Universidade do Minho
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Assessment and identification of concrete box-girder bridges properties using surrogate model calibration. Case study: El Tablazo Bridge.

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Abstract

This work consists in identifying and assessing the properties in a pre-stressed concrete bridge related to material, geometry and physic sources, through a surrogate model. The participation of this mathematical model allows to generate a relationship between bridge properties and its dynamic response, with the purpose of creating a tool to predict the analytical values of the studied properties from measured eigenfrequencies; in this case, it is introduced the identification of damage scenarios, giving the application for validate the generated metamodel (Artificial Neural Network - ANN). A FE model is developed to simulate the studied structure, a Colombian bridge called El Tablazo, one of the higher in the country of this type (box-girder bridge), with a total length of 560 meters, located on the Sogamoso riverbed in the region of Santander - Colombia. Once the damage scenarios are defined, this work allows to indicate the basis for futures plans of structural health monitoring.

KEYWORDS: Artificial Neural Network, Structural Performance, Damage Scenarios, Dynamic Behavior, Bridge Assessment.

Resumo

Este trabalho consiste em identificar e avaliar as propriedades de uma ponte em betão pré-esforçado em relação ao material, geometria e características físicas através de um metamodelo. A participação deste modelo matemático permite gerar uma relação entre as propriedades da ponte e sua resposta dinâmica, com o objetivo de criar uma ferramenta para prever os valores analíticos das propriedades estudadas a partir de frequências próprias medidas; neste caso, é introduzida a identificação de cenários de dano, dando uma aplicação para validar o metamodelo (Rede Neural Artificial - ANN). Um modelo de elemento finito é desenvolvido para simular a estrutura estudada, uma ponte colombiana chamada El Tablazo, uma das que apresenta maior altura do país em seu tipo (pontes em viga-caixão), com um comprimento total de 560 metros, localizada no rio Sogamoso, na região de Santander - Colômbia. Uma vez que os cenários de dano são definidos, a tese permite indicar a base para os planos futuros de monitoramento da saúde estrutural.

PALAVRAS-CHAVE: Rede Neural Artificial, Desempenho Estrutural, Cenários de Dano, Comportamento Dinâmico, Avaliação de Ponte.

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CHAPTER 1

INTRODUCTION AND METHODOLOGY

1.1 Introduction

The investment in the development of a country infrastructure like Colombia is reflected in its competitive capacity in this new globalized era. To bring economic and social growth to the regions it is necessary a good condition of the road infrastructure [23], therefore, bridges construction is rising and it is important to assure the suitable service of the road infrastructure [48]. This is why, it arises the need to assess the performance of already built structures, to anticipate and identify any damage. These evaluations will help to know their life time, and if it is required any rehabilitation or strengthening [32, 33, 42].

Structural assessment begins understanding the behavior of the bridge and how it is influenced by its physical properties (i.e. materials and geometry). These properties define the dynamic behavior and its characteristic values (natural frequencies, modal damping, mode shapes, modal curvatures, modal strain energy and modal flexibility), as well it is necessary the development of a finite element model (FE) used to simulate, in theory, the real behavior of the bridge [36].

The identification and assessment of bridges properties and its relationship with the dynamic behavior allow to detect damages through abrupt changes of its eigenfrequencies (first level proposed by Rytter [45]). Due to this complex issue, is necessary the use of machine learning algorithms [36], for this reason, a surrogate model is developed as a tool to identify damages due to frequency changes.

To validate the surrogate model (or metamodel) is necessary to use a set of damage scenarios in order to test the prediction capacity based on an algorithm (Frequency-based damage detection - FBDD). Some authors describe this method [12, 25] using eigenvalues before and after damage for its identification, therefore any data changing in future measurements and monitoring of the bridge generate an update in the FE model, allowing to control the structure safety [43].

1.2 Objectives

The objectives of this work are:

- Develop a base of a monitoring plan on the "*El Tablazo*" Bridge.
- Generate a finite element model of the studied structure.
- Assess and identify properties of the bridge sensitive to the dynamic response (eigenfrequencies).
- Define a set of damages scenarios on the bridge to validate a surrogate model capable to identify the damage.
- Compare the optimization algorithms used in the surrogate model.

1.3 Methodology

In order to reach the objectives of this work, as a base for future plans of monitoring the structural health of the El Tablazo bridge, the next procedure will be followed:

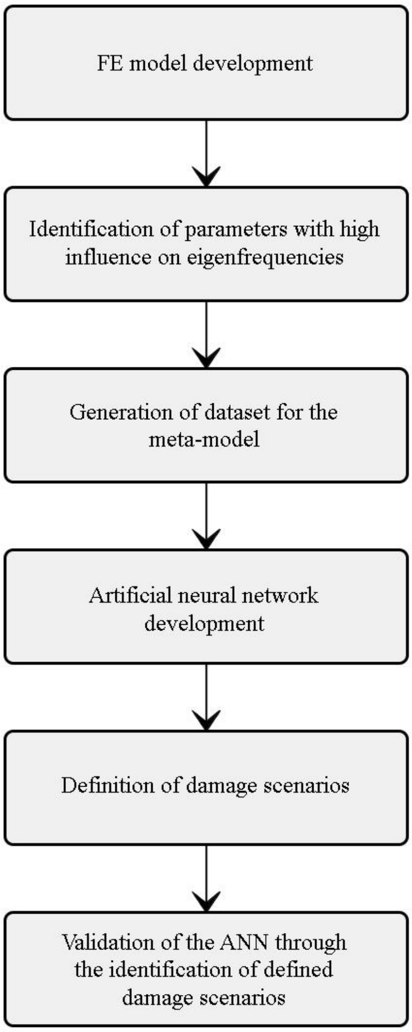

Figure 1. Flowchart of the methodology used in this work.

Firstly, a finite element model should be developed using the software MIDAS Civil©. The bridge is idealized from blueprints, besides, the missing data is assumed based on bibliography and reference consulted (Colombian norms) [1, 2]; the reference FE model should be composed by beam elements for all the structural parts such a girders, columns and foundation; besides, it must be considered the joints and boundary conditions.

To develop an ANN, it is necessary to define the components of the meta-model structure; therefore, a sensitivity analysis should be developed to identify which parameters have a higher influence on the bridge eigenfrequencies; once the critical parameters are defined, a dataset must be generated to fill the ANN; therefore, 112 FE models are randomly developed using combinations of critical parameters.

The training process of the ANN is optimized through a genetic algorithm; however, it is not discarded the traditional methods (backpropagation) used to train neural networks, a comparison between them is necessary to select the most appropriated for this research; the algorithm is developed using tools of the software MATLAB©.

Damage scenarios are defined in the FE model to affect the dynamic behavior of the structure, therefore this step considered a reduction of stiffness in the structural elements (e.g. girders, columns and foundations), representing scenarios related with material and geometry (e.g. cracking, bad concrete).

The last step is the validation of the ANN through a parameter assessment procedure. Once the eigenfrequencies are obtained for each defined scenario, the network is used to determine the value of the critical parameters. Therefore, an error is computed between the reduced parameters (defined by damage scenario) and those calculated with ANN.

CHAPTER 2

STATE OF THE ART REVIEW

2.1 Surrogate models

It is verified that in many areas of engineering, surrogate models are used to replace the traditional way to obtain data and verify the performance of the particular engineering products. However, the investment in computation time using analytical models in some cases is impractical (a simulation could take minutes, hours or even days). To deal with the problem, a construction of approximation models (surrogate models) to predict the performance through the generation of a relationship between the analyzed parameters (Inputs - Outputs) is recommended [14].

There exist different types of approximation methods, D. Gorissen [14] define the global approximations using compact surrogate model, highlighting methods such a rational function, Kriging models, ANN, Splines, and Support Vector Machines (SVM).

In civil engineering problems, the assessment of complex structures, such as bridges, requires the involvement of metamodels to reduce the computational time, J. Ghosh [24] presents a research with a large number of nonlinear dynamic analyses of FE models to obtain a seismic response; therefore, the approximation is generated from different types of surrogate models and are compared to each other, such as polynomial response surface models with stepwise regression (PRSM), multivariate adaptive regression splines (MARS), radial basis function networks (RBFN) and support vector machines for regression (SVMR).

2.1.1 Artificial Neural network

According to Bakhary [4], the artificial neural network (ANN) is used by many researchers in structural problems, one of them is the identification of damage location and its severity through variations inputs – outputs parameters; besides, Bakhary [4] presented a review of studies about the use of ANN, such a detection of damage in a 3-storey frame, a detailed treatment of the network architecture to identify damage in a bar bridge truss, the use of a counter-propagation neural network in the damage analysis in a continuous beam, and a procedure for the damages assessment of steel structures.

In [27] is presented a research where is used the back-propagation neural network (BPN), in the estimation of depth scour in bridges the performances of the network is validated by using measured data of the bridge; [10] use the same training method to develop the network and estimate a prediction of local scour around bridge piers, the validation of the network is through laboratory tests.

Bakhary [4] conclude, most of the studies developed using an artificial neural network provide a correct damage identification, besides, it is specified the probability of error in different stages of the health monitoring by modelling error in the FE model due to the uncertainties of parameters, and the possible error in measured data; for this reasons, it is presented a developed meta-model considering the uncertainties [4].

2.2 Structural health monitoring

The actual structural dynamics, allows the development of accurate models with many fields of application in civil engineering, such as damage detection, structural control, structural assessment and health monitoring [5]. Structural health monitoring (SHM) is implemented to extend the lifetime of the already build structures, through the identification of changes in the dynamic response [7].

G. Roeck [16] describes the vibration monitoring plan developed in a Swiss bridge named Z24, defining damage scenarios such as settlement of foundation, tilt of foundation, landslide, among others. This research identified only damage scenarios produced by a stiffness reduction; the measured data is obtained through ambient vibration testing.

Hinterding, R. [20] describes the introduction of a multi-channel dynamic monitoring system in a long span concrete arch bridge located in Porto, Portugal. The Infante D. Henrique bridge is studied through the continuous data received by the installed monitoring system, allowing to identify modal parameters (natural frequencies, mode shapes). Magalhaes, F., Cunha, A., & Caetano, E. [30, 31] describes the automatic monitoring plan developed, which combine high-quality equipment and powerful identification algorithms after months of measurements.

2.2.1 Structural damage identification

Rytter [45] proposed four levels for assessment being the aim for the damage identification methods, which are: (1) detection level (to detect damages in the structure), (2) location level (to locate the detected damage), (3) assessment level (to quantify the damage severity) and (4) prediction level (to estimate the remaining service life). According to [36] the levels 1 to 3 can be develop with vibration data only, but the level 4 requires the incorporation of a sophisticated numerical model.

According to [25] the damage assessment has an important role in any decision to repair, rehabilitate, or replace a structure; an early intervention could have a significant saving resources. The damage could be detected and localized using change in eigenfrequencies; however, the localization is limited due to the fact that natural frequencies presents low changes despite of the significant damage. Two methods are presented in the research, the frequency-based damage detection (FBDD) and the mode-shape-based detection (MBDD) method, making a comparison between them.

On the other hand, C. R. Farrar [12] shows a comparison of damage identification algorithms, the case study developed is an interstate highway bridge (the I-40 bridge), located in Albuquerque, NM (United States of America), as it is expected, a FE model is updated by measured vibration testing, and the damage scenarios are introduced; the purpose of the review algorithms are to identify and locate the damage in the numerical model.

Table 1. Description of damage identification algorithms based in [9, 12].

Algorithm	**Description**
Damage index method	Based in measured mode shapes before and after damage, could identify and locate damages (is enough with a few modes to obtain good results).
Mode shape curvature method	Characterize damage related with loss of stiffness and increase in structural damping. Therefore, eigenfrequencies and mode shapes are used to detect and locate damage. (by assuming a damage effect in stiffness)
Change in flexibility method	As an alternative method, using the flexibility of the structure to identify changes, however, dynamic measurement is required, but is not precise as an analytical model.
Change in uniform load surface curvature method	A combination between mode shape curvature and change in flexibility methods.
Change in stiffness method	Using the eigenvalues before and after damage, developed a method for identify damages.

2.2.2 Dynamic measurements

To characterize the dynamic properties with field measurement, different kind of vibration test have been developed; the target of these tests is collecting voltage data produced by the vibration of the structure. However, it is necessary a numeric process of modal identification to get acceleration of the structure, as this is the base of the searched dynamic properties [7].

According to [48] the vibration tests could be divided in two types, experimental modal analysis (EMA) and operational modal analysis (OMA). The main difference between them is the excitation on the structure; EMA induces it by a known excitation force, on the other hand, OMA is based on ambient excitation (e.g. wind forces, vehicular traffic), these forces can not be measured, therefore are assumed as white noise.

There exist two types of EMA tests; forced vibration and free vibration. The first test, [48] must produce a controlled excitation on the studied structure, this parameter is considered as an input signal; thus, the output data is recorded by a special equipment (acceleration data). This kind of tests represent a bigger complexity due to the high cost of the equipment and the structure safety. The second test, the authors in [7, 48] define the used excitation as an initial deformation rapidly release, producing a free movement. this test can be performed using strain cables or an impact force in a determined zone.

As was mentioned, OMA is based on environment excitation, only output parameters are recorded, because it is not necessary an artificial excitation, and is known as ambient vibration testing (AVT). According to [5] and [7] the advantage of AVT is the inexpensive equipment and the uninterrupted service state while testing is executed.

2.2.3 Finite element models calibration

The basic purpose of model calibration is to modify the numerical model properties (e.g. mass, stiffness), in order to obtain results close to reality based on field measured tests [5]. This is the case of [47] where it is checked the difference between experimental and FE model initial data, having a maximum percentage error of 42.72% (sixth mode of vibration); the calibration was performed manually, changing the boundary condition and thickness of the deck, getting a less value of error in the vibration mode (6.2%). The research in [28] opted for the same method of calibration due to the structure complexity, the calibrated model is checked after successive iterations using different FE models with variations in material properties, connections between structural elements and boundary conditions.

The calibration process developed in [3] is achieved by the variation of one parameter (elasticity modulus) and the process is considered finished when the percentage error is smaller than 0.5%. However, more complex methods are used by others authors, able to change and update large number of structural parameters. The model calibration in [21] is performed by multi-objective optimization methods.

The optimization problem in [5] has an objective function for the updating (based in a combination of eigenvalue residual, mode shape and modal flexibility residual); and [11] use the Bayesian methodology as a tool to calibrate the bridge parameter of FE model.

Among all these research, it is important to highlight two methods, the first one used by C. Costa [8] which applies a genetic algorithm (GA) to optimize their objective function related to natural frequencies and modal assurance criterion (MAC) values; and the second one [19] which uses a surrogate model (Artificial Neural Network) to develop an interaction between selected parameters of the bridge and the dynamic response (frequencies).

Table 2. Comparison between calibration methods.

Updating method	**Maximum error percent**	**Dynamic parameters**	**Bridge properties calibration**
Manually [13]	18.50%	6 frequencies	Boundary condition, deck thickness
Manually [14]	17.58%	4 frequencies	Elasticity modulus, boundary conditions, joints in structural elements
Manually [15]	0.31%	1 frequency	Elasticity modulus
Objective function optimization [16]	5.00%	20 frequencies	Geometry, materials properties
Objective function optimization [12]	4.18%	6 frequencies	10 parameters (geometric and material properties)
Artificial neural network [10]	4.59%	3 frequencies	Material properties, boundary condition
Genetic algorithm [18]	7.80%	9 frequencies	Material properties

CHAPTER 3

CASE STUDY: EL TABLAZO BRIDGE

3.1 General description

The "El Tablazo" bridge presented in Figure 2, located in the Sogamoso riverbed was the result of one of the planned phases (e.g. road-infrastructure restitution) in the complex project of Hidrosogamoso (hydroelectric power station of Colombia) [22].

Figure 2. "El Tablazo" bridge Panoramic view. Panoramio.com/photo/113559860.

The new box-girder bridge started operation in 2014 as the connection between the capital of the Santander department (Bucaramanga) and the municipality of San Vicente de Chucuri. Figure 3 shows the new routes in the region due to the filling of a reservoir.

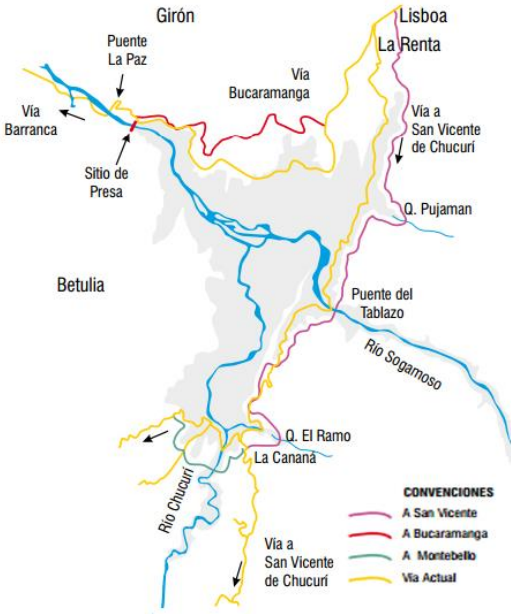

Figure 3. Infrastructure to be replaced.

3.1.1 Geometric description

El Tablazo is considered one of the highest bridges in Colombia, built using the balanced cantilever method, it has 560 meters in horizontal length constituted by 4 spans, presented in Figure 4; the two extreme spans have 92 meters and the others (central spans) 186 meters. Figure 5 shows the longitudinal section blueprint.

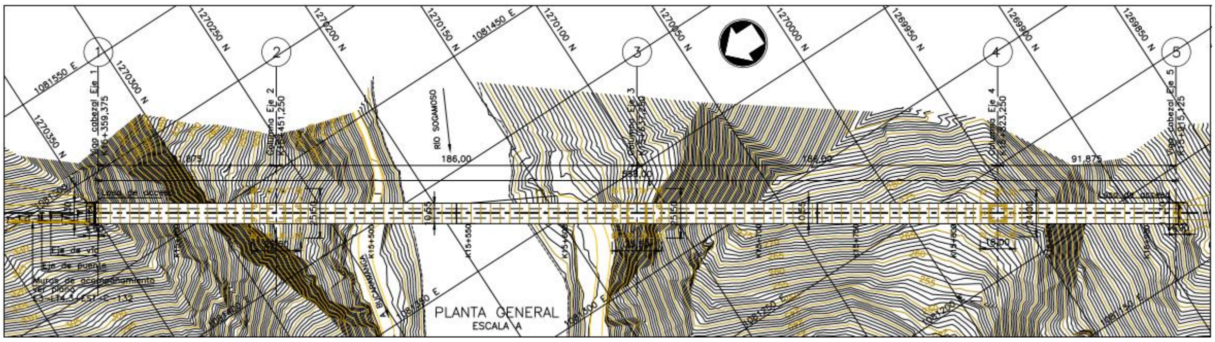

Figure 4. Deck plan El Tablazo bridge.

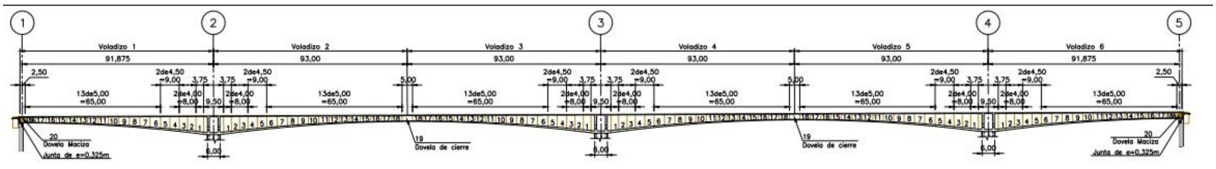

Figure 5. Longitudinal section El Tablazo bridge.

The bridge deck is supported by three reinforced concrete columns with a variable rectangular hollow cross-section, existing a monolithic union between them. The heights varies between 57 and 109 meters, a sketch of the bridge column can be seen in Figure 6.

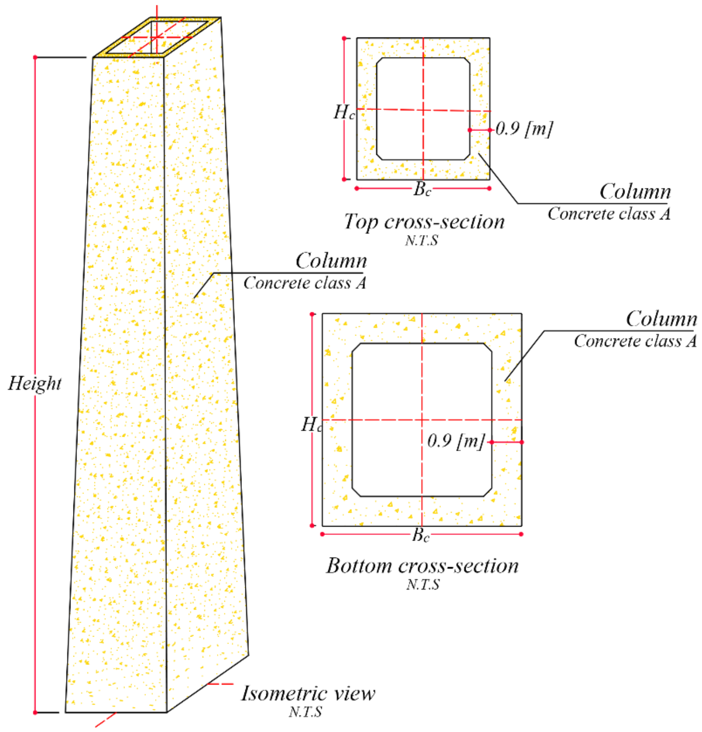

Figure 6. Type column, cross-section.

The deck has a box-girder type cross-section, tapered in its edges and bottom (see appendix A), the box-girder has a 10.55 meters' width, variation of 3 to 9 meters high as shown in Figure 7.

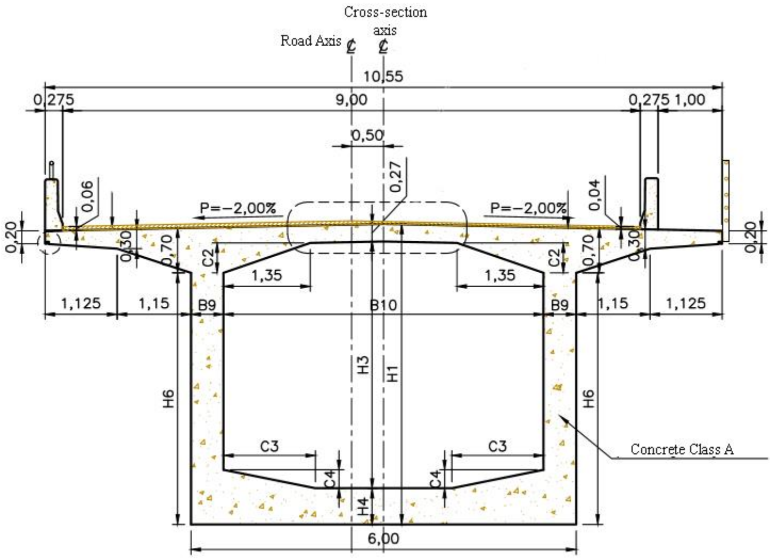

Figure 7. Box-girder cross-section.

The highest columns are supported by foundations of 25.5 x 25.5 x 5.0 meters and the short column, 18.0 x 24.0 x 4.5 meters, at the same time, the foundations are sustained by 12 and 8 piles of 2.5 meters' dimeter respectively, the piles height vary between 25 to 35 meters. See figure 8 and 9; all the bridge construction blueprints used in this research are in appendix A.

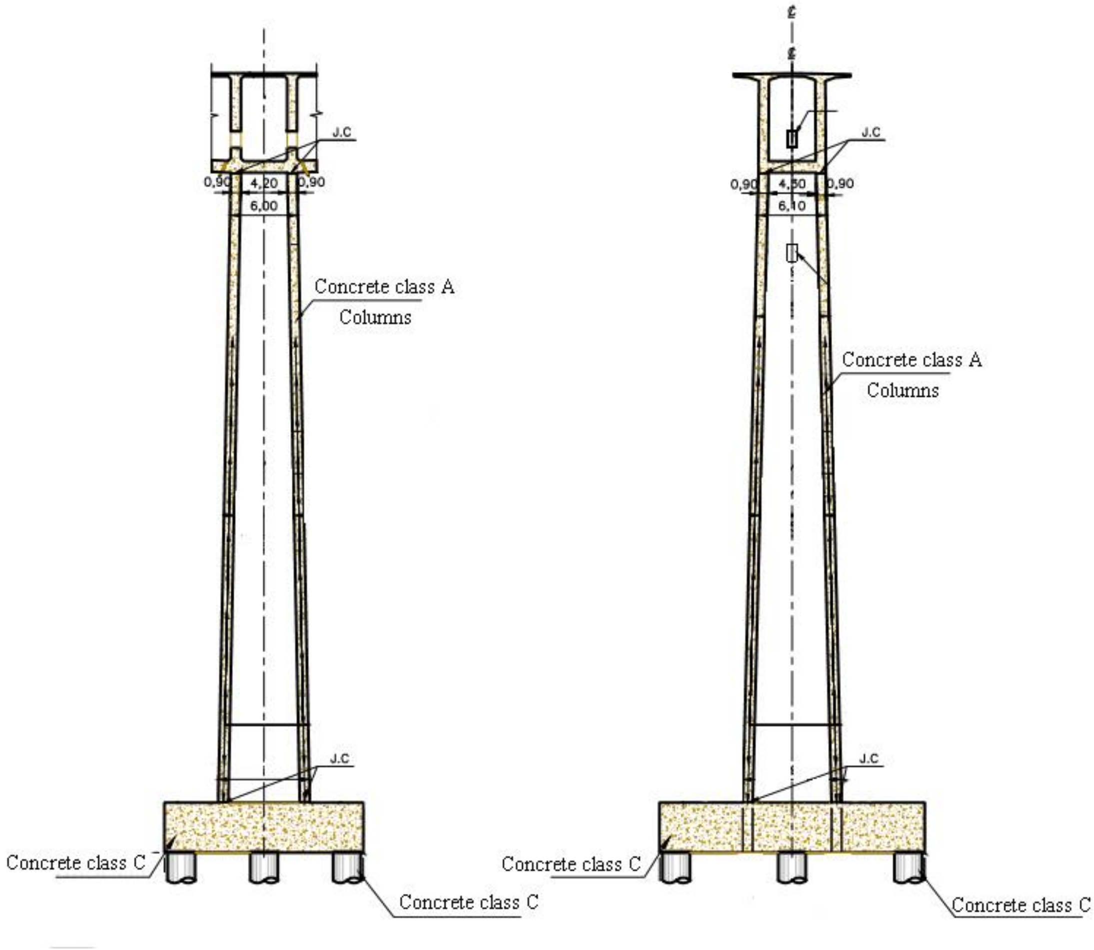

Figure 8. Column lateral view.

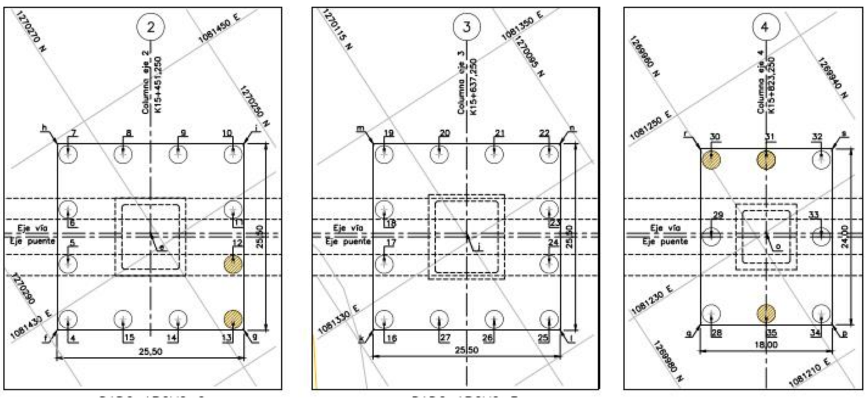

Figure 9. Foundations plan.

3.2 Finite element model description

The bridge dynamic parameters are obtained through a finite element (FE) model, generated using the structure analysis software MIDAS Civil© developed by MIDAS Information Technology Co, Ltd., the FE model is an idealization based in the "as build" blueprints. The modelling details are specified in appendix B.

Figure 10 shows a typical FE model of the bridge, which consists of line elements representing the superstructure and substructure, joined by six degrees of freedom (DOF) (3 translational, 3 rotation) nodes. The FE model is composed by 211 frame elements, 215 nodes and 1250 DOF.

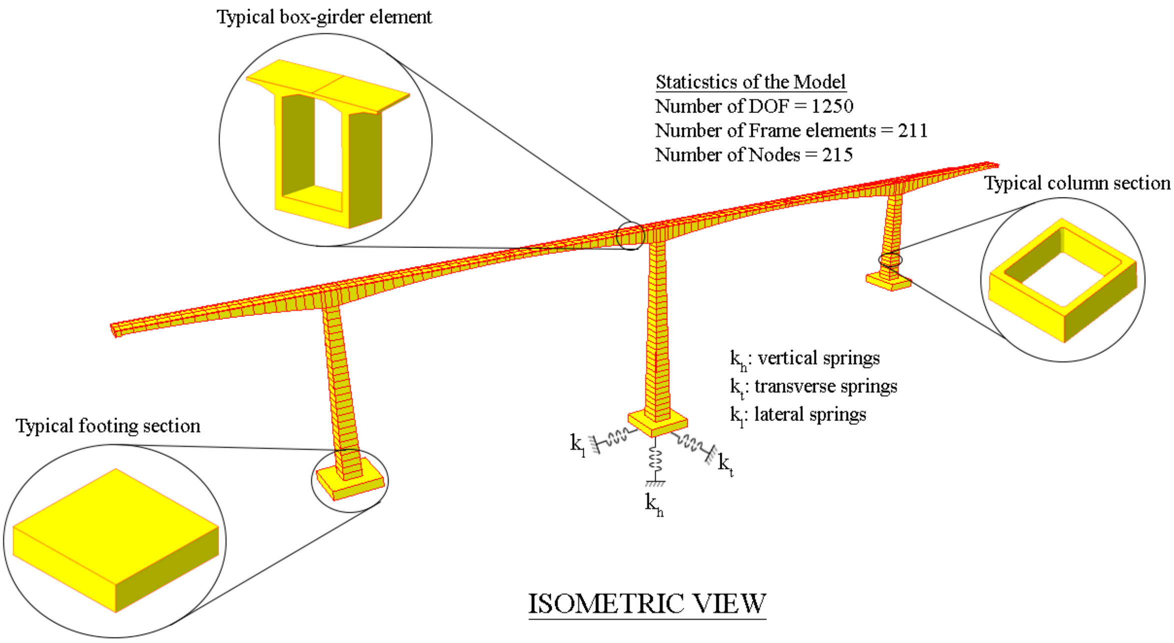

Figure 10. General description of numerical model.

3.2.1 Material properties

The definition of the material properties is based on the "as build" blueprints and Colombia seismic design codes [1, 2]. Table 3 shows the information required by the software, their values and the corresponding specification. All the calculation and used Colombian norms are specified in appendix C.

Table 3. Material properties.

Material Properties	**Symbol**	**Concrete class A**	**Concrete Class C**	**Specification**
Compressive Strength	f_c [MPa]	35	28	See figure A1
Modulus of Elasticity	E_c [GPa]	22.93	20.64	See figure A1
Poisson's Ratio	v	0.2	0.2	See equation C.2
Thermal Coefficient	T [1/°C]	1.08E-05	1.08E-05	See table C2
Weight Density	ρ [kN/m3]	22.70	22.70	See table C1
Damping Ratio	ξ	0.05	0.05	See equation C.3

3.2.2 Boundary conditions

Boundary conditions of the footings are usually idealized by fix supports in this kind of bridges [7]. However, the objective of this research implies to simulate the springs stiffness (K – see Figure 10) to generate damage scenarios related with loss stiffness, therefore, the boundary conditions are represented by linear springs (translational); rotational degrees of freedom are assumed to be fixed, based in the bibliography review about used parameters to calibrate FE models with surrogate model [19]. About the boundary conditions of the superstructure, the longitudinal direction of the bridge is released due to the existence of a neoprene support (Slide-Flon type). The transversal direction is restrained, due to the existence of neoprene supports, and the vertical direction is modelled fixed due to a pre-stress cable (see Figure 11).

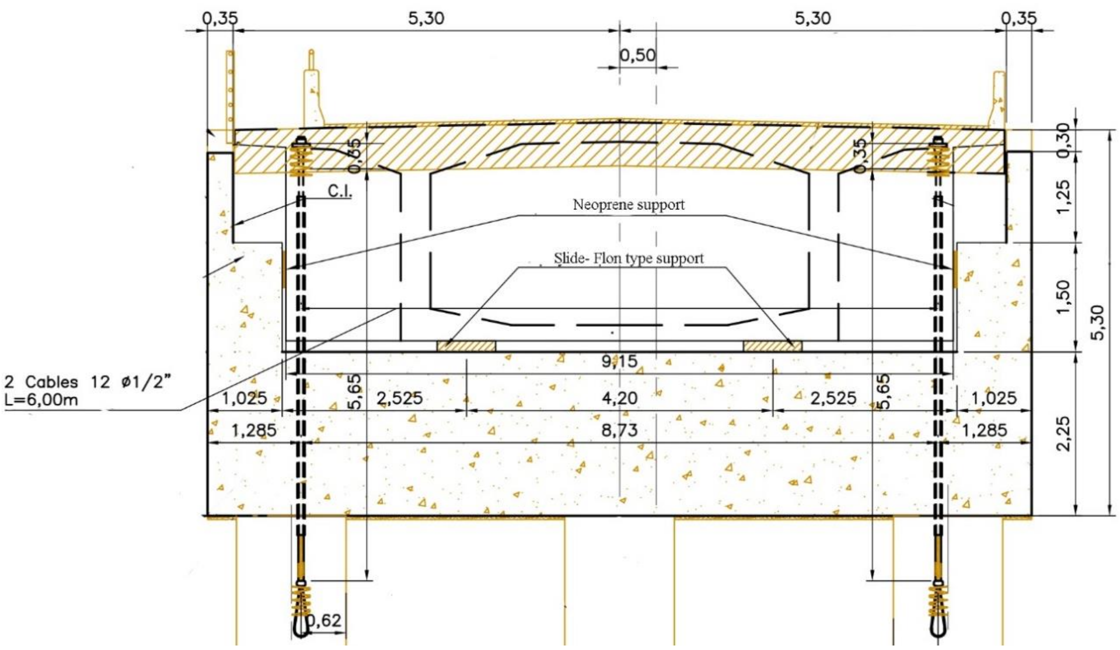

Figure 11. Boundary conditions in the superstructure.

3.2.3 Loads

The FE model is loaded using two principal dead loads: (1) self-weight of the structural elements; (2) self-weight of the non-structural elements (e.g. parapets, handrails, asphalt), being both modelled by a uniform linear load along the beam elements of the deck. A uniform torsion is modelled, due to the eccentricity between the girder centroid and the real position of the loads. Appendix C shows the calculated loads based on the blueprints. See Figure 12.

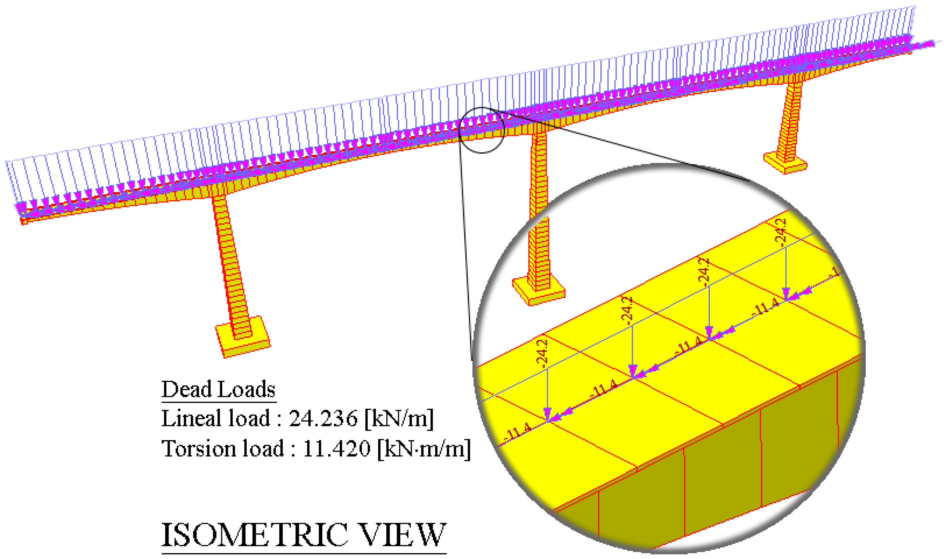

Figure 12. Non-structural loads defined.

3.2.4 Special considerations

The pre-stress steel is also modelled in this work. Figure 13 shows the model details; however, the computed frequencies with or without pre-stress forces are the same. This is the reason why the FE model is developed ignoring those forces, and therefore, the running time is reduced.

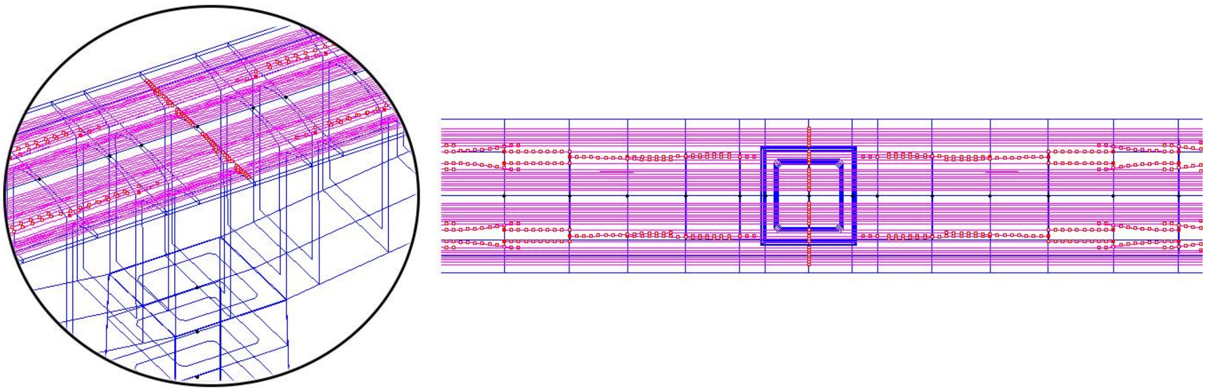

Figure 13. Modeling of prestressed.

To sustain this affirmation, according to [18], it was developed a numerical model considering the non-linear behavior of the pre-stressed beams, the equations of motion, the boundary and continuity conditions are derived using the variational principle of virtual work following Hamilton's principle. The model proves then that pre-stress does not affect the dynamic behavior of the beams; Hamed E. & Frostig Y. [18] considered as well the change in the cable eccentricity, the cable force during the vibration of the beam, the effect of the compressive axial load caused by the prestressed tendons on the vibrations of the beam.

The connection between the superstructure and substructure is modelled as rigid in all directions (X, Y and Z), therefore, all degrees of freedom (rotational and translational) of the joint nodes are the same, thus, guaranteeing a monolithic union. Figure 14 shows the rigid link between the nodes.

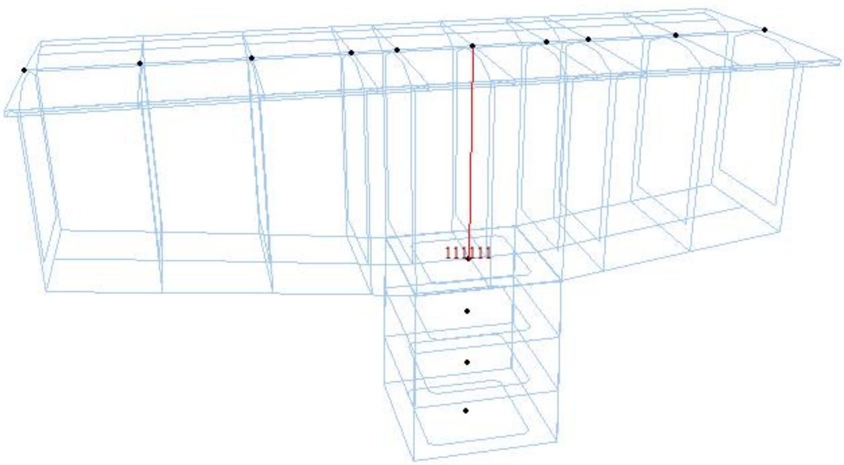

Figure 14. Superstructure and substructure rigid link.

The springs stiffness of the footings are evaluated through a sensitivity analysis, the bridge natural frequencies are recorded as stiffnesses are increased from 10^2 kN/m to a very high value (10^{13} kN/m) [19, 35]. To identify the appropriate value for each parameter (*K*) on the reference FE model.

3.2.5 Eigenfrequencies of the reference FE model

The dynamic behavior of the finite element model is analyzed by using the first twelve modes of vibration of the structure, the natural frequencies are computed and used as reference for the objectives of this thesis, Figure 15 shows the dynamic details of the model.

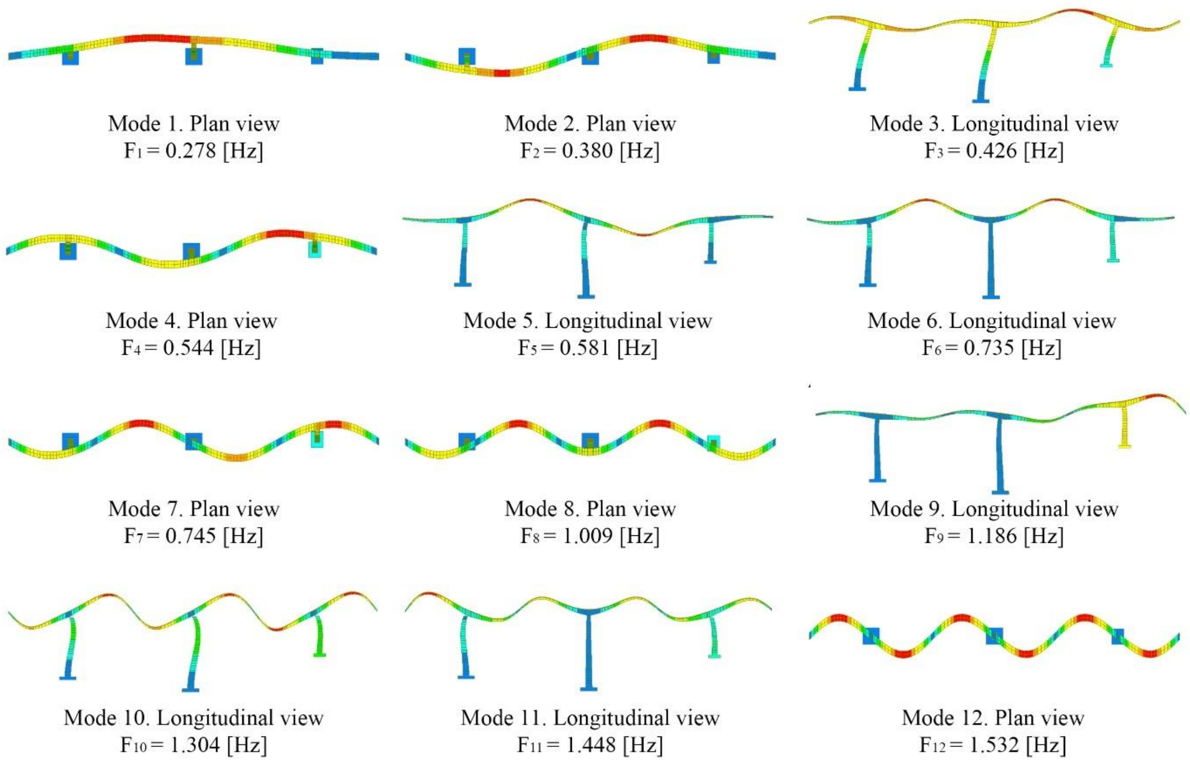

Figure 15. Eigenfrequencies of the reference FE model.

3.3 Parameter selection and sensitivity analysis

Identification of critical bridge parameters is an important step to reduce the computational cost due to reduction on the number of used input parameters, those parameters should present a high influence in the dynamic response of the bridge. Studied parameters are associated to material, geometry and physic sources.

J. C. Matos [32] recommends a procedure to the development of sensitivity analysis: (1) use mean values for inputs parameters to develop the numerical model; (2) use categories to divide the structural parameters (e.g. material, geometry); (3) each parameter should have a respective coefficient of variation (CV); (4) each mean value of input parameters are varied by adding or subtracting a standard deviation value, it is important keep all the other parameters fixed; (5) apply the equation 1:

$$b_k = CV * \sum_{i=1}^{n} \left(\frac{\Delta y_k}{y_m}\right) / \left(\frac{\Delta x_k}{x_m}\right) \ [\%] \tag{1}$$

being b_k the importance measure of parameter k, Δy_k the variation in the output parameter due to a deviation of the input parameter Δx_k in relation to the mean value of the input parameter x_m, y_m is the average response and n the number of generated parameters; (6) identify the maximum b_k; (7) to normalize all values in relation to $b_{k,max}$ (maximum importance measured value); (8) if the normalize b_k is equal or higher that b_{lim} (limit for the importance measure), the parameter will be considered as critical.

The analyzed parameters are 23, divided into six groups, corresponding to: (1) concrete class A - elasticity modulus (class A) ($E_{c,a}$), density weight (class A) (ρ_a), Poisson's ratio (class A) (v_a) and damping ratio (class A) (ξ_a); (2) concrete class C - elasticity modulus (class C) ($E_{c,c}$), density weight (class C) (ρ_c), Poisson's ratio (class C) (v_c) and damping ratio (class C) (ξ_c); (3) geometry of box-girders at middle span - (H_{01}), (H_3), (B_1), (B_9), (H_4) and (C_2); figure 16 shows the details about such dimensions. (4) geometry of columns - column height (H_c), column width (B_c) and column thickness (t_c); (5) foundation - thickness of the first footing (H_{f1}), thickness of the second footing (H_{f2}) and thickness of the third footing (H_{f3}); (6) static dead load - pavement weight density (ρ_{pav}), dead load of the nonstructural elements (W_{sob}) and uniform torsion, produced by the nonstructural elements (M_{sob}), see Figure 17 and 18.

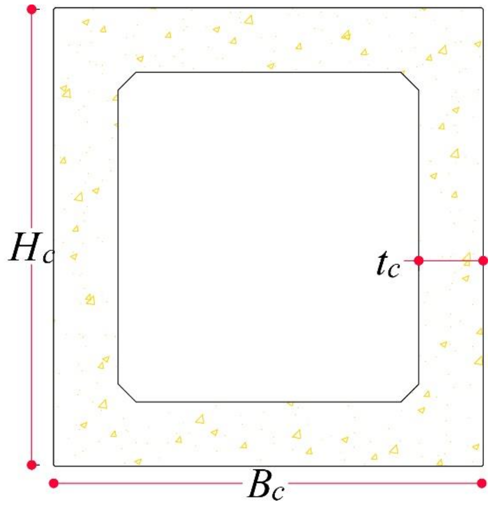

Figure 16. Columns parameters.

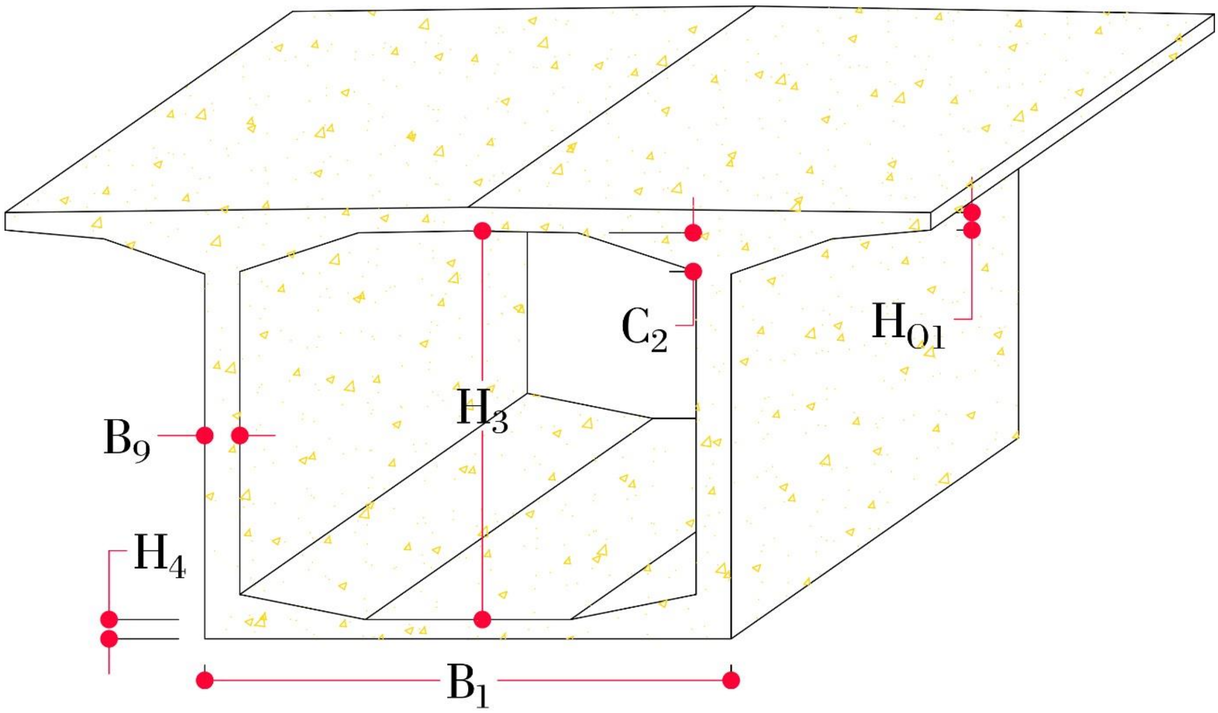

Figure 17. Box-girder middle span parameters.

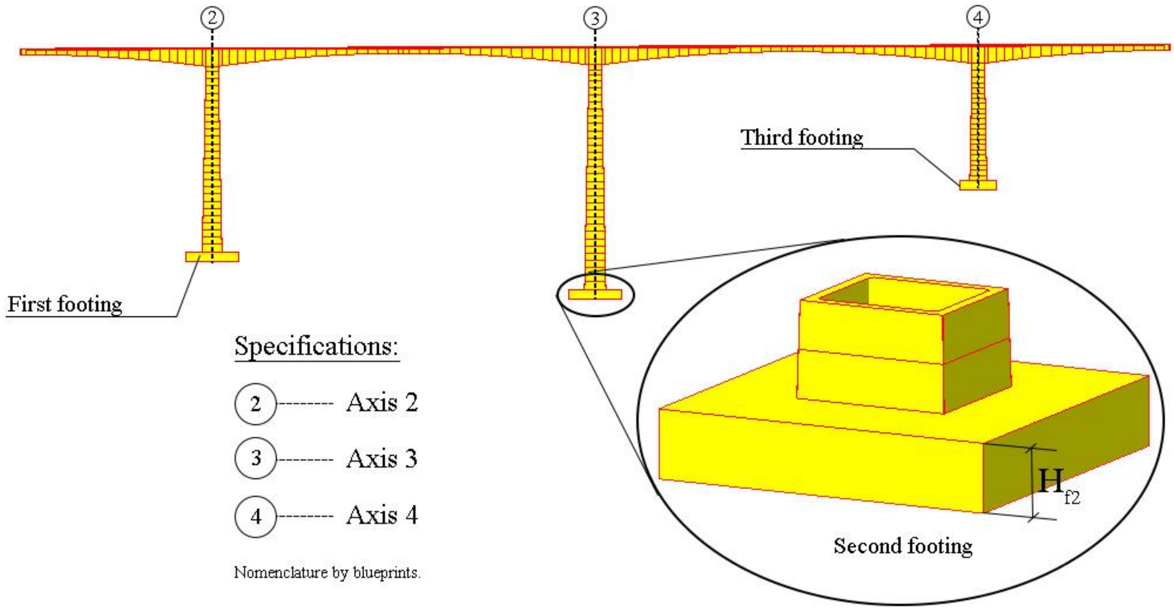

Figure 18. Footings parameters.

Table 4 presents the evaluated parameters and their respective coefficient of variation (CV) used to calculate the importance measures (b_k). The values are obtained based on bibliography [32]. Table 5 shows the objective parameters in the dynamic analysis.

Table 4. Parameter variation in sensitivity analysis.

Parameter	**CV [%]**	**Parameter**	**CV [%]**	**Parameter**	**CV [%]**	**Parameter**	**CV [%]**
$E_{c,a}$ [GPa]	10%	B_1 [m]	5%	t_c [m]	5%	ν_c	10%
ρ_a [kN/m3]	10%	B_9 [m]	5%	h_{f1} [m]	10%	ξ_c	5%
ν_a	10%	H_4 [m]	5%	h_{f2} [m]	10%	ρ_{pav} [kN/m3]	10%
ξ_a	5%	C_2 [m]	5%	h_{f3} [m]	10%	W_{sob} [kN/m]	10%
H_{O1} [m]	5%	H_C [m]	5%	$E_{c,c}$ [Mpa]	20%	M_{sob} [kN/m*m]	10%
H_3 [m]	5%	B_c [m]	5%	ρ_c [kN/m3]	20%		

Table 5. Considered output parameters for sensitivity analysis (eigenfrequencies).

Mode No	**Parameter**
1	F_1
2	F_2
3	F_3
4	F_4
5	F_5
6	F_6
7	F_7
8	F_8
9	F_9
10	F_{10}
11	F_{11}
12	F_{12}

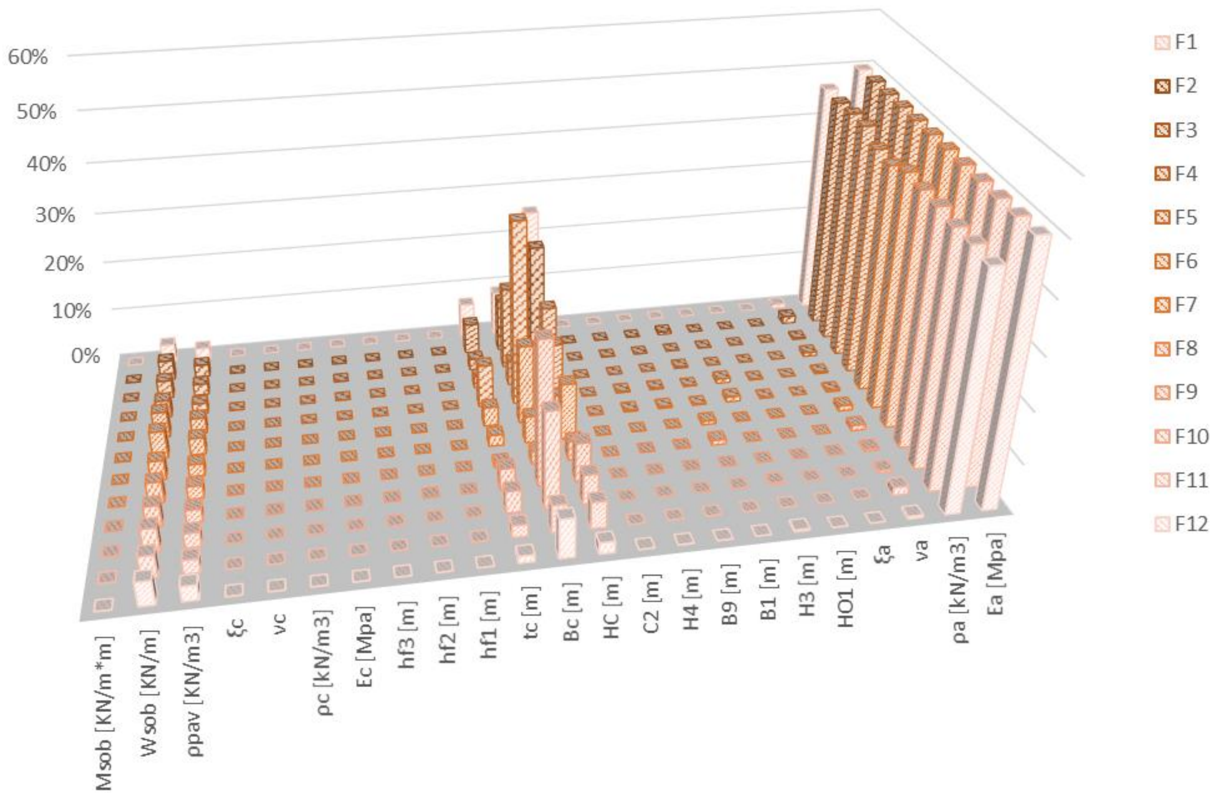

Figure 19. Importance measures (b_k) for each evaluated parameter.

The obtained data in the sensitivity analysis, their variations and the importance measures for each input parameter, besides, computed interactions between b_k parameters, are presented in Appendix D. Figure 19 shows the results of the computed b_k parameters and Figure 20 shows the critical parameters obtained in the sensitivity analysis.

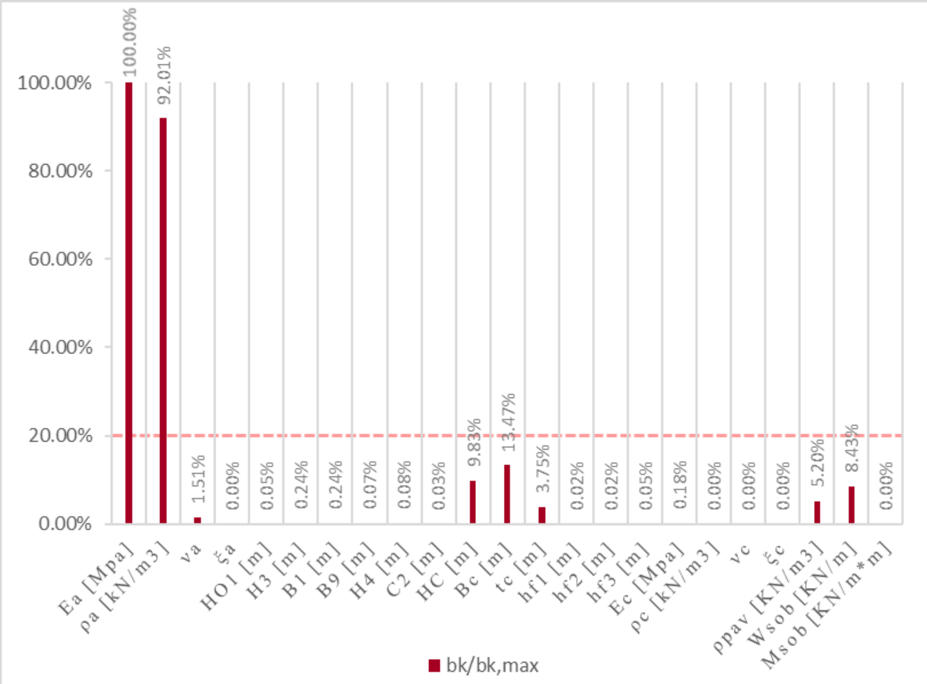

Figure 20. Sensitivity analysis results.

It is important to highlight some parameters, which are giving a high percent in the first frequency mode, however, the interaction between them is too low to be considered a critical parameter; the b_{lim} is recommended from other authors with a value of 20% [34], b_k related to the parameter must be higher to this limit to be considered critical. In this situation, the critical parameters are: (1) concrete elasticity modulus (class A) ($E_{c,a}$); and (2) concrete density weight (class A) (ρ_a).

After critical parameters selection, it becomes necessary to determine an effective range for the spring stiffness influence on the dynamic response of the bridge. In this case, the sensitivity analysis is computed with the specifications in section 3.2.4 and the variable description are defined in Figure 10 and 18. The results are presented in nine graphics using a logarithmic scale (see appendix D), besides it can be observed a significant impact on frequencies.

The interpreted results provide an effective range with a high sensitivity to the dynamic behavior of the bridge ([10^4 - 10^6] kN/m2), therefore the springs stiffness for the reference FE model should be assumed as the maximum value (10^6 kN/m2) in the obtained range due to the large decrease in eigenfrequencies (Choosing a lower value means a significant change in dynamic behavior). After the specified ranges, the behavior of the frequencies is constant, in other words, the results are the same if they are considered as fixed; all the computed FE models on this analysis are presented in appendix D.

CHAPTER 4

DEFINITION OF DAMAGE SCENARIOS

4.1 Introduction

The early detection of any damage in the structures has been frequently used and implemented to avoid a bad behavior of the structures or even its collapse [7]. A good monitoring plan has to define a set of damage scenarios based on the subject conditions; however, the defined scenarios in this work are based on the ones commonly generated for the bridge type studied [6, 17, 39, 40].

4.1.1 Damage scenarios at mid span

As damage scenarios for mid span it were considered the following: using the FE model of reference (based in construction blueprints) is introduced a loss of stiffness in the first interior span of the bridge, located between the axis 2 and 3, and as is specified in chapter 3, this reduction is possible changing the elasticity modulus of the concrete in the box-girder sections located at mid span (see Figure 21), this kind of scenario is related with concrete problems such a cracking. The reduction of stiffness to simulate the scenario is recommended from other authors with a value of 20% [15, 37, 38, 46].

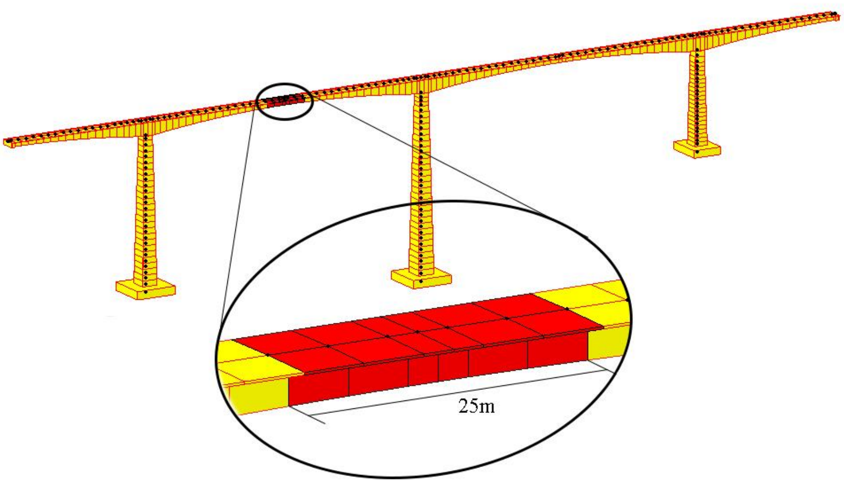

Figure 21. Location of 1st damage scenario.

The stiffness reduction introduced in the second span of the bridge, between the axis 3 and 4 is considered with the same terms on the previous scenario. The Figure 22 shows the details about position and the specific pre-cast segments affected.

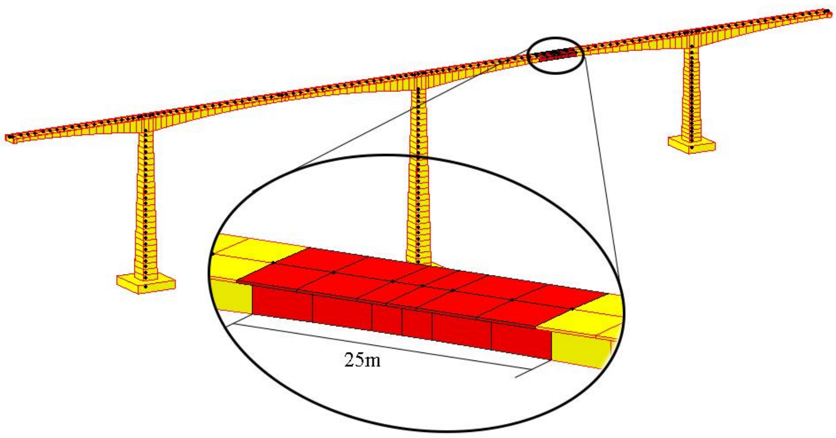

Figure 22. Location of 2nd damage scenario.

4.1.2 Damage scenarios at columns

As damage scenarios for the bridge columns it were considered the following: defined as the mid span scenarios, the reduction now is applied in the middle of the studied affecting four segments of 3.5 meters each one, besides the same reduction of twenty percent (20%) is used as well [37, 38, 46]. Each scenario introduces the specified damage in one column at a time. The details are presented in Figure 23.

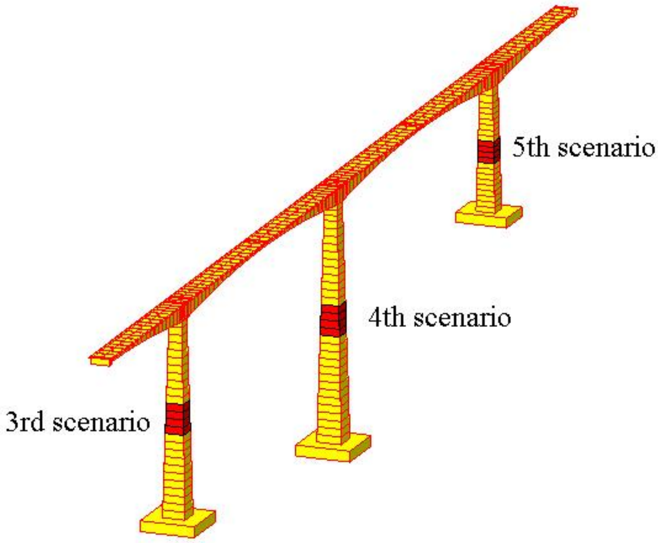

Figure 23. Damage scenarios in columns.

4.1.3 Damage scenarios at foundations

This type of damage is located in the structure footings are defined considered the following: the loss of stiffness is introduced in the first, second and third footing (see Figure 18). These scenarios are simulated in the FE model of reference changing the values of the boundaries conditions, in other words, the springs stiffness is reduced and these considerations are related with scour (this phenomenon is the result of the erosive action of flowing water, which remove material around the foundations, causing a loss of stiffness in the supports) [13]. To create the simulation the stiffness support has to be reduced by fifty percent (50%) [15].

4.1.4 Obtained results

All the scenarios (See Table 6) are developed one at time through the FE model, changing the specified properties is obtained the structure response for each case, all the results and descriptions are presented in Table 7.

Table 6. Resume of damage scenarios defined.

	Structural element affected	**Location**	**Changed parameters**	**Percent of reduce**
1st Scenario	Girder	Mid span between axis 2 and 3.	Elasticity modulus	20%
2nd Scenario	Girder	Mid span, between axis 3 and 4.	Elasticity modulus	20%
3rd scenario	Column	Axis 2.	Elasticity modulus	20%
4th scenario	Column	Axis 3.	Elasticity modulus	20%
5th scenario	Column	Axis 4.	Elasticity modulus	20%
6th scenario	Foundation	Axis 2.	Spring stiffness of footings	50%
7th scenario	Foundation	Axis 3.	Spring stiffness of footings	50%
8th scenario	Foundation	Axis 4.	Spring stiffness of footings	50%

Table 7. Obtained frequencies for different damage scenarios.

	FE model	**1st Scenario**	**2nd Scenario**	**3rd Scenario**	**4th Scenario**	**5th Scenario**	**6th Scenario**	**7th Scenario**	**8th Scenario**
F_1	0.278	0.284	0.285	0.276	0.275	0.277	0.276	0.274	0.277
F_2	0.380	0.389	0.388	0.377	0.379	0.378	0.375	0.379	0.377
F_3	0.426	0.438	0.437	0.425	0.426	0.424	0.422	0.420	0.412
F_4	0.544	0.559	0.557	0.543	0.543	0.539	0.540	0.542	0.532
F_5	0.581	0.587	0.591	0.581	0.580	0.581	0.568	0.573	0.569
F_6	0.735	0.747	0.742	0.735	0.735	0.735	0.719	0.681	0.710
F_7	0.745	0.759	0.762	0.744	0.744	0.740	0.741	0.744	0.729
F_8	1.009	1.028	1.027	1.008	1.008	1.007	0.876	0.892	1.003
F_9	1.186	1.216	1.217	1.186	1.186	1.186	1.002	0.967	1.031
F_{10}	1.304	1.336	1.336	1.304	1.304	1.303	1.070	1.041	1.146
F_{11}	1.448	1.486	1.486	1.447	1.447	1.446	1.108	1.076	1.271
F_{12}	1.532	1.568	1.569	1.531	1.531	1.531	1.269	1.244	1.448

4.2 Artificial neural network

An objective of this research is to develop a relationship between the bridge properties and its dynamic response, an Artificial Neural Network (ANN), based on the neurons of the biological brain, is proposed. The model is able to obtain predictions with self-learning.

The procedure to develop a neural network can be numerate as follow:

(1) Definition of the set of bridge parameters to be estimated by the ANN; therefore, the outputs are $E_{c,a}$, ρ_a, K_{h1}, K_{t1}, K_{l1}, K_{h2}, K_{t2}, K_{l2}, K_{h3}, K_{t3}, K_{l3} (See Figure 10); (2) Definition of the bridge response parameters (i.e. F_1, F_2, F_3, F_4, F_5, F_6, F_7, F_8, F_9, F_{10}, F_{11}, F_{12}. See Figure 15) to be the inputs in the network; (3) Generate a dataset for training the network; however, there are necessary enough FE models, developed with arbitrary values for the bridge parameters within the defined ranges, to obtain the corresponding analytical structure response; (4) Choose the network architecture; (5) Train the network using the generated data set to learn the inverse relationship between bridge response (input parameters) and bridge properties (output parameters); (6) run the network with analytical structure response (F_1, F_2, F_3, F_4, F_5, F_6, F_7, F_8, F_9, F_{10}, F_{11}, F_{12}) obtained from damage scenarios (see Table 6), in order to identify the input parameters.

4.2.1 Generating dataset for ANN

Once the critical parameters of the bridge are identified in the sensitivity analysis, the construction of the meta-model could start. Therefore, following the specified steps, it is necessary to generate FE models for network training; the parameters ($E_{c,a}$, ρ_a, K_{h1}, K_{t1}, K_{l1}, K_{h2}, K_{t2}, K_{l2}, K_{h3}, K_{t3}, K_{l3}) will vary in different scenarios with their specific ranges: the critical parameters by their own CV (see Table 4) and the spring stiffness of foundation [10^4 - 10^6] kN/m2. The dataset is generated randomly to simulate enough scenarios. For better behavior of the network, between the sensitivity analysis data and the random scenarios are obtained a total of 112 FE models to fill the network. According to [19], this total of FE models are enough for generate an input - output relationship with the same analyzed parameters. The dataset is specified in appendix E.

4.2.2 Network architecture

Once the dataset is obtained from the FE model, it is defined the used architecture, first, it is necessary to choose the typology, in this research is used a feed-forward network (see Figure 24); therefore, this network need the definition of a few parameters (i.e. Number of training patterns, number of input layer neurons, number of output layer neurons, number of hidden layer neurons, error minimization algorithm and termination rule) to allow the development architecture.

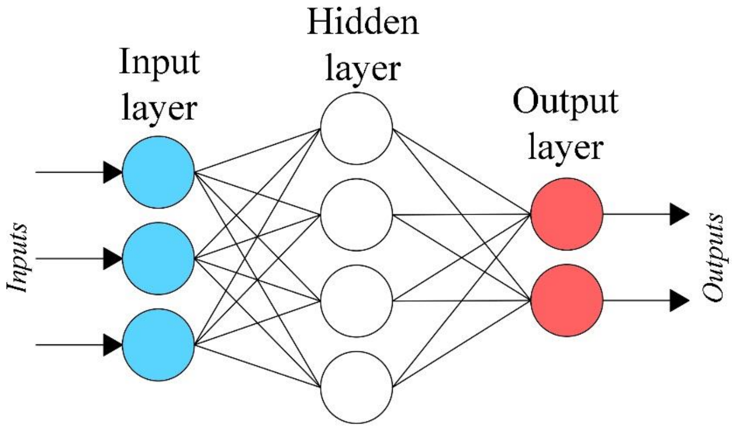

Figure 24. Feed-forward network architecture.

4.2.3 Training process

The training of a network is the process to adjust the weight connections between neurons, in another words, the outputs are estimates by a set of weights, subsequently, the computed data is compared and recalculated until it reaches the maximum error allowed. At this point, the network is trained using two different algorithms (i.e. backpropagation, genetic) comparing both and analyze which one predict better the outputs.

The backpropagation algorithm (BP) also designated as backward propagation of errors is an optimization method and it is the most commonly used algorithm for optimizing networks due to its predefinition in commercial software as MATLAB©; the process for developing this algorithm could be represented in a scheme, see Figure 25.

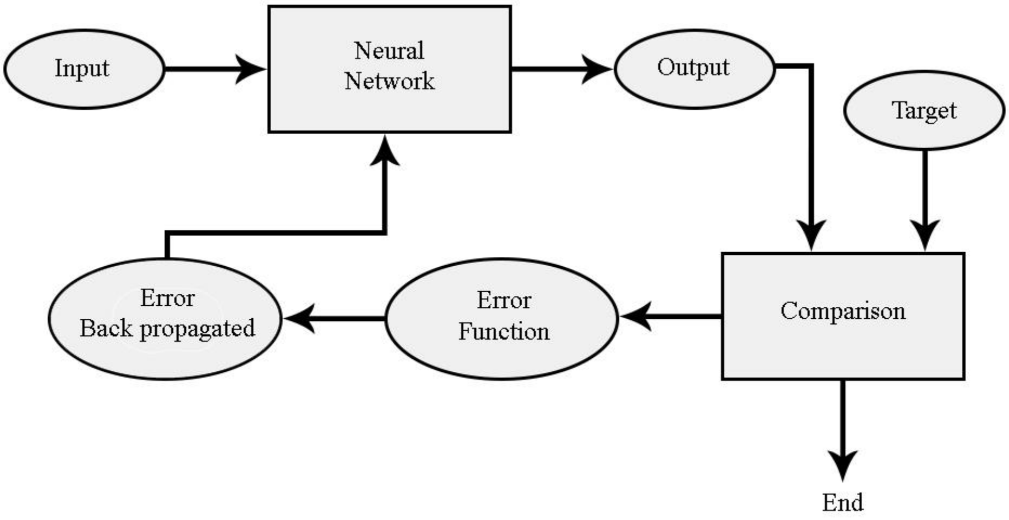

Figure 25. Backpropagation algorithm process.

The network is developed with the architecture specified in Table 8, the Bayesian Regularization method (BP algorithm), described as the better for solving challenging problems. The 80% of the dataset is used for the network training and the rest for validations and testing, to prevent the over-fitting which causes a loss of ability to make good predictions [19]. The representation of the developed network trained by BP is presented in Figure 26.

Table 8. Options of the BP algorithm training.

Network parameters	
Number of training patterns	112
Number of input layer neurons	12
Number of output layer neurons	11
Number of hidden layer neurons	10
Error minimization algorithm	Bayesian Regularization
Termination rule	maximum epoch: 1000

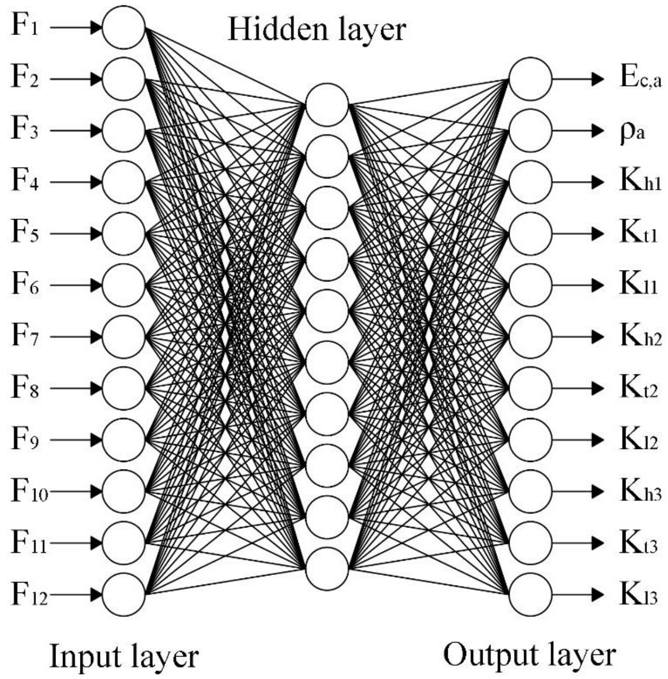

Figure 26. Network architecture used for BP training.

The error function defined to develop the algorithm is the mean squared error (MSE), used for compute the performance between the generated outputs (calculated by the network) and the targets (output data used to fit the ANN) defined by the Equation 2, being N the number of training examples, x_i the vector of input parameters, y_i the vector of targets and f is the neural network function.

$$MSE = \frac{1}{N}\sum_{i=1}^{N}(f(x_i) - y_i)^2 \quad (2)$$

BP uses these error values to calculate the gradient of the error function (MSE). Then, this gradient is fed to the optimization method (backpropagated), which in turn uses it to update the vector of weights (variable to be modified, generating an optimal relationship between the studied parameters), in an attempt to minimize the error function (MSE) [44].

The results of the trained network by BP are presented in Figure 27 using a plot to represent the comparison between computed outputs and targets, a correlation (R) of 0.99 is obtain reaching the maximum epoch.

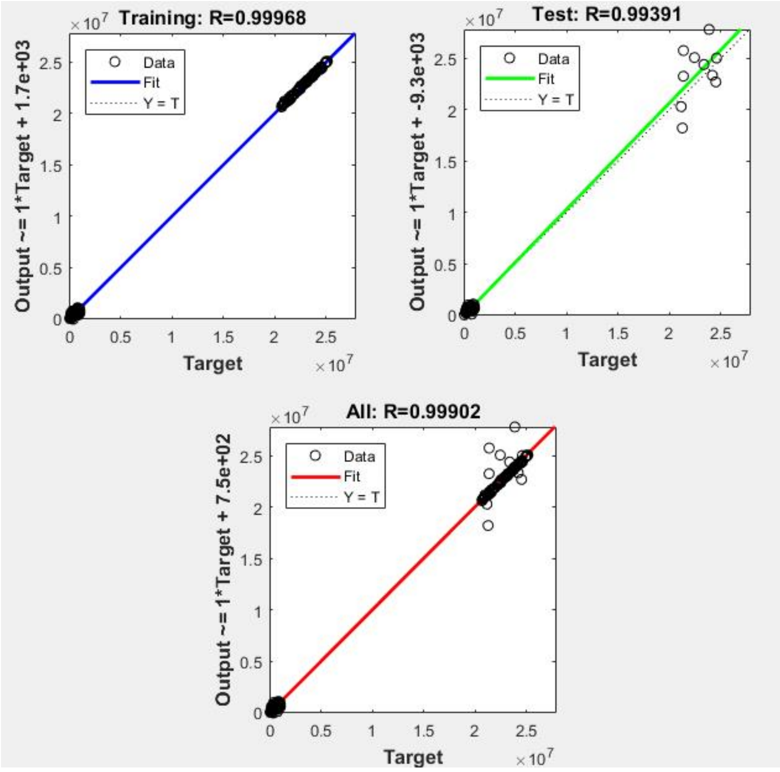

Figure 27. Regression plot of training network using BP.

The next process executed is the neural network trained with a genetic algorithm (GA), defined as a global method to optimize functions. The main difference from the previous process is that the BP algorithm find the optimum value locally. For introduce the GA in the network developed is necessary to define the objective function to be optimized, the used model based in [29] and its graphical representation is presented in Figure 28.

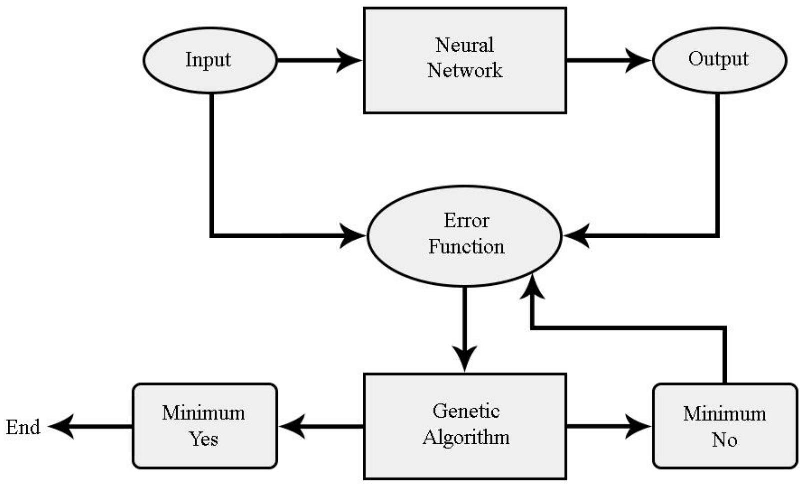

Figure 28. Proposed model for network training with GA.

The GA models are inspired in natural selection, in this sense, it is generated an initial population who is evaluated by the fitness function (objective function), then a new population is generated through a process of selection, crossover and mutation of the "population gens". In that way, is possible to generate a loop until the termination criteria is achieved. For this work, the fitness function is that which controls the error in the difference between the computed outputs and the desired outputs or "targets". At the end, the weights vector is optimized and the network completed. In Figure 29 the GA is represented in a flowchart [26].

Starting with the same architecture of the previous method, the network is developed using a MATLAB© code (see appendix F), the configuration of the code is a simple application of the two numerical models, firstly, the hidden neurons and the weights vector are defined, then, it is possible to generate the new network (fitting by the dataset of the FE model and the hidden neurons).

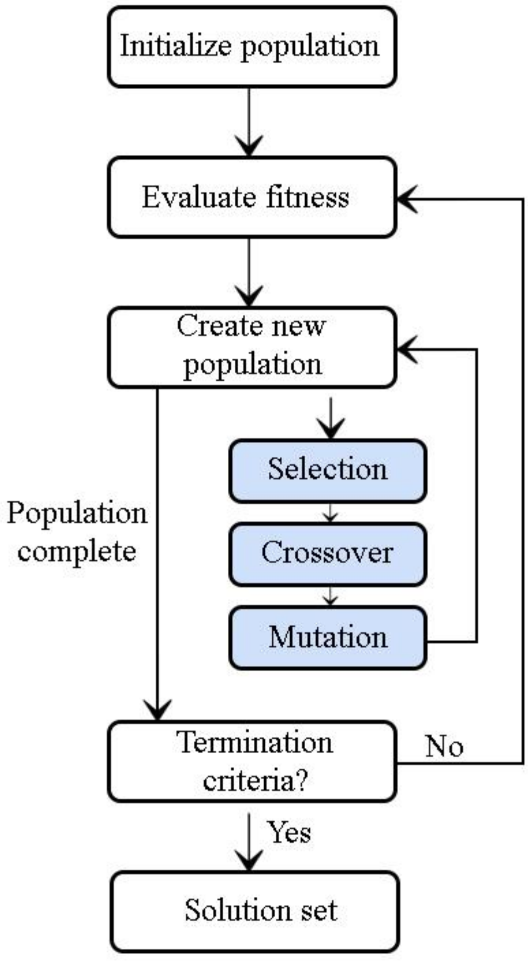

Figure 29. Flowchart of basic GA.

Following the Figure 28, the error function has to be defined. However, the error function is also the fitness function for the GA, being the function developed in another code and called by the principal program.

Now, the GA has to be configured, a variety of options are used to understand the behavior or the network, as the allowed error, the population size, the number of generations, type of mutation and crossover function used. The chosen options developed in the code are defined in the Table 9.

Table 9. Options of the genetic algorithm training.

Genetic algorithm options	
Allowed error	1.00E-07
Generations	1000
Population size	150
Fitness function	MSE
Mutation function	@mutationgaussian [44]
Crossover function	@Crossoverscattered [43]

The training is developed and it is obtained the regression plot of the network (see Figure 31), it was necessary to adjust the architecture of the ANN to obtain a better correlation. Therefore, it is varied all the network options and after many simulations it is found the used configuration (see Table 8). Figure 30 shows the new ANN architecture.

Table 10. ANN architecture definition for GA training.

Network parameters	
Number of training patterns	112
Number of input layer neurons	12
Number of output layer neurons	11
Number of hidden layer neurons	2
Error minimization algorithm	Genetic algorithm
Termination rule	Minimum error: 1.00E-07 maximum epoch: 1000

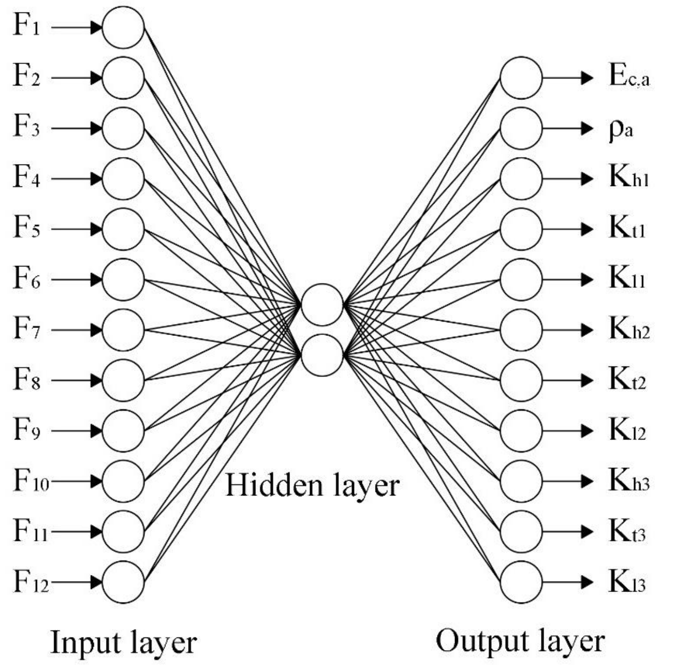

Figure 30. Network architecture used for GA training.

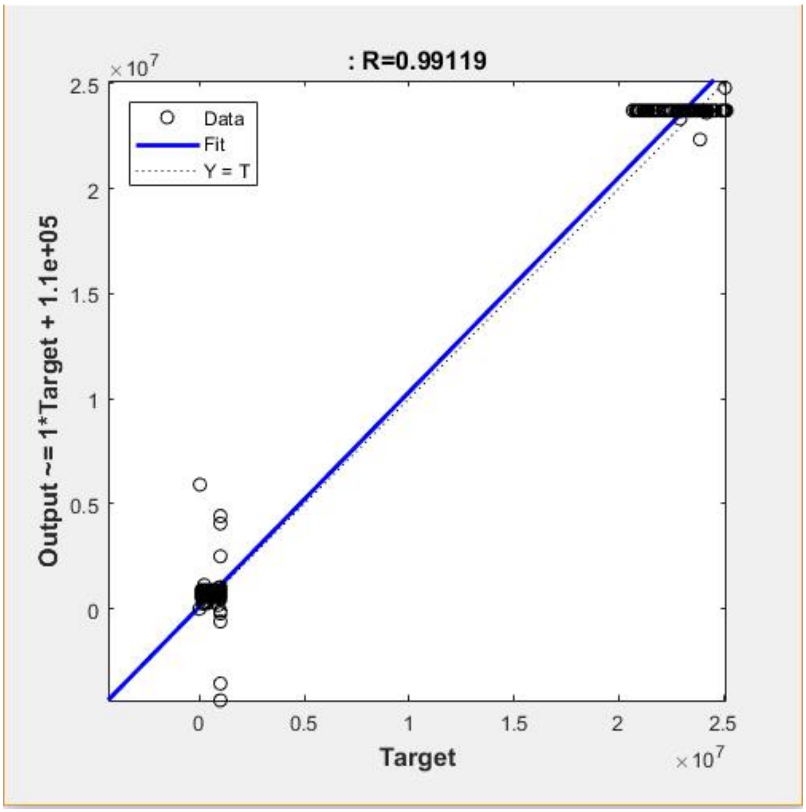

Figure 31. Regression plot of training network using GA.

4.3 Results

Once the ANN are correctly trained, the generated frequencies based on the damage scenarios are used to test the networks, the objective, is to try to obtain the bridge changed parameters, and to determine the performance in the damage identification. Therefore, it is established a comparison between outputs of the two trained networks and the FE model referenced properties (see Table 11), for each defined scenario (see Table 6).

Table 11. Comparison of ANNs results and reference FE model for first scenario.

1st Scenario					
FE model referenced properties		Backpropagation algorithm		Genetic Algorithm	
Parameter	Value	Predicted	Error percent	Predicted	Error percent
E [GN/m2]	18.346	16.701	10%	22.017	20%
ρa [kN/m3]	22.7	22.701	0%	21.946	3%
K_{l2} [kN/m]	1000000	1096377.741	9%	862529.263	14%
K_{t2} [kN/m]	1000000	1024148.462	2%	1091195.31	9%
K_{h2} [kN/m]	1000000	1083058.345	8%	964280.06	4%
K_{l1} [kN/m]	1000000	984768.4784	2%	885212.792	11%
K_{t1} [kN/m]	1000000	1003690.742	0%	518616.02	48%
K_{h1} [kN/m]	1000000	993147.565	1%	197538.524	80%
K_{l3} [kN/m]	1000000	1104006.517	9%	547769.972	45%
K_{t3} [kN/m]	1000000	856514.182	17%	755538.411	24%
K_{h3} [kN/m]	1000000	1036221.772	3%	168373.516	83%

Table 12. Comparison of ANNs results and reference FE model for second scenario.

2nd Scenario					
FE model referenced properties		Backpropagation algorithm		Genetic Algorithm	
Parameter	Value	Predicted	Error percent	Predicted	Error percent
E [GN/m2]	18.346	16.592	11%	22.936	25%
ρa [kN/m3]	22.7	22.701	0%	22.650	0%
K_{l2} [kN/m]	1000000	1092911.877	9%	852185.750	15%
K_{t2} [kN/m]	1000000	1033828.873	3%	901033.492	10%
K_{h2} [kN/m]	1000000	1081234.995	8%	787519.826	21%
K_{l1} [kN/m]	1000000	986015.135	1%	792838.004	21%
K_{t1} [kN/m]	1000000	1004139.326	0%	553854.533	45%
K_{h1} [kN/m]	1000000	993819.057	1%	457202.677	54%
K_{l3} [kN/m]	1000000	1107372.26	10%	591384.197	41%
K_{t3} [kN/m]	1000000	853831.576	17%	832747.988	17%
K_{h3} [kN/m]	1000000	1030950.882	3%	380545.696	62%

Table 13. Comparison of ANNs results and reference FE model for third scenario.

3rd Scenario					
FE model referenced properties		Backpropagation algorithm		Genetic Algorithm	
Parameter	Value	Predicted	Error percent	Predicted	Error percent
E [GN/m2]	18.346	22.730	19%	23.588	29%
ρa [kN/m3]	22.7	22.701	0%	23.151	2%
K_{l2} [kN/m]	1000000	1137467.535	12%	844839.030	16%
K_{t2} [kN/m]	1000000	1004780.94	0%	765966.637	23%
K_{h2} [kN/m]	1000000	1070102.863	7%	661971.760	34%
K_{l1} [kN/m]	1000000	1063107.48	6%	727226.664	27%
K_{t1} [kN/m]	1000000	990549.250	1%	578883.507	42%
K_{h1} [kN/m]	1000000	1022788.807	2%	641635.179	36%
K_{l3} [kN/m]	1000000	1090241.266	8%	622362.214	38%
K_{t3} [kN/m]	1000000	826819.211	21%	887587.885	11%
K_{h3} [kN/m]	1000000	1010072.851	1%	531245.927	47%

Table 14. Comparison of ANNs results and reference FE model for fourth scenario.

4th Scenario					
FE model referenced properties		Backpropagation algorithm		Genetic Algorithm	
Parameter	Value	Predicted	Error percent	Predicted	Error percent
E [GN/m2]	18.346	22.609	19%	23.440	28%
ρa [kN/m3]	22.7	22.701	0%	23.037	1%
K_{l2} [kN/m]	1000000	1140753.951	12%	846503.935	15%
K_{t2} [kN/m]	1000000	971608.693	3%	796575.324	20%
K_{h2} [kN/m]	1000000	1074486.845	7%	690423.310	31%
K_{l1} [kN/m]	1000000	1063065.834	6%	742095.426	26%
K_{t1} [kN/m]	1000000	1000257.351	0%	573211.471	43%
K_{h1} [kN/m]	1000000	1022372.352	2%	599839.309	40%
K_{l3} [kN/m]	1000000	1084993.695	8%	615342.013	38%
K_{t3} [kN/m]	1000000	851811.278	17%	875160.134	12%
K_{h3} [kN/m]	1000000	1011921.307	1%	497094.41	50%

Table 15. Comparison of ANNs results and reference FE model for fifth scenario.

5th Scenario					
FE model referenced properties		Backpropagation algorithm		Genetic Algorithm	
Parameter	Value	Predicted	Error percent	Predicted	Error percent
E [GN/m2]	18.346	22.800	20%	22.921	25%
ρa [kN/m3]	22.7	22.701	0%	22.639	0%
K_{l2} [kN/m]	1000000	1125335.414	11%	852352.960	15%
K_{t2} [kN/m]	1000000	1011396.37	1%	904107.583	10%
K_{h2} [kN/m]	1000000	1078650.053	7%	790377.272	21%
K_{l1} [kN/m]	1000000	1051235.68	5%	794331.303	21%
K_{t1} [kN/m]	1000000	993981.080	1%	553284.879	45%
K_{h1} [kN/m]	1000000	1019363.475	2%	453005.034	55%
K_{l3} [kN/m]	1000000	1089134.245	8%	590679.144	41%
K_{t3} [kN/m]	1000000	832580.069	20%	831499.844	17%
K_{h3} [kN/m]	1000000	1013711.369	1%	377115.793	62%

Table 16. Comparison of ANNs results and reference FE model for sixth scenario.

6th Scenario					
FE model referenced properties		Backpropagation algorithm		Genetic Algorithm	
Parameter	Value	Predicted	Error percent	Predicted	Error percent
E [GN/m2]	22.932	29.078	21%	23.726	3%
ρa [kN/m3]	22.7	22.701	0%	23.256	2%
K_{l2} [kN/m]	1000000	923612.104	8%	843293.451	16%
K_{t2} [kN/m]	1000000	684464.062	46%	737551.707	26%
K_{h2} [kN/m]	1000000	777726.747	29%	635559.362	36%
K_{l1} [kN/m]	500000	716768.133	30%	713423.561	43%
K_{t1} [kN/m]	500000	669732.810	25%	584149.021	17%
K_{h1} [kN/m]	500000	769886.182	35%	680435.495	36%
K_{l3} [kN/m]	1000000	789957.859	27%	628879.27	37%
K_{t3} [kN/m]	1000000	551170.919	81%	899124.926	10%
K_{h3} [kN/m]	1000000	694293.184	44%	562949.749	44%

Table 17. Comparison of ANNs results and reference FE model for seventh scenario.

7th Scenario					
FE model referenced properties		Backpropagation algorithm		Genetic Algorithm	
Parameter	Value	Predicted	Error percent	Predicted	Error percent
E [GN/m2]	22.932	27.068	15%	23.726	3%
ρa [kN/m3]	22.7	22.701	0%	23.256	2%
K_{l2} [kN/m]	500000	952640.129	48%	843293.451	69%
K_{t2} [kN/m]	500000	441897.69	13%	737551.703	48%
K_{h2} [kN/m]	500000	749870.780	33%	635559.359	27%
K_{l1} [kN/m]	1000000	734240.903	36%	713423.559	29%
K_{t1} [kN/m]	1000000	771700.665	30%	584149.022	42%
K_{h1} [kN/m]	1000000	749423.896	33%	680435.500	32%
K_{l3} [kN/m]	1000000	694329.682	44%	628879.270	37%
K_{t3} [kN/m]	1000000	707722.552	41%	899124.927	10%
K_{h3} [kN/m]	1000000	661853.353	51%	562949.753	44%

Table 18. Comparison of ANNs results and reference FE model for eighth scenario.

8^{th} Scenario					
FE model referenced properties		Backpropagation algorithm		Genetic Algorithm	
Parameter	Value	Predicted	Error percent	Predicted	Error percent
E [GN/m2]	22.932	24.962	8%	23.725	3%
ρa [kN/m3]	22.7	22.701	0%	23.256	2%
K_{l2} [kN/m]	1000000	1049885.326	5%	843295.828	16%
K_{t2} [kN/m]	1000000	843718.445	19%	737595.401	26%
K_{h2} [kN/m]	1000000	979651.720	2%	635599.977	36%
K_{l1} [kN/m]	1000000	912784.879	10%	713444.786	29%
K_{t1} [kN/m]	1000000	957230.904	4%	584140.925	42%
K_{h1} [kN/m]	1000000	930970.079	7%	680375.831	32%
K_{l3} [kN/m]	500000	840663.854	41%	628869.248	26%
K_{t3} [kN/m]	500000	771043.709	35%	899107.185	80%
K_{h3} [kN/m]	500000	949636.457	47%	562900.998	13%

The obtained results are interpreted in four parts: (1) According to the computed percent error in each trained network, the backpropagation method had a great behavior predicting the introduction of the damages in the 1^{st} and 2^{nd} scenarios, however, the related scenarios with the damage located in the columns, presents a high percent of error, this can be due to the use of the same parameter (elasticity modulus – *E*) to define all the structural elements; (2) the supports are idealized fixed when the value is highest than 10^6 , therefore the reduction of 50% in the defined scenarios involve just the range of [10^5 - 10^6] where some of the *K* parameters are insensitive to some mode of vibration (see appendix D), in this case the training of the meta-model is affected due to the insensitivity parameters which needs more data to learn its behavior; (3) the GA algorithm results compared with BP have a highest percent error, but the behavior in the 6^{th}, 7^{th} and 8^{th} scenarios are better at predicting the changes in the spring stiffness; (4) in all the scenarios the identification of the parameter reduction is positive using both training algorithms.

CHAPTER 5

CONCLUSIONS AND FUTURE DEVELOPMENTS

5.1 Conclusions

The main conclusions from this thesis we have:

- As a base for the structural health monitoring of the El Tablazo bridge, it was proposed a numerical model developed by a structural software of finite elements MIDAS Civil© and a metamodel based in the dynamic behavior of the bridge and its critical parameters.

- The prestress of the box girder was modeled to verify the dynamic behavior including those forces, the result was the same and consequently was not taken in count and discarded due to the computational cost to run the FE model.

- According to the results of the sensitivity analysis developed it was possible to identify the parameters with more influence on eigenfrequencies of the studied bridge (El Tablazo), being the elasticity modulus of the concrete class A and the density weight of the concrete class A, however, more parameters could be take in count based in the decrease of the vibration-modes studied.

- The assessment and identification of the bridge parameters are made through the definition of damage scenarios, allowing to know the dynamic response for future testing, necessary to validate and to update the FE model according to the real response.

- The neural network generated in this research allows to identify damage scenarios related with loss of stiffness on the structural elements through vibration data measured in field before and after damages.

- The comparison between training algorithms (BP and GA) for the ANN, allows to conclude that a global optimization of the error function develop a better behavior to predict the output parameters, but the GA computational cost is a disadvantage compared with the local optimization.

- The analysis developed to the foundations spring stiffness determines the effective range where these parameters are sensitive to the natural frequencies of the bridge, a value (10^6) is adopted in the reference model based in the constant behavior of the frequencies if the parameter takes highest values and abrupt changes if is a lowest.

5.2 Future developments

As suggestions for future research to complement and continue this thesis we have:

- For future plans of monitoring the El Tablazo bridge it is recommended:

To calibrate a numerical model based on the combination of static and dynamic data measurement in field through vibration testing (to obtain vibration data) and load testing (to obtain deflections) using the Artificial Neural Network to generate the relationship between critical parameters (obtained by a sensitivity analysis) and the measured data.

To perform an assessment in the bridge, using modal-values to develop methods of damage detection in order to obtain information about the bridge state.

- In order to optimize the performance of the metamodel, it is necessary a large dataset, to increase the learning in insensitivity ranges of the spring stiffness.

- The combination of both training methods used in this work (BP algorithm and GA) to use as a hybrid algorithm training the metamodel and performing a comparison between the predictions.

- To obtain structural assessment detailed is necessary to create the FE model using: data from soil-structure interaction, to model the spring stiffness in the foundation, different type of element (frame) in the software (i.e. plates, solid), the boundary condition on the superstructure and the reinforcement steel in the structural elements.

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Appendix A: Construction blueprints

The necessary information about geometry and materials of the El Tablazo bridge to generate the FE model using the structure analysis software MIDAS Civil©, was provided by the co-advisor of this research PhD. Alvaro Viviescas Jaimes. The blueprints presented contain details about the deck plan of the bridge, the longitudinal section.

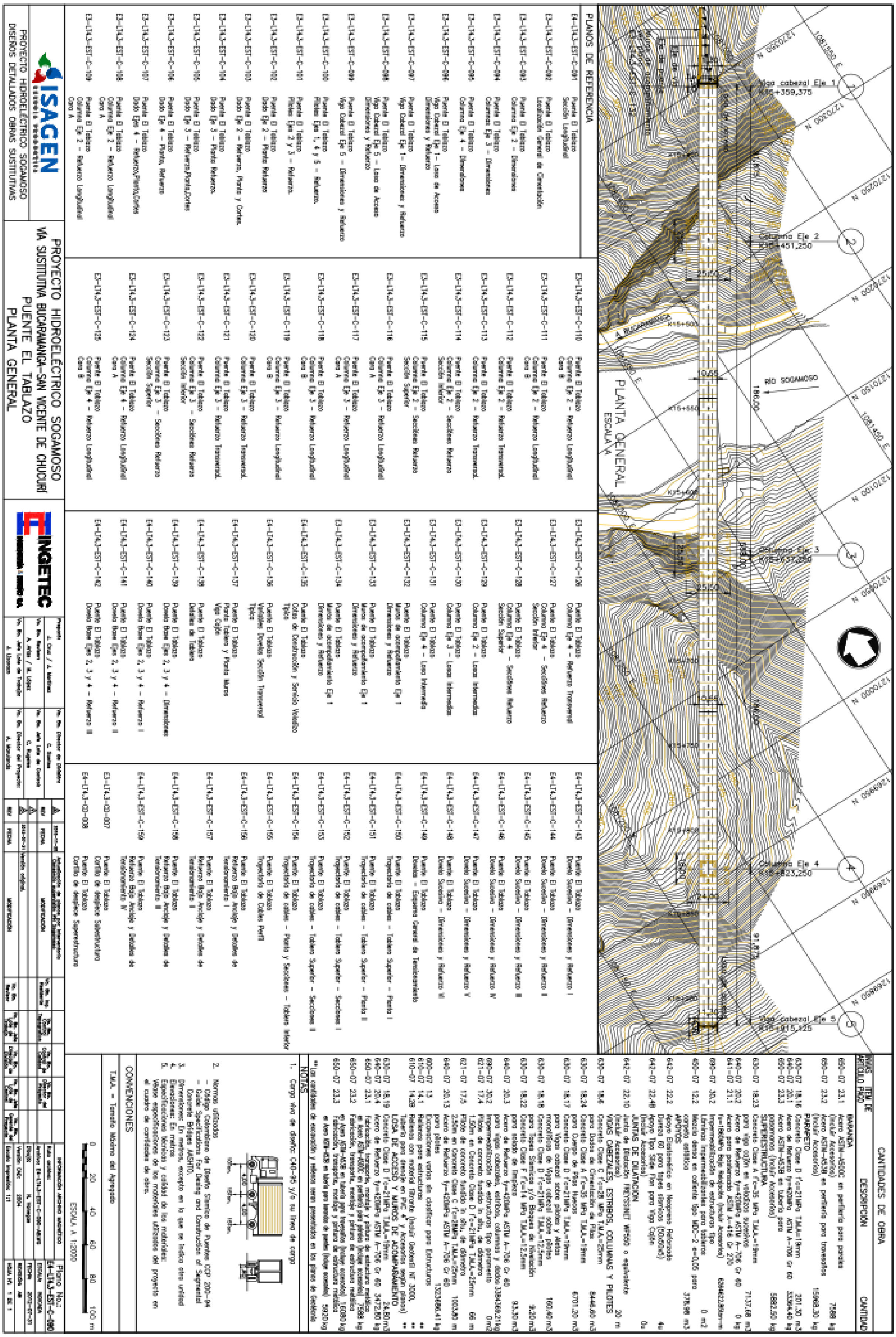


Figure A.1. Deck plan blueprint.

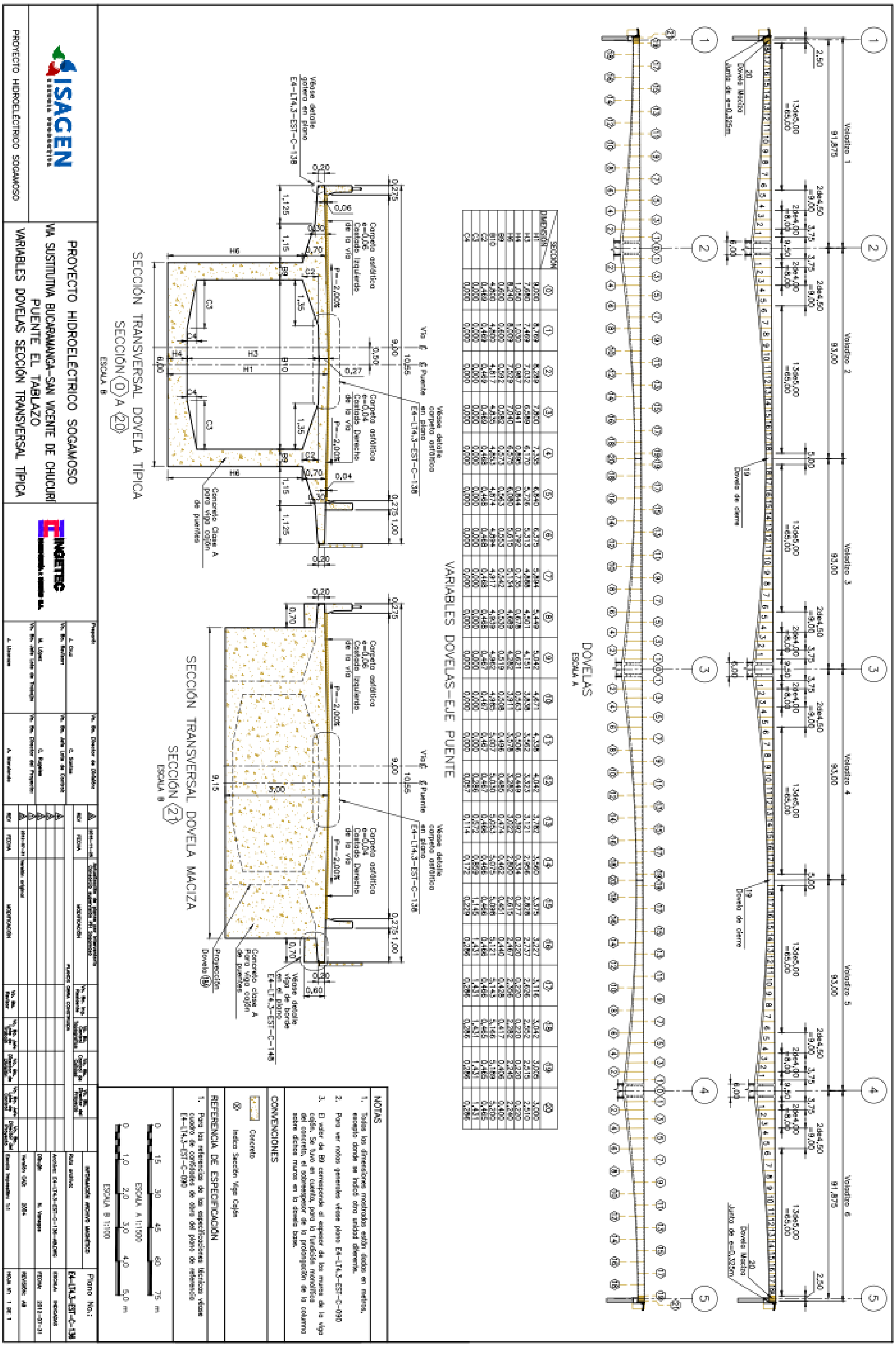


Figure A.2. Box girder blueprints.

Appendix B: Modeling procedure

The bridge model is developed in the software MIDAS Civil©; this appendix defines the methodology used.

(1) The procedure begins with a new project creation, followed by the definition of the unit system employed; The steps and the units used are specified in figure B.1.

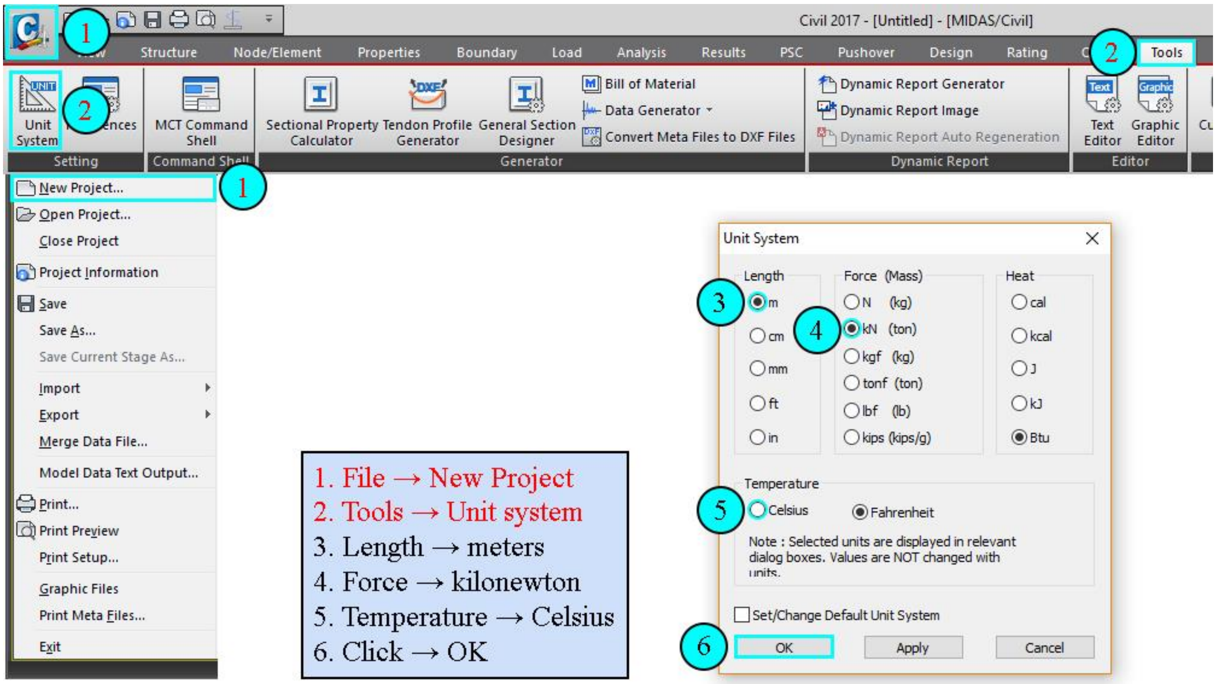


Figure B.1. New project and Units definition.

(2) To define the bridge materials are used the information in appendix A and C, the steps to create a new material and fill the information is presented in figure B.2.

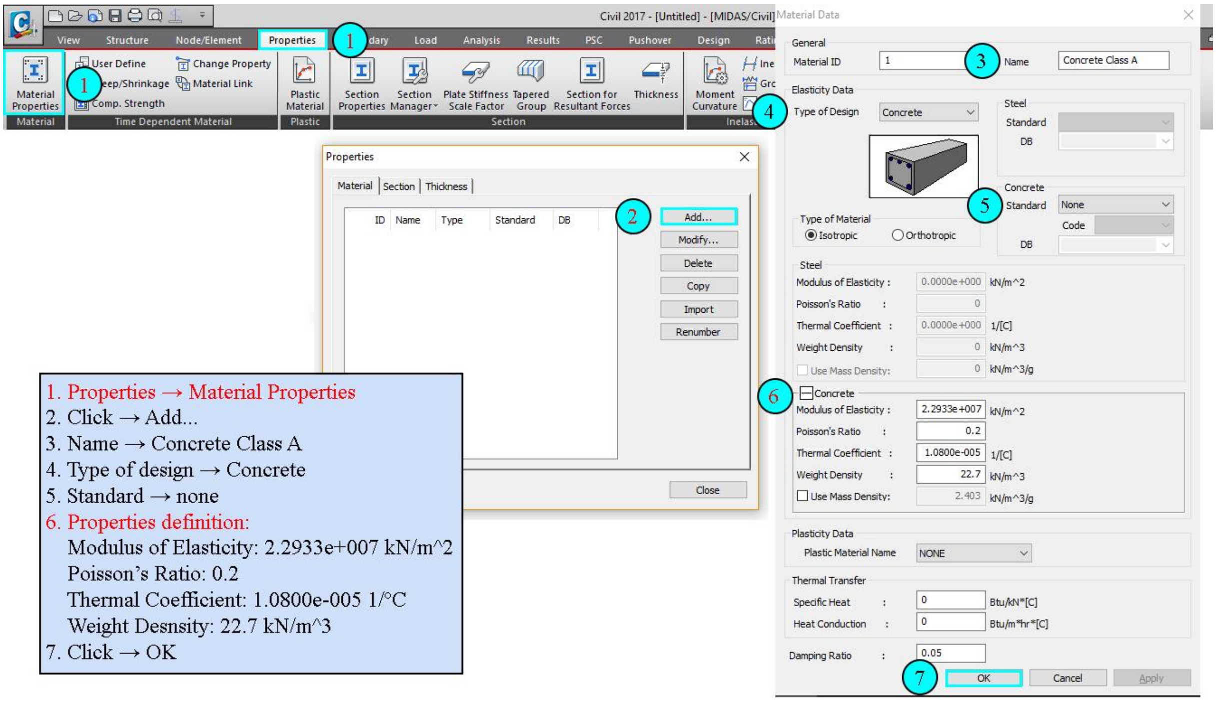


Figure B.2. Definition of material properties.

(3) The procedure to define the cross section of all the structural elements is presented in figure B.3. an B.4, however, to developed correctly the geometry defined in appendix A, was necessary the use of complementary software's (MIDAS Civil tools).

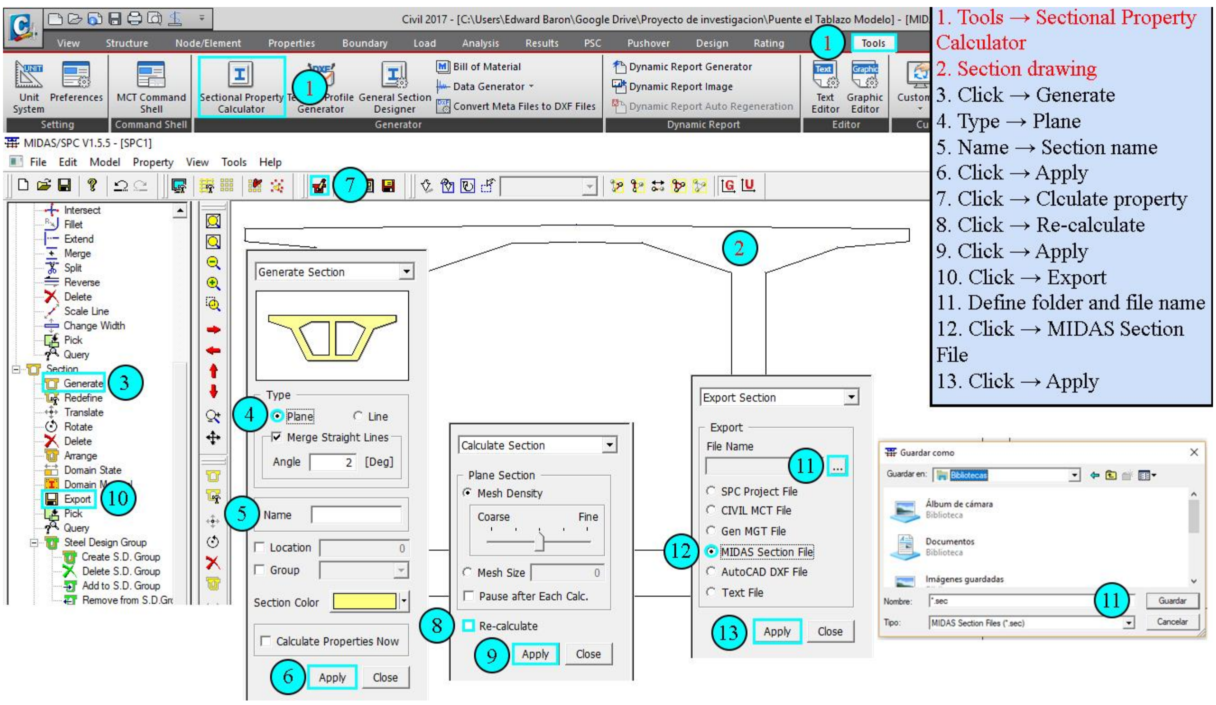


Figure B.3. Cross-section definition.

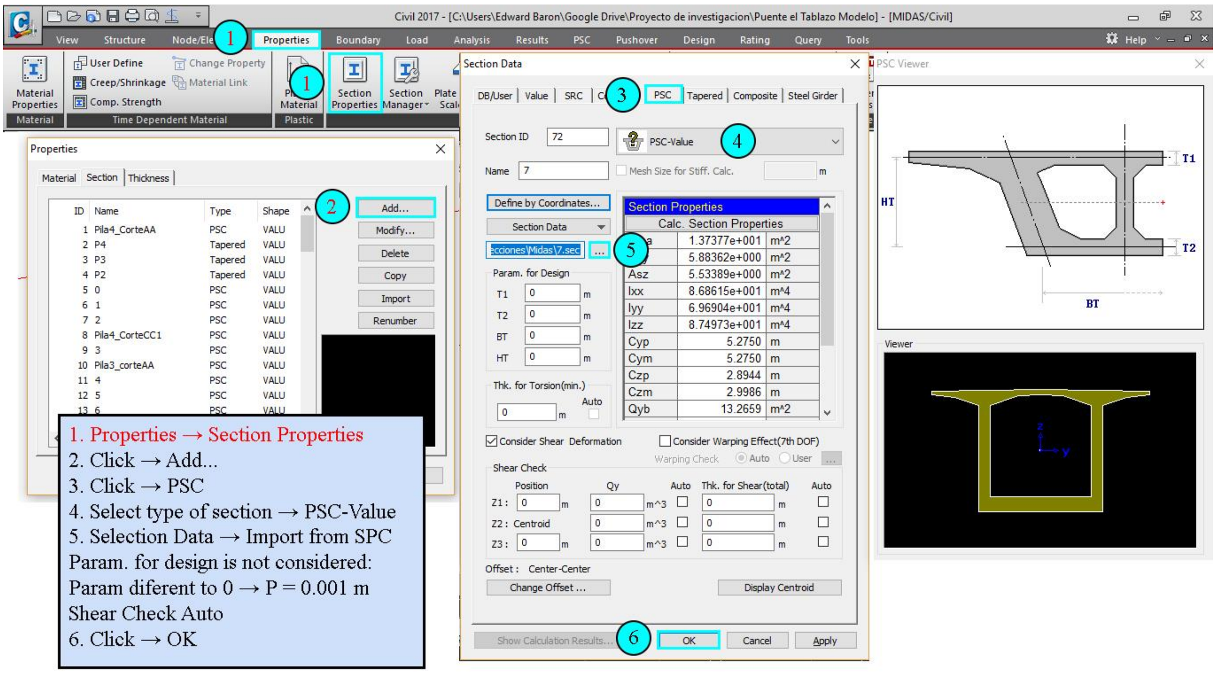


Figure B.4. import SPC file to MIDAS Civil©.

(4) Before to generate the tapered section of the box-girders and the columns is necessary the creation of the starting and final sections, figure B.5. shows the steps to develop tapered section.

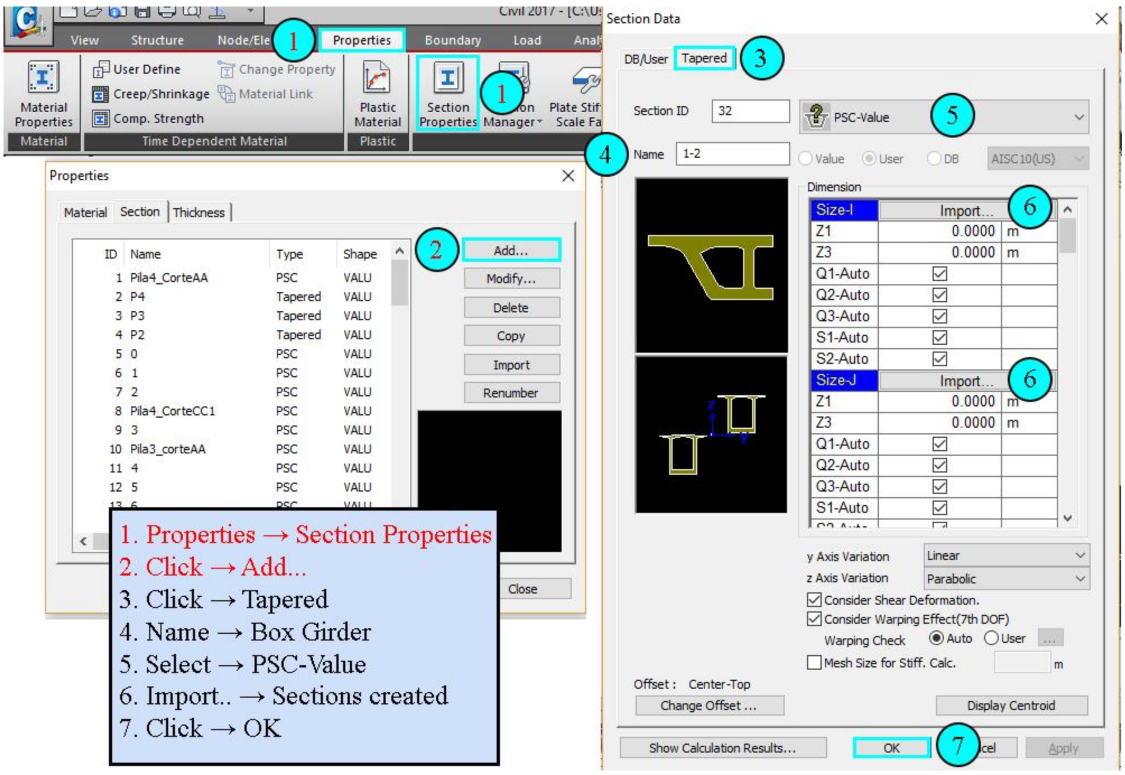


Figure B.5. Creation of tapered sections

(5) The nodes creation is specified in figure B.6, it's important to highlight the nodes coordinates are based in the construction blueprints, using the altitudes and longitudinal measurements.

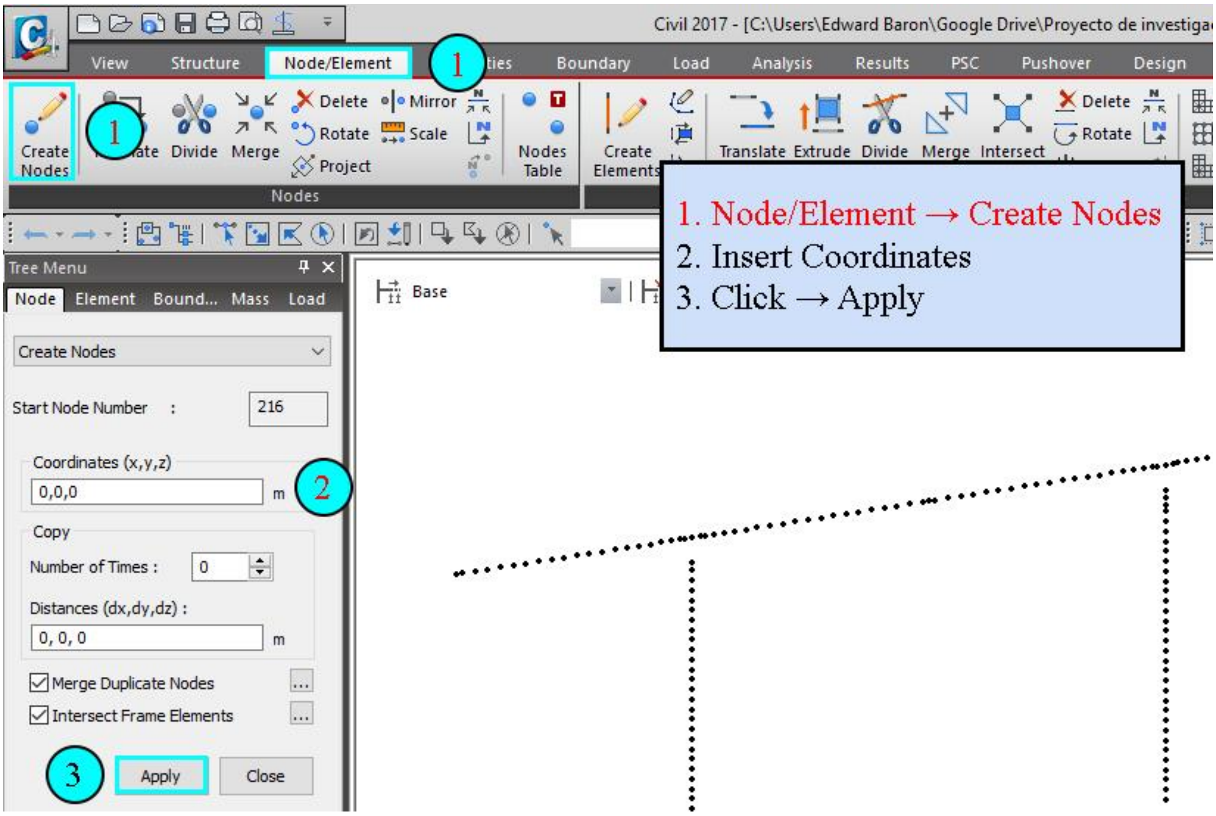


Figure B.6. Nodes insertion by coordinates.

(6) The element creation is presented in figure B.7, all the element of the superstructure and substructure are conformed by elements type General Beam.

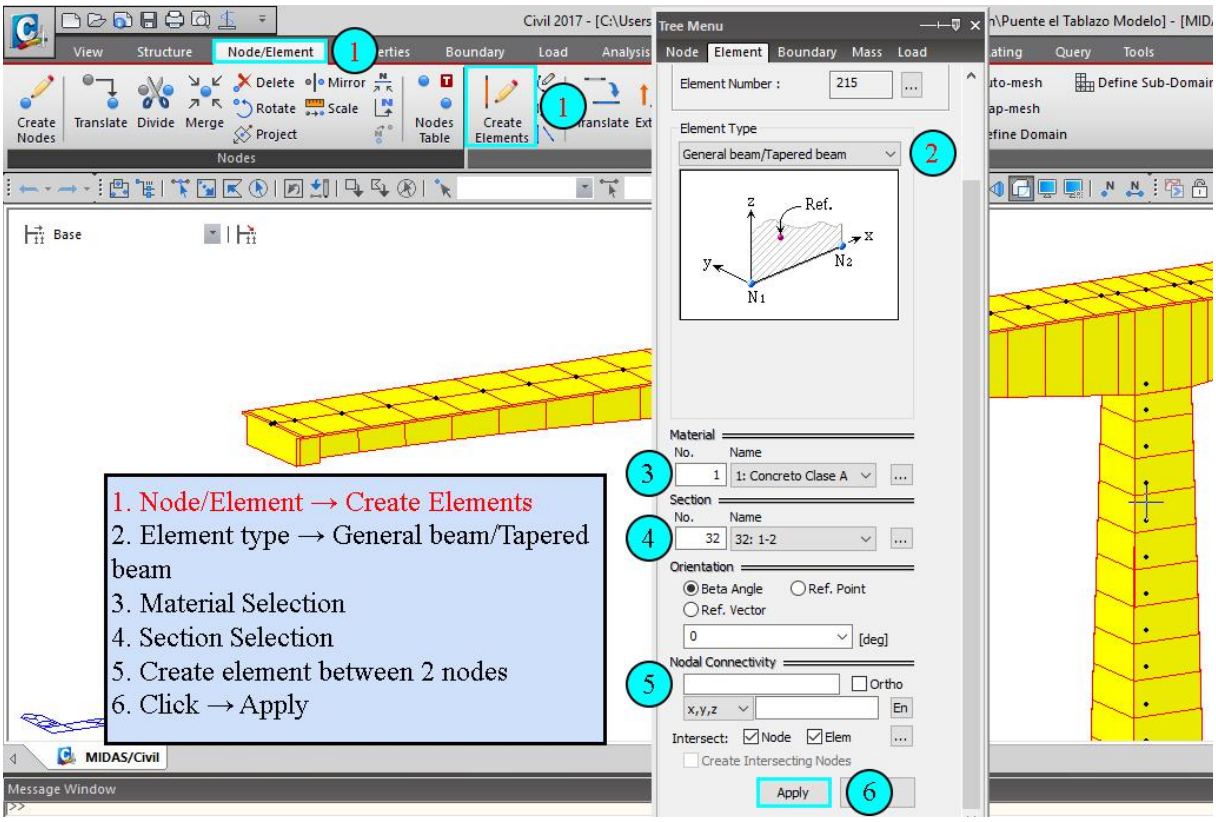


Figure B.7. Elements creation.

(7) For the columns are created one tapered section for each one, therefore, every column is divided in elements between 2.5 and 3.5 meters following the construction specifications in appendix A, however, to generated correctly the tapered section is necessary follow the steps in figure B.8.

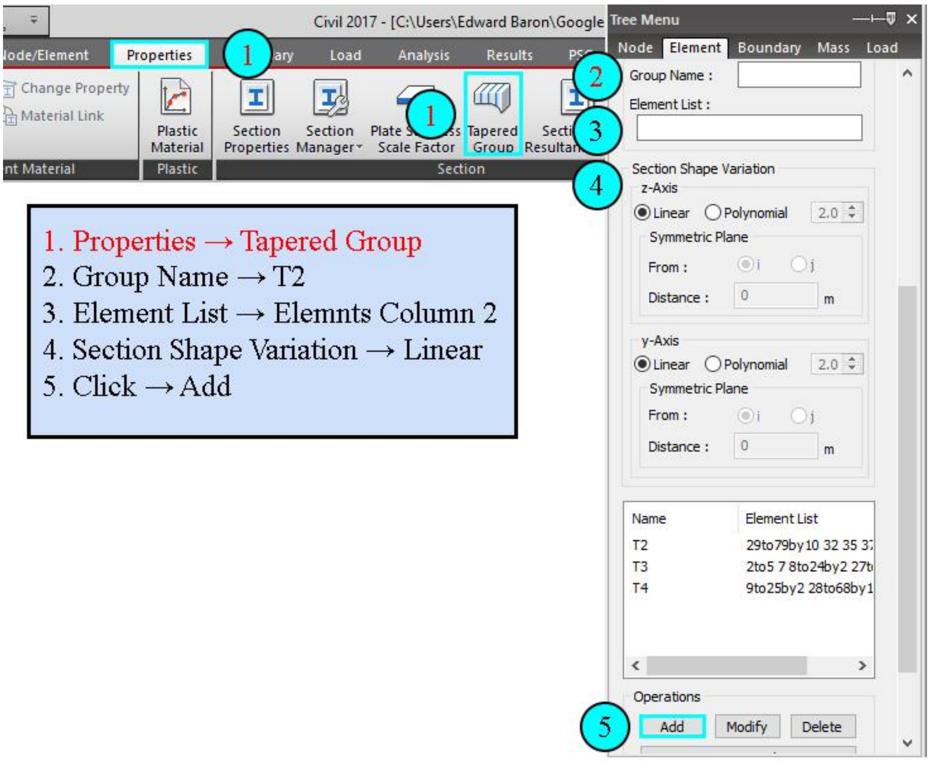


Figure B.8. Tapered group definition for the bridge columns.

(8) The boundary condition of the foundation is specified in chapter 3, however, the steps for modelling those conditions are presented in figure B.9.

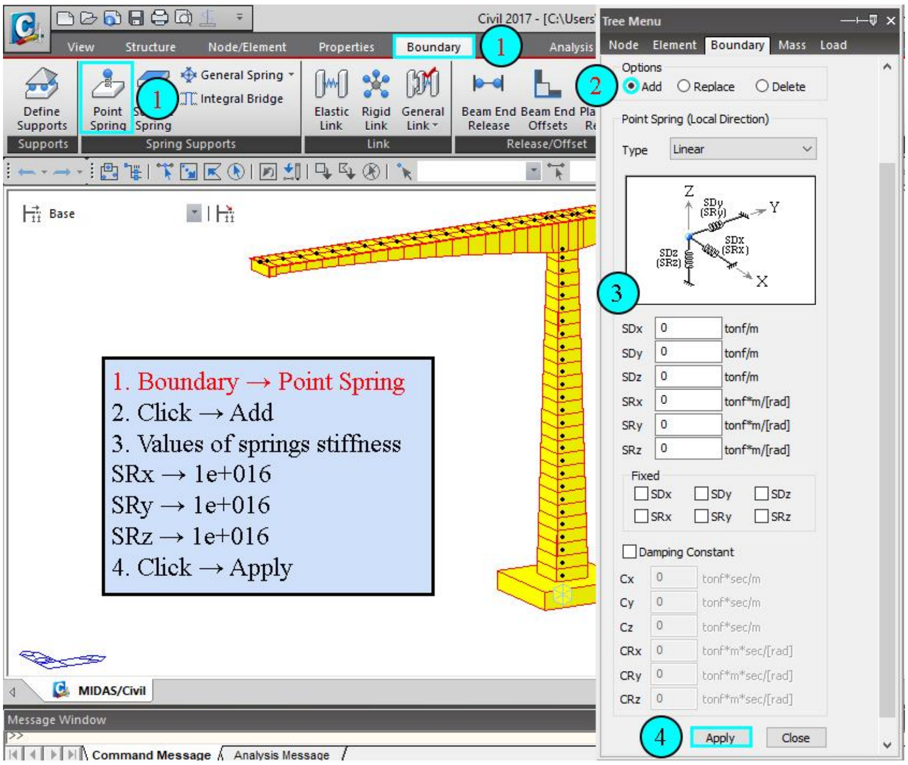


Figure B.9. Modelling the springs stiffness foundation.

(9) The boundary condition in the box girder is specified in chapter 3 based in the construction blueprints; figure B.10 shows the procedure to idealize the supports.

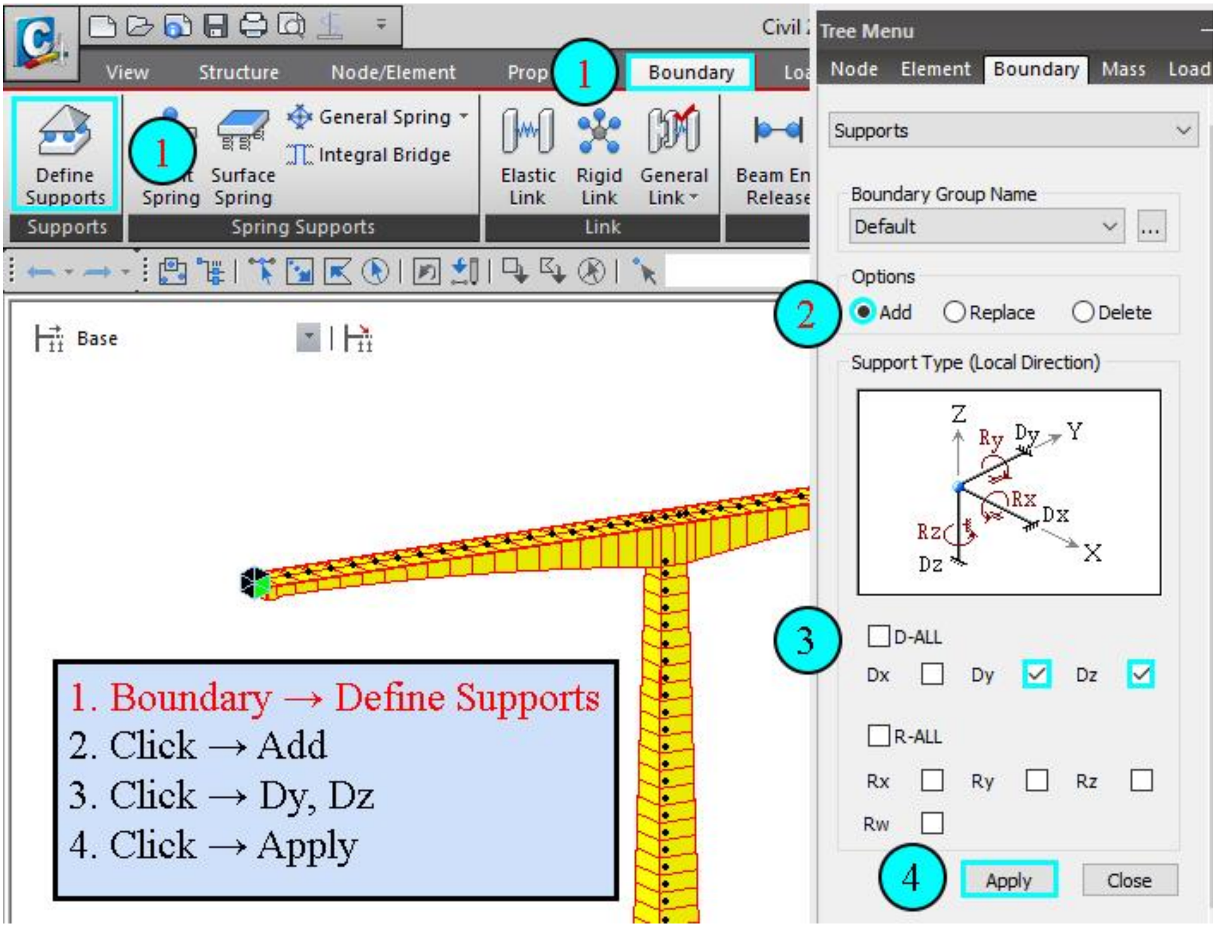


Figure B.10. Box Girder supports.

(10) The monolithic union between the superstructure and the substructure are modelling follow the procedure in figure B.11, the description of this consideration is specified in chapter 3.

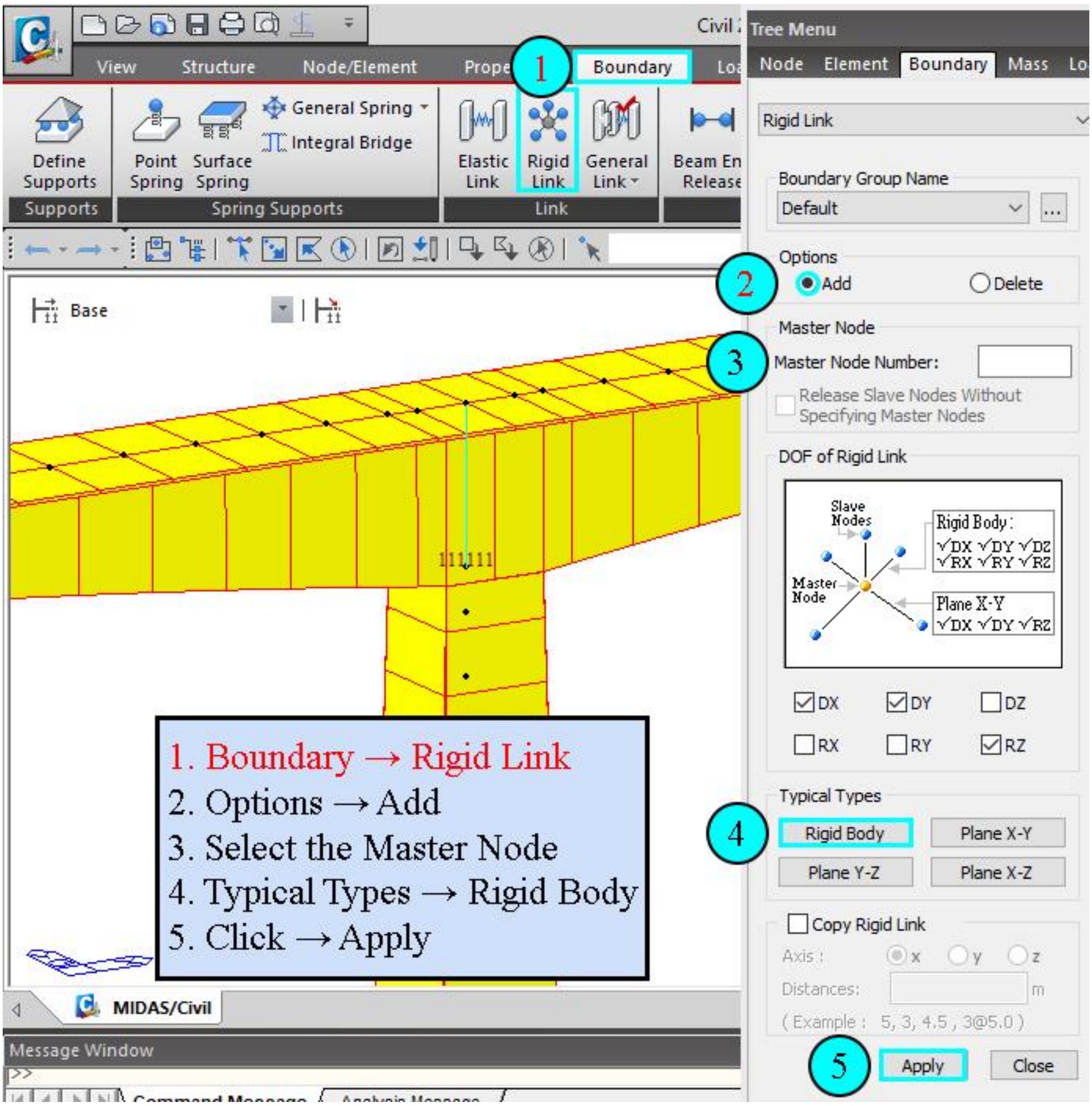


Figure B.11. Rigid body modelling.

(11) Dead load in the model is applied follow the steps in figure B.12; all the calculation about it are specified in appendix C; this loads are produced by the non-structural elements in the bridge (e.g. parapets, pavement).

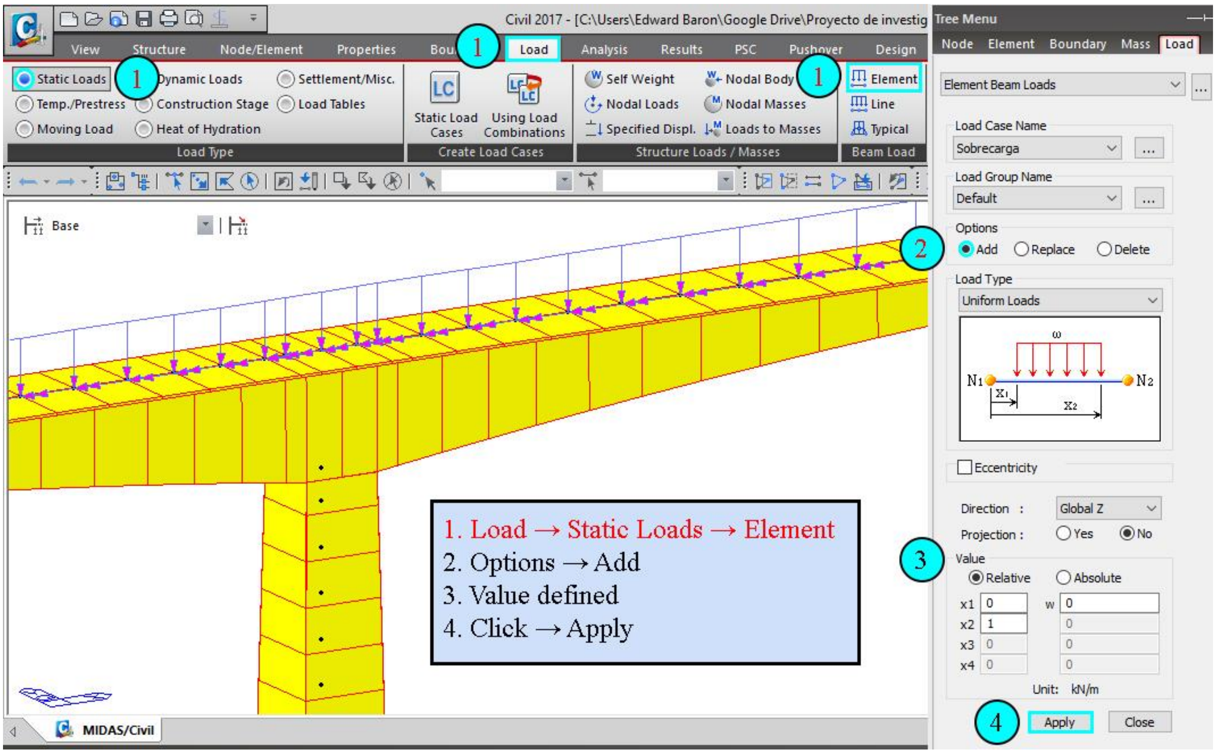


Figure B.12. Dead Load defined.

(12) The calculation of the self-weight is developed by the software, figure B.13 shows the steps for introduce this load case.

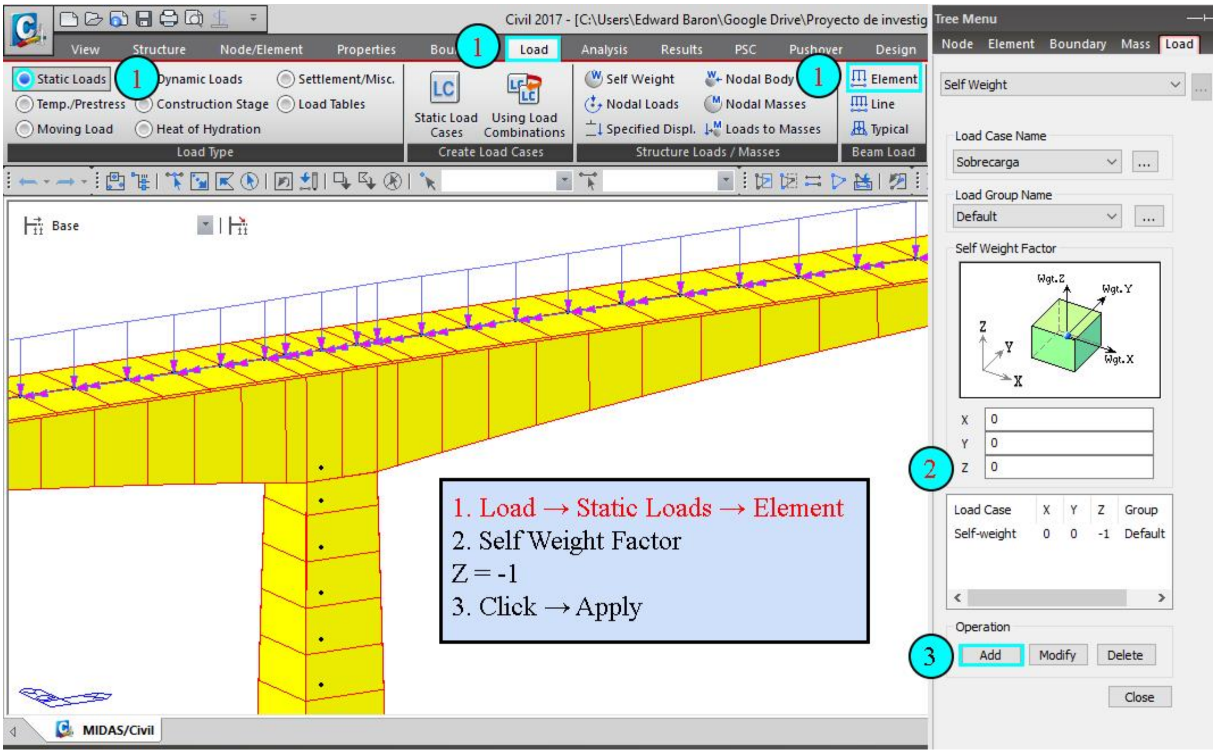


Figure B.13. Self-weight defined.

(13) The analysis of the model response is carry out by the eigenvalue tool, this analysis allows to known the natural parameters of the FE model such a frequencies, period and modes of vibration; therefore, the procedure to developed is presented in figure B.14.

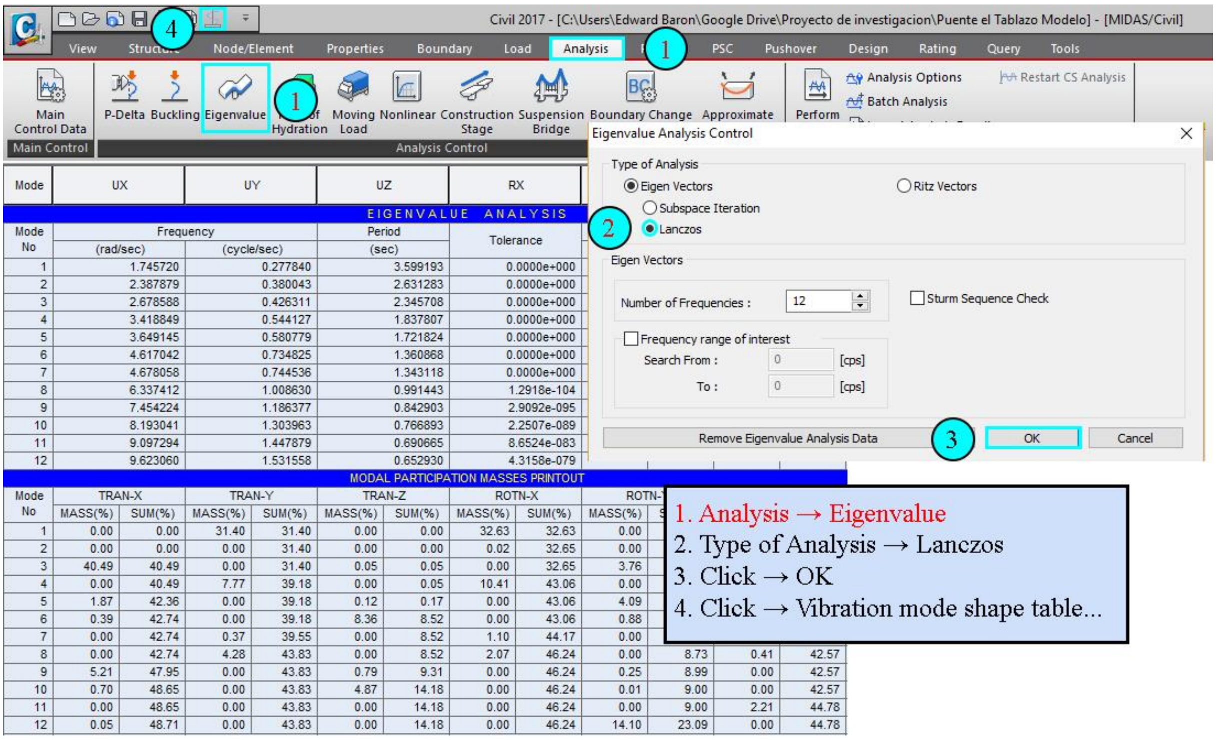


Figure B.14. Eigenvalue analysis procedure.

Appendix C: Expressions and calculations

Modelling de FE model of the structure in MIDAS Civil© is necessary computed the required parameters, obtained by equations and norms founds in the bibliography consulted.

Elasticity modulus of the concrete (E_c)

To compute this value is used the seismic norm of Colombia (NSR-10 – C.8.5), due to the non-specification in the blueprints about this property, the following expression could be used, when is unknown the unitary mass of the concrete:

$$E_c = 3900\sqrt{f_c'} \quad [MPa] \tag{C.1}$$

The previous expression doesn't need the type of aggregate and is a mean value for all the experimental information in Colombia.

Poisson's ratio (ν)

This property of the concrete is based in the seismic norm of Colombia (NSR-10) Title C – CR. 8.5; is used in cases when is not available an experimental value, the Poisson's ratio could be taken:

$$\nu = 0.2 \tag{C.2}$$

Damping ratio (ξ)

Based in the NSR-10 Title A – A-2.6.1, the critical damping value then should be uses in the structure design is:

$$\xi = 5\% \tag{C.3}$$

Wight density (ρ)

According to the bridge Colombian code, the weight density could be determinate by the information in table C1

Table C1. Weight density

	Material	Peso unitario kN/m3
Concreto	Liviano	17.4
	De arena liviana	18.9
	Normal con $f'_c \leq 35$ Mpa (5.0 ksi)	22.7
	Normal con $35 < f'_c \leq 105$ Mpa	22.0 +0.022f'c

Thermal Coefficient

This property is based in the bridge Colombian code, this approximation could be used if this property is not determined by laboratory tests, therefore, the values are presented in table C2

Table C2. Thermal Coefficient

Concreto	Coeficiente [1/°C]
Peso normal	1.08x10-5
Peso liviano	1.7x10-5

Dead loads

The loads applied in the FE model is based in the construction blueprints and are caused by the non-structural elements supported for the deck (i.e. parapet, pavement, etc.) (see appendix A); in table C3 is presented the computed loads.

Table C3. Dead loads

	Quantity	Distance	Density	Moment	Load
Railing					
Steel ASTM a-500C	7588	-5.275		-0.70369521	0.13340194
Steel ASTM-A53B	15968.3	-5.275		-1.48086666	0.28073302
Parapet					
Concrete	201.3		22.7		
Left	100.65	5.1375		21.0357147	4.09454301
Right	100.65	-4.1375		-16.9411717	4.09454301
Reinforcement steel (parapet)	33364.4				
Left	16682.2	5.1375		1.50674572	0.29328384
Right	16682.2	-4.1375		-1.21346188	0.29328384
Steel ASTM	5882.5	5.1375		0.53131072	0.10341815
Pavement					
Left	217.4	2.5	22.7	22.1101254	8.84405018
Right	161.6	-2		-13.1481004	6.57405018
Total				11.6966008	24.7113072

Appendix D: Sensitivity analysis results

The computed data for the sensitivity analysis is presented in this appendix, the properties parameters of the bridge are change it in the FE model to obtain the structure response, the first figure shows the mean value of each parameter and its corresponding variation, the second and third specified the structure response, and the last table shows the computed measurement coefficient for each frequency and the interaction between it.

		CV [%]	X_m	$X_m+\Delta X$	$X_m-\Delta X$
Concrete Class A	E_a [Mpa]	10%	22932.840	25226.124	20639.556
	ρ_a [kN/m3]	10%	22.700	24.970	20.430
	ν_a	10%	0.200	0.220	0.180
	ξ_a	5%	0.050	0.053	0.0475
Box-girder middle span	H_{O1} [m]	5%	0.200	0.210	0.190
	H_3 [m]	5%	2.510	2.636	2.385
	B_1 [m]	5%	6.000	6.300	5.700
	B_9 [m]	5%	0.400	0.420	0.380
	H_4 [m]	5%	0.220	0.231	0.209
	C_2 [m]	5%	0.465	0.488	0.442
Columns	H_C [m]	5%	6.100	6.405	5.795
	B_c [m]	5%	6.000	6.300	5.700
	t_c [m]	5%	0.900	0.945	0.855
Foundation	h_{f1} [m]	10%	5.000	5.500	4.500
	h_{f2} [m]	10%	5.000	5.500	4.500
	h_{f3} [m]	10%	4.500	4.950	4.050
Concrete Class C	E_c [Mpa]	20%	20636.860	24764.232	16509.488
	ρ_c [kN/m3]	20%	22.700	27.240	18.160
	ν_c	10%	0.200	0.220	0.180
	ξ_c	5%	0.050	0.053	0.0475
Static dead load	ρ_{pav} [KN/m3]	10%	22.000	24.200	19.800
	W_{sob} [KN/m]	10%	24.236	26.659	21.812
	M_{sob} [KN/m*m]	10%	11.420	12.562	10.278

Modo 1			Modo 2			Modo 3			Modo 4			Modo 5			Modo 6		
F_m	F_1	F_2	F_m	F_1	F_2	F_m	F_1	F_2	F_m	F_1	F_2	F_m	F_1	F_2	F_m	F_1	F_2
0.27784	0.291378	0.263603	0.380043	0.398568	0.360562	0.426311	0.447063	0.404484	0.544127	0.570646	0.516239	0.580779	0.609107	0.550994	0.734825	0.770671	0.697134
0.27784	0.265807	0.29167	0.380043	0.363675	0.398839	0.426311	0.407807	0.447585	0.544127	0.520419	0.571402	0.580779	0.55634	0.608752	0.734825	0.704066	0.770003
0.27784	0.277635	0.278049	0.380043	0.37947	0.380628	0.426311	0.426172	0.42645	0.544127	0.543707	0.544555	0.580779	0.580583	0.580976	0.734825	0.73455	0.735101
0.27784	0.27784	0.27784	0.380043	0.380043	0.380043	0.426311	0.426311	0.426311	0.544127	0.544127	0.544127	0.580779	0.580779	0.580779	0.734825	0.734825	0.734825
0.27784	0.27783	0.27785	0.380043	0.380041	0.380044	0.426311	0.426298	0.426323	0.544127	0.544131	0.544123	0.580779	0.580732	0.580827	0.734825	0.734753	0.734898
0.27784	0.277832	0.277845	0.380043	0.380038	0.380033	0.426311	0.426324	0.426294	0.544127	0.544133	0.544111	0.580779	0.581026	0.580508	0.734825	0.735176	0.734439
0.27784	0.277848	0.277829	0.380043	0.380141	0.379933	0.426311	0.426296	0.426324	0.544127	0.544204	0.544041	0.580779	0.580745	0.580811	0.734825	0.734769	0.734877
0.27784	0.277829	0.277851	0.380043	0.380032	0.380053	0.426311	0.426298	0.426323	0.544127	0.544122	0.544131	0.580779	0.580707	0.580852	0.734825	0.734713	0.734938
0.27784	0.277864	0.277847	0.380043	0.380063	0.380051	0.426311	0.426339	0.426313	0.544127	0.544127	0.544129	0.580779	0.580925	0.580764	0.734825	0.735049	0.73481
0.27784	0.277836	0.277844	0.380043	0.380039	0.380047	0.426311	0.426306	0.426315	0.544127	0.544125	0.544128	0.580779	0.580755	0.580803	0.734825	0.734788	0.734861
0.27784	0.281323	0.274219	0.380043	0.384023	0.375904	0.426311	0.42696	0.42562	0.544127	0.548267	0.539843	0.580779	0.584047	0.577383	0.734825	0.736566	0.732993
0.27784	0.279048	0.276583	0.380043	0.382003	0.377996	0.426311	0.429723	0.422678	0.544127	0.545428	0.54279	0.580779	0.591206	0.569854	0.734825	0.740345	0.728828
0.27784	0.278764	0.276861	0.380043	0.381159	0.37886	0.426311	0.426763	0.425813	0.544127	0.544756	0.543457	0.580779	0.583159	0.578209	0.734825	0.736159	0.733377
0.27784	0.277835	0.277845	0.380043	0.38003	0.380055	0.426311	0.426302	0.426318	0.544127	0.544118	0.544135	0.580779	0.580774	0.580785	0.734825	0.734822	0.734828
0.27784	0.277823	0.277856	0.380043	0.38004	0.380046	0.426311	0.4263	0.426321	0.544127	0.544122	0.544132	0.580779	0.580774	0.580785	0.734825	0.734825	0.734825
0.27784	0.277839	0.277842	0.380043	0.380034	0.380051	0.426311	0.426272	0.426349	0.544127	0.5441	0.544153	0.580779	0.580769	0.58079	0.734825	0.734807	0.734843
0.27784	0.277877	0.277785	0.380043	0.380081	0.379986	0.426311	0.426399	0.426178	0.544127	0.544189	0.544034	0.580779	0.580811	0.580732	0.734825	0.734857	0.734777
0.27784	0.27784	0.27784	0.380043	0.380043	0.380043	0.426311	0.426311	0.426311	0.544127	0.544127	0.544127	0.580779	0.580779	0.580779	0.734825	0.734825	0.734825
0.27784	0.27784	0.27784	0.380043	0.380043	0.380043	0.426311	0.426311	0.426311	0.544127	0.544127	0.544127	0.580779	0.580779	0.580779	0.734825	0.734825	0.734825
0.27784	0.27784	0.27784	0.380043	0.380043	0.380043	0.426311	0.426311	0.426311	0.544127	0.544127	0.544127	0.580779	0.580779	0.580779	0.734825	0.734825	0.734825
0.27784	0.277207	0.278477	0.380043	0.379113	0.380979	0.426311	0.425368	0.427259	0.544127	0.542988	0.545274	0.580779	0.57896	0.582616	0.734825	0.73241	0.737265
0.27784	0.276816	0.278875	0.380043	0.378539	0.381564	0.426311	0.424786	0.427852	0.544127	0.542284	0.545991	0.580779	0.577838	0.583768	0.734825	0.73092	0.738795
0.27784	0.27784	0.27784	0.380043	0.380043	0.380043	0.426311	0.426311	0.426311	0.544127	0.544127	0.544127	0.580779	0.580779	0.580779	0.734825	0.734825	0.734825

Modo 7			Modo 8			Modo 9			Modo 10			Modo 11			Modo 12		
F_m	F_1	F_2	F_m	F_1	F_2	F_m	F_1	F_2	F_m	F_1	F_2	F_m	F_1	F_2	F_m	F_1	F_2
0.744536	0.780842	0.706359	1.00863	1.05784	0.956889	1.186377	1.244236	1.125537	1.303963	1.367581	1.237072	1.447879	1.518544	1.373583	1.531558	1.60629	1.452982
0.744536	0.712357	0.781515	1.00863	0.965267	1.058424	1.186377	1.135374	1.244933	1.303963	1.248476	1.367573	1.447879	1.38594	1.518942	1.531558	1.466481	1.606142
0.744536	0.743784	0.745306	1.00863	1.007573	1.009708	1.186377	1.185874	1.18688	1.303963	1.303379	1.304548	1.447879	1.44566	1.450145	1.531558	1.530803	1.532313
0.744536	0.744536	0.744536	1.00863	1.00863	1.00863	1.186377	1.186377	1.186377	1.303963	1.303963	1.303963	1.447879	1.447879	1.447879	1.531558	1.531558	1.531558
0.744536	0.744551	0.74452	1.00863	1.008644	1.008615	1.186377	1.186373	1.18638	1.303963	1.303959	1.303964	1.447879	1.447896	1.447863	1.531558	1.531544	1.53157
0.744536	0.744534	0.744508	1.00863	1.008608	1.008595	1.186377	1.186423	1.186328	1.303963	1.304155	1.303746	1.447879	1.447867	1.447892	1.531558	1.531813	1.531279
0.744536	0.744775	0.744272	1.00863	1.009075	1.008136	1.186377	1.186368	1.186387	1.303963	1.30398	1.303943	1.447879	1.447899	1.44786	1.531558	1.531566	1.531546
0.744536	0.744531	0.74454	1.00863	1.008608	1.00865	1.186377	1.186379	1.186373	1.303963	1.303969	1.303954	1.447879	1.447881	1.447878	1.531558	1.531535	1.531579
0.744536	0.744542	0.744545	1.00863	1.00868	1.008658	1.186377	1.186393	1.186374	1.303963	1.303972	1.303918	1.447879	1.447838	1.447876	1.531558	1.531614	1.531518
0.744536	0.744533	0.744539	1.00863	1.00862	1.008641	1.186377	1.186375	1.186379	1.303963	1.303957	1.303971	1.447879	1.447881	1.447876	1.531558	1.531545	1.531571
0.744536	0.748665	0.740303	1.00863	1.01052	1.006674	1.186377	1.19089	1.181689	1.303963	1.307759	1.300042	1.447879	1.451607	1.444049	1.531558	1.533203	1.529866
0.744536	0.746479	0.742526	1.00863	1.009873	1.007312	1.186377	1.203027	1.169264	1.303963	1.316296	1.291408	1.447879	1.451771	1.443878	1.531558	1.537389	1.525699
0.744536	0.745323	0.743695	1.00863	1.008189	1.009054	1.186377	1.188735	1.183754	1.303963	1.306655	1.301063	1.447879	1.44952	1.44617	1.531558	1.532531	1.530486
0.744536	0.744531	0.744541	1.00863	1.008628	1.008633	1.186377	1.186361	1.186392	1.303963	1.303961	1.303965	1.447879	1.447878	1.44788	1.531558	1.531552	1.531563
0.744536	0.744535	0.744537	1.00863	1.008621	1.008639	1.186377	1.186361	1.186392	1.303963	1.30396	1.303966	1.447879	1.447878	1.44788	1.531558	1.531549	1.531566
0.744536	0.744507	0.744564	1.00863	1.008621	1.00864	1.186377	1.186359	1.186394	1.303963	1.303938	1.303987	1.447879	1.447877	1.447882	1.531558	1.531548	1.531567
0.744536	0.744589	0.744456	1.00863	1.008663	1.008582	1.186377	1.18645	1.186267	1.303963	1.304006	1.303899	1.447879	1.447886	1.447869	1.531558	1.531591	1.531507
0.744536	0.744536	0.744536	1.00863	1.00863	1.00863	1.186377	1.186377	1.186377	1.303963	1.303963	1.303963	1.447879	1.447879	1.447879	1.531558	1.531558	1.531558
0.744536	0.744536	0.744536	1.00863	1.00863	1.00863	1.186377	1.186377	1.186377	1.303963	1.303963	1.303963	1.447879	1.447879	1.447879	1.531558	1.531558	1.531558
0.744536	0.744536	0.744536	1.00863	1.00863	1.00863	1.186377	1.186377	1.186377	1.303963	1.303963	1.303963	1.447879	1.447879	1.447879	1.531558	1.531558	1.531558
0.744536	0.742796	0.74629	1.00863	1.006113	1.011169	1.186377	1.183411	1.189365	1.303963	1.300306	1.307652	1.447879	1.444047	1.451742	1.531558	1.527195	1.535958
0.744536	0.741722	0.747387	1.00863	1.004559	1.012758	1.186377	1.181578	1.191235	1.303963	1.298048	1.309962	1.447879	1.44168	1.454161	1.531558	1.524501	1.538714
0.744536	0.744536	0.744536	1.00863	1.00863	1.00863	1.186377	1.186377	1.186377	1.303963	1.303963	1.303963	1.447879	1.447879	1.447879	1.531558	1.531558	1.531558

$b_{i,f1}$	$b_{i,f2}$	$b_{i,f3}$	$b_{i,f4}$	$b_{i,f5}$	$b_{i,f6}$	$b_{i,f7}$	$b_{i,f8}$	$b_{i,f9}$	$b_{i,f10}$	$b_{i,f11}$	$b_{i,f12}$	b_k	$b_k/b_{k,max}$
49.98%	50.00%	49.94%	49.99%	50.03%	50.04%	50.02%	50.04%	50.03%	50.04%	50.06%	50.05%	60.02%	100.00%
46.54%	46.26%	46.65%	46.85%	45.12%	44.87%	46.44%	46.18%	46.17%	45.67%	45.93%	45.59%	55.23%	92.01%
0.75%	1.52%	0.33%	0.78%	0.34%	0.37%	1.02%	1.06%	0.42%	0.45%	1.55%	0.49%	0.91%	1.51%
0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
0.07%	0.01%	0.06%	0.01%	0.16%	0.20%	0.04%	0.03%	0.01%	0.00%	0.02%	0.02%	0.03%	0.05%
0.05%	0.01%	0.07%	0.04%	0.89%	1.00%	0.03%	0.01%	0.08%	0.31%	0.02%	0.35%	0.14%	0.24%
0.07%	0.55%	0.07%	0.30%	0.11%	0.15%	0.68%	0.93%	0.02%	0.03%	0.03%	0.01%	0.15%	0.24%
0.08%	0.06%	0.06%	0.02%	0.25%	0.31%	0.01%	0.04%	0.01%	0.01%	0.00%	0.03%	0.04%	0.07%
0.06%	0.03%	0.06%	0.00%	0.28%	0.33%	0.00%	0.02%	0.02%	0.04%	0.03%	0.06%	0.05%	0.08%
0.03%	0.02%	0.02%	0.01%	0.08%	0.10%	0.01%	0.02%	0.00%	0.01%	0.00%	0.02%	0.02%	0.03%
25.57%	21.36%	3.14%	15.48%	11.47%	4.86%	11.23%	3.81%	7.76%	5.92%	5.22%	2.18%	5.90%	9.83%
8.87%	10.54%	16.53%	4.85%	36.76%	15.67%	5.31%	2.54%	28.46%	19.09%	5.45%	7.63%	8.09%	13.47%
6.85%	6.05%	2.23%	2.39%	8.52%	3.79%	2.19%	0.86%	4.20%	4.29%	2.31%	1.34%	2.25%	3.75%
0.02%	0.03%	0.02%	0.02%	0.01%	0.00%	0.01%	0.00%	0.01%	0.00%	0.00%	0.00%	0.01%	0.02%
0.06%	0.01%	0.02%	0.01%	0.01%	0.00%	0.00%	0.01%	0.01%	0.00%	0.00%	0.01%	0.01%	0.02%
0.01%	0.02%	0.09%	0.05%	0.02%	0.02%	0.04%	0.01%	0.01%	0.02%	0.00%	0.01%	0.03%	0.05%
0.08%	0.06%	0.13%	0.07%	0.03%	0.03%	0.04%	0.02%	0.04%	0.02%	0.00%	0.01%	0.11%	0.18%
0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
2.29%	2.45%	2.22%	2.10%	3.15%	3.30%	2.35%	2.51%	2.51%	2.82%	2.66%	2.86%	3.12%	5.20%
3.71%	3.98%	3.60%	3.41%	5.11%	5.36%	3.80%	4.06%	4.07%	4.57%	4.31%	4.64%	5.06%	8.43%
0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%

In this part of the appendix is presented the analysis of the springs stiffness, the values are changed using the range [10^2 kN/m - 10^{13} kN/m] as specified in chapter 3.

	K_{h1}											
	100	1000	10000	100000	1000000	10000000	1E+08	1E+09	1E+10	1E+11	1E+12	1E+13
F_1	0.106792	0.112723	0.160288	0.27784	0.27784	0.27784	0.27784	0.27784	0.27784	0.27784	0.27784	0.27784
F_2	0.27784	0.27784	0.27784	0.380043	0.380043	0.380043	0.380043	0.380043	0.380043	0.380043	0.380043	0.380043
F_3	0.380043	0.380043	0.380043	0.387998	0.426308	0.42631	0.42631	0.42631	0.426311	0.426311	0.426311	0.426311
F_4	0.426544	0.426545	0.426564	0.427513	0.544127	0.544127	0.544127	0.544127	0.544127	0.544127	0.544127	0.544127
F_5	0.544127	0.544127	0.544127	0.544127	0.580596	0.580761	0.580761	0.580778	0.580779	0.580779	0.580779	0.580779
F_6	0.588582	0.588612	0.588922	0.594073	0.734615	0.734805	0.734805	0.734823	0.734825	0.734825	0.734825	0.734825
F_7	0.740714	0.740729	0.740885	0.743028	0.744536	0.744536	0.744536	0.744536	0.744536	0.744536	0.744536	0.744536
F_8	0.744536	0.744536	0.744536	0.744536	1.00863	1.00863	1.00863	1.00863	1.00863	1.00863	1.00863	1.00863
F_9	1.00863	1.00863	1.00863	1.00863	1.186371	1.186376	1.186376	1.186377	1.186377	1.186377	1.186377	1.186377
F_{10}	1.186429	1.18643	1.18643	1.186435	1.303341	1.303908	1.303908	1.303958	1.303962	1.303963	1.303963	1.303963
F_{11}	1.308142	1.308145	1.308174	1.308487	1.447879	1.447879	1.447879	1.447879	1.447879	1.447879	1.447879	1.447879
F_{12}	1.447879	1.447879	1.447879	1.447879	1.526209	1.531104	1.531104	1.531513	1.531553	1.531557	1.531558	1.531558

	K_{t1}											
	100	1000	10000	100000	1000000	10000000	1E+08	1E+09	1E+10	1E+11	1E+12	1E+13
F_1	0.100788	0.107617	0.15817	0.267397	0.277124	0.277771	0.277833	0.277839	0.27784	0.27784	0.27784	0.27784
F_2	0.2928	0.293052	0.295997	0.349958	0.377843	0.379831	0.380022	0.380041	0.380043	0.380043	0.380043	0.380043
F_3	0.414998	0.415407	0.419914	0.426311	0.426311	0.426311	0.426311	0.426311	0.426311	0.426311	0.426311	0.426311
F_4	0.426311	0.426311	0.426311	0.497282	0.542294	0.543959	0.54411	0.544125	0.544127	0.544127	0.544127	0.544127
F_5	0.560517	0.560651	0.562103	0.580779	0.580779	0.580779	0.580779	0.580779	0.580779	0.580779	0.580779	0.580779
F_6	0.580779	0.580779	0.580779	0.603758	0.734825	0.734825	0.734825	0.734825	0.734825	0.734825	0.734825	0.734825
F_7	0.734825	0.734825	0.734825	0.734825	0.743174	0.744425	0.744525	0.744535	0.744536	0.744536	0.744536	0.744536
F_8	0.749907	0.749931	0.750176	0.75475	1.007562	1.008564	1.008624	1.00863	1.00863	1.00863	1.00863	1.00863
F_9	1.010181	1.010184	1.010219	1.010679	1.186377	1.186377	1.186377	1.186377	1.186377	1.186377	1.186377	1.186377
F_{10}	1.186377	1.186377	1.186377	1.186377	1.303963	1.303963	1.303963	1.303963	1.303963	1.303963	1.303963	1.303963
F_{11}	1.303963	1.303963	1.303963	1.303963	1.447873	1.447879	1.447879	1.447879	1.447879	1.447879	1.447879	1.447879
F_{12}	1.44788	1.44788	1.44788	1.44788	1.487694	1.531558	1.531558	1.531558	1.531558	1.531558	1.531558	1.531558

	K_{l1}											
	100	1000	10000	100000	1000000	10000000	1E+08	1E+09	1E+10	1E+11	1E+12	1E+13
F_1	0.220423	0.223962	0.255773	0.27784	0.27784	0.27784	0.27784	0.27784	0.27784	0.27784	0.27784	0.27784
F_2	0.27784	0.27784	0.27784	0.380043	0.380043	0.380043	0.380043	0.380043	0.380043	0.380043	0.380043	0.380043
F_3	0.380043	0.380043	0.380043	0.39125	0.424143	0.426105	0.42629	0.426308	0.42631	0.42631	0.426311	0.426311
F_4	0.454507	0.454795	0.457946	0.517756	0.544127	0.544127	0.544127	0.544127	0.544127	0.544127	0.544127	0.544127
F_5	0.544127	0.544127	0.544127	0.544127	0.577776	0.580506	0.580752	0.580777	0.580779	0.580779	0.580779	0.580779
F_6	0.604211	0.604371	0.606085	0.641778	0.732418	0.73462	0.734805	0.734823	0.734825	0.734825	0.734825	0.734825
F_7	0.744536	0.744536	0.744536	0.744536	0.744536	0.744536	0.744536	0.744536	0.744536	0.744536	0.744536	0.744536
F_8	0.746396	0.746453	0.747055	0.758794	1.00863	1.00863	1.00863	1.00863	1.00863	1.00863	1.00863	1.00863
F_9	1.00863	1.00863	1.00863	1.00863	1.162844	1.185138	1.186259	1.186365	1.186375	1.186377	1.186377	1.186377
F_{10}	1.207539	1.207572	1.207905	1.211877	1.299891	1.303763	1.303944	1.303961	1.303963	1.303963	1.303963	1.303963
F_{11}	1.307394	1.3074	1.307456	1.30814	1.447879	1.447879	1.447879	1.447879	1.447879	1.447879	1.447879	1.447879
F_{12}	1.447879	1.447879	1.447879	1.447879	1.5131	1.531251	1.531529	1.531555	1.531557	1.531558	1.531558	1.531558

	K_{h2}											
	100	1000	10000	100000	1000000	10000000	1E+08	1E+09	1E+10	1E+11	1E+12	1E+13
F_1	0.082725	0.089552	0.140547	0.27784	0.27784	0.27784	0.27784	0.27784	0.27784	0.27784	0.27784	0.27784
F_2	0.27784	0.27784	0.27784	0.365402	0.380043	0.380043	0.380043	0.380043	0.380043	0.380043	0.380043	0.380043
F_3	0.380043	0.380043	0.380043	0.380043	0.426274	0.426307	0.42631	0.42631	0.426311	0.426311	0.426311	0.426311
F_4	0.426539	0.42654	0.426556	0.427107	0.544127	0.544127	0.544127	0.544127	0.544127	0.544127	0.544127	0.544127
F_5	0.544127	0.544127	0.544127	0.544127	0.580648	0.58077	0.580778	0.580779	0.580779	0.580779	0.580779	0.580779
F_6	0.581111	0.581112	0.581123	0.58129	0.719036	0.733759	0.734722	0.734815	0.734824	0.734825	0.734825	0.734825
F_7	0.744536	0.744536	0.744536	0.744536	0.744536	0.744536	0.744536	0.744536	0.744536	0.744536	0.744536	0.744536
F_8	0.76073	0.760785	0.761345	0.768443	1.00863	1.00863	1.00863	1.00863	1.00863	1.00863	1.00863	1.00863
F_9	1.00863	1.00863	1.00863	1.00863	1.136736	1.186148	1.186356	1.186375	1.186376	1.186377	1.186377	1.186377
F_{10}	1.188243	1.188245	1.18826	1.18843	1.201472	1.302419	1.303825	1.303949	1.303962	1.303963	1.303963	1.303963
F_{11}	1.314981	1.314989	1.315068	1.315928	1.347219	1.447879	1.447879	1.447879	1.447879	1.447879	1.447879	1.447879
F_{12}	1.447879	1.447879	1.447879	1.447879	1.447879	1.53155	1.531557	1.531558	1.531558	1.531558	1.531558	1.531558

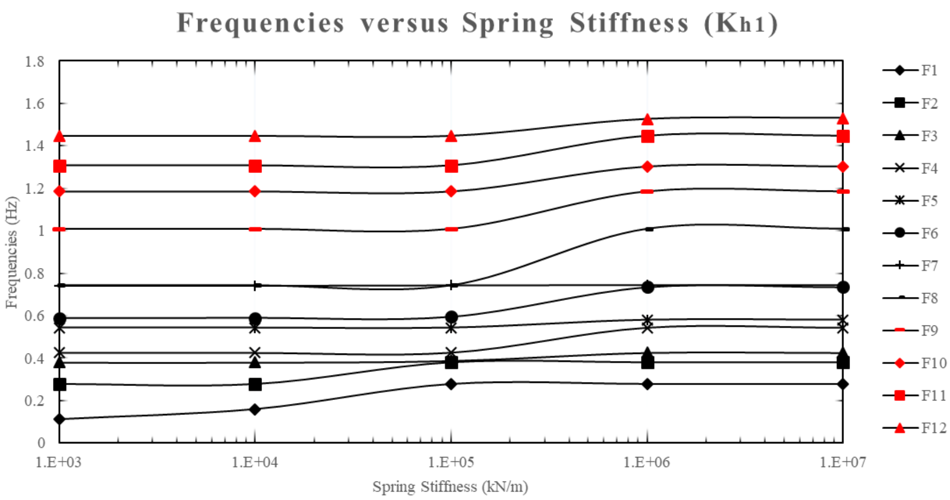
	K_{t2}											
	100	1000	10000	100000	1000000	10000000	1E+08	1E+09	1E+10	1E+11	1E+12	1E+13
F_1	0.0768	0.084677	0.137963	0.254111	0.275854	0.277646	0.277821	0.277838	0.27784	0.27784	0.27784	0.27784
F_2	0.324684	0.325375	0.3329	0.374961	0.379689	0.380009	0.380039	0.380042	0.380043	0.380043	0.380043	0.380043
F_3	0.388468	0.388639	0.390751	0.426311	0.426311	0.426311	0.426311	0.426311	0.426311	0.426311	0.426311	0.426311
F_4	0.426311	0.426311	0.426311	0.491729	0.543159	0.54404	0.544118	0.544126	0.544127	0.544127	0.544127	0.544127
F_5	0.551492	0.551547	0.552146	0.573254	0.580779	0.580779	0.580779	0.580779	0.580779	0.580779	0.580779	0.580779
F_6	0.580779	0.580779	0.580779	0.580779	0.734825	0.734825	0.734825	0.734825	0.734825	0.734825	0.734825	0.734825
F_7	0.734825	0.734825	0.734825	0.734825	0.744361	0.744522	0.744535	0.744536	0.744536	0.744536	0.744536	0.744536
F_8	0.745088	0.74509	0.745112	0.745477	1.002823	1.008303	1.008599	1.008627	1.00863	1.00863	1.00863	1.00863
F_9	1.015184	1.015196	1.015321	1.016883	1.186377	1.186377	1.186377	1.186377	1.186377	1.186377	1.186377	1.186377
F_{10}	1.186377	1.186377	1.186377	1.186377	1.303963	1.303963	1.303963	1.303963	1.303963	1.303963	1.303963	1.303963
F_{11}	1.303963	1.303963	1.303963	1.303963	1.37182	1.447878	1.447879	1.447879	1.447879	1.447879	1.447879	1.447879
F_{12}	1.447888	1.447888	1.447888	1.447888	1.447933	1.531558	1.531558	1.531558	1.531558	1.531558	1.531558	1.531558

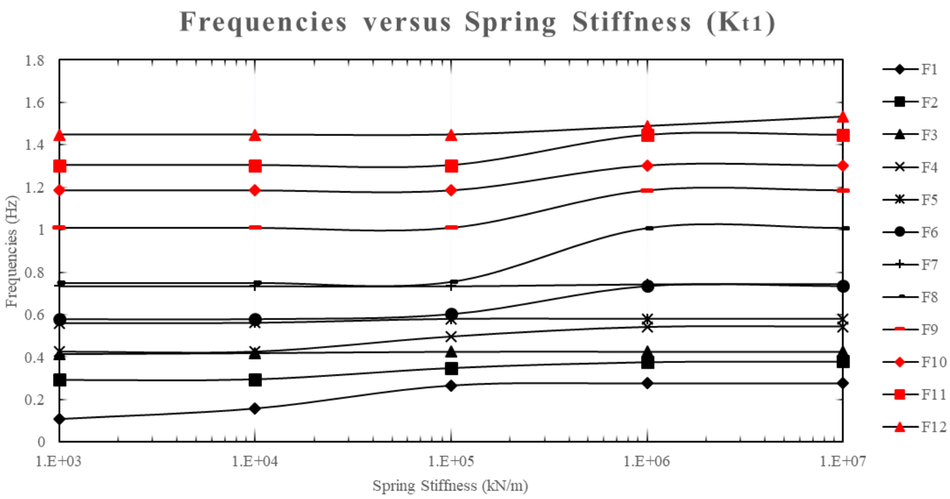
	K_{l2}											
	100	1000	10000	100000	1000000	10000000	1E+08	1E+09	1E+10	1E+11	1E+12	1E+13
F_1	0.176021	0.180338	0.218	0.27784	0.27784	0.27784	0.27784	0.27784	0.27784	0.27784	0.27784	0.27784
F_2	0.27784	0.27784	0.27784	0.378035	0.380043	0.380043	0.380043	0.380043	0.380043	0.380043	0.380043	0.380043
F_3	0.380043	0.380043	0.380043	0.380043	0.423487	0.426045	0.426284	0.426308	0.42631	0.42631	0.426311	0.426311
F_4	0.457582	0.457856	0.460821	0.514527	0.544127	0.544127	0.544127	0.544127	0.544127	0.544127	0.544127	0.544127
F_5	0.544127	0.544127	0.544127	0.544127	0.577542	0.580485	0.58075	0.580776	0.580779	0.580779	0.580779	0.580779
F_6	0.605457	0.605624	0.607406	0.64601	0.734783	0.734822	0.734825	0.734825	0.734825	0.734825	0.734825	0.734825
F_7	0.735008	0.735009	0.735018	0.735217	0.744536	0.744536	0.744536	0.744536	0.744536	0.744536	0.744536	0.744536
F_8	0.744536	0.744536	0.744536	0.744536	1.00863	1.00863	1.00863	1.00863	1.00863	1.00863	1.00863	1.00863
F_9	1.00863	1.00863	1.00863	1.00863	1.164521	1.185391	1.186284	1.186367	1.186376	1.186377	1.186377	1.186377
F_{10}	1.201206	1.201227	1.201437	1.203905	1.299081	1.303782	1.303946	1.303961	1.303963	1.303963	1.303963	1.303963
F_{11}	1.306486	1.306489	1.306523	1.306923	1.447879	1.447879	1.447879	1.447879	1.447879	1.447879	1.447879	1.447879
F_{12}	1.447879	1.447879	1.447879	1.447879	1.466933	1.53089	1.531497	1.531552	1.531557	1.531558	1.531558	1.531558

	K_{h3}											
	100	1000	10000	100000	1000000	10000000	1E+08	1E+09	1E+10	1E+11	1E+12	1E+13
F_1	0.129166	0.136231	0.192898	0.27784	0.27784	0.27784	0.27784	0.27784	0.27784	0.27784	0.27784	0.27784
F_2	0.27784	0.27784	0.27784	0.380043	0.380043	0.380043	0.380043	0.380043	0.380043	0.380043	0.380043	0.380043
F_3	0.380043	0.380043	0.380043	0.42028	0.426175	0.426298	0.426309	0.42631	0.42631	0.426311	0.426311	0.426311
F_4	0.427808	0.427824	0.428009	0.467696	0.544127	0.544127	0.544127	0.544127	0.544127	0.544127	0.544127	0.544127
F_5	0.544127	0.544127	0.544127	0.544127	0.579565	0.580675	0.580769	0.580778	0.580779	0.580779	0.580779	0.580779
F_6	0.587174	0.587209	0.587582	0.596123	0.730915	0.734514	0.734795	0.734822	0.734825	0.734825	0.734825	0.734825
F_7	0.744536	0.744536	0.744536	0.744536	0.744536	0.744536	0.744536	0.744536	0.744536	0.744536	0.744536	0.744536
F_8	0.747672	0.747719	0.74821	0.756074	1.00863	1.00863	1.00863	1.00863	1.00863	1.00863	1.00863	1.00863
F_9	1.00863	1.00863	1.00863	1.00863	1.182165	1.186239	1.186364	1.186375	1.186377	1.186377	1.186377	1.186377
F_{10}	1.188185	1.188187	1.188211	1.188481	1.292187	1.303798	1.303948	1.303961	1.303963	1.303963	1.303963	1.303963
F_{11}	1.305623	1.305625	1.305641	1.305828	1.35317	1.447879	1.447879	1.447879	1.447879	1.447879	1.447879	1.447879
F_{12}	1.447879	1.447879	1.447879	1.447879	1.447879	1.526807	1.531133	1.531516	1.531553	1.531557	1.531558	1.531558

	K_{t3}											
	100	1000	10000	100000	1000000	10000000	1E+08	1E+09	1E+10	1E+11	1E+12	1E+13
F_1	0.126263	0.133505	0.18788	0.274504	0.277598	0.277598	0.277598	0.277598	0.277598	0.277598	0.277598	0.27784
F_2	0.284797	0.284972	0.287278	0.355877	0.37842	0.37842	0.37842	0.37842	0.37842	0.37842	0.37842	0.380043
F_3	0.406033	0.406351	0.409884	0.426311	0.426311	0.426311	0.426311	0.426311	0.426311	0.426311	0.426311	0.426311
F_4	0.426311	0.426311	0.426311	0.47215	0.538167	0.538167	0.538167	0.538167	0.538167	0.538167	0.538167	0.544127
F_5	0.580779	0.580779	0.580779	0.580779	0.580779	0.580779	0.580779	0.580779	0.580779	0.580779	0.580779	0.580779
F_6	0.605606	0.60602	0.610349	0.663821	0.734825	0.734825	0.734825	0.734825	0.734825	0.734825	0.734825	0.734825
F_7	0.734825	0.734825	0.734825	0.734825	0.737406	0.737406	0.737406	0.737406	0.737406	0.737406	0.737406	0.744536
F_8	0.809394	0.809818	0.814285	0.885681	1.00621	1.00621	1.00621	1.00621	1.00621	1.00621	1.00621	1.00863
F_9	1.019995	1.020051	1.020638	1.03229	1.186377	1.186377	1.186377	1.186377	1.186377	1.186377	1.186377	1.186377
F_{10}	1.186377	1.186377	1.186377	1.186377	1.303963	1.303963	1.303963	1.303963	1.303963	1.303963	1.303963	1.303963
F_{11}	1.303963	1.303963	1.303963	1.303963	1.447878	1.447878	1.447878	1.447878	1.447878	1.447878	1.447878	1.447879
F_{12}	1.447881	1.447881	1.447881	1.447881	1.531558	1.531558	1.531558	1.531558	1.531558	1.531558	1.531558	1.531558

	K_{l3}											
	100	1000	10000	100000	1000000	10000000	1E+08	1E+09	1E+10	1E+11	1E+12	1E+13
F_1	0.232566	0.235142	0.258193	0.27784	0.27784	0.27784	0.27784	0.27784	0.27784	0.27784	0.27784	0.27784
F_2	0.27784	0.27784	0.27784	0.360908	0.380043	0.380043	0.380043	0.380043	0.380043	0.380043	0.380043	0.380043
F_3	0.380043	0.380043	0.380043	0.380043	0.419275	0.425617	0.426241	0.426304	0.42631	0.42631	0.426311	0.426311
F_4	0.511052	0.511456	0.515477	0.544127	0.544127	0.544127	0.544127	0.544127	0.544127	0.544127	0.544127	0.544127
F_5	0.544127	0.544127	0.544127	0.546362	0.576719	0.580378	0.580739	0.580775	0.580779	0.580779	0.580779	0.580779
F_6	0.634405	0.634736	0.638122	0.672392	0.726543	0.734006	0.734743	0.734817	0.734824	0.734825	0.734825	0.734825
F_7	0.744536	0.744536	0.744536	0.744536	0.744536	0.744536	0.744536	0.744536	0.744536	0.744536	0.744536	0.744536
F_8	0.833729	0.834335	0.840617	0.921467	1.00863	1.00863	1.00863	1.00863	1.00863	1.00863	1.00863	1.00863
F_9	1.00863	1.00863	1.00863	1.00863	1.173218	1.185447	1.186287	1.186368	1.186376	1.186377	1.186377	1.186377
F_{10}	1.209348	1.209395	1.209864	1.215558	1.283708	1.302259	1.303796	1.303946	1.303961	1.303963	1.303963	1.303963
F_{11}	1.363171	1.363323	1.36488	1.384607	1.447879	1.447879	1.447879	1.447879	1.447879	1.447879	1.447879	1.447879
F_{12}	1.447879	1.447879	1.447879	1.447879	1.524645	1.531075	1.531511	1.531553	1.531557	1.531558	1.531558	1.531558





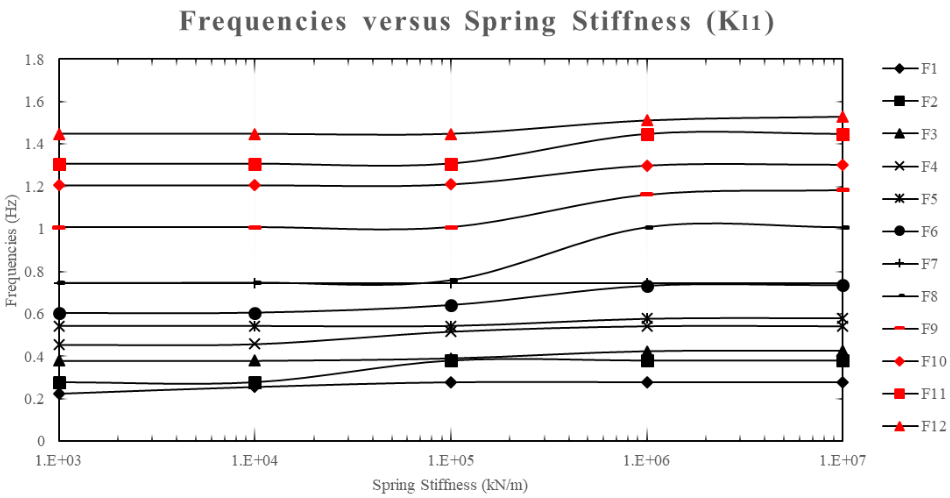


Figure D.1. The change of the natural frequencies based on the spring stiffnesses of the 1st footing. AUTHOR.

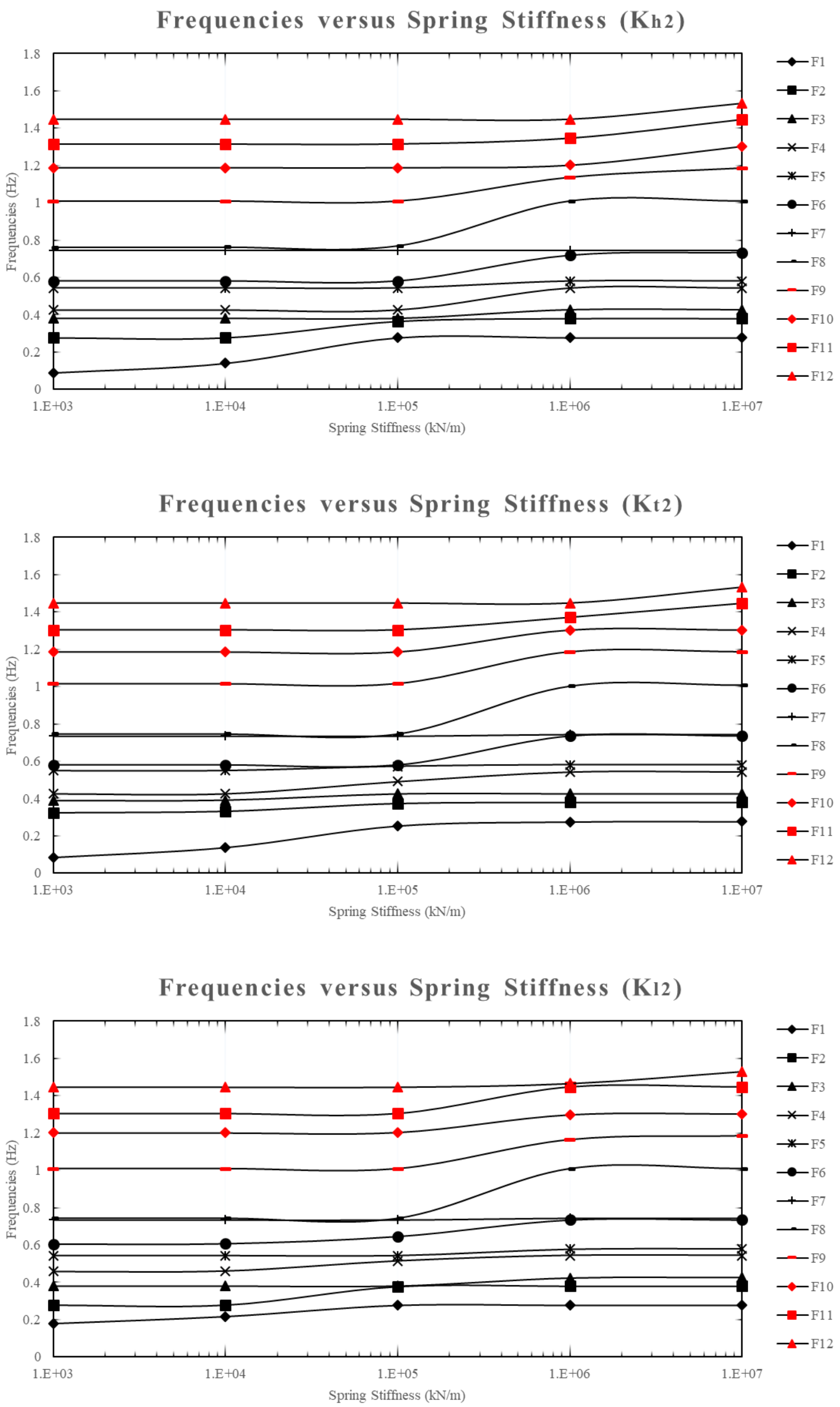


Figure D.2. The change of the natural frequencies based on the spring stiffnesses of the 2nd footing. AUTHOR.

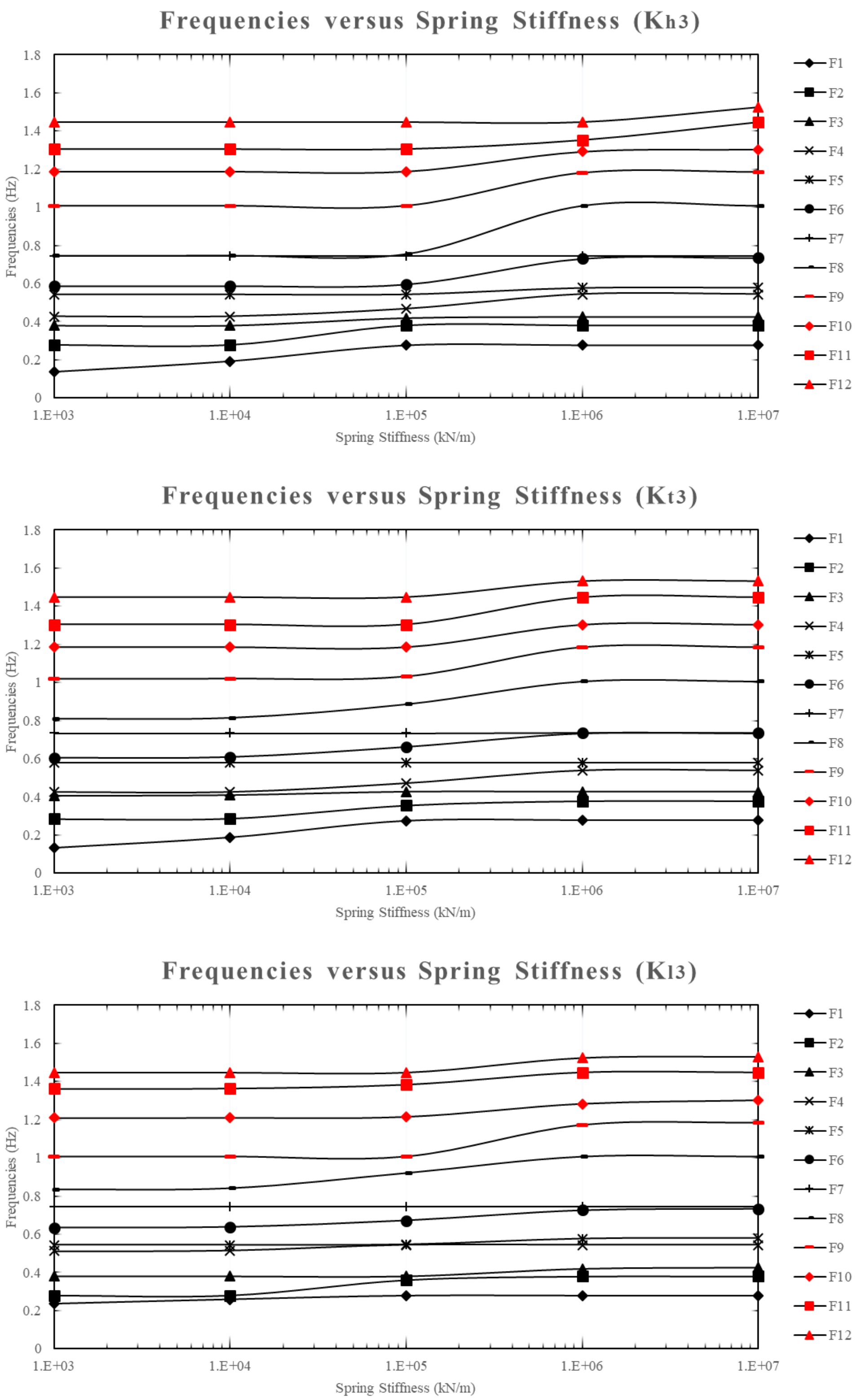


Figure D.3. The change of the natural frequencies based on the spring stiffnesses of the 3rd footing. AUTHOR.

Appendix E: Artificial Neural Network dataset

In this appendix is specified the FE model dataset generated to training the meta-model (Artificial Neural Network – ANN); composed by the FE models developed for the sensitivity analysis and a random FE models varying the critical parameters of the bridge, then in total is 213 scenarios defined.

Input data

FE models	Concrete Class A: E [kN/m2]	Concrete Class A: ρa [kN/m3]	Boundary Conditions: K_{l2} [kN/m]	K_{t2} [kN/m]	K_{h2} [kN/m]	K_{l1} [kN/m]	K_{t1} [kN/m]	K_{h1} [kN/m]	K_{l3} [kN/m]	K_{t3} [kN/m]	K_{h3}
1	22220165	23.5488	552693	586940	571489	344845	177303	553426	880158	543868	884953
2	25072040	21.5575	897329	383293	898602	223330	800517	565249	257091	470106	592088
3	23727006	22.7209	567406	613976	855287	862316	311184	372859	345590	917015	574054
4	21271790	20.9498	586298	105789	589592	846305	352653	896184	752837	716736	815215
5	24441414	20.7370	685399	943854	304350	627483	512866	881394	749278	249521	146899
6	20809983	20.7058	355596	591435	960463	553253	688923	879880	128735	870292	708892
7	21359202	21.3428	429340	671425	178218	806845	836749	581894	524697	185287	756222
8	24111955	23.3216	535038	764456	693858	954092	560920	777296	283869	850716	440735
9	24474533	20.7544	251333	635678	394019	148150	236551	162093	451465	165296	510415
10	23881461	22.4963	508853	942679	123960	392707	908248	144494	860264	712252	182284
11	20691868	21.1909	915214	136810	871977	341310	300498	979403	266190	847078	182158
12	22539775	22.9895	886383	120496	463575	884926	176449	591536	804597	465361	299440
13	23499219	23.3511	614784	709019	203882	655203	571847	518296	531694	612111	246325
14	23177612	23.6841	451030	523136	902009	216850	325776	340331	100210	882468	651912
15	21952693	21.6539	414293	514263	604050	115278	799775	152007	425126	689154	162175
16	21291249	22.4517	504692	438349	408113	710707	910580	150000	220831	503421	799467
17	23084393	23.9639	694772	261921	360589	991359	121704	519559	968780	947889	436932
18	23830072	21.4876	245558	810498	572035	274116	918833	430720	979059	672648	257550
19	21157182	22.3520	551756	444819	320443	772463	509173	434483	880464	782867	329316
20	23570971	23.8766	266020	689114	310014	429834	384346	711024	768229	650583	688433
21	22775932	22.3765	663132	191846	390973	342001	546895	644598	437797	545986	923144
22	21372190	23.9931	950699	497129	640462	478021	294488	539505	640078	243612	543105
23	23951257	22.5727	685979	374263	888612	709514	987344	866909	259076	863403	683922
24	22544697	24.5305	755429	766917	233865	409021	991965	984546	463877	894798	882374
25	21910381	23.1396	248190	683266	712866	342403	610906	788092	360894	631948	858852
26	23759732	21.3465	106543	702224	833932	419864	913022	411099	820445	976999	511260
27	24584274	24.7016	222268	138984	349144	246595	212862	981615	775844	879586	160156
28	21192262	21.7595	265099	780361	805936	600988	442999	634462	958456	714715	141723
29	22955452	20.8531	853763	799381	593523	717462	623045	788312	640190	831031	882508
30	21600410	23.5337	861855	493417	329907	256844	143971	794153	490935	478334	560339
31	23053703	22.1191	964122	322428	690185	938634	160507	906931	193559	565097	470940
32	24486142	20.5700	563845	554780	949221	404036	138461	382879	125217	966043	491840
33	23813967	20.8735	357542	175034	750591	378909	702879	743204	728689	438344	558190
34	24113036	24.5230	103201	734653	256224	662128	415398	760814	593788	955328	175034
35	21365929	24.6520	536051	792894	213744	623642	795036	328985	721309	191174	906648
36	24260847	20.7876	243298	743637	167127	561536	292595	303736	994354	920949	910096
37	25019100	21.6111	785097	626444	798642	128237	351395	631302	451949	166978	631055
38	24210735	22.1956	840251	699987	644617	399518	867918	391318	682600	717744	676760
39	25069678	24.3837	833231	197898	726153	752248	858635	538991	157916	720248	188554
40	25077908	23.5010	380930	567080	642716	418589	401735	719876	910762	902161	245295
41	22516780	23.1303	644379	634313	643739	721987	330050	932604	351385	245794	103849
42	24196844	21.7308	970745	536959	114393	488770	973038	461849	739039	150053	929897
43	21584491	23.4490	779656	732832	434055	677269	834339	823047	327448	275726	521292
44	24605766	22.3712	435618	167610	824576	173640	550049	485784	261141	841821	935910
45	21030434	23.9401	792529	509924	130757	899580	252747	974109	370038	417153	825495

Output data

FE models	Modo 1 F_1	Modo 2 F_2	Modo 3 F_3	Modo 4 F_4	Modo 5 F_5	Modo 6 F_6	Modo 7 F_7	Modo 8 F_8	Modo 9 F_9	Modo 10 F_{10}	Modo 11 F_{11}	Modo 12 F_{12}
1	0.261067	0.350658	0.393652	0.499645	0.537126	0.654164	0.665211	0.745069	0.88235	0.889896	0.956263	0.967963
2	0.289595	0.398387	0.405735	0.560205	0.57569	0.689929	0.774825	0.833572	0.914885	0.946433	1.074197	1.092582
3	0.276279	0.375719	0.403969	0.536881	0.556534	0.672976	0.73813	0.78207	0.898128	1.007834	1.052011	1.069067
4	0.255371	0.366833	0.410472	0.494227	0.562285	0.569856	0.683003	0.72925	0.946657	0.947237	1.02231	1.079501
5	0.293856	0.395085	0.437552	0.546367	0.548085	0.603969	0.661148	0.756718	0.850296	1.038437	1.138199	1.142727
6	0.271945	0.36522	0.372514	0.530716	0.531817	0.655769	0.730136	0.896747	0.962383	0.98056	1.085718	1.133431
7	0.271045	0.364943	0.401892	0.481902	0.501436	0.552759	0.693735	0.741747	0.943822	0.949086	0.988386	1.067775
8	0.276922	0.378038	0.397294	0.538271	0.556031	0.673949	0.741345	0.957087	0.983233	1.002475	1.023415	1.060301
9	0.289993	0.383122	0.40463	0.483672	0.521175	0.57979	0.652798	0.715294	0.725616	0.829054	0.83101	0.890313
10	0.281282	0.384014	0.399384	0.416155	0.462003	0.545161	0.56408	0.655924	0.75005	0.800848	0.997831	1.020815
11	0.255859	0.360606	0.384924	0.50472	0.527124	0.60116	0.619313	0.715938	0.722446	0.885784	0.934258	1.000839
12	0.251618	0.352207	0.40583	0.48793	0.550645	0.570661	0.640095	0.675095	0.753029	0.805967	0.872606	0.942815
13	0.273041	0.372446	0.403491	0.498027	0.528331	0.547717	0.69457	0.72819	0.802689	0.888691	0.986689	1.066825
14	0.26755	0.33897	0.364801	0.514679	0.520808	0.606648	0.717328	0.720795	0.809788	0.893647	0.901439	0.968946
15	0.272926	0.374429	0.37746	0.459241	0.531487	0.53253	0.580287	0.671982	0.732143	0.743943	0.959684	0.970476
16	0.26382	0.36224	0.376349	0.458555	0.511764	0.553217	0.634346	0.705882	0.808147	0.92012	0.981293	1.025094
17	0.257093	0.344865	0.403442	0.493076	0.547926	0.609091	0.610036	0.734741	0.763598	0.778597	0.880513	0.964921
18	0.28661	0.391583	0.413445	0.552674	0.555536	0.665571	0.764448	0.778853	0.821749	0.832964	0.88386	0.989696
19	0.263243	0.361094	0.397755	0.513925	0.537804	0.591706	0.708513	0.747502	0.826378	0.884039	0.923081	1.026603
20	0.269915	0.367152	0.395347	0.522155	0.539615	0.586247	0.719702	0.76966	0.816211	0.952252	0.985053	1.005207
21	0.26586	0.372024	0.400081	0.521565	0.553956	0.632613	0.678873	0.73228	0.805886	0.936486	0.993796	1.013169
22	0.254844	0.343789	0.384477	0.47952	0.526574	0.640516	0.663439	0.853632	0.878879	0.919563	0.929452	1.006414
23	0.277752	0.383951	0.399887	0.544534	0.564594	0.690485	0.751505	0.893063	1.043694	1.053329	1.082919	1.106164
24	0.262394	0.359375	0.384285	0.511086	0.516672	0.535563	0.703189	0.725689	0.952521	0.973376	1.092224	1.162977
25	0.265069	0.36197	0.377471	0.513921	0.523387	0.655082	0.707919	0.791573	0.922139	0.959326	0.99446	1.059409
26	0.286765	0.386806	0.392948	0.519035	0.559454	0.653809	0.70741	0.769598	0.82447	1.021454	1.036311	1.057759
27	0.255941	0.359271	0.387828	0.503778	0.511091	0.567159	0.59677	0.641932	0.705564	0.749264	0.786403	0.795069
28	0.268308	0.365377	0.395785	0.513427	0.520533	0.58264	0.715093	0.716445	0.824637	0.969101	0.973962	1.046301
29	0.28501	0.389079	0.425307	0.55417	0.584056	0.702827	0.762398	0.95711	1.032486	1.088	1.145192	1.234649
30	0.255949	0.34215	0.381866	0.486655	0.527102	0.587269	0.628072	0.719404	0.742905	0.85325	0.929783	1.042248
31	0.269967	0.363926	0.390188	0.51768	0.55643	0.666662	0.670552	0.764863	0.843806	0.963855	0.990989	1.039234
32	0.288543	0.383543	0.386411	0.545704	0.569684	0.668537	0.677987	0.807404	0.809954	0.960061	1.019285	1.043097
33	0.27875	0.392251	0.424654	0.543298	0.579768	0.671414	0.718018	0.76715	0.931407	0.990481	1.044051	1.066923
34	0.269967	0.363593	0.367937	0.490894	0.525067	0.532895	0.593359	0.661971	0.723345	0.785577	0.966028	1.023026
35	0.253998	0.341986	0.379653	0.471048	0.493181	0.519144	0.652854	0.669709	0.748299	0.901675	0.996026	1.037049
36	0.291339	0.394864	0.427693	0.475236	0.564613	0.572405	0.694228	0.772859	0.820176	0.827306	0.897487	1.066905
37	0.288846	0.383978	0.403424	0.521413	0.552635	0.664475	0.72469	0.762706	0.942171	0.984767	1.003107	1.075831
38	0.284072	0.388711	0.420786	0.55164	0.571014	0.6881	0.759577	0.796187	0.954869	1.025671	1.033755	1.176522
39	0.267671	0.376572	0.376847	0.526334	0.528068	0.620981	0.6821	0.707242	0.742181	0.885495	0.941502	1.026385
40	0.279588	0.381746	0.414287	0.544299	0.558752	0.660492	0.750294	0.782508	0.930289	0.965756	0.976565	0.997104
41	0.266796	0.359101	0.390857	0.460042	0.500111	0.569728	0.68734	0.691505	0.903479	0.950489	0.970348	1.047108
42	0.284138	0.379889	0.393556	0.428767	0.511773	0.583398	0.713633	0.75884	0.86777	0.952824	1.054233	1.083411
43	0.261519	0.354748	0.381699	0.495075	0.532053	0.621077	0.683654	0.814279	0.939791	1.021795	1.030304	1.058227
44	0.273312	0.38651	0.388926	0.539739	0.54898	0.654602	0.661954	0.764519	0.769926	0.876715	0.99787	1.052149
45	0.253234	0.342953	0.376368	0.404972	0.485949	0.52455	0.668938	0.695173	0.809039	0.919304	1.027823	1.046358

	Input data											*Output data*											
FE models	Concrete Class A		Boundary Conditions									Modo 1	Modo 2	Modo 3	Modo 4	Modo 5	Modo 6	Modo 7	Modo 8	Modo 9	Modo 10	Modo 11	Modo 12
	E [kN/m2]	ρa [kN/m3]	K_{l2} [kN/m]	K_{t2} [kN/m]	K_{h2} [kN/m]	K_{l1} [kN/m]	K_{t1} [kN/m]	K_{h1} [kN/m]	K_{l3} [kN/m]	K_{t3} [kN/m]	K_{h3}	F_1	F_2	F_3	F_4	F_5	F_6	F_7	F_8	F_9	F_{10}	F_{11}	F_{12}
46	23089052	24.3028	150930	610347	823343	848058	870696	919898	529505	338823	348823	0.265383	0.361562	0.375643	0.506293	0.511712	0.660638	0.675978	0.699452	0.873919	0.95168	1.032493	1.100645
47	22805226	20.5485	394407	801130	516383	689114	591110	228166	686827	575482	867706	0.285837	0.389313	0.423037	0.546137	0.552343	0.630587	0.715093	0.760311	0.920763	0.974017	1.033463	1.195769
48	23182916	22.5180	851519	691916	853592	215510	528778	513743	662867	343591	909930	0.275355	0.373591	0.403122	0.524409	0.550629	0.679463	0.723989	0.802811	0.895529	0.988003	1.086919	1.135112
49	22843570	20.7772	106327	866530	600975	190846	521943	883302	872044	621462	817185	0.284674	0.381753	0.387363	0.5058	0.550432	0.645013	0.694219	0.757578	0.793008	0.974796	1.029009	1.145652
50	24978138	20.6173	767730	319115	479340	744439	829472	979220	778182	565319	748654	0.293826	0.406463	0.4462	0.572661	0.611467	0.706768	0.792094	0.859486	0.90977	1.107689	1.177815	1.197285
51	20958087	20.5092	168129	575323	238418	878521	572944	894998	769310	144867	113585	0.272296	0.363845	0.394209	0.490665	0.493845	0.541091	0.583122	0.685403	0.715741	0.792473	0.922452	1.032572
52	22233680	23.8223	792405	949358	196453	943227	573723	925038	954571	985473	815631	0.2642	0.360448	0.397768	0.487055	0.514132	0.546139	0.707361	0.736088	0.960277	1.081985	1.10719	1.159326
53	24979746	22.1294	952600	645214	319821	596859	950344	598885	532303	310059	727516	0.288029	0.391492	0.426816	0.545556	0.586493	0.612913	0.753742	0.807389	0.959729	1.024658	1.126526	1.187287
54	20872164	21.1760	540309	961991	598584	631990	198225	729127	332006	590979	464094	0.267924	0.36	0.392932	0.513744	0.545165	0.660281	0.691827	0.77192	0.936505	0.987008	1.004684	1.029467
55	24006629	24.7133	194081	660462	424833	330490	476597	395022	604913	281825	828429	0.267708	0.362076	0.382702	0.504814	0.519173	0.624899	0.698879	0.720704	0.76541	0.815771	0.947467	0.954893
56	23372765	23.6964	986547	400424	651089	242573	664078	900620	702860	258736	704377	0.267779	0.365033	0.397875	0.506219	0.546465	0.665235	0.702376	0.825448	0.89425	0.965796	1.013838	1.105712
57	23642594	21.8992	920983	229749	781051	625173	695834	545742	841521	582287	549836	0.275461	0.384072	0.423882	0.539931	0.577191	0.704672	0.731758	0.758021	0.917089	1.043303	1.045771	1.081491
58	23721775	24.6330	921074	174925	243741	847740	778900	697576	531889	655064	289999	0.258211	0.364464	0.397822	0.509384	0.527158	0.544259	0.645868	0.715442	0.717565	0.828328	0.98719	0.993079
59	23126249	23.8736	734779	987832	242809	383430	757131	574617	754772	763833	459682	0.269354	0.367537	0.398643	0.522466	0.52913	0.548262	0.718948	0.736138	0.907501	0.961613	0.978704	1.007079
60	24904234	22.2041	902031	953318	381404	617887	354178	208460	960556	282790	880719	0.286735	0.384978	0.432239	0.529667	0.537289	0.625779	0.663097	0.741323	0.851383	0.950067	1.043317	1.138594
61	23304029	23.6688	660047	582799	974903	981545	344866	355357	223443	174502	799177	0.267651	0.357659	0.383588	0.489306	0.538802	0.649499	0.680361	0.758014	0.905997	0.960076	1.003405	1.049405
62	22115595	22.0286	913128	817232	596240	838916	860306	552046	215274	872597	576928	0.273294	0.373767	0.387468	0.532055	0.547791	0.655352	0.731631	0.905709	0.932982	0.991974	1.022431	1.087503
63	24478170	24.9436	930104	778261	356649	440181	809116	316513	630040	337212	548647	0.270348	0.367176	0.400571	0.51365	0.538953	0.600406	0.688412	0.709737	0.795298	0.973668	1.006113	1.055684
64	20679007	22.4646	186393	264682	907764	396158	539262	194202	425202	136207	999811	0.255533	0.345593	0.372587	0.466041	0.48748	0.562797	0.652768	0.676483	0.72096	0.762042	0.904482	0.97495
65	24912758	22.3761	922127	295840	305108	624315	308586	197192	722560	326928	982334	0.280011	0.382407	0.428095	0.517306	0.534276	0.603808	0.626966	0.739499	0.815651	0.823323	0.905113	1.064264
66	22352023	21.8370	721904	510357	960379	763106	546057	339390	603738	815986	514526	0.273851	0.375239	0.410196	0.534139	0.554955	0.671886	0.736057	0.76398	0.971306	1.047356	1.08499	1.100529
67	21291994	22.7989	816911	130101	429802	941624	911121	497370	419328	826668	444107	0.250866	0.358583	0.389647	0.495784	0.536014	0.586713	0.624497	0.707912	0.810208	0.874599	0.974927	0.975309
68	23828542	23.8268	674650	159918	876246	124084	273931	984382	220383	599652	750246	0.259805	0.362202	0.363639	0.50971	0.520841	0.611841	0.628726	0.718616	0.72698	0.850345	1.001662	1.015452
69	22226241	20.9113	803264	115032	329267	377254	593963	444407	408180	210022	935636	0.261866	0.373039	0.408646	0.491981	0.562919	0.580665	0.616494	0.716959	0.780912	0.854956	0.996976	0.999872
70	23334729	23.9387	253308	221019	600872	651085	532591	617172	839170	978553	228013	0.262301	0.366251	0.394513	0.518516	0.532128	0.634026	0.703253	0.730235	0.743823	0.809156	0.918388	0.954643
71	23111538	24.7273	129521	532533	575456	145519	885953	737630	649566	897892	651725	0.263364	0.358206	0.361931	0.478322	0.514387	0.611929	0.655192	0.708764	0.7184	0.903665	0.942359	1.014679
72	24531365	24.1265	573340	802940	555569	173799	559326	549546	484287	749777	983950	0.274965	0.374997	0.390151	0.533143	0.537456	0.648914	0.734593	0.748527	0.876523	0.9371	0.995817	1.078219
73	22886523	24.4859	778163	438142	401777	911688	416975	994008	878333	119126	876055	0.259607	0.343967	0.397529	0.459842	0.545374	0.622789	0.647667	0.80622	0.861961	0.950292	0.99902	1.040356
74	23512498	23.0738	655377	837010	127765	424997	114208	389178	899961	705307	236143	0.267961	0.352864	0.401295	0.414992	0.500961	0.54824	0.613897	0.674679	0.750599	0.767344	0.823459	1.003339
75	21707195	24.2346	684530	156244	390988	990770	152934	852430	306417	977931	953237	0.24548	0.338261	0.376223	0.481914	0.526774	0.601442	0.604821	0.650011	0.721292	0.784171	0.962575	1.01387
76	20958076	21.8871	975126	567239	480368	560687	985897	743177	614014	338838	197960	0.265724	0.362493	0.397587	0.508759	0.535215	0.617457	0.7016	0.722654	0.870073	0.951871	1.03387	1.072778
77	22503500	20.6224	841241	222746	854446	695646	721187	454150	932952	167055	106552	0.27532	0.377046	0.424003	0.487683	0.509184	0.596826	0.713438	0.731217	0.739182	0.863787	0.982521	1.093547
78	23902906	23.0495	659444	881183	764567	355768	845447	979932	598585	427344	572005	0.277787	0.377944	0.407799	0.532246	0.561823	0.688337	0.733629	0.943261	1.004466	1.032783	1.089662	1.125899
79	24057443	22.6171	704806	105963	409372	644715	477061	261163	181067	767294	272767	0.259563	0.377169	0.387948	0.499724	0.537581	0.580454	0.599162	0.680852	0.750057	0.779666	0.837195	1.017109
80	25056363	22.7999	688009	212724	760787	625123	753642	651036	266262	995913	641166	0.276962	0.388972	0.405658	0.548145	0.572075	0.69268	0.711339	0.768932	0.977784	1.017346	1.058726	1.067855
81	20877794	23.0175	707319	251744	855354	420999	288553	336638	263919	502486	187138	0.253771	0.34896	0.372495	0.494084	0.507586	0.594229	0.68268	0.697273	0.740388	0.748891	0.859597	0.967145
82	22525518	23.806	546176	912756	321711	527502	474367	638787	551383	484298	236239	0.265384	0.36032	0.39159	0.510148	0.532215	0.579623	0.703171	0.705543	0.800147	0.954521	0.959532	1.017954
83	24625934	22.1576	927256	496772	821829	642939	841171	220512	862186	676139	452267	0.285218	0.391722	0.429712	0.539117	0.554915	0.627492	0.736207	0.765345	0.989509	0.999058	1.071086	1.103082
84	20827303	23.716	319139	361178	855637	312277	423632	362638	351004	254401	487483	0.252468	0.343824	0.366886	0.479166	0.505131	0.624707	0.664364	0.745208	0.846907	0.847256	0.909787	0.955804
85	24487152	20.9683	916255	854160	318127	862391	608407	256029	831808	819782	354084	0.293467	0.400266	0.440213	0.566665	0.569841	0.61851	0.672895	0.78417	0.82302	0.943626	1.062347	1.196524
86	21649162	21.9415	206071	940873	136643	911808	711478	123637	288556	631233	242829	0.271008	0.369431	0.380754	0.413235	0.440471	0.524672	0.539415	0.691693	0.721774	0.755365	0.802159	0.98363
87	22449203	24.0691	868494	227101	544445	520448	683215	980655	828859	922097	736778	0.257577	0.35961	0.395157	0.50887	0.542235	0.651366	0.699262	0.723663	0.882875	0.980082	1.039177	1.120451
88	21823244	20.5257	205023	620339	526461	385601	462273	370736	309525	169605	205339	0.27777	0.371896	0.392407	0.508667	0.531037	0.631612	0.704612	0.723296	0.772416	0.795129	0.906742	0.954037
89	21619190	23.9104	154752	492077	320971	613744	602328	676558	473605	304295	466486	0.257938	0.351321	0.367657	0.491661	0.501897	0.579872	0.673492	0.679949	0.750024	0.914724	0.973447	0.982596
90	20975802	23.5949	785085	185684	161309	837222	611259	568857	439268	957517	918724	0.249942	0.350545	0.381095	0.446332	0.495259	0.527192	0.655269	0.694687	0.702115	0.909446	0.954526	1.040891

FE models	Concrete Class A		Boundary Conditions									Modo 1	Modo 2	Modo 3	Modo 4	Modo 5	Modo 6	Modo 7	Modo 8	Modo 9	Modo 10	Modo 11	Modo 12
	Input data											*Output data*											
	E [kN/m2]	ρa [kN/m3]	K_{l2} [kN/m]	K_{t2} [kN/m]	K_{h2} [kN/m]	K_{l1} [kN/m]	K_{t1} [kN/m]	K_{h1} [kN/m]	K_{l3} [kN/m]	K_{t3} [kN/m]	K_{h3}	F_1	F_2	F_3	F_4	F_5	F_6	F_7	F_8	F_9	F_{10}	F_{11}	F_{12}
91	23750404	20.6282	658320	339046	495268	611670	308996	621398	903097	908339	433823	0.285932	0.392036	0.435689	0.559131	0.591978	0.694293	0.768705	0.875172	0.895344	0.917661	0.985973	1.01742
92	25146636	21.2113	397039	335014	359846	260627	391156	632212	751978	966285	705783	0.290412	0.399863	0.428796	0.569382	0.5829	0.647875	0.785447	0.807311	0.87211	0.900801	0.972013	1.004126
93	24201531	23.9614	248535	874505	237841	292636	236083	601226	899954	908265	161486	0.272177	0.367213	0.39539	0.518365	0.524367	0.525603	0.613153	0.715354	0.760626	0.81109	0.813842	0.886396
94	21648877	20.9598	226895	290497	154401	183179	339722	350847	472509	151757	707728	0.269658	0.363328	0.38839	0.454181	0.492581	0.527915	0.687067	0.689857	0.734427	0.781292	0.80629	0.847892
95	23513763	20.4586	624010	752007	506908	443244	447103	593640	636748	439444	529433	0.28991	0.393321	0.429821	0.555781	0.587428	0.694633	0.765739	0.901502	0.969229	1.030508	1.052256	1.08875
96	23894911	21.1019	937876	613392	813576	876038	574510	144575	693081	348972	125999	0.287595	0.390785	0.429994	0.467202	0.51966	0.548089	0.641313	0.756581	0.765836	1.02313	1.084364	1.171312
97	21447322	23.7045	496701	538261	752527	329128	999726	514943	453665	788959	252074	0.259173	0.35595	0.378484	0.505599	0.517223	0.624303	0.696188	0.768832	0.860302	0.905298	0.931592	1.002956
98	24144591	24.5795	700847	707051	620588	911055	873019	975547	806993	199972	570383	0.269582	0.362991	0.405583	0.49795	0.556425	0.671348	0.691415	0.928929	0.94982	1.05291	1.082441	1.106138
99	23267417	24.6861	725941	885478	603339	698720	596237	786581	589730	114220	285785	0.263254	0.345664	0.39396	0.461501	0.538234	0.641075	0.649573	0.798455	0.877204	0.918624	1.001446	1.032385
100	22456073	20.8713	838139	306040	759211	685437	698766	361284	335405	944042	889459	0.277718	0.384631	0.41041	0.546335	0.565688	0.680882	0.754123	0.786826	0.835114	1.041792	1.048933	1.098542
101	22932840	22.7000	1000000	1000000	1000000	1000000	1000000	1000000	1000000	1000000	1000000	0.27784	0.380043	0.426311	0.544127	0.580779	0.734825	0.744536	1.00863	1.186377	1.303963	1.447879	1.531558
102	25226.124	22.7000	1000000	1000000	1000000	1000000	1000000	1000000	1000000	1000000	1000000	0.291378	0.398568	0.447063	0.570646	0.60911	0.770671	0.780842	1.05784	1.244236	1.367581	1.518544	1.60629
103	20639.556	22.7000	1000000	1000000	1000000	1000000	1000000	1000000	1000000	1000000	1000000	0.263603	0.360562	0.404484	0.516239	0.550994	0.697134	0.706359	0.956889	1.125537	1.237072	1.373583	1.452982
104	22932840	22.7000	1000000	1000000	1000000	1000000	1000000	100000	1000000	1000000	1000000	0.27784	0.380043	0.387998	0.427513	0.544127	0.594073	0.743028	0.744536	1.00863	1.186435	1.308487	1.447879
105	22932840	22.7000	1000000	1000000	1000000	1000000	100000	1000000	1000000	1000000	1000000	0.267397	0.349958	0.426311	0.497282	0.580779	0.603758	0.734825	0.75475	1.010679	1.186377	1.303963	1.44788
106	22932840	22.7000	1000000	1000000	1000000	100000	1000000	1000000	1000000	1000000	1000000	0.27784	0.380043	0.39125	0.517756	0.544127	0.641778	0.744536	0.758794	1.00863	1.211877	1.30814	1.447879
107	22932840	22.7000	1000000	1000000	100000	1000000	1000000	1000000	1000000	1000000	1000000	0.27784	0.365402	0.380043	0.427107	0.544127	0.58129	0.744536	0.768443	1.00863	1.18843	1.315928	1.447879
108	22932840	22.7000	1000000	100000	1000000	1000000	1000000	1000000	1000000	1000000	1000000	0.254111	0.374961	0.426311	0.491729	0.573254	0.580779	0.734825	0.745477	1.016883	1.186377	1.303963	1.447888
109	22932840	22.7000	100000	1000000	1000000	1000000	1000000	1000000	1000000	1000000	1000000	0.27784	0.378035	0.380043	0.514527	0.544127	0.64601	0.735217	0.744536	1.00863	1.203905	1.306923	1.447879
110	22932840	22.7000	1000000	1000000	1000000	1000000	1000000	1000000	1000000	1000000	100000	0.27784	0.380043	0.42028	0.467696	0.544127	0.596123	0.744536	0.756074	1.00863	1.188481	1.305828	1.447879
111	22932840	22.7000	1000000	1000000	1000000	1000000	1000000	1000000	1000000	100000	1000000	0.274504	0.355877	0.426311	0.47215	0.580779	0.663821	0.734825	0.885681	1.03229	1.186377	1.303963	1.447881
112	22932840	22.7000	1000000	1000000	1000000	1000000	1000000	1000000	100000	1000000	1000000	0.27784	0.360908	0.380043	0.544127	0.546362	0.672392	0.744536	0.921467	1.00863	1.215558	1.384607	1.447879

Appendix F: MATLAB© code

The used algorithm wrote in the software MATLAB© is presented in this appendix, are used two different codes for each optimization algorithm, however, the backpropagation algorithm is the predefine for optimize the neural network tool in the software; the genetic algorithm is developed by a basic code, using a simple network and optimize the MSE using the GA tool in MATLAB©.

(1) ANN training with backpropagation.

```
% Solve an Input-Output Fitting problem with a Neural Network
% Script generated by Neural Fitting app
% Created 23-Jun-2017 00:23:55
%
% This script assumes these variables are defined:
%
%   Inputs - input data.
%   Targets - target data.

x = Inputs;
t = Targets;

% Choose a Training Function
% For a list of all training functions type: help nntrain
% 'trainlm' is usually fastest.
% 'trainbr' takes longer but may be better for challenging problems.
% 'trainscg' uses less memory. Suitable in low memory situations.
trainFcn = 'trainbr';  %backpropagation.

% Create a Fitting Network
hiddenLayerSize = 10;
net = fitnet(hiddenLayerSize,trainFcn);

% Choose Input and Output Pre/Post-Processing Functions
% For a list of all processing functions type: help nnprocess
net.input.processFcns = {'removeconstantrows','mapminmax'};
net.output.processFcns = {'removeconstantrows','mapminmax'};

% Setup Division of Data for Training, Validation, Testing
% For a list of all data division functions type: help nndivide
net.divideFcn = 'dividerand';  % Divide data randomly
net.divideMode = 'sample';  % Divide up every sample
net.divideParam.trainRatio = 80/100;
net.divideParam.valRatio = 10/100;
net.divideParam.testRatio = 10/100;

% Choose a Performance Function
% For a list of all performance functions type: help nnperformance
net.performFcn = 'mse';
```

```
% Mean Squared Error
% Choose Plot Functions
% For a list of all plot functions type: help nnplot
net.plotFcns = {'plotperform','plottrainstate','ploterrhist', ...
    'plotregression', 'plotfit'};

% Train the Network
[net,tr] = train(net,x,t);

% Test the Network
y = net(x);
e = gsubtract(t,y);
performance = perform(net,t,y)

% Recalculate Training, Validation and Test Performance
trainTargets = t .* tr.trainMask{1};
valTargets = t .* tr.valMask{1};
testTargets = t .* tr.testMask{1};
trainPerformance = perform(net,trainTargets,y)
valPerformance = perform(net,valTargets,y)
testPerformance = perform(net,testTargets,y)

% View the Network
%view(net)

% Plots
% Uncomment these lines to enable various plots.
%figure, plotperform(tr)
%figure, plottrainstate(tr)
%figure, ploterrhist(e)
%figure, plotregression(t,y)
%figure, plotfit(net,x,t)

% Deployment
% Change the (false) values to (true) to enable the following code blocks.
% See the help for each generation function for more information.
if (false)
    % Generate MATLAB function for neural network for application
    % deployment in MATLAB scripts or with MATLAB Compiler and Builder
    % tools, or simply to examine the calculations your trained neural
    % network performs.
    genFunction(net,'myNeuralNetworkFunction');
    y = myNeuralNetworkFunction(x);
end
if (false)
    % Generate a matrix-only MATLAB function for neural network code
    % generation with MATLAB Coder tools.
    genFunction(net,'myNeuralNetworkFunction','MatrixOnly','yes');
    y = myNeuralNetworkFunction(x);
end
if (false)
    % Generate a Simulink diagram for simulation or deployment with.
    % Simulink Coder tools.
    gensim(net);
end
```

(2) ANN training with GA

```
%Create a fitting network
%Hidden layers
H = 2;
% This script assumes these variables are defined:
%   Inputs - input data.
%   Targets - target data.
i=inputs;
t=targets;
net1 = feedforwardnet(H);
s = rng('default');
net1= configure(net1, i, t);
% Vector of weights
x = getwb(net1)';
% objective fuction
m = @(x) mse_test(i,net1,x,t);
%GA optimization
ga_opts=gaoptimset('TolFun',1e-
20,'display','iter','Generations',1000,'PopulationSize',150,'MutationFcn',@
mutationgaussian,'CrossoverFcn',@crossoverscattered,'UseParallel', true);
[x_ga_opt] = ga(m, length(x), ga_opts);
net1 = setwb(net1, x_ga_opt');
% plot of regression
plotregression(t,net1(i))
%scenarios prediction
a=net1(scenarios);
```

```
%mse_test
%objective function of the GA
function mse_calc = mse_test(i,net1,x,t)
net1 = setwb(net1, x');
y=net1(i);
mse_calc = mse( t - y );
end
```