
WORKING PAPER

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**“A Time-Frequency Analysis of
Sovereign Debt Contagion in Europe”**

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A Time-Frequency Analysis of Sovereign Debt Contagion in Europe*

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Abstract

This paper adopted a wavelet approach to investigate the financial contagion in the Eurozone debt market during various crisis-ridden periods in the zone. We used weekly 10-year bond yield data and showed that until the onset of the financial crisis of 2007/2008, bond yields were highly synchronised among all countries. However, the bond yields in Greece, Ireland, Italy, Spain, and Portugal became unsynchronised with core countries after 2008. Similarly, there was no synchronisation among the periphery countries during this period, except for Italy and Spain.

We found evidence of contagion emanating from Ireland during the first part of the sovereign debt crisis until around 2010, and from Greece afterwards. We also established that contagion spread to Portugal, Greece and Ireland, and can be observed at high frequencies. However, Italy and Spain were not affected. At business cycle frequencies, we found that the Greek crisis propelled a flight-to-quality flow to Belgium, Finland, France and Germany.

Keywords: Contagion; European Sovereign Debt; Cross-market Co-movements; Wavelet Partial Coherency; Partial Phase-Difference; Wavelet Distance

JEL classification:

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1 Introduction

The increasing interconnectedness of the global economy and the rapid integration of global financial markets promote global economic growth, increase volume and velocity of international financial transactions, and improve capital flows to many countries (Gereffi, 2005; Kenc and Dibooglu, 2010; Rajan, 2006). On the flip side, it poses challenges to global economic and financial architectures. For instance, global financial markets have witnessed many financial and currency crises in the last four decades. One feature of these crises is the snowballing effect from one market or geographical location to another. While this undesired domino effect of financial market crises is generally called contagion, there is no consensus on the definition or measurements of contagion.

Some prominent definitions include: a substantial increase in the conditional probability of a crisis in one country relative to another country; volatility asset-price spillover from a crisis-ridden country to another; cross-country co-movements of asset prices that cannot be explained by fundamentals; substantial increases in co-movements of prices and quantities across markets conditioned on the occurrence of a crisis in one market or collection of markets; intensified shock transmissions or changes after a shock in one market; substantial increases in the cross-market correlation during turmoil (see Dungey et al. (2005), Forbes and Rigobon (2002) and Pericoli and Sbracia (2003) for details).

Overall, four fundamental considerations are vital in defining contagion: significant cross-market correlation, measures of shock transmission across markets, differentiate between contagion and interdependence and distinguish between normal and excessive co-movements across financial markets. Numerous studies incorporated these features of contagion in their definitions. For example, Dornbusch et al. (2000) define it as a significant increase in cross-market linkages after a shock to at least one country. Forbes and Rigobon (2002) expanded on this definition and argued that contagion implies a fundamental difference in cross-market difference after a shock to one market while interdependence emphasises no significance change in cross-market relationships.

Despite a proliferation of studies on contagion, they adopted different methodologies and the

results are mixed. For instance, Masih and Masih (1999) examined the short- and long-term dynamic linkages among stock markets in OECD countries and Asia using a VAR model and found that regional markets rather than the OECD markets explain stock market fluctuations in Asia. Similarly, Edwards and Susmel (2001) utilised switching volatility models to analyse the evolution of volatility in Latin American countries and found short-lived high-volatility episodes, lasting between two and twelve weeks, and accompanied by volatility co-movements across countries in the sample. Also, Schwert (1990) found a dramatic jump in stock return volatility during and after the crash while such market-return volatility can bias correlation coefficients and induce heteroscedasticity (also see Forbes and Rigobon, 2002).

Furthermore, Bekaert et al. (2014) analysed the crisis transmission to country-industry equity portfolio using a factor model in 55 countries and found limited evidence of contagion from both US markets and the global financial sector. However, the study found substantial evidence of contagion from domestic equity markets to individual domestic equity portfolios, but its severity is inversely related to the quality of economic policies and fundamentals of the countries. Similarly, there is a strand of literature exploring the recent sovereign debt crisis in Europe. For example, Missio and Watzka (2011) used dynamic conditional correlation models to assess contagion during the European debt crisis and found that yield returns in Belgium, Italy, Portugal, and Spain increased as Greece experienced increasing yield spread with Germany.

Similarly, Arghyrou and Kontonikas (2012) explored the European sovereign debt crisis and found a shift in market pricing behaviour from a convergence-trade model to one propelled by macro-fundamentals and international risk. Specifically, the study found that other EMU countries experienced contagion from Greece but found no significant speculation effects from CDS markets. Giordano et al. (2013) investigated the link between a sharp increase in the sovereign spread of Eurozone countries and deterioration of macroeconomic and fiscal fundamentals or financial contagion after the Greek crisis and found evidence of wake-up contagion, but not pure contagion. Martins and Amado (2018) found long-run contagion effects across peripheral countries while Broto and Pérez-Quirós (2015) and Mink and de Haan (2013) found Greece, Portugal and Ireland as sources of contagion.

One major takeaway from the definition provided by Forbes and Rigobon is that transmission of shocks beyond what fundamentals can explain drives contagion. With fundamentals unable to explain contagion, some authors view it as a short-run phenomenon due to the typical rigidity of fundamentals in this time horizon. This view galvanises a strand of literature utilising either spectral analysis or wavelet analysis to explore contagion. This literature considers contagion as a temporary and significant shift in cross-market linkages and interdependence as a permanent shift in cross-market linkages after a shock (Bodart and Candelon, 2009). With the established link between period (time horizon) and frequency, several authors — such as Bodart and Candelon, 2009, Gallegati, 2012 and Orlov, 2009 — associate contagion with high frequencies and interdependency with low frequencies.

While our work builds on the papers mentioned in the previous paragraph, we propose a different way to distinguish contagion from interdependence. As stressed by Martins and Amado (2018), an increase in the correlation between financial series during times of turmoil is not sufficient evidence of contagion. It may be merely the result of higher volatility accompanied by stable and substantial interdependence. Contagion is the change in market interdependence during periods of high volatility. Similarly, our paper contrasts with most of the existing literature and relates to Martins and Amado (2018) in two different ways. One, we do not impose a pre-defined date for the turmoil; instead, we allow the dynamics of the bond-yield data to speak for itself. Two, our wavelet approach allows us to simultaneously work in different frequencies and automatically enables us to distinguish between the short- and the long-run.

Our approach relies mainly on the concept of partial wavelet coherence proposed by Aguiar-Conraria and Soares (2014). Aguiar-Conraria, Soares and Sousa (2018), Ko and Funashima (2019), and Verona (forthcoming) have recently applied this concept to financial time series. In this paper, we use a set of control countries, called the core countries, to control for structural interdependence. To identify the core countries, we rely on a wavelet dissimilarity measure proposed by Aguiar-Conraria and Soares (2011), and also applied by Aguiar-Conraria, Magalhães, and Soares (2013), Aguiar-Conraria, Martins and Soares (2013), and Flor and Klarl (2017). The core countries have been mostly immune to the turmoil that affected the sovereign yields. The

coherency between the core countries and each peripheral country captures ordinary market interdependence and synchronization between countries across frequencies. This allows us to identify precise time and frequencies that the highly affected countries cease anchoring to the core countries.

Furthermore, we explore the highly affected countries for evidence of contagion, and if any, the source of such contagion. We identify contagion as the leftover coherence — partial coherency — between countries. Therefore, when studying the contagion between two countries, say Portugal and Greece, we estimate the partial coherency between the yields of these two countries after controlling for what the core countries can explain.

The paper proceeds as follows. Section 2 describes the wavelet tools used in the paper. Section 3 identifies the core and the peripheral countries, analyses the time-frequency relationship between the core and the peripheral countries, and tests for evidence of contagion between peripheral countries while Section 4 concludes.

2 The Continuous Wavelet Transform

The determination of the most relevant cyclical components of a time series typically requires using the Fourier transform to transform the time series from the time domain to the frequency domain. However, the time information is lost under the Fourier transform, and this makes it impossible to view the evolution of the frequency contents of the series over time. For this reason, Fourier analysis is only appropriate for stationary time-series. For non-stationary series, which are predominant in Economics and Finance, a time-frequency representation is needed. Such type of representation can be obtained, in an efficient manner, with the wavelet analysis. The importance of this technique was recognized in 2017, when the French mathematician Yves Meyer received the 2017 Abel Prize, also known as the Nobel Prize for mathematics, ‘for his pivotal role in the development of the mathematical theory of wavelets’ — e.g., see Meyer (1993).

Wavelets appeared in the mid-1980s (Grossmann and Morlet 1984; Goupillaud et al. 1984) to solve problems in geophysics. Virtually all fields of engineering and applied science rapidly adopted them.

In Economics, the first studies used discrete versions of the wavelet transform — Crowley (2007) is a good starting point, and Gallegati et al. (2011) provide a very nice application of this literature. In the last ten years, several studies have adopted the Continuous Wavelet Transform (CWT). It is almost impossible to keep track of papers applying CWT to economic data. Flor and Klarl (2017), Bekiros et al. (2017), Ko and Funashima (2019), and Verona (forthcoming) are just a few recent examples of this literature.

We provide a brief description of the continuous wavelet tools used in our analysis in this section. We refer the readers to Aguiar-Conraria and Soares (2014), Aguiar-Conraria, Soares, and Sousa (2018), Aguiar-Conraria, Martins, and Soares (2018) for more technical details. Readers interested in more intuitive explanations may rely on Aguiar-Conraria, Magalhães, and Soares (2012 and 2013). For a broader view on the digital signal processing and spectral analysis, including the Continuous Wavelet Transforms, Alessio (2016) provides an excellent read. For all practical purposes, a *wavelet* is simply a small wave: a *wave*, in the sense that it is a function $\psi(t)$ whose graph oscillates up and down the t -axis (integrating to zero) and *small* meaning that it rapidly decays as $t \rightarrow \pm\infty$.

A prototype wavelet ψ (the so-called *mother wavelet*) is used to generate a family $\psi_{\mathbf{t},\mathbf{s}}$ of wavelets (wavelet daughters) by scaling and translation operations:

$$\psi_{\mathbf{t},\mathbf{s}}(t) = \frac{1}{\sqrt{|\mathbf{s}|}} \psi\left(\frac{t - \mathbf{t}}{\mathbf{s}}\right), \mathbf{t}, \mathbf{s} \in \mathbb{R}, \mathbf{s} \neq 0.$$

The scaling parameter \mathbf{s} controls the width of the wavelet¹ and the translation parameter \mathbf{t} controls the location of the wavelet along the t -axis.

Given a function (time-series) $x(t)$, its *continuous wavelet transform* (CWT) with respect to the wavelet ψ is a function of two-variables, $W_x(\mathbf{t}, \mathbf{s})$, obtained by “comparing” x with all the wavelet daughters $\psi_{\mathbf{t},\mathbf{s}}$, as follows:

$$W_x(\mathbf{t}, \mathbf{s}) = \int_{-\infty}^{\infty} x(t) \bar{\psi}_{\mathbf{t},\mathbf{s}}(t) dt = \frac{1}{\sqrt{|\mathbf{s}|}} \int_{-\infty}^{\infty} x(t) \bar{\psi}\left(\frac{t - \mathbf{t}}{\mathbf{s}}\right) dt, \mathbf{t}, \mathbf{s} \in \mathbb{R}, \mathbf{s} \neq 0. \quad (1)$$

¹Since ψ is an oscillatory function, it makes sense to consider it as a function of a given frequency; when $|\mathbf{s}| < 1$, $\psi_{\mathbf{t},\mathbf{s}}$ becomes a compressed version of ψ , i.e. is a function of higher frequency, and when $|\mathbf{s}| > 1$, $\psi_{\mathbf{t},\mathbf{s}}$ becomes a dilated version of ψ and thus corresponds to a function of lower frequency.

Remark 1 *In the above formula, and in what follows, the over-bar is used to denote complex conjugation.*

Remark 2 *As we proceed from here, all the quantities we intend to introduce are functions of scale and time. To simplify the notation, we will describe the procedure to obtain these quantities for a specific value of the argument (\mathbf{t}, \mathbf{s}) which, unless duly necessary, will be omitted in the formulas.*

There are several types of wavelet functions available with different characteristics, and these include Morlet, Mexican hat, Haar and Daubechies. We need a complex wavelet, as it yields a complex transform, with information on both the amplitude and phase, and this is essential to study the business cycle synchronisation between different time-series. The wavelet we used in our computations is a particular member of the so-called *Morlet family*, defined by

$$\psi(t) = \pi^{-\frac{1}{4}} e^{i\omega_0 t} e^{-t^2/2}, \quad (2)$$

obtained by considering $\omega_0 = 6$; this is the complex wavelet mostly used in Economics, due to its interesting properties. In particular, the Morlet wavelet has optimal joint time-frequency concentration. When such wavelet is used, one can consider that the Fourier frequency f satisfies $f \approx \frac{1}{s}$, and this greatly facilitates the interpretation of the results.

2.1 The Wavelet Power Spectrum and the Wavelet Phase

Given the choice of a complex wavelet, the wavelet transform W_x is also complex-valued and can, therefore, be expressed in polar form as $W_x = |W_x| e^{i\phi_x}$, $\phi_x \in (-\pi, \pi]$. The angle ϕ_x is referred to as the *(wavelet)-phase* and the square of the modulus of W_x is called the (local) *wavelet power spectrum* and is denoted by $(WPS)_x$, i.e.

$$(WPS)_x = |W_x|^2. \quad (3)$$

We can interpret the wavelet power spectrum as depicting the local variance of a time-series

in the time-scale (or time-frequency) plane.

2.2 Wavelet Coherency and Wavelet Phase-Difference

To deal with the time-frequency dependencies between two non-stationary time-series, we may use the so-called cross-wavelet power, wavelet coherency and wavelet phase-difference, which naturally generalised the basic wavelet analysis tools to the bivariate case. Given two time-series, $x(t)$ and $y(t)$, their *cross-wavelet transform*, W_{xy} , is simply defined as $W_{xy} = W_x \overline{W_y}$, where W_x and W_y are the wavelet transforms of x and y , respectively. We also define their *cross-wavelet power*, as $|W_{xy}|$. The cross-wavelet power of two time-series depicts the local covariance between these functions at each time and frequency. In analogy to the concept of coherency used in Fourier analysis, one can define the *complex wavelet coherency* ϱ_{xy} of series x and y by:

$$\varrho_{xy} = \frac{S(W_{xy})}{[S(|W_x|^2) S(|W_y|^2)]^{1/2}}, \quad (4)$$

where $S(\cdot)$ denotes a smoothing operator in both time and scale; smoothing is necessary, because, otherwise, coherency would have modulus one at all scales and times. Time and scale smoothing can be achieved by convolution with appropriate windows.

As with the wavelet transform, the complex wavelet coherency can be written in polar form, as $\varrho_{xy} = |\varrho_{xy}| e^{i\phi_{xy}}$. The absolute value of the complex wavelet coherency is called the *wavelet coherency* and it is denoted by R_{xy} and the angle ϕ_{xy} of the complex coherency is called the (*wavelet*) *phase-difference*. The angle ϕ_{xy} is obtained from both the real part $\Re(\varrho_{xy})$ and the imaginary part $\Im(\varrho_{xy})$ of ϱ_{xy} by using the formula

$$\phi_{xy} = \text{Arctan} \left[\frac{\Im(\varrho_{xy})}{\Re(\varrho_{xy})} \right], \quad \phi_{xy} \in (-\pi, \pi], \quad (5)$$

together with the information on the signs of $\Re(\varrho_{xy})$ and $\Im(\varrho_{xy})$ to determine the quadrant that the angle belongs to.²

²The phase-difference is sometimes defined using the spectra without smoothing, i.e. as the phase of the cross-wavelet transform. In this case, it is easy to see that $\phi_{xy} = \phi_x - \phi_y$ (or, to be more precise, $\phi_{xy} = (\phi_x - \phi_y) \bmod 2\pi$), justifying its name. The definition we adopted here corresponds to the one we used in our computer programs.

A phase-difference of zero indicates that the time series move together at the specified time-frequency value; if $\phi_{xy} \in (0, \frac{\pi}{2})$, then the series move in phase, but the time-series x leads y ; if $\phi_{xy} \in (-\frac{\pi}{2}, 0)$, then y leads; a phase-difference of π indicates an anti-phase relation; if $\phi_{xy} \in (\frac{\pi}{2}, \pi)$, then y leads; time-series x leads if $\phi_{xy} \in (-\pi, -\frac{\pi}{2})$.

2.3 Contagion and the Partial Wavelet Coherency and Partial Wavelet Phase-Difference

To estimate the contagion between two countries, say Portugal and Greece, we need to estimate the interdependence, in the time-frequency domain, between the yields of these two countries after eliminating the effect of yield returns of other countries. To do this, we rely on the concept of partial coherency. If we find that, after controlling for effects for other countries, the (partial) coherency Portugal and Greece is significant in some region of the time-frequency space, we conclude that there is contagion. The Partial Phase-difference identifies the originating country of such contagion.

The *complex partial wavelet coherency* between x and y after controlling for z , denoted by $\varrho_{xy,z}$, is the quantity given by

$$\varrho_{xy,z} = \frac{\varrho_{xy} - \varrho_{xz}\overline{\varrho_{yz}}}{\sqrt{(1 - R_{xz}^2)(1 - R_{yz}^2)}}.$$

The *partial wavelet coherency* of x and y after controlling for z , denoted by $R_{xy,z}$, is simply the absolute value of the complex partial wavelet coherency, and the *partial phase-difference* of x over y , given z , denoted by $\phi_{xy,z}$, is the phase-angle of $\varrho_{xy,z}$. The reader can find the general formula for n variables in Aguiar-Conraria and Soares (2014).

2.4 Core Countries and the Wavelet Spectra Dissimilarity

We rely on a wavelet-based dissimilarity index proposed by Aguiar-Conraria and Soares (2011) to identify the core countries, and this index has applied in various studies, such as Aguiar-Conraria, Martins and Soares (2013), Aguiar-Conraria, Magalhães, and Soares (2013), Aguiar-

Conraria et al. (2017) and Flor and Klarl (2017). The basic idea is to find a suitable way to compare the wavelet spectra of two time-series. If their wavelet spectra are very similar, then the dissimilarity index will be very close to zero. A dissimilarity of zero between the yields of two countries implies that they share the same high-power regions and their phases precisely aligned. That is, the contribution of cycles at each frequency to the total variance is similar, and this contribution occurs at the same time while the ups and downs of each cycle coincide. It is in this sense that we say the two countries are synchronised. The core countries will be a group of countries that are highly synchronised while the synchronisation between peripheral countries should be substantially lower.

To measure the dissimilarity between countries x and y , we start by computing the Singular Value Decomposition (SVD) of the matrix $W_x W_y^H$, where W_y^H is the conjugate transpose of W_y , to focus on the common high power time-frequency regions.³ With this method extracting the components that maximise covariances, the first extracted K components correspond to the most important common patterns of the wavelet transforms.⁴ The wavelet distance between the wavelet spectra of two countries x and y , denoted by $\text{dist}(W_x, W_y)$, is computed as:

$$\text{dist}(W_x, W_y) = \frac{\sum_{k=1}^K \sigma_k^2 [d(\mathbf{l}_x^k, \mathbf{l}_y^k) + d(\mathbf{u}_k, \mathbf{v}_k)]}{\sum_{k=1}^K \sigma_k^2}. \quad (6)$$

In the above formula, \mathbf{u}_k and \mathbf{v}_k are the singular vectors and σ_k the singular values obtained in the SVD and \mathbf{l}_x^k and \mathbf{l}_y^k are the so-called leading patterns, given by $\mathbf{l}_x^k = \mathbf{u}_k^H W_x$ and $\mathbf{l}_y^k = \mathbf{v}_k^H W_y$. We compute the distance $d(\mathbf{u}, \mathbf{v})$ between two vectors \mathbf{u} and \mathbf{v} (leading vectors or leading patterns) by measuring the angle between each pair of corresponding segments, defined by the consecutive points of the two vectors, and take the mean of these values (see Aguiar-Conraria (2011) for more details).

³In practice, the CWT of a time-series is computed only for a finite number of values of the time and scale parameters, so the computed wavelet spectrum of a series ends up being simply a matrix.

⁴The value of K is, in general, an integer much smaller than the rank of the matrix $W_x W_y^H$; in our case, we considered $K = 3$.

3 Empirical Findings

We utilised weekly data from January 2001 to June 2019. We extracted daily data of 10-year sovereign bond yields from Eurostat for nine European countries: Belgium, Finland, France, Germany, Greece, Ireland, Italy, Portugal, and Spain. However, the data were converted to weekly data to alleviate the computational burden.

We present the empirical results in this section. First, we describe the data estimating the wavelet power spectrum of the weekly returns for each country, which is akin to presenting the descriptive statistics. We then identify the core countries by exploring the wavelet spectra dissimilarity of yields for various countries. Subsequently, we estimate the wavelet coherency between the core and each of the peripheral countries. Finally, we use partial coherency and phase-difference to evaluate evidence of contagion between selected countries.

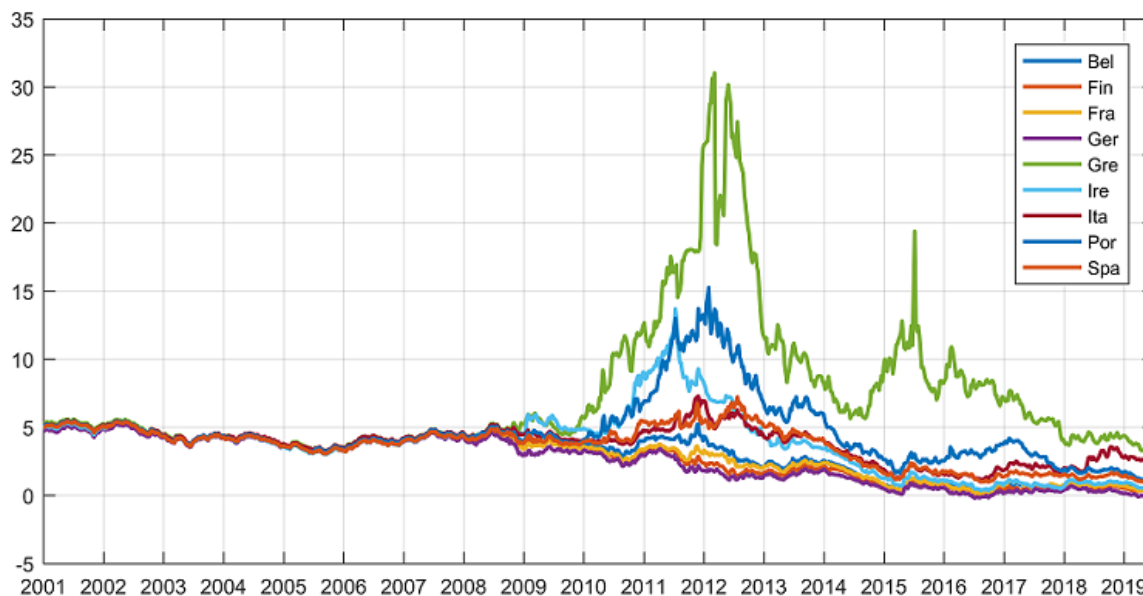


Figure 1: Weekly data on 10-year government bond yields for nine Eurozone countries from January 2001 to June 2019

3.1 The Wavelet Power Spectrum

Figure 1 shows the weekly 10-year government benchmark bond yields for the nine countries in our sample. It should be noted that the turmoil in the sovereign debt started in late 2008 or early 2009. Greek bonds spiked the most, followed by Portugal and Ireland. However, looking

at the dynamics, it is not completely obvious which one is the leading country. While it is also apparent that Italy and Spain have similar dynamics, the turmoil did not affect Belgium, Finland, France, and Germany. However, it is unclear if Italy and Spain are closer to the former or the latter group.

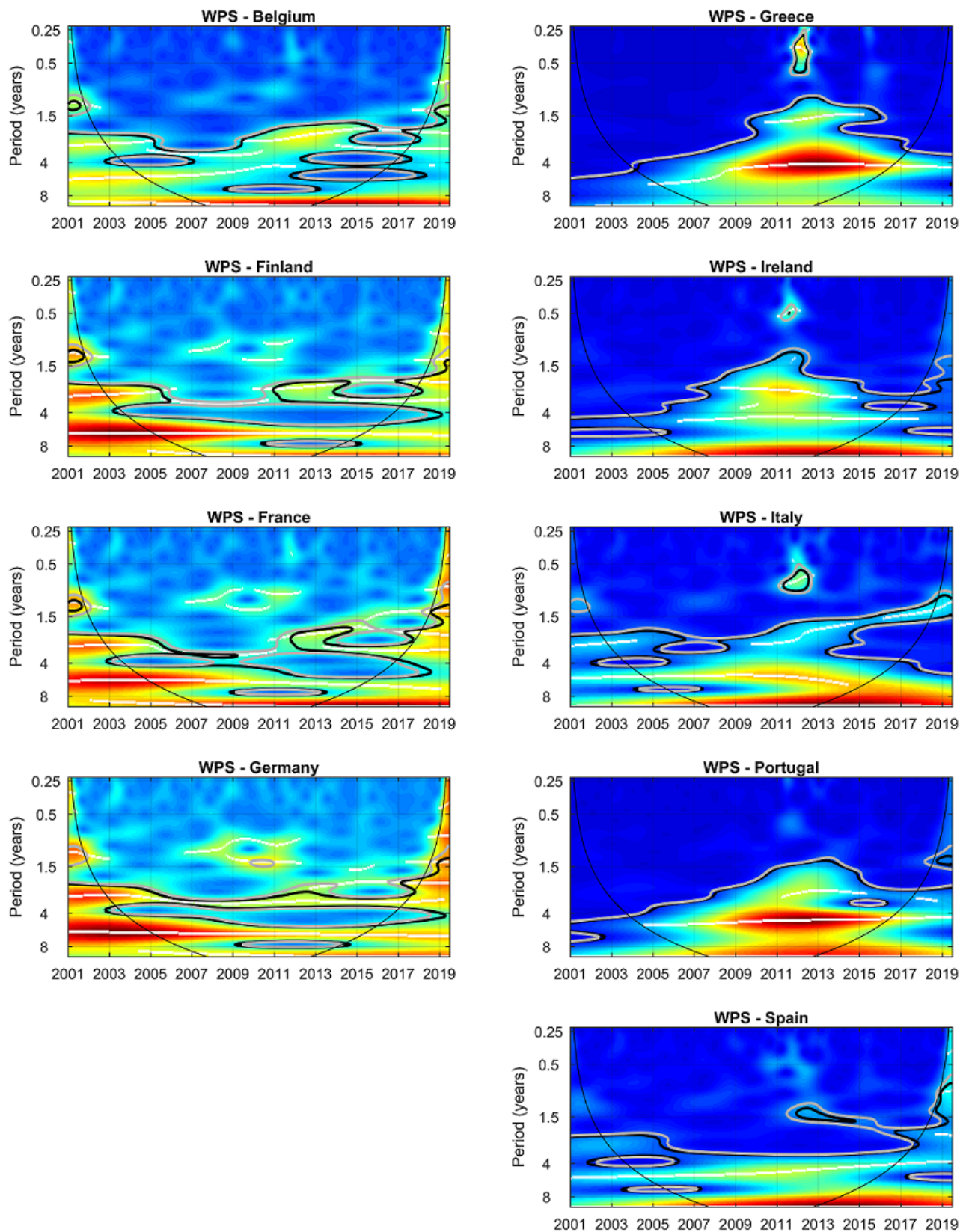


Figure 2: The wavelet power spectrum of each country's government yield. The black/grey contour designates the 5%/10% significance level. The cone of influence, which indicates the region affected by edge effects, is shown with a parabola-like black line. The colour code ranges from blue (low power) to red (high power). The white lines show the local maxima of the wavelet power spectrum.

Figure 2 shows the wavelet power spectrum of the sovereign bond yields. In the power spectra, the colours reflect the degree of volatility, with the blue colour depicting low variability and the red colour depicting high volatility. A thick black/grey contour identifies the regions of 5/10% significance against the null of a flat power spectrum. The white stripes identify local maxima and are, therefore, an estimation of the period of the most relevant cycles.

For the four countries on the left (Belgium, Finland, France and Germany), it is apparent that the dominant cycles are at the lower end of business cycle frequencies — frequencies corresponding to a period between four and eight years. One can also observe that volatility increased in the 2 ~ 4-year frequency band between 2011 and 2012, except for Germany. However, the dynamic is different for the five countries on the right (Greece, Ireland, Italy, Portugal and Spain). In general, the power spectrum for these countries is much higher after 2008 and, mainly, after 2010. This applies to both the upper and lower end of business cycle frequencies, but it is especially visible at the 2 ~ 4-year frequency band.

Among these five countries, the power spectrum of Spain is the closest to the power spectrum of the four countries on the left of Figure 2. However, the power spectrum at 1.5 years frequency becomes statistically significant in 2012. On the other hand, Greece exhibits the most peculiar characteristic. We can see a very predominant five-year cycle starting in 2009 and extending to the end of the sample, a shorter cycle, between 2010 and 2015 at the frequency of about 1.5 years, and very high frequencies around 2012. For Italy and Ireland, we observe a (smaller) spike at very high frequencies in 2012. However, Ireland is not statistically significant. Ireland is also peculiar for another reason as it exhibits high volatility at all business cycle frequencies between 2008 and 2015. Portugal, like Greece, has a very salient cycle at a frequency slightly below four years while volatility also increased at higher frequencies, but with some delay, when compared to Greece and Ireland.

3.2 Core Countries

In Table 1, we show the pairwise dissimilarity index between countries. It is based on the comparison between the wavelet transforms of the yields for each country. Note that this is

not exactly the same as comparing the wavelet power spectra of Figure 2. When one computes the wavelet power spectrum, one takes the absolute value, which implies that complex numbers disappear. By focusing on the wavelet transform, one preserves the information provided by the complex numbers. To be more precise, one retains the information about the phase of the cycle. A dissimilarity index between two countries very close to zero means that the two countries have a very similar wavelet transform. In turn, this implies that both countries share the same high-power regions and that their phases are aligned. Therefore, (1) the contribution of cycles at each frequency to the total variance is similar between both countries, (2) this contribution happens at the same time, and, finally, (3) the ups and downs of each cycle coincide in both countries.

	Belgium	Finland	France	Germany	Greece	Ireland	Italy	Portugal	Spain
Belgium		0.115	0.064	0.103	0.323	0.212	0.245	0.338	0.183
Finland	0.115		0.061	0.031	0.298	0.246	0.278	0.344	0.241
France	0.064	0.061		0.053	0.311	0.248	0.262	0.354	0.208
Germany	0.103	0.031	0.053		0.301	0.252	0.285	0.344	0.237
Greece	0.323	0.298	0.311	0.301		0.207	0.248	0.120	0.225
Ireland	0.212	0.246	0.248	0.252	0.207		0.196	0.193	0.139
Italy	0.245	0.278	0.262	0.285	0.248	0.196		0.210	0.098
Portugal	0.338	0.344	0.354	0.344	0.120	0.193	0.210		0.204
Spain	0.183	0.241	0.208	0.237	0.225	0.139	0.098	0.204	
Color code:				p < 0.005		p < 0.05		p < 0.10	

Table 1: Pairwise dissimilarities. p-values obtained by Monte Carlo simulation (10,000 replications) against the null that the cycles are not synchronized.

As we intend to consider all range of high frequencies to business cycle frequencies, the wavelet transform was computed from frequencies with two weeks period to eight years. Regarding the upper end, given that our data is weekly, the highest frequency that we can use is bi-weekly. The lower end, eight years, is just the typical lower end of business cycle frequencies.⁵

The results are entirely in line with what we can observe in Figure 1. Namely, there is a core of countries that have their cycles well aligned with each other (Belgium, Finland, France, and Germany), and a group of countries that are autonomous (Greece, Ireland, Italy Portugal, and

⁵Given that, traditionally, we associate the frequency band of 1.5 to 8 years to business cycles, in our analysis, we will separate the business cycle frequencies from short-run frequencies (frequencies higher than 1.5 years).

Spain). Any doubt about whether Spain and Italy should be part of the core should disappear when one looks at Figure 2.

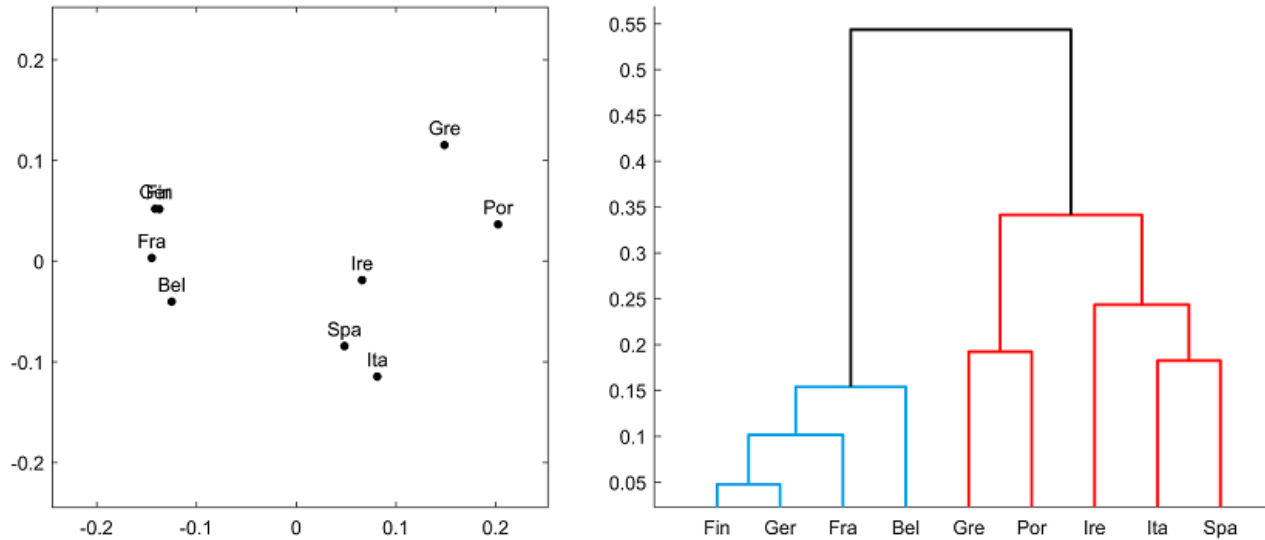


Figure 3: Multidimensional Scaling Map

To visualize Table 1, we reduced the dissimilarity matrix to a two-column matrix called the configuration matrix, which contains the position of each country in two orthogonal axes. This, of course, cannot be performed with perfect accuracy because the dissimilarity matrix does not represent Euclidean distances. From that configuration matrix, Figure 3 was produced with a multidimensional scaling map (on the left), and a dendrogram (on the right).

With highly synchronised cycles, it should be clear that Germany, Finland, France, and Belgium form a group of its own. Thus, they are referred to as the core countries while Greece, Ireland, Italy, Portugal and Spain are the periphery countries.

3.3 Wavelet Coherency and Phase-Differences between the Core Countries and the Peripheral Countries

The interpretation of our econometric results proceeds as follows. First, we check the time-frequency regions where the coherency between the variables is statistically significant, implying that, in those episodes, we may confidently say that there is a significant co-movement of the variables for cycles at the indicated period. Then, for the statistically significant time-frequency locations, we analyse the phase differences, to detect whether the co-movement has been pos-

itive or negative and determine the leading and lagging variable. When coherency is low and statistically non-significant, it makes little sense to look at the phase-differences in detail as they should be erratic given the absence of a meaningful relationship.

In Figure 5, we have the wavelet coherency and the wavelet phase-differences between the yield of the core countries and each peripheral country.⁶ Just for comparison, we included Figure 4, which makes the same computations for two core countries, Belgium and France.

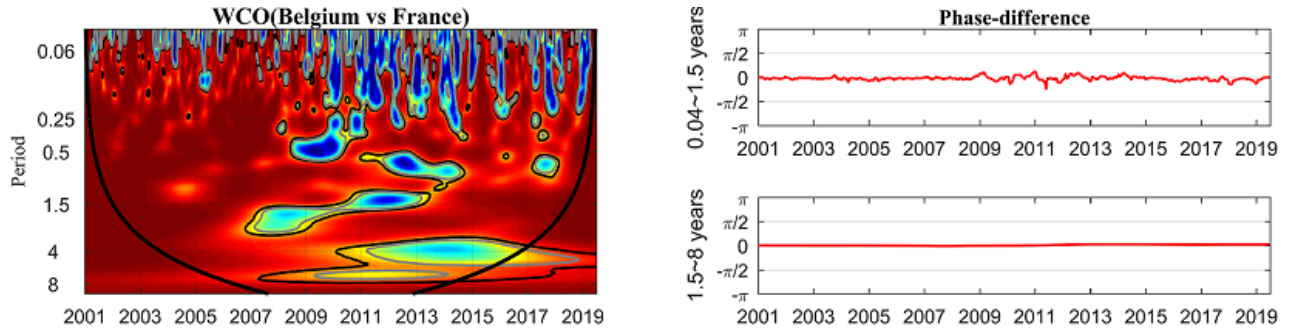


Figure 4: **On the left** – wavelet coherency between the yield of Belgium and France. The black contour designates the 5% significance. The colour code for coherency ranges from blue (low coherency – close to zero) to red (high coherency – close to one). **On the right** – phase-differences between Belgium and France.

We can observe that the wavelet coherency between two core countries is red and statistically significant almost everywhere. Note that the phase-differences are almost zero for the whole sample period, meaning that the yields are positively correlated at all frequencies, and they simultaneously co-move. In this case, there is no leading or lagging country.

⁶The yield of the core countries is just the average yield of Belgium, France, Finland, and Germany.

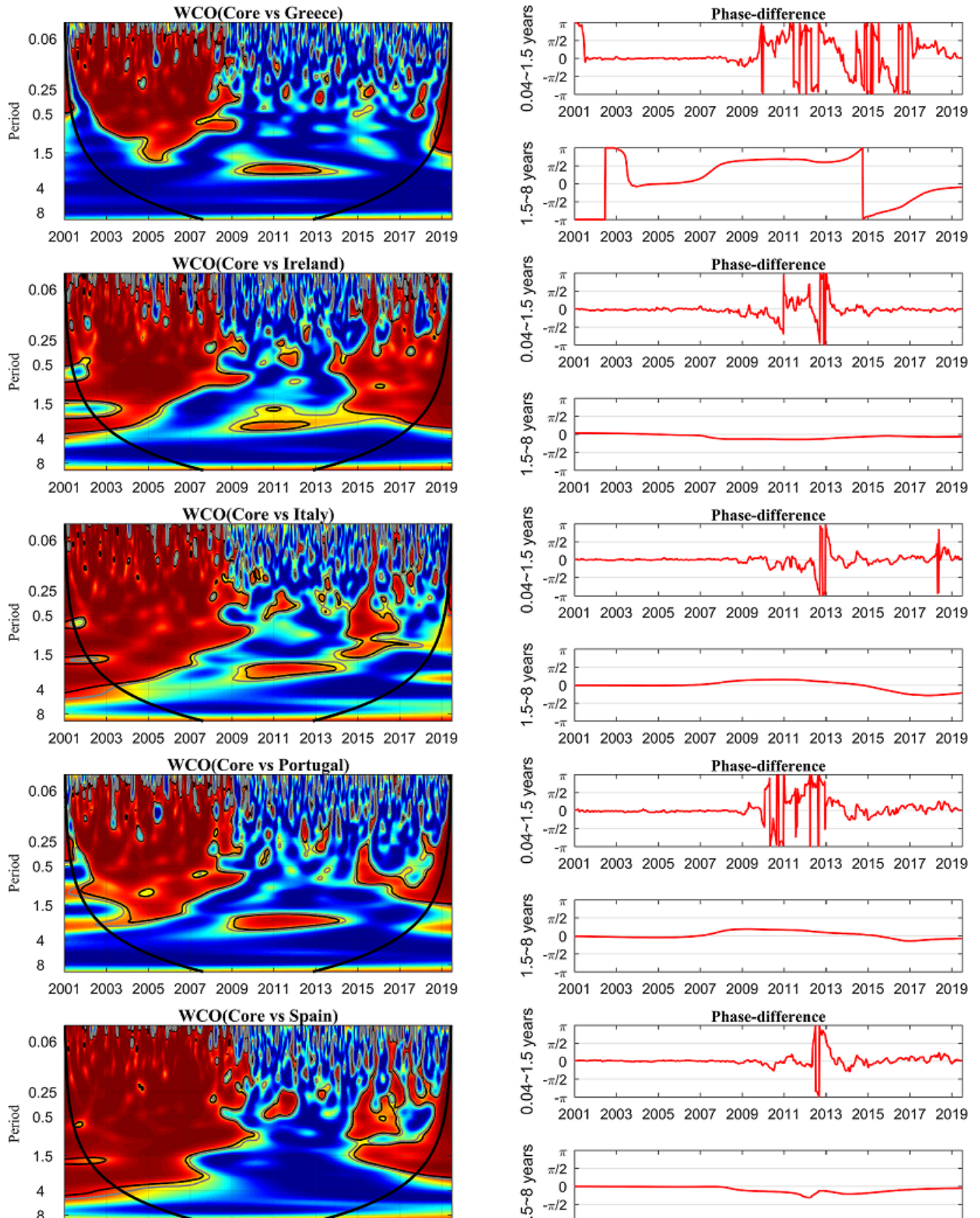


Figure 5: **On the left** – wavelet coherence between the yield of the core countries and each of the peripheral countries. The black contour designates the 5% significance. The colour code for coherence ranges from blue (low coherence – close to zero) to red (high coherence – close to one). **On the right** – phase-differences between the core countries and each peripheral country.

The scenario is quite different when one looks at the relationship between the core countries and the peripheral countries. Greece is the most peculiar case. It is highly coherent at high frequencies until 2008. After that, the picture turns blue, indicating low coherencies and stays blue until the end. At high frequencies, the phase-difference becomes erratic in 2009. This signifies that the Greek bond yields detach from others afterwards. In contrast to other countries, the coherency is mostly blue at low frequencies at the beginning of the sample. This suggests that the long-run co-movement between Greece and the core countries has always been weak. The most notable exception is between 2009 and 2013. During this period, we observe an island of high coherency at the frequency band of about 2-3 years. The phase-difference lies between $\pi/2$ and π , signifying an anti-phase (or negative) relation, with Greece leading. We do not observe this with any other country. It suggests that in this period, there was a flight-to-quality flow, with investors taking refuge in core countries.

For other periphery countries, except Spain, we observe the same region of high coherency. However, the phase-difference is close to zero, suggesting that at business cycle frequencies, in the middle of the turmoil, these countries remained tied to the core countries. Ireland, Italy, and Spain were highly coherent for a longer period at both lower and higher frequencies. In this regard, Portugal is somewhere between Greece and the other periphery countries. Regarding shorter-run phase differences, they became erratic in Portugal and Ireland around 2010, and only much later, between 2012 and 2013, in Spain and Italy.

Finally, Ireland became highly synchronized with the core countries in 2014. While Spain and Italy were the only periphery countries whose governments did not seek external assistance, Ireland was the first to re-align with the core countries. After Ireland, there is some evidence that the same happened in Portugal and Spain. Regarding Italy, we observe high coherency at several frequencies between 2015 and 2017, but they vanish after that.

3.4 Wavelet Partial Coherency and contagion between Greece, Ireland and Portugal

In the previous subsection, we saw that Greece, Ireland, and Portugal were the first countries to show signs of stress in their sovereign bond markets. In this subsection, we explore those results a little bit further. To check the evidence of contagion, we estimate the partial wavelet coherency and phase-differences between the yields of these countries, after controlling for the yields of Spain, Italy, and the Core. We interpret the existence of leftover (significant) coherency at high frequencies as evidence of contagion. The partial phase-difference will then inform us about the direction.

If we first consider Portugal, we can see that Portugal consistently lagged both Greece and Ireland between 2008 and 2014 at higher frequencies. i.e. at the 0.25 to 1.5-year frequency band. Similarly, the partial phase-differences between Greece and Portugal, and Ireland and Portugal are consistently between 0 and $\pi/2$. Therefore, Portugal is not the source of contagion.

Regarding Greece and Ireland, we observed a switch in 2010. Until then, Greece lagged Ireland. After that, Greece became the leader. This evidence suggests that until 2010, the primary source of contagion was Ireland while Greece became the originating country afterwards. The Irish banking crisis forced its government to issue a broad state guarantee of Irish domestic banks in September 2008, and the subsequent public finance crisis in Greece, which led to a bailout programme in 2010 are the most obvious culprits.

At business cycle frequencies, phase relations are much more stable. Ireland consistently leads both Portugal and Greece. The phase relation between Portugal and Greece is also very stable, with Portuguese yields slightly leading Greek yields for most of the period. However, the phase-difference becomes zero between 2009 and 2011, suggesting that the yields were highly synchronized in the peak of the crisis at business cycle frequencies.

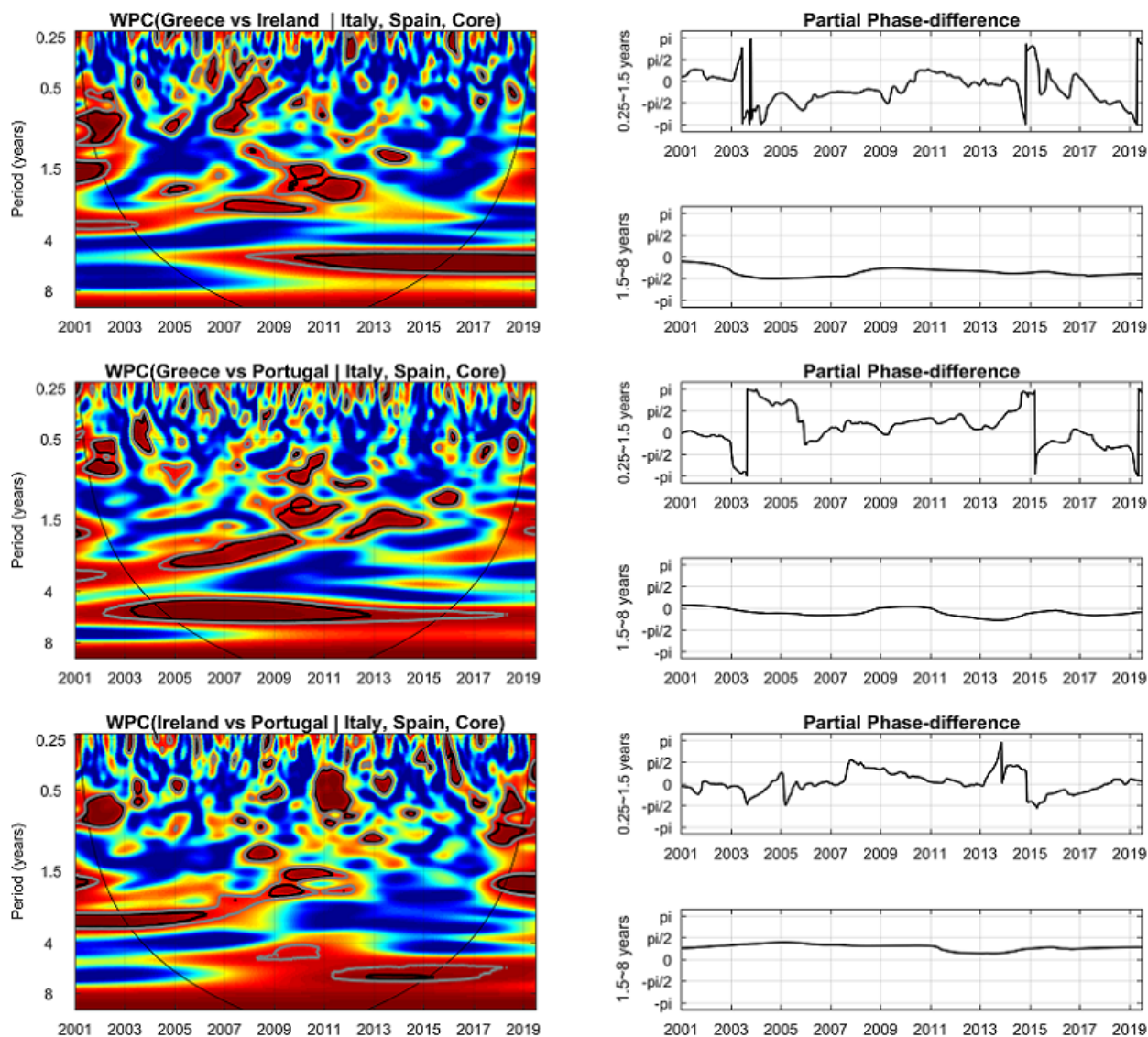


Figure 6: **On the left** – wavelet partial coherence between the Greece, Ireland and Portugal, after controlling for Italy, Spain and the Core countries. The black/grey contour designates the 5/10% significance. The colour code for coherence ranges from blue (low coherence – close to zero) to red (high coherence – close to one). **On the right** – the partial phase-differences.

4 Conclusions

We applied the Continuous Wavelet Transform to study the yields of the sovereign debt in nine Eurozone countries. We used weekly data from 2001 to June 2019. We first applied the dissimilarity index, proposed by Aguiar-Conraria and Soares (2011), to identify and separate the core (Belgium, Finland, France and Germany) from the periphery countries, also known as the GIIPS (Greece, Ireland, Italy, Portugal and Spain).

After identifying the core countries, we estimated the wavelet coherency and the wavelet phase-difference between the core and each peripheral country. We saw that Greece had a significant coherency with the core countries until 2008 and it became erratic in 2009. For the other periphery countries, their detachment from the core happened later (in the case of Ireland and Portugal) and much later in the case of Spain and Italy. In the case of Ireland, we also observed that it re-aligned with the core countries in 2014. An interesting result that we established was the evidence of the flight-to-quality flow, with investors taking refuge in core countries due to the instability in Greece.

Finally, we analysed the possibility of contagion between the first three countries that detached from the core: Greece, Ireland, and Portugal. We concluded that until 2010, Ireland was probably the main source of contagion while Greece took over later. We can connect these timings to the Irish banking crises, which started in September 2008, and the Greek public finance crisis, which led to a bailout programme in 2010.

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