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A dynamic programming model for designing a quality control plan in a manufacturing process

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Abstract

Process quality planning should establish the quality control plan to achieve the desired quality level with the minimum quality cost (appraisal and failure costs) for the final product. This plan sets out the critical quality variables, the control stations in the process, and the control method at each control station. The quality costs associated with quality control and defective products can be greater than or less than ideal regarding the required quality level. The purpose of this paper is to provide a stochastic dynamic programming model for designing the quality control plan in a manufacturing process, which allows obtaining the desired level of control with the lowest cost. Inputs to the model are, in particular, control stations in the process, levels of quality, control methodologies (no control, statistical process control, 100% inspection), probabilities of changing the quality level and quality costs. The output of this model is the quality control plan that satisfies the desired level of quality at the lowest cost. This plan establishes the control stations, the methodology used in each control station, the desired quality level for the final product, and the estimated quality costs. Finally, an illustrative example based on a manufacturing process demonstrates the applicability of this approach and several considerations are reported about future research directions.

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Keywords: Dynamic programming model; Optimization; Quality control plan; Quality costs.

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1. Introduction

The combination of high quality products at low cost has been a critical issue for manufacturers to remain currently competitive in global markets. Companies use a variety of methodologies (sampling inspection, 100% inspection, re-inspection, control charts ...) that allows them to achieve the desired quality levels at competitive costs. Such methodologies focus on operations where there is a greater probability of occurrence of failures or its impact is more significant. In this way, an integrated approach to the quality of the manufacturing system is not done, resulting in a misuse of human, financial and material resources and increased quality costs. This situation is more relevant with the increasing rate of introduction of new products in companies [1].

Manufacturing systems are generally composed of several workstations (WS) or stages, in which raw materials pass through various operations and are transformed into finished products. This type of systems is called multi-station (or multi-stage) manufacturing systems (MMS) [2]. In MMS, each WS will produce a proportion of defective items [3]. To economically maintain the product quality level is a critical issue for the MMS, in which each station may inevitably shift to the out-of-control condition resulting in higher nonconforming rate and larger quality loss [4, 5].

Quality planning should establish the quality control plan (QCP) of the MMS to achieve the desired quality level for the final product. This plan sets out the critical quality variables, the location of quality control stations (QCS) in the MMS, and the control method at each QCS. After defining the QCP, some control parameters may be changed, for example, defining different control limits to change the probability of getting false warnings. This second level of analysis is not part of this study, but several studies deal in depth with this subject [6, 7].

The implementation of the QCP results in quality costs (appraisal and failure costs) that can be bigger than ideal. Greater process control can result in a reduction of failure costs but also an increase in appraisal costs, while reducing the control can result in a reduction of appraisal costs but also an increase of failure costs. Several studies deal with this subject using, fundamentally, dynamic programming models [6, 8], non-linear programming models [9], or Monte Carlo simulation methods [10, 11].

The importance of QCP as an element of management of MMS is recognized by academics and industrial engineers, however the poor practical application of the theoretical models can be difficult [12]. These models are complex, given the characteristics of the MMS, the high number of parameters/variables and the depth level of the models. The literature presents several barriers-difficulties [13, 14] for this situation of which are highlighted: lack of knowledge/information of how measure the high number of parameters/variables of the models, the ignorance of the benefits of a suitable QCP, the company culture which do not promote rigor and evaluation, the difficulty in identifying quality cost elements, and difficulty in collection of quality data.

Considering technological developments and powerful data collection, processing and analysis, communication, and decision support systems available in industry 4.0 era, some of these barriers may be mitigated or eliminated. Assuming that the industry 4.0 context provides more data on process parameters/variables, these data can used in real time to establish the appropriate QCP, that minimize quality costs.

The purpose of this paper is to provide a stochastic dynamic programming (SDP) model for designing the QCP in a MMS, which allows obtaining the desired quality level with the lowest cost. The remaining of this paper is organized as follows. Section 2 presents the QCS allocation problem, the description of MMS and the quality control process decision of a WS. The introduction to SDP is presented in Section 3. The proposed model, with its assumptions and dynamic programming formulation for the QCP, is presented in Section 4. The last two sections present an illustrative example and conclusions.

2. Problem statement

2.1. Quality Control Station Allocation Problem

The allocation of QCS in MMS has been studied extensively over the decades. Shetwan *et al.* [6] present a review the existing approaches, models comparison and solution techniques applied in allocation of QCS.

The problem when designing the QCP for a new product is to know in which WS to implement quality control, what or which quality characteristics to control, and what type inspection / control should be implemented. As extreme QCP, we can have: no QCS in the whole process, or else; for each workstation WS_n ($n = 1, 2, ..., N$) implement a QCS_n

control station (Fig. 1). For QCP, the cost of quality can be estimated. Practice has shown that the best solutions are between these two extremes. The procedure of making decisions of whether or not to inspect a product at every processing workstation is shown schematically in Fig. 2.

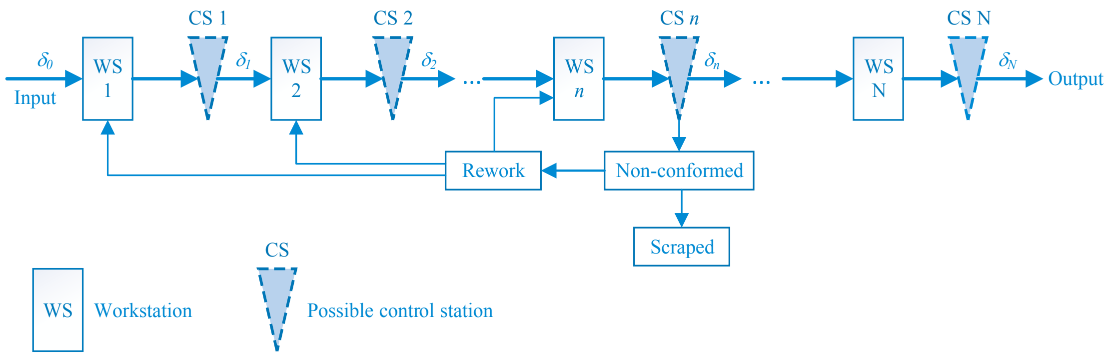


Fig. 1. Multi-station manufacturing system

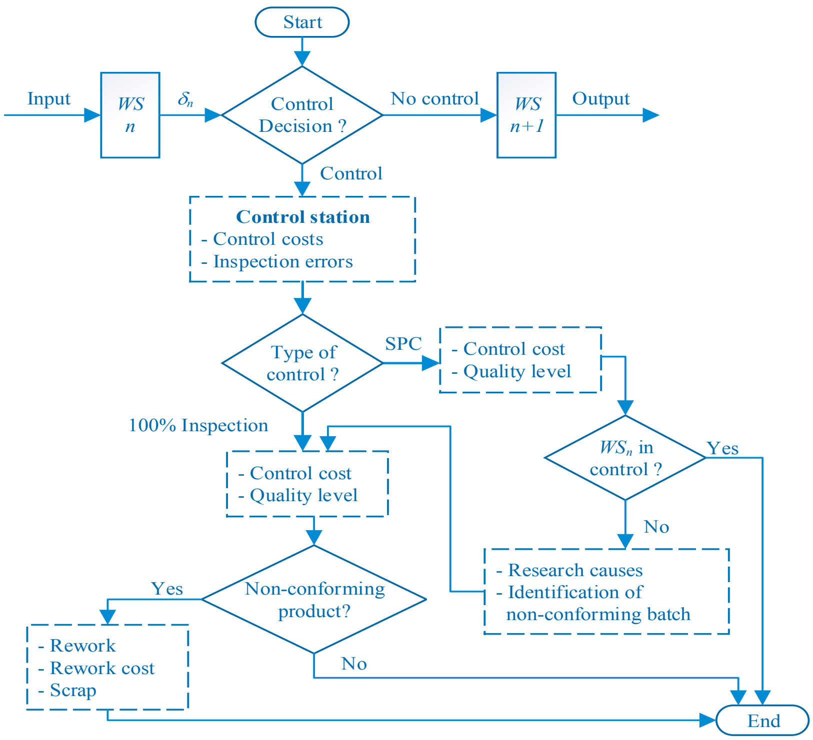


Fig. 2. Quality control process decision of workstation n

2.2. Description of the system

The type of MMS considered in this work is characterized by the following general considerations:

- MMS is constituted by N WS and N possible QCS, where in each WS the items undergo transformations until the final product is obtained. Only one final product is considered in the system. The raw material can present a nonconforming product rate δ_0, and in WS_n nonconforming product rate is δ_n. Each QCS can be assigned to perform inspections of critical quality variables, influenced by one or more upstream WS.

- QCS can determine conforming and non-conforming items. Conforming items are sent to the next stage and non-conforming items may be repaired or replaced by a conforming item and sent to the next stage.
- For WS_n, different values of δ_n correspond to different quality levels outputs (QL_n^O). Typical values for QL_n are: 99% or 95% which corresponds to the nonconforming product rate, δ_n of 1% or 5%, respectively.
- Every WS_n is either in-control or out-of-control due to the equipment condition and the operating environment. The out-of-control condition has a higher nonconforming product rate [4] and, therefore, higher quality cost.
- To reduce the quality loss of MMS, different decisions regarding control mechanisms in each WS_n can be made. For example, the output of WS_n may have no control, sampling control using control charts, or 100% inspection. Different control decisions result in different levels of product quality and have different quality costs.

In production lines, decisions are made among alternative inspection strategies for any possible QCS, sometimes depending on each other. To find out the cheapest QCP of a MMS this study proposes to use the SDP method [15].

3. Stochastic dynamic programming

Dynamic programming is a mathematical optimization [16] and has found applications in numerous fields, from aerospace engineering to economics. It refers to simplifying a complicated problem by breaking it down into simpler sub-problems in a recursive manner. The problem presented in this work regarding the MMS can be divided in decisions made at each QCS. Each decision will influence quality costs and output product quality levels, but it is also influenced by received quality level. So, the problem can be divided into smaller overlapping sub-problems and an optimum solution can be achieved by using an optimum solution of smaller sub-problems and memorization.

In SDP the state at the next stage is not completely determined by the state and policy decision at the current stage. There is a probability distribution for what the next stage will be, which is completely determined by the state and the policy decision at the current stage. The resulting basic structure for SDP is described in Fig. 3.

Given the state s_n and decision x_n at stage n, the system goes to state i with probability p_i^{n,x_n} ($i =1, 2, ..., S$). If the system goes to state i, C_i^{n,x_n} is the contribution of stage n with policy decision x_n to the objective function (S – denote the number of possible states at stage $n+1$ and label these states on the right side as 1, 2, …, S). When Figure 3 is expanded to include all the possible states and decisions at all the stages, it is sometimes referred to as a decision tree. If the decision free is not too large, it provides a useful way of summarizing the various possibilities. Because of the stochastic structure, the relationship between $f_n(s_n, x_n)$ and $f_{n+1}^*(s_{n+1})$ is complicated, the precise form of this relationship will depend upon the form of the overall objective function.

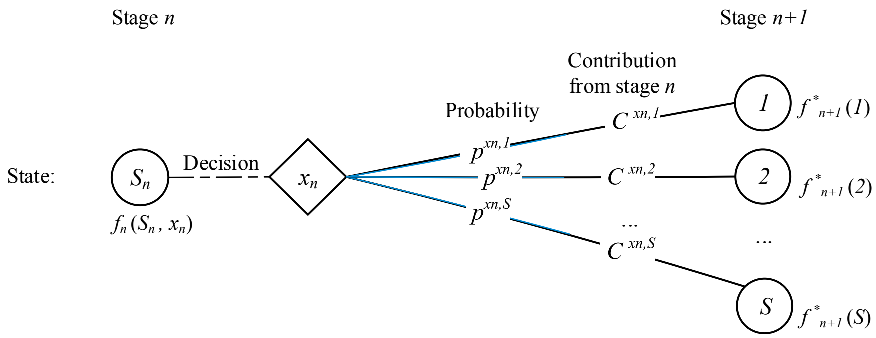


Fig. 3. Basic structure for stochastic dynamic programming

Since it is intended to obtain the optimal control plan of a MMS, the objective for this problem is to minimize the total expected sum of contribution (costs) from the all individual stages of the MMS. In this case, $f_n(s_n, x_n)$ represents the minimum expected sum from stage n forward, given that the state and decision at stage n. Thus,

$f_n(s_n,x_n)=\sum_{k=1}^{S} p_k\left[C_k+f_{n+1}^*(k)\right]$ with $f_{n+1}^*(k)=\min f_{n+1}(k,x_{n+1})$ where this minimization is taken over the feasible values of x_{n+1}.

4. Model formulation

4.1. Assumptions

1. For WS_n ($n=1, 2,..., N$) the quality level output QL_n^O will be equal to its quality level input QL_n^I (with a probability p) or reduced one level, $QL_n^O=QL_n^I-1$ (with a probability $q=1-p$). In these conditions, for an MMS with N workstation there are $N+1$ admissible quality levels (states).
2. After WS_n can be placed a QCS_n, as shown in Fig. 1. In each QCS_n one of three alternative decisions can be made:
    a. do not do any quality control ($x_n = 0$). In practice, there will be no *QCS* and, consequently, the control costs will be zero;
    b. to do a sample inspection using control charts ($x_n = 1$), and incurring control costs;
    c. perform 100% inspection of the items produced in WS_n ($x_n = 2$), and incurring control costs.
3. With policy decision $x_n = 0$, the output quality of WS_n is identical to the quality of input of WS_{n+1} ($QL_n^O=QL_{n+1}^I$)
4. Adopting policy decision $x_n = 1$, two situations may occur with a known probability distribution:
    a. QL_n^O is identical to QL_{n+1}^I in case the operation performed in the WS_n is under control (stable). The probability of this result is given by the probability that the operation is not under control (the α error is considered negligible);
    b. QL_{n+1}^I improves one level compared to QL_n^O if WS_n is out of control. Under these circumstances a 100% inspection will be performed on the last items produced in WS_n.
5. The policy decision $x_n=2$ will also allow two distinct situations with a known probability distribution:
    a. QL_{n+1}^I passes to the maximum quality level (MQL) considered in this study;
    b. QL_{n+1}^I passes to the quality level immediately below MQL, e.g. due to inspection errors in QCS_n.
6. At the end of the MMS, each level of quality of the final product will have an associated cost (related with potential of external failures), represented by vector F_0.

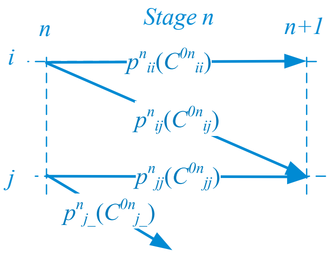
4.2. Dynamic programming formulation for the quality control plan

Based on the assumptions presented in subsection 4.1 and the generic SDP model presented in section 3, the SDP formulation for the problem of QCS allocation in MMS is modelled by the following elements:

- *Stage n*: *QCS* in a MMS (n=1, 2, 3, ..., N);
- *Policy decision x_n*: type of control for stage n (no control (x_n=0); SPC (x_n=1); 100% inspection (x_n=2))
- *State s_n*: Quality Level at beginning of stage n (s_n ={1, 2, …N+1}).
- $f_n=f_n(s_n,x_n)$: minimum expected cost eared from the stages n, ... , N, given the state in stage n is s_n and a decision x_n is made in stage n;
- $f_n^*(s_n)$: minimum total expected cost for stages n, ... , N, given the state in stage n is s_n;
- $x_n^*(s_n)$: optimal decision in stage n, given that the state in stage n is s_n.

Policy decision: No control (x_n=0)

In Fig. 4 it is shown, for the policy decision $x_n = 0$, all possible transitions between states i and j, of stage n, □$_{ijk} \in S$. The matrix P^n represents the probabilities of the WS_n to maintain the quality level of its input ($QL_n^I=QL_n^O$) or degrade quality of the output relative to the quality of the input to the level immediately below ($QL_n^O=QL_n^I-1$) . We consider that this matrix does not change with the different policy decisions that can be taken for stage n. With this policy decision, since no control action is considered for the WS_n output, $QL_n^O = QL_{n+1}^I$ ($n=1, 2, \ldots N$). The matrix C^{0n} presents the contributions of stage n to $x_n=0$ (quality costs associated with every possible transition between states). In this particular case, the appraisal costs in stage n are null.

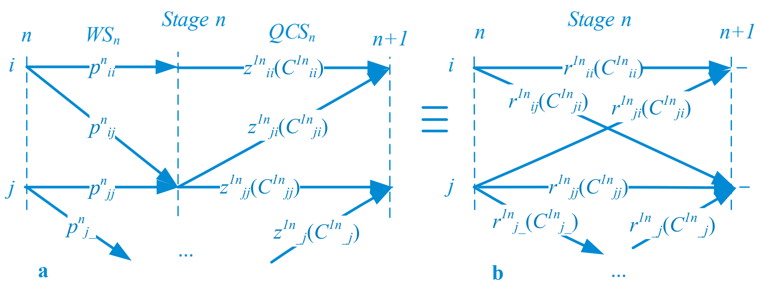


$$P^n = Z_n^0 = \begin{bmatrix} p_{ii}^n & p_{ij}^n & - & - \\ - & p_{jj}^n & p_{j\_}^n & - \\ - & - & \ldots & \ldots \end{bmatrix}$$

Fig. 4. Graph of the possible transitions between states i, j and k of stage n for a policy decision $x_n=0$

Policy decision: SPC control ($x_n=1$)

Fig. 5 shows, for policy decision $x_n=1$, the possible transitions between states i and j of stage n, □$_{ij} \in S$. The matrix P^n represents the probabilities of maintaining or changing among quality levels. The matrix Z^{1n} represents the probabilities of maintaining or changing among quality levels due to the quality policy (SPC) and the matrix C^{1n} represents the contributions of stage n (SPC costs). By combining the admissible transitions for the WS_n with the allowable transitions for the CS_n (Fig. 5-a) we obtain the graph represented on Fig. 5-b.



$$Z_n^1 = \begin{bmatrix} z_{ii}^{1n} & - & - & - \\ z_{ji}^{1n} & z_{jj}^{1n} & - & - \\ - & \ldots & \ldots & - \end{bmatrix}$$

$$R_n^1 = \begin{bmatrix} r_{ii}^{1n} & r_{ij}^{1n} & - & - \\ r_{ji}^{1n} & r_{jj}^{1n} & r_{j\_}^{1n} & - \\ - & r_{\_j}^{1n} & \ldots & \ldots \end{bmatrix}$$

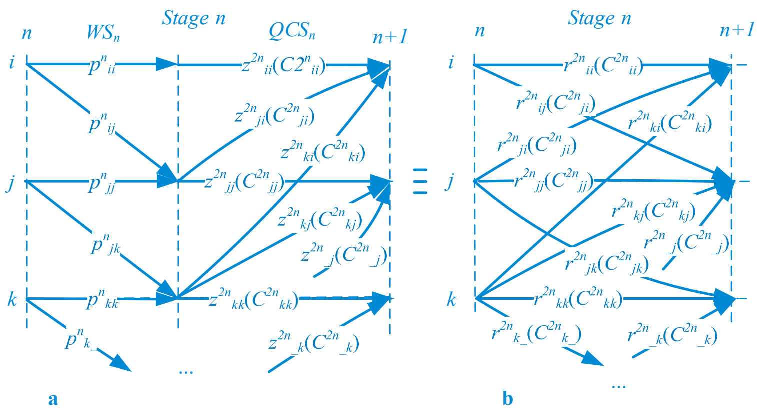
Fig. 5. Graph of the possible transitions between states i, j and k of stage n for the policy decision $x_n=1$

The transition probabilities of R_n^1 are given by $r_{ii}^{1n} = p_{ii}^n z_{ii}^{1n} + p_{ij}^n z_{ji}^{1n}$; $r_{ij}^{1n} = p_{ij}^n z_{jj}^{1n}$; $r_{ji}^{1n} = p_{jj}^n z_{ji}^{1n}$; $r_{jj}^{1n} = p_{jj}^n z_{jj}^{1n} + p_{jk}^n z_{kj}^{1n}$.

Policy decision: 100% Inspection ($x_n=2$)

For policy decision $x_n=2$, all the possible transitions between states i, j and k of stage n, □$_{ijk} \in S$ are represented in Fig. 6, taking into account the possibilities of transitions referred in section 4.1.

Thus, we obtain the transition probabilities R_n^2 with: $r_{ii}^{2n} = p_{ii}^n z_{ii}^{2n} + p_{ij}^n z_{ji}^{2n}$; $r_{ji}^{2n} = p_{jj}^n z_{ji}^{2n}$; $r_{ki}^{2n} = p_{kk}^n z_{ki}^{2n}$ $r_{ij}^{2n} = p_{ij}^n z_{jj}^{2n}$ $r_{jj}^{2n} = p_{jj}^n z_{jj}^{2n} + p_{jk}^n z_{kj}^{2n}$; $r_{kj}^{2n} = p_{kk}^n z_{kj}^{2n}$; $r_{jk}^{2n} = p_{jk}^n z_{kk}^{2n}$; $r_{kk}^{2n} = p_{kk}^n z_{kk}^{2n} + p_{k\_}^n z_{\_k}^{2n}$.



$$Z_n^2=\begin{bmatrix} z_{ii}^{2n} & - & - & - & - \\ z_{ji}^{2n} & z_{jj}^{2n} & - & - & - \\ z_{ki}^{2n} & z_{kj}^{2n} & z_{kk}^{2n} & - & - \\ - & \cdots & \cdots & \cdots & - \end{bmatrix}$$

$$R_n^2=\begin{bmatrix} r_{ii}^{2n} & r_{ij}^{2n} & - & - & - \\ r_{ji}^{2n} & r_{jj}^{2n} & r_{jk}^{2n} & - & - \\ r_{ki}^{2n} & r_{kj}^{2n} & r_{kk}^{2n} & - & - \\ - & - & \cdots & \cdots & \cdots \end{bmatrix}$$

Fig. 6. Graph of the possible transitions between states i, j and k of stage n for the policy decision x_n=2

5. Illustrative example

To illustrate the practical application of the proposed SDP model, let us consider an MMS under conditions described in section 4, with N=3 and S=4. Thus, the SDP model has 3 stages ($n=\{1, 2, 3\}$), 3 policy decisions ($x_n=\{0, 1, 2\}$) and 4 states (S=$\{QL_1, QL_2, QL_3, QL_4\}$). In order to limit state space, it is assumed that the state QL_1 holds all states $\geq QL_1$ and state QL_4, all states $\leq QL_4$. Without loss of generality, let us also consider that S={≥99,]99, 95],]95, 09], ≤90}, $F_0^T=\begin{bmatrix}0 & 100 & 1000 & 10000\end{bmatrix}$ and that for each x_n, $R_1^{x_n}=R_2^{x_n}=R_3^{x_n}$ and $C_1^{x_n}=C_2^{x_n}=C_3^{x_n}$, with:

$$R_1^0=\begin{bmatrix} 0.8 & 0.2 & - & - \\ - & 0.8 & 0.2 & - \\ - & - & 0.8 & 0.2 \\ - & - & - & 1 \end{bmatrix},\ R_1^1=\begin{bmatrix} 0.95 & 0.05 & - & - \\ - & 0.95 & 0.05 & - \\ - & - & 0.95 & 0.05 \\ - & - & - & 1 \end{bmatrix},\ R_1^2=\begin{bmatrix} 0.99 & 0.01 & - & - \\ 0.9 & 0.1 & - & - \\ 0.8 & 0.2 & - & - \\ 0.75 & 0.25 & - & - \end{bmatrix}$$

$$C_1^0=\begin{bmatrix} 0 & 0 & - & - \\ - & 0 & 0 & - \\ - & - & 0 & 0 \\ - & - & - & 0 \end{bmatrix},\ C_1^1=\begin{bmatrix} 25 & 25 & - & - \\ - & 25 & 25 & - \\ - & - & 25 & 25 \\ - & - & - & 25 \end{bmatrix}\ C_1^2=\begin{bmatrix} 150 & 150 & - & - \\ 250 & 250 & - & - \\ 500 & 500 & - & - \\ 1500 & 500 & - & - \end{bmatrix}$$

Solving the example using the SDP method [16] we obtain the optimal solution of the problem i.e., the policy decision to be made at each stage n, knowing the state s_n (state at the beginning of stage n). For example, if the quality level of the WS_1 input is QL_2 then the optimal QCP is as follows:

Stage n	1	2		3		
State s_n at the beginning of stage n	QL_2	QL_2	QL_3	QL_1	QL_2	QL_3
Policy decision x_n	*1*	1	2	0	1	2
Cost of optimal QCP						254,4

6. Conclusion

Modern MMS requires significantly larger number of inspections to be considered. The inspection allocation problem has been studied using several analytical optimization and simulations methods. Although a number of methods are available to address this problem, there is still a considerable gap between theoretical methods and their practical application. The focus of this work is related to reducing this gap because we consider that obtaining a good QCP for a MMS is a key element in achieving the desired quality level with the lowest control cost.

The proposed approach makes it possible to obtain the QCP of an MMS using an SDP model. As shown by the example the approach is simple to understand and to implement. Some obstacles to its implementation may arise from the difficulty of obtaining the inputs of the model. Here we suggest the use of other studies to obtain the probabilities of transition and inspection costs in each operation, as well as the use of estimations made by operators and experts deep knowledge of the MMS under analysis

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