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Stabilization of Model-Based Networked Control Systems

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Abstract. A class of networked control systems called Model-Based Networked Control Systems (MB-NCSs) is considered. Stabilization of MB-NCSs is studied using feedback controls and simulation of stabilization for different feedbacks is made with the purpose to reduce the network traffic. The feedback control input is applied in a compensated model of the plant that approximates the plant dynamics and stabilizes the plant even under slow network conditions. Conditions for global exponential stabilizability and for the choosing of a feedback control input for a given constant time between the information moments of the network are derived. An optimal control problem to obtain an optimal feedback control is also presented.

Keywords: Model-Based Networked Control Systems, stabilization, feedback control, global exponential stabilizability.
PACS: 02.30.Yy

INTRODUCTION

With the increasing of the use of networks in the industrial control systems, many works about networked control systems have been developed and different approaches for stability and stabilization have been presented in the last years. The increasing of the use of networks on industry to transfer information is due to its flexibility and easy maintenance of a system. In systems using network is very easy to modify the control strategy by rerouting signals, and the control strategies can be activated automatically when component failure occurs. The use of a network on a control system is desirable when there is a large number of distributed sensors and actuators.

One of the problems of the use of networks in a control system is the limitation of bandwidth that is necessary to the communication network in a networked control system. Two different methods to solve this problem can be used. One of them is to minimize the transfer of information between the sensor and the controller/actuator. The second method is to compress or reduce the size of the data transferred at each transaction. As the data compression by reducing the size of the data transmitted has negligible effects on the system performance, reduce the number of transmitted packets brings better benefits than data compression. Also any delay in an information transaction is usually due to network access contention. That is, the sensor with a fast sampling rate can send through the network the latest data available resulting in an insignificant information transfer delay. But there will be contention in the network so that, even if the delay is small, the sensor data will not be available at all times for the controller/actuator. This brings us back to the idea of reducing the data transfer rate as much as possible, that is, increasing as much as possible the time between the information moments sent by the sensor to controller/actuator. Thus more bandwidth will be available to allocate more resources without sacrificing stability and overall system performance.

Many works about this subject were published, for example, [1]–[5]. Other important works about necessary and sufficient conditions for stability have been presented and optimal control problems to obtain an optimal control have been developed (see [6]–[17]). The work presented here complements the results obtained in the literature. First, we analyse if for a given Model-Based Networked Control System (MB-NCS) with a fixed feedback gain there is always a constant time between the information moments sent by the sensor to controller/actuator in order that there exists stabilization. Secondly, for a fixed constant time between the information moments sent by the sensor to controller/actuator, we obtain sufficient conditions for global exponential stabilizability. This second purpose is preceded by a simple example, where we simulate the stabilization of the control system for different fixed constant times between the information moments sent by the sensor to controller/actuator using correct feedback controls. These results help us to find the smallest frequency with that the network must update the state in the controller. Finally, we present an optimal control problem to maximize the transmission intervals and to optimize the performance of the system, but considering the required control effort.

We will consider the linear time-invariant (LTI) continuous-time systems case and the problem of having a sensor

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that is connected to the controller/actuator by a network. A plant model is used to recreate the plant behavior so that the sensor can delay sending data once the model can provide an approximation of the plant dynamics. The idea is to perform a feedback (a linear state feedback control law) by updating the model's state using the actual state of the plant that is provided by the sensor. In the rest of the time, the control action is based on a plant model that is incorporated in the controller/actuator and is running open loop for a period of h seconds. We will also assume that the plant and the model are controllable, the transportation delay is insignificant that is justifiable in most of the popular network standards like CAN bus or Ethernet, and the frequency with that the network must update the state in the controller is constant, h. The plant may be unstable.

CHARACTERIZATION OF THE CONTROL SYSTEM

In the control system described in the Introduction section, the plant, the model and the feedback control input are given by

$$\begin{array}{ll} \text{Plant:} & \dot{x} = Ax + Bu, \\ \text{Model:} & \dot{\hat{x}} = \hat{A}\hat{x} + \hat{B}u, \\ \text{Feedback control input:} & u = K\hat{x}, \end{array}$$

where $x, \hat{x} \in \mathbb{R}^n$, $A, \hat{A} \in \mathbb{R}^{n\times n}$, $B, \hat{B} \in \mathbb{R}^{n\times m}$, and $K \in \mathbb{R}^{m\times n}$. The state error is defined as $e = x - \hat{x}$, and represents the difference between the plant state and the model state. The modeling error matrices that represent the difference between the plant and the model are $\bar{A} = A - \hat{A}$ and $\bar{B} = B - \hat{B}$. Once the sensor has the full state vector available, the sensor can send the state information through the network every h seconds. Then, the update time instants are t_k, where $t_{k+1} - t_k = h$, $k = 0, 1, 2, \ldots$, and h is a constant. Since the model state is updated every t_k seconds, $e(t_k) = 0$, $k = 0, 1, 2, \ldots$. This condition for the state error is the crucial key in the characterization of the control system. Therefore, for $t \in [t_k, t_{k+1})$ and $u = K\hat{x}(t)$, we have the overall system described by

$$\begin{pmatrix} \dot{x}(t) \\ \dot{\hat{x}}(t) \end{pmatrix} = \begin{pmatrix} A & BK \\ 0 & \hat{A} + \hat{B}K \end{pmatrix} \begin{pmatrix} x(t) \\ \hat{x}(t) \end{pmatrix}$$

with initial conditions $\hat{x}(t_k) = x(t_k)$, $k = 0, 1, 2, \ldots$. Introducing the error $e(t) = x(t) - \hat{x}(t)$, we see that the dynamics of the overall system can be described by

$$\begin{pmatrix} \dot{x}(t) \\ \dot{e}(t) \end{pmatrix} = \underbrace{\begin{pmatrix} A + BK & -BK \\ \bar{A} + \bar{B}K & \hat{A} - \bar{B}K \end{pmatrix}}_{\Theta} \begin{pmatrix} x(t) \\ e(t) \end{pmatrix}, \quad \begin{pmatrix} x(t_k) \\ e(t_k) \end{pmatrix} = \begin{pmatrix} x(t_k^-) \\ 0 \end{pmatrix}, \quad t \in [t_k, t_{k+1}), \quad \text{with} \quad t_{k+1} - t_k = h. \tag{1}$$

Define by M^T the transpose of a matrix M. In work [9], as well as in other works cited here, system (1) with initial condition $\begin{pmatrix} x^T(t_0) & e^T(t_0) \end{pmatrix}^T = \begin{pmatrix} x^T(t_0) & 0^T \end{pmatrix}^T = \begin{pmatrix} x_0^T & 0^T \end{pmatrix}^T$ has the solution

$$\begin{pmatrix} x(t) \\ e(t) \end{pmatrix} = e^{\Theta(t-t_k)} \Bigg(\underbrace{\begin{pmatrix} I & 0 \\ 0 & 0 \end{pmatrix} e^{\Theta h} \begin{pmatrix} I & 0 \\ 0 & 0 \end{pmatrix}}_{\Psi} \Bigg)^k \begin{pmatrix} x_0 \\ 0 \end{pmatrix}, \quad t \in [t_k, t_{k+1}), \quad \text{with} \quad t_{k+1} - t_k = h,$$

where I is the identity matrix. In the same works (e.g., [9]) is presented the following result:

Theorem 1 ([9]) *System (1) is globally exponentially stable around the solution* $\begin{pmatrix} x^T & e^T \end{pmatrix}^T = \begin{pmatrix} 0^T & 0^T \end{pmatrix}^T$ *if and only if the eigenvalues of* Ψ *are strictly inside the unit circle.*

Applying the transformation $P = \begin{pmatrix} I & 0 \\ I & -I \end{pmatrix}$ with inverse $P^{-1} = \begin{pmatrix} I & 0 \\ I & -I \end{pmatrix}$ over Θ we easily obtain

$$\Psi = \begin{pmatrix} I & 0 \\ 0 & 0 \end{pmatrix} e^{\tilde{\Theta} h} \begin{pmatrix} I & 0 \\ I & 0 \end{pmatrix} = \begin{pmatrix} \Psi_{11} & 0 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} \sum\limits_{j=0}^{\infty} \frac{A^j + \sum\limits_{l=0}^{j-1} A^{j-l-1} BK (\hat{A} + \hat{B}K)^l}{j!} h^j & 0 \\ 0 & 0 \end{pmatrix},$$

where $\tilde{\Theta} = P\Theta P^{-1} = \begin{pmatrix} A & BK \\ 0 & \hat{A} + \hat{B}K \end{pmatrix}$. Therefore, in Theorem 1 we can change Ψ by Ψ_{11}.

STABILIZATION OF THE CONTROL SYSTEM

First, consider a first-order plant $\dot{x} = ax + bu$, $a, b \in \mathbb{R}$, where the model is $\dot{\hat{x}} = \hat{a}\hat{x} + \hat{b}u$, $\hat{a}, \hat{b} \in \mathbb{R}$, and the feedback control input is $u = k\hat{x}$, for a fixed $k \in \mathbb{R}$. For the first-order case we have

$$|\Psi_{11}| = \left| e^{ah} + \frac{bk(e^{ah} - e^{(\hat{a}+\hat{b}k)h})}{a - \hat{a} - \hat{b}k} \right| \leq 2\alpha e^{\beta h},$$

where $\alpha = \max\{|1 + bk/(a - \hat{a} - \hat{b}k)|, |bk/(a - \hat{a} - \hat{b}k)|\} > 0$ and $\beta = \max\{a, \hat{a} + \hat{b}k\}$. Consider that $\beta = 0$. If $\alpha < 1/2$, from Theorem 1, we have global exponential stability around the zero solution. The first impression is that is necessary to restrict α to obtain global exponential stability, but this is a sufficient condition and not a necessary condition. Therefore, it is necessary to study this case in more detail. Suppose that $\alpha = |1 + bk/(a - \hat{a} - \hat{b}k)|$. So we have $bk/(a - \hat{a} - \hat{b}k) = -(\alpha + 1)$ or $bk/(a - \hat{a} - \hat{b}k) = \alpha - 1$. The first equality contradicts the definition of α, because $|bk/(a - \hat{a} - \hat{b}k)| = \alpha + 1 > \alpha$. Then we have that $bk/(a - \hat{a} - \hat{b}k) = \alpha - 1$. Suppose that $a = 0$ and $\hat{a} + \hat{b}k < 0$ (these considerations respect the condition $\beta = 0$). Therefore we only have global exponential stability if and only if there exists $h > 0$ such that $|\alpha - (\alpha - 1)e^{(\hat{a}+\hat{b}k)h}| < 1$. This happens if and only if $0 < bk/(\hat{a} + \hat{b}k) < 1$. So, for the general case (n-order plant), fixed a feedback gain K, the existence of a period $h > 0$ in order that system (1) is globally exponentially stable around the zero solution depends of the matrices A, $\hat{A}$, B and $\hat{B}$.

Now, in the following example, we will present simulations of stabilization of an MB-NCS using different feedback gains for different fixed periods h. The main purpose is to verify in that conditions we can increase the period h.

Consider the plant and the model with

$$A = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \quad \hat{A} = \begin{pmatrix} -0.5 & 0.5 \\ -0.5 & -0.5 \end{pmatrix}, \quad B = \hat{B} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad \text{and} \quad x_0 = \hat{x}_0 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}.$$

Simulations of stabilization of the plant for different periods and different feedback gains are shown in Fig. 1.

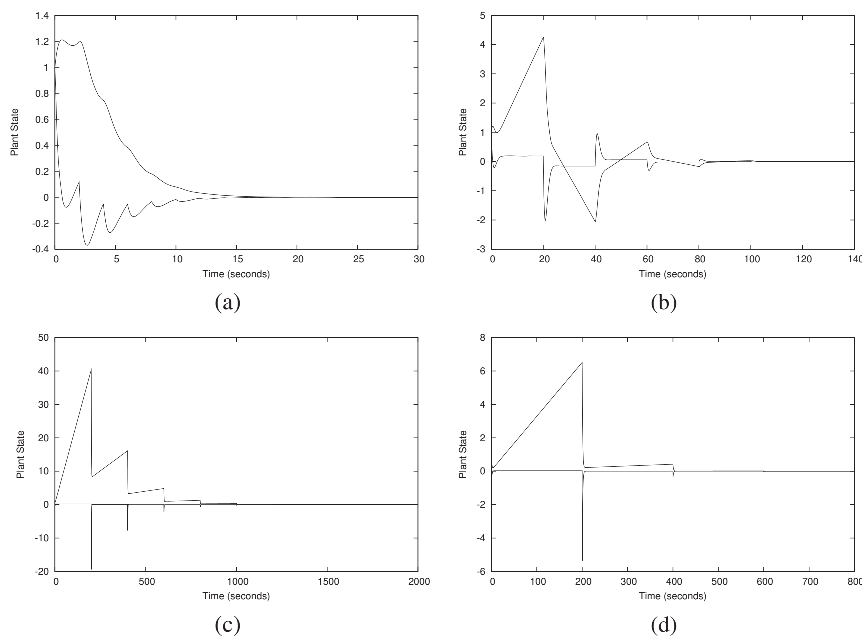

FIGURE 1. Plant States. (a) Feedback gain $K = (-1.5 \quad -2)$ and period $h = 2$ seconds. (b) Feedback gain $K = (-1.7 \quad -1.5)$ and period $h = 20$ seconds. (c) Feedback gain $K = (-2 \quad -2)$ and period $h = 200$ seconds. (d) Feedback gain $K = (-15 \quad -15)$ and period $h = 200$ seconds.

From Fig. 1, we can see that for this example we increased the period h until 200 seconds and we always obtained stabilization of the plant. For this purpose, we just chose the correct feedback gains. In Fig. 1 (a), for a period of 2 seconds, we have chosen the feedback gain $K = \begin{pmatrix} -1.5 & -2 \end{pmatrix}$. The matrix Ψ_{11} has the maximum of eigenvalue magnitude less than $0.4 < 1$, so it verifies the condition of Theorem 1 and system (1) is globally exponentially stable.

In Fig. 1 (b), we have chosen the feedback gain $K = \begin{pmatrix} -1.7 & -1.5 \end{pmatrix}$ for a period of 20 seconds. The matrix Ψ_{11} has the maximum of eigenvalue magnitude less than $0.3 < 1$. So, system (1) is also globally exponentially stable. In Fig. 1 (c), for a period of 200 seconds, we used the feedback gain $K = \begin{pmatrix} -2 & -2 \end{pmatrix}$. In this case, the maximum of eigenvalue magnitude of the matrix Ψ_{11} is approximately equal to $0.2 < 1$, so we also have global exponential stability.

Inspired in the previous example, we will investigate what is the relation that there must be between the period h and the feedback gain K for we have stabilization. From Gerschgorin circle theorem (see [18], [19]), we know that a complex $n \times n$ matrix C, with entries c_{ik}, $i,k = 1,2,\ldots,n$, has its eigenvalues inside the union of the closed discs $D(c_{ii}, R_i)$ centered at c_{ii} with radius $R_i = \sum_{k=1,k\neq i}^{n} |c_{ik}|$ for all $i \in \{1,2,\ldots,n\}$. Thus, for system (1) to be globally exponentially stable, it is sufficient that $D(\psi_{ii}, R_i) \subset D(0,1)$ for all $i \in \{1,2,\ldots,2n\}$, where ψ_{ii} are the diagonal entries of the matrix Ψ, $R_i = \sum_{k=1,k\neq i}^{2n} |\psi_{ik}|$ for ψ_{ik}, $i,k = 1,2,\ldots,2n$, the entries of Ψ, and $D(0,1)$ is the unit circle (centered at 0 with radius 1). That is, the absolute values of the entries of Ψ_{11} must be sufficiently small. As $\sum_{j=0}^{\infty} \varepsilon/j! = \varepsilon e$ for a positive constant ε, we just make $|(A^j + \sum_{l=0}^{j-1} A^{j-l-1} BK(\hat{A} + \hat{B}K)^l)_{ik}| h^j \leq \varepsilon$, for $i,k = 1,2,\ldots,n$ and ε sufficiently small. This is the same to make $\|A^j + \sum_{l=0}^{j-1} A^{j-l-1} BK(\hat{A} + \hat{B}K)^l\| = 1/(\delta(j)h^j)$, for $\delta(j)$ sufficiently large for all j, where $\|\cdot\|$ is a matrix norm. Therefore we have the result that follows.

Theorem 2 *Fixed a period $h > 0$, system (1) is globally exponentially stable around the solution $\begin{pmatrix} x^T & e^T \end{pmatrix}^T = \begin{pmatrix} 0^T & 0^T \end{pmatrix}^T$ if there exists a feedback gain K such that $\|A^j + \sum_{l=0}^{j-1} A^{j-l-1} BK(\hat{A} + \hat{B}K)^l\| = 1/(\delta(j)h^j)$, for $\delta(j)$ sufficiently large for all j.*

The above study helps us to increase the time between the information moments sent by the sensor to controller/actuator, that is, helps us to reduce the network traffic. But we can see in Fig. 1 that this increasing also increases the time of stabilization. So, the performance of the system decreases. A solution to reduce this problem is to use a better feedback gain. We can see an example to reduce this problem in Fig. 1 (d). Using a new feedback gain $K = \begin{pmatrix} -15 & -15 \end{pmatrix}$ we also obtain global exponential stability for system (1) with a period of 200 seconds, but with a much more fast stabilization. The matrix Ψ_{11} has the maximum of eigenvalue magnitude less than $0.04 < 1$. However, this solution can be an obstacle to the required control effort. Therefore, we should also consider this parameter in our stabilization process. To obtain an optimal period h and an optimal feedback gain K we must solve an optimal control problem. For this purpose, we will present an optimal control problem when the model is nominal. Once the complete nominal error dynamics can be represented by $\dot{e} = (1-\gamma(h))\hat{A}e$, with $\gamma(h) = 1$ for $t = t_k$, and $\gamma(h) = 0$ for $t \in (t_k, t_k + h)$, $k = 0,1,2,\ldots$, we present an optimal control problem as follows:

$$\begin{aligned} & \max_{K \in \eta B_{m\times n}} \; h, \\ \text{subject to} \quad & \left\| \hat{A}^j + \sum_{l=0}^{j-1} \hat{A}^{j-l-1} \hat{B}K(\hat{A} + \hat{B}K)^l \right\| = \frac{1}{\delta(j)h^j}, \\ & e \to 0, \quad \dot{e} = (1-\gamma(h))\hat{A}e, \quad \gamma(h) \in \{0,1\}, \\ & \hat{x}(t_0) = \hat{x}_0, \quad \hat{x}(T) = 0, \quad T > t_0, \end{aligned}$$

for $\delta(j)$ sufficiently large for all j, and where η is a positive constant and $B_{m\times n} = \{X \in \mathbb{R}^{m\times n} : \|X\| \leq 1\}$.

CONCLUSION

The main purpose of this work was to reduce the network traffic in the stabilization of MB-NCSs because of the limitation of bandwidth that is necessary to the communication network. Then, we presented sufficient conditions to find the smallest frequency with that the network must update the state in the controller, that is, sufficient conditions that help us to find a feedback control input that increases the time between the information moments sent by the sensor to controller/actuator. However, we have seen that for large times between the information moments, the performance of the system decreases. So, we presented an optimal control problem to maximize the transmission intervals and to optimize the performance of the system, but considering the required control effort. Therefore, we conclude that we can consider large times between the information moments sent by the sensor to controller/actuator since the conditions presented are satisfied. No less important was the proof that, for a fixed feedback gain, the existence of a time between the information moments in order to have stabilization depends of the matrices of the plant and the model. Equivalent considerations can be derived for LTI discrete-time systems, output feedback plants, and for network delay cases.

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