]. Redundancy can be achieved, amongst others, by absence of critical components whose failure would cause collapse of the structure [27]. A non-redundant structure is statically determinate,

while a redundant structure is statically indeterminate [27]. While the degree of redundancy allows to determine the degree to which the structure is statically determinate, redundancy index allows to measure redundancy (and is required in calculation of the robust index). Redundancy is also related to damage propagation [28]. Damage propagation dictates the involvement of local damage or failure of a member in the collapse of the whole structure [28]. Agency based mechanism consists of the mechanism in which damage propagates from the local damaged member to the other members which are connected to the damaged one [28].

It is important to note that, while robustness is closely related with redundancy, a robust structure does not automatically classify as redundant [29]. Two examples can be observed, Ballerup Super Arena (a robust, yet not redundant structure), and Bad Reichenhall Icehall (a not robust, and not redundant structure) [29]. Ballerup Super Arena contained a roof which was made of a robust system, since it limited the propagation of damage from the initially damaged elements to the rest of the roof structure. However, the roof collapsed due to a human error in the design of the joints in the trusses of the roof [29]. On the other hand, Bad Reichenhall Icehall, collapsed progressively, the damage propagating from the initially damaged elements into the rest of the roof [29].

2.7.3.2 VULNERABILITY

By defining vulnerability, robustness can be explained as the immediate damage tolerance. Vulnerability is known as the reciprocal of damage tolerance, defined as the probability of withstanding failure [30]. Vulnerability is dependant on reliability. In fact, when reliability decreases, (and number of components in a structure increases), vulnerability increases [30]. Therefore, when reliability increases, (and number of components decreases), robustness increases. Vulnerability (V) may be denoted as (7) :

$$V = 1/P_0 \quad (7)$$

where P_0 is defined as probability of failure when no damage has occurred [30] (if the system is damaged following the failure of one component, $P_0=1$).

Following the statement that robustness can be denoted as the opposite of vulnerability, robustness can be defined, in that case, as directly proportional to P_0 (probability of failure). A structure that has a lesser probability of failure is deemed more robust.

2.7.3.3 PROGRESSIVE COLLAPSE

According to the American Society of Civil Engineers: “Progressive collapse is the spread of initial local failure of a member” [10]. Progressive collapse is the result of disproportionate collapse, where damage begins from a local damaged member but triggers successive failures in the rest of the structure [25]. Progressive damage is dependant on collapse resistance, while collapse resistance is dependant on robustness (**Figure 14**).

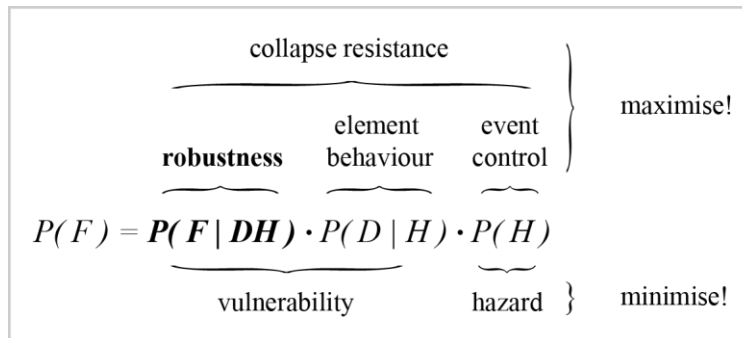


Figure 14 - Terms in context of progressive collapse [31].

2.8 Means to calculate robustness

2.8.1 Deterministic

2.8.1.1 STIFFNESS-BASED ROBUSTNESS METHOD

The stiffness-based method is an easy-to-calculate method to calculate the robustness index. However, the method is only appropriate to measure robustness for zipper-like collapse; it is unsuitable for pancake-like or domino-like collapse. Zipper-like collapse occurs when a single load bearing member fails, the force is redistributed to the members oriented transversely to the failure direction, and the resistance of those transversal members is exceeded [32]. Pancake-like collapse occurs when the upper part of structure fails, accumulating kinetic energy, that is then forced upon the floor below, that cannot resist the oncoming energy [32]. The collapse continues from one floor at a time, from upper level to lowest level [32]. Domino-like collapse occurs with initial overturning of one element, that damages or destabilizes subsequent members [32]. The stiffness-based method lacks expressiveness [31], which is an essential concept in pancake-like or domino-like collapses. Disregarding expressiveness, the method focuses on the static load redistribution capability of the structure. Stiffness-based robustness index is measured using the following expression (8):

$$R_s = \min_j \frac{\det K_j}{\det K_0} \quad (8)$$

where R_s consists of the stiffness-based robustness measure, K_0 and K_j consist of the active system stiffness matrix of the intact structure, and after removal of element respectively.

2.8.1.2 DETERMINISTIC BASED REDUNDANCY

Deterministic Based redundancy consists of a structural measure of the structure. It is calculated using the following equation (9):

$$RIF_f = \frac{RSR_{fail,f}}{RSR_{intact}} \quad (9)$$

Where RIF_f consists of Damaged Strength Ratio, RSR_{intact} $RSR_{fail,f}$ is the RIF value of the intact structure and of the structure where a member has failed. The RIF values range between 0 to 1, where 0 indicates a least robust structure, 1 indicates a more robust structure [2].

2.8.2 Probabilistic

2.8.2.1 DAMAGE-BASED ROBUSTNESS METHOD

The damage-based method measures robustness by incorporating expressiveness and quantification of damage propagation. Expressiveness of damage progression refers to the assumed initial damage and acceptable progression in terms of the structure and environment. Three damage progression scenarios exist; a not robust building (A), a medium robust building (B), and a most robust building (C). The scenarios are plotted on an initial damage-progressed damage graph [31].

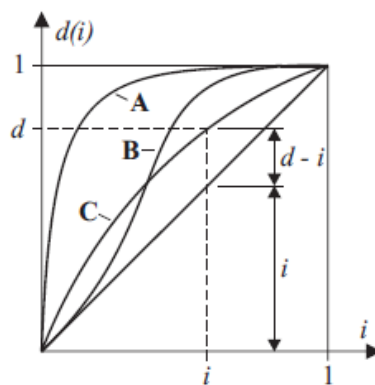


Figure 15 - Damage-Progressed Damage Graph [31]

Damage-based method measures robustness with the following expression (10):

$$R_d = 1 - \frac{p}{p_{lim}} \quad (10)$$

where R_d consists of damage-based robustness measure, p consists of the maximum damage progression caused by assumed initial damage (i_{lim}) and p_{lim} consists of the acceptable damage progression [31].

2.8.2.2 PROBABILISTIC BASED REDUNDANCY

Probabilistic based redundancy focuses on the probability of failure of the structure. Probabilistic procedures require high computational resources, which is a drawback. A better option consists of gathering the most important parameters, which reduces the number of variables, which in turn has a lower computational effort [23]. Probabilistic Based Redundancy is calculated with the following equation (11) [2]:

$$RI = \frac{P_{f(damaged)} - P_{f(intact)}}{P_{f(intact)}} \quad (11)$$

Where RI consists of redundancy index, $P_{f(damaged)}$ and $P_{f(intact)}$ consist of probability of failure of damaged and intact structure [2]. The index varies from 0 to infinity, with 0 indicating larger robustness, while infinity indicating lower robustness [2].

2.8.3 Risk Based

2.8.3.1 ENERGY-BASED ROBUSTNESS METHOD

The energy-based method focuses on initial energy level for fracture of a structural element (which is still available for the fracture of the next structural element), and the energy required for failure of the next element. The method is not only easy to calculate; it is also expressive. While this method does not consider acceptable damage progression, it verifies the possibility of total collapse. The method is not always applicable, since the initial energy level for the fracture of a structural element can be easily over or under estimated. Therefore, the method is best used for pancake-like and domino-like collapses [31]. The formula used for the energy-based method is as follows (12):

$$R_e = 1 - \max_j \frac{E_{r,j}}{E_{s,k}} \quad (12)$$

where R_e consists of the energy based robustness measure, $E_{r,j}$ consists of the energy released by the initial failure of a structural element j —available for the damage of the next structural element, and $E_{s,k}$ consists of the energy required for failure of the next structural element [31].

2.8.3.2 RISK BASED ROBUSTNESS

Risk based robustness divides consequences following the damage between direct and indirect risks. It is calculated with the following equation (13):

$$I_{rob} = \frac{R_{Dir}}{R_{Dir} + R_{Ind}} \quad (13)$$

Where I_{rob} consists of robustness index, R_{Dir} and R_{Ind} consist of direct and indirect risks [2]. The index value varies from zero to 1, where zero indicates no robustness, while 1 indicates a robust structure [2]

2.8.4 Degree of Redundancy

Degree of redundancy allows to determine to what degree a structure is statically determinate [27]. It can be represented by the following equation, where R_1 consists of the degree of redundancy, E consists of the number of independent equilibrium equations, and F consists of the number of unknown reactive forces [27](14):

$$R_1 = F - E \quad (14)$$

2.8.5 Frangopol and Curley

During the 1980's Frangopol and Curley [23] analyzed the effects of damage with deterministic and probabilistic measures. Frangopol and Curley suggest that redundancy is a representative parameter of robustness, suggesting that robustness is explicitly translated by redundancy in the following equation (15) [23]:

$$R = \frac{L_{intact}}{L_{intact} - L_{damaged}} \quad (15)$$

where R consists of robustness, and L_{intact} and $L_{damaged}$ consists of load capacity of the intact and damaged structure. R varies from 1 to infinity, with the structure offering no resistance

when R equals to 1, and the structure experiencing no loss in its capacity to withstand damage when R approaches infinity [23].

2.8.6 Cavaco

In the 2010's Cavaco describes robustness as the representation of tolerance to damage, as opposed to previous statements which regarded robustness as the representation of the response to damage [23]. Cavaco states that robustness depends on the most likely occurrence of damage. Robustness is represented as a function of a performance indicator [23] (16):

$$R = \int_{D=0}^{D=1} f(x) dx \quad (16)$$

Where R consist of robustness, D consists of damage (where 0 consists no damage, and 1 consists fully damaged), $f(x)$ consist of the function describing the behavior of the structure's resistance. Robustness varies from 0 to 1, when nearing 0 representing no robustness, while approaching 1, representing maximum robustness [23]. In fact, three possible scenarios emerge (in graph bellow), consisting of least robust structure (left) to most robust structure (right). Cavaco's approach allows to compare structures with different levels of robustness which have been exposed to the same amount of damage (**Figure 16**).

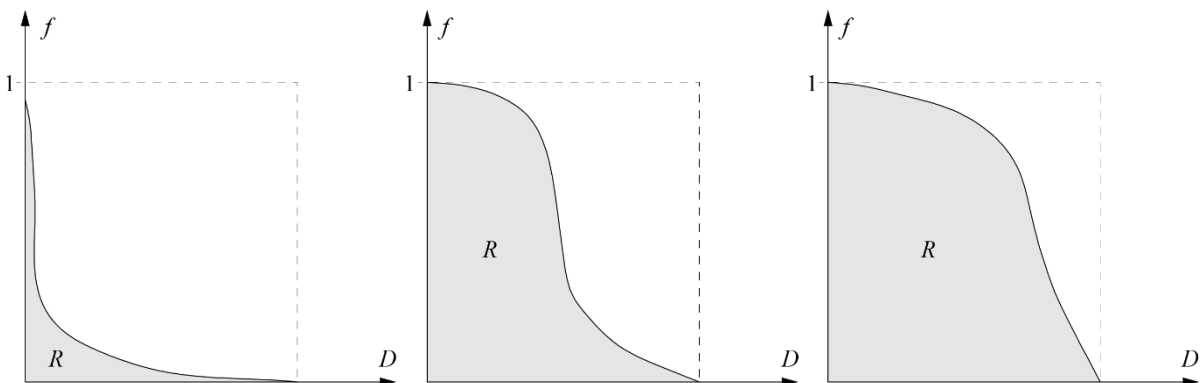


Figure 16 - Standard curve of robustness adapted from Cavaco [23]

3. THE CASE STUDY OF VILA MEA BRIDGE

3.1 History

Vila Mea bridge is situated in Vila Mea (**Figure 17**), district of Porto, Portugal. The viaduct was built as part of the Douro line in 1878 [33]. It is situated at 49.9 kilometers from the start of the Douro line. Vila Mea bridge consists of a granite stone masonry bridge, with single row masonry rings on the arch barrels.



Figure 17 - Vila Mea Bridge [21]

3.2 Geometry

As stated in **Chapter 2** the structural behavior of a MAB is dependant on its geometrical and material properties. When performing a numerical analysis, the first task to be performed consists of the geometric characterization of the structure. While conventional measuring methods have been applied to capture the geometrical parameters of structures over the years, technological advancements allow for newer, possibly more effective modern methods. Most common modern measuring methods used to capture the geometry of a structure include the computation of photogrammetric models as well as scanning the structure with 3D laser scanners [34]. It is best to collect data from various sources of information, and then conduct a comparison between the data to ensure greater accuracy. For inaccessible geometrical parameters, non-destructive testing (NDT) should be employed. NDT include, amongst others, geo-radar, sonic tomography or endoscopy [35] [11]. If NDT is not available, consultation of

the design project, maintenance reports, in-situ surveys or even consultation of NDT performed on similar structures should be performed.

The geometry parameters of the Vila Mea bridge were based solely on the drawings of the bridge of 1930. The bridge displays a total length of around 165 meters and a width of 5 meters. The bridge bears 5 piers along its length, with an arch barrel in between each pier. The piers vary in height from 11.5 to around 12 meters. The number of stone blocks forming the piers varies between 34 to 36 blocks. It is important to note that the bridge is symmetrical, with the middle pier being the tallest, and the two edge piers being the shortest. The width of each of the piers equals 3.5 meters, while the height of the abutment over each pier equals 5 meters. Each of the arch barrels between the piers is 21 meters wide. The rise at mid arch consists of around 10.5 meters. The single ring of each arch is made up of 127 stone blocks of a thickness of 1.1 meters. Around 1 meter of backfill exists over each of the arches. On top of the backfill, there is another 0.9 meter of bearings (ballast and track). The crucial geometrical parameters for the modeling of the bridge are displayed in **Figure 18**, while the crucial parameters and corresponding means (μ) and CV required for the development of the numerical model are displayed in **Table 1**.

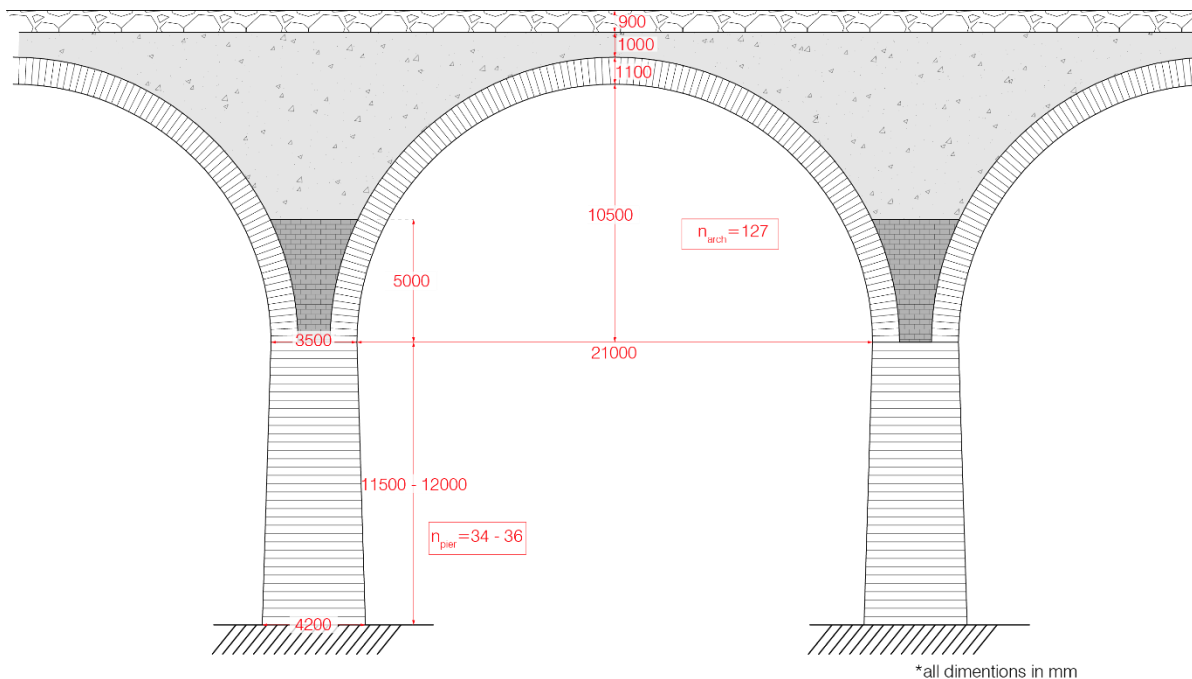


Figure 18 - Crucial geometrical parameters of the Vila Mea Bridge

Table 1. Geometry of Vila Mea bridge

ELEMENT	PARAMETER	PDF	μ	CV (%)
Backing	Height, h_h (m)	Normal	5,00	5
Piers	Thickness, t_p (m)	Normal	3,50	5
Arch	Thickness, t_a (m)	Normal	1,10	10
Ballast	Height, h_b (m)	Normal	0,90	10
Fill material	Depth at crown, h_f (m)	Normal	1,00	10

3.3 Material Properties

As mentioned several times, material property of a MAB along with its geometrical properties are necessary for the construction of a numerical model of the MAB. Contrarily to modern bridge design, which is code-based, old MAB are constructed with uncontrolled and not manufactured material. Therefore, properties of the material are unknown, resulting in the need of tests to define material properties. A range of various destructive, semi-destructive and non-destructive techniques are available to collect material data of masonry [11]. Each method is most appropriate for specific characteristics of the material. For example, to obtain the masonry's compressive strength, the most proper approach consists of the destructive uniaxial compressive tests [11]. Destructive tests are unfavorable, but may be necessary when no other alternative in masonry characterization. Furthermore, sometimes the performance of tests cannot be performed due to large costs or inaccessibility to the structure. In that case, material data is based on literature and engineering judgement [36].

Generally, masonry is known to be heterogenous, composed of masonry units and mortar joints. Masonry displays brittle behaviour resulting from moderate-to-high compressive strength and low-to-null tensile strength. Concerning the modeling of any masonry structure, the most relevant material property of masonry consists of the compressive strength normal to the bed joints due to its compressive properties, once these structures are conceived to be under compressive stresses. As mentioned in **Chapter 2**, one of the most common failure mechanisms of a MAB consist of the development of plastic hinges in the arches of the bridge. The formation of plastic hinges (as seen in **Figure 2** from **Chapter 2**) in an arch is directly dependant on the compressive strength normal to the bed joint of masonry.

Vila Mea bridge consists of a stone masonry construction. The piers and spandrel walls are made of granite stone and mortar. The infill consists of consists of agglomerates of various particle size materials placed over the masonry vaults [36] [37]. Material property and in-situ

testing were out of the scope of the present work; but have not been performed so far due to inaccessibility to the site, therefore material data is fully based on literature as well as engineering judgement. The following material properties are assumed (Table 2).

Table 2. Material Parameters for Vila Mea Bridge [36] [38] [39]

ELEMENT	PARAMETER	PDF	μ	CV (%)
Masonry	Density, γ_m (kN/m ³)	Normal	25,00	5
	Compressive strength, f_c (MPa)	Normal	8,00	20
	Friction coefficient, μ (-)	Log-Normal	0,58	20
Fill material	Density, γ_f (kN/m ³)	Normal	20,00	10
	Friction angle, ϕ (°)	Normal	30,00	20
	Cohesion, c (kPa)	Log-Normal	0,00	40
Ballast	Density, γ_b (kN/m ³)	Normal	18,00	10
Track	Track load, T_l (kN/m ²)	Normal	1,49	10

3.4 Numerical Modeling

3.4.1 Assessment of Masonry Arch Bridges

Many old MAB are continued to be used in the modern era. Not only are they continuously in use, but the loads that the bridges are subjected to surpass the magnitude of load to which the bridges were originally designed to withstand. However, all structures deteriorate with time—with structure deterioration, the load carrying capacity is reduced. Therefore, as old MABs become older with time, their condition deteriorates. To ensure safety of MABs it is necessary to complete an assessment of the MAB in question. The assessment ensures that the load capacity of the MAB can withstand current and future applied loads, while maintaining serviceability. Currently, the assessment of a masonry arch bridge has no widely accepted structural assessment procedure. However, the assessments proposed require a numerical model.

3.4.2 Modelling a MAB

The geometrical and material parameters are crucial elements in numerical modeling the bridge Vila Mea. Several options exist in the type of model to be constructed. The options depend on the requirements as well as timing necessary to conduct the requirements. The modeling options also depend on the type of analysis conducted. As discussed in **Chapter 2**, the possible analyses include: i) Thrust Line Analysis method ii) Limit Analysis method; iii) Finite Element Method (FEM); and iv) Discrete Element Method (DEM) [36]. The Limit

Analysis method is the method used to assess the bridge of Vila Mea, due to the ability to control the complexity of the model with the amount of input parameters [40].

3.4.3 Available Software

There exist many different software available to compute an assessment of a MAB. The choice in software used largely depends on the type of analysis conducted. While it could have been possible to apply the FEM for the bridge of Vila Mea, the Limit Analysis method is chosen instead due to its lower computation requirements. DIANA is a software which could have been used to perform the FEM analysis [41]. However, since a limit state analysis is performed, the software LimitState RING is used instead [3].

As described in **Chapter 2**, FEM is a numerical method which allows to perform a structural assessment in a 3-dimensional, non-linear manner [4]. The method consists of a division of a structure into smaller sections, called *finite elements*. A greater number of finite elements allows for greater accuracy in the solution [4], but also a greater computational effort.

In the case of robustness assessment of the bridge of Vila Mea, each damage simulation requires 100 simulations to account for the uncertainties, based on Schuyler rule, with an overall average error of 10%, and an uncertainty of 1% [42]. The global average error is computed using the average of the CV of several PDF associated with critical parameters, and the uncertainty value obtained guarantees reliability in the obtained results [13]. As a result, the tests on the numerical model must be performed 100 times for each damage. Each of the tests performed require greater computational effort and time compared to simpler software, such as LimiteState: RING. Due to the large number of tests required, DIANA is not used for the analysis of the bridge of Vila Mea.

3.4.3.1 LIMIT STATE: RING software

LimitState: RING [3] is a software meant to perform analysis specifically for MAB [3]. The main purpose of RING is to analyze the ultimate load capacity of MABs. The software is largely based on the formation of plastic hinges as failure mechanism, a method of analysis pioneered by Heyman [3], who's theory is discussed in **Chapter 2**, RING simplifies the numerical model of the bridge, by incorporating its 3D aspects into a 2D model. RING software creates a MAB model as an in-plane assemblage of rigid units, separated by interfaces along the masonry joints, presenting a rigid-plastic constitutive behavior [36]. The spandrel walls are

not explicitly modeled into the numerical model. Instead, the spandrel walls are modelled indirectly, through horizontal passive pressure [3]. Furthermore, the fill's influence on the structure is also indirectly modeled, through density, dispersion of the loads applied at the surface as well as a reduction in horizontal passive pressure [3]. In fact, the passive pressure provided by the fill is based on the Rankine theory. It is important to note that passive pressure is only fully exerted on high displacements of section of arch. To take that into account, the passive pressure is reduced by 50% [3]. The live loads applied to the model are dispersed in both the longitudinal and transversal direction. In both cases, the model considers the Boussinesq theory when distributing the loads through the elements of the MAB. The Boussinesq theory is based on performed laboratorial tests—through which a limiting distribution angle of 30 degrees was determined to be correct [3]. The possible failure mechanism of MAB through RING software include collapse due to crushing, sliding along interfaces, or a combination of both failure mechanisms [36].

3.5 Model of the Bridge of Vila Mea

Numerical models allow to simulate damages in existing structures, while leaving the existing structures intact. Therefore, to perform a robustness assessment, the bridge of Vila Mea required to be numerically modelled. Due to the nature of the assessment, further discussed in **Chapter 5**, and following a comparison of the task to be performed to similar tasks completed by professional engineers [36], it is decided that LimitState: RING should be used to perform the robustness assessment of the bridge of Vila Mea. In fact, LimitState: RING consists of a reliable and simple software. The software requires a limited number of parameters; namely the geometry of the bridge, material property, and load cases applied. The geometry of the bridge and material properties are discussed in **section 3.2** and **section 3.3**. The load cases applied are described in **section 3.6**.

The Vila Mea bridge is modeled in its entity. The bridge is specified as a railway bridge, with an effective width corresponding to the obtained width of the bridge, namely 5 meters. All the partial factors of safety applied to loads affecting elements of the bridge are set as 1, since the uncertainties of the bridge are considered during the reliability based assessment. The spandrel walls are not modeled. The backfill is modelled by filling elements, through the consideration of the generated passive horizontal pressures by the arch. Each of the arches are modeled as segmental in shape. The resulting model is represented in **Figure 19**.

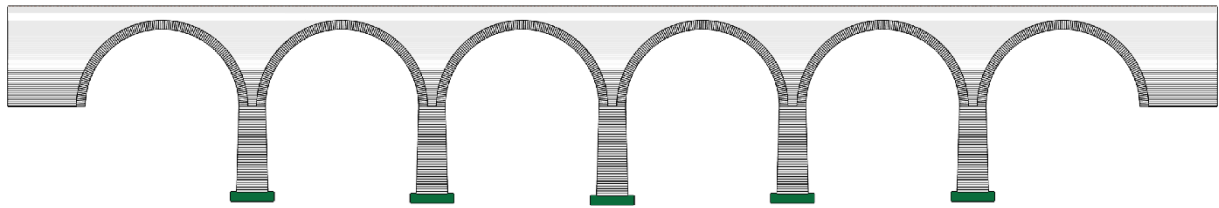


Figure 19 - LimitState:RING numerical model

3.6 Load Model

There are many purposes to building numerical models. In the case of the bridge of Vila Mea, the numerical model is constructed to determine the ultimate loading capacity of the structure. Since the bridge is currently in use as a railway bridge, it is subjected to train traffic. In that case, the train traffic to which the bridge is exposed should be quantified. To quantify the overloading of the bridge of Vila Mea, the European standard was consulted [43]; it is determined that the load model LM71 is composed of four concentrated forces and two uniform distributed forces [36], see **Figure 20** [43].

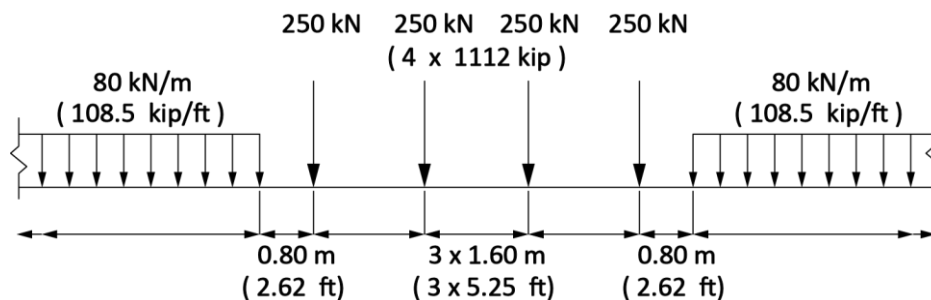


Figure 20 - Load model 71 (LM71) [43].

Both uniformly distributed loads are optional, whether designing or assessing a bridge. In fact, the two uniform distributed loads may yield favorable effects, which is not a desired effect when testing the safety of MAB. In fact, the effects of the two uniform distributed loads on MABs have been observed by Santis [44]. Santis [44] tested three load combinations; i) four concentrated loads and no uniform distributed load, ii) four concentrated loads and one uniform distributed load, and iii) four concentrated loads and two uniform distributed loads. From those tests, Santis [44] determined that the two distributed loads increase the compression exerted on the arch. This additional compression prevents the formation of the collapse mechanism of the arch which, in absence of the two uniform loads, would be created when parts of the arch move upwards [44]. Therefore, when performing a safety assessment of a MAB, the model used should only compromise the four concentrated loads for a more effective assessment.

Consequently, the load model used in the robustness assessment of the bridge of Vila Mea consists of the LM71 model, without the two uniformly distributed loads (**Figure 21**).

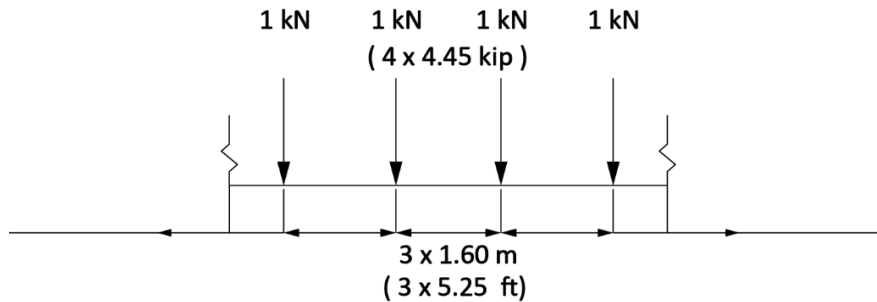


Figure 21 - Modified Load Model 71

The load model is used to simulate a train moving across the bridge of Vila Mea. In order to mimic the movement of a train across the bridge, the load model must be applied multiple times in separate load cases. The position of each load cases is moved from the last one with a distance from its predecessor called spacing. The spacing is determined through the length of the bridge and the amount of computational effort desired. A spacing that is too small would require a large amount of load cases applied on the bridge to fill its full length. With a large number of load cases, the safety assessment of the bridge would require large computational effort, with little benefit in the result. For the assessment of the bridge of Vila Mea, engineering judgment and consultation of previous works [36] lead to a decision to set the spacing of each load case as 250 mm, resulting in a total 654 load cases filling the length of the bridge. At each new position of the load application, a collapse load factor is computed. The lowest value from each of the 654 load cases applied on the bridge of Vila Mea consists the adequacy factor (failure load factor) of the bridge.

3.7 Dynamic effects

Serviceability of a MAB may not only be reduced through failure due to loading, but may also be reduced through failure due to dynamic effects. In fact, a MAB offering high load capacity may fail due to its sensitivity to vibrations. Vibrations of the structure can lead to dismantling of masonry, joint opening or arch barrel cracking [36]. Such occurrences in a MAB can lead to an implicit reduction of the load capacity of the bridge. With movement of traffic over the bridge, as well as environmental factors, such as strong winds, contribute to the dynamic effects exerted on the bridge of Vila Mea. The most significant dynamic effects exerted on a railway MAB are caused by the traffic of the trains moving along the bridge. Such

movement in load application results of an amplification of the structural response compared to a static loading [36].

To counter the amplification of the structural response due to the moving loads, in the assessment of railway MAB, were designed to resist the live loads are increased by an impact a dynamic factor [36]. Recent codes of practice address problem of dynamic effects by recognizing that the resonance of a bridge may occur due to the train or bridge characteristics, as well as irregularities found in the train tracks [36]. Therefore, it can be stated that the most significant parameters dictating the dynamic effects on MABs consist of the frequency and damping of both the bridge and the train, the train's velocity as well as the track irregularities [43].

In the case of the assessment of the bridge of Vila Mea, the dynamic effects considered, EN 1991-2 [43] is considered to quantify the static response amplification due to dynamic effects, through the use of a dynamic factor. The dynamic factor is obtained through expression (17) [43].

$$\phi_3 = \frac{2.16}{\sqrt{L_\phi} - 0.2} + 0.73 \quad (17)$$

Where ϕ_3 consists of the dynamic factor, and L_ϕ consists of the determinant length. According to EN 1991-2 [43], the determinant length for MAB is twice the clear opening between arches. In the case of the bridge of Vila Mea, the dynamic factor is computed in (18)

$$\begin{aligned} \phi_3 &= \frac{2.16}{\sqrt{42} - 0.2} + 0.73 \\ \phi_3 &= 1.07 \end{aligned} \quad (18)$$

Additionally, in the case of MAB, a reduction factor may be applied in order to reduce the dynamic effects [43]. The reduction factor is obtained through expression (19).

$$red\phi_3 = \phi_3 - \frac{h - 1}{10} \quad (19)$$

Where h consists of the height of the cover, in meters including the ballast from the top of the deck to the top of the sleeper or, in case of arch bridges, from the crown of the extrados. It is important to not that $red\phi_3$ must be greater or equal to 1.0 for the dynamic factor to be

considered. In the case of the bridge of Vila Mea, $red\phi_3$ is lesser than 1.0, therefore it is not applied during the assessment of the bridge (20).

$$red\phi_3 = 1.07 - \frac{1.9 - 1}{10} \quad (20)$$

$$red\phi_3 = 0.98$$

3.8 Probabilistic based assessment

Current codes state that structural assessments must take uncertainties into account [18]. Two methods exist that take the uncertainties into consideration; the use of calibrated partial safety factors or the inputting and outputting PDF which take into account the randomness in selected model parameters. The use of partial safety factor is applied in a deterministic approach, while the use of PDFs is applied in a probabilistic approach. Both methods are described in **Chapter 2**. Since the deterministic approach obtains calibrated partial safety factors based on current design code, the approach is not always the most effective tactic to be used when assessing older MAB, since those bridges were not design through current code. Furthermore, the uncertainty predicted by codes may differ from the one of the assessed structure. Therefore, the probabilistic based approach is used instead, as it is a more suitable approach that considers uncertainties in older MABs.

To perform a probabilistic analysis of the bridge of Vila Mea, the basic variables involved are statistically defined first. In the case of the bridge of Vila Mea, the basic variables are based on available literature [8] [36] [45]. Variability exists in the geometry of the bridge, the material properties of the bridge as well as the loading of the bridge [46].

3.8.1 Geometrical Uncertainties

In the geometry of a MAB, the most representative parameters consist of the arch thickness, fill material height in the crown zone of the arch, the arch deflection, the span length and piers geometry [36]. While the shape of a segmental arch is also important, it is considered through an implicit way [36]. Pier geometry is also an important parameter, dictating the spreading of the damage. In fact, a slender pier allows for easier damage propagation compared to a stockier pier. Therefore, a slender pier tends to lead to a global collapse, while a stocky pier tends to lead to a local collapse [36].

While some sources, such as the Probabilistic Model Code (PMC) [46], state that geometrical uncertainties can be neglected when compared to other parameters, it is not the case with MAB assessment. First, the geometrical parameters are based on drawings of the bridge from 1930. The dimensions on the drawings may not be accurate, or may be outdated. Secondly, variability in geometry—a reduction of the cross section of the arch, for example—may be crucial in dictating the structural response of a MAB. Therefore, it is of utmost importance to consider geometrical uncertainty when assessing MAB [4] [38] [39].

By consulting measurements performed by several authors [8] [47] [45], it has been confirmed that the use of a CV of 10% is a common approach to consider the uncertainty in the thickness of the arch—the most influential parameter. Regarding the rest of the parameters, previous examples lead to CVs determined through engineering judgement, since there is limited literature that covers the CVs of other parameters. The CVs for all of the geometrical parameters considered during the assessment of the bridge of Vila Mea are presented in **Table 1**, in **section 3.2**.

3.8.2 Material Uncertainties

Material properties most representative of material strength of masonry arch bridges (MAB)s consist of : i) masonry density, γ_m ; ii) masonry compressive strength, f_c ; iii) masonry tensile strength, f_t ; iv) cohesion of the masonry mortar-interface, s_0 ; v) fill density, γ_f ; vi) fill internal friction angle, ϕ [8] [45]. Most of these material properties are presented in **Table 2** from **section 3.3**. LimitState:RING [3], the software used for the assessment of the bridge of Vila Mea, follows an updated version of the Heyman approach [36] as explained in **Chapter 2**. By following the Heyman's approach, LimitState:RING considers null masonry tensile strength, f_t , and considers no cohesion of masonry mortar-interference, s_0 . Therefore, the two parameters were not considered for the assessment of the bridge of Vila Mea. Additionally, following experience and results obtained by material characterization measurements, it is decided to add more representative material properties, namely; i) masonry friction coefficient, μ ii) fill friction cohesion, c ; iii) ballast density, γ_b , iv) Track load, T_l .

The material properties used for the bridge of Vila Mea are based on literature and data obtained from similar bridges [10] [38], since the possibility of material testing of the bridge of Vila Mea was not possible. Material variance, as well as material randomness, is considered

through the computation of PDF. The values considered for each material parameter, along with the CV for each parameter are defined in **Table 2** from **section 3.3**.

3.8.3 Loading Uncertainties

As mentioned in **section 3.6**, the numerical model of the bridge of Vila Mea is loaded with the Load Model 71 (LM71) based on the Eurocode [43]. The model is not only employed in the design of new bridge, but also for the assessment of existing bridges. However, a characteristic value of load magnitude must be defined for the assessment of the bridge of Vila Mea, since a PDF must be computed. The characteristic values for load magnitude are defined for the 98th percentile of a normal PDF, for a return period of 50 years [48], through which the value of 207.40 kN is obtained. To determine the CV to be used for the load PDF, previous works are consulted through which a CV of 10% is obtained [49]. The resulting load curve parameter is represented by expression (21) [7].

$$LM71 \sim N(207.40; 20.74^2) \quad (21)$$

Based on the expression above, a load PDF for the bridge of Vila Mea is computed. The load PDF takes into consideration the randomness in loading which occurs, following the probabilistic approach for the bridge assessment.

3.9 Methodology in Probabilistic Based Assessment for the Bridge Vila Mea

The probabilistic approach is used instead of the deterministic approach for the robustness assessment of the bridge of Vila Mea since the probabilistic approach considers the uncertainty of the involved variables—geometry, material, loading—in an explicit manner. The probabilistic approach is fully described under **Chapter 2**. To assess the robustness of the bridge of Vila Mea, the reliability index of the bridge, through failure probability, is computed first. A probabilistic approach in computation of the reliability index allows to consider the basic nature and randomness of variables which influence the resistance of the masonry arch bridges [36].

By taking into consideration the randomness in the response of the bridge, a PDF is computed for the structural resistance of the bridge of Vila Mea. A PDF is also computed for the load application onto the bridge of Vila Mea. As described in **Chapter 2**, when both the resistance and load PDF are plotted on the same graph, the area under which the two PDF

overlap consists of the failure probability. Through the failure probability, the reliability index can be obtained [18].

As mentioned in **Chapter 2**, several methods exist in order to compute the PDF for both the resistance and the load of a MAB. Since the common method of MCS requires great computational efforts, a sensitivity analysis was performed in order to distinguish which considered parameters are most influential in terms of the structural behavior of the bridge of Vila Mea. The most influential parameters were then used during the computation of the resistance PDF. After performing the sensitivity analysis, the probabilistic application for the computation of the resistance PDF is performed through the method of LHS. As described in **Chapter 2**, LHS is a variant of the MCS. LHS method applies a sample scheme to the data, that reduces the number of samples, which in turn reduces the amount of computational effort. Therefore, LHS was used for the resistance PDF of the bridge of Vila Mea, since it allowed for a computation of accurate results with a lower number of samples. Since the variables of the resistance and load PDFs for the bridge of Vila Mea were Normal, and the expression was linear, it has been decided to apply the Basler Cornel method, expression (6) from **Chapter 2**, to calculate the reliability index based on the characteristics of the PDFs obtained.

4. SENSITIVITY AND RELIABILITY ANALYSIS

4.1 Sensitivity Analysis

4.1.1 General Methodology

A probabilistic approach in robustness assessment of any structure may prove to require a lot of computational effort, depending on the method used. There are methods—regarding the computation process of the solution—that reduce the amount of computational resources. However, the computational effort will also be reduced when the number of variables considered is reduced. A sensitivity analysis permits to identify which parameters of the problem are most critical in the overall structural response [50] [51]. The analysis should be preceding the LHS application, to limit the number of unnecessary overlap of samples at the mean of the solution. The computation of the importance measure of each parameter is achieved through the following equation [50] (22) :

$$b_k = \sum_{i=1}^n \frac{\Delta y_{i,k}}{y_{m,k}} / \frac{\Delta x_{i,k}}{x_{m,k}} \cdot CV \quad (22)$$

Where b_k consists of the importance measure of parameter k , $\Delta y_{i,k}$ consists of the variation in structural response due to a deviation of $\Delta x_{i,k}$ in relation to the parameter mean value $x_{m,k}$, $y_{m,k}$ consists of the average response and n is the number of generated parameter values. The obtained values of importance measure, b_k , should then be standardized to the highest importance measure achieved. An importance measure limit, b_{lim} , is defined by the user. The parameters which are greater than the importance measure limit, b_{lim} , are considered to be critical [36].

This method is applied for the bridge of Vila Mea; by performing a sensitivity analysis a sample scheme is applied to the data, reducing the number of samples required to complete the assessment. The sensitivity analysis determines which parameters are most influential during the structural response of the bridge of Vila Mea [50] [51]. The random variables used for the sensitivity analysis consist of both the geometrical and material property parameters presented in **Table 1** and **Table 2** from **Chapter 3**. Each parameter is defined through consultation of previous work presented by several authors [8] [45] [47], as well as engineering judgement. The variables used for the sensitivity analysis are summarized again as: i) backing height, h_h ; ii) piers thickness, t_p ; iii) arch thickness, t_a ; iv) ballast height, h_b ; v) depth at crown, h_f ; vi) masonry density, γ_m ; vii) masonry compressive strength, f_c ; viii) masonry friction coefficient, μ ; ix) fill material density, γ_f ; x) fill material friction angle, ϕ ; xi) fill material cohesion, c ; xii)

ballast density, γ_b ; xiii) track load, T_l . Each of the listed parameters is assigned a specified mean (μ) and coefficient of variance (CV).

4.1.2 Deterministic Model

In order to perform a sensitivity analysis, it is necessary to build a deterministic numerical model of Vila Mea. The numerical model of the bridge is then subjected to the loading described in **Chapter 3**; the load model LM71 is applied in succession, with a spacing of 250 mm, throughout the length of the bridge. At each load case, the collapse load factor is defined, with the lowest consisting of the adequacy factor for the bridge. In the case of the bridge of Vila Mea, the adequacy factor is determined to be 6.37 ($207.41\text{kN} * 6.37 = 1322.14\text{kN}/\text{single load}$). Thus, each pointed load is multiplied by this value, resulting a total failure load of 5288.56kN ($1322.14\text{kN} * 4$).

This signifies that the bridge of Vila Mea can be loaded with the LM71 used in the analysis 6.37 times before failing. At the moment of failure, the bridge experiences the formation of 8 plastic hinges, resulting in a global collapse mechanism following Heyman's approach (**Figure 22**).

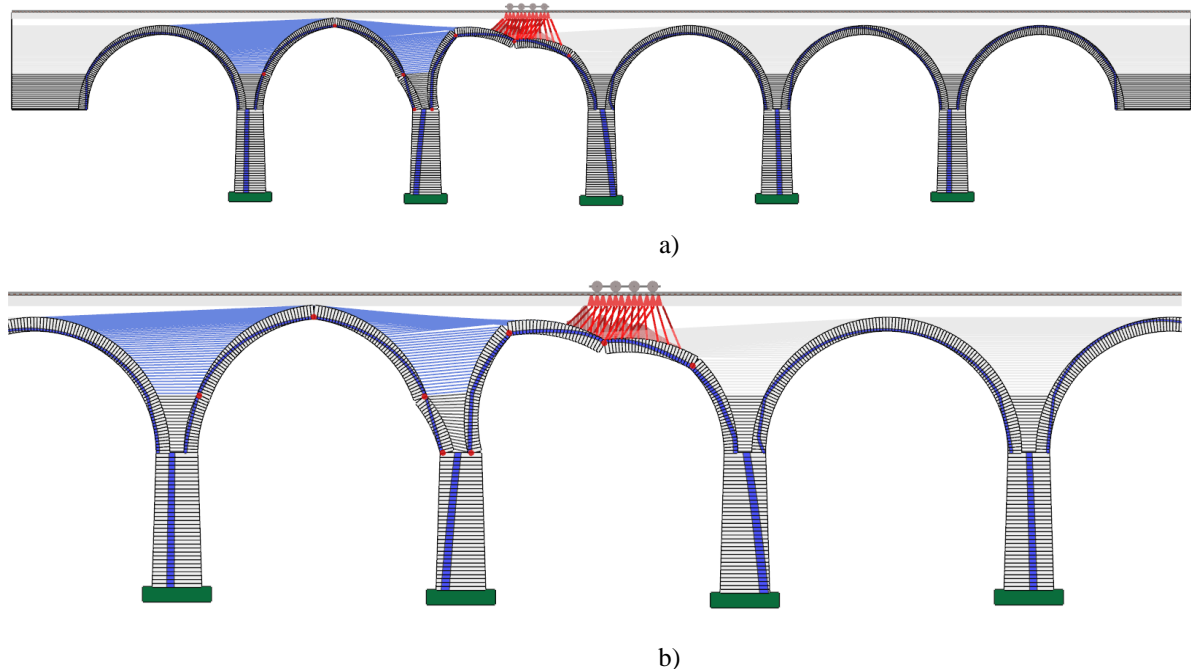


Figure 22 - a) collapse Mechanism of the bridge of Vila Mea; b) close view of the formation of the hinges of the bridge of Vila Mea.

The deterministic model is then used to perform the sensitivity analysis to determine which bridge parameters are the most relevant ones in the structural behaviour. Each of the analysed parameters are then modified and imputed into the numerical model accordingly, and the resulting adequacy factor for each modification is recorded and used for the sensitivity analysis.

4.1.3 Importance Measure

The importance measure, b_k , for each parameter is computed through the definition of the output function (y_i) for each parameter, the correction of the defined functions, and comparison of the results in a bar diagram. First, each parameter is multiplied individually by its CV to determine its deviation (σ), through the following expression (23):

$$\sigma = \mu \times CV \quad (23)$$

where σ consists of the deviation, μ consists of the mean value for the parameter, and CV consists of the coefficient of variance.

Once the deviation is determined, the output function (y_i), for each parameter analysed is computed. In order to compute the output function (y_i), each parameter is varied through its deviation, as well as twice its deviation, to include all possible variations of the parameter. Then, for each parameter, the numerical model of the bridge of Vila Mea is deterministically analysed 5 times—once for each of the 5 values defined by deviation for the considered parameter. For each deterministic computation, the resulting adequacy factor is considered as one of the output values for the analysed parameter. By combining the 5 outputted values, an output function (y_i), is defined. The output function (y_i) for each parameter is similar to the function presented as an example for the parameter of masonry density in **Figure 23**. The rest of the output functions (y_i) are present in **Annexe 1**. The output functions obtained for each parameter are used in the expression of importance measure, b_k (22).

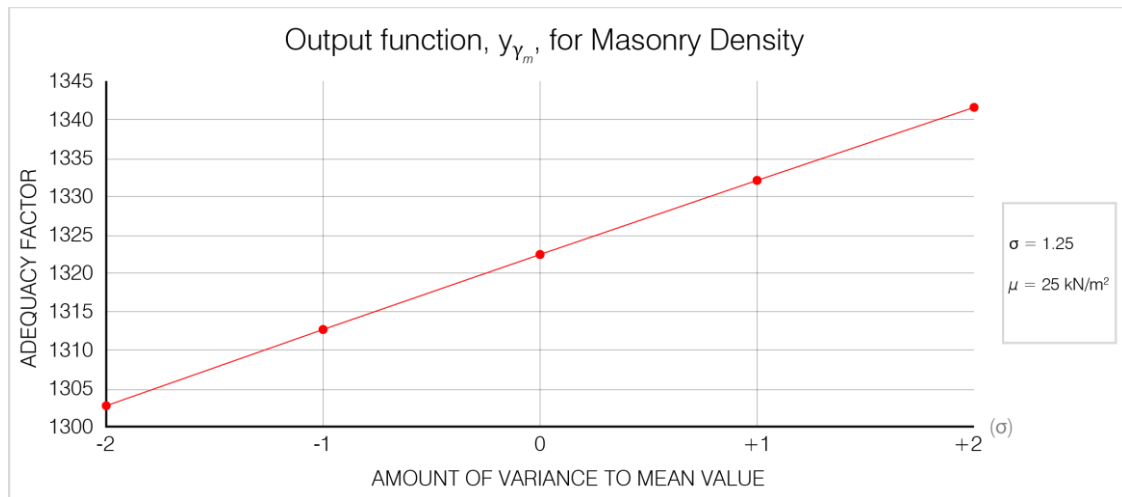


Figure 23 - Output function for Masonry Density

Since each of the parameters analysed in the sensitivity analysis consists of different units, the importance measure, b_k , obtained from the parameters cannot be directly compared. Therefore, the resulting importance measures are first normalized, then compared in a bar diagram (Figure 24). An importance measure limit, b_{lim} , is set as 35%. The value for the importance measure limit is derived through engineering judgement and consultations of works done previously by other authors [36].

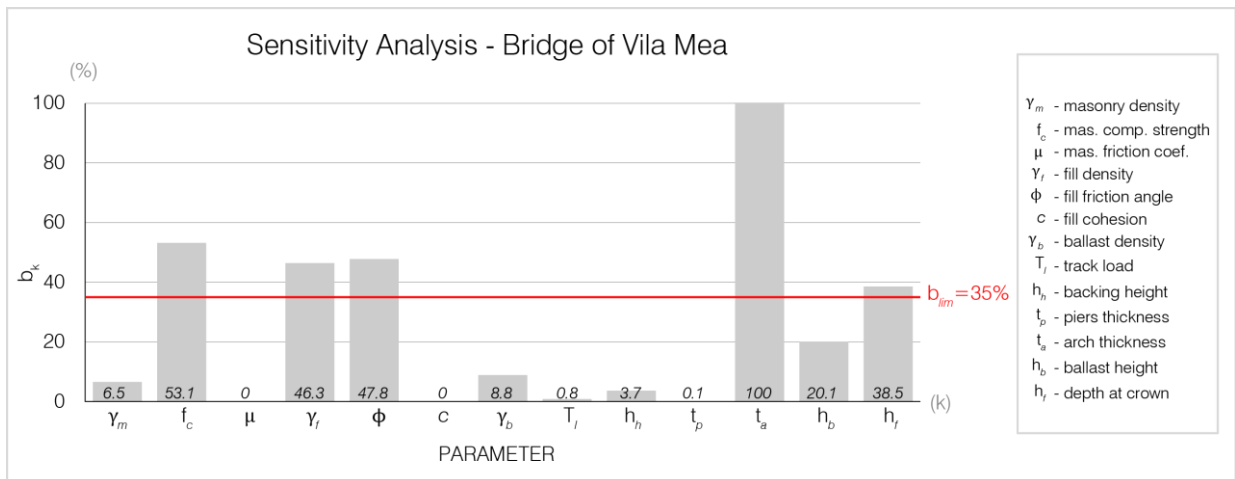


Figure 24 - Sensitivity Analysis results for the Bridge of Vila Mea

By analysing the computed sensitivity analysis for the bridge of Vila Mea, it is determined that only 5 out of the 13 originally chosen parameters are greater than the importance measure limit of 35%. Those 5 parameters consist of: i) masonry compressive strength, f_c ; ii) fill material density, γ_f ; iii) fill friction angle, ϕ ; iv) arch thickness, t_a ; and finally, v) depth at crown, h_f .

The most influential parameter obtained consists of the arch thickness. In fact, the arch thickness has an importance measure almost twice as large, or greater, than the rest of the parameters. This is expected, since the arch thickness sustains the material's self-weight and live loads in the longitudinal direction, playing an important role in the structural resistance of a bridge [38]. The masonry compression strength, fill material density, as well as fill friction angle are considered quite critical, ranging from 46-53% in importance measure compared to the arch thickness. The obtained results were expected, since MAB are gravity structures, that achieve stabilization through the mass provided by masonry and fill material [38]. The last parameter presenting an importance measure greater than the importance measure limit consists of the depth at crown, with a value of 39%. The depth at crown of the fill is an important factor in dispersing the live loads, diminishing the stresses applied on the arch, increasing the ultimate load-carrying capacity of a bridge.

By looking at the sensitivity analysis for the bridge of Vila Mea, it is also determined that some of the original parameters do not have any, or very little influence, on the structural response of the bridge. In fact, both the masonry friction coefficient as well as the fill cohesion have an importance measure of 0%, therefore neither the masonry friction coefficient nor the fill cohesion have an impact on the structural response of the bridge of Vila Mea. Pier thickness closely follows suit, with an importance measure of only 0.1%. Therefore, the performance of the sensitive analysis, prevents the use of random variables that do not have any impact in the accuracy of results.

4.2 Reliability Analysis

4.2.1 Reliability Index

Reliability Index can be defined in various ways. As mentioned in previous chapters, reliability index, β , is obtained from the failure probability, p_f . Mathematically, reliability index is defined as the shortest distance from the origin of the reduced load and resistance distributions to the Limit State function [18]. Reliability index is defined through a probabilistic approach. In fact, reliability accounts for the uncertainties of the solution through the use of random variables, which in turn may be reproduced by PDFs. The purpose of reliability consists of quantifying a structure's safety.

Historically, a structure's safety was enforced even when the laws of constructions were not yet fully understood. Before the development of code, structures were built intuitively, with

rejections of designs that failed and adaptations of designs that stood the test of use and time. Innovations were made to designs that were known to work, with the innovator attempting a more effective structure. With a process of trial and error over the passing of centuries, structures evolved from modified nature-made environmental structures, to post and lintel constructions, to the use of arches, the use of truss systems, and use of framed constructions. The knowledge of construction was passed down from one generation to another. While the next generation recognized which designs were reliable, they did not always fully understand the reasons for that reliability.

However, just as structures evolved, the understanding of laws of nature also grew. Mathematical formulations and theories were developed, reducing the amount of trial and error in structural design. In fact, mathematical theories provided a more rational basis in structural design [18]. Nevertheless, it was not until the 20th century, that the first mathematical formulation of the structural safety problem was developed by Mayer (1926), Wierzbicki (1936) and Streletskii (1947) [18]. It was then recognized that load and resistance parameters are random variables. As a result, each structure presents a finite probability of failure [18]. At the time of those formulations, the probability of failure was represented by convolution functions that were too complex to for practical applications [18]. It was not until Cornell proposed a second moment reliability index in 1969, and Hasofer and Lind [18] presented a format invariant reliability index in 1974, that the probability of failure could be represented and calculated through reliability, which allowed to overcome the invariant problem [18]. Other pioneers further developed the concept of reliability, leading to the *reliability-based design*. Reliability based code can be applied to the design of structural members or structural systems. The code presents a reliability target, β_T , as a base for reliability-based design, further described in **section 4.2.3**.

It is important to distinguish reliability of a member and reliability of a system. In fact, a failure of a member does not necessary lead to failure of the system [18]. Therefore, reliability of a member of a system does not necessarily represent reliability of the entire system. The type of members found in a system may be classified as brittle and ductile members. A brittle member is completely ineffective once it yields, while a ductile member retains its load-carrying capacity after yield point [18]. The type of member dictates the way damage will propagate through the entire system. Furthermore, the type of system of a structure will affect the structure's reliability. A structure can either be a series system, parallel system or hybrid

system. Failure of a series system corresponds to failure of the weakest element of the system [18]. Failure of a parallel system depends on load distribution and type of members that form the system, failing when all members fail [18]. A hybrid system contains both series and parallel system [18]. Therefore, reliability of a structure depends on the type of elements from which the structure is made from, as well as the type of system in which the structure is assembled.

4.2.2 Methodology

To conduct a robustness assessment of the bridge of Vila Mea, the reliability of the undamaged bridge model, β_u , and the reliability of the bridge model being subjected to a specific damage, $\beta_{d,j}$, are computed.

As discussed in **Chapter 2** and **Chapter 3**, in order to compute the damaged and undamaged reliability, of the bridge of Vila Mea, a probabilistic approach must be applied. The probabilistic approach takes into consideration the uncertainties of the structural parameters, which are considered random variables, resulting in the representation of the structural response of the bridge of Vila Mea through a PDF, that represents its resistance. Different methods exist through which a probabilistic approach can be applied, as described in **Chapter 2**). In the case of the assessment of the bridge of Vila Mea, LHS is performed, followed by the Basler Cornel method. LHS is a variance-reduction technique, which achieves a uniform sampling through a Latin Hypercube scheme, reducing the number of required samples to compute the failure probability.

For the damaged and undamaged system, the PDF for the structural response is created, defining the mean and standard deviation for the response function of the bridge of Vila Mea when it is damaged, and when it is not damaged. Once the mean and standard deviation for the response function are defined, the mean and standard deviation for the load function is defined. Thus, the reliability index, and the correspond failure probability, may be computed by the application of Basler Cornel method.

As explained in **Chapter 2**, Basler Cornel method measures the shortest distance between the origin of the reduced response and load function and the Limit state function. The expression for limit state is explained in **Chapter 2**, where the Limit State, LS , consists of the difference between the response function, R , and load function, S .

$$R - S \leq 0 = LS \quad (1)$$

The computation of the load PDF is described in **Chapter 3**, where the mean value of the function is set as 207.40 kN, and the CV is set at 10%, resulting in a standard deviation of 20.74 kN [38]. Since both the response and load functions are observed to be normal, and the expression itself is linear, Basler Cornell method is applied to determine the reliability, β , of the bridge of Vila Mea. The obtained value for reliability, β , is then compared to a target reliability, β_T , of 3.8 [52]. The value chosen for the target reliability, β_T , is based on the Eurocode design for 50 years [52].

4.2.3 Target Reliability

Reliability index, β , allows to quantify the safety of a structure. Once the reliability of a structure is determined, it must be compared with a base value, called the target reliability, β_T . Target reliability presented in the code consists of an aid in design optimization. Reliability can be determined in elements, connections, or in the entire system. The code offers target reliability for various design situations. Selection of target reliability involves structural safety analysis, economic analysis and political decisions [18]. A reliability lesser than the target reliability is unacceptable [18]. However, sometimes values of reliability which fall below the target reliability may be justified in some special cases, such as maintaining the simplicity of the format [18]. Reliability indexes that are higher than the target reliability are accepted and justified [18]. In fact, if a reliability index is greater than the target reliability, the structure is technically over-designed. The design may be modified to reduce its cost, lowering the unnecessary high reliability index.

In selecting a target reliability for a design, current code should be consulted, evaluation of performance of existing structures should be conducted, and engineering judgement should be used [18]. It is important to note that there exists a different cost for safety in various structures and parts [18]. For example, increasing the safety level in a beam connection is less costly than the increase of safety in the beam itself [18]. Therefore, a connections target reliability should be greater than a member's target reliability, which in turn should be greater than a system's target reliability.

While the code offers target reliability values for various connections, members and structures, the code is intended as a basis for new structure design. The code can be used for reliability assessment of existing structures. Usually, the reliability of existing structures that were not designed by code will be quite greater than the target reliability offered in code. In

fact, pre-code designs were not optimized the way modern designs are optimized [18]. The importance laid in making sure the structure performed its function well, for a long period, without experiencing collapse. Techniques in safety measurement of a structure did not exist until the 20th century. Therefore, safety was ensured in an intuitive manner, following previous successful design. While material optimization was important, and lead to development of certain techniques—such as the use of arches—material optimization was not as emphasized and perfected as it is in the current period. By current codes, most, if not all, ancient structure is overdesigned. In fact, it is not unusual to encounter ancient structures that support greater loads than the ones they were originally designed for. Consequently, structures that were constructed before the creation of code present a greater reliability than the target reliability found in code.

4.2.4 Undamaged Reliability

The Reliability, β , for the bridge of Vila Mea is computed through the Basler Cornel method, which requires the computation of a PDF for the structural response of the bridge. Accordingly, the PDF is created by the generation of 100 samples by the LHS method. The PDF for the structural response is defined with 5 random parameters—defined through the sensitivity analysis—taken into consideration.

The resulting PDF is defined as the structural response of the bridge of Vila Mea when it is not subjected to any additional damages, i.e., when the system is undamaged (**Figure 25**). The resulting mean of the structural response, μ_R , consists of 1370.2, the standard deviation, σ_R , consists of 211.24. As mentioned in **Chapter 3**, the loading PDF presents a mean, μ_S , of 207.4, and a standard deviation, σ_S , of 20.74. (**Table 3**)

Table 3. Data from resistance and load PDF

DATA	RESISTANCE, R	LOAD, S
μ	1370.2	207.4
σ	211.24	20.74

The response function presents a normal and symmetrical PDF; therefore, Basler Cornel method may be used to compute the reliability of the structure. Basler Cornel Method requires only the mean and standard deviation of both the response and load distributions. By imputing data from both the response and load functions into the Basler Cornel method expression, a reliability index of 5.48 is obtained (24).

$$\beta = \frac{\mu_z}{\sigma_z} = \frac{1370.2 - 207.4}{\sqrt{211.24^2 + 20.74^2}} \quad (24)$$

$$\beta = 5.48$$

Comparing the result to the target reliability, β_T , of 3.8 from the Eurocode [52], the structure is considered reliable when it is not subjected to any additional damages. In fact, the obtained reliability is quite greater than the target reliability (expression (25)).

$$\beta \geq \beta_T \quad (25)$$

$$5.48 > 3.8$$

It was expected for the reliability of the bridge of Vila Mea to present a greater reliability than the target reliability found in code, since it was observed that MABs have a higher ultimate-load carrying capacity based on previous assessments [10] [36] [38]. The obtained results from reliability analysis of an undamaged bridge of Vila Mea allow for a comparison of the structural behavior of the bridge when it is exposed to different types of damages. To compare the reliability of the undamaged and damaged bridge, damage simulation is performed on the numerical model, leading to the robustness assessment of the bridge of Vila Mea.

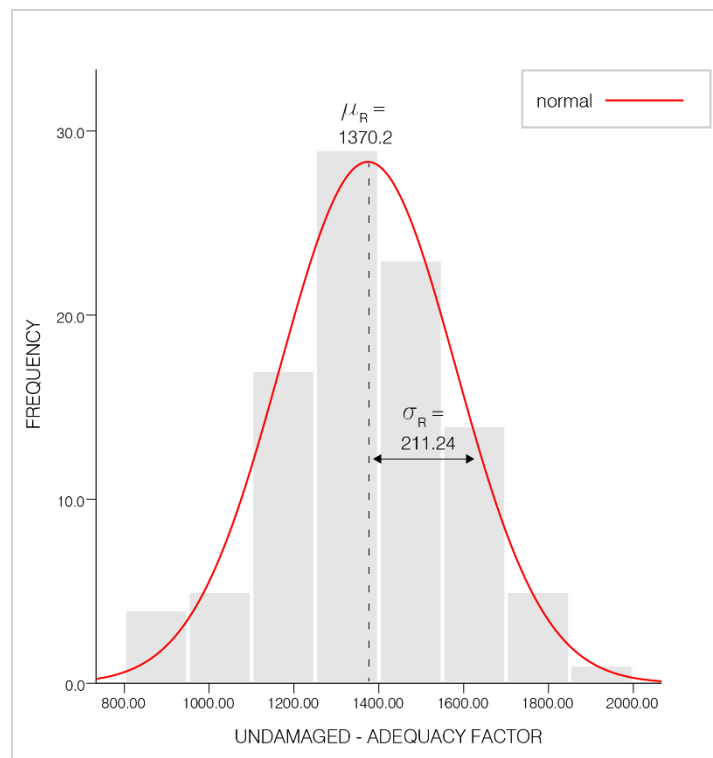


Figure 25 - PDF for the undamaged bridge of Vila Mea

4.2.5 Damaged Reliability

In order to assess the robustness of the bridge of Vila Mea, the bridge undergoes 5 damage simulations; i) transversal cracking; ii) longitudinal cracking due to spandrel wall detachment; iii) longitudinal cracking due to bi-block sleepers load concentration; iv) mortar loss; v) delamination.

For each damage scenario, a PDF for the bridges structural resistance is defined. To compute the PDF, each damage scenario is modeled individually in the numerical model of the bridge of Vila Mea. Once a damage is modeled into the bridge, the bridge undergoes loading simulation (described in **Chapter 3**) 100 times, which in turn returns 100 adequacy factors for each damage. The adequacy factors obtained are filtered to eliminate any data which does not make logical sense with the rest of the values, that are designated outliers. Through the obtained adequacy factors, a structural resistance PDF is plotted for each damage scenario. Each plotted PDF represents the damaged reliability index, $\beta_{d,j}$, specific to each case of damaged structure (j). It is then that the damaged reliability is compared to the undamaged reliability of the bridge, computed in **section 4.2.4**, to determine the robustness of the bridge of Vila Mea.

4.3 Damage scenario 1: Transversal Cracking of Masonry

4.3.1 Methodology

The maximum damage due to transversal cracking is set at a reduction of compressive strength by 10%. In order to model the damage, the compressive strength of masonry is reduced by 10% [53] [54].

4.3.2 Results

The PDF for the resistance of the bridge of Vila Mea, after the bridge undergoes transversal cracking, is presented in **Figure 26**. Data obtained from the damaged resistance following transversal cracking, as well as data from the load curve are presented in **Table 4**.

Table 4. Data when bridge undergoes transversal cracking

DATA	RESISTANCE, R	LOAD, S
μ	1327.4	207.4
σ	207.45	20.74

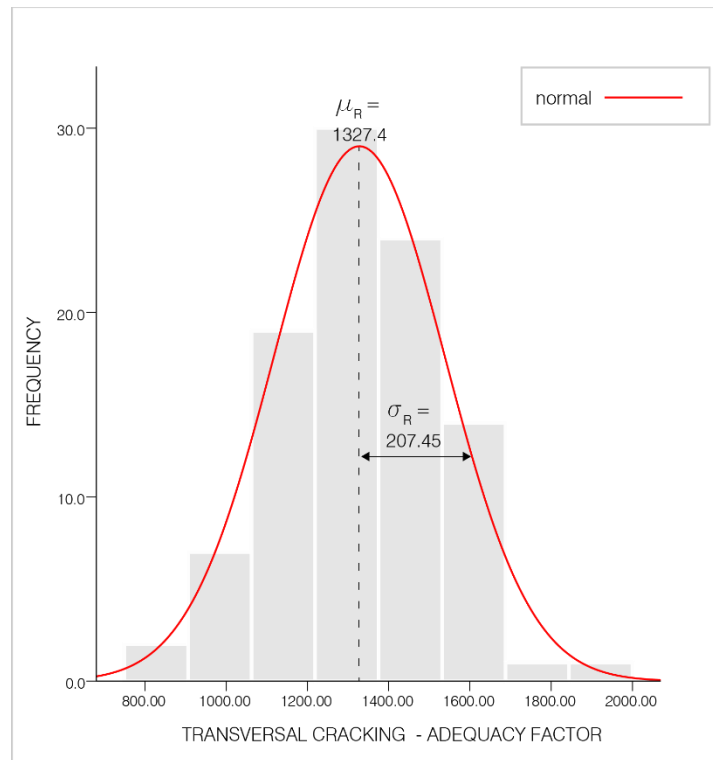


Figure 26 - Resistance function when the bridge is subjected to transversal cracking

The data is then imputed into the expression proposed by Basler Cornel to compute reliability analysis, and then the data is compared with the target reliability of 3.8, from European standard [52], established **section 4.2.2** (24)(25). The reliability obtained is compared with the rest of the damage reliabilities in **section 4.2.4**.

$$\beta_{d,1} = \frac{\mu_{z,1}}{\sigma_{z,1}} = \frac{1327.4 - 207.4}{\sqrt{207.45^2 + 20.74^2}} \quad (26)$$

$$\beta_{d,1} = 5.37$$

$$\begin{aligned} \beta_{d,1} &\geq \beta_T \\ 5.37 &> 3.8 \end{aligned} \quad (27)$$

4.4 Damage scenario 2: Longitudinal Cracking of the Arch (Detachment of Spandrel Walls)

4.4.1 Methodology

The maximum damage by longitudinal cracking due to detachment of spandrel walls is set at 30 cm from each side of the bridge. This value was obtained from the design project. To

model the detachment of spandrel walls, the width of the bridge is reduced by 600 mm [53] [8] [55] .

4.4.2 Results

The PDF for the resistance of the bridge of Vila Mea, after the bridge is subjected to detachment of spandrel walls, is presented in **Figure 27**. Data obtained from the damaged resistance following detachment of spandrel walls, as well as data from the load curve are presented in **Table 5**.

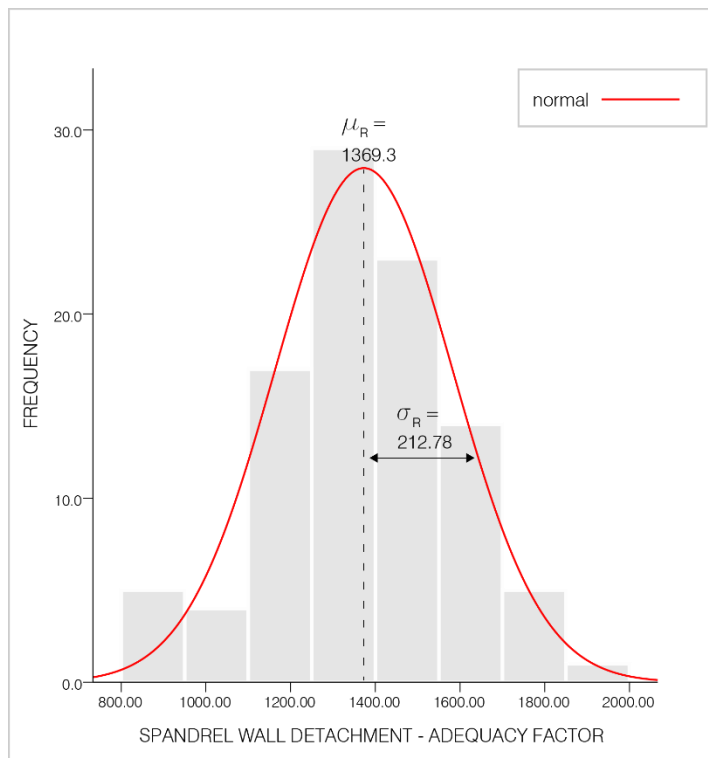


Figure 27 - Resistance function when the bridge is subjected to detachment of spandrel walls

Table 5. Data when bridge undergoes detachment of spandrel walls

DATA	RESISTANCE, R	LOAD, S
μ	1369.3	207.4
σ	212.78	20.74

Through the Basler Cornell method, the damaged reliability analysis is computed, and then the result is compared with the target reliability of 3.8 [52], established **section 4.2.4** (28)(29). The reliability obtained is compared with the rest of the damage reliabilities in **section 4.8.2**.

$$\beta_{d,2} = \frac{\mu_{z,2}}{\sigma_{z,2}} = \frac{1369.3 - 207.4}{\sqrt{212.78^2 + 20.74^2}} \quad (28)$$

$$\beta_{d,2} = 5.43$$

$$\beta_{d,2} \geq \beta_T \quad (29)$$

$$5.43 > 3.8$$

4.5 Damage scenario 3: Longitudinal Cracking of the arch (Bi-Block Sleepers)

4.5.1 Methodology

The maximum damage caused by bi-block sleepers load concentration is a reduction of the effective bridge to their width to 2600 mm. Therefore, to model cracking due to bi-block sleepers, the maximum effective bridge width is changed to 2600 mm [53].

4.5.2 Results

The PDF for the resistance of the bridge of Vila Mea, after the bridge is subjected to damage from bi-block sleepers, is presented in **Figure 28**. Data obtained from the damaged resistance following cracking to addition of bi-block sleepers, as well as data from the load curve are presented in **Table 6**.

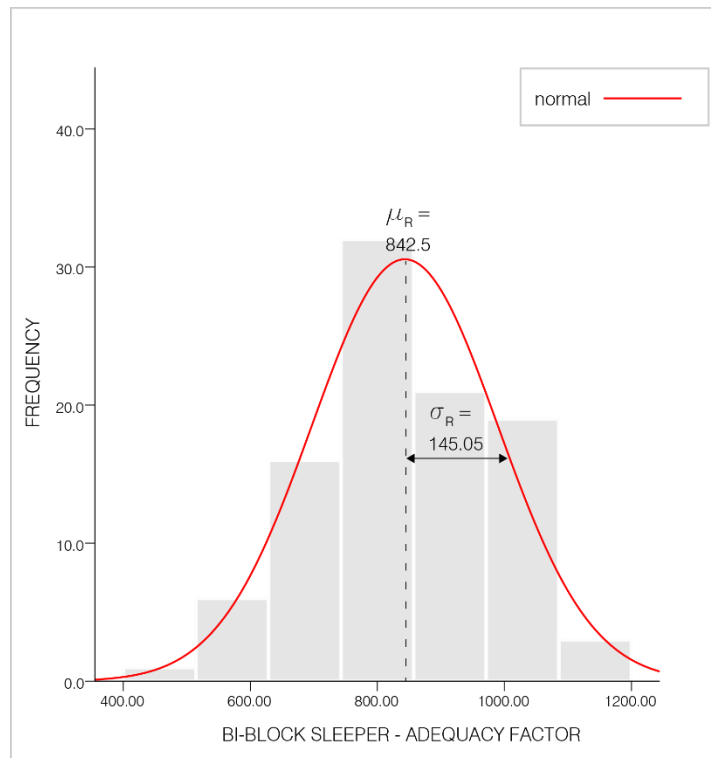


Figure 28 - Resistance function when the bridge is subjected to bi-block sleepers

Table 6. Data when bridge undergoes longitudinal cracking due to bi-block sleepers

DATA	RESISTANCE, R	LOAD, S
μ	842.5	207.4
σ	145.05	20.74

Through the Basler Cornell method, the damaged reliability analysis is computed, and then the result is compared with the target reliability of 3.8 [52]. The reliability obtained is compared with the rest of the damage reliabilities in **section 4.8.2**.

$$\beta_{d,3} = \frac{\mu_{z,3}}{\sigma_{z,3}} = \frac{842.5 - 207.4}{\sqrt{145.05^2 + 20.74^2}} \quad (30)$$

$$\beta_{d,3} = 4.33$$

$$\begin{aligned} \beta_{d,3} &\geq \beta_T \\ 4.33 &> 3.8 \end{aligned} \quad (31)$$

4.6 Damage scenario 4: Mortar-Loss at the Joints

4.6.1 Methodology

The maximum damage caused by mortar loss is a reduction of the mortar joints in the arch, from the intrados, by 10cm. Therefore, to model the damage, the mortar joints in the intrados are reduced by 100 mm [53] [54].

4.6.2 Results

The PDF for the resistance of the bridge of Vila Mea, after the bridge undergoes mortar loss, is presented in **Figure 29**. Data obtained from the damaged resistance following mortar loss, as well as data from the load curve are presented in **Table 7**.

Table 7. Data when bridge is subjected to mortar loss

DATA	RESISTANCE, R	LOAD, S
μ	1183.0	207.4
σ	194.41	20.74

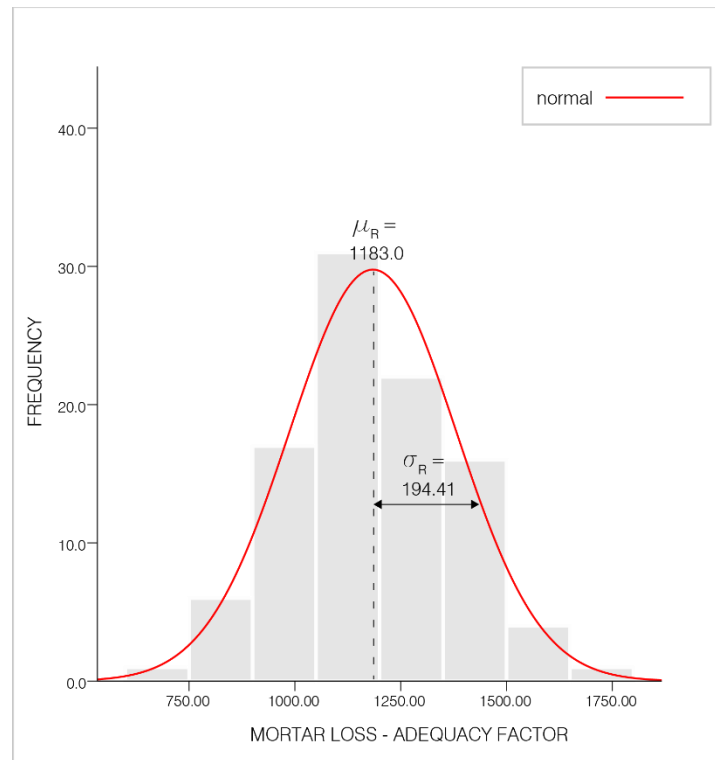


Figure 29 - Resistance function when the bridge is subjected to mortar loss

Through the Basler Cornel method, the damaged reliability analysis is computed, and then the result is compared with the target reliability of 3.8 [52], (32)(33). The reliability obtained is compared with the rest of the damage reliabilities in **section 4.8.2**.

$$\beta_{d,4} = \frac{\mu_{z,4}}{\sigma_{z,4}} = \frac{1183.0 - 207.4}{\sqrt{194.41^2 + 20.74^2}} \quad (32)$$

$$\beta_{d,4} = 4.99$$

$$\begin{aligned} \beta_{d,4} &\geq \beta_T \\ 4.99 &> 3.8 \end{aligned} \quad (33)$$

4.7 Damage scenario 5: Delamination of Granite Stone

4.7.1 Methodology

The maximum damage caused by delamination in the bridge consists of a delamination of 2cm. To model delamination, the arch thickness is reduced by 20 mm [56] [54].

4.7.2 Results

The PDF for the resistance of the bridge of Vila Mea, after the bridge undergoes delamination, is presented in **Figure 30**. Data obtained from the damaged resistance following transversal cracking, as well as data from the load curve are presented in **Table 8**.

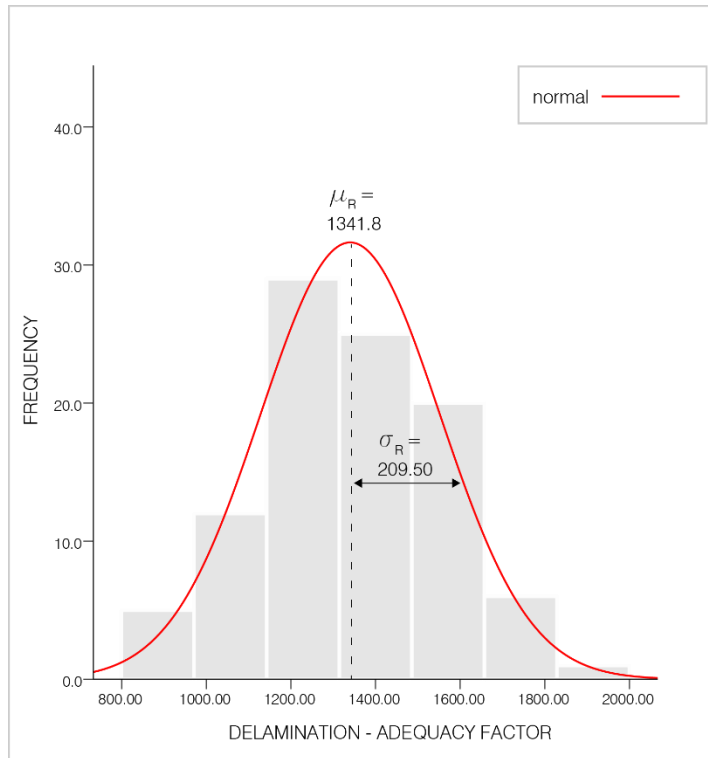


Figure 30 - Resistance function when the bridge is subjected to delamination

Table 8. Data when bridge is subjected to delamination

DATA	RESISTANCE, R	LOAD, S
μ	1341.8	207.4
σ	209.50	20.74

Through the Basler Cornell method, the damaged reliability analysis is computed, and then the result is compared with the target reliability of 3.8 [52]. The reliability obtained is compared with the rest of the damage reliabilities in **section 4.8.2**.

$$\beta_{d,5} = \frac{\mu_{z,5}}{\sigma_{z,4}} = \frac{1341.8 - 207.4}{\sqrt{209.50^2 + 20.74^2}} \quad (34)$$

$$\beta_{d,5} = 5.39$$

$$\beta_{d,4} \geq \beta_T$$

$$5.39 > 3.8$$
(35)

4.8 Obtained Results of Reliability Analysis

4.8.1 Goodness of Fit: Chi-Square Test

To determine test the goodness of fit between theoretical and experimental data, the chi-square test is performed with the results. The chi-square test depends on the degree of freedom in the problem, the expected values, as well as the observed values. The following expression is used to perform the chi-square test (36):

$$\chi^2 = \sum_{i=1}^k \frac{(O - E)^2}{E}$$
(36)

where O consists of observed value, E consists of expected value, and k consists of number of possible possible outcomes. The resulting χ^2 is compared with a critical χ^2 . For each damage, the critical χ^2 is set as 11.0705, as there are 5 degrees of freedom, with a percentage of rejecting the null hypothesis when it should be retained, (α), of 5%. Each damage presents a χ^2 much lower than the critical χ^2 , as it is presented in **Table 9**.

Table 9. Chi Test results for damage

DAMAGE	CRITICAL χ^2	RESULTING χ^2
1. Transversal Cracking	11.0705	5.9479
2. Spandrel Wall Detachment	11.0705	5.3185
3. Bi-Block Sleeper	11.0705	4.9736
4. Mortar Loss	11.0705	6.4948
5. Delamination	11.0705	6.0524

4.8.2 Comparison of Undamaged and Damaged Reliability

Reliability of the structure following each damage is computed and compared with the target reliability of 3.8 set by the European code [43] (**Figure 31**).

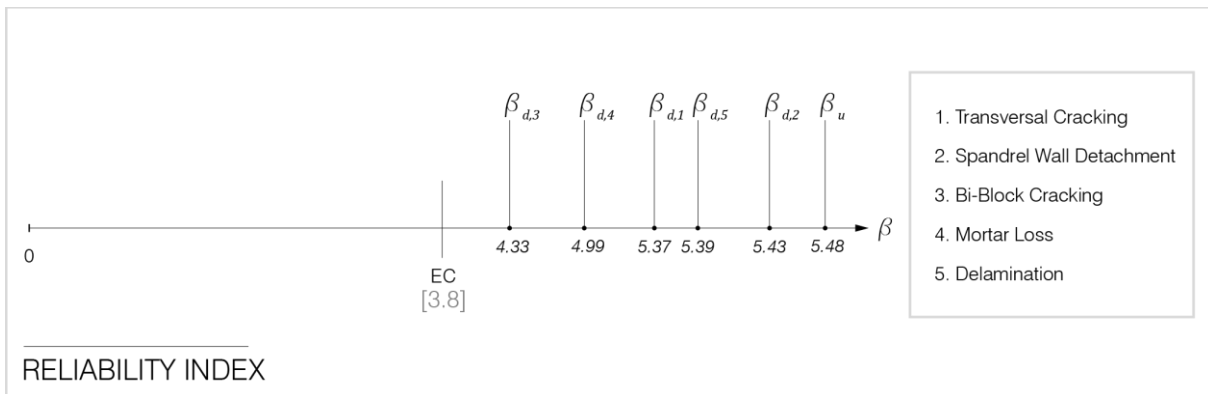


Figure 31 - Reliability Index of undamaged and damaged bridge of Vila Mea

Each computed reliability is greater than the target reliability set by code. Reliability of the undamaged structure is quite larger than the target reliability, with a value of 5.48. The lowest reliability is presented by the damaged structure due to bi-block sleepers, with a value of 4.33. Following cracking due to bi-block sleepers, the second lowest reliability is obtained through mortar loss, with a value of 4.99. Next, the structure undergoing delamination and transversal cracking follow suit, with reliability values of 5.37 and 5.39 respectively. Finally, the damaged structure closest in reliability value to the undamaged structure consists of the structure undergoing spandrel wall detachment, with a reliability of 5.43. This being said, 3 out of the 5 damaged structures have reliabilities greater than 5.0, while the target reliability consists of 3.8. The rest of the two damaged structures have reliabilities greater than 4.0. The order of the obtained values, from lowest to greatest, follow the same order as the order of the obtained robustness in **Chapter 5**. The obtained values demonstrate that even when the bridge experiences damage, it presents acceptable reliability based on the European standard.

5. ROBUSTNESS ANALYSIS

5.1 Notion of Robustness

The notion of robustness has been developed only recently. Current code states that a structure is robust when proportionality exists between damage scenario and consequence [24] [57]. However, as described in detail in **Chapter 2**, robustness is described by different authors in various ways [1] [24] [25] [26] [28]. For the sake of the thesis, robustness can be defined as the prevention of disproportional damage spreading and propagation into a structure—by eliminating failure consequences following a local damage—rendering a structure collapse resistant.

The study of robustness of ancient structures that are still in use is essential in determining the safety of such structures. In fact, such retrofitted structures were often designed to resist much lower loads to which the structures are exposed in their modern use [36] [58]. In some cases, the historical structure structures may need a strengthening intervention, while in other cases the original design of the structure will suffice in resisting the current loading. It is robustness that allows existing structure to withstand a new increase in loads without significant damage [1]. Since MAB are examples of ancient structures retrofitted to modern needs, it is important to understand how robustness impacts those bridges. The bridge of Vila Mea is an example of such MAB. The bridge has been constructed at the end of the 19th century, but continues to serve as a means of train transportation in the beginning of the 21st century. The robustness of the bridge of Vila Mea is assessed through its previously calculated reliability indexes of undamaged (β_u) and damaged ($\beta_{d,j}$) structure (where j consists of numbered damage).

5.2 Applied Methods to Calculate Robustness

As described in **Chapter 2**, many techniques were developed in order to compute robustness. Robustness of the bridge of Vila Mea is assessed through 3 different probabilistic-based approaches all described in detail in **Chapter 2**, namely: i) Probabilistic based redundancy; ii) Frangopol and Curley approach; iii) Cavaco method. Each of the method computes robustness through the reliability index, β , of the undamaged (β_u) and damaged ($\beta_{d,j}$) bridge of Vila Mea (computed in **Chapter 4**).

5.3 Robustness Assessment

5.3.1 Damage scenario 1: Transversal Cracking of the Masonry

Reliability data that is used to assess robustness when the bridge of Vila Mea is subjected to transversal cracking is presented in **Table 10**.

Table 10. Data when bridge undergoes transversal cracking

UNDAMAGED RELIABILITY, β_u	DAMAGED RELIABILITY, $\beta_{d,1}$
5.48	5.37

5.3.1.1 PROBABILISTIC BASED REDUNDANCY – Damage scenario 1

Robustness of the bridge of Vila Mea, when it is subjected to transversal cracking, is assessed through the modified probabilistic passed redundancy (37). In probabilistic based redundancy method, the closer the result is to 0, the more robust the structure is. The resulting robustness is very close to 0, therefore the structure is deemed very robust when it is subjected to transversal cracking. The result obtained is compared with the rest of the results from the rest of the damages, in **section 5.4**.

$$R_1 = \frac{5.37 - 5.48}{5.48}$$

$$R_1 = 0.019 \quad (37)$$

5.3.1.2 FRANGOPOL AND CURLEY – Damage scenario 1

Robustness of the bridge of Vila Mea, when the bridge undergoes transversal cracking, is assessed through the method proposed by Frangopol and Curley (38). Through this method, when the result approaches 0, the structure is not robust. The expression is unbounded in the positive range; therefore it is hard to state if the results are robust or not. The result obtained is compared with the rest the results from the rest of the damages, in **section 5.4**.

$$R_1 = \frac{5.48}{5.48 - 5.37}$$

$$R_1 = 51.6 \quad (38)$$

5.3.1.3 CAVACO – Damage scenario 1

The behavior of the damaged and undamaged bridge of Vila Mea, when the bridge is subjected to transversal cracking, is represented through an approximate function proposed by Cavaco (**Figure 32**)(37). The area under the function is computed through expression (39). It is a bounded method to compute robustness, as the closer the expression is to 1, the more robust

the structure is. The results demonstrate that the bridge of Vila Mea is very robust when it is subjected to transversal cracking.

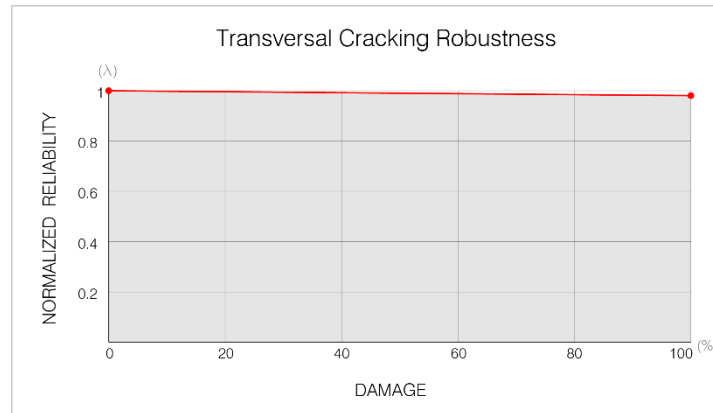


Figure 32 - Cavaco function for damage from transversal cracking

$$R_1 = \int_{D=0}^{D=1} f(x) dx \quad (39)$$

$$R_1 = 0.990$$

5.3.2 Damage scenario 2: Longitudinal Cracking of the Arch (Detachment of Spandrel Walls)

Reliability data that is used to assess robustness when the bridge of Vila Mea is subjected to longitudinal cracking due to spandrel wall detachment is presented in **Table 11**.

Table 11. Data when bridge undergoes spandrel wall detachment

UNDAMAGED RELIABILITY, β_u	DAMAGED RELIABILITY, $\beta_{d,2}$
5.48	5.43

5.3.2.1 PROBABILISTIC BASED REDUNDANCY – Damage scenario 2

Robustness of the bridge of Vila Mea, when the bridge undergoes longitudinal cracking due to spandrel wall detachment, is assessed through the modified probabilistic passed redundancy (40). In probabilistic based redundancy method, the closer the result is to 0, the more robust the structure is. The resulting robustness is very close to 0, therefore the structure is deemed very robust when it is subjected to spandrel wall detachment. The result obtained is compared with the rest of the results from the rest of the damages, in **section 5.4**.

$$R_2 = \frac{5.43 - 5.48}{5.48}$$

$$R_2 = 0.008 \quad (40)$$

5.3.2.2 FRANGOPOL AND CURLEY – Damage scenario 2

Robustness of the bridge of Vila Mea, when the bridge is subjected to spandrel wall detachment, is assessed through the method proposed by Frangopol and Curley (41). Through this method, when the result approaches 0, the structure is not robust. The expression is unbounded in the positive range. Since the result is far from 0, the bridge is found to be robust when it is subjected to spandrel wall detachment. The result obtained is compared with the rest the results from the rest of the damages, in **section 5.4**.

$$R_2 = \frac{5.48}{5.48 - 5.43}$$

$$R_2 = 125.0 \quad (41)$$

5.3.2.3 CAVACO – Damage scenario 2

The behavior of the damaged and undamaged bridge of Vila Mea, when the bridge undergoes spandrel wall detachment, is represented through an approximate function proposed by Cavaco (**Figure 33**)(37). The area under the function is computed through expression (42). The results demonstrate that the structural capacity of the bridge of Vila Mea is almost intact when it is subjected to spandrel wall detachment.

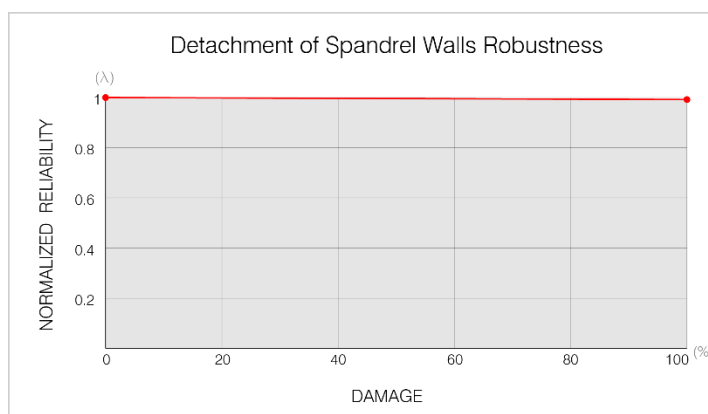


Figure 33 - Cavaco function for damage from spandrel wall detachment

$$R_2 = \int_{D=0}^{D=1} f(x)dx \quad (42)$$

$$R_2 = 0.996$$

5.3.3 Damage scenario 3: Longitudinal Cracking of the arch (Bi-Block Sleepers)

Reliability data that is used to assess robustness if the bridge of Vila Mea is subjected to the installation of bi-block sleepers, that in turn results in longitudinal cracking is presented in **Table 12**.

Table 12. Data when bridge undergoes cracking due to bi-block sleepers

UNDAMAGED RELIABILITY, β_u	DAMAGED RELIABILITY, $\beta_{d,3}$
5.48	4.33

5.3.3.1 PROBABILISTIC BASED REDUNDANCY – Damage scenario 3

Robustness of the bridge of Vila Mea, when the bridge experiences longitudinal cracking due to bi-block sleepers, is assessed through the modified probabilistic passed redundancy (43). In probabilistic based redundancy method, the closer the result is to 0, the more robust the structure is. The resulting robustness is not as close to 0 as the other results, therefore the structure is not as robust when the bridge is subjected to bi-block sleepers. The result obtained is compared with the rest of the results from the rest of the damages, in **section 5.4**.

$$R_3 = \frac{4.33 - 5.48}{5.48} \quad (43)$$

$$R_3 = 0.21$$

5.3.3.2 FRANGOPOL AND CURLEY – Damage scenario 3

Robustness of the bridge of Vila Mea, when the bridge undergoes longitudinal cracking resulting from bi-block sleepers, is assessed through the method proposed by Frangopol and Curley (44). Through this method, when the result approaches 0, the structure is not robust. The expression is unbounded in the positive range. Since the result is close to 0, the structure is not that robust when it is subjected to cracking due to bi-block sleepers. The result obtained is compared with the rest the results from the rest of the damages, in **section 5.4**.

$$R_3 = \frac{5.48}{5.48 - 4.33} \quad (44)$$

$$R_3 = 4.79$$

5.3.3.3 CAVACO – Damage scenario 3

The behavior of the damaged and undamaged bridge of Vila Mea, when the bridge is subjected to cracking due to bi-block sleepers, is represented through an approximate function proposed by Cavaco (**Figure 34**). The area under the function is computed through expression (45)(39). It is an accurate method to compute robustness, as the closer the expression is to 1, the more robust the structure is. The results demonstrate that the bridge of Vila Mea is still quite robust when it suffers from cracking resulting from bi-block sleepers.

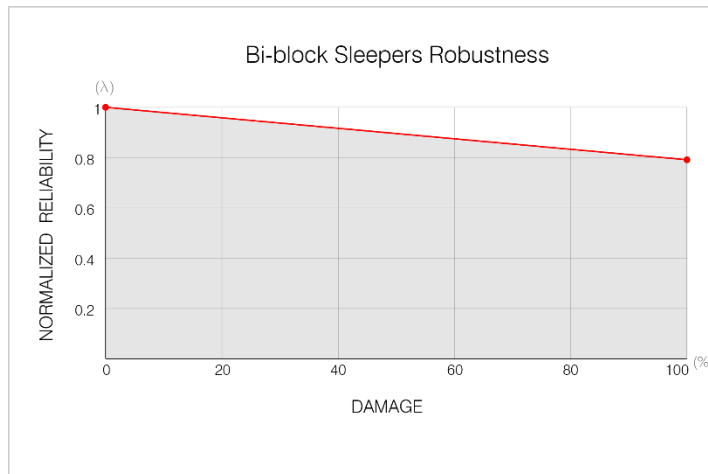


Figure 34 - Cavaco function for damage from bi-block sleepers

$$R_3 = \int_{D=0}^{D=1} f(x) dx \quad (45)$$

$$R_3 = 0.86$$

5.3.4 Damage scenario 4: Mortar-Loss at the Joints

Reliability data that is used to assess robustness when the bridge of Vila Mea is subjected to mortar loss is presented in **Table 13**.

Table 13. Data when bridge undergoes mortar loss

UNDAMAGED RELIABILITY, β_u	DAMAGED RELIABILITY, $\beta_{d,4}$
5.48	4.99

5.3.4.1 PROBABILISTIC BASED REDUNDANCY – Damage scenario 4

Robustness of the bridge of Vila Mea, when the bridge experiences mortar loss, is assessed through the modified probabilistic passed redundancy (46). In probabilistic based redundancy method, the closer the result is to 0, the more robust the structure is. The resulting robustness is still close to 0, therefore the structure is deemed robust when it is subjected to mortar loss. The result obtained is compared with the rest of the results from the rest of the damages, in **section 5.4**.

$$R_4 = \frac{4.99 - 5.48}{5.48}$$
$$R_4 = 0.089 \quad (46)$$

5.3.4.2 FRANGOPOL AND CURLEY – Damage scenario 4

Robustness of the bridge of Vila Mea, when the bridge undergoes mortar loss, is assessed through the method proposed by Frangopol and Curley (47). Through this method, when the result approaches 0, the structure is not robust. The expression is unbounded in the positive range. Since the result is relatively close to 0, the bridge is found to be not as robust when it is subjected to mortar loss. The result obtained is compared with the rest the results from the rest of the damages, in **section 5.4**.

$$R_4 = \frac{5.48}{5.48 - 4.99}$$
$$R_4 = 11.2 \quad (47)$$

5.3.4.3 CAVACO – Damage scenario 4

The behavior of the damaged and undamaged bridge of Vila Mea, when the bridge is subjected to mortar loss, is represented through an approximate function proposed by Cavaco (**Figure 32**)(37). The area under the function is computed through expression (48). It is an accurate method to compute robustness, as the closer the expression is to 1, the more robust the structure is. The results demonstrate that the bridge of Vila Mea is quite robust when it is subjected to mortar loss.

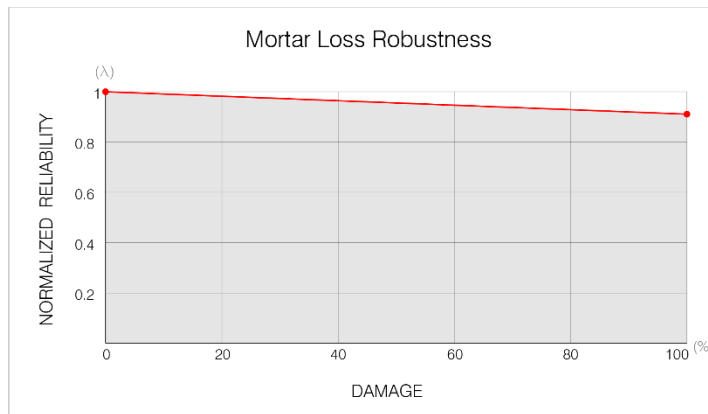


Figure 35 - Cavaco function for damage due to mortar loss

$$R_4 = \int_{D=0}^{D=1} f(x) dx \quad (48)$$

$$R_4 = 0.96$$

5.3.5 Damage scenario 5: Delamination of Granite Stone

Reliability data that is used to assess robustness when the bridge of Vila Mea is subjected to delamination is presented in **Table 14**.

Table 14. Data when bridge undergoes delamination

UNDAMAGED RELIABILITY, β_u	DAMAGED RELIABILITY, $\beta_{d,5}$
5.48	5.39

5.3.5.1 PROBABILISTIC BASED REDUNDANCY – Damage scenario 5

Robustness of the bridge of Vila Mea, when the bridge suffers from delamination, is assessed through the modified probabilistic passed redundancy (49). In probabilistic based redundancy method, the closer the result is to 0, the more robust the structure is. The resulting robustness is very close to 0, therefore the structure is deemed very robust when the bridge undergoes delamination. The result obtained is compared with the rest of the results from the rest of the damages, in **section 5.4**.

$$R_5 = \frac{5.39 - 5.48}{5.48} \quad (49)$$

$$R_5 = 0.016$$

5.3.5.2 FRANGOPOL AND CURLEY – Damage scenario 5

Robustness of the bridge of Vila Mea, when the bridge undergoes delamination, is assessed through the method proposed by Frangopol and Curley (50). Through this method, when the result approaches 0, the structure is not robust. The expression is unbounded in the positive range. Since the result is not close to 0, the bridge is found to be robust when it is subjected to delamination. The result obtained is compared with the rest the results from the rest of the damages, in section 5.4.

$$R_5 = \frac{5.48}{5.48 - 5.39} \quad (50)$$

$$R_5 = 60.9$$

5.3.5.3 CAVACO – Damage scenario 5

The behavior of the damaged and undamaged bridge of Vila Mea, when the bridge is subjected to delamination, is represented through an approximate function proposed by Cavaco (Figure 36). The area under the function is computed through expression (51). It is an accurate method to compute robustness, as the closer the expression is to 1, the more robust the structure is. The results demonstrate that the bridge of Vila Mea is very robust when it is subjected to delamination.

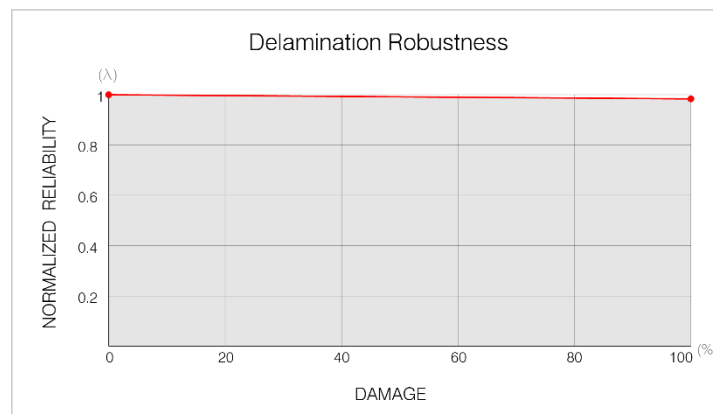


Figure 36 - Cavaco function for damaged due to delamination

$$R_5 = \int_{D=0}^{D=1} f(x) dx \quad (51)$$

$$R_5 = 0.992$$

5.4 Robustness Analysis of the Bridge of Vila Mea

Robustness of the bridge of Vila Mea is assessed by analysing the reliability of the undamaged (β_u) and damaged ($\beta_{d,j}$) bridge. The structure's robustness is compared when it is subjected to 5 different damage scenarios; i) transversal cracking, ii) longitudinal cracking due to spandrel wall detachment, iii) longitudinal cracking due to bi-block sleepers, iv) mortar loss, v) delamination.

By definition, a robust structure is collapse resistant as it does not suffer failure consequences following a local damage. The robustness index for each damage is computed and compared through probabilistic based analysis (**Figure 37**), Frangopol and Curley approach (**Figure 38**), and Cavaco method (**Figure 39**). While the value of each of the results from each method cannot be directly compared, the order of the results is comparable.

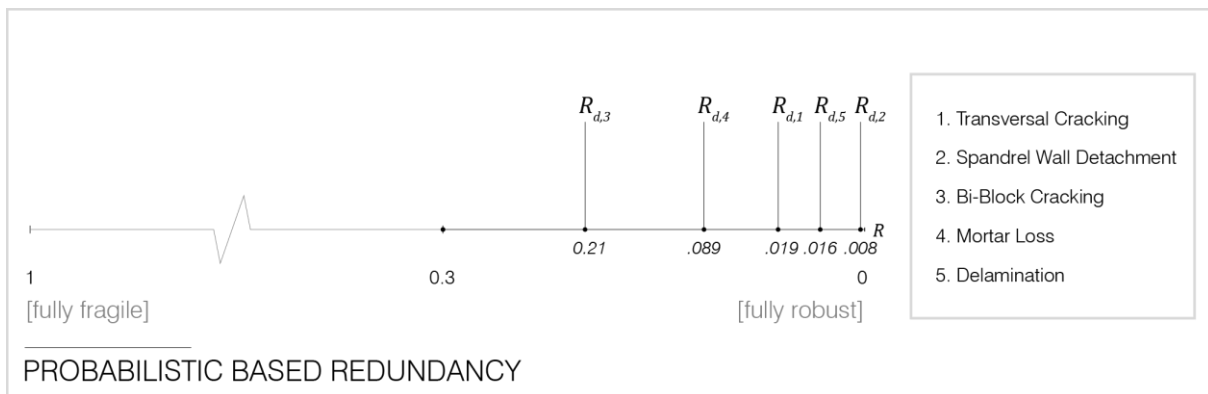


Figure 37 - Comparison of all results for probabilistic based redundancy

In the probabilistic based redundancy, the results range from 1 (fully fragile), to 0 (fully robust). The results for the bridge of Vila Mea all consist of values close to 0. Robustness once the bridge experiences spandrel wall detachment depicts almost no damage consequences, with a value of 0.008, nearing the value of 0. Robustness of the bridge, once the bridge is subjected to delamination or transversal cracking closely follow suit, with values close to 0; 0.016 and 0.019 respectively. The bridge is least robust when it is subjected to cracking due to bi-block sleepers, with a value of 0.21, as it is the value farthest from 0. Compared to the range of possible values, a value of 0.21 is still a very robust result. The order of least robust to most robust bridge once it is subjected to a type of damage is the same all throughout the three methods used to assess the robustness of the bridge of Vila Mea.

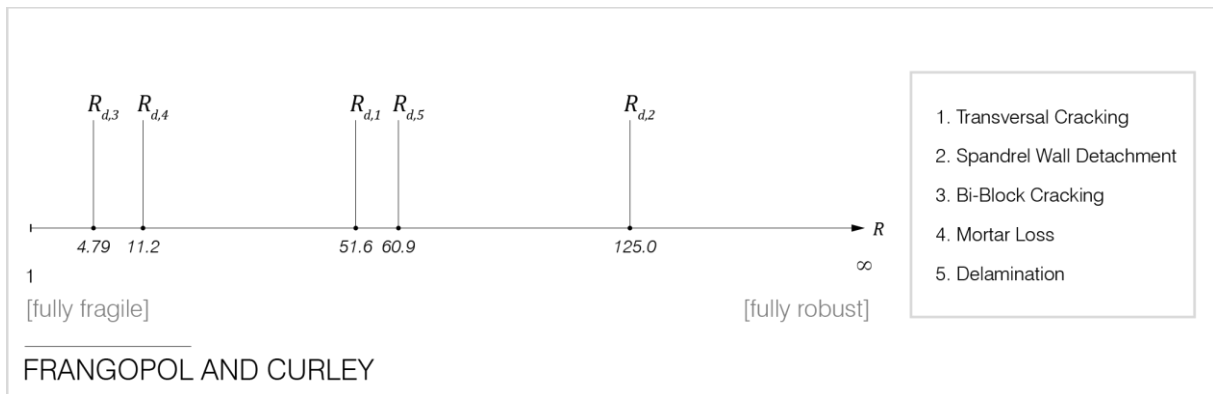


Figure 38 - Comparison of all results for the method by Frangopol and Curley

Frangopol and Curley propose a method in which the possible results are bounded in the lower end by 1 (fully fragile) and unbounded in the upper end (fully robust). This method offers less precision in comparing values of robustness. In fact, it is important to note that if, for example, a first resulting value is twice as great as a second resulting value, it does not necessarily signify that the first structure is twice as robust as the second one. The results of the robustness assessment of the bridge of Vila Mea range from 4.79 to 125.0—the lowest value consists of robustness once the bridge experiences cracking due to bi-block sleepers, while the highest value consists of robustness once the bridge is subjected to spandrel wall detachment. By comparing the relatively low value of the robustness when the bridge is subjected to cracking due to bi-block sleeper with the rest of the results, one may believe that the structure is not robust when it undergoes the latter damage. However, due to the unbounded properties of this method, the value of the results is not proportional to the robustness expressed by the structure. Nevertheless, this method still produced the same order of robustness when the bridge of Vila Mea is subjected to damage scenarios as the other two methods.

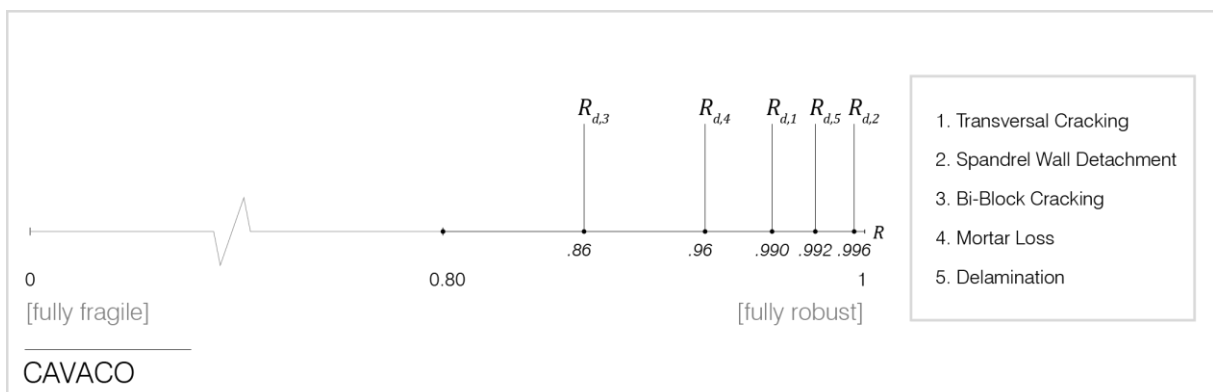


Figure 39 - Comparison of all results for the method by Cavaco

Cavaco's method offers a range of results from 0 to 1, but contrarily to the probabilistic based redundancy method, the value of 0 represents full fragility of the structure, while the value of 1 represents full robustness of the structure. All results from the robustness assessment of the bridge of Vila Mea are located near the value of 1. The lowest value, 0.86, consists of the robustness when the bridge is subjected to cracking due to bi-block sleepers. This value, farthest from 1, indicates that the bridge is least robust when experiencing load concentration from bi-block sleepers. Robustness values when the bridge experiences transversal cracking or delamination, or spandrel wall detachment all are equal or greater than 0.99—namely 0.96, 0.996, 0.992, respectively—representing most robust structure scenarios, as these are very close to the value of 1. As with the other methods, the most robust value consists of the detachment of spandrel wall, with a value of 0.996. By analysis, it can be said that the bridge structure of Vila Mea experiences negligible damage consequence when the bridge undergoes transversal cracking or delamination or spandrel wall detachment. Mortar loss presents the bridge with low damage consequences. Cracking due to bi-block sleepers presents the highest damage consequence—however compared by the possible range of results, the damage consequences is still relatively low.

All methods applied to assess the robustness of the bridge of Vila Mea offer the same order of least to greatest robustness. Cavaco's method, as well as the probabilistic based redundancy approach offer greater precision in their results, while Frangopol and Curley offer a more abstract approach. Therefore, Cavaco's method and probabilistic based redundancy offer a better understanding of robustness from different damage scenarios of the bridge of Vila Mea. The order of robustness, from least robust to most robust, is as follows: i) longitudinal cracking due to bi-block sleepers, ii) mortar loss, iii) transversal cracking, iv) delamination, v) longitudinal cracking due to spandrel wall detachment. It is interesting that both scenarios presenting longitudinal cracking—longitudinal cracking due to spandrel wall detachment, and cracking due to bi-block sleepers—offer the most robust and least robust damage scenario. The results demonstrate the importance of understanding the cause of presented damages when structurally assessing a structure. Cracks are only symptoms of issues experienced by the structure. While different types of longitudinal cracks may seem similar, their location and pattern is a strong indicator of the seriousness of the issue experienced by the structure.

Often, in an attempt to increase resistance of older bridges to the increase of subjected loads, mono-block sleepers are replaced by bi-block sleepers. This leads to a concentration of

longitudinal loads onto the arch barrel. As a result, the increase of stresses at the centre of the arch may result in a longitudinal cracking pattern at the crown of the arch. Since the arch barrel consists of the main distributor of longitudinal loads, it is logical that the robustness of the overall bridge be greatly affected when the arch is cracked in such a way that it cannot distribute loads properly. Cracking at the crown of the arch due to bi-block sleepers prevents proper distribution of loads. The robustness assessment of the bridge indicates that, compared to the damages analysed, the bridge is least robust when it is subjected to cracking due to bi-block sleepers.

The second least robust damage scenario analysed in the bridge of Vila Mea consists of mortar loss. There exist two general types of masonry structure assembly; dry jointed and wet jointed masonry. In wet jointed masonry, the masonry units are bounded by some sort of binder, usually mortar. Mortar adds flexibility into the structure by adding some tensile strength to a non-tensile type of assemblage of mortar units. Mortar also allows small deformation of the masonry component during the loading and unloading of stresses in the component. By allowing some deformation, mortar limits stress propagation into adjacent mortar units. When a masonry bridge experience mortar loss, the bridge's flexibility is lowered and stress propagation is no longer limited [9]. This is presented through the robustness assessment of the bridge of Vila Mea where, due to mortar loss, the structure experiences overall weakening, leading to lower robustness.

The damage set in the middle of the assessed robustness consists of transversal cracking. Even as being in the middle of the least to most robust order, transversal cracking presents great robustness, with limited damage consequences. This is expected since the arch barrel works in compression. Therefore, transversal cracking does not directly impact structural integrity of the bridge. In fact, since the arch works in compression, the load travels perpendicular to the transversal cracking, the compression keeping the cracks closed. However, transversal cracking may lead to further damage of structure, such as loss of fill material. Therefore, an assessment of the damages resulting from transversal cracking may give a better representation of robustness of the bridge after it is subjected to transversal cracking.

The second most robust damaged scenario consists of delamination. As with transversal cracking, delamination does not usually have a direct strong impact on the structural integrity of the bridge. However, it may result in mortar loss or mortar wash-out. As discussed above, mortar loss does have an impact on the robustness of the structure. Furthermore, a great amount

of delamination may reduce the arch thickness of the bridge to the point that the reduced thickness leads to a decreased carrying capacity of the bridge. Since delamination does not have a direct impact on the structure's robustness if it is not excessive, it is logical that a bridge remains quite robust following delamination. In fact, it is important to note that each damage scenario and related robustness is highly dependent on the maximum considered damage.

Lastly, the most robust damage scenario assessed for the bridge of Vila Mea consists of longitudinal cracking due to spandrel wall detachment. Spandrel walls contain the fill over the arches. Most older MAB experience spandrel wall detachment. The wall detaches itself from the rest of the arch, leading to an opening between the spandrel wall and the arch barrel [11]. When the wall is detached from the rest of the arch, the arch support is reduced, preventing the arch to bear as much applied loads as it could before the detachment occurred. Not only does this reduce the lateral forces on the fill over the arch, the detachment of spandrel walls reduces the effective maximum width of the bridge [11]. However, after performing the robustness assessment of the bridge of Vila Mea, it is determined that the reduction of load capacity due to spandrel wall detachment is very slight. In fact, from all the damages assessed, spandrel wall detachment has the least negative impact on the bridge of Vila Mea.

6. CONCLUSIONS

6.1 Concluding remarks

While the current codes define robustness as proportionality between damage scenario and consequence [24], robustness should also be understood through the different components to which it is related. Robustness is directly related to redundancy, vulnerability as well as progressive collapse. Redundancy is often interchanged with robustness by several authors [26]. However, a redundant structure is not necessarily robust. Through redundancy, robustness is understood as the inhabitation of damage propagation. On the other hand, vulnerability is inversely proportionate to robustness—when a structure is more vulnerable, it is less robust. Finally, robustness is also related to progressive collapse. Progressive collapse consists of the spread of initial local failure of a member to the rest of the system. Progressive damage is dependant on collapse resistance, which in turn is dependant on robustness.

Three categories of methods exist to calculate robustness index: the deterministic approach, the probabilistic approach and the risk approach. The deterministic approach is easy to calculate; however, since the method focuses on the static load redistribution capability of the structure, it lacks expressiveness. Expressiveness of damage progression refers to the assumed initial damage and acceptable progression in terms of the structure and environment. Probabilistic methods include expressiveness in their solutions. Probabilistic methods account for the uncertainties of the solution by including randomness in selected variables. As a drawback, probabilistic procedures require high computational efforts. Reliability theory can be applied to the design of new structures or the evaluation of existing ones through a probabilistic method [18]. Robustness of the bridge of Vila Mea is assessed through probabilistic methods through the application of the reliability theory.

First, a numerical model is computed to perform the robustness assessment of the bridge of Vila Mea. To construct the numerical model, the geometry is based on drawings of the bridge while material properties are based on literature and engineering judgement. A variety of types of analysis can be conducted to assess the numerical model. Limit Analysis is performed to assess the bridge of Vila Mea due to the ability to control the complexity of the model. In fact, by controlling the complexity of the model, the computational effort is moderated. The limit state analysis software LimitState: RING [3] is used to perform the analysis. The numerical model allows to compute the ultimate load capacity of the structure of Vila Mea in different scenarios.

The reliability index of the undamaged system of the bridge is computed first, since it allows to consider the basic nature and randomness of variables impacting the structure of the bridge. To compute the reliability index, the bridge is assessed in a probabilistic manner. Various approaches exist in probabilistic assessment of a structure. Due to the nature of the assessment, the bridge undergoes LHS to reduce the computational effort. To reduce the number of random variables considered in LHS, a sensitivity analysis is performed. Lastly, the mean and standard deviation obtained from the probabilistic analysis and the load function are imputed into the *Basler Cornell expression*, that presents the reliability index.

To perform the sensitivity analysis, a deterministic model of the bridge of Vila Mea is created. To determine which parameters have greatest influence on the structural resistance of the bridge, various defined parameters are modified respectively in the deterministic model and the model's resistance capacity is computed at each instance. The original 13 variables are reduced to 5 variables for the reliability analysis.

The reliability of the undamaged and damaged bridge model of Vila Mea is computed. There are 5 assessed damaged bridge models namely: bridge undergoing i) transversal cracking, ii) longitudinal cracking due to spandrel wall detachment, iii) longitudinal cracking due to bi-block sleepers, iv) mortar loss, v) delamination. The computed reliability indexes allow to define the robustness of the bridge when it is subjected to specific damages. For reliability/safety analysis, the reliability index consists mathematically of the shortest distance from the origin of the reduced load and resistance distributions to the LS function. The purpose of measuring reliability consists of quantifying the safety of a structure by recognizing that each structure presents a finite probability of failure. A target reliability is defined, and the obtained reliability of a specific structure is compared to the target reliability. In the case of the bridge of Vila Mea, the undamaged structure as well as all the damaged structures present a greater reliability index than the target reliability of 3.8, obtained from the European code [43]. When arranged in a chronological order from lowest to greatest reliability index, the order is the same as the order of the robustness index later obtained.

The robustness index of the damaged bridge of Vila Mea is assessed through the previously obtained reliability indexes. Robustness of the bridge of Vila Mea is assessed through 3 different probabilistic-based approaches, namely: i) Probabilistic based redundancy; ii) Frangopol and Curley approach; iii) Cavaco method. While the resulting values from each method may not always be directly comparable, the order of the results can be compared. All

the methods resulted in the same order of least to most robust damaged structure. It is noted that the approach proposed by Frangopol and Curley offers unbounded results. On the other hand, both the probabilistic based redundancy method as well as Cavaco's method resulted in robustness values that lay closely to the fully robust bound of possible outcomes. All methods rendered different results, therefore a consensual definition of robustness is not achieved, making the comparison difficult.

The structure with the lowest robustness index consist of the bridge when it is subjected to longitudinal cracking due to bi-block sleepers load concentration. This is explained through the cracking at the crown of the arch, that prevents proper distribution of loads. The second least robust damage scenario consists of the structure that undergoes mortar loss. Mortar is an extremely important element in any masonry composition since it adds flexibility to the composition and limits damage propagation. The next three damaged structures present almost fully robust results. The structure experiencing transversal cracking displays the lowest robustness of the three-almost-fully-robust structures. Transversal cracking does not directly impact the structural integrity of a bridge, since the arch barrel works in compression. Next, the structure undergoing delamination produces an almost perfect robustness. Similarly, to transversal cracking, it is known that delamination does not have a strong direct impact on the structure's resistance capacity. Lastly, the greatest robustness is presented by the damaged structure undergoing longitudinal cracking due to spandrel wall detachment. Spandrel walls contain the fill over the arches, reducing the arch support once the spandrel wall is detached. The results demonstrate that after a certain value, the bridge's width is not fully used. Comparably, in the case of bi-block sleeper load concentration, the full width is always used, affecting the load carrying capacity of the bridge.

6.2 Further considerations

Interest in the measure of robustness did not get much attention until the second half of the 20th century. Increased interest in accurate measures of robustness appeared only in the beginning of the 21st century. It is clear that the attempt to quantify robustness, especially when dealing with specific structures, is still under development. The definition of robustness is still defined variously by several authors, with no unified theory being achieved. A clear method to calculate robustness is still not enforced. However there exists various effective methods proposed to quantify robustness. These methods require the undamaged and damaged reliability

index of a structure. To assess the robustness of a specific bridge, damages are introduced into a model, providing a reliability index of the damaged structure.

When assessing the bridge of Vila Mea, 5 damage scenarios were introduced into the model, resulting in 5 numerical models respectively. The number of models was derived from the amount of time available to perform the assessment. A larger number of damage scenario would have allowed for a better understanding of the bridge's robustness. It is also recommended to assess a specific damage scenario more in depth. In fact, the location and size of a damage will have an impact on a bridge's robustness. Therefore, the location and size of a specific damage can be varied, and the robustness assessed at each situation. This would provide an understanding on the impact the location of the damage has on the structural integrity of the bridge.

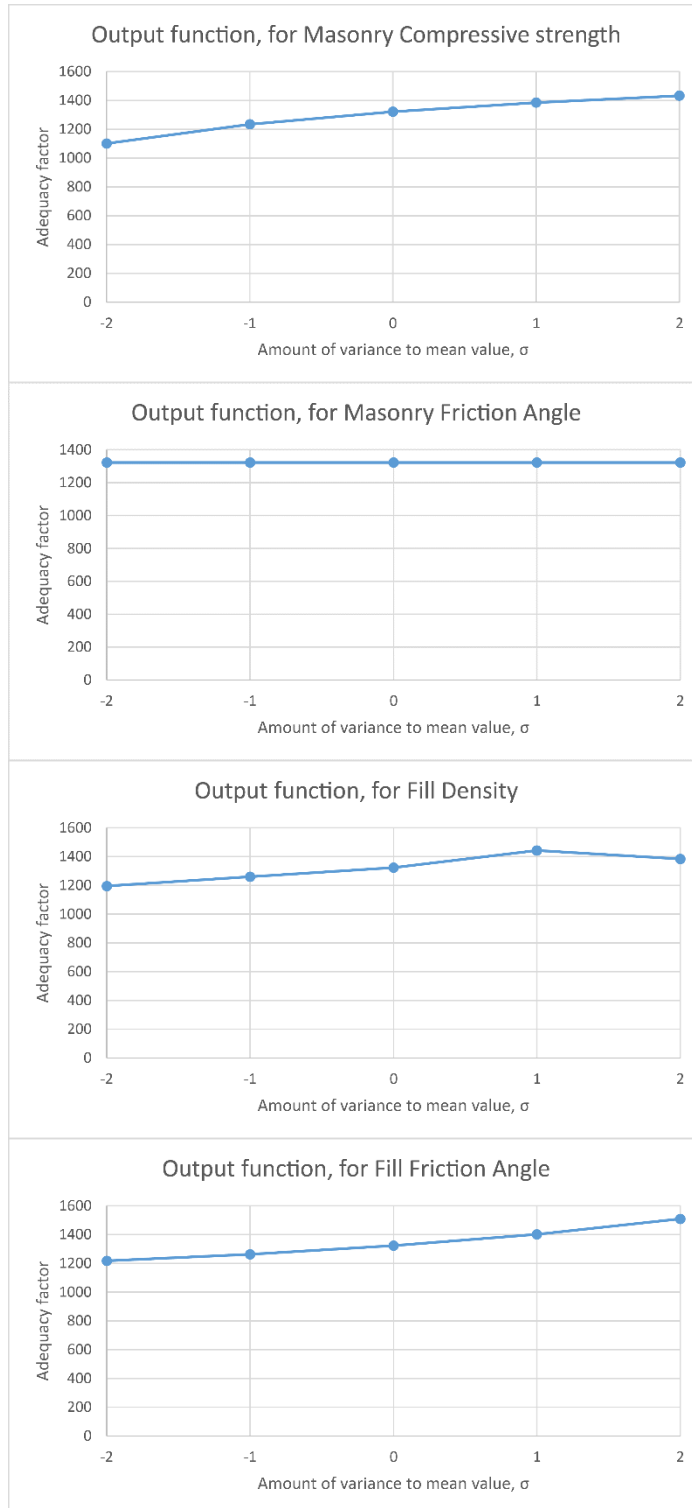
It is also important to note that a seemingly harmless damage scenario usually leads to more serious deteriorations in a bridge. Transversal cracking, as well as flooding, can lead to fill washout while delamination can lead to mortar loss. By themselves, transversal cracking and delamination seem to have almost no negative impact on the bridge's structural capacity. On the other hand, fill washout and mortar loss are both damages that have a strong negative influence on the resistance of a structure. To perform a more accurate robustness assessment, different damage cases should be combined, especially when one damage is known to lead to a second type of more serious deterioration.

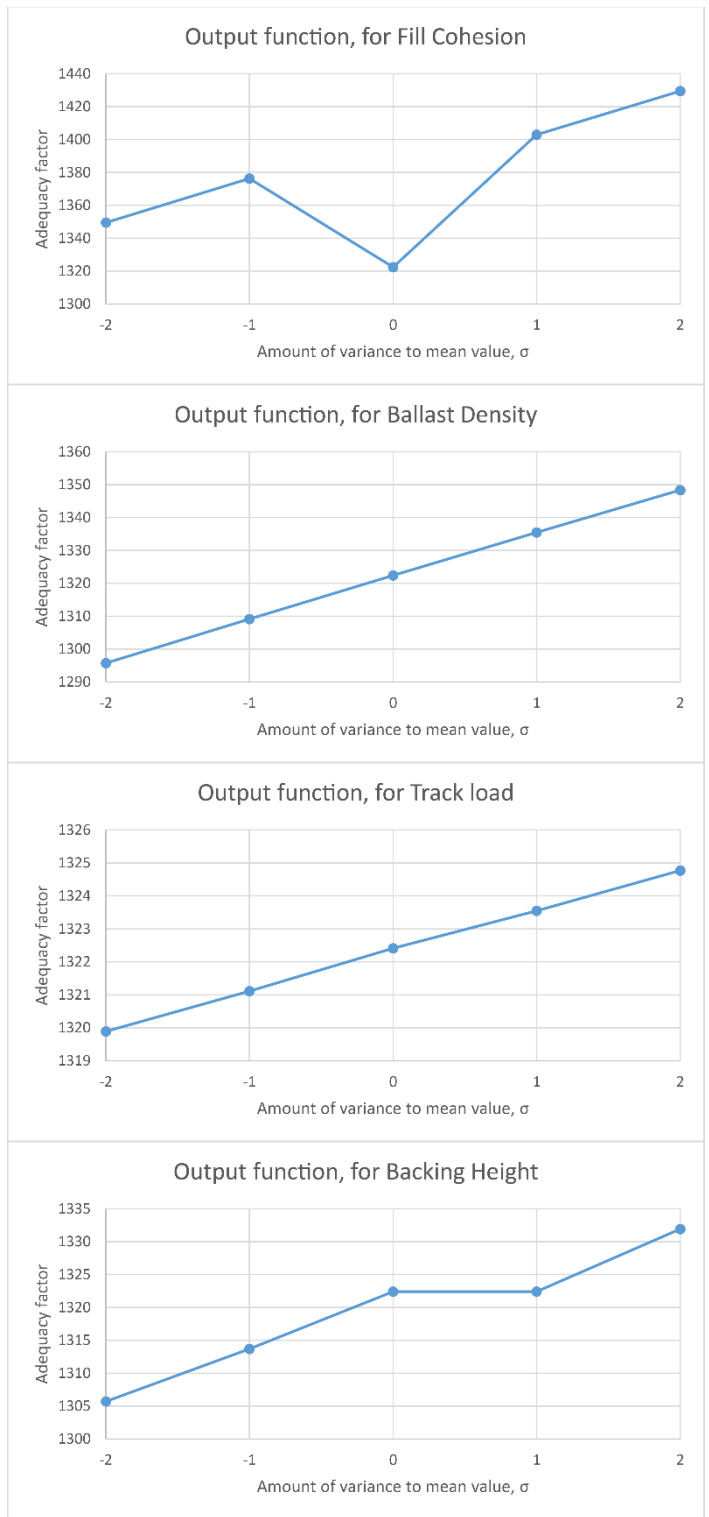
A variation in size, location, and combination of damages would allow to further the probabilistic approach in terms of robustness assessment of a bridge. Size, location, and combination of damages would become randomized variables. As a result, the model will not only acknowledge the uncertainties in terms of geometry, material and loading, but it will also recognize that uncertainties are also present in the damage to which the bridge is subjected. This would refine the robustness indexes obtained, leading to a further understanding of a structure's failure probability. A more refined robustness assessment can also be achieved by varying the method used to assess the structure. In the thesis, the structure was assessed through limit state analysis due to the control on the computational efforts. The structure could be assessed through FEM, which considers three-dimensional, non-linear or linear cases.

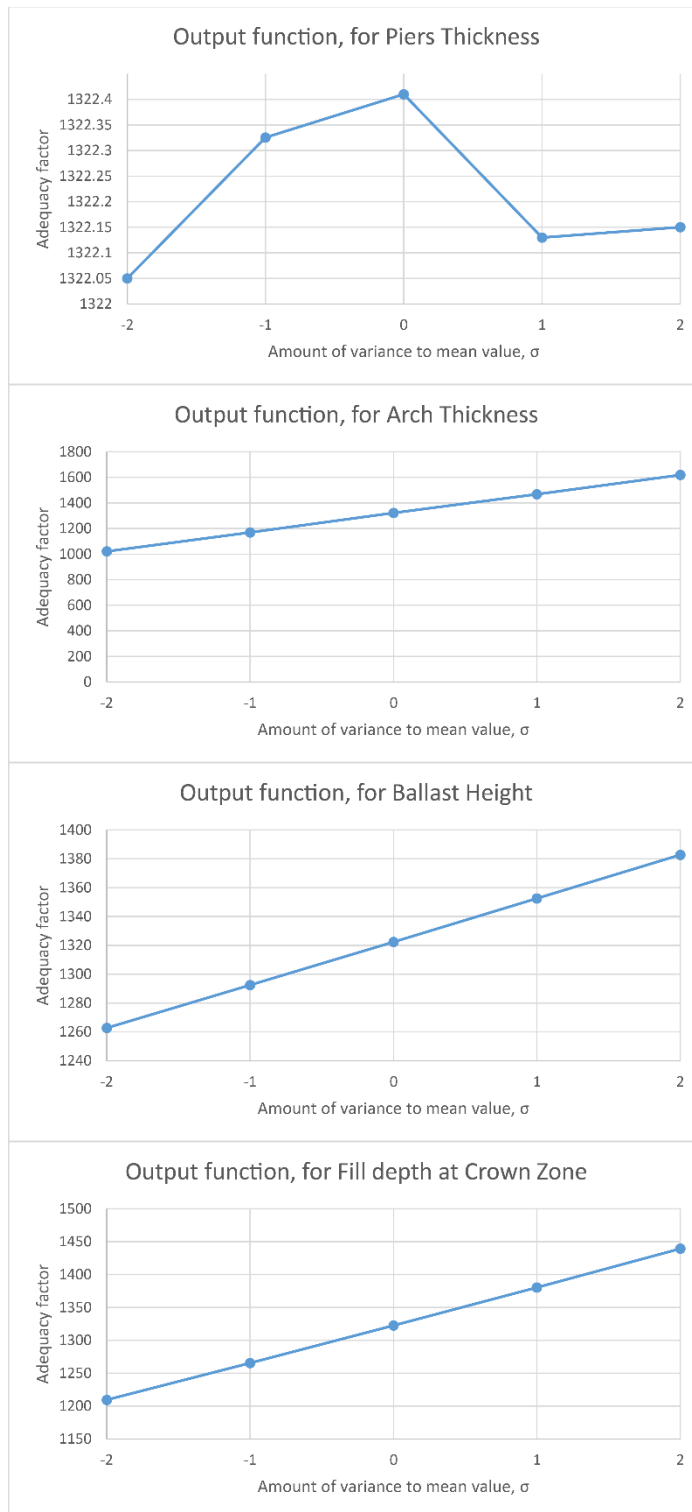
The methods used in assessing the robustness of the bridge of Vila Mea can also be expanded into different types of structures. By assessing the robustness of the bridge of Vila

Mea, the severity of each potential damage is understood. Such understanding would lead to a greater respect towards heritage structures.

7. ANNEXE 1: OUTPUT FUNCTION OF SENSITIVITY ANALYSIS







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