

Numerical Modeling of the Tension Stiffening in Reinforced Concrete Members via Discontinuum Models

Bora Pulatsu, Ece Erdogmus, Paulo B. Lourenço, José V. Lemos, and Kagan Tuncay

Abstract: This study presents a numerical investigation on the fracture mechanism of tension stiffening phenomenon in reinforced concrete members. A novel approach using the discrete element method (DEM) is proposed, where three-dimensional randomly generated distinct polyhedral blocks are used, replicating concrete and one-dimensional truss elements are utilized, representing steel reinforcements. Thus, an explicit representation of reinforced concrete members is achieved, and the mechanical behavior of the system is solved by integrating the equations of motion for each block using the central difference algorithm. The inter-block interactions are taken into consideration at each contact point with springs and cohesive frictional elements. Once the applied modeling strategy is validated, based on previously published experimental findings, a sensitivity analysis is performed for bond stiffness, cohesion strength, and the number of truss elements. Hence, valuable inferences are made regarding discontinuum analysis of reinforced concrete members, including concrete-steel interaction and their macro behavior. Results demonstrate that the proposed phenomenological modeling strategy successfully captures the concrete-steel interaction and provides an accurate estimation of the macro behavior.

Keywords: DEM, Discontinuum Analysis, Tension Stiffening, Contact Mechanics

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21 **1 Introduction**

22 The cracking phenomenon in reinforced concrete (RC) structures has a critical role not only under
23 the serviceability conditions but also in the structural behavior leading to collapse, which is directly
24 influenced by the steel-concrete interface (or bond). The tensile behavior and strength of
25 reinforcement are essential to calculate the ultimate strength of RC members subjected to bending
26 since the tensile strength of concrete is quite limited compared to its compression capacity. On the
27 other hand, the tensile resistance and stiffness of concrete can change the load-deflection
28 characteristics, member stiffness, and deformation of reinforced concrete members. For instance,
29 the intact concrete part of the RC beam carries significant forces under typical in-service loads,
30 which directly contributes to the stiffness of the member. Moreover, a direct tension test of a
31 reinforced concrete tie exhibits gradual stiffness degradation at the macro level with successive
32 primary cracks in concrete when it is subjected to incremental tensile forces. Once a stabilized
33 crack pattern appears in concrete, the stiffness of the cross section is much lower and stresses
34 transfer to steel reinforcement through the steel-concrete interface. This phenomenon is called
35 tension stiffening, and it should be taken into account in computational models to assess reinforced
36 concrete members behavior accurately [1].

37 As a matter of fact, the tension stiffening in reinforced concrete structures is a complex problem,
38 mostly affected by the local behavior (cracking of concrete and bond quality). Different numerical
39 approaches have been developed in the literature to analyze this problem using both continuum-
40 and discontinuum-based finite element modeling (or Finite Element Analysis, FEA), where
41 continuous or discrete zero-thickness interface elements are utilized, respectively [2–6].
42 Reinforcements are usually modeled as truss or beam elements associated with solid elements in
43 FEA. In general, a perfect bond condition is assumed by using embedded reinforcement elements
44 in concrete elements [7, 8]. In the case of weak (or deteriorated) bond conditions, more
45 sophisticated models can be considered using interface elements to simulate the bond-slip
46 interaction between steel and concrete surfaces [9–12].

47 Discontinuum based modeling techniques have been also employed to explore the structural
48 behavior of reinforced concrete members under service and extreme loading conditions since the
49 mid-1970s. Pioneer work in DEM was published by Lorig and Cundall [13] to explore the behavior

50 of reinforced concrete beams under static and dynamic loading conditions. The concrete texture
51 was represented via 2D Voronoi polygons, discretized into elastic finite-difference zones, whereas
52 truss (or link) elements were used to replicate steel rebars, assuming fully bonded (or perfect bond)
53 condition. Hence, failure in reinforced concrete beams under bending were analyzed within the
54 framework discrete element formulation, considering brittle contact-displacement behavior among
55 the adjacent Voronoi polygons. Note that progressive contact openings, slippage, and eventual
56 diagonal tension cracks, were obtained successfully. Similarly, in the finite-discrete element
57 method (FDEM), fracture, and fragmentation processes of deformable bodies are simulated via
58 contact elements that are implemented within the finite element mesh. The applications of FDEM
59 on RC members can be found in [14, 15]. Furthermore, rigid-body-spring model, proposed by
60 Kawai [16], was applied to analyze the crack propagation in reinforced concrete structures, where
61 the concrete was represented via rigid bodies based on Voronoi diagrams, interconnected by
62 springs [17]. Recently, a lattice-based approach was presented where the steel, concrete, and bond
63 behaviors are explicitly modeled using overlapping truss elements to explore the structural
64 response of reinforced concrete members [18, 19]. In this research, an alternative computational
65 modeling strategy is proposed by further extending the pioneering work of Lorig and Cundall [13],
66 in which the tension stiffening phenomenon is numerically investigated via discontinuum models
67 composed of large polyhedral blocks in 3D. The fracture mechanism of the RC members is studied
68 by dissecting the interaction between polyhedral blocks based on the appropriate softening contact-
69 constitutive laws.

70 The present study focuses on two essential points: 1) the influence of model parameters on the
71 macro behavior, and 2) the fracture mechanisms of reinforced concrete members. Also, the
72 research further extends the application of recently implemented user-defined contact constitutive
73 models executed in a commercial discrete element modeling software, 3DEC, by adopting the one-
74 dimensional truss elements to simulate reinforced concrete behavior. It is worth to mention that
75 there are very few applications of discontinuum models to simulate RC members using DEM. In
76 this context, this study presents a novel approach to explore the composite action of reinforced
77 concrete structures. The goals of this research are, thus, as follows:

- 78 • To provide a discontinuum based approach for the analysis of reinforced concrete members
79 consisting of randomly generated polyhedral blocks interacting with cohesive contact
80 models and one-dimensional truss elements for the reinforcement.
- 81 • To simulate the fracture mechanism of tension stiffening by explicitly considering the
82 concrete, the steel reinforcement, and the bond between the two materials.

83 Two experimental studies that have been previously analyzed by other researchers [20–23] are
84 taken into consideration as benchmark studies. The obtained numerical results are compared with
85 the experimental findings, and relevant inferences are made. In the first part, the tensile test of an
86 RC prism, (also called RC tie, indicating a square cross section with a rebar at the center), tested
87 by [24], is simulated, and parametric research is performed on the validated model. Then, a four-
88 point bending test of a singly reinforced concrete beam, presented in [25], is investigated utilizing
89 the outcomes and suggestions derived from the tension test.

90 Next, the mathematical formulation of DEM and the proposed discrete models are presented. Note
91 that in the present research, cyclic response (e.g., bar slip history), time-dependent material and
92 bond degradations (i.e., creep and shrinkage) are not considered.

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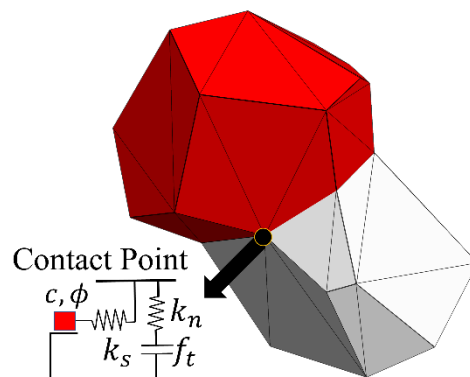
94 **2 Theoretical Background**

95 In the present research, the discrete element method (DEM), proposed by Cundall [26], is applied
96 to explore the cracking and fracture mechanism of reinforced concrete members using the
97 commercial software 3DEC developed by ITASCA [27]. DEM falls into the category of
98 discontinuum type of analysis, used to analyze the structural behavior of blocky systems. The
99 discontinuous formulation of DEM provides a great advantage for the simulation of crack
100 localization and propagation phenomena in quasi-brittle materials computationally since it does
101 not require special crack tracking algorithms, remeshing, and material property updates.

102 In this study, the mechanical interaction between the distinct polyhedral blocks is used to represent
103 the heterogeneous structure of plain concrete. The internal structure of concrete is represented by
104 a tessellation into polyhedral blocks generated via the Neper software package. There is no attempt
105 made to model the actual shape or number of aggregates in the examined concrete models.

106 Recently, quasi-brittle construction materials (e.g., concrete and masonry) were analyzed with the
107 same strategy, where the Laguerre tessellation was generated by the weighted points optimized to
108 obtain specified morphological properties of the polyhedral blocks [28–30]. The objective was,
109 therefore, to obtain a phenomenological approach in which the observed mechanisms are
110 represented in such a fashion that simulations are in reasonable agreement with experiments. In a
111 similar fashion to continuum mechanics, no attempt is made to formulate constitutive models that
112 fully incorporate all the interacting mechanisms of concrete. But an important difference is that
113 failure is discontinuous, and the discrete element method is used, thus avoiding the problems
114 related to mesh sensitivity and convergence of the solution procedure. Further information about
115 the software and the mathematical background of the tessellation algorithm can be found in [31–
116 34]. The mechanical interaction between the blocks occurs through their contact points via
117 linear/nonlinear spring and cohesive frictional elements. Furthermore, the deformability of each
118 block is considered by internally subdividing them into constant strain tetrahedral elements
119 (denoted as finite-difference zones), as shown in Figure 1. Approximately 20 tetrahedral elements
120 are used for each polyhedral block. The interpenetration of the blocks is allowed assuming a soft
121 contact approach based on the spring stiffness of the assigned contact in three orthogonal
122 directions.

123



124 Figure 1. Polyhedral blocks generated via Neper software and representation of point contact.

125

126 The core part of the numerical procedure of DEM relies on the integration of the equations of
127 motion via a central difference formulation to calculate the nodal velocities for each block in the

128 discrete system. Quasi-static calculations are performed by means of a dynamic relaxation
129 algorithm adopting artificial damping. The compact form of the equations of motion written for a
130 deformable block is given in Equation 1. Note that, in the given expression, nodal velocities are
131 evaluated at the mid-intervals of the time step (Δt) considering as $t - \Delta t/2$ and $t + \Delta t/2$, which
132 are denoted by t^- and t^+ , respectively.

133

$$\begin{aligned}\dot{\mathbf{u}}_i^{t^+} &= \dot{\mathbf{u}}_i^{t^-} + [\Sigma \mathbf{F}_i^t - (\mathbf{F}_d)_i] \Delta t / m_n \\ (\mathbf{F}_d)_i &= \gamma |\Sigma \mathbf{F}_i^t| \text{sgn}(\dot{\mathbf{u}}_i^{t^-})\end{aligned}\tag{1}$$

134

135 where $\dot{\mathbf{u}}$, \mathbf{F}^t , and m_n are the velocity vector, net nodal force vector, and lumped nodal mass,
136 respectively, calculated for each node (or so called gridpoint). The force vector \mathbf{F} includes external
137 loads, contact forces (only for gridpoint along the surface of the block), gravity forces and the
138 contribution to the internal stress in the zones adjacent to the gridpoint that can be obtained as

139

$$\mathbf{F} = \int_S \sigma_{ij} n_j ds\tag{2}$$

140

141 where n_j is the outward normal to the surface S (closed polygonal surface) and σ_{ij} indicates the
142 zone stress tensor. Here, a local form of damping is used by defining the force (\mathbf{F}_d) with damping
143 constant, γ (default value is 0.8). The damping force is proportional to the magnitude of
144 unbalanced force which opposes the motion considering the velocity vector where $\text{sgn}(\zeta) = 1$, if
145 $\zeta \geq 0$; $\text{sgn}(\zeta) = -1$, if $\zeta < 0$ [35]. The damping constant remains unchanged during the analysis.
146 A detailed explanation of the implemented local damping algorithm can be found in [36].
147 Moreover, scaled masses are used to speed up the calculations, since, in quasi-static analysis,

148 inertial forces have minimal effect as long as they are small compared to the other forces in the
149 system.

150 Once the nodal velocities are calculated, the block positions are updated, and relative contact
151 displacements are found. It is important to note that through the dynamic solution scheme of DEM,
152 contact conditions are updated at each time step using a contact detection algorithm based on the
153 common plane concept [37]. The contact-detection algorithm requires a unit normal vector to
154 define a plane that changes its direction during the analysis as blocks move relative to each other.
155 In 3DEC, the contact detection algorithm provides uniquely defined “*common-plane*” bisecting
156 the space between the two convex polyhedral blocks based on the geometrical configuration that
157 maximizes the gap or minimizes the overlap between the blocks [38]. It is worth noting that the
158 conditionally stable solution scheme of DEM necessitates sufficiently small time steps to capture
159 inter-block displacement. To obtain a stable solution, the required critical time step (Δt_{cr}) can be
160 estimated as

161

$$\Delta t_{cr} = 2 \sqrt{m_n/k_{gp}} \quad (3)$$

162

163 where k_{gp} denotes nodal stiffness obtained by adding zone and contact (only the gridpoint on the
164 faces) stiffness [39]. The employed numerical model has the ability to capture large deformations
165 and rotations of the blocks as well as to take into account the interaction forces between the blocks
166 by tracing relative geometrical configuration during the analysis in an updated Lagrangian
167 approach.

168

169 2.1 Contact Constitutive Law

170 Through this research, relatively high elastic stiffness (i.e., ten times the value of the macro elastic
171 stiffness) is assigned for continuum blocks, denoted as *semi-rigid*, to determine the governing
172 deformation and nonlinear behavior of the material predominantly via discontinuities, as shown in

173 [29]. Contact stress increments (normal $\Delta\sigma$ and tangential $\Delta\tau$) are calculated based on the relative
 174 deformation increments between the blocks both in the normal, Δu_n , and shear, Δu_s , directions,
 175 as

$$\begin{aligned}\Delta\sigma &= k_n \Delta u_n \\ \Delta\tau &= k_s \Delta u_s\end{aligned}\tag{4}$$

177
 178 where k_n and k_s denote, respectively, the contact normal and shear stiffness. Once the contact
 179 stress increments are calculated, stresses are updated and corrected based on the stress-
 180 displacement law assigned to the contact through the explicit solution scheme of DEM. Here, the
 181 mechanical response of plain concrete is simulated using softening functions (linear or
 182 polynomial) in tension, whereas elastic perfectly plastic behavior is assumed in compression. The
 183 Coulomb-slip joint model is employed in shear direction, which requires cohesion and friction
 184 angle parameters. The mathematical formulations of the contact models are given in Equation 5.
 185 Furthermore, the graphical representation of the contact behaviors both in normal and shear
 186 directions are given in Figure 2.

$$\begin{aligned}187 \\ \text{if } \sigma_{tension} \geq f_T \text{ then } \left\{ \begin{array}{l} f_t = f_T \left(1 - \frac{u_n - \eta}{\zeta - \eta} \right) \\ \text{or} \\ f_t = f_T \left(\frac{\zeta}{\zeta - \eta + u_n} \right)^\alpha ; \zeta = \frac{2G_f^I}{f_T}, \eta = \frac{f_T}{k_n} \end{array} \right. \tag{5} \\ |\sigma_{compression}| \leq f_c\end{aligned}$$

$$\text{if } \tau \geq c_0 + \sigma_n \tan \phi_0 ; \text{ then } \tau = c_{res} + \sigma_n \tan \phi_{res}$$

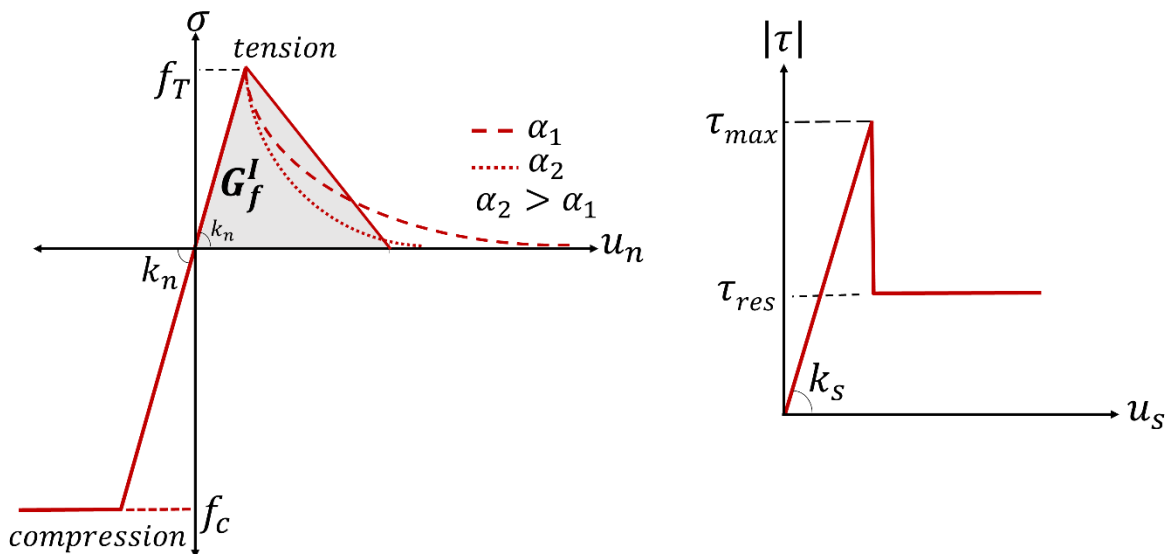
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 189 where c_0 , ϕ_0 and α are the initial cohesion, friction angle, and the user-defined power for the
 190 polynomial softening function. Also, the residual cohesion and frictional angles are shown as c_{res} ,

191 and ϕ_{res} , respectively. Note that tension and shear forces utilized in the proposed contact failure
 192 criterion are implemented in a decoupled manner, similarly to [40]. The adopted contact models
 193 are written in C++ and compiled as DLL (dynamic link library) into 3DEC via the user-defined
 194 constitutive model option. Furthermore, contact stiffnesses are defined in Equation 6 [41], where
 195 t indicates the average thickness of the fracture zone.

196

$$\begin{aligned} k_n &= E/t \\ k_s &= G/t \end{aligned} \tag{6}$$

197



198 Figure 2. Contact behavior in the normal ((elasto-plastic in compression and linear or power-law
 199 softening in tension as a function of the polynomial power α , left) and shear (brittle behavior
 200 with residual friction, right) directions.

201

202 2.2 Reinforcement Elements in DEM

203 The reinforcement embedded in concrete provides remarkable tensile capacity to the material;
 204 however, the quality and contribution to the tensile strength mainly depend on the type of the rebar
 205 and performance of the bond between the two materials. To replicate the action of reinforcement,
 206 truss elements are utilized that may pass through the existing discontinuities. Truss elements use a
 207 force-displacement relationship to simulate axial and bond behavior of the reinforcement

208 implemented into the finite-difference algorithm in 3DEC. The axial (ΔF_{axial}) and bond (ΔF_{bond})
209 force increments are calculated using relative normal (Δu_{axial}) and shear (Δu_{bond}) nodal
210 displacements of a truss element as

211

$$\begin{aligned}\Delta F_{axial} &= K_{axial} \Delta u_{axial} \\ \Delta F_{bond} &= K_{bond} \Delta u_{bond}\end{aligned}\tag{7}$$

212

213 where K_{axial} and K_{bond} denote axial and bond stiffness, respectively. The axial stiffness is
214 determined in terms of the reinforcement cross-sectional area (A), the elastic modulus (E) and the
215 length of the truss element (L_{el}), given in Equation 8. Elasto-plastic behavior with a certain yield
216 force is adopted both in tension and compression. There is no rupture limit assigned to truss
217 elements meaning that the yielding deformation continues with no restriction.

218

$$K_{axial} = \frac{(AE)_{steel}}{L_{el}}\tag{8}$$

219

220 The bond stiffness, K_{bond} (or shear resistance) is considered using a shear spring that is parallel to
221 the reinforcement. The ratio between the bond stiffness and the axial stiffness of reinforcement is
222 denoted by ξ and referred to as bond stiffness ratio throughout the article, which directly influences
223 the bond performance and overall behavior of the structural member (Equation 9). The adopted
224 bond-slip constitutive law is similar to the Coulomb-slip joint model with an elastic-fully plastic
225 response, which is readily available in 3DEC. Hence, only the cohesion (c_{bond}) and friction
226 coefficient (ϕ_{bond}) are required to define the shear failure criteria for the bond.

227

$$\xi = \frac{K_{bond}}{K_{axial}} \quad (9)$$

228

229 Additionally, each reinforcement node passing through a constant strain tetrahedral element is
230 linked with a particular tetrahedron (denoted as host zone) to calculate the axial displacement of
231 the reinforcement node considering an interpolation scheme using moment equilibrium. Initially,
232 the host zone is subdivided into tetrahedron based on the geometrical configuration of the
233 reinforcement nodal point to calculate the weighting factors, which is the ratio of an individual
234 tetrahedron to the total volume of the host zone, V_i/V_T , as presented in Figure 3. Then, the
235 displacement increments for reinforcement node Δu_{rn} are obtained as,

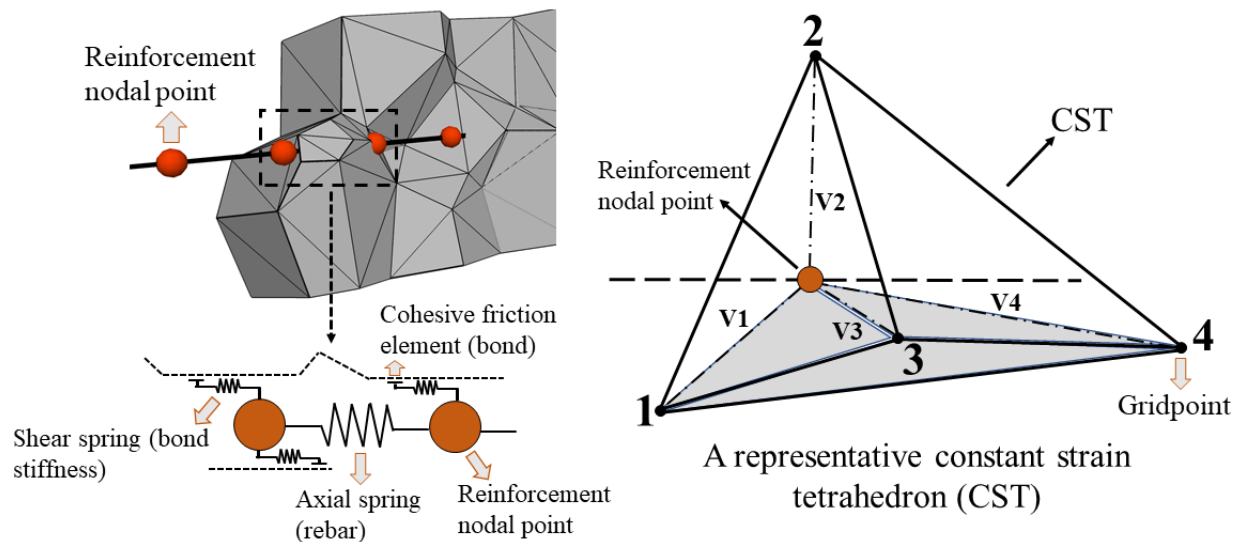
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$$\Delta u_{rn} = \frac{V_1}{V_T} \Delta u_1 + \frac{V_2}{V_T} \Delta u_2 + \frac{V_3}{V_T} \Delta u_3 + \frac{V_4}{V_T} \Delta u_4 \quad (10)$$

237

238 where Δu_i denotes the incremental gridpoint displacements, including x, y, and z components that
239 are utilized to find out the new local position of the rebar node and the axial deformation. In the
240 end, forces calculated for the steel-concrete interface are distributed back to gridpoints applying
241 the weighting factors mentioned earlier. Note that confining stress, acting in the plane
242 perpendicular to the reinforcement axis is computed along the reinforcement for each
243 reinforcement node (see Figure 3), depending on the stress developing in the zone to which the
244 nodal point is associated. Then the confining stresses are utilized in the bond strength calculation.

245



246 Figure 3. *Left*: Representation of a truss (reinforcement) element embedded into polyhedral
247 blocks. *Right*: Illustration of reinforcement nodal point passing through a tetrahedron element.

248

249 Different applications of reinforcement elements (truss or beam) in discrete and combined finite-
250 discrete element methods were made in the literature to simulate reinforced concrete and masonry
251 structures [15, 42–44].

252

253 3 Validation of Discontinuum Modeling of an RC Tie

254 The direct tension test of a concrete prism is analyzed to ensure the replication of experimental
255 data. Quasi-brittle materials, such as concrete, masonry, and rock, have a heterogeneous material
256 structure, exhibiting micro- and macro-cracks that cause failure of the material. Specifically, the
257 disordered structure of plain concrete includes various defects such as inclusions, flaws, stiffness,
258 and geometrical differences of its constituents, leading to a highly nonlinear response and crack
259 localization. Furthermore, concrete reveals a softening post-peak response (tension softening),
260 implying a gradual decrease of the mechanical resistance together with an increasing crack opening
261 after reaching its strength [45]. It is important to note that to capture this response, including the
262 transition from micro- to macro-crack, a displacement-controlled test setup is required such that it
263 eliminates any unnecessary snap-back behavior [46]. Moreover, boundary conditions in the test

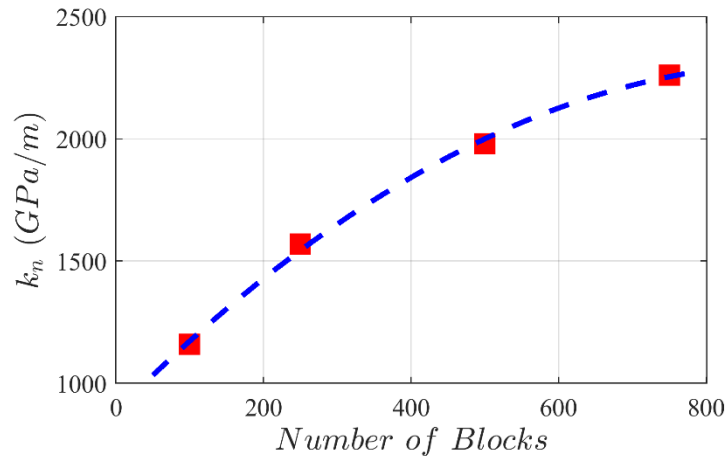
264 setup (fixed or rotating) should be defined carefully since they directly influence the fracture
265 mechanism and energy obtained from the experiments [47].

266 First, the direct tension behavior of plain concrete prism without a notch is analyzed to demonstrate
267 the unreinforced response of concrete obtained via discontinuum models using the same
268 geometrical and material properties of the benchmark experimental study [24]. The contact
269 properties used in the analyses are given in Table 1. The macro elastic properties of the concrete
270 specimens (E , G , and ν) are taken from the experiment as $E = 28 \text{ GPa}$ and $\nu = 0.2$, given in [24].

271 Note that when the number of blocks is increased, the average thickness of the fracture zone gets
272 smaller due to the decrease in the block sizes. For instance, in coarse and fine discrete models, the
273 average thicknesses are taken as 24 and 14 mm, corresponding to 100 and 500 blocks, respectively.

274 Thus, the contact stiffnesses (elastic contact parameters) are updated for the different number of
275 blocks using Equation 6, as depicted in Figure 4. It can be seen that higher contact stiffnesses are
276 utilized for higher number blocks. The average block edges are 12, 18, 14, and 12 mm for 100,
277 250, 500, and 750 blocks, respectively. However, identical cohesion, tensile strength, and fracture
278 energy values are used for all concrete prism models. Therefore, the same energy dissipation at the
279 contact point is achieved for different number blocks during the analysis. Moreover, the bottom
280 gridpoints are fixed, whereas roller supports are used at the top gridpoints to eliminate any
281 excessive rotation in the specimen. Polynomial softening functions satisfying the appropriate
282 mode-I fracture energy are used whereas, there is no compression limit assigned to the contact,
283 given the nature of the problem. The cohesion strength is assumed two times of the tensile strength,
284 which does not have drastic influence when it is larger than f_T as previously presented by Pulatsu
285 et. al [29].

286



287 Figure 4. Contact stiffness variation based on the number block in the model.

288

289 Table 1. Contact properties for concrete (contact stiffnesses are given for 100, 250, 500, and 750
290 blocks, respectively).

k_n (GPa / m)	1160, 1570, 1980, 2260
k_s (GPa / m)	482, 654, 824, 943
f_T (MPa)	2.15
c_0, c_{res} (Pa)	$2f_T, 0.01c_0$
ϕ_0, ϕ_{res} (degrees)	35, 30
G_f^I (N/m)	60

291

292 The results of the analyses are presented in Figure 5, revealing the stress-displacement curves of
293 discrete element-models with different numbers of blocks. Although similar pre- and post-peak
294 trends are obtained in each analysis, results vary slightly due to the irregular and random generation
295 of the blocks, which is also observed in the experiments. In order to better represent this variation,
296 the results are expressed in the form of a computational model envelope (CME), as shown in Figure
297 5. Once the first crack occurs in concrete, it causes redistribution of the stresses in the material,
298 yielding a rough surface of fracture pattern. Similar behavior can be observed in the computational
299 model, in which the crack occurs at the contact point that has reached the capacity and presents an
300 irregular cracking pattern under incremental tensile forces (see Figure 5). For further details on the
301 discontinuum analysis of plain concrete subjected to direct tension, the readers are referred to [29].

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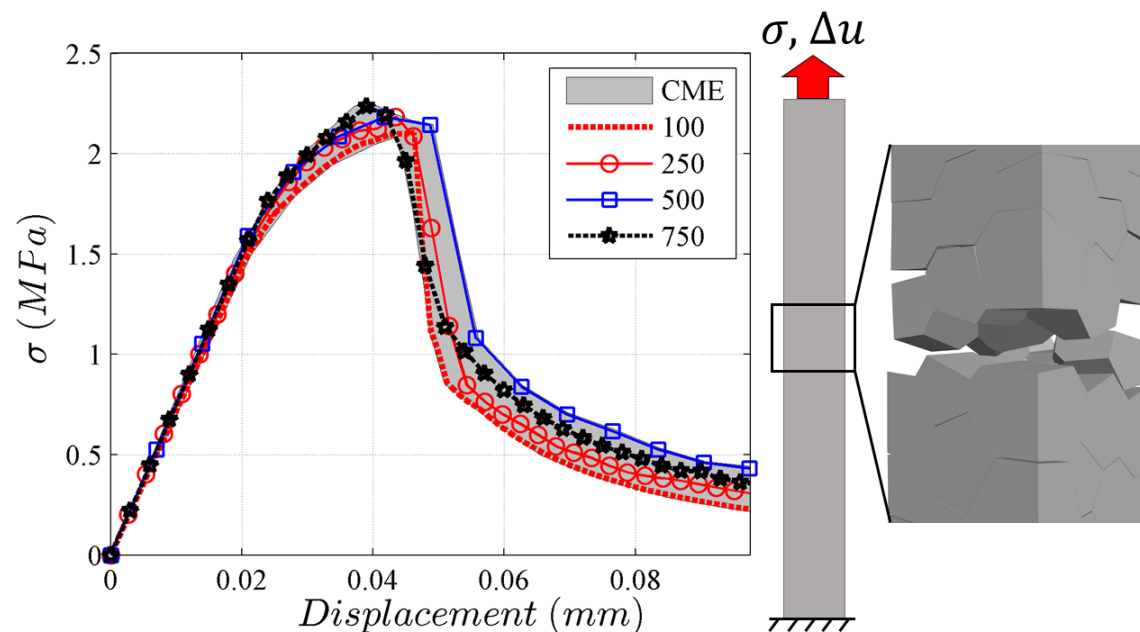


Figure 5. Stress-displacement curves obtained for different number of polyhedral blocks representing the plain concrete (CME: Computational Model Envelope)

303
304
305

306 Hence, the results show that the tension-softening response of concrete can be captured through
307 *semi-rigid* discrete polyhedral blocks with tensile softening contact models based on the dynamic
308 stress update scheme of DEM. It is worth noting that the number of blocks has a negligible role in
309 the capacity ($\pm 5\%$) and causes rational variation at the post-peak behavior regarding the
310 morphology of the discontinuum representation. Also, it should be noted that the present research
311 aims to simulate the fracture mechanism of concrete in a relatively cost-effective way with larger
312 polyhedral blocks to capture progressive crack localization and gradual strength degradation. Next,
313 the proposed modeling approach is further extended to explore the tension stiffening behavior of
314 a reinforced concrete tie.

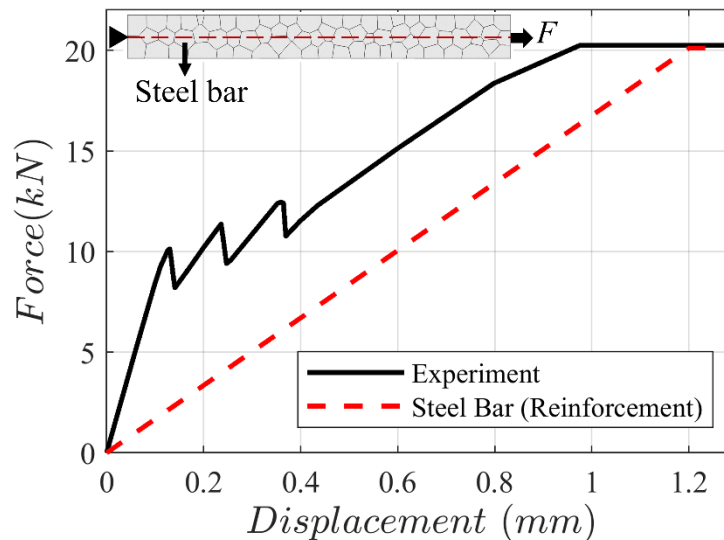
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316 3.1 Tension Stiffening

317 A typical tension stiffening curve obtained from the experiment [24], as well as the bare steel rebar
318 behavior, are given in Figure 6, where the stress drops and stiffness degradation of the section can
319 be observed due to primary cracks in concrete. In the benchmark study, a 0.6 m long RC prism

320 with a cross-section of 68×68 mm and a single 8 mm diameter rebar embedded at the center is
321 tested under direct tension.

322



323 Figure 6. Macro behavior of RC prism [24] and bare still rebar subjected to uniaxial tension.

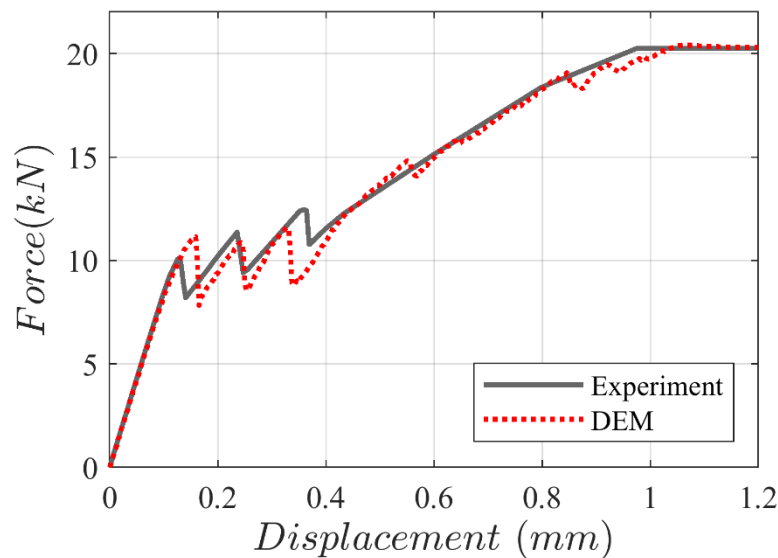
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325 To replicate the test setup, the first nodal point of the reinforcement is restricted in all directions,
326 and roller supports are assigned to the right and left gridpoints in the numerical model. Then, the
327 right reinforcement node is subjected to a fixed velocity boundary condition by setting a
328 displacement rate (v_n) of 5 mm/s to elongate the steel reinforcement. During the analysis, the
329 reaction forces are recorded from the left node of the rebar, extracted at each time step by the
330 implemented subroutine in the software based on FISH functions (an executable programming
331 language in 3DEC).

332 In Figure 7, the numerical solution is compared with the experimental result for the perfect bond
333 (no-slip) condition considering a relatively high cohesive strength ($c_{bond} = 10^9$ N/m) to make
334 sure that no sliding can occur at the bond. Furthermore, the number of truss elements and the bond
335 stiffness ratio are assumed as 96 and 4, respectively. Therefore, approximately four truss elements
336 can be located for each polyhedral block along the steel reinforcement, which yields $K_{Axial} =$
337 1.55 GPa. Accordingly, from the bond stiffness ratio of four, K_{Bond} can be calculated as 6.19 GPa.
338 The contact and material properties are mentioned earlier (Table 1). The steel elastic modulus and

339 yield stress are defined as 192.3 GPa and 400 MPa , respectively. Overall, there is a good
340 agreement found between the results from the proposed discontinuum model consisting of 100
341 blocks and the experiment. The computational model captures the stress drops and the convergence
342 to yielding the force of rebar in a similar fashion with the experiment. Thus, the presented
343 numerical model and its input parameters are considered as the validated baseline in the following
344 section, where a parametric study is conducted.

345

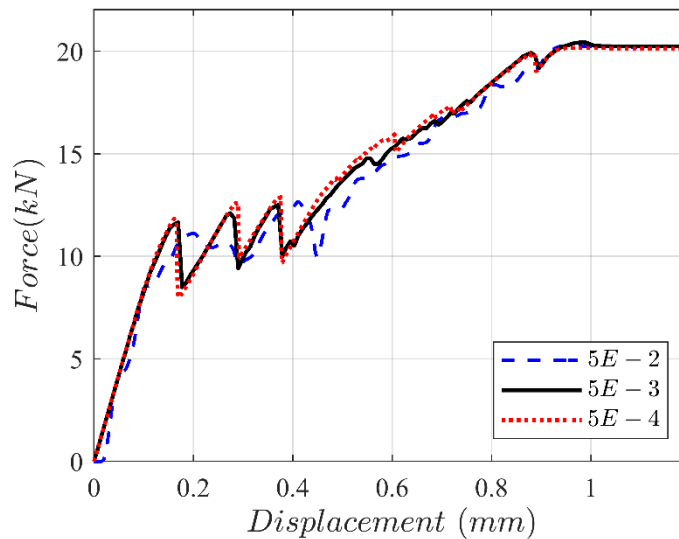


346 Figure 7. Comparison between the experiment and discrete element model (100 Blocks).

347

348 Furthermore, the velocity boundary condition, considered at the reinforcement node to extend the
349 rebar, is varied to validate the influence of loading rate on the force-displacement response of the
350 numerical model. As shown in Figure 8, the results reveal a convergence trend when the applied
351 displacement rate gets smaller (e.g., $v_n \leq 5 \text{ mm/s}$).

352



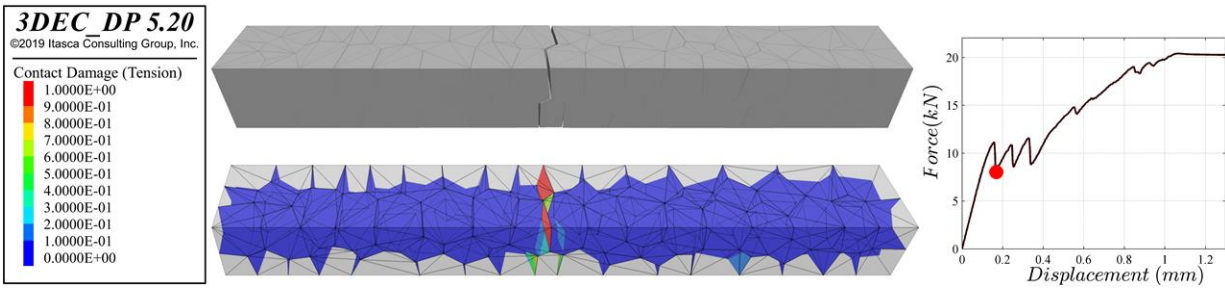
353 Figure 8. Influence of the displacement rate applied to pull the rebar (m/s) on the macro response
354 of the RC specimen.

355

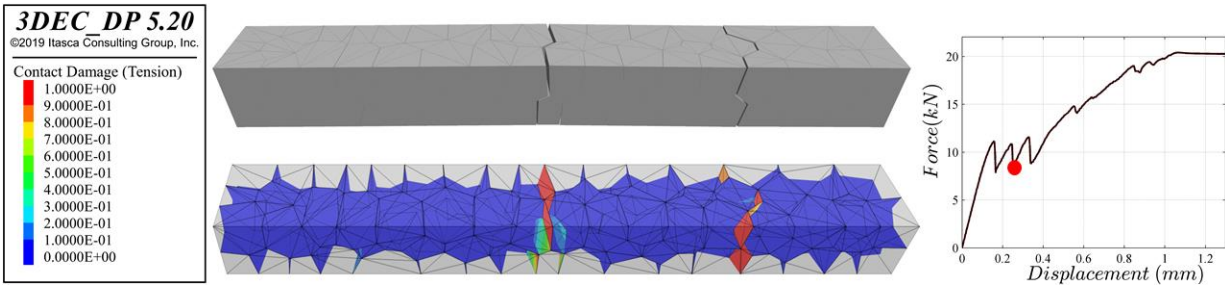
356 It is worth noting that three local peak points are obtained, representing the crack localization in
357 the specimen that occurs sequentially under incremental forces. Each stress drop exhibits the loss
358 of tensile strength in concrete as a local phenomenon, where the stresses gradually transfer to
359 reinforcing rebar through the bond. The progressive cracking mechanism is presented in Figure 9,
360 in which the cracking in concrete and the tensile damage at the contact surfaces can be observed
361 with the corresponding location on the force-displacement curve, denoted by a red circle. At the
362 end of the analysis, three fully opened cracks are observed in the specimen, each one corresponding
363 to a load drop in the response.

364

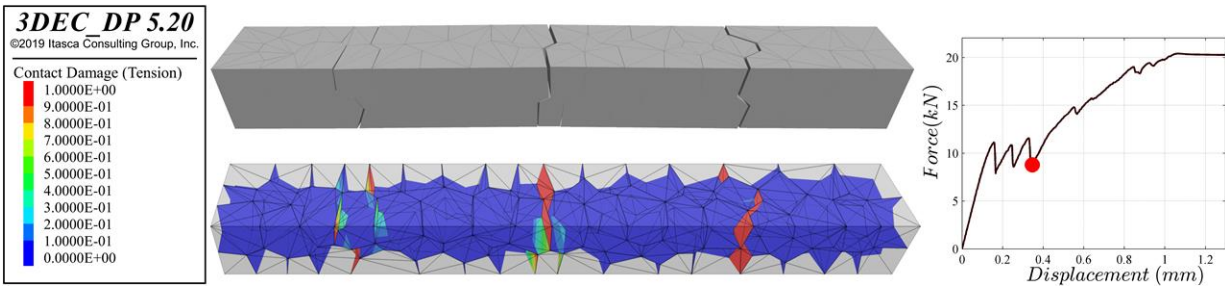
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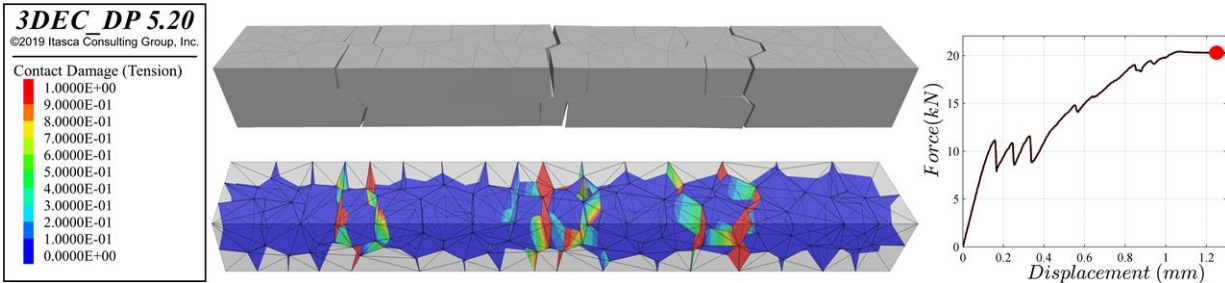
Crack localization at the first stress drop and the corresponding contact damage state in tension.



Crack localizations at the second stress drop and the corresponding contact damage state in tension.



Crack localizations at the third stress drop and the corresponding contact damage state in tension.



Final crack pattern of the specimen at steel yielding

Figure 9. Numerical simulation revealing the fracture mechanism of an RC prism.

366

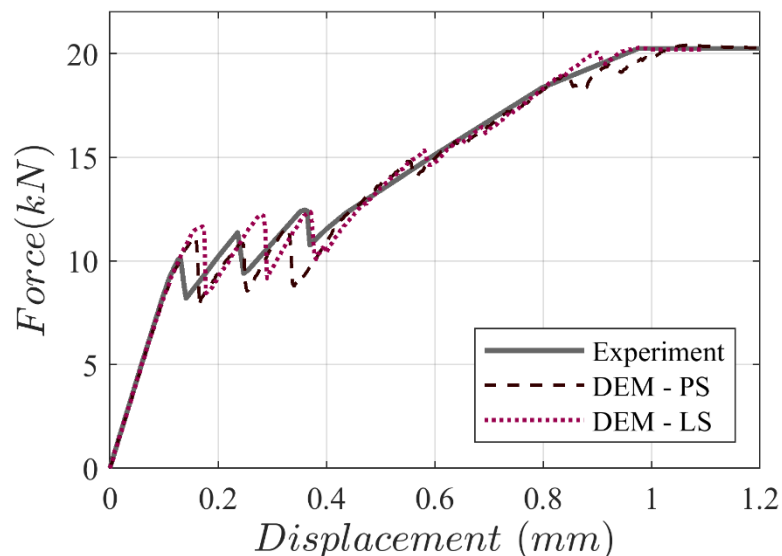
367

368 3.2 Sensitivity Analyses

369 In this section, a comprehensive parametric study is presented, including contact models (tensile
370 softening type), bond stiffness ratio, number of truss elements, number of polyhedral blocks, and
371 bond cohesion strength. It is aimed at providing practical information compromising the
372 computational cost and the accuracy obtained via the proposed modeling strategy with respect to
373 each parameter.

374 First, two different tensile softening regimes, namely linear softening (LS) and polynomial
375 softening (PS) at the contact points between the discrete polyhedral blocks, are investigated to
376 demonstrate their influence on the macro response of the RC specimen. It is also important to note
377 the identical contact parameters (f_T, c and G_f^I) are employed in both computational models, and
378 the polynomial function power (α) is taken as 3. As shown in Figure 10, both contact constitutive
379 laws provide close results to each other with small differences at the local peak points. However,
380 since the linear softening contact model requires fewer input parameters, it may be a favorable
381 solution in cases where experimental data are lacking.

382

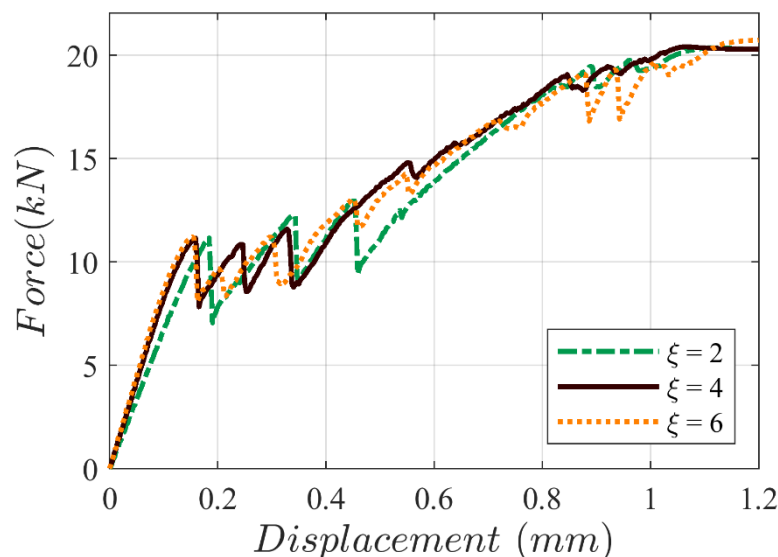


383 Figure 10. Applications of different softening models (PS: Polynomial softening, LS: Linear
384 softening) in reinforced discontinuum model of concrete.

385

386 Another critical aspect of the proposed modeling strategy is to estimate the appropriate bond
387 stiffness ratio (ξ). In the present research, the bond stiffness (K_{bond}) is taken to be proportional
388 to the stiffness of the truss elements (K_{axial}), as previously presented in [48], using the rigid
389 circular particles in the framework of DEM. The bond stiffness ratio is varied as 2, 4, and 6, and
390 the sensitivity of the numerical model to the bond stiffness ratio is shown in Figure 11. According
391 to the parametric study, lower values of the bond stiffness ratio (e.g., $\xi = 2$) decreases the initial
392 macro stiffness by reducing force transfer between the concrete and rebar. On the other hand, a
393 higher bond stiffness ratio (e.g., $\xi = 6$) may yield unnecessary limitations in the numerical model
394 by reducing the bond-slip displacement excessively. Thus, the best numerical estimation is
395 obtained considering a bond stiffness ratio of four ($\xi = 4$), which provided the perfect bond as
396 well as the required force-displacement response from the system when compared to experimental
397 behavior (see Figure 11).

398



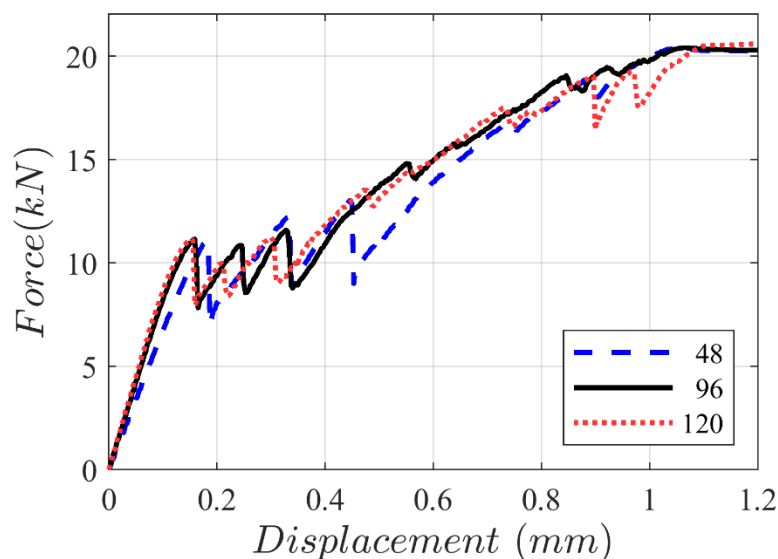
399 Figure 11. Influence of the bond stiffness ratio (ξ) on the macro behavior of the specimen.

400

401 Furthermore, the number of truss elements representing the steel reinforcement is analyzed
402 considering the same contact parameters and bond stiffness ratio, as shown in Figure 12. The total
403 truss element number of 48, 96, and 120 is taken into consideration along the rebar that corresponds
404 roughly to 2, 4, and 6 truss elements passing through a single polyhedral block, respectively. The

405 results demonstrate that a lower number of truss elements (e.g., 48) cause poor force transfer
406 between the concrete and rebar yielding higher deformation and a decrease in stiffness (see Figure
407 12). According to the results of analyses, it is recommended to employ approximately four truss
408 elements though each polyhedral block to make sure that there is a perfect bonding and proper
409 force transfer between the two materials. However, in the case of finer mesh, more truss elements
410 would be advisable. Also, it is observed that an increase in the number of truss elements (e.g., 126)
411 converges to a particular initial stiffness at the macro level; however, it may cause more brittle
412 response as noticed from the stress-drop at the second peak and small contact losses just before
413 reaching to yielding force of steel (Figure 12).

414



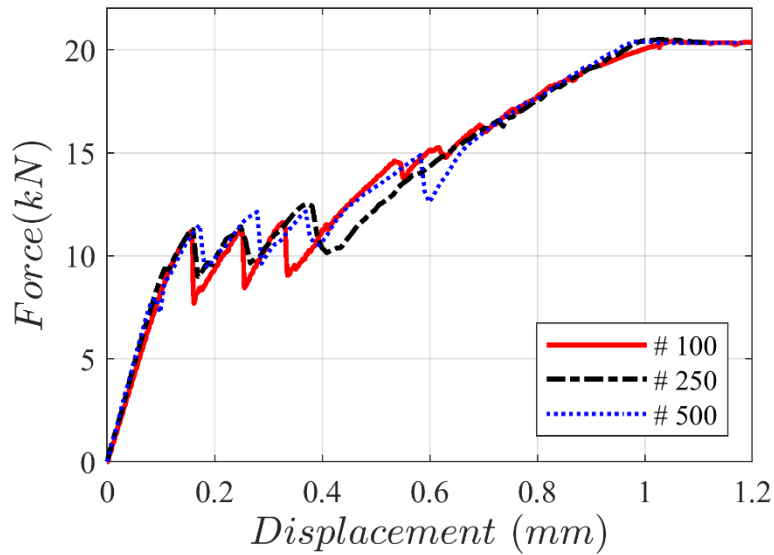
415 Figure 12. Influence of the number of truss elements on the macro behavior of the RC tie ($\xi =$
416 4).

417

418 In order to increase the resolution in the cracking pattern in the computational model, a higher
419 number of blocks are employed considering the same nonlinear contact parameters but different
420 elastic stiffnesses that are predicted based on the average thickness of the fracture zone, as
421 mentioned earlier. The results indicate that the proposed discontinuum models do not exhibit
422 severe block size dependency on the macro behavior of the composite material, as shown in Figure
423 13. However, due to the random generation of the blocks and orientation differences at the fracture

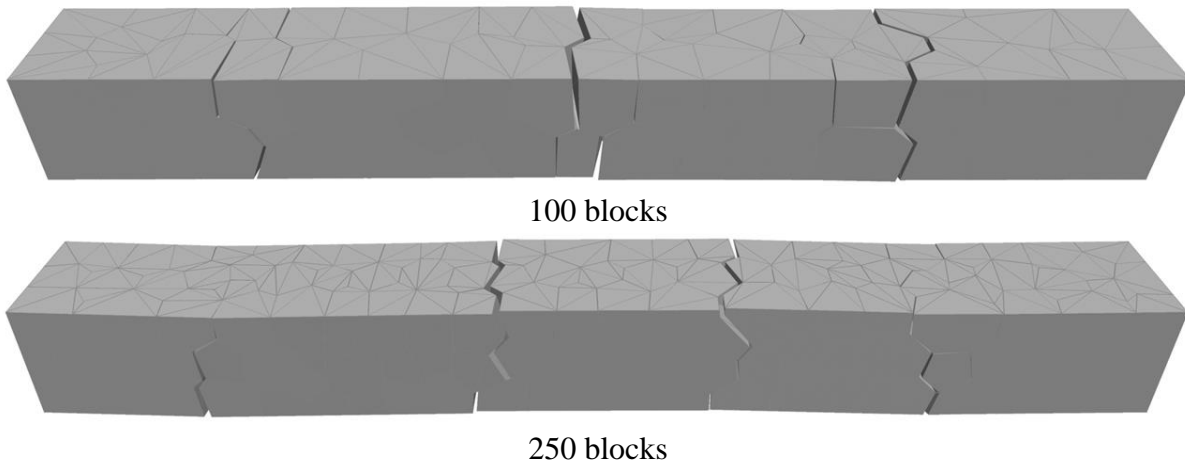
424 surfaces, some small variation is noted for different block numbers. Furthermore, it is worth noting
425 that the crack distribution along the RC prism converges to a particular stabilization trend
426 appearing at the four distinctive locations along the RC tie for higher number blocks (e.g., 250 and
427 500) that can be noticed from Figure 14.

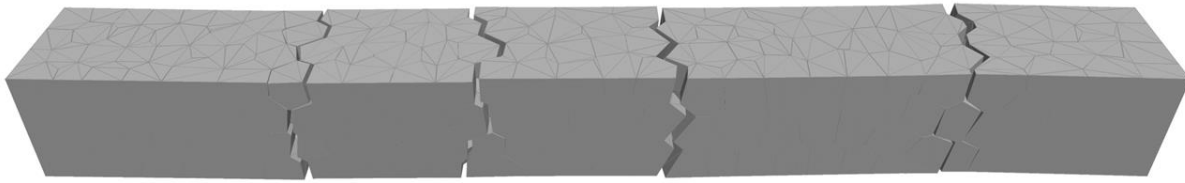
428



429 Figure 13. Influence of the number of polyhedral blocks on the macro behavior of the RC tie.

430





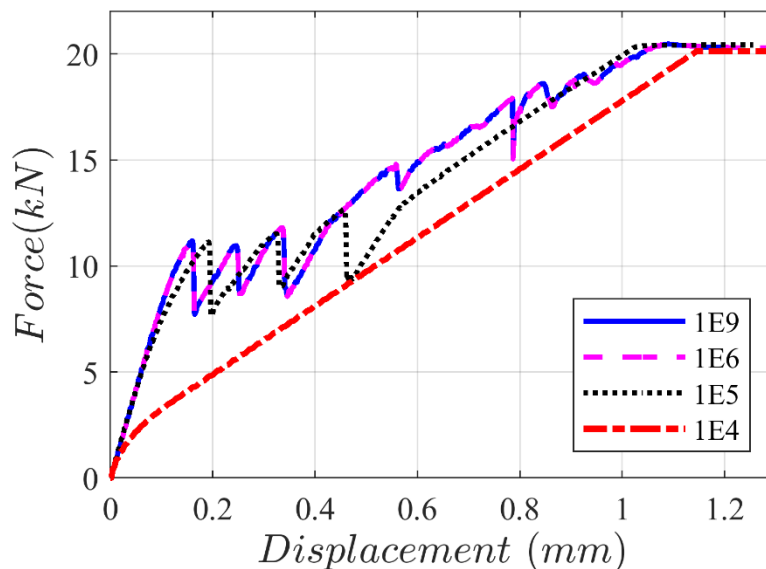
500 blocks

431 Figure 14. Crack pattern of the discrete model at the time of steel yielding for the different
432 numbers of blocks.

433

434 Finally, the bond cohesive strength (c_{bond}) is gradually reduced from 1E9 to 1E4 N/m to explore
435 the weak bond effect on the macro response of the numerical model, as shown in Figure 15. It is
436 important to note that the adopted bond-slip contact law only depends on the constant value of
437 cohesion. Thus, it should be considered as the most straightforward representation of the
438 interaction between concrete and rebar. The results reveal that perfect and weak bond conditions
439 are successfully captured via applied discontinuum models composed of large blocks. Moreover,
440 it can be noticed that once the cohesive strength gets smaller than 1E5 N/m, bond slip failure
441 becomes dominant, and the response gets close to the bare steel behavior.

442



443 Figure 15. Influence of the bond cohesive strength, c_{bond} (N/m) on the macro behavior (100
444 Blocks).

445

446 Through this research, the models are calculated using a computer system with Intel(R) Xeon(R)
447 CPU @ 2.1 GHz processor and 128 GB memory RAM. As shown in Table 4, a significant increase
448 in the computational cost occurs related to the number of blocks and the number of contact points
449 used in the discontinuum system.

450

451 Table 2. Computational time required to perform the direct tension test of RC prisms.

Number of blocks (Number of contact points)	Computational Time (min)
100 (3808)	20
250 (11564)	54
500 (25960)	130

452

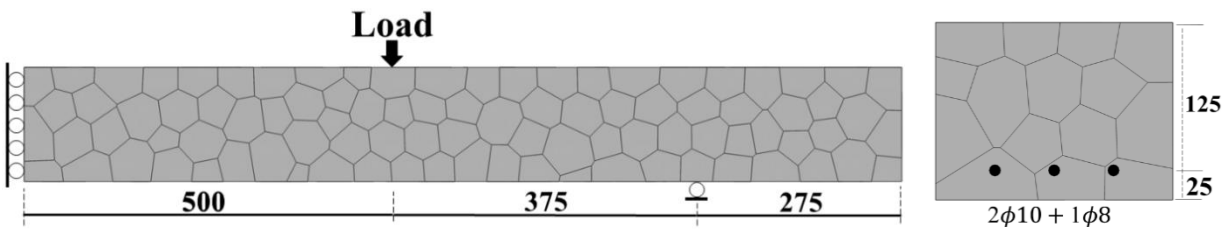
453 **4 Discrete Element Modeling of Bending Test of an RC Beam**

454 To extend the application of the proposed modeling strategy from direct tension to flexural
455 behavior, a four-point bending test of a singly reinforced concrete beam is analyzed, tested by
456 Walraven [25]. Detailed information about geometrical properties, boundary, and loading
457 conditions in the original testing can be found in [25], which are adopted in the numerical work
458 here as appropriate. The same discontinuum modeling technique made up of semi-rigid polyhedral
459 blocks is used, and the outcomes of the comprehensive sensitivity analysis provided in the previous
460 section are utilized. Two discontinuum models consisting of 250 and 500 blocks are generated,
461 where approximately four truss elements are employed for every single polyhedral block along the
462 rebar in both models. Furthermore, to obtain an accurate stress distribution along the beam cross
463 section, a minimum of 10 contact points are considered between the bottom and top surface of the
464 model, as discussed in [49, 50]. A perfect bond condition is assumed during the analysis using a
465 relatively high bond cohesion strength (e.g., 1E9 N/m). Also, to reduce the computational cost,
466 only the half-symmetric geometry of the RC beam is modeled considering roller supports, as
467 shown in Figure 16, together with its dimensions. It is noted that this forces cracks to be symmetric,
468 thus slightly overestimating post-peak shear controlled inelastic failures. Similarly, nodes are
469 subjected to a fixed velocity boundary condition by setting a displacement rate of 5 mm/s to apply
470 flexural forces, represented in Figure 16. During the analysis, the reaction forces are recorded from

471 upper nodes, extracted at each time step by the implemented subroutine in the software based on
472 FISH functions.

473 Additionally, elasto-plastic contact behavior for compression with linear tensile softening is
474 assumed in the normal direction, whereas the Coulomb-slip joint model is defined in the shear
475 direction to simulate the interaction between the blocks replicating concrete. The beam thickness
476 is taken as 200 mm, and the elastic modulus of concrete is defined as 25 GPa. The linear contact
477 stiffness (k_n) is predicted as 500 GPa/m and 625 GPa/m for 250 and 500 blocks, respectively. The
478 additional nonlinear contact parameters can be found in Table 3. Note that the bond stiffness ratio
479 is assumed as 4. The modulus of elasticity and yield stress of the longitudinal rebars are defined
480 based on the reference study as 210 GPa and 440 MPa, respectively.

481



482 Figure 16. *Left*: Dimension of the RC beam (all in mm) and boundary conditions; *Right*: Cross-
483 section including reinforcements.

484

485 Table 3. Nonlinear contact parameters used for blocks representing concrete at the RC beam.

f_T (MPa)	c (MPa)	ϕ (°)	f_c (MPa)	G_f^I (N/m)
2.5	$2f_T$	35	35	60

486

487 The results indicate that there is a good agreement between the numerical analyses and experiment,
488 where the macro behavior of the tested RC beam is captured successfully, given in Figure 17a. It
489 is important to note that results do not depend on the number of blocks. Initially, flexural tensile
490 cracks are observed, which are developed along the constant moment region of the beam.
491 Extensive yielding of reinforcement is found with wide cracks. Since there is a perfect bond
492 assumed during analysis, the tensile forces are carried by the steel reinforcement, which yields
493 new cracks closer to the support predominantly in the constant moment zone. Finally, the failure

494 of the RC beam is obtained due to diagonal tension crack in the discontinuum model similar to the
495 experiment, as shown in Figure 17b. Note that the obtained failure mechanism is typical for
496 reinforced concrete beams without stirrups, as discussed in [8, 13]. The average spacing of cracks
497 in the pure bending zone of the beam is 90 mm, which agrees well with the experimental value.
498 The number of cracks in both models (250 and 500 blocks) is the same. In Table 4, the
499 computational times for the analyses are given.

500

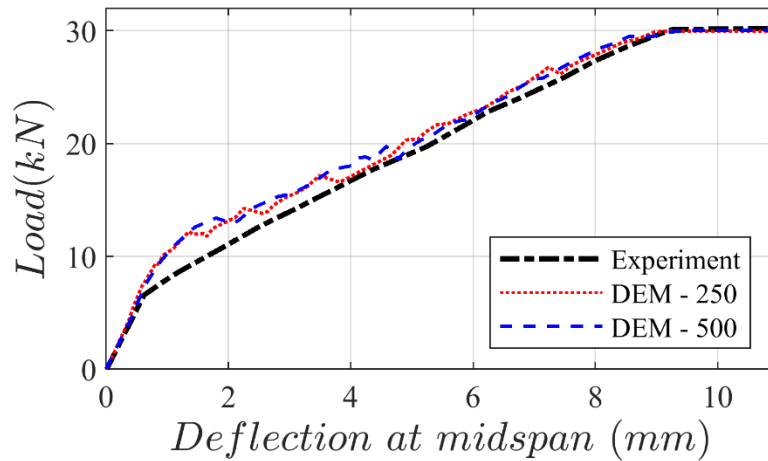
501 Table 4. Computational time required to perform flexural analysis.

Number of blocks (Number of contact points)	Computational Time (min)
250 (27274)	420
500 (55228)	990

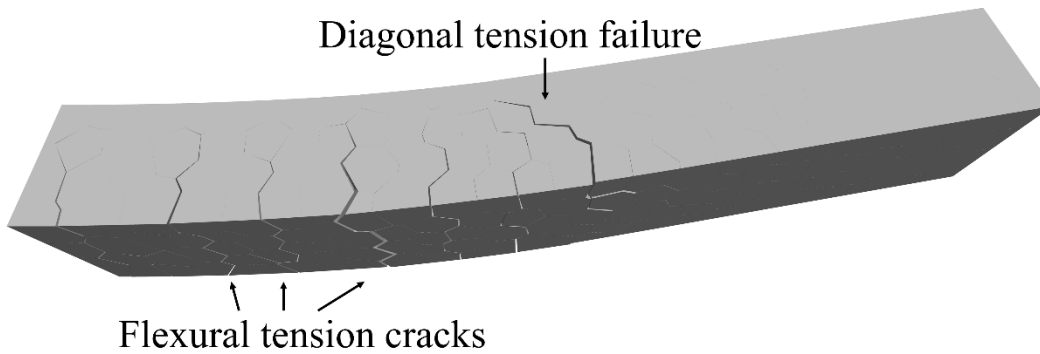
502

503 Finally, linear elastic contact constitutive behavior is assumed instead of the elasto-plastic model
504 in compression as a simple alternative solution. Although the results do not exhibit a considerable
505 difference in the macro behavior in this case (concrete crushing is not relevant for the global beam
506 behavior), neglecting the plasticity in concrete results in slightly higher stiffness on the macro
507 behavior, as shown in Figure 18.

508



a) Load-deflection response of RC beam; Experiment vs. DEM



b) Cracking pattern of RC beam at failure (500 Blocks)

Figure 17. Load-deflection response of the RC beam and failure mechanism.

509

510

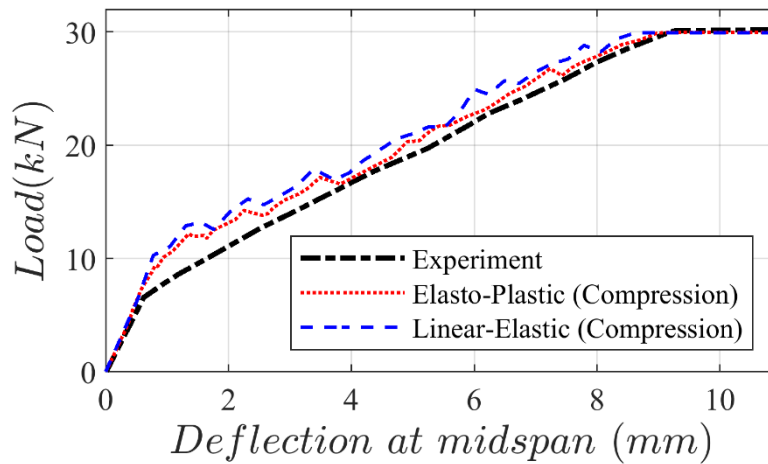


Figure 18. Influence of different contact-constitutive models in the compression regime on the macro-response of the RC beam.

511

512

Preprint version, Reference: Pulatsu, B., Erdogmus, E., Lourenço, P.B. *et al.* Numerical modeling of the tension stiffening in reinforced concrete members via discontinuum models. *Comp. Part. Mech.* (2020). <https://doi.org/10.1007/s40571-020-00342-5>

513

514

515 **5 Conclusions**

516 The primary contribution of this paper lies in combining randomly generated three-dimensional
517 polyhedral blocks with one-dimensional truss elements to simulate the fracture mechanism of RC
518 members within a discrete element modeling framework. The results clearly show the great
519 potential of the applied modeling strategy to simulate the composite action of reinforced concrete
520 members, including tension stiffening phenomenon. From the numerical results, the following
521 conclusions are derived.

- 522 • The proposed phenomenological discontinuum approach captures the fundamental fracture
523 mechanism of RC members based on the inter-block interaction and defined elasto-plastic
524 contact constitutive laws. Moreover, the applied contact models, including the mode-I
525 fracture energy, provide an accurate estimation of the macro behavior, and numerical
526 solutions do not exhibit a severe block size dependency.
- 527 • It is shown that discrete element models have a clear advantage compared to the smeared
528 crack approach since the discrete representation of concrete provides physical crack
529 localizations and realistic fracture patterns that are in line with the experimental
530 observations.
- 531 • From the many numerical simulations carried out, practical inferences are made about the
532 essential parameters in the discontinuum model (e.g., number of truss elements and bond
533 stiffness ratio) to obtain a perfect bond action between concrete and reinforcement in the
534 applied modeling strategy.
- 535 • It is possible to estimate the capacity and corresponding failure patterns efficiently with a
536 rather small number of discrete blocks. Hence, proposed models can be used as a robust
537 research tool to better understand the brittle failure mechanisms of plain concrete subjected
538 to tension and composite behavior of reinforced concrete members under tension and
539 bending.
- 540 • The proposed discontinuum approach can be used for various types of RC members to
541 understand their cracking mechanism and predict their capacity.

542 In future studies, more elaborate bond-slip constitutive laws may be developed and implemented.
543 Furthermore, the time-dependent material degradation effects, such as creep, shrinkage, etc., may
544 be considered to better estimate the current performance of structural RC members.

545 **Conflict of Interest**

546 The authors declare that they have no conflict of interest

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