



Universidade do Minho
Escola de Psicologia

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The influence of local reinforcement rate on local response rate under different types of interval schedules with pigeons

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**The influence of local reinforcement rate on
local response rate under different types of
interval schedules with pigeons**

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Trabalho efetuado sob a orientação do
Professor Doutor Marco Vasconcelos
e do
Professor Doutor Armando Machado

novembro de 2020

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STATEMENT OF INTEGRITY

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A influência da taxa de reforço local na taxa de resposta local em diferentes tipos de programas de intervalo com pombos

RESUMO

O timing é tipicamente visto como a capacidade de se comportar de acordo com durações fixas, mas os animais são sensíveis a contingências temporais, mesmo quando estão perante pistas associadas a durações variáveis. Usando diferentes esquemas de intervalo variável (VI), Catania e Reynolds (1968) mostraram que os perfis da taxa local de resposta ao longo do tempo se relacionavam com os perfis da taxa local de reforço. Com base nisso, foi elaborada a concepção comportamental de timing, de acordo com a qual a taxa local de reforço controla a taxa local de resposta. No entanto, dados mais recentes obtidos por Swanton, Gooch e Matell (2009) demonstraram uma função de resposta em forma de pico num procedimento de pico desenvolvido a partir de um programa de intervalo variável (VI) que os autores explicaram como sendo resultado da média de várias memórias temporais, uma interpretação incompatível com a concepção comportamental de timing. Na primeira parte desta tese, explicamos como um modelo comportamental de percepção temporal, o modelo Learning-to-Time (LeT), pode de fato explicar o pico de resposta observado no momento correspondente ao intervalo médio de Swanton et al. (2009). Depois, com uma experiência que estende a manipulação feita por Swanton et al. (2009) mostramos que, consistente com o modelo LeT e com a ideia de que a taxa local de reforço se traduz em taxa local de resposta mas inconsistente com a teoria da média, a largura do pico de resposta nos procedimentos de pico está relacionada com o intervalo de intervalos. Além disso, mostramos que nenhuma das versões da Scalar Expectancy Theory (SET, o principal modelo cognitivo de percepção temporal) pode explicar os nossos resultados. Após este primeiro estudo, apresentamos um novo VI, inspirado nos VIs do procedimento de pico, que alcançam uma taxa constante de reforço num intervalo de tempo finito. Em seguida, no segundo estudo, investigamos a questão da constância de resposta sob VIs de probabilidade constante e os limites dos processos de percepção temporal. Numa experiência comparamos o nosso novo método, o VI exponencial uniforme, ao popular VI de Fleshler & Hoffman e mostramos que, se ambos os VI sustentam uma taxa de resposta aproximadamente constante, o VI exponencial uniforme induz uma função de resposta mais plana se excluirmos a aceleração inicial. Numa outra experiência investigamos a evolução das funções de resposta de pombos treinados em três condições com intervalos máximos diferentes, com o VI exponencial uniforme e observamos que, apenas numa minoria dos casos, os padrões de resposta no tempo apresentaram invariância escalar, sugerindo uma situação limite para processos de percepção temporal ocorrerem. Apesar disso, na discussão geral argumentamos que os processos que traduzem a taxa local de reforço em taxa local de resposta continuam a existir num VI de probabilidade constante mas estão parcialmente ocultos por um efeito de teto do reforço local e / ou por um efeito da resposta motivacional. Concluímos no sentido da validade do princípio da conversão de taxas proposto por Catania e Reynolds para dar conta do padrão de respostas nos programas intervalo no geral e, portanto, no sentido da validade da concepção comportamental de timing.

Palavras-chave: teoria de averaging; modelo Learning-to-Time; timing; uniform exponential VI; esquema de intervalo variável.

The influence of local reinforcement rate on local response rate under different types of interval schedules with pigeons.

ABSTRACT

Timing is commonly seen as the ability to behave in accordance with fixed durations, but animals are sensitive to temporal contingencies even when facing cues associated to variable durations. Using different variable interval (VI) schedules, Catania and Reynolds (1968) showed that profiles of local rate of responding in time related to profiles of local rate of reinforcement in time. On this basis was elaborated the behavioral conception of timing according to which local rate of reinforcement controls local rate of responding. However, more recent data obtained by Swanton, Gooch and Matell (2009) came to show a peak-shaped response function under a peak-procedure made of a VI schedule, which the authors have explained by the averaging of temporal memories, an interpretation incompatible with the behavioral conception of timing. In the first part of this thesis we explain how a behavioral model of timing, the Learning to Time (LeT) model, can in fact account for the peak of responding observed at the time corresponding to the mean interval in Swanton et al. (2009). Then, with an experiment stretching Swanton et al. (2009) manipulation we show that, consistently with the LeT model and the idea that local rate of reinforcement translates into local rate of responding, but at odd with the averaging theory, the width of the peak of responding under peak-procedures relates to the range of the intervals. Moreover, we show that none of the versions of the Scalar Expectancy Theory (the leading cognitive model of timing), can account for our results. After this first study we present a new VI, inspired by the peak-procedure VIs, which achieves constant rate of reinforcement in a finite time range. Then, in a second study we investigate the question of the constancy of responding under constant-probability VIs and the limits of timing processes. In an experiment we compare our new method, the uniform exponential VI, to the popular Fleshler & Hoffman VI and show that if both VI sustain roughly constant response rate, the uniform exponential VI induces a flatter response function when excluding initial acceleration. In another experiment we investigate the evolution of the response functions of pigeons trained across three different maximum interval conditions with the uniform exponential VI and observe that only in a minority of cases patterns of responding in time presented scalar invariance, suggesting a limit situation for timing processes to happen. Though, in a general discussion we argue that the processes translating local rate of reinforcement into local rate of responding still exist under constant-probability VI but that there are partially hidden by a ceiling effect of local reinforcement and/or by an effect of motivational responding. We conclude in the sense of the validity of the rate translation principle brought by Catania and Reynolds to account for responding in interval schedules in general, and thus, in the sense of the validity of the behavioral conception of timing.

Key-words: averaging theory; Learning-to-Time model; timing; uniform exponential VI; variable interval schedule.

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Acronyms

ANOVA: analysis of variance

FI: fixed interval

LeT: Learning to Time

Pdf: probability density function

Peak-FI: peak-procedure FI

Peak-VI: peak-procedure VI

IRT: inter-response time

ITI: inter-trial interval

SET: Scalar Expectancy Theory

VI: variable interval

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GENERAL INTRODUCTION

Timing is commonly defined as the ability of animals to behave in accordance with temporal regularities in the environment but there is no consensus about the processes involved. There are two main psychological conceptions of timing. One relates to the cognitive approach and sees timing as the action to time events, dependent upon operations on temporal memories, the other relates to the behavioral approach and sees timing as the control of behavior by temporal contingencies through associative mechanisms.

In a fixed interval schedule (FI), the simplest timing task, animals are reinforced for the first response emitted after a given temporal criterion from trial onset; this procedure elicits no responding or low responding on the early segment of the trial and high responding near the time of the availability of reinforcement. From the cognitive perspective represented by the Scalar Expectancy Theory (SET; Gibbon, 1977, 1991), animals possess an internal clock with a pacemaker that emits pulses as the trial interval elapses. These pulses amass in an accumulator until reinforcement, at which time the count in the accumulator is saved in memory. Once reinforcement in time has been experienced, animals retrieve at the beginning of an FI trial, a memory of pulses to food, which is compared to the current count of pulses. And, animals respond whenever a comparator module finds a given degree of similarity between the memory and the current count. At early times into the trial interval, the count of pulses in the accumulator is significantly lower than the count in memory and no responding occur, but as time elapses, the count of pulses in the accumulator gets closer to the count in memory and responding is emitted.

From the behavioral perspective represented by the Learning to Time (LeT; Machado, Malheiro and Erlhagen, 2009) model,¹ animals traverse time from behavioral state to behavioral state, which offers a basis for associations to structure in time. At the onset of an FI trial, a series of behavioral states is triggered, and, when reinforcement is delivered, the association between the operant response and the current behavioral state is strengthened while the association between the operant response and all the states that were activated at earlier times decreases. Responding during subsequent trials is determined by the strength of association that the behavioral states have with the operant response.

¹ The model described in Machado, Malheiro and Erlhagen (2009) is the second version of LeT. We do not consider in this thesis the original LeT from Machado 1997.

Behavioral states of the series that are activated early in the trial acquire weak associations with the operant response, and thus, in turn, sustain little or no responding on the early part of the trial interval; behavioral states of the series that are activated late in the trial acquire strong associations with the operant response, and thus, in turn, sustain responding on the later part of the trial interval.

Timing expresses with one major feature which is that the dispersion of the responses around the criterion time is proportional to that criterion time. In the FI, this scalar property, appears in the fact that the response function obtained with an animal trained under a given criterion time will superimpose with the response function obtained with the same animal trained under a different criterion time, when the two functions are plotted on normalized axis.² SET generates the scalar property from the pacemaker module that produces pulses at a rate that is variable between trials but constant within trials. These two elements make that repeated exposition to the same FI interval generates memories of counts of pulses that form a distribution with standard deviation proportional to the interval being timed and also make that responding, in turn, form a distribution with standard deviation proportional to the FI interval. Similarly, LeT obtains the proportionality to the interval being timed, of the diffusion of the strength of associations and, in turn, of the variance of the response distribution, from the fact that the speed with which activation proceeds across behavioral states is variable between trials but constant within trials.

Historically, the FI is a procedure that comes from the behaviorists and the pattern of responding caused by the FI was in a first place interpreted with the concepts of the behaviorists. The shape of the response function, with a mode at the time of the temporal criterion and with a response magnitude ordered with the distance to the temporal criterion, was first conceived as a gradient of temporal generalization (Dews, 1962; Catania and Reynolds, 1968). It is only later that Gibbon (1977) made the demonstration of the scalar property and proposed the information-processing theory SET with a mathematical construct to account for this property of what got called timing. Then followed the reply of the defenders of the behavioral conception, the behavioral model of timing of Killeen and Fetterman (1988), with an alternative mathematical account of timing and its scalar property, which inspired LeT.

Premises of the discovery of the scalar property were in fact already present in Catania and Reynolds (1968), noting from an experiment with a two values FI (or mixed FI) that the distribution of

² We call response function the function relating the response rate to the time into the trial interval obtained from the average of several trials and sessions.

the responses around each temporal criterion was large relative to the time since reinforcement. In this important paper made of several experiments investigating response rate maintained by different interval schedules of reinforcement, including variable interval (VI) schedules of reinforcement, Catania and Reynolds pointed to what could be the other major feature associated with the timing phenomenon. Comparing VIs made of different distributions of intervals yielding different profiles of rate of reinforcement in time, they discovered a relation which they described as the control of local rate of responding by the local rate of reinforcement.

This relation between local rate of reinforcement and local rate of responding is easy to conceive when taking responding under an FI as a gradient of temporal generalization around a point in time where reinforcement occurs, which gradient will naturally extend and adjust to the regularities of the rate of reinforcement if reinforcement is made variable in time like in a VI. On the other hand, this relation is more difficult to conceive from the cognitive perspective that comes with the term timing which from common usage bears the meaning of an action done accurately in regard to a fixed duration. Thus, as the cognitive conception prevailed in the literature since the seventies, what could be an important feature of the so-called timing phenomenon has been rather neglected and has not been considered in the design of SET. Yet, recent experimental results seem to contradict the findings of Catania and Reynolds (1968).

Comparing rats trained under peak-procedures,³ made of an FI or made of a VI, but with mean interval corresponding to the FI value, Matell, Kim and Hartshorne (2014), found two near similar response patterns characterized by a peak at the time of the FI criterion. The authors interpreted these results using the averaging theory, an alternative version of SET in which the memory of the time to reinforcement is not be maintained as a distribution but as an average. From the averaging theory viewpoint, animals in the two conditions of Matell et al. (2014) would have form the same memory, corresponding to the FI temporal criterion, and therefore would have behave in the same manner.

It is critical to determine whether the interpretation of Matell, Kim and Hartshorne (2014) is correct. If it is, if animals operate averaging of temporal memories, it contradicts the idea that local rate of reinforcement controls local rate of responding because the manipulation of the profile of local rate of reinforcement would not cause systematic changes in the response function. If this feature is

³ The peak-procedure (Catania, 1970; Roberts 1981), consists in alternating FI (or VI) reinforced trials, with long unreinforced trials that last typically four times the FI criterion (or four times the mean of the VI intervals). This procedure was created to observe the expectation of the animals after the temporal criterion, which is not possible under a traditional FI wherein the first response after the temporal criterion ends the trial.

rejected, it would mean that the behavioral conception of timing which incorporates it, is probably wrong. On the other hand, if the feature is real, it is the cognitivist conception of timing, which does not incorporate it, that may be wrong.

In Study 1 we examine how LeT, whose mechanism translates local rate of reinforcement into local rate of responding, could in fact account for the apparent averaging of temporal memory phenomenon. We show how the surprising peak shape of the response function obtained by Matell, Kim and Hartshorne (2014) in their peak-VI condition can be explained by the dispersal of associative strength, compensating the attribution of associative strength to a large pool of behavioral states. Then, we replicate the experiment of Matell, Kim and Hartshorne (2014), including conditions with larger interval ranges. While the tenants of the averaging theory should expect that this manipulation would not modify the shape of the response function, we predict that the response pattern should systematically broaden and narrow with the increase and the decrease of the range of the intervals.

After this first study we present a new VI. Inspired by the peak-procedures associated to VIs used in Study 1, our method alternates reinforced and unreinforced trials to obtain constant reinforcement rate on a finite range from an overall uniform distribution of intervals. The uniform exponential VI, as it sustains constant reinforcement rate on a finite range of intervals without the need for truncation or approximation may constitute a better control condition for temporal contingencies compared to the constant-probability VIs based on approximations of the exponential distribution.

Having presented this procedure that allows to suppress temporal contingencies, we come to question how animals behave in such condition. From a theory-free reasoning, if there is no clue in time about the likelihood of reinforcement, animals should respond at a constant rate, or, if they do not, it could only be because of issues related to responding in time in itself (e.g., a warm up effects causing an acceleration at the beginning of the trial). On the other hand, from the principles of Catania & Reynolds (1968), and from our own conclusion in Study 1, we should expect that the process translating local reinforcement rate into local response rate, would still be working. And, because this translation process is imperfect, with a diffusion of the effect of the rate of reinforcement that is proportional to the time within the interval, we should expect a distortion of the constancy of the response function that would be scalar invariant.

In Study 2, we investigate responding under constant-probability VIs. This class of VI is often thought to sustain constant rate of responding, but, closer examination of the few studies which have

reported data shows some discordance. If all studies report roughly constant response functions, some find a slight acceleration (Catania and Reynolds, 1968; Harzem, Lowe and Priddle-Higson, 1978) where other find a slight deceleration (Leslie, 1981; Church and Lacourse, 2001). In a first experiment of Study 2 we compare the uniform exponential VI to the popular Fleshler & Hoffman with the idea that if imperfections in the constancy of the response rate in constant-probability VIs relate to the imperfection of the method, the uniform exponential VI should sustain a flatter response rate than the Fleshler & Hoffman VI. In a second experiment we investigate whether the timing processes, or the processes that translate local rate of reinforcement into local rate of responding are still working by looking for scalar invariance in the shape of the response functions of pigeons exposed to different maximum interval conditions of the uniform exponential VI.

STUDY 1: Local response rate under fixed and variable interval peak-procedures

Introduction

The study of timing with animals started with the invention of the FI schedule and the observation that the duration of the post-reinforcement pause depended on the temporal criterion for reinforcement (Ferster and Skinner, 1957; Felton and Lyon, 1966), showing adjustment of behavior to time itself. Investigations on timing have been mostly concerned with the study of FI schedules, modified versions of the FI such as the peak-procedure, or other tasks based on fixed temporal relationships such as the bisection task. Yet, from the early work on schedules of reinforcement by Ferster and Skinner (1957) it was seen that, even when the reinforcement criterion in an interval schedule was made variable, there were signs of the influence of temporal contingencies. Indeed, when in VI schedules, more intervals associated with short temporal criteria of reinforcement were added, responding increased at earlier times within the trial interval, suggesting a relationship between responding and reinforcement dependent upon temporal information. It is possible that the use of the term timing, which from common sense suggests the idea of an action done with precision in regard to a fixed duration, came to influence the way researchers conceived their object of study and thus came to orient research, though, the field of timing should certainly include the study of behavioral phenomena related to variable temporal contingencies.

In line with the work of Ferster and Skinner (1957), Catania and Reynolds (1968) pursued the investigations on responding under interval schedules with pigeons. In order to study more precisely the relation between the allocation of responses in time and the allocation of reinforcers in time, they introduced the concept of local rates, shifting the analyzes from the perspective of responding across entire sessions to the perspective of responding across the trial interval (we will come back to this methodological evolution in the introduction of Study 2). They defined the local rate of responding, considered under a certain time segment into the trial interval, as the number of responses emitted in the segment, divided by the time spent in that segment (that is, the duration of the segment itself multiplied by the number of times VI intervals lasted at least as long as the latest time point defining the segment). Likewise, they defined the local rate of reinforcement, as a ratio between the number of

reinforcers on a certain segment and the time spent under that segment. The local rate of reinforcement is determined by the distribution of intervals used to build the schedule.⁴

Catania and Reynolds (1968) compared the profiles of local rate of responding obtained with schedules presenting different profiles of local rate of reinforcement. They observed an increasing rate of responding in VI schedules constituted of uniform distributions of intervals, wherein the local rate of reinforcement increased as time elapsed into the interval (see explanation for the profile of local rate of reinforcement under VIs made of uniform distributions pages 24-25). In VI schedules using approximations of exponential distributions presenting roughly constant rate of reinforcement in time (see explanation for the profile of local rate of reinforcement under VIs made of exponential distributions pages 24-25), they observed a roughly constant rate of responding. From this correspondence between the pattern of responding in time and the profile of reinforcement in time they concluded that the local rate of responding was under the control of the local rate of reinforcement. Regarding the results under the FI schedule, where responding is maximal around the time criterion but ramps in the last third of the interval, they concluded that the effect of local probability of reinforcement spreads to adjacent times. And, in regard to the results that they obtained under mixed-FIs, wherein the concentration of the responses around each criterion was proportional to the length of the interval criterion, they concluded that this spread of the effect of the local probability of reinforcement was proportional to the length of the interval to food. Although devoid of explanations in terms of processes, this study of Catania and Reynolds (1968) had the great interest of establishing relationships between temporal variables (i.e. local rate of reinforcement, and time elapsed in the interval) and local responding that were consistent across FIs, VIs and their variants. But later studies seemed to contradict these findings.

Matell, Kim, and Hartshorne (2014) compared two groups of rats trained in peak-procedures with the same mean reinforced interval (30s) but associated to different schedules. One group of rats was exposed to a 30s FI (peak-FI30) and the other group of rats to a VI with a uniform distribution of intervals ranging from 15 to 45s (peak-VI15/45). Figure 1 presents the response rate in function of time obtained with the two conditions. As typically observed during the long empty trials of the peak-procedure, the rats trained in the peak-FI30 showed an increasing rate of responding until about the time of the temporal criterion, followed by a more or less symmetrical decrease. At issue but consistent

⁴ The profile of local rate of reinforcement should not be mistaken with the shape of the distribution of the intervals of the VI. The profile of local rate of reinforcement follows the hazard function of that distribution. In mathematics, the hazard function gives the instantaneous risk that the event of interest happens.

with observations of Church, Lacourse, and Crystal (1998), the rats trained in the peak-VI15/45 showed a near similar pattern. From these results it seems that the local rate of reinforcement would not be the variable controlling the pattern of responding since two different profiles of local rate of reinforcement (all reinforcement opportunities at end of the same interval in one case, and the opportunities spread along a range of intervals in the other) had produced close forms of response functions.

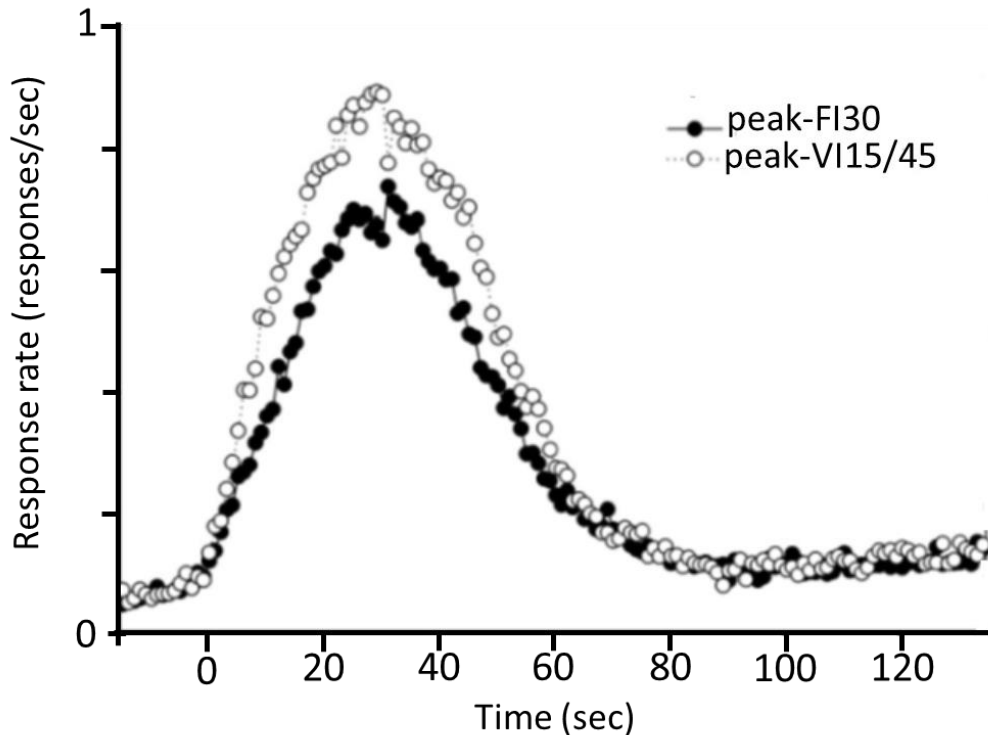


Figure 1: Response rate in function of time under peak-FI30 and peak-VI15/45, replicated from Matell, Kim, and Hartshorne (2014).

Matell, Kim, and Hartshorne (2014) interpreted their result using the framework of the averaging theory. This theory relies on the principles of SET, except that it assumes that, rather than to keep in memory a distribution of samples, animals would retain a unique memory, an average of the experienced intervals to reinforcement. In this view, rats on the peak-VI15/45 schedule would have computed a mean time to food similar to the time to food remembered by the rats of the peak-FI30 schedule, and therefore, would have behaved in the same manner.

But, although in Matell, Kim, and Hartshorne (2014) the patterns in the two conditions were two resembling peaks, the spread of responding was larger in the peak-VI than in the peak-FI, which was also found by Church, Lacourse, and Crystal (1998) comparing rats trained with peak-VI30/60

and peak-FI45. It is possible that these response patterns were indeed sustained by unique, averaged, temporal memories and that differences between peak-FI and peak-VI conditions were due to variations of the threshold for responding (the resemblance that the perception of the current interval has to have with the memory sample in order for responding to be emitted) as hypothesized by Matell et al. (2014). On the other hand, from the perspective of Catania and Reynolds (1968), there is diffusion in the way reinforcement at a particular time affects responding around that time, and this spread of the effect of reinforcement could potentially smooth differences in responding under distinct profiles of reinforcement in time. Hence, the difference that can be found between the patterns of responding obtained with the peak-FIs and peak-VIs conditions might in fact reflect control of the local rate of responding by the local rate of reinforcement. Below we develop this account using LeT (Machado Malheiro, and Erlhagen, 2009) which proposes a mechanism translating local rate of reinforcement into local rate of responding.

LeT postulates an associative structure upon which temporal contingencies come to control behavior. This structure comprises a series of time dependent behavioral states, an operant response, and associative links connecting them. The model assumes that a stimulus signaling an interval to food (a time marker) recurrently triggers a behavioral chain along which behavioral states get activated serially, one after the other. The model functions according to the principles that a behavioral state active concomitantly with reward gets more strongly associated with the operant response while a behavioral state active at any other moment loses association with the operant response, with, in turn, the strength of the association that links a behavioral state to the operant response determining whether a behavioral state will sustain responding or not. Because each behavioral state correlates to a certain placement in time, the rate of reinforcement at each of these times can control responding around these times. The effect of reinforcement or extinction at a certain time diffuses to adjacent times because the speed with which activation jumps from one behavioral state to the next is variable. And, because this speed of the spread of activation is constant within a trial but variable across trials, the biasing effect of low or high speeds increases across the interval, making the diffusion of the rate of reinforcement at time t proportional to t . That is, the model conforms to the relationships pointed out by Catania and Reynolds (1968), the rate translation principle and the scalar property.

The model accounts well for many timing phenomena and in particular for the results of the peak-procedure (Machado et al. 2009). At the beginning of each trial of a peak-FITs, the time marker

triggers the series of behavioral states. In the case of a reinforced trial, the behavioral state that is active around T seconds, when a response produces a reward, sees its association with the response strengthened while all the behavioral states that had been activated previous to that moment see their association with the operant response weakened. During an empty trial more behavioral states of the series get activated and all of them loose association with the operant response. Because the speed with which activation proceeds across behavioral states varies, many behavioral states will have the opportunity of concurrent activation with the operant response while extinction will modulate every link of the vector. If on average the rate of activation is of one state per second, the succession of the trials yields a gradient in which the strength of association is the greatest for the behavioral states surrounding behavioral states $n^\circ T$ and decreases with the distance to these behavioral states. The model assumes a threshold according to which a certain associative strength between a behavioral state and the operant response is needed in order for the behavioral state to sustain responding, which implies that only a pool of strong states of the middle of the series is capable of sustaining responding. Because the speed with which activation spreads varies, the pool of states will be placed at different locations in time, and due to the fact that this variation follows a Gaussian random variable, the responses emitted at the different locations across trials yield a smooth peaked averaged pattern of responding.

The principles of the LeT can be applied to schedules containing variable intervals with no additional hypotheses. In the case of a peak-VI with a range of intervals of mean T_s , the variability of the time to food makes that more behavioral states have the opportunity of concurrent activation with reinforcement than in a peak-FITs. At the same time, this makes that less associative strength is attributed to each of the behavioral states susceptible of concurrent activation with the reward. Thus, counting also on the modulation of the associations by extinction when behavioral states are activated before reinforcement, or activated during empty trials, a peak-VITs generates a pool of strong behavioral states that is broader, but not necessarily much broader than that generated by peak-FITs. Hence, according to the model, a peak-VITs would yield a broader pattern of responding than a peak-FITs, but not necessarily a much broader one.

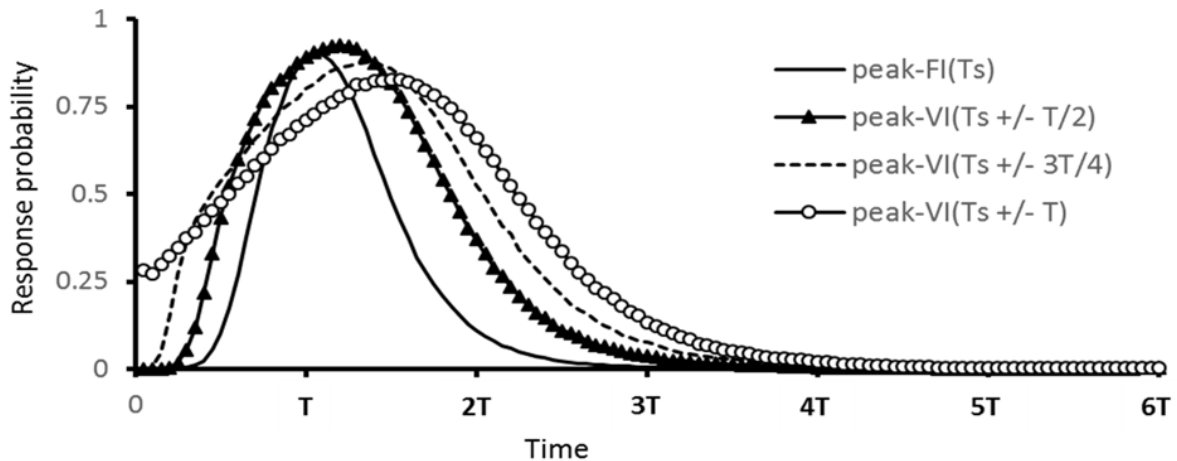


Figure 2: LeT simulations of peak-procedures made of different distributions of intervals sharing the same mean interval of T_s . Time and intervals are expressed in proportion of T .

Figure 2 presents simulations of LeT, using the rules and parameters described in Machado et al. (2009), with peak-procedures associated with FI and with VIs of different ranges, with mean interval T_s (because the spread of the effect of local rates of reinforcement is proportional to the distance to the onset of the interval, the outcomes of the simulations are independent of the absolute duration of the mean interval: i.e. scalar property). Here, the comparison peak-FITs - peak-VITs($\pm T/2$), corresponds to the comparisons peak-FI30 - peak-VI15/45 of Matell et al. (2014). Consistent with the data, the simulation generates a pattern of responding for peak-VI- $T_s(\pm T/2)$ resembling to the pattern obtained with peak-FITs, but broader. That is, the model can stand as an alternative account for the phenomena observed by Matell et al. (2014).

The aim of the following experiment was to confront the accounts of the averaging theory and of the LeT model, and to determine whether the idea that the local rate of reinforcement control the local rate of responding holds, or whether timing entails complex operations on temporal memories. To do so we adapted the peak-FI/VI experiment with pigeons and manipulated the range of the intervals.

Although it is not possible to circumscribe the predictions of the averaging theory since its threshold for responding is not systematized, the basic logic of the theory leads to the expectation that the pattern of the response function should not change much as long as the mean interval is maintained constant. On the other hand, LeT predicts (see in Figure 2 the conditions with different ranges of intervals around the mean T_s) that the pattern will systematically broaden when the range of the interval increases and narrow when the range of the interval decreases. But also, LeT predicts that

the median of the responses will be displaced to later times when the range of the intervals increases and displaced to earlier times when the range of the intervals decreases. This second prediction relates to the use of uniform distributions of intervals, making that the probability of reinforcement increases from the left bound to the right bound of the range of the intervals: as the range increases and the reinforcement probability increases across a longer segment around the mean, more associative strength is given to later behavioral states, which, will sustain responding at later times.

Methods

Subjects

Eight pigeons (*Columba livia*) were maintained at approximately 85% of their free-feeding body weight. The animals were kept in individual home cages, where water and grit were freely available. The pigeon room was maintained in a 13:11-h light/dark cycle, with lights on at 08:00, and its temperature was maintained between 20 and 22 °C. The experiment was conducted once a day, at approximately the same time for each pigeon, 5 days a week. All birds had experience with peak-procedures.

Apparatus

Five LVE (Lehigh Valley Electronics) were used. The LVE chambers measured 34 × 35 × 31 cm (height × length × width). On the response panel, three circular response keys with diameter of 2.5 cm, were arranged horizontally 9 cm apart, center to center, and, vertically 22.5 cm above the wire mesh floor. Below the response panel, a food hopper was accessible through a 6-cm wide × 5-cm high opening centered horizontally and 8.5 cm above the floor. When the hopper was raised to provide grain to the pigeon, a 28-V, 0.04-A light illuminated the opening. On the opposite wall and 30 cm above the floor, a 28-V, 0.1-A houselight provided general illumination.

Procedure

At first, all birds were placed in a peak-procedure associated with an FI (peak-FI20A). Reinforced trials followed a FI20s and were randomly alternated with empty trials (a session comprised

twelve blocks of two trials of each type). A trial started with the simultaneous illumination of the central key and houselight. If the trial was a reinforced one, the first peck after the criterion raised and illuminated the feeder, giving access to a mix of grain; at the same time, the houselight was turned off. Otherwise, the trial ended independently of responding at a variable time between 120s and 160s. Trials were separated by an inter-trial interval (ITI) lasting between 20s and 40s, spent in darkness. Except for the distribution of the reinforced intervals all following conditions were identical.

Table 1: Order of conditions for the two groups of the experiment of Study 1.

Condition rank	Group 30-40	Group 40-30
	P123; P452; P974; P709	P451; PG40; P595; PG37
1	peak-FI20A	peak-FI20A
2	peak-VI10/30	peak-VI0/40
3	peak-VI0/40	peak-VI10/30
4	peak-FI20B	peak-FI20B
5	peak-VI5/35	peak-VI5/35

Table 1 shows the order with which birds went across five conditions. After the initial peak-FI20 condition (peak-FI20A), birds were exposed to peak-procedures associated with VIs of different ranges (corresponding to the ranges of the VIs in the different conditions of Figure 2), all built from uniform distributions with spacing of 1s between intervals and with mean 20s. The ranges were 10s to 30s (peak-VI10/30), 5s to 35s (peak-VI5/35) and 0s to 40s (peak-VI0/40). For the four birds of Group 30-40, peak-VI10/30 was the second condition while peak-VI0/40 was the third, this order was reversed for the four birds of Group 40-30. As a fourth condition all birds were placed again in peak-FI20 (peak-FI20B) before going in peak-VI5/35 as fifth and last condition. Each condition lasted for twenty-five sessions.

Analyses

Responses during empty trials in the last ten sessions of each condition were collected in bins of 1 second. Quartiles were computed from the responses on the interval 0-80s cumulated in the last ten sessions.

Results

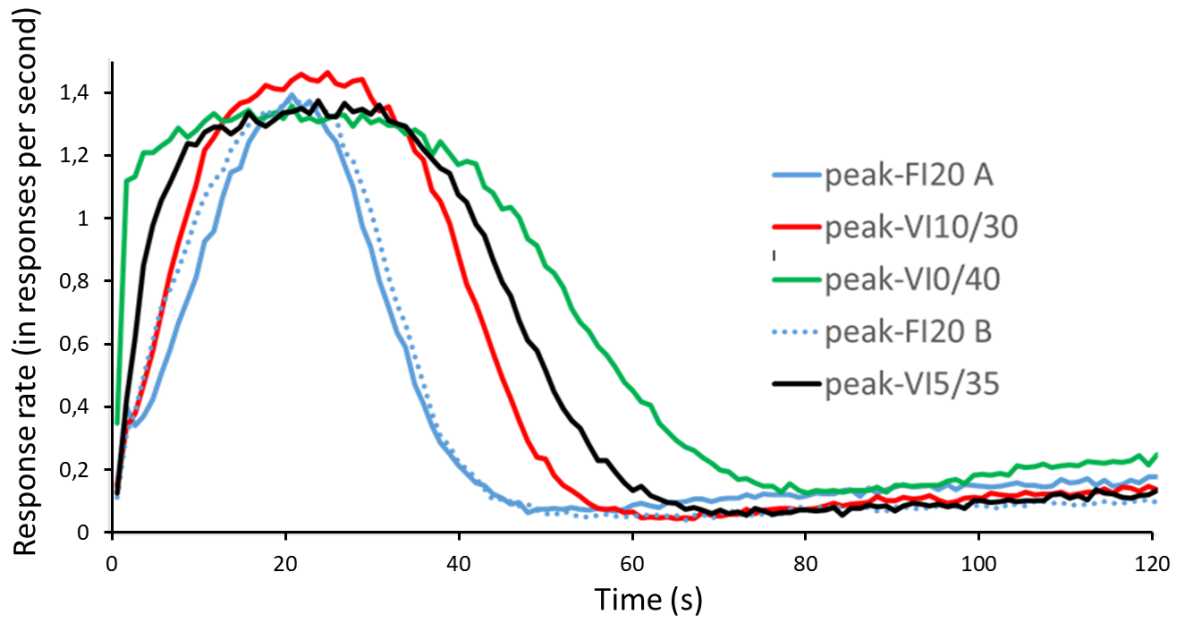


Figure 3: Response rate in function of time into the trial interval averaged across birds in the five conditions of peak-procedure of the experiment of Study 1.

Figure 3 presents the response rate in function of time into the trial interval averaged across birds in the five conditions. From visual inspection we see that, compared to the narrow peaked patterns observed in the peak-FI conditions, the patterns in the peak-VI conditions broadened consistently with the order of the range. Peak-VI10/30 presents a broader peak than peak-FI20, peak-VI5/35 a broader peak than peak-VI10/30 and peak-VI0/40 a stretched peak or plateau-like response function larger than all others. The mass of the responses seems to be pushed to the right as the range of the interval increases.

In order to characterize the broadness of the patterns of responding in time, interquartile ranges were extracted from the quartiles. An ANOVA comparing the five conditions on interquartile range shows a difference between all conditions ($F(4,28)=38.64, p < .001$). And, another ANOVA, run on the medians to assess the displacement of the mass of the responses, shows a difference between all conditions ($F(4,28)=31.02, p < .001$).

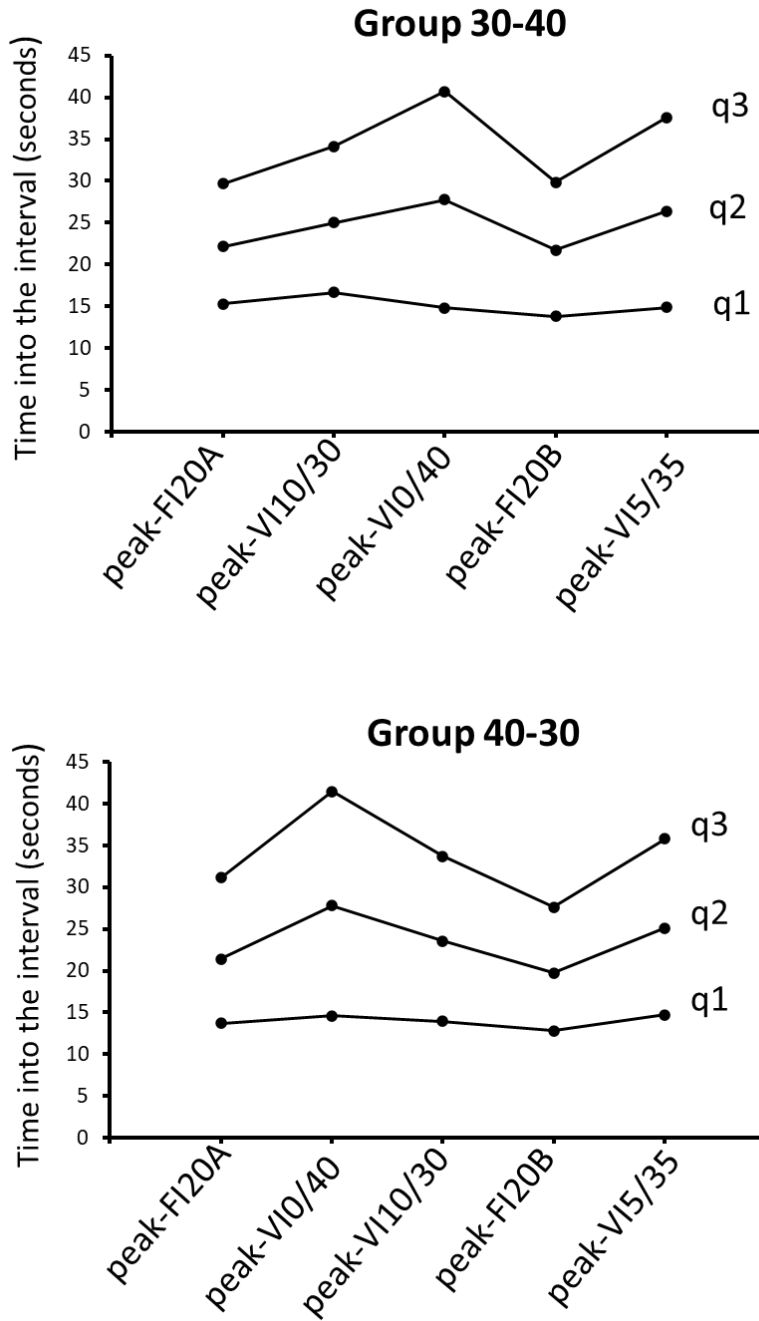


Figure 4: Evolution of the quartiles of the responses across the five conditions in the two groups of the experiment of Study 1. Conditions are ordered from the first one, on the leftmost to the last one on the right most. The top graph gives the quartiles of Group 30-40, the bottom graph, the quartiles of Group 40-30. Each line correspond the quartile indicated next to it, noted q1 for first quartile, q2 for median and q3 for third quartile.

Figure 4 shows the evolution of the position of the quartiles across the five conditions. In Group 30-40, going from peak-FI20A to peak-VI10/30 increased the interquartile range (the distance between q1 and q3) while displaced the median (q2), to a later position; going from peak-VI10/30 to peak-

VI0/40 also increased the interquartile range and displaced the median to a later position; going from peak-VI0/40 to peak-FI20B decreased the interquartile range and displaced the median to an earlier position; going from peak-FI20B to peak-VI5/35 increased the interquartile range and displaced the median to a later position. In Group 40-30, going from peak-FI20A to peak-VI0/40 increased the interquartile range and displaced the median to a later position; going from peak-VI0/40 to peak-VI10/30 decreased the interquartile range and displaced the median to an earlier position; going from peak-VI10/30 to peak-FI20B decreased the interquartile range and displaced the median to an earlier position; going from peak-FI20B to peak-VI5/35 increased the interquartile range and displaced the median to a later position. In every instances, going from a condition with a smaller interval range to a condition with a larger interval range increased the interquartile range and displaced the median to a later position, while, going from a condition with a larger interval range to a condition with a smaller interval range, decreased the interquartile range and displaced the median to an earlier position.

Post-hoc testing following the ANOVA on the interquartile range using the Bonferroni correction rejected significant differences for two pairs of comparisons, peak-FI20A – peak-FI20B ($t(7)=0.46$, ns), and peak-FI20A – peak-VI10/30 ($t(7)=2.72$, $p=0.03$), while finding significant differences for the other eight comparisons (for all, $p < .01$). Although significance of the difference was rejected for the comparison peak-FI20A – peak-VI10/30 by the post-hoc analyses, its p (0.03) was only slightly greater than 0.025 (alpha/2 for the penultimate comparison of the Bonferroni testing), moreover, the post-hoc analyses found a difference for the pair peak-FI20B – peak-VI10/30 ($t(7)=3.47$, $p=0.017$). Hence, with caution, a difference may be admitted between the conditions peak-FI20 and peak-VI10/30. Mean interquartile ranges were 15.90s and 15.43s in the first and second peak-FI20 respectively; it increased with the increase of the range of the interval, it was 18.60s in condition peak-VI10/30, 21.89s in the condition peak-VI5/35 and, 26.35s in the condition peak-VI0/40.

Post-hoc testing on medians using the Bonferroni correction revealed that all pairs of comparisons were significantly different (for all, $p < .01$) except the pair peak-VI10/30 - peak-VI5/35 ($t(7)=1.99$, $p=0.09$) and the pair of the two peak-FI20 conditions ($t(7)=1.65$, $p=0.14$). The median was at its earliest in peak-FI20, at 21.76s in the first one and 20.73s in the second one, it was delayed at 24.30s in peak-VI10/30, later at 25.75s in peak-VI5/35 (but not significantly later than in the preceding condition) and at the latest at 27.75s in peak-VI0/40, altogether with the conclusions on the interquartile range index confirming visual analysis.

Table 2: Quartiles and interquartile ranges obtained from simulation (left table) and data (right table) in the different condition of the experiment of Study 1.

SIMULATIONS					DATA				
	q1	q2	q3	q3-q1		q1	q2	q3	q3-q1
peak-FI20	17.74	22.44	27.94	10.20	peak-FI20A	14.49	21.77	30.39	15.90
					peak-FI20B	13.29	20.73	28.72	15.43
peak-VI10/30	17.59	24.86	32.86	15.27	peak-VI10/30	15.30	24.31	33.90	18.60
peak-VI5/35	17.35	26.50	35.92	18.56	peak-VI5/35	14.81	25.75	36.70	21.89
peak-VI0/40	17.94	28.75	39.53	21.59	peak-VI0/40	14.70	27.75	41.05	26.35

Table 2 presents the quartiles ($q1$, $q2$, $q3$) and the interquartile ranges ($q3-q1$) for all conditions, from the data and from the simulations, showing trends for agreements and disagreements. For all conditions, the interquartile range appears greater in the data than in the simulations. $q1$ appears systematically earlier in the data than in the simulations. Symmetrically, $q3$ appears latter in the data than in the simulations. Medians ($q2$) obtained from the data, show good correspondence with the simulations, although they are always slightly earlier in the data than in the simulations. Importantly, order relations across conditions for the different indices were similar in the data and simulations. That is, the first quartile remained stable across conditions while the median and the third quartile moved to later times – and so, the interquartile range increased – as the range of the intervals increased.

Figure 5 presents the individual response functions in the five conditions, with response rate given in proportion of the rate after 20s into the trial interval (this limitation was used to exclude the early high rate discussed below). Individual functions show homogeneity, with only one case in which the width of the response function does not covary with the range of the intervals (peak-VI10/30 is slightly broader than peak-VI5/35 for PG40). However, individual functions show an intermittent phenomenon of early high rate, displayed by some birds in some conditions (peak-FI20B and peak-VI0/40 for P709; peak-FI20A for P451; peak-VI0/40 for P595).

Overall, the data and the LeT simulations present a certain agreement in regard to the shape of the response functions and order relationships of quartiles indices. But there is a difference between the data and the simulations regarding the broadness of the response functions, systematically overestimated by the simulations. And, there is an unexpected early high rate phenomenon.

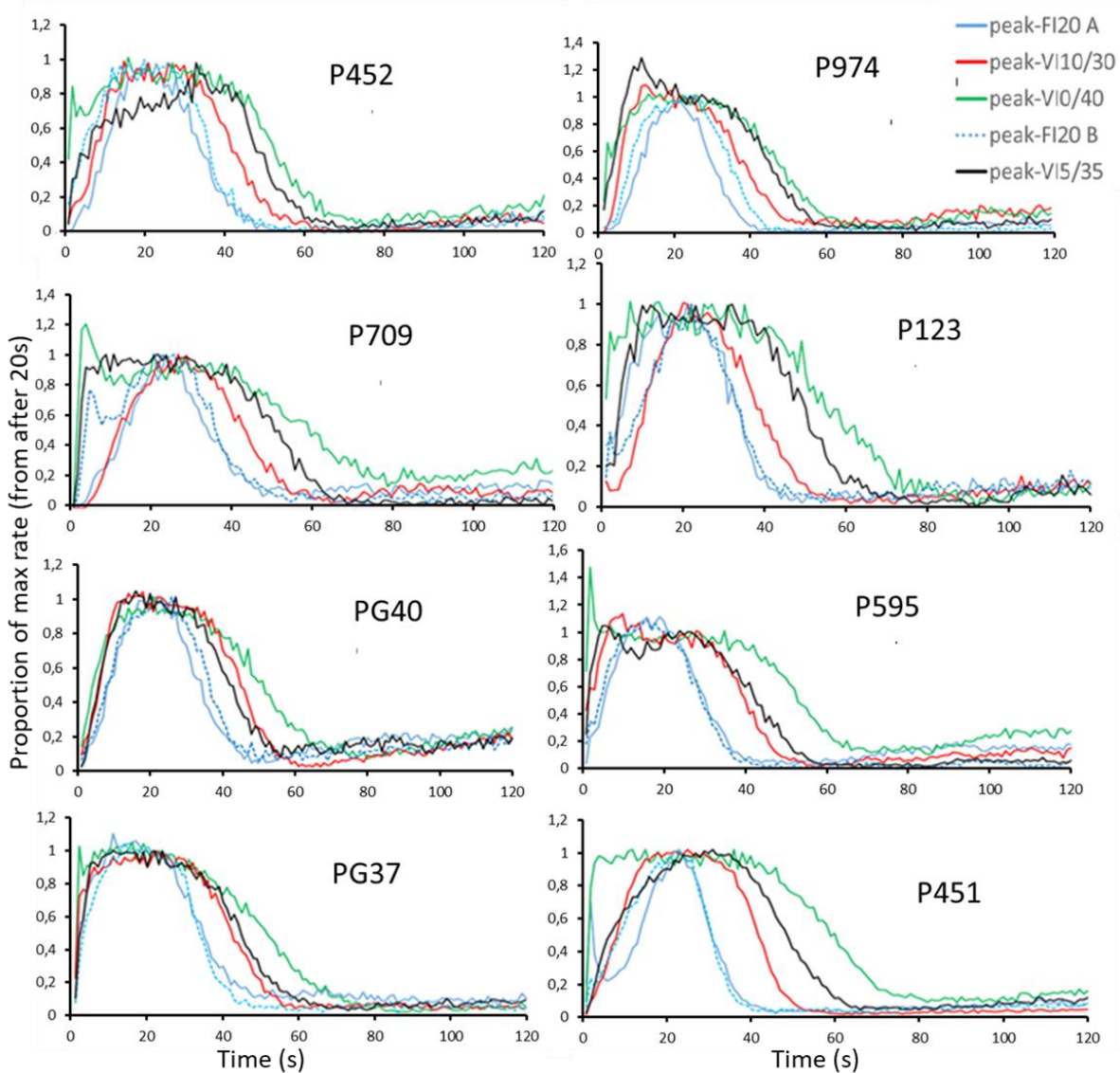


Figure 5: Individual graphs of the response rate in function of time in the five conditions of the experiment of Study 1.

Discussion

The results show that the pattern of responding in peak-procedures depends on the variability of the criterion interval around its mean. Systematically, the increase of the range of the intervals broadened the pattern of the response function, as well as displaced to later times the median of the responses, while, the decrease of the range of the intervals narrowed the response pattern and moved the median of the responses to earlier times.

This could hardly be accounted for by the averaging theory according to which animals should have had the same, single, average memory of the time to food in all the present conditions, and thus, should have behaved in a similar way in all conditions. Though, certain configurations of the averaging model in regard to the threshold for responding may enable it to account for the broadening of the response function if the threshold for responding is made dependent upon the variability of the reinforced interval (Matell, et al., 2014), but, systematic rules have yet to be explicated.

It is noteworthy, however, that our observations may be partially accounted for using the framework of the expectancy theory (SET), to which the averaging theory relates. As discussed in Church, Lacourse, and Crystal (1998) and Brunner, Fairhurst, Stolovitzky, and Gibbon (1997), SET, which was not originally designed to deal with variable time to food may be expected to do so in two different ways. Following the Minimax model (Brunner, Kacelnik, and Gibbon, 1996), or, two samples version of SET, in situation of variable time to food, animals would only maintain two memories, one of the shortest experienced reinforced interval, which would determine the onset of responding, and one of the longest experienced reinforced interval, which would determine the offset of responding. Differently, following the Complete-memory model (Gibbon, Church, Fairhurst, and Kacelnik, 1988), or, single sample version of SET, animals would maintain all reinforced intervals in memory, and responding during a trial would be determined by a sample drawn from the compound memory at the beginning of the trial.⁵ With these two versions of SET, responding will happen at earlier and at later times when the animal is placed under a VI with mean T_s than when it is placed under an FI T_s , because the memories of short and long intervals will diverge in one case (Minimax model) or because the compound memory will broaden in the other (Complete-memory model). Thus, under a peak-VI, the two versions of SET predict the broadening of the pattern of the response function from the increase of range of the intervals.

In both the single sample and the two samples SET, the memories of reinforced intervals are represented as normal distributions with standard deviation proportional to their duration. In the Minimax model, the memory for the long interval can thus be retrieve as a more extreme duration than can be the memory for the short interval, which creates an imbalance of the runs of responses towards longer duration with a magnitude dependent on the range of the intervals experienced. Thus, the

⁵ Next to these sub-theories, that are single sample SET and two samples SET, the averaging theory which is basically a version of SET in which the memory distribution is replaced by an average memory, could have been called average sample SET.

Minimax model predicts the shift of the median of the responses to later times from the increase of the range of the intervals in a peak-VI. In the Complete-memory model, the length of a run of responses in a trial is proportional to the duration of the sample extracted from memory at the beginning of the trial. As the compound memory is skewed because of the summation of the memories of shorter intervals with small standard deviation and of longer intervals with larger standard deviation, there is an imbalance of the runs of responses towards longer duration, which magnitude depends on the range of the intervals experienced. Thus, similarly to the Minimax model, the Complete-memory model predicts the shift of the median of the responses to longer durations from the increase of the range of the intervals in a peak-VI.

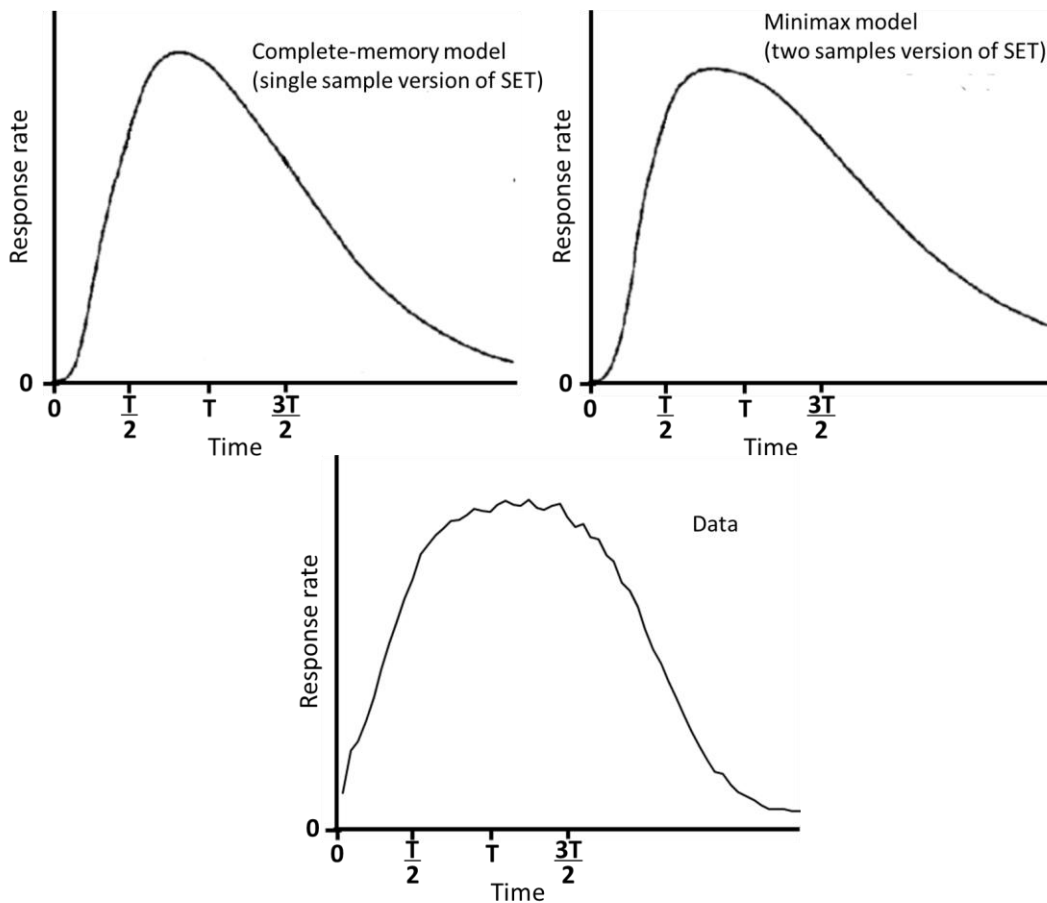


Figure 6: Reproductions of simulations of the complete-memory model and of the minimax model from Brunner, Fairhurst, Stolovitzky, and Gibbon (1997), contrasted with the data of experiment 1. The procedure simulated is a peak-procedure associated to a VI made of intervals from 5 to 15s in the case of the simulation, whereas the procedure of experiment 1 was a peak-procedure associated with a VI made of intervals from 10 to 30s; in absolute time these VIs with mean T s are made of intervals from $T/2$ to $(3T)/2$ s.

Notwithstanding the capacity of the Minimax model and of the Complete-memory model to account for important features of our results, the shape of the response functions that they predict clearly differ from the data. The top graphs of Figure 6 replicate the simulations of the Minimax model and of the Complete memory model found in Brunner, Fairhurst, Stolovitzky, and Gibbon (1997), which correspond to our peak-VI10/30. The response function obtained in the peak-VI10/30 (bottom graph of Figure 6) is bell shaped with an axis of symmetry, at the mode, at a time slightly later than the mean duration T_s . Contrarily, the response functions simulated from the two different versions of SET present a strong positive skew with the mode appearing earlier than T_s . This skew of the response function relates to the skew in the memory distribution. In the Minimax model, there are only two memory distributions with very different standard deviations which translates in its simulation of the response function by an even stronger skew than that of the complete-memory model, which depends on a compound memory containing memories of many intermediate duration intervals.

LeT accounts for the broadening-narrowing and displacement of the median phenomena. Furthermore, the study of the shape of responding by visual inspection and using indices based on quartiles, indicates a good correspondence between the model and the data. However, if order relations for the different indices, in the different conditions, were similar in simulations and data, the patterns of the data show more broadness than expected from the simulations using LeT.

In part, the magnitude of the broadness comes from steady early bursts that affected some birds in some conditions. These early bursts appeared and disappeared abruptly regardless of changes of conditions; therefore, we suspect that the phenomenon is unrelated to temporal contingencies. Yet, most of the broadness is unrelated to the bursts. From our perspective the fact that the broadness was more important in the data than in our simulation would suggest that our current parameters underestimate the diffusion of the effect of local reinforcement, which could be due to an overestimation of the effect of extinction. We replicated the simulations using a smaller extinction parameter.

As can be seen in Figure 7 comparing the novel simulation (with extinction parameter set to 0.3; see Machado et al. 2009 for original settings) with the data, the decrease of the extinction parameter allows a much finer fitting of the data, and it does so for every condition at the same time. The only marked mismatch can be seen on the left limb of the peak-FI20, where the data response functions are more to the left than the one from the simulation. But note that considering the usual

low rate in the early part of the peak-FI20 condition early bursts have had proportionally an important effect.

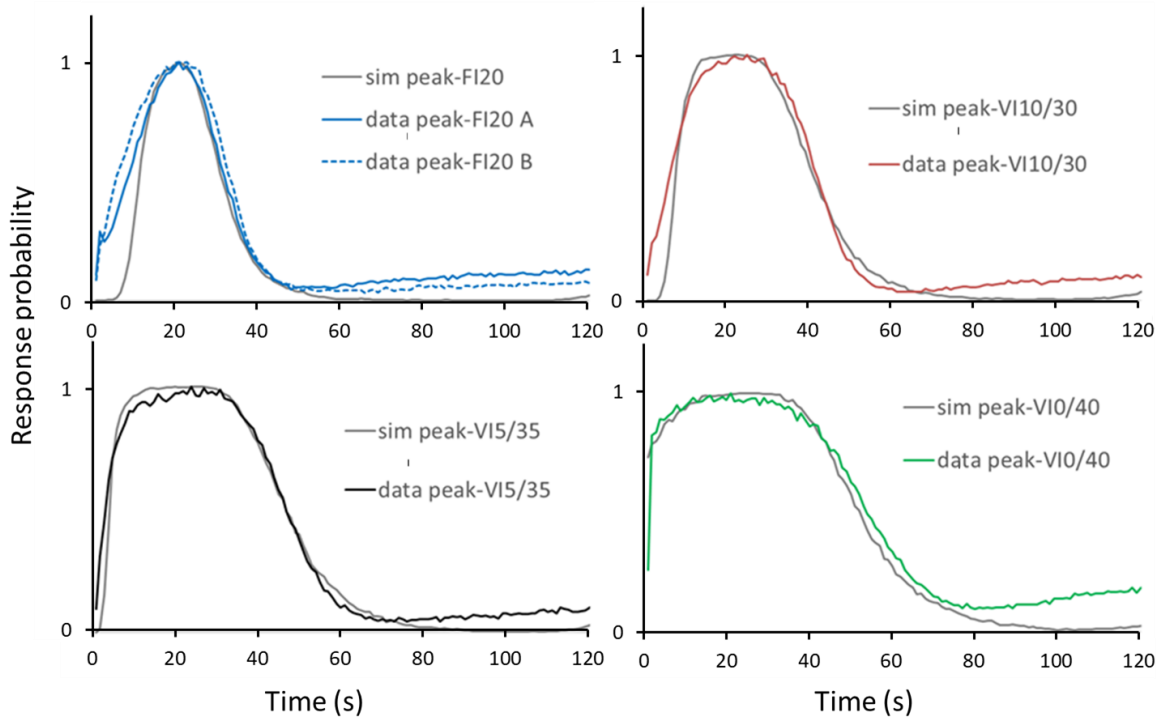


Figure 7: Comparisons simulation-data (response function averaged across birds), for the four different conditions of variability of the experiment of Study 1, with extinction parameter reduced to 0.3.

The best way to account for the distributions of responses induced by peak-procedures associated to an FI, and to VIs of different ranges, was by using the LeT model which translates local rate of reinforcement into local rate of responding from an associative structure. That is, our results support the conclusions of Catania and Reynolds (1968) according to which responding in FIs, VIs, and interval schedules on general, can be apprehended as the control of the local rate of reinforcement on the local rate of responding, and supports the idea that this rate translation is mediated by associative mechanisms.

It sounds odd to talk of timing when describing behavioral phenomena produced by schedules in which the temporal criterion is variable. In fact, the Cambridge Dictionary gives the following definition for timing: the ability to do something at exactly the right time. It does not make sense to do something at exactly the right time in a VI. Thus, the term timing does not suit well for the description of the behavioral phenomena related to variable temporal contingencies, and, this inadequacy may disturb conceptual thinking and hinder the development of research in the line of the work of Catania

and Reynolds (1968). Furthermore, the noun timing goes with the verb *to time* which is an action verb suited to describe a cognitive process performed by an agent, but certainly less suited to describe the control of behavior by the environment through associative processes, whereas, we argue in the favor of the latter conception.

UNIFORM EXPONENTIAL VI

The work on the previous study, concerned with local rate of reinforcement under different peak-VIs, led us to think about the fact that schedules made of reinforced and unreinforced intervals could offer possibilities regarding the design of profiles of local rate of reinforcement not offered by traditional VIs. Consider an experimenter who wants to create a VI with a profile of constant rate of reinforcement (e.g., as a control condition for the influence of temporal contingencies), he has to get the intervals of this VI from a distribution presenting a constant hazard function. But, the only distribution with the desired mathematical property is the exponential distribution, which is necessarily defined on zero to infinity and is as such impossible to implement as an interval structure without alterations (e.g., by truncation, or by using other methods that we will discuss in Study 2). Our interest was to find a combination of reinforced and unreinforced intervals that would allow constant rate of reinforcement within a finite interval range, with no need for truncation or approximation. In what follows we present this search and the method that we found.

In a traditional VI, wherein all intervals are reinforced, the instantaneous rate of reinforcement at any time t into the trial interval corresponds to the ratio of the probability that an interval ends within the thin slice of time from t to $t+\Delta$ and of the probability that an interval lasts longer than t .⁶ The probability that an interval ends within the thin slice of time from t to $t+\Delta$ can be grasped as the relative likelihood that an interval sample will equal t in the distribution of the intervals. That is, the instantaneous rate of reinforcement at t under a VI build with a distribution of intervals defined by $f(x)$ is

$$R(t) = \frac{f(t)}{\int_t^{+\infty} f(x)dx}$$

In order to achieve a constant rate of reinforcement, the nominator and the denominator of this ratio must decrease at the same rate as time passes in the interval.

The top graphs of Figure 8 illustrate the evolution of the instantaneous rate of reinforcement under a VI made of a uniform distribution of intervals. From the time t_1 (left graph) to the time t_2 (right

⁶ While the local rate of reinforcement, in its definition by Catania and Reynolds (1968), calculated on the basis of segments of time, is adapted to characterize the reinforcement rate in function of time under VIs made of discrete distributions of intervals, the instantaneous rate of reinforcement, calculated at any point in time, is adapted to characterize the reinforcement rate in function of time under VIs made of continuous distributions of intervals.

graph), the relative likelihood of the sample, the height to the curve, remains the same, while, the probability that an interval will last longer than the time considered, the area under the curve from that time, decreases: under a VI made of a uniform distribution of intervals, the rate of reinforcement increases as time elapses since the beginning of the interval. The bottom graphs of Figure 8 illustrate the evolution of the instantaneous rate of reinforcement under a VI made of an exponential distribution of intervals. From the time t_1 (left graph) to the time t_2 (right graph), the relative likelihood of the sample decreases, and, the probability that an interval will last longer than the time considered decreases at the same rate: under a VI made of an exponential distribution of intervals, the rate of reinforcement remains constant at any time into the interval, from 0s to infinity.

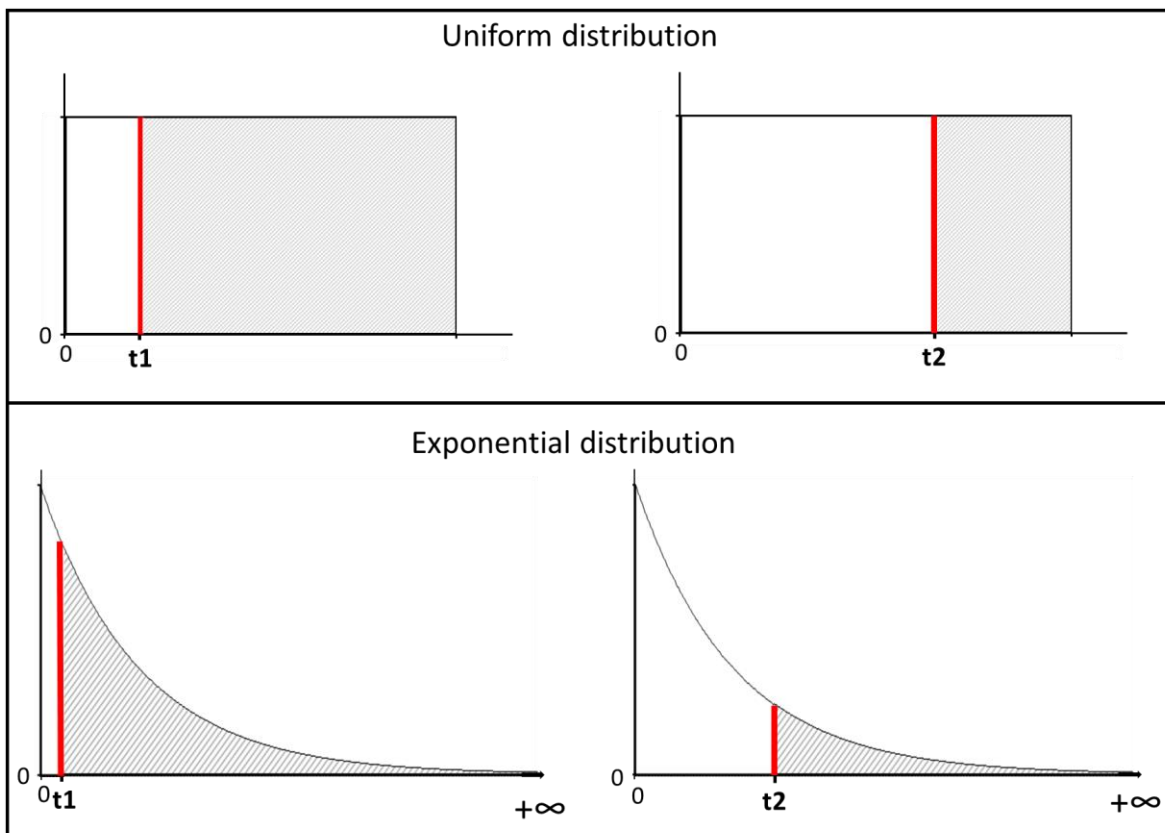


Figure 8: Evolution of the relative likelihood of an interval of length t and of the probability that an interval is longer than t , in the uniform distribution and in the exponential distribution, at two different times. The top panel shows that, within a uniform distribution, from t_1 to t_2 , the relative likelihood (red height) remains the same, while the probability that an interval last longer than that the considered t (shaded area) decreases, which makes increase the ratio of the two elements. The bottom panel shows that, within an exponential distribution, from t_1 to t_2 , the relative likelihood (red height) decreases as does, and at the same rate, the probability that an interval last longer than that the considered t (shaded area), which makes the ratio of the two constant.

The exponential distribution is the only distribution within which, as t increases, the relative likelihood of an interval of length t will decrease at the same rate as the probability that an interval lasts

longer than t . But, if we introduce unreinforced trials, we make that these two elements will depend on two distinct distributions and so open new possibilities.

In a VI made of a combination of a distribution of reinforced intervals and of a distribution of unreinforced intervals, an instantaneous rate of reinforcement at a time t , will be determined by the relative likelihood of an interval of length t , in the distribution of the reinforced intervals, divided by the probability that an interval will last longer than t , within the overall distribution of intervals, comprising reinforced and unreinforced intervals. In this new framework it becomes not impossible to maintain constant the ratio of the instantaneous rate of reinforcement from intervals of a finite range.

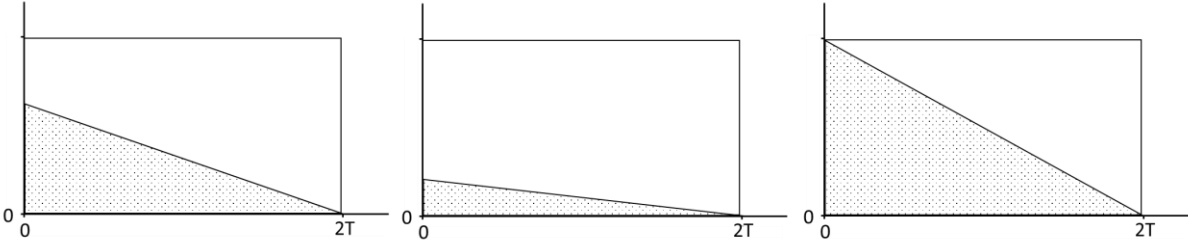


Figure 9: Examples of combinations of distributions of reinforced and unreinforced intervals sustaining constant rate of reinforcement. While the overall distribution is rectangular and, necessarily remains of the same mass (1), the sub-distribution of the reinforced intervals, the triangle with dotted area, can take any mass between 0, excluded, and 0,5.

Let us consider a combination of reinforced and unreinforced intervals with an overall distribution of intervals that would be the simplest: uniform. If the overall distribution of the intervals is uniform (with mean interval T and maximum interval $2T$), then the probability that an interval will last longer than t , as t increases, decreases in a linear manner from a maximum at $t=0$, down to zero at $t=2T$. From there, to have the ratio of the instantaneous rate of reinforcement to be constant from 0 to $2T$, we need to have the relative likelihood in the distribution of the reinforced intervals to decrease in a linear manner from a maximum at $t=0$, down to zero at $t=2T$. This is achieved simply by a distribution having a right triangle shape with right angle on the origin of the axes.

Figure 9 presents three examples of combinations of distributions that achieve constant rate of reinforcement. The triangular sub-distribution of the reinforced intervals is of maximal mass, 0,5, when it matches the diagonal of the rectangle formed by the overall distribution intervals (see the rightmost graph of Figure 9), but it can be of any smaller mass. That is, under a VI made of such combination of distributions, in which reinforced trials would be alternated with unreinforced trials, the probability of reinforced trials would be maximal when reinforced and unreinforced trials would be equally likely. The probability for reinforced trials has to be comprised between 0, excluded, and 0,5

and a probability for unreinforced trials, complementing, has to be comprised between 0.5 and 1, excluded.

Figure 10 illustrates the parallel decrease of the relative likelihood of an interval of length t , within the triangular distribution of the reinforced intervals, and of the probability that an interval is longer than t , within the rectangular overall distribution of intervals, creating the constant ratio of the instantaneous rate of reinforcement. At $t=0$, the relative likelihood of an interval of length t , within the distribution of the reinforced intervals, matching the side of the triangle, is at its maximum as well as is the probability that an interval is longer than t , within the overall distribution of intervals, matching the complete area of the rectangle. At $t=T$, the relative likelihood of an interval of length t , within the distribution of the reinforced intervals, represented in the figure by the red height, is half of its maximum, as is the probability that an interval is longer than t within the overall distribution of intervals, represented in the figure by the shaded area. At $t=2T$, both elements reach zero.

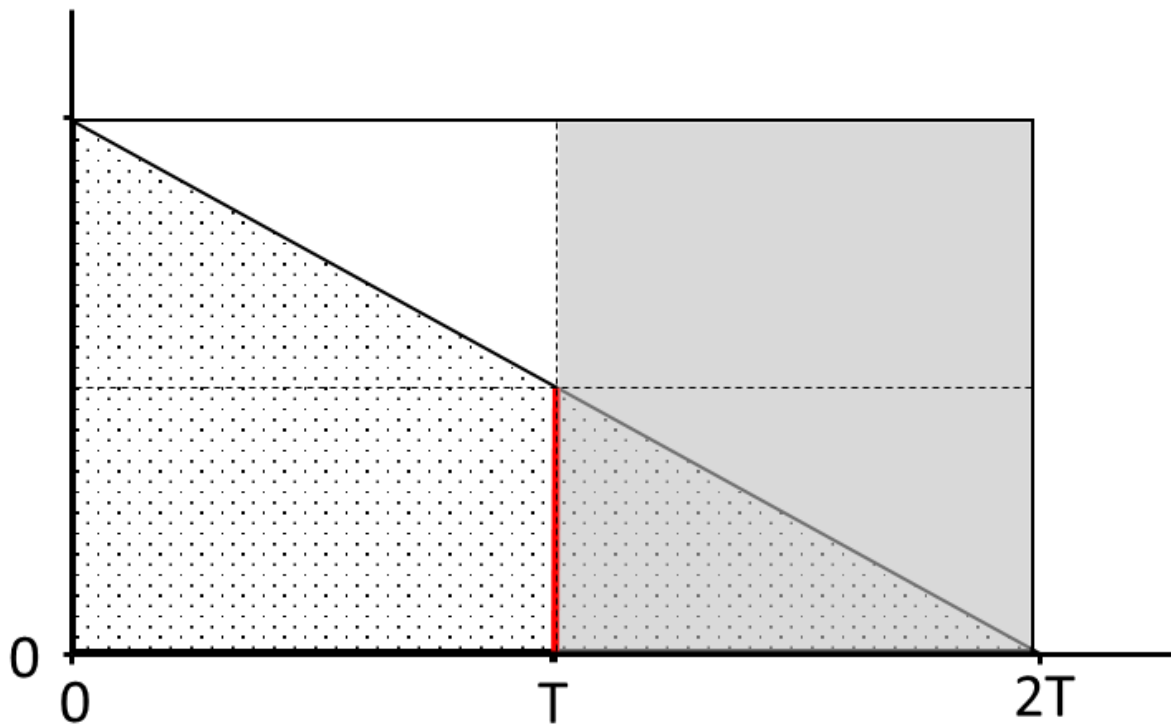


Figure 10: Combination of distributions of reinforced and unreinforced intervals with parallel decrease of the relative likelihood of an interval of length t , and of the probability that an interval lasts longer than t , as t increases. The big rectangle corresponds to the overall distribution of intervals and the big triangle with dotted area corresponds to the distribution of the reinforced intervals. The height in red represents the relative likelihood of an interval of length t and the shaded area represents of the probability that an interval lasts longer than t , when $t=T$ (the mean interval of the overall distribution). At $t=0$, the height of the triangle and the area of the rectangle are at their maximum. At $t=T$, the height in the triangle is at half its maximal length and the shaded area is half the area of the big rectangle. At $t=2T$, the height is null, as is the area.

Guidelines to operationally construct a VI based on this combination of distributions can be found in Bugallo, Machado and Vasconcelos (2018). Bugallo et al. (2018) also provides complete mathematical description of the method and demonstrates that, while our combination of reinforced and unreinforced intervals forms a uniform distribution, the same intervals, when laid end to end until reinforcement follow an exponential distribution. From this relation we extracted an original representation of the number e , which we present in Appendix 1. And, because of this relation we call the VI obtained from our method, uniform exponential VI.

STUDY 2: Local response rate under constant-probability VI schedules

Introduction

The VI schedule is often casually presented as sustaining constant rate of responding, in opposition to the FI which is known to induce break and run patterns. As we have seen in Study 1 this belief is generally wrong, as it is only under certain settings that VIs engenders flat response functions (Catania and Reynolds, 1968).

A certain misunderstanding about the behavioral effect of the VIs can be traced back to the influential book, *Schedules of Reinforcement* by Ferster and Skinner (1957), wherein the authors investigated the problematic of responding under VI schedules while stating from the introduction of the dedicated chapter that “*The VI schedule is designed to produce a constant rate*”. The chapter in question has an awkward structure since it proposes the investigation of local responses rates produced by schedules which were designed to promote a certain type of responding; constant responding. Rather than to assess the behavioral effects of VIs made of regular distributions of intervals (e.g, using true arithmetic and geometric series), the authors used modified versions of arithmetic and geometric series (while still calling them arithmetic and geometric VIs; the only accurately named VI was their Fibonacci VI), which they apparently already knew from previous observation to sustain flatter response patterns.

Besides, Ferster and Skinner (1957) based their analyzes on cumulative records of responding from entire sessions, a tool that is appropriate to study the evolution of response rate across sessions but much less appropriate for the study of the regularities within the reinforcer-to-reinforcer interval (or time into the trial interval). If the study of the evolution of the response rate within sessions could reveal information about acquisition, other relevant observations were to be expected from the analyzes of the response rate within the reinforcer-to-reinforcer intervals along which existed relations between reinforcement and time. The relatively constant response rate observed across sessions at the steady state and the initial statement of the authors on the purpose of the VI may have imprint the idea that

the VI sustain constant rate of responding. Yet, Ferster and Skinner (1957) also suggested patterns in the reinforcer-to-reinforcer intervals differing between VIs.

According to Fleshler & Hoffman (1962), in an ideal VI, a VI that could sustain constant rate of responding, the probability of reinforcement would not be correlated with any temporal variables. The VIs used by Ferster and Skinner (1957) differed from such VI in three ways. First, they used limited numbers of intervals, and reinforcement could be obtained only at certain points in time. Second, the intervals were arranged as series repeated over and over again. These two limitations, due to the use of tape-reading devices, could have led animals to learn patterns of the reinforcer-to-reinforcer-intervals. Third, regardless of the order and the gaps in the distributions of the intervals, the series of intervals used yielded uneven profiles of local reinforcement rate; as we have seen in Study 1, animals can learn the profile of density of reinforcement within a temporal interval.

An ideal VI, in the sense of Fleshler & Hoffman (1962), should pick at random intervals, from a continuous distribution presenting a constant hazard rate, that is, from an exponential distribution. Methods have been proposed that only approach the exponential distribution, only approach the ideal VI. The first method proposed was that of Fleshler & Hoffman (1962), which consists in the segmentation of the exponential distribution in chunks of equal areas and in the extraction as VI intervals of the mean of each of these chunks. This method presents the advantage of approximating the exponential distribution in a finite time range and was of convenient use for tape-reading systems. But it presents the limitations of only approximating a constant probability of reinforcement in time and of a discrete distribution of intervals which could permit temporal discrimination, especially regarding the spacing of the longest intervals. The second method was presented at the same time by Millenson (1963) and Farmer (1963); they proposed the use of the geometric distribution (not to be confounded with the geometric progression also used in VIs): the discrete approximation of the exponential distribution, which can be obtained by sampling every $\bar{T}x$ seconds a probability p of reinforcement. With a sufficiently small $\bar{T}x$ and a sufficiently small p , this method allows a fair approximation of constant probability of reinforcement in time, but it can only approach the continuous distribution of intervals and faces the problem of the infinite time range of the exponential distribution (an experimenter using this method would sometimes deal with very long intervals).⁷

⁷ Back at the time where computer programs were not available for the construction of experiments, the Millenson/Farmer system, with the only need of a random generator and of a timer, was an alternative to the use of the tape-reading device; nowadays, this method as well as the Fleshler & Hoffman

These methods, constituting a family of VIs referred to as random-interval or as constant-probability VIs, are frequently used, but it exists only a rather scarce number of studies which have investigated their behavioral effect. Millenson (1963) and Farmer (1963), both with pigeons, were the first ones to investigate the response profile induced by such schedules, but like Ferster and Skinner (1957), their analyzes were restricted to the visual inspection of cumulative records of responding. Thus, they could not precisely address the question of the constancy of the response rate, and their conclusions differed. While Farmer (1963) suggested that the random-interval schedule could sustain flatter response rate than other VIs, Millenson (1963) came to the conclusion that it had an effect similar to other VIs.

Catania and Reynolds (1968) initiated the study of VIs with the use of graphs relating the rate of responding to the time into the trial interval, which permitted to reveal response patterns within that interval. As we have already seen in the introduction of Study 1, they were able to show with pigeons that VIs made of uniform distributions of intervals, wherein the probability of reinforcement increases with the time elapsed, induced accelerating response patterns, whereas, VIs approximating constant probability of reinforcement in time induced flat response patterns.

However, a closer look at Catania and Reynolds (1968) results with the constant-probability VI reveals a slight acceleration in the response rate, which could have been caused by the imperfection of their schedule using an adaptation of the Millenson (1963) method with a tape-reading device. That is, their schedule used discrete intervals but more importantly it presented a longest interval causing a necessary increase of the probability of reinforcement at the end of the interval. In the same paper Catania and Reynolds (1968) presented the results of an unpublished thesis by Chorney which investigated the response rate maintained by a constant-probability VI using the Fleshler & Hoffman method which found a roughly constant response rate.

Three other studies have examined the rate of responding under constant-probability VIs using rats. Harzem, Lowe and Priddle-Higson (1978), with the Fleshler & Hoffman method obtained roughly flat response patterns with a slight tendency to an acceleration. Leslie (1981) using the Millenson (1963) VI, and Church and Lacourse (2001), using an unspecified approximation of the exponential distribution, found patterns of responding characterized by an early acceleration followed by a slight

method, may not be better than the use of a computer program picking randomly intervals from a truncated exponential; yet, these methods are still popular.

deceleration (a similar inconsistency has been found in pavlovian settings using constant-probability variable time schedules; see Kirkpatrick and Church, 2000; and Harris, Gharaei and Pincham, 2011).

From these few studies and their inconsistency, the degree of confidence with which we can assert what is the pattern of responding sustained by constant-probability VIs, is weak. The differences observed between studies have three main possible sources, the use of different species (pigeon or rat), of different amounts of training (from 30 sessions in Church and Lacourse, 2011, up to 127 sessions in Catania and Reynolds, 1968), and of different methods to approximate the exponential distribution (Millenson method, Fleshler & Hoffman method or unspecified method). Taken simply, in any of these studies, the divergence from constant response rate could be primarily due to the divergence from constant probability of reinforcement in time inherited from the limitations of the method employed to approach the ideal VI.

Previously, we presented the uniform exponential VI (Bugallo, Machado and Vasconcelos; 2018) which can be taken as an ideal VI since it actually sustains constant probability of reinforcement in time over a finite time range. To observe responding under the uniform exponential VI would thus inform us on what really is the pattern of responding maintained by a constant-probability VI.

Experiment 1: Comparison of the uniform exponential VI with the Fleshler & Hoffman VI

From a reasoning devoid of theory, the suppression of the temporal contingencies should induce a constant rate of responding. Or, if patterns still emerge in the response function despite the suppression of temporal contingencies, these patterns would then have to relate to constraints that animals encounter in responding in time (e.g., animals may need a few seconds to reach their steady rate when the response key gets turned on, which would cause an acceleration in the early part of the response function).

In this experiment we compare the uniform exponential VI to the popular Fleshler & Hoffman VI, in their capacity to sustain constant rate of responding. Whereas the latter only approximates constant reinforcement rate in time, the former truly achieves it. The observation of a flatter rate of responding in the uniform exponential VI would suggest that the divergences from constant rate of

responding found under constant-probability VIs relate to the limitations of the methods used to obtain constant rate of reinforcement.

We trained two groups of pigeons, both exposed to the two VIs, in opposite orders. We inserted an extinction phase between the two conditions, to minimize the influence of the first condition on the second condition. And we took the opportunity of this extinction phase to look for potential different resistance to extinction after the two VIs that differ as to the inclusion of unreinforced trials.

Methods

Subjects

Ten pigeons (*Columba livia*) participated in the experiment, all of them had previous experience with timing tasks. The birds were maintained at 85% of their free-feeding body weight, with mixed grain provided mostly during experimental sessions. Water and grit were freely available in their home cages. The colony room was maintained in a 13:11 light/dark cycle, with the lights on at 08:00, and its temperature was kept between 20 and 22 °C. The experiment was conducted five days per week at approximately the same time of day for each pigeon.

Apparatus

Five Lehigh Valley Electronics Skinner boxes were used. The boxes measured 34 x 35 x 31 cm (h x l x w). Three circular response keys, 2.5 cm in diameter, were arranged horizontally on the response panel. The bottom edge of each key was 22.5 cm above the wire mesh floor, and the keys were 9 cm apart, center to center. Each key was equipped with a 12-stimulus IEE (Industrial Electronics Engineers) in-line projector. The food hopper was accessible through a 6-cm wide x 5-cm high opening that was centered horizontally on the response panel, 8.5 cm above the floor. When the hopper was raised, a 28-V, 0.04-A light illuminated its opening and grain became accessible to the pigeon. On the wall opposite the response panel, 30 cm above the floor, a 28- V, 0.1-A houselight provided general illumination. The boxes were enclosed in outer boxes equipped with exhaust fans. The fans circulated air through the boxes and masked outside noises.

Softwares

The experiment was run using ABET II. Randomization of the intervals was made using Microsoft Excel and then pasted as lists in each session program.

Procedure

Table 3 shows the order of the experimental phases for each bird. Half of the birds were first exposed to the uniform exponential VI (Group 1) and the other half to the Fleshler & Hoffman VI (Group

Table 3: Organization of Experimental phases for the birds of the two groups of Study 2 Experiment 1. Birds were trained under each VI condition for 30 sessions which were separated by a 5 sessions extinction phase. P458 was stopped at the end of its first condition; P960 was stopped at the beginning of its second condition.

		GROUP 1				
condition rank		P088	P449	P501	P724	PG39
1		uniform exponential VI				
2		extinction phase				
3		Fleshler & Hoffman VI				
		GROUP 2				
condition rank		P157	P444	P918	P960	P458
1		Fleshler & Hoffman VI				
2		extinction phase				
3		uniform exponential VI				

2). Then all birds went through an extinction phase, using the same schedule in which they were engaged but with reinforcers removed. Finally, they switched to the other VI condition. The first and the last conditions lasted for thirty sessions each and the intermediate extinction phase lasted for five sessions. Two pigeons of Group 2 broke their beaks and did not complete the entire experiment. One completed only 25 sessions of the first condition (P458), another one completed the first condition and the extinction phase (P960).

Because the implementation of the uniform exponential VI requires the insertion of ITIs, in order to make the comparison of the VIs meaningful, both VIs had trials separated by ITIs. A trial started by the illumination of both the central key and the houselight; if the trial was a reinforced one (all trials in the Fleshler & Hoffman VI; half of the trials in the uniform exponential VI), the first peck on the central key after the interval criterion gave access to the feeder and ended the trial, if the trial was an unreinforced trial (half of the trials in the uniform exponential condition), once the interval criterion was

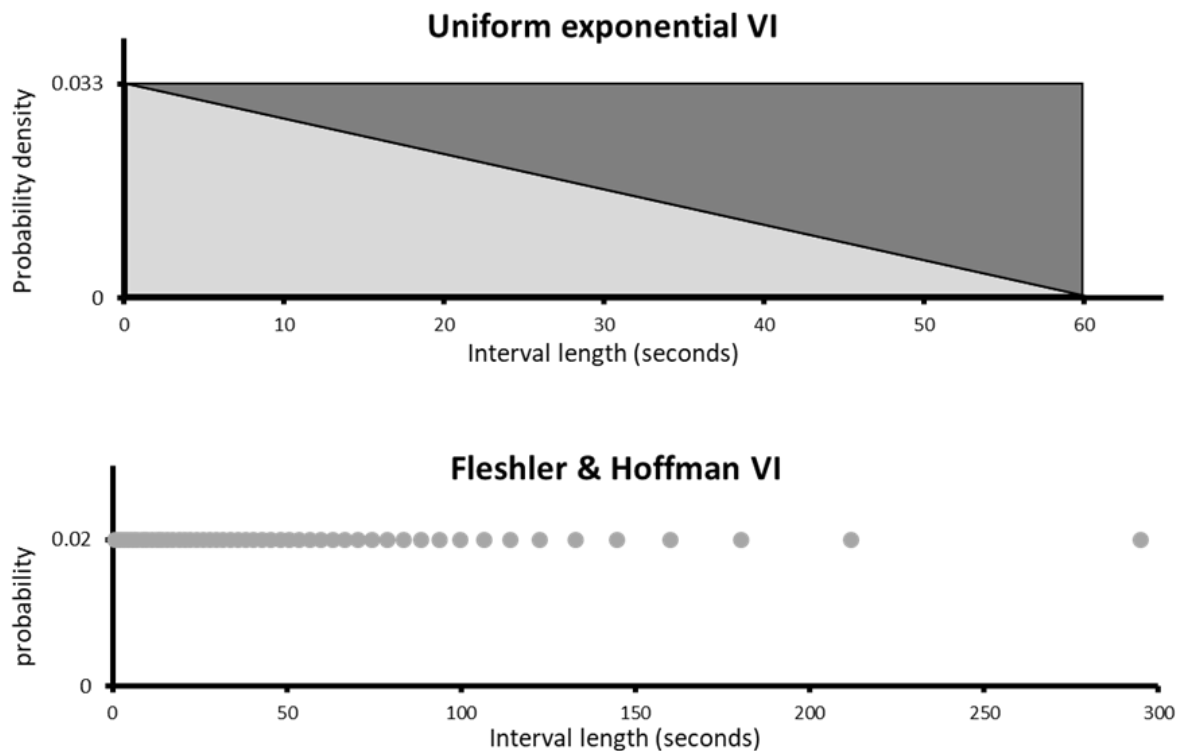


Figure 11: Distributions of the intervals of the two VI conditions of Study 2 Experiment 1. The top graph shows the probability density function (pdf) associated to the intervals of the uniform exponential VI. This pdf combines two different types of trial intervals, reinforced interval (light gray area) and unreinforced interval (dark grey area). The bottom graph shows the probability mass function associated to the intervals of the Fleshler & Hoffman VI.

reached the trial ended independently of the bird's behavior. ITIs were spent in darkness. Access time to the feeder was adjusted for each bird in order to minimize extra feeding outside of the experiment.

Each VI was defined by two parameters. For the uniform exponential VI we set the maximum interval length to $2T = 60s$ and the probability of reinforcement to $p = 0.5$. For the Fleshler & Hoffman VI we set the mean interval $T = 60s$ and the number of intervals $N = 50$ (longest interval: 294.7s). With these parameters both VI conditions had a mean time to reinforcement during the signal (i.e., key light on) of 60s. Figure 11 shows the two distributions from which intervals were drawn.

In order to equate the number of reinforcers obtained, there were two times more trials in the uniform exponential VI (100 trials per session) than in the Fleshler & Hoffman VI (50 trials per session). And, to equate the average duration of sessions, ITIs were made half as short in the uniform exponential VI (20s) as in the Fleshler & Hoffman VI (40s), thereby also equating the mean time to reinforcement taking into account ITIs.

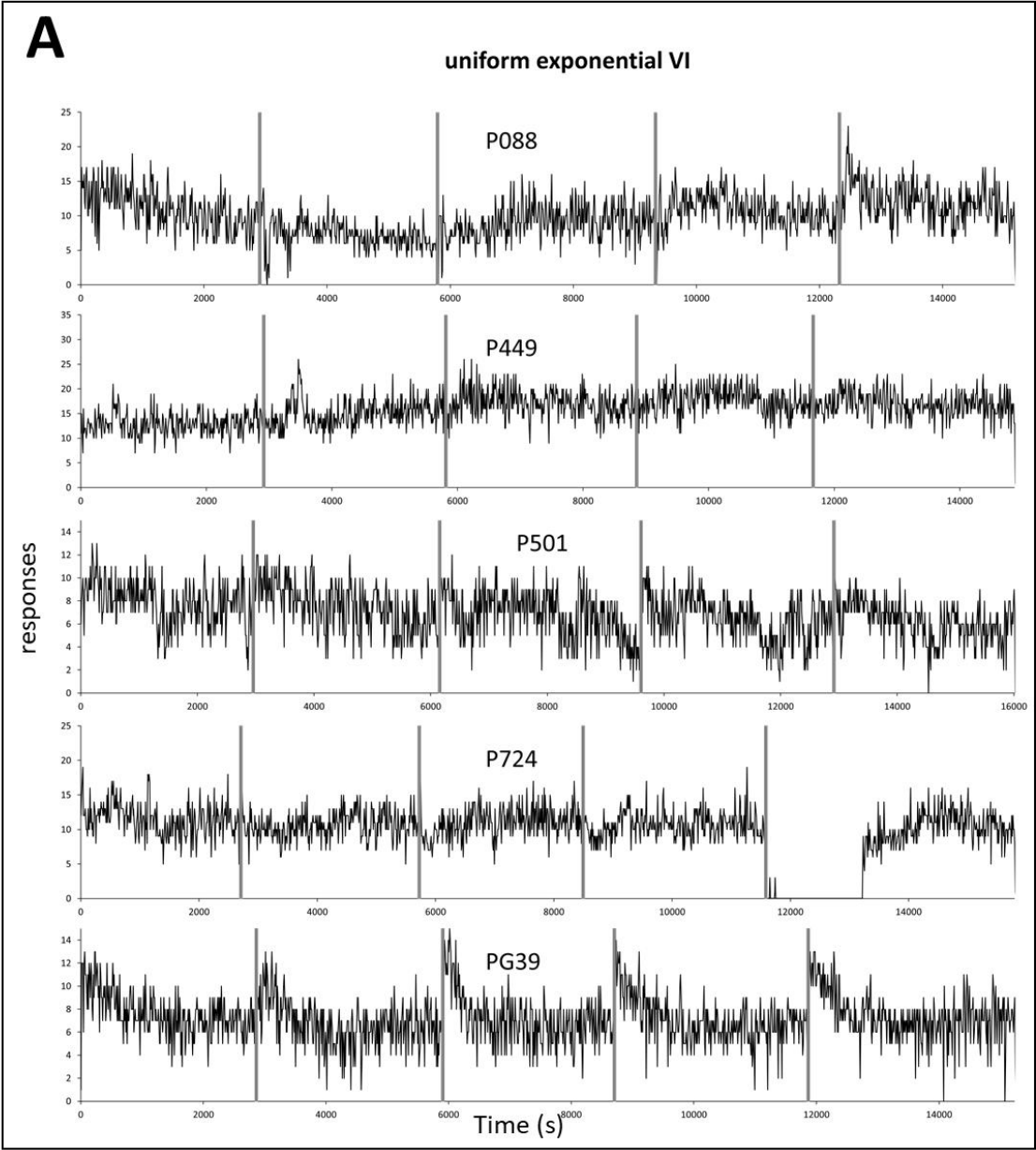
Results

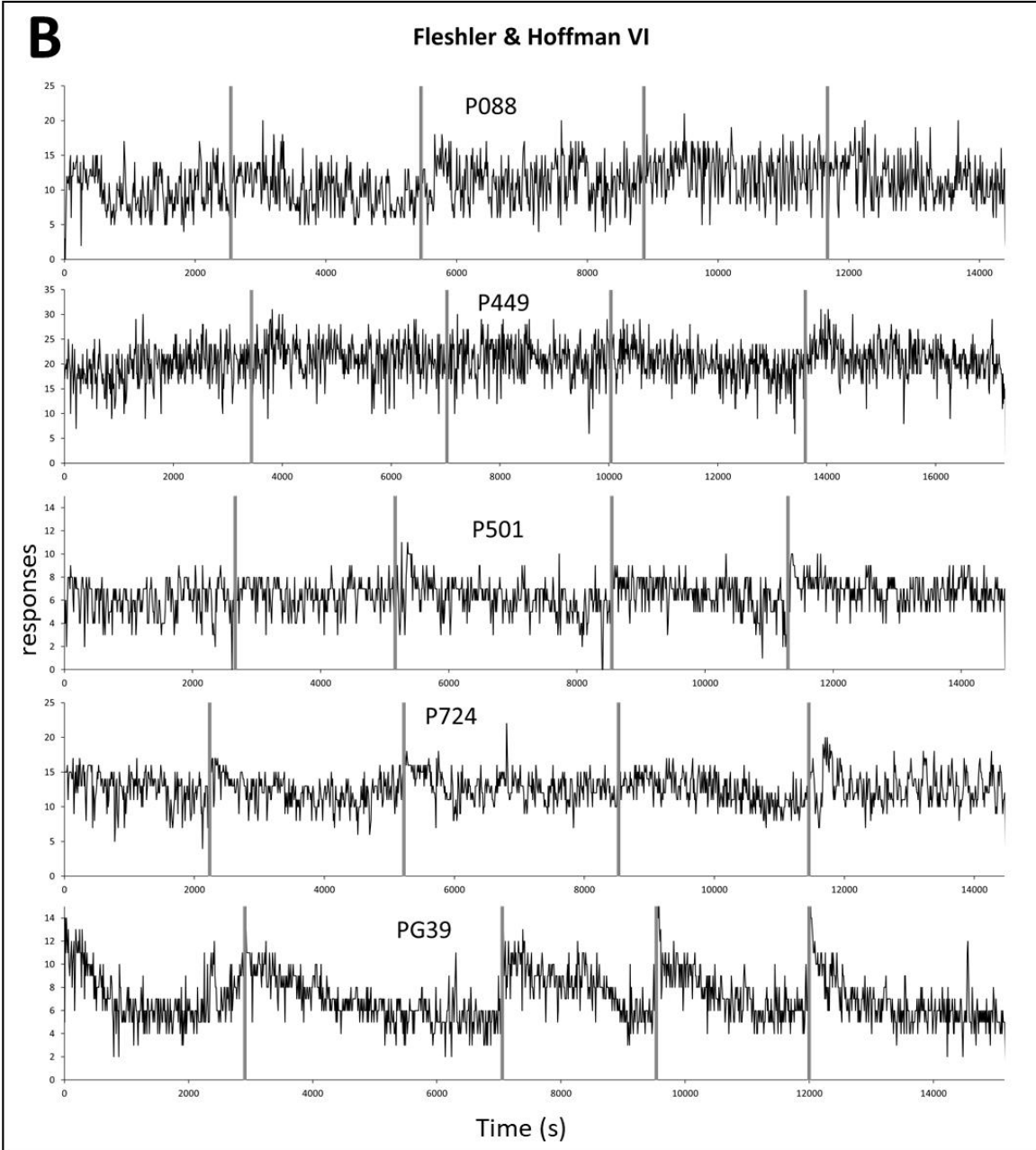
We examine the response rate in function of time from the perspective of responding across sessions and from the perspective of responding as a function of time into the trial interval. First, we look at the response rate across sessions, laying end to end all trials of the last five sessions (i.e., we remove the ITIs) and pooling responses in 10-second bins, for each bird, in the two VI conditions. Figure 12 shows on pages 37 to 40, the response rate in function of time across the five last sessions for each bird in each VI condition. Panel A shows the data of Group 1 in the uniform exponential VI, panel B, the data of Group 1 in the Fleshler & Hoffman VI, panel C, the data of Group 2 in the Fleshler & Hoffman VI, and, panel D, the data of Group 2 in the uniform exponential VI. From visual inspection we can see that across the last five sessions of each VI conditions, most birds show a rather stable mean response rate with a stable magnitude of oscillations (exceptions are P088 and P449 in the uniform exponential VI). Yet, some birds show session patterns. PG39, in the two conditions, P458 in the Fleshler & Hoffman VI, P501 in the uniform exponential VI, and to a lesser extent, P918 and P157, both in the two conditions, show a tendency to decline of responding as time into the session elapse.

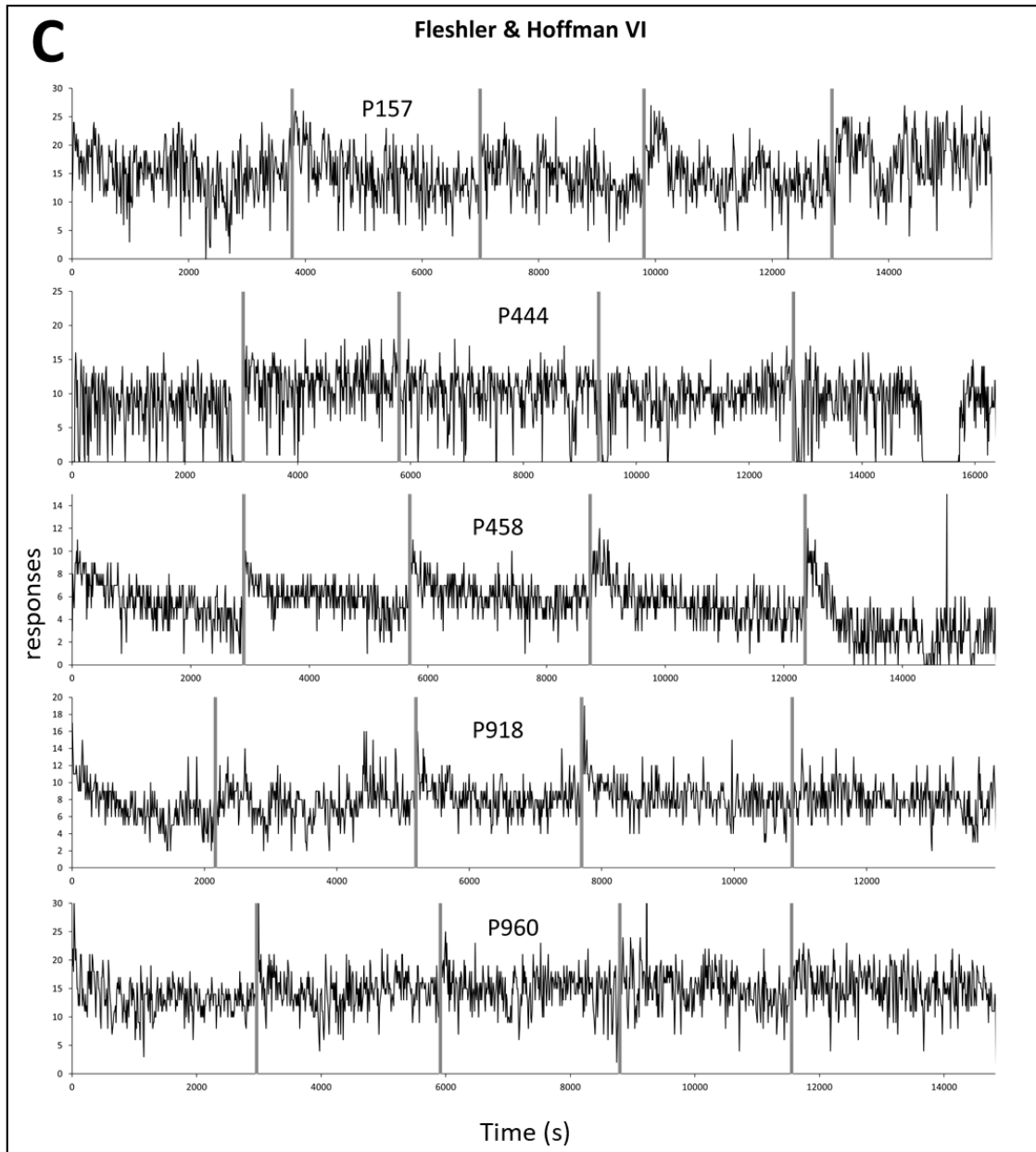
For certain birds (P157, P444), the mean response rate seems to change from one condition to the other, but none of the VI consistently sustained a higher rate than the other. In the same way the variability of the response rate, changed for certain birds (P449, P501), from one condition to the next but we do not observe consistent changes across birds.

It can be seen that two birds, P444 (in the two conditions) and P724 (in the uniform exponential VI condition), stopped to respond at certain moments. Whereas we do not know the reason of the unique period observed with P724, we were able to see that P444 froze when noise was produced in the experimental room in which two or three experiments were conducted at the same time (the segments of no responding longer than 20 seconds were removed from quantitative

analyses). We also note that P458 shows an important decline of the response rate, which revealed, at the end of session 25 of the first condition, that its beak was broken.







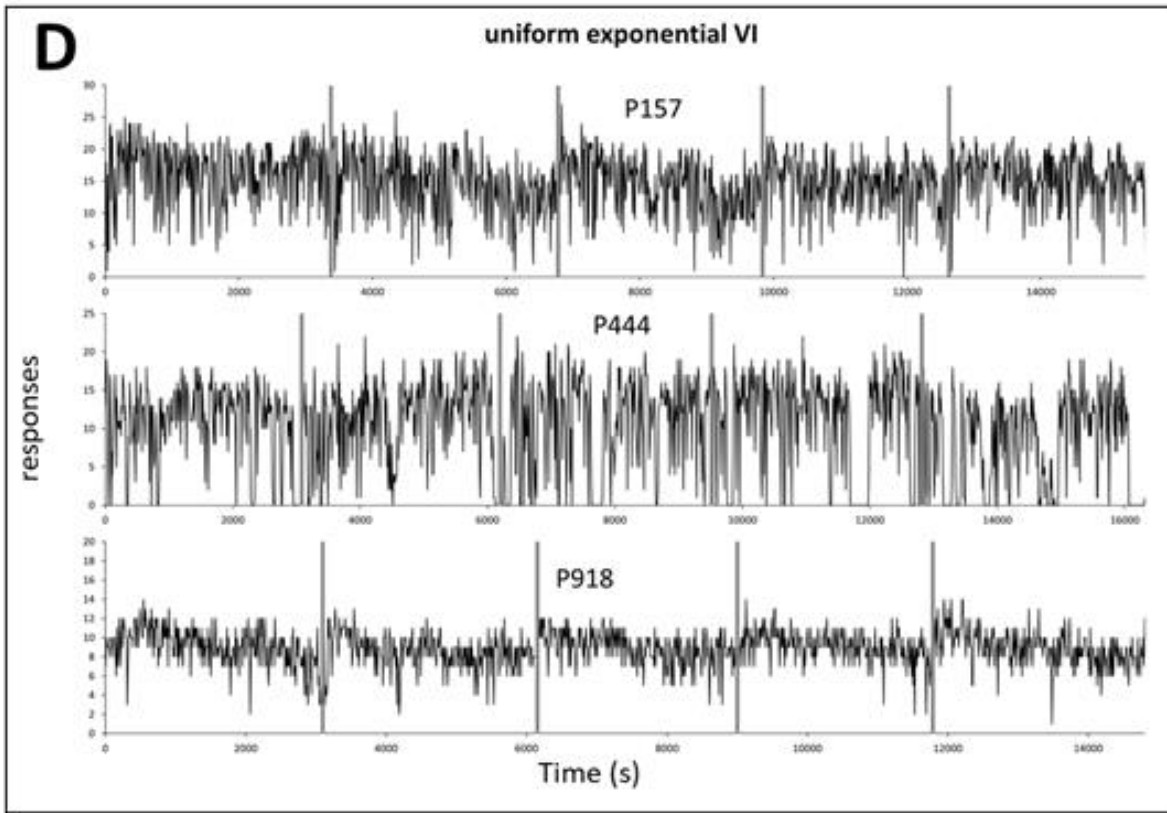


Figure 12: Response rate in function of time along sessions, from the beginning of the 26th session (20th in the case of P458), all trials laid end to end. The number of responses is given per bins of 10 seconds. Panel A presents the data of group 1 in the uniform exponential VI condition, panel B presents the data of group 1 in the Fleshler & Hoffman VI condition, panel C presents the data of group 2 in the Fleshler & Hoffman condition VI and panel D presents the data of group 2 in the uniform exponential VI condition.

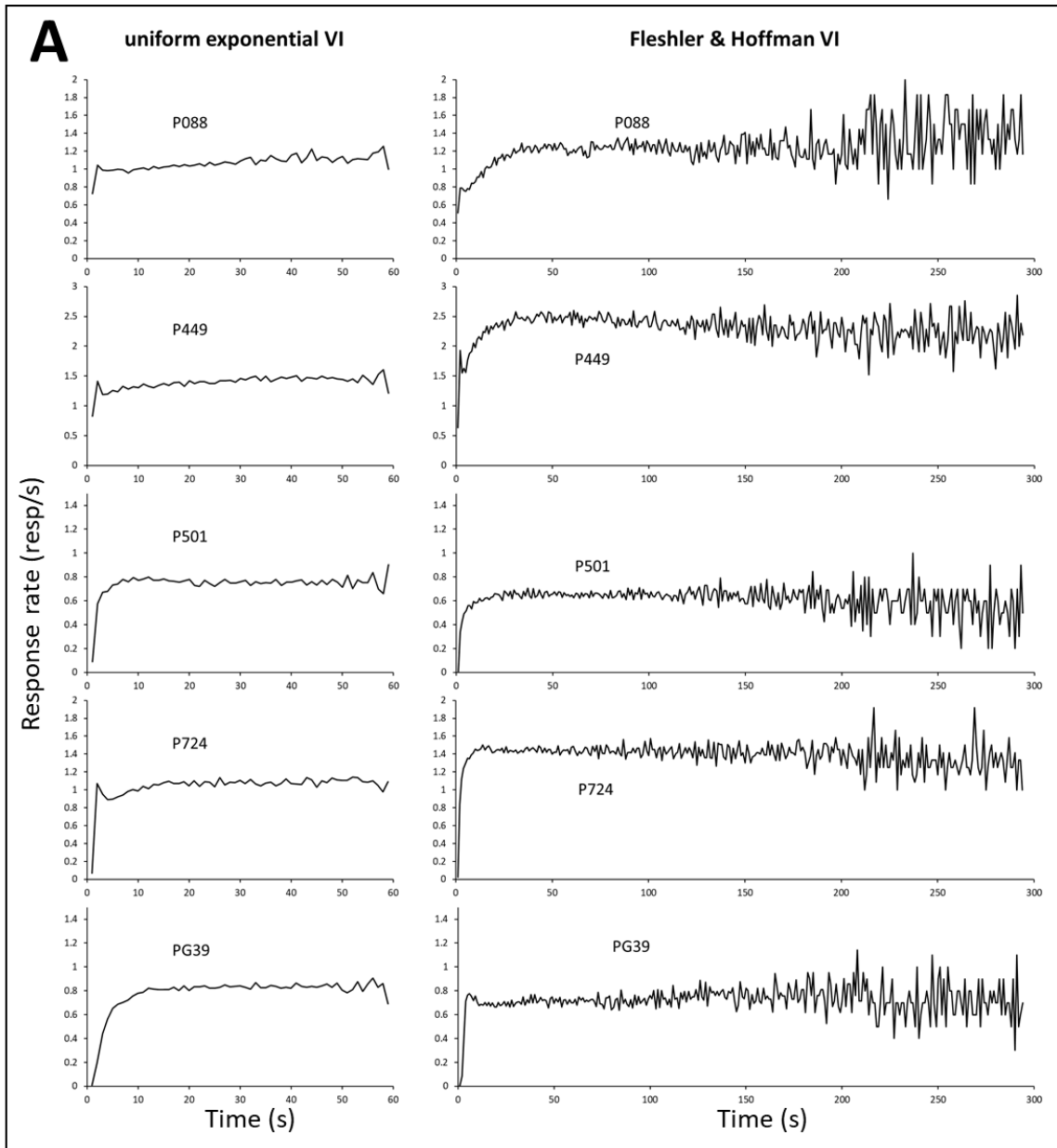
Table 4: Means and coefficients of variation of the number of responses across 10s bins of the last 5 sessions of each condition for each bird in Study 2 Experiment 1.

birds	Uniform exponential VI		Fleshler & Hoffman VI	
	m	CV	m	CV
P088	9.96	0.31	11.04	0.28
P449	15.98	0.20	20.61	0.18
P501	6.96	0.30	6.43	0.22
P724	10.78	0.21	12.60	0.18
PG39	7.21	0.29	7.15	0.30
P157	15.10	0.30	15.27	0.29
P444	12.01	0.37	9.98	0.32
P458			5.29	0.39
P918	8.92	0.21	7.98	0.27
P960			14.69	0.24

Table 4 presents the means and coefficients of variation computed from the number of responses in 10-seconds bins, across the five last sessions of each condition, for each bird. The mean indexes overall response rate, and the coefficient of variation indexes the variability of the response rate. Consistently with visual analysis, differences between conditions in neither overall response rate nor variability could be shown from statistical analysis ($t(7)=0.71$ and $t(7)=1.15$, respectively).

Now looking at responding from the trial interval perspective, we extract for every 1-second bin (60 bins in the uniform exponential VI, 295 bins in the Fleshler & Hoffman VI), the number of responses emitted in the bin and the occurrences of intervals ending after that bin. Then, we sum across the last fifteen sessions, for each bird, both the number of responses and the number of occurrences per bin, and divide the first sum by the second sum, bin per bin. This variable maximizes the amount of data considered (only the responses emitted between the time of the last passed 1-second bin and the termination of the current interval are discarded; hence, no data points on the 60th bin in the uniform exponential VI and on the 295th bin in the Fleshler & Hoffman VI). Figure 13 shows for each bird, in each condition, the response rate in function of the time into the trial interval. Panel A of Figure 13 shows the data of Group 1 with the uniform exponential VI condition on the left and the Fleshler & Hoffman VI condition on the right, panel B of Figure 13 shows the data of Group 2 with the inverse mapping. It can be seen that all birds, present a rather constant rate of responding, though with a

tendency to acceleration or a tendency to deceleration. In the uniform exponential VI, birds of Group 1 – except P501 which shows a flat response rate – show a slight increase of response rate, whereas birds of Group 2 show a slight decrease of the response rate. In the Fleshler & Hoffman VI, the tendency to the deceleration dominates in the two groups.



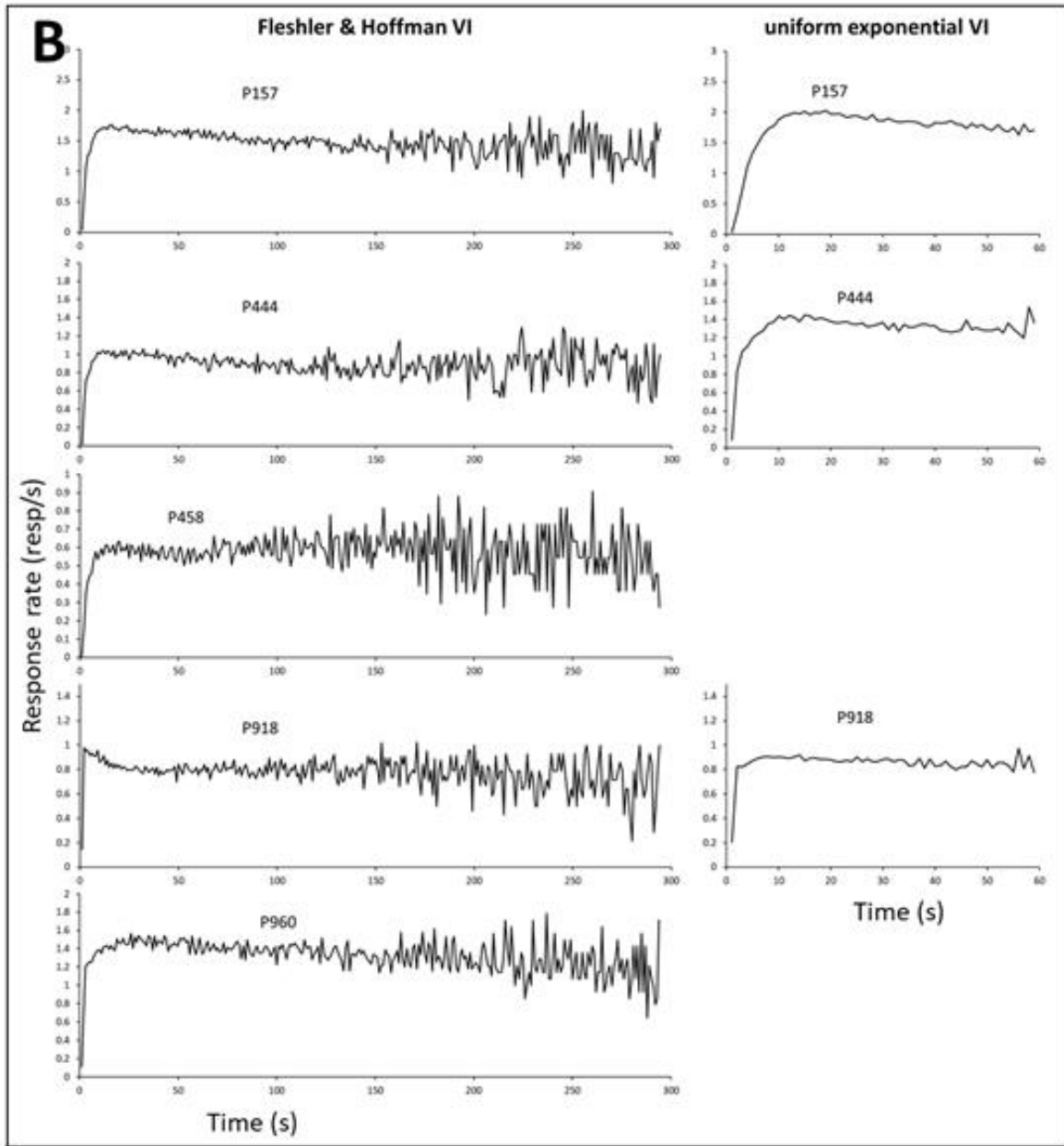


Figure 13: Response rate in function of time into the trial interval. Panel A shows the data of group 1, in the uniform exponential VI (on the left) and in the Fleshler & Hoffman VI (on the right). Panel B shows the data of group 2 in the Fleshler & Hoffman VI (on the left) and in the uniform exponential VI (on the right).

The important oscillations observed in the second half of the trial interval in the graphs of the Fleshler & Hoffman VI condition stem from the fact that the response rate on that segment was computed from few data. In the uniform exponential condition, this lowest of measurements was only

approximately matched in the last 4 seconds into the trial interval, which can be seen affecting the final segments of the response functions.

Table 5A shows the mean and coefficients of variation computed from a variable made of the

Table 5: Means and coefficients of variation on the number of responses per 5 seconds bins on the trial interval, averaged across the 3 last long trials in each condition, for each bird in Study 2 Experiment 1. Table A shows the indexes from all 5 seconds bins and table B shows the indexes computed without the first 10 seconds bins.

A

birds	Uniform exponential VI		Fleshler & Hoffman VI	
	m	CV	m	CV
P088	4.65	0.15	6.15	0.19
P449	7.71	0.15	10.92	0.15
P501	3.05	0.16	3.17	0.13
P724	5.25	0.12	6.36	0.13
PG39	3.65	0.22	3.23	0.17
P157	7.97	0.26	6.89	0.19
P444	7.03	0.14	4.53	0.25
P458			3.09	0.21
P918	4.34	0.08	3.94	0.18
P960			6.92	0.21

B

birds	Uniform exponential VI		Fleshler & Hoffman VI	
	m	CV	m	CV
P088	4.75	0.15	6.26	0.16
P449	7.48	0.15	11.04	0.14
P501	3.09	0.12	3.18	0.13
P724	5.43	0.09	6.40	0.13
PG39	3.88	0.10	3.27	0.16
P157	8.69	0.08	6.91	0.18
P444	7.04	0.14	4.58	0.25
P458			3.09	0.21
P918	4.34	0.09	3.96	0.18
P960			6.97	0.20

same amount of measures for each point of response rate as a function of time in the trial interval for the two conditions: the average number of responses in 5-seconds bins on the last 3 long intervals (longer than 59 s in the uniform exponential VI; equal to 294.7 s in the Fleshler & Hoffman VI). No difference in mean response rate nor difference in variability could be found between the two VI

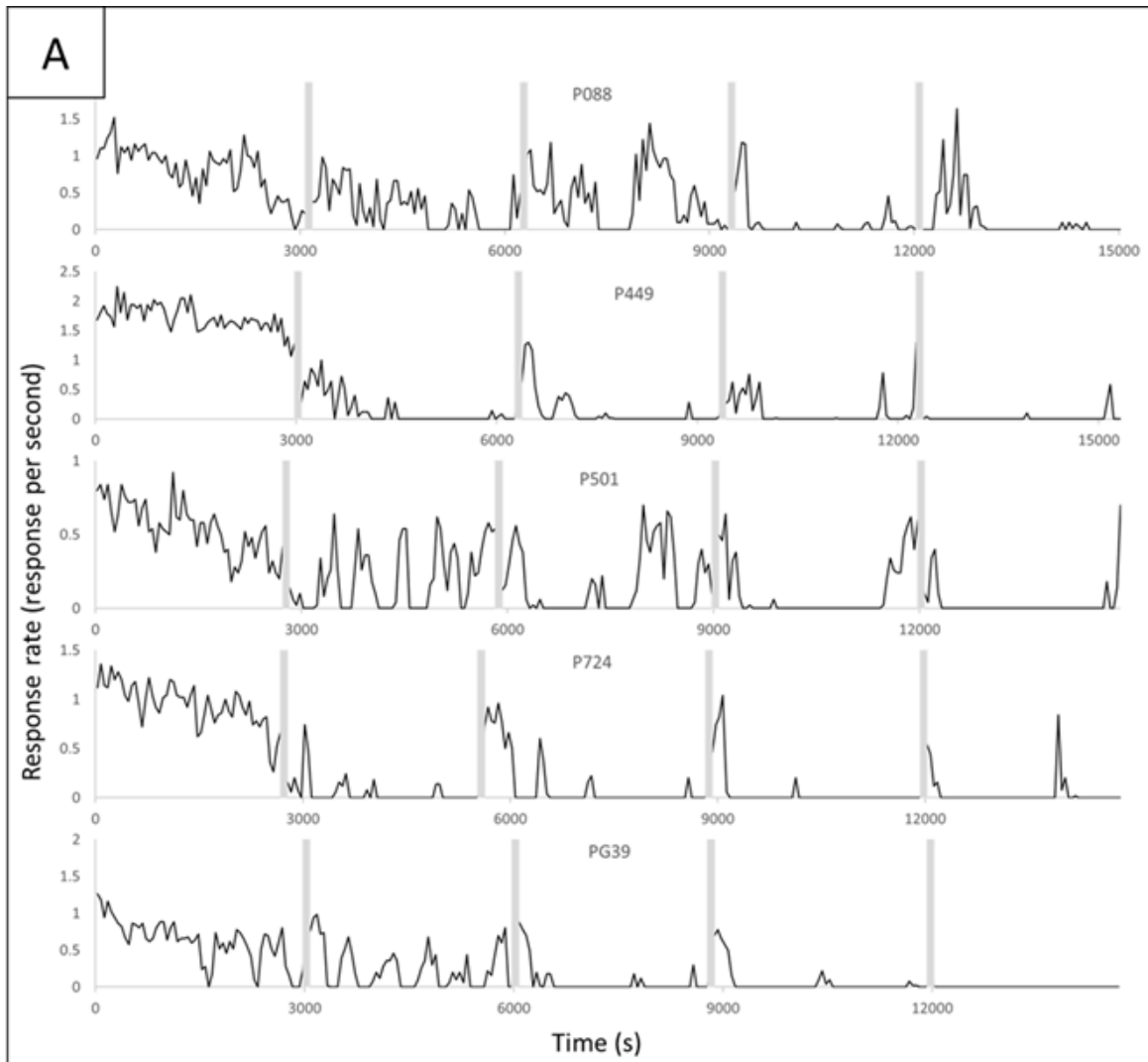
conditions from the eight birds that completed the entire experiment (mean comparison: $t(7)=0.31$; cv comparison: $t(7)=0.56$).

We can see in Figure 13 that in the two VIs, the main changes in rate happen early in the trial. In fact, it seems that birds need a few seconds to reach a certain rate which they will approximately maintain during the rest of the trial; there is also a tendency of early burst of responding which then let place to a steadier rate which can be seen with P449 in the two VI conditions, with P724 in the uniform exponential VI, and, with PG39 and P918 in the Fleshler & Hoffman VI.

Table 5B presents the means and coefficients of variation from the average number of responses in the 5-seconds bins on the last 3 long intervals, but, excluding the first two bins (corresponding to the first 10 seconds in the trial). Comparison on this new variable for the means still finds no difference ($t(7)=0.16$, ns), but coefficients of variation now appear to be significantly different ($t(7)=3.23$, $p=0.01$). When excluding the first 10 seconds, the uniform exponential VI sustains a response rate less variable, or flatter, than that sustained by the Fleshler & Hoffman VI.

In the same way that we looked at responding under the VI conditions, we look at responding during the extinction phase from the perspective of the sessions and from the perspective of the trial interval. Figure 14, on the two next pages, presents the response rate across the five extinction sessions. Panel A of Figure 14 shows the data of Group 1, for which extinction trials structure followed the uniform exponential VI condition. Panel B of Figure 14 shows the data of group 2, for which extinction trials structure followed the Fleshler & Hoffman VI condition. It can be seen that although responding dropped for all birds, little responding still happened in the 5th session of extinction (except with PG39, which did not respond in this last session). the evolution of the response rate across the sessions does not suggest differences dependent on the group and on the VI that preceded the extinction phase.

Figure 15, on pages 48 and 49, presents the same data as Figure 14 but with response rate given in function of time into the interval, and with sessions displayed as separate curves, along with the data from the last ten sessions of the previous condition (unique multi-session average curve of the graph), so to allow comparison of the shape of the response function as the extinction phase goes on. Because sessions were made of a rather small number of trials, running means were used to smooth



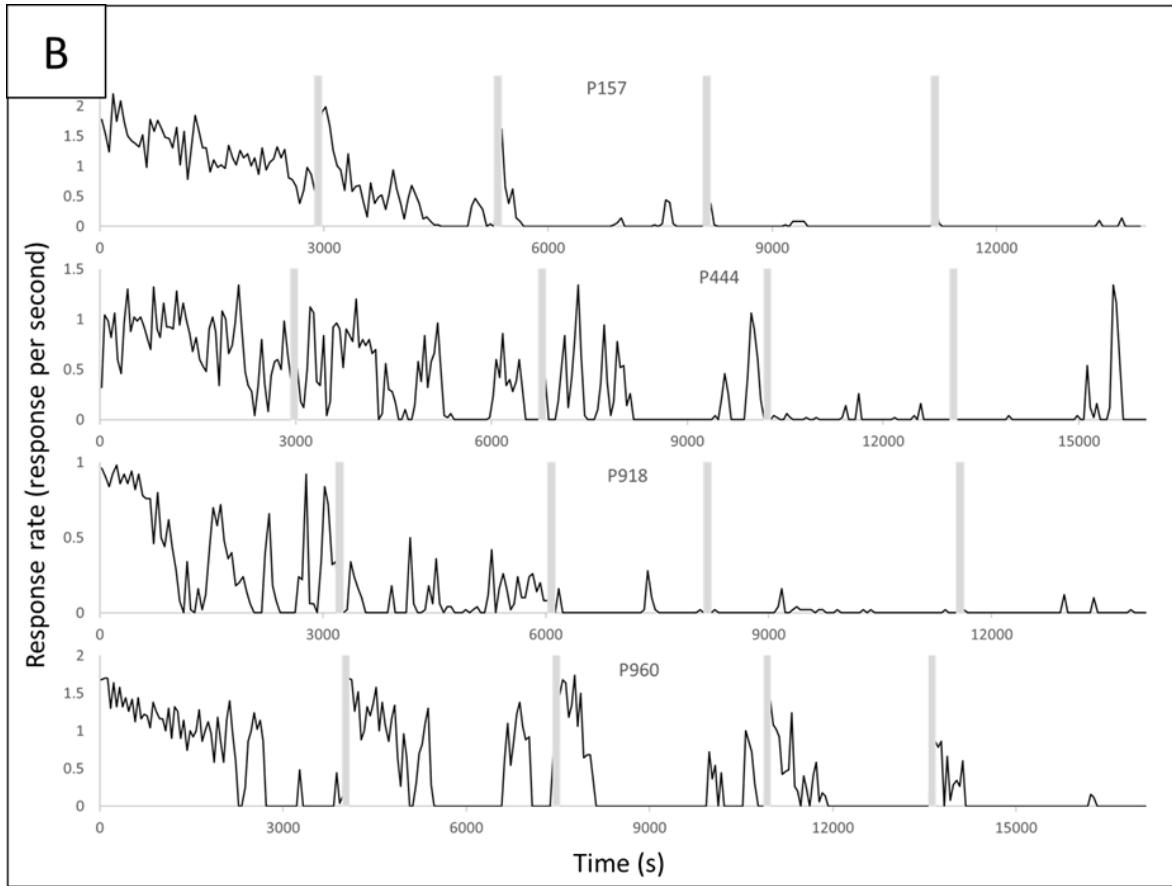
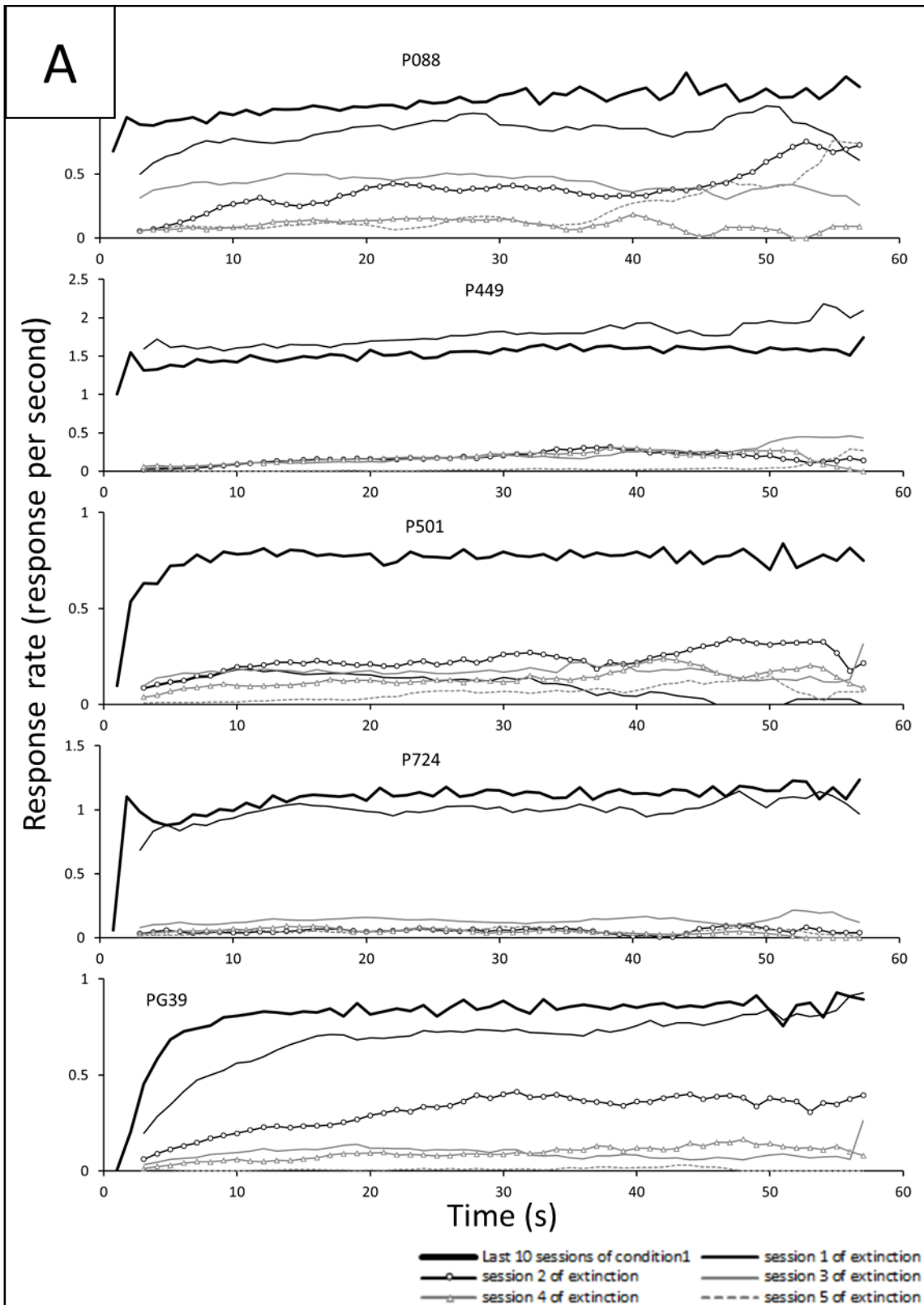


Figure 14: Response rate in function of time across the interval made from the trials of the 5 extinction sessions laid end to end, for each bird of Study 2 Experiment 1. Panel A shows the data of group 1, for which extinction trials structure followed the uniform exponential VI condition. Panel B shows the data of group 2, for which extinction trials structure followed the Fleshler & Hoffman condition. Rate is calculated from 10s bins. Gray vertical lines separate sessions.

undue variability. Despite this effort, the lack of data on the last third of the interval under the Fleshler & Hoffman VI extinction condition precludes firm interpretation. Nevertheless, it can be noted that the shapes of the response functions remained flat in Group 1 under uniform exponential VI extinction whereas the response functions in Group 2 under Fleshler & Hoffman VI extinction let appear bumps.



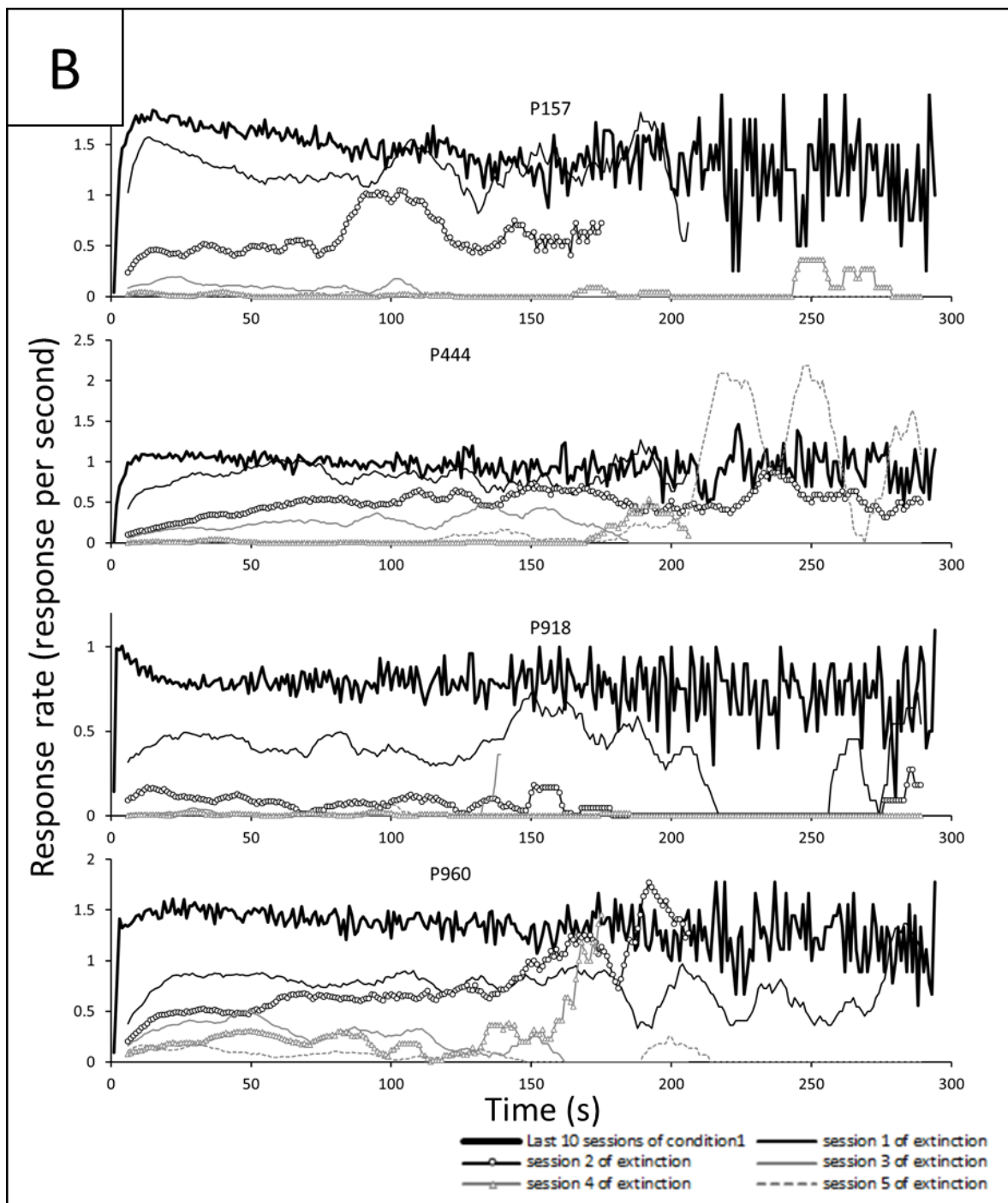


Figure 15: Response rate in function of time into the interval during the last 10 session of the first condition (thick line) and during each of the 5 extinction sessions, for each bird of Study 2 Experiment 1. Panel A shows the data of group 1, which extinction phase followed uniform exponential VI condition. Panel B shows the data of group 2, which extinction phase followed Fleshler & Hoffman VI condition. Empirically appreciated running means were applied to each single session curve: a 5s running mean in the case of extinction trials structure based on uniform exponential VI distribution (group 1 data) and a 11s running mean in the case of extinction trials structure base on Fleshler & Hoffman VI (group 2 data).

Discussion

No difference could be found between the uniform exponential VI and the Fleshler & Hoffman VI in regard to the behavior they maintain across sessions. Under both VIs, there is a tendency for response rate to decrease as time into the session elapses. Such within session pattern with operant responding and repeated reinforcement can be attributed to sensitization and habituation (McSweeney, Hinson and Cannon; 1996).

Within the trial interval however, the two VIs appear to sustain different behavioral patterns. While the uniform exponential VI, when presented at first, induced a rather flat pattern of responding characterized by a slight acceleration, the Fleshler & Hoffman VI, induced in most cases a roughly flat response function with a slight deceleration, which emerged even when the Fleshler & Hoffman VI was the second condition, and that seemed to have impacted the behavior of the birds of Group 2 in the uniform exponential VI despite the extinction phase. The deceleration in the Fleshler & Hoffman VI, could stem from the important gaps between the long intervals of that VI (see bottom graph of Figure 11). If we ignore the initial accelerative segment of the function relating the rate of response to the time into the interval, it appears that the uniform exponential VI sustains a flatter rate of responding than the Fleshler & Hoffman VI.

In regard to the observation made during the extinction phase, the Fleshler & Hoffman VI and the uniform exponential VI do not seem to induce different resistance to extinction despite the difference they present in their inclusion or not of unreinforced trials. We note on the side that It is possible that the bumps observed in the panel B of Figure 15, under the Fleshler & Hoffman VI extinction reveal expectations of reinforcement at specific times.

The slight advantage of the uniform exponential VI over the Fleshler & Hoffman VI may relate to the better constancy of the probability of reinforcement in time that the new schedule creates. Yet, although the uniform exponential VI is based on a distribution of intervals with truly constant rate of reinforcement, the response functions it sustains let appear deviations from constant response rate. Whereas the early acceleration that most birds present under the uniform exponential VI can certainly be attributed to a sort of warm up period, the quasi systematic slight and constant acceleration observed when the uniform exponential VI was the first condition can hardly be accounted for using an explanation of the constrain-on-responding type.

Experiment 2: Effect of different mean time to reinforcement in the uniform exponential VI

In the previous experiment we observed that, when presented as initial condition, the uniform exponential VI sustained a response function characterized by a small acceleration. This pattern is surprising when considering that under the uniform exponential VI the temporal contingencies are suppressed, and, from a theory-free reasoning there seem to be no reason to respond more as time passes in the trial interval. Though, we may try to account for this phenomenon using the theoretical framework that we used to account for the results of the experiment of Study 1.

From our conclusions of Study 1, supporting the idea that responding in interval schedules corresponds to the control of local rate of responding by local rate of reinforcement, we can suppose that the behavior of animals under the uniform exponential VI is still determined by the translation of reinforcement rates into response rates. And, from this perspective, because the translation process is not perfect, the response function obtained under the uniform exponential VI should present a divergence from constancy. In fact, the LeT which makes the translation of the rate of reinforcement into rate of responding, predicts a response function under the uniform exponential VI with a moderate acceleration resembling our results.

From the perspective of LeT, under the uniform exponential VI, all behavioral states among those that have a position in the series sufficiently early to be activated during training, will receive associative strength because reinforcement can happen at every time into the trial interval. On the other hand, there is considerable extinction pressed on the earlier behavioral states from the fact that extinction occurs at each trial for each behavioral states that had been activated previous to the behavioral state that gets activated at the time of reinforcement. This makes that, with the repetition of trials, emerges a pool of strong behavioral states comprising most behavioral states of the series but excluding a part of the early behavioral states of the series. As during training, reinforcement is constantly distributed at different point in time, the fulcrum point between behavioral states that are below and above the threshold for responding is very variable. This, associated to the variability of the spread of the activation of the behavioral states when considering, in turn, responding, makes that the model predicts a smoothly accelerative response function. And, because the mean and the variance

of the fulcrum point within the behavioral state series are proportional to the maximal interval length parameter, the shape of the response function is scalar invariant.

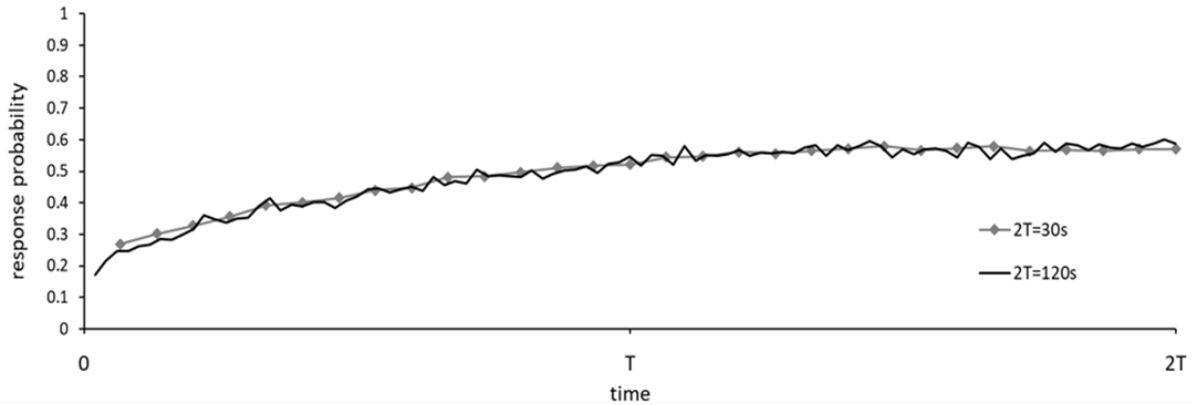


Figure 16: LeT simulations of responding under the uniform exponential VI. One simulation was made with a maximum interval equal to 30s (gray line with markers) and the other was made with a maximum interval equal to 120s (black line), in both cases the probability of reinforcement, p , equals 0.5. Simulations consisted in 1000 stat birds ran for 1200 trials; parameters were the same as in Machado, Malheiro & Ernhagen, 2009.

Figure 16, presents two simulations of LeT under the uniform exponential VI, using different maximum interval parameter ($2T= 30s$ and $2T=120s$), but the same probability of reinforcement $p=0.5$, and, plotted on normalized x axis. We observe the scalar conservatism with the superimposition of the two curves (early deviations and difference in noise in the curves relate to the difference in the number of behavioral states considered)⁸.

If the pattern observed in the uniform exponential VI condition of Group 1 in the previous experiment is due to an imperfect translation of the local rate of reinforcement into local rate of responding, as predicted by LeT, this pattern should remain unchanged, in regard to normalized time axis, when the maximum interval is changed.

To verify this, we trained four pigeons with three different maximum interval conditions of the uniform exponential VI. Because of the preliminary nature of part of this experiment, two pigeons ended having massive amount of training under a same condition; we take this as an opportunity to study the evolution of the behavior across long training under a same uniform exponential VI condition.

⁸ In all simulations we kept the same mean rate for the spread of activation: one behavioral state per second, like in Machado, Malheiro and Ernhagen 2009.

Methods

Subjects

Four adult pigeons (*Columba livia*) participated in the experiment, only two of them (P229 and P068) had previous experience with timing tasks, the two others (P785 and P746) were naïve. P746 and P229 broke their beaks a couple of times each and received less training, notably P746 was not trained in the last condition.

Apparatus

Four Med Associates Skinner boxes were used for this experiment. Each chamber was enclosed in an outer box equipped with a fan to circulate air and mask extraneous noises. In each chamber, a 7.5-W houselight located in the back panel provided general illumination. In the front panel, a 6 x 5 cm feeder opening, 3.5 cm above the floor and centered horizontally along the wall, provided access to mixed grain. When the feeder was activated, a 7.5-W light illuminated the grain. The front panel also included three keys, each 2.5 cm in diameter, arranged in a row, 9 cm apart, center-to-center, and 18.5 cm above the floor.

Softwares

The experiment was run using ABET II. Randomization of the intervals was made using Microsoft Excel and then pasted as lists in each session's program.

Procedure

Each bird went across three conditions of uniform exponential VI only varying according to the maximum interval parameter (the trial probability of reinforcement, p , was maintained at 0.5). 2T was set at 30s in the first condition, at 60s in the second condition, and at 120s in the third condition. Table 6 shows the number of sessions for each bird in each condition.

Table 6: Number of sessions of each bird in each condition of study 2 experiment 2.

birds	condition 1 (2T=30s)	condition 2 (2T=60s)	condition 3 (2T=120s)
P068	88	40	40
P229	38	30	28
P746	57	10	
P785	86	40	40

Trial structure was the same as in the uniform exponential VI condition of the previous experiment except for the ITI which is here 45s. That is, except for the duration of the ITI and the number of sessions, the second condition (2T=60s) of the present experiment corresponds to the uniform exponential VI condition of the previous experiment.

Analyzes

In this experiment we only look at the response rate in function of time from the trial interval perspective, computed in the same way as in the previous experiment.

Results

Figure 17 shows the response rate as a function of time into the interval for the four birds in each condition. Response rate was computed from the last twenty sessions of a condition (except in the case of P746, whose response function in the second condition was calculated from ten sessions it was exposed to). The gray curves of the second condition (2T=60s) can directly be compared to the response functions obtained in the uniform exponential VI of the previous experiment. Again, we find response functions that are roughly flat, though, one bird (P785) presents a steady acceleration more pronounced than that of any bird of the previous study. P068 and P229, present an acceleration on the early segment of the interval, followed by a steady rate in the case of P229 and by a slight deceleration in the case of P068. P746 shows an early burst, a fast acceleration followed by a deceleration, before reaching a steady rate. In the first condition, when the maximum interval was two times shorter (2T=30s), the patterns of P068 and P785 were more pronounced, the pattern of the

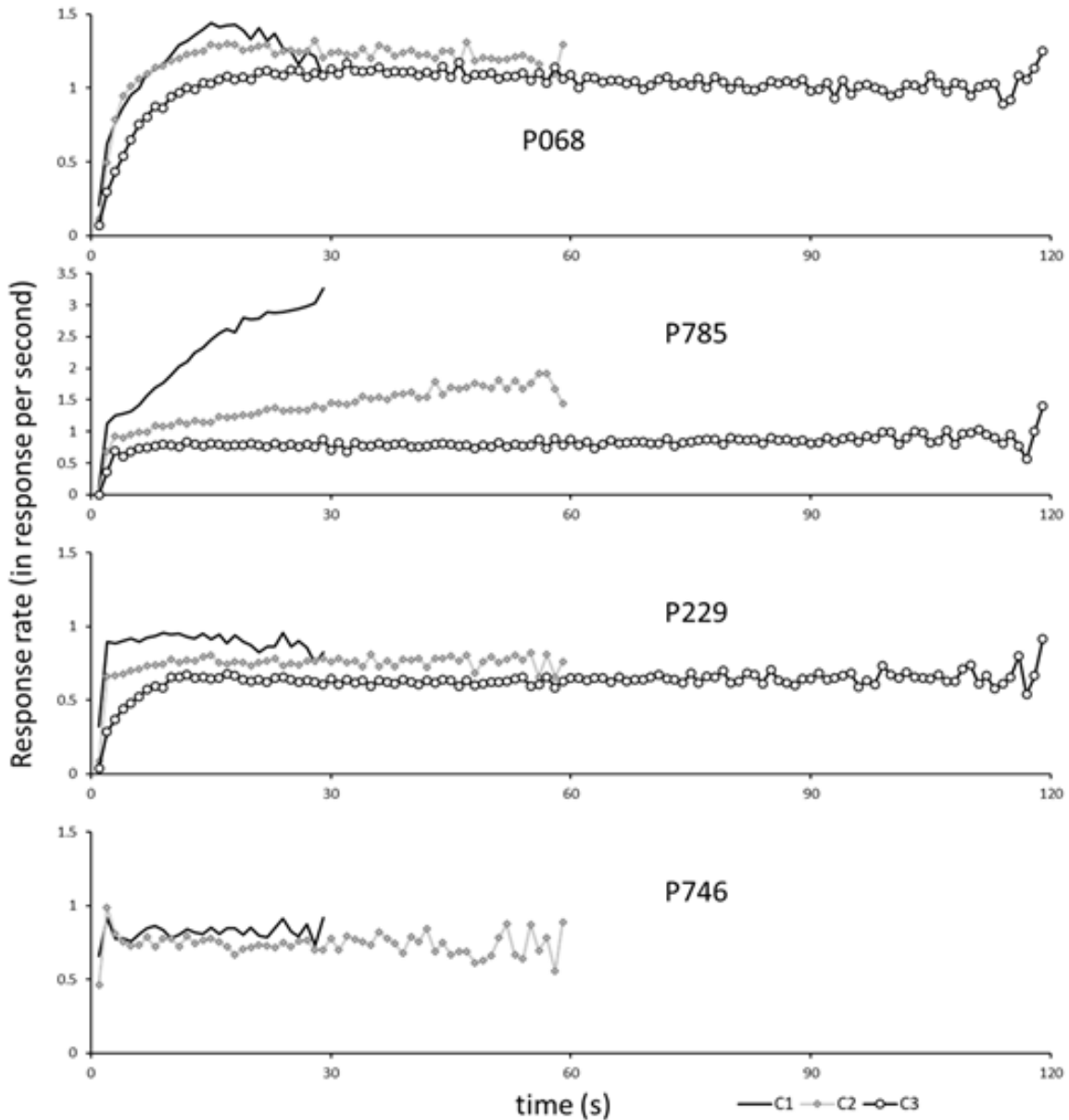


Figure 17: Response rate in function of time into the trial in the 3 conditions of uniform exponential VI in Study 2 Experiment 2: C1 ($2T=30s$), C2 ($T=60s$) and C3 ($2T=120s$).

response function of P229 showed a slight deceleration in the second half of the trial interval, while, the response function of P746 was already flat with the exception of an already present early burst. In the last condition (not completed by P746), where the maximum interval was 120s, response functions kept their overall shapes as compared to the second condition although with a much weaker acceleration in the case of P785. In all graphs, small but steep increase of the response rate at the end of the trial interval very likely corresponds to oscillations due to the lesser amount of data samples used to compute the rate on late segments.

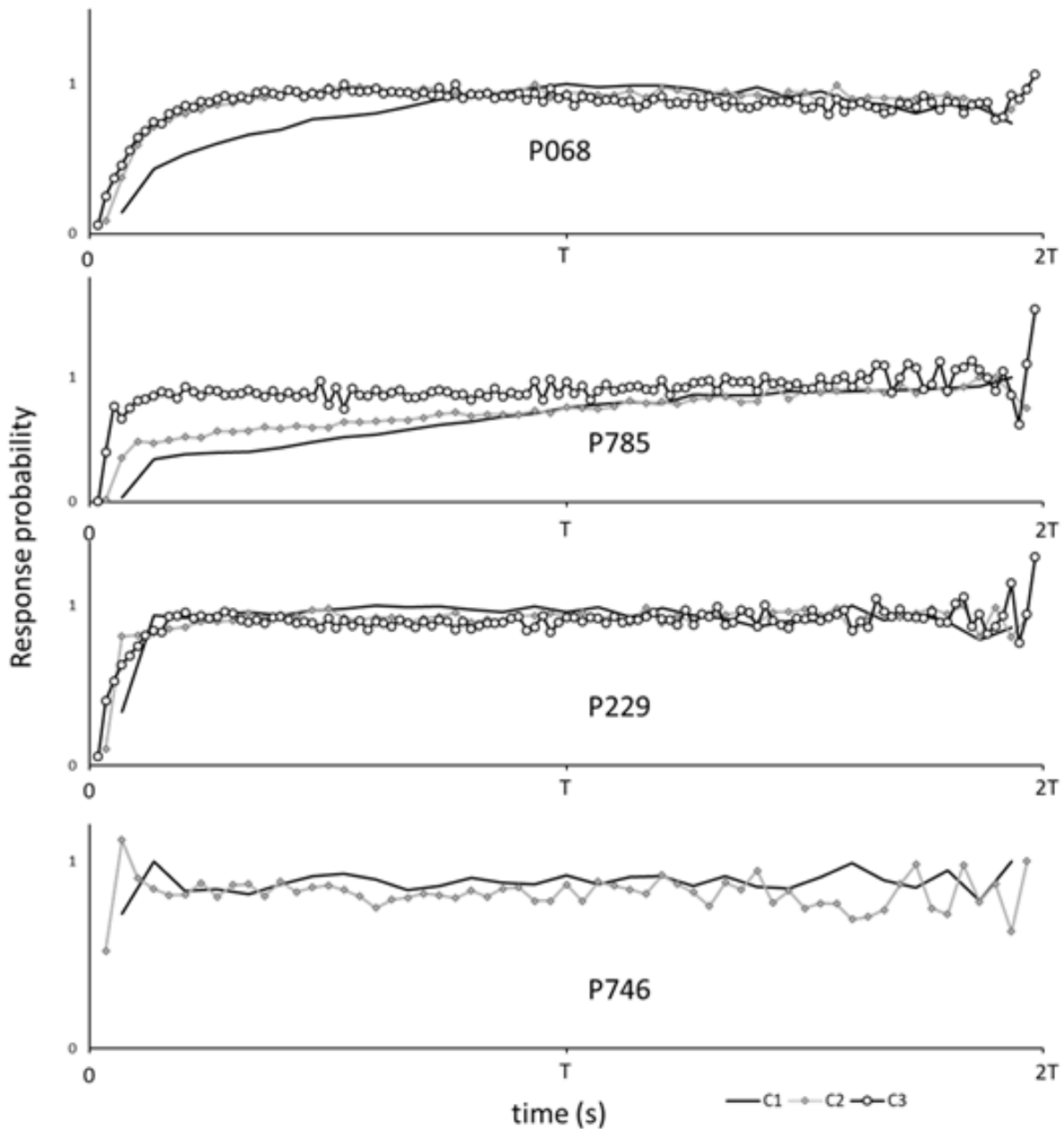


Figure 18: Response rate in function of time into the trial in the three conditions of uniform exponential VI in Study 2 Experiment 2: C1 ($2T=30s$), C2 ($T=60s$) and C3 ($2T=120s$), on normalized axes. Maximum rate was considered, excluding the first 10 seconds and the last quarter of the maximum interval, in order to prevent the influence of early high rate and late oscillations.

Figure 18 presents the same curves as Figure 17 but plotted in normalized x and y axes (the normalization of the y axis on the maximum rate basis was made with the exclusion of the data points from the first 10s and the last fourth of the trial intervals to neutralize the influence of early bursts). For P068 there is a particularly good match between the curves of the second and third conditions,

but that of the first condition presents a longer accelerative segment. Curves of P785 are dissimilar, while the curve of the first condition presents a steep acceleration, that of the second condition presents a less pronounced acceleration, and that of the third condition is almost flat. For P229, the three response functions obviously superimpose on their flat segments, and, they are not far apart on the early segments of acceleration. In the case of P746, the flat segments of the response functions of first and second conditions superimpose, while the early bursts, which superimposed in the previous figure in the absolute time, are now separated.

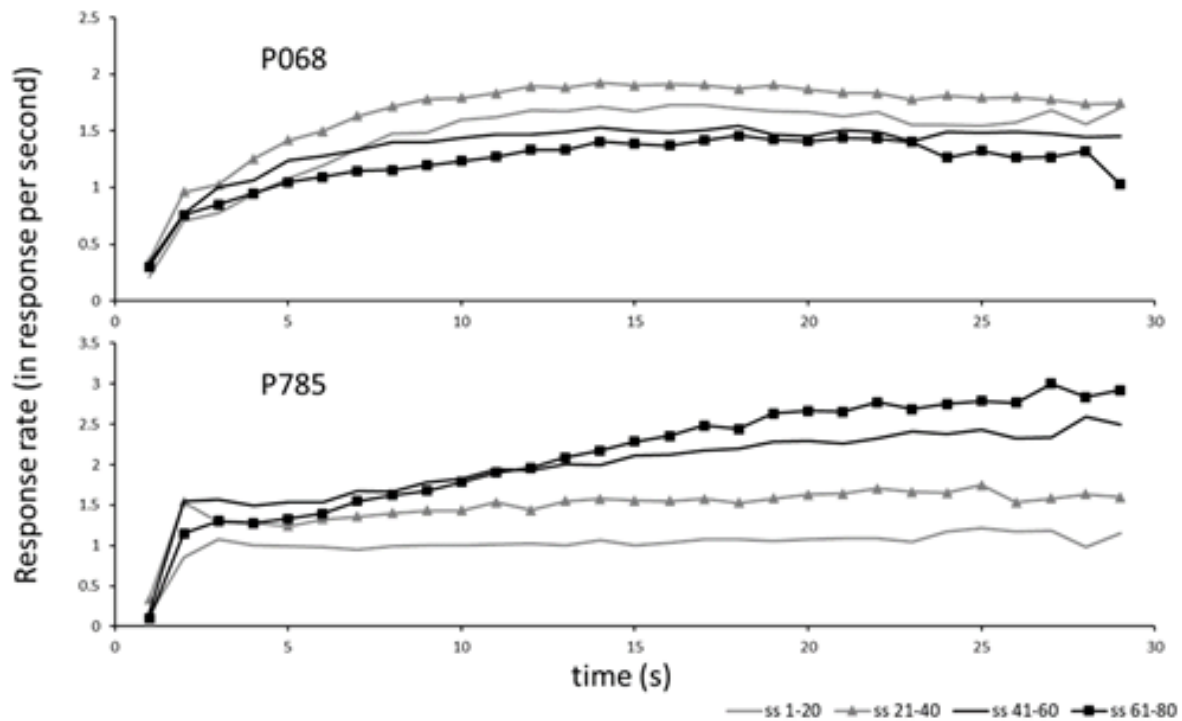


Figure 19: Response rate in function of time into the trial interval, for P068 (top graph) and P785 (bottom graph), in four periods of training along the first condition (uniform exponential VI; $2T=30s$) of Study 2 Experiment 2. Each curve corresponds to data averaged over 20 sessions (sessions 1 to 20, 21 to 40, 41 to 60 and 61 to 80).

Figure 19 depicts the evolution of the response function in the course of the first condition for P068 and P785, the two birds which were in that condition for more than 80 sessions. The pattern of responding of P068 shows a progressive overall decrease, with an elongation of the initial accelerative segment and an accentuation of the deceleration in the final part. The pattern of P785 changed more considerably: whereas it was flat in the first twenty sessions, it became an accelerative response function in the second block of twenty sessions, which became more and more pronounced in the two last sessions blocks.

Discussion

This experiment is, after the first experiment of this study, another demonstration that the uniform exponential VI sustains a roughly constant rate of responding. But whereas we expected to find, like with Group 1 of the previous experiment, slight accelerations, we found – at least in the second condition that was directly comparable to the exponential VI condition of the previous experiment –, with one bird a more pronounced acceleration, with two birds a flat response function, and with the last bird a deceleration. The decelerative pattern had only been found with birds of Group 2 in the previous experiment, which suggested a carryover from the Fleshler & Hoffman VI condition, we find here that it well may not have been the case. It so appears that, in the details, the uniform exponential VI sustains patterns of responding that differ across birds. But also, as we see in this experiment, the pattern of a bird can vary considerably during training within a condition. This variability is one difficulty concerning the question we wanted to address in this experiment as to whether the shapes of the response functions would present scalar invariance across different conditions of maximum interval.

P785, presents important changes within and across conditions, because of which no superimposition of the response functions can be observed in Figure 18. Yet, the strong accelerative pattern could be seen as a transient phenomenon since the bird started with a very flat response function which slowly became a steep increase during the first condition, reduced during the second condition and was again almost flat by the end of the third condition. P785 could so be compared with P746 and P229 which response function are very flat. This flatness of the response functions sort of dismiss our question regarding scalar invariance as those could be considered as shapeless, though patterns on the early segments can be examined. In the case of P785, in all conditions there is an early maximal acceleration on the three first seconds, which tend to match on the absolute time axis figure and so mismatch on the normalized scales figure. P746, presents a different pattern, an early burst, which also did not change in absolute time. On the other hand, the early acceleration of P229 in the three conditions gives a rather good match on the normalized axis figure. It is the case of P068, with a more apparent shape, which is the most intriguing, since it showed a perfect match in the normalized axis figure, but, for only two conditions.

Overall, the response functions observed in this experiment are largely flat whereas LeT simulations had led us to we expect a systematic deviation from constant responding. These flat response functions seem compatible with the theory-free assumption that constant rate of reinforcement, the unpredictability of the occurrence of reinforcement, would yield a constant rate of responding. At the same time, there are shapes and traces of scalar invariance in some of these response functions that could reflect the translation of local rate of reinforcement into local rate of responding. Could it be that the uniform exponential VI schedule reveals a limit for the “timing” processes to operate?

GENERAL DISCUSSION

In Study 1, we investigated the effect of the range of the intervals in peak-procedures on the shape of the function relating response rate to the time into the trial interval. From the initial pattern in peak that we replicated in a condition with fixed reinforced interval, we found that the response function broadened or narrowed systematically with increase or decrease of the range of the reinforced intervals. Furthermore, we found that the median of the responses was shifted to the right or to the left systematically with increase or decrease of the range of the reinforced intervals. These facts could not be accounted for by the averaging theory which predicted that the peak of the response function should have remained stable as long as the mean time to food was kept the same. The Minimax model (or two samples version of SET) and the Complete-memory model (or single sample version of SET) could account for the broadening of the pattern of the response function and for the shift of the median of the responses. But these two versions of SET both predicted – at least in peak-VI condition with intermediate interval range from $T/2$ to $(2T)/3$ –, an important positive skew of the response function with an early mode, where our observations found a bell-shaped response function with a mode slightly later than the mean interval T .

Only LeT predicted the broadening/narrowing and the shift of the median altogether with the shape of the response functions under the different conditions of peak-FI and peak-VIs. However, original parameter settings of the model underestimated the width of the response functions. But, by decreasing the extinction rate, we obtained simulations which accurately fitted the response functions of all the different conditions at the same time, showing the ability of LeT to account for the results. The LeT model is based on the principles expressed by Catania and Reynolds (1968), and, using an associative structure, it functions as a translator of local rate of reinforcement into local rate of responding. Thus, the ability of LeT to account for our results supports the view of Catania & Reynolds (1968) that in interval schedules the local rate of reinforcement controls the local rate of responding, and, supports the view that this translation is governed by associative mechanisms.

In Study 2 we investigated the shape of responding under constant-probability VIs. In the first experiment we compared the Fleshler & Hoffman VI and the uniform exponential VI and found that both sustained rather flat responding in function of time into the interval but that the former was associated in most cases with a deceleration while the latter was associated in most cases with an

acceleration. When excluding the initial 10s within the trial interval, the uniform exponential VI was shown to produce a steadier response rate than the Fleshler & Hoffman VI. We suggest that when an experimental setting allows ITIs, the uniform exponential VI should be preferred over the other constant-probability VIs if constancy in responding is wished.

If the flatness of the response functions under the uniform exponential VI in the two experiments of Study 2 could seem to strengthen the conclusion of Study 1 – the response functions were flat because birds tracked the probability of reinforcement, which is constant –, the interpretation of this result is in fact more complicated. Indeed, as we have seen in Study 1, the translation of the profile of reinforcement in time into a profile of responding in time would not be perfect, and, as shown in Figure 16, such process, under a constant-probability VI would yield an accelerative curvature. Although we found that response functions from Group 1 of Experiment 1 presented a shape compatible with the LeT simulation, in a majority of cases, the response functions from Experiment 2 seemed too flat to be compatible with the LeT simulation.

Besides the accelerative curvature, the LeT model predicted that this shape of the response function would remain scalar invariant under different maximum interval conditions. In the second experiment of Study 2, we manipulated the maximum interval in the uniform exponential VI and saw that the accelerative segment presented scalar invariance, but in only a few instances. Hence, it was not possible to firmly conclude whether the same process that we described in Study 1, with translation of local rate of reinforcement into local rate of responding in peak-FI and peak-VI schedules, was still in action with the constant-probability VIs. It rather seemed that under constant-probability VIs the process may reach a limit and that some birds may simply respond at a constant rate.

It is an attractive idea that the “timing” processes would reach a limit in conditions under which reinforcement was made unpredictable. But the instalment of the unpredictability of reinforcement relies on a whole distribution of intervals, of which birds experience only one at a time, one after the other. That is, even under the uniform exponential VI birds experience irregularities in the constancy of the rate of reinforcement. From this point of view, it would be surprising that the processes that we claim are working under other important variability conditions (e.g., peak-VI/40), would have cease to work under the uniform exponential VI.

Differently, the idea of a limit to the “timing” processes reveals an inconsistency when considering the problem of the account of the overall response rate, or mean rate. In Figure 17, it can

be seen that magnitudes of the response functions are ordered relative the overall rate of reinforcement (or mean rate of reinforcement) associated to the condition. With the all birds, responding is consistently higher in the first condition ($2T=30s$; 2 rf/min) than in the second condition ($2T=60s$; 1 rf/min), that itself sustained higher responding than the third condition ($2T=120s$; 0.5 rf/min). This observation of a relation between the overall response rate and the overall reinforcement rate in constant-probability VIs was also found by Millenson (1963). In the view of Catania and Reynolds (1968), the overall rate of responding is accounted for by the local rate of responding which is determined by the local rate of reinforcement. In constant-probability VIs (and in all VIs), the overall rate of reinforcement would determine the overall rate of responding because the profile of local rate of reinforcement is bound to this overall rate of reinforcement and that the profile of local rate of reinforcement determines the profile of local rate of responding, which makes the overall rate of responding. If there is no longer translation of the local rate of reinforcement into local rate of responding there could no longer be the adjustment of the overall rate of responding to the overall rate of reinforcement. From any other reasoning it is not possible to account for the relation between overall rates, of reinforcement and of responding, if the amount of time it takes to obtain a reinforcer is not “timed” in some way.

An account for the constancy of the rate of responding observed with part of the birds under the uniform exponential VI may in fact be conceived with the LeT framework. Consistently with what we noticed in Study 1, it is possible that our simulation of the uniform exponential VI using the parameters of Machado, Malheiro and Erlhagen (2009), overestimated the extinction rate. With a lower extinction rate, it is possible that the uniform exponential VI condition leads almost all the behavioral states to acquire an association with the operant response strong enough to sustain responding, which would produce flat response functions. The variability of the sensitivity to extinction among pigeons may explain that we found under the same uniform exponential VI schedule, response functions characterized by an acceleration and response functions that were almost perfectly flat. Actually, individual response functions from Study 1 under the peak-VI conditions with large interval range varied in the same way, with some birds showing a defined left limb (an acceleration to a mode) where some other birds showed a plateau, a flat responding segment. This plateau pattern is particularly striking with P451 in the peak-VI0/40 condition (see below, the top graph of Figure 20), it affected the response function until about the time of the maximal interval while the rest of the response function and the

response functions of P451 in the other conditions show evident shapes from control by temporal contingencies (e.g., the right limbs of all the response functions of P451 are ordered consistently with the ranges of intervals associated to the different conditions). It is possible that flat segments relate to a ceiling effect of the effect of local reinforcement.

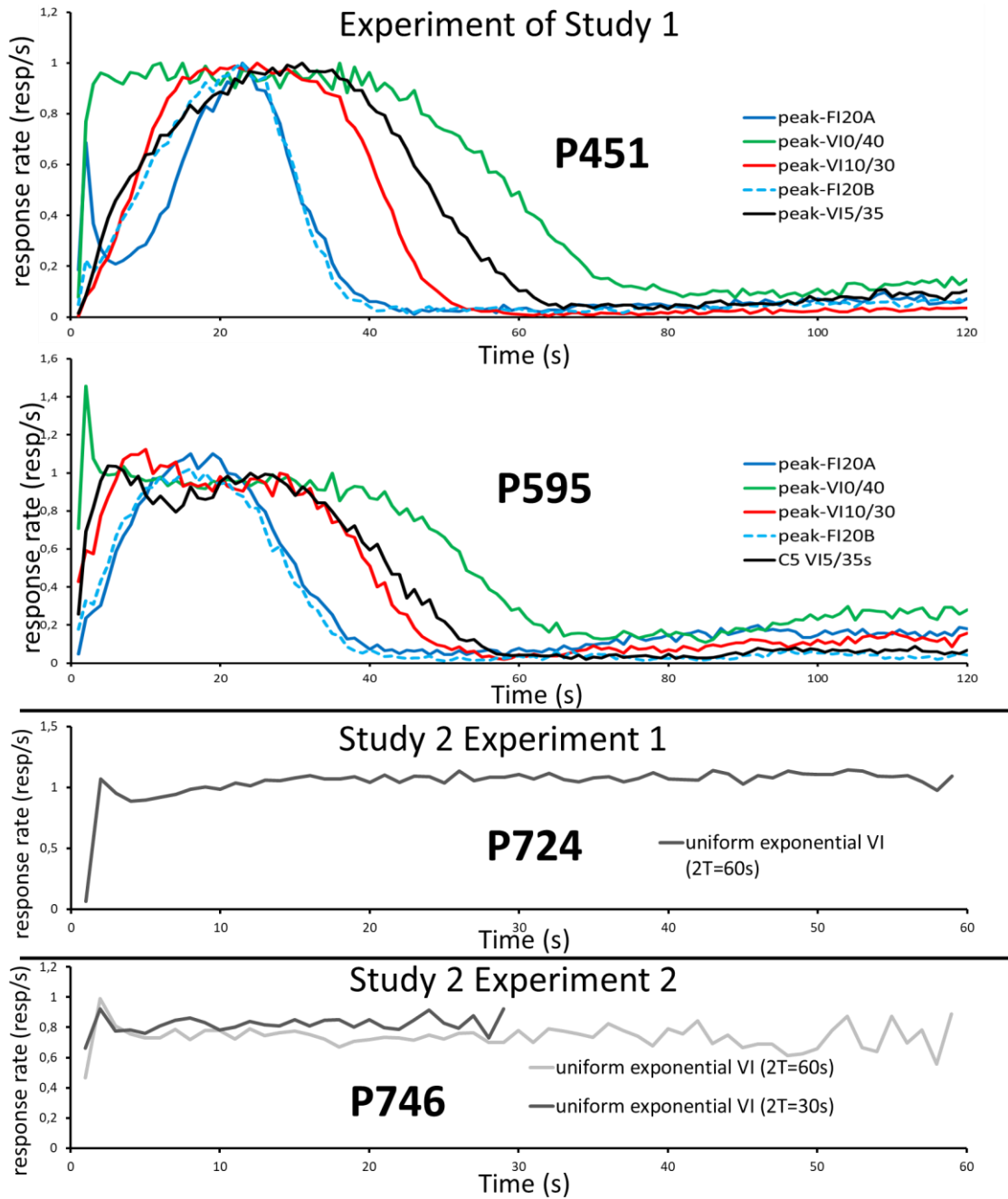


Figure 20: Instances of individual response functions presenting flat responding along with early high rate of responding, from the three experiments of this thesis. Graphs are reproductions from previous figures.

An alternative explanation to the flat segments could be that birds were anticipating the forthcoming high rate of reinforcement, leading them to respond at a rate close to the response rate that sustain the higher reinforcement rate expected, from the onset of the trial until the time of the end of the high local rate of reinforcement. This possibility is supported by the fact that flat segments appear more often with birds that also present the early burst phenomenon. Figure 20 shows such cases from the three experiment of this thesis. P451 (top graph of Figure 20), presented a very flat segment from the beginning of the trial time in peak-VI04/40, and, presented a strong early burst in the initial peak-FI20A; P595 (second graph from the top in Figure 20), presented early high rates in peak-VI0/40 as well as in peak-VI5/35 and peak-VI10/30, which were followed by flat segment in peak-VI0/40; P724 (third graph from the top in Figure 20), in the uniform exponential VI with $2T=60s$, showed an early burst and then a flat rate; P746 (bottom graph of Figure 20), in two conditions of uniform exponential VI ($2T=30s$ and $2T=60s$), presented an early burst followed by a flat rate. These phenomena are consistent with the observations of Ludvig, Balci and Spetch (2011) showing that the left limb of the response function in the peak-procedure could be altered by motivational factors without changing the peak time. This motivational responding, also discussed by Galtres, Marshall, Kirkpatrick (2012) and Daniels & Sanabria (2017), is not incompatible with the translation of the local rate of reinforcement into local rate of responding, but, it would come to partially override this rate translation when it occurs.

Under the uniform exponential VI we did not only observe flat or accelerative response functions, in some cases we also observed patterns characterized by a deceleration. To the exception of P068 of the second experiment of Study 2, every noticeably decelerative response functions were those of pigeons of the first experiment of Study 2 which went in the Fleshler & Hoffman VI prior to the uniform exponential VI. The deceleration could have in a first place affected responding under the Fleshler & Hoffman VI from the fact that under this VI there were important gaps between reinforcement opportunities associated with the longest intervals. And it seems that the pattern remained in the following uniform exponential VI condition. Likewise, it is possible that the curvy aspect of the response function of P068 relates to the history of past experiences of the bird which had been trained under a peak-procedure. Although we could not find studies in the literature about the effect of experience history on the profile of local response rate in VIs, it exists evidence of the influence of history on response rate in VIs (Ono and Iwabuchi, 1997). In the particular instance of P785 of the second experiment of Study 2, the bird evolved from a stable flat response pattern to a stable steep

acceleration, before changing back to the stable flat response pattern. The clue that we have to discuss this strange performance, is that the bird changed its responding fashions, with the accelerative response function relating to the habit of the bird to direct its head towards the feeder after every peck, but only at the beginning of the trial, before switching to a very high rate of responding. All these deviations from flat responding or smooth accelerations, we thus believe, were not determined by the temporal contingencies that we were manipulating.

At the end, the present thesis makes a strong case for the validity of the rate translation principle introduced by Catania and Reynolds (1968). Although we could only determine the reality of the translation of local rate of reinforcement into local rate of responding in schedules with limited variability of the time to food, we also have seen that the principle still allows an account of responding under constant-probability VIs. More investigations of the local rate of responding under VIs associated with diverse profiles of local rate of reinforcement should refine our conclusions. Though, with our current knowledge, the ability of the rate translation principle to offer an account of responding in interval schedules in general leads us to argue that it might be considered, next to the scalar property, as the other main feature of “timing”.

The rate translation principle is equivalent to the idea of gradient of temporal generalization and takes roots in the behaviorist realm. Whereas the rate translation principle is at the core of the behavioral conception that explains the control of behavior by temporal contingencies with associative mechanisms, it is absent from the cognitive conception that takes timing as the action to time events dependent upon operations on temporal memories. At the same time that we argue in the sense of the validity of the rate translation principle of Catania and Reynolds (1968) we argue in the sense of the validity of the behavioral conception and we argue against the validity of the cognitive conception. Consistently, we argue against the use of the term timing as it relates to the cognitive conception and as it bears a meaning of exact time hardly compatible with the study of the phenomena related to variable temporal contingencies as those that have been the object of our investigations.

In the studies of this thesis we have not consider the question of the influence of the contingencies created by the schedules upon the inter-response times (IRTs), whereas, there are findings in the literature suggesting that VI performances would be partly determined by the effect of differential reinforcement of IRTs (Skinner, 1938; Anger, 1956; Morse, 1966; Shimp, 1967). Yet, these

studies which have shown impact of schedule contingencies over IRTs distributions were all using modified VIs having reinforcement dependent upon specific IRT categories.

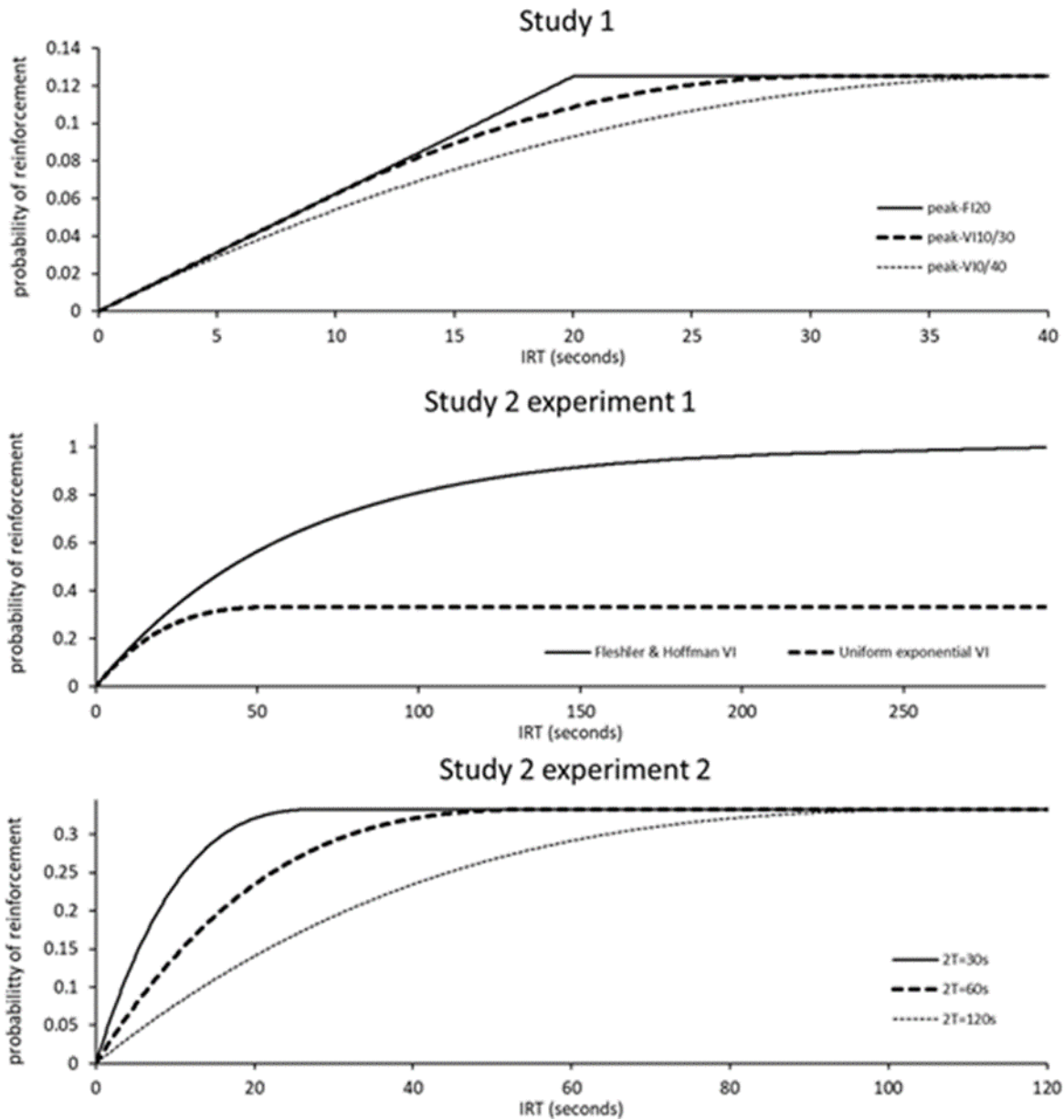


Figure 21: Probability of reinforcement in function of IRT length in the three experiments of the thesis. Within each experiment the maximal IRT length considered correspond to the maximum reinforced interval, all conditions taken together. In the condition where there are no empty trials (Fleshler & Hoffman VI), the probability of reinforcement reaches 1 when IRT length crosses the duration of the maximal interval, in the conditions where there are empty trials (peak-FI/VIs and uniform exponential VIs), the probability of reinforcement reaches an asymptote corresponding to the proportion of time spent in reinforced trials.

In interval schedules, in general, the longer a subject waits since the last response, the more likely is the next response to be reinforced. In other words, in all interval schedules, longer IRTs are associated with higher probabilities of reinforcement than are shorter IRTs. But the differential of the

probability of reinforcement of IRTs depends on the distribution of the intervals of the schedule. Figure 21 shows the functions relating IRTs to their probability of reinforcement for each condition (except the intermediate peak-VI5/35) in our three experiments (for the methods of calculation, see Appendix 2). Within each experiment, each condition is associated with a different profile of the probability of reinforcement in function of IRT length.

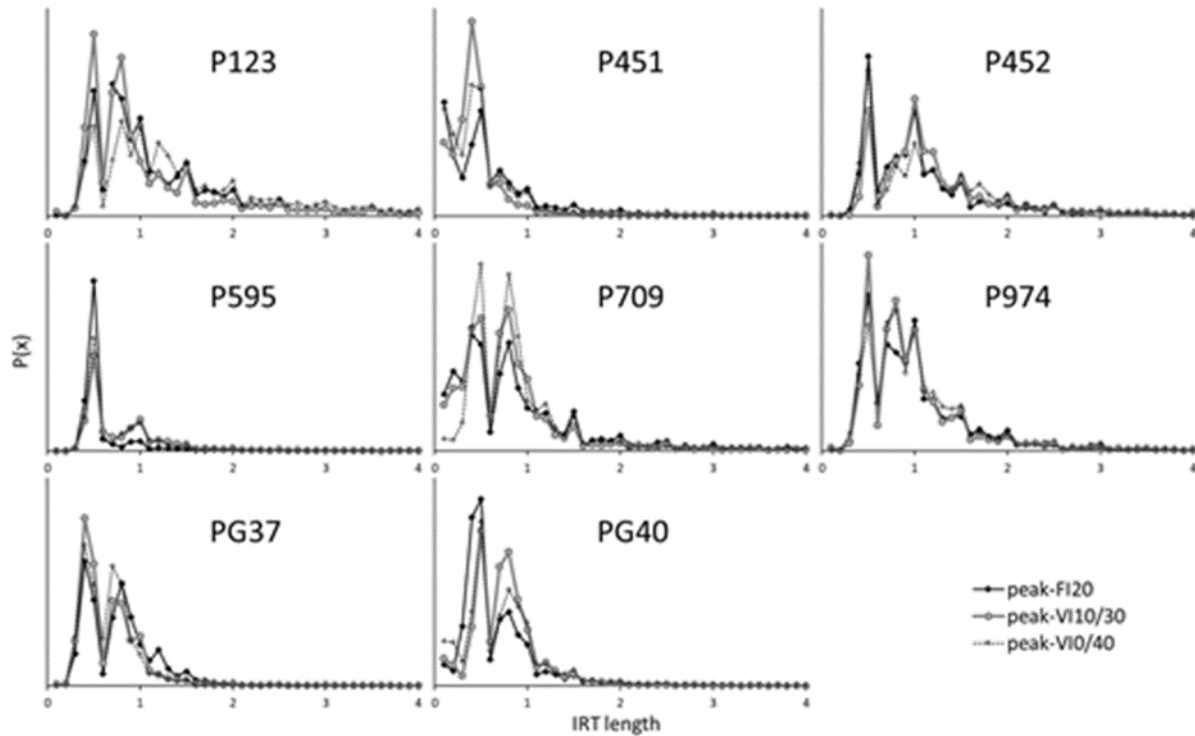


Figure 22: Distributions of the IRTs emitted during the last five sessions of the three first conditions (peak-FI20, peak-VI10/30 and peak-VI0/40), for each bird of Study 1. IRTs are gathered in 0.1s bins

Figure 22 for the experiment of Study 1, Figure 23 for the first experiment of Study 2 and Figure 24 for the second experiment of Study 2, show the distribution of the IRTs of each bird, in each condition, in each experiment (the only condition not represented is the intermediate peak-VI5/35 from Study 1). Except those of P785 which had shown dramatic evolution in its responding, the IRTs distributions remained stable across conditions. That is, despite the changes of the contingencies as to the differential probability of reinforcement of IRTs, IRTs distributions remained stable. In the experiments of this thesis, consisting of different types of interval schedules, responding was determined by the local rate of reinforcement, and was not noticeably affected by the other contingencies of the schedules related to the differential reinforcement of IRTs.

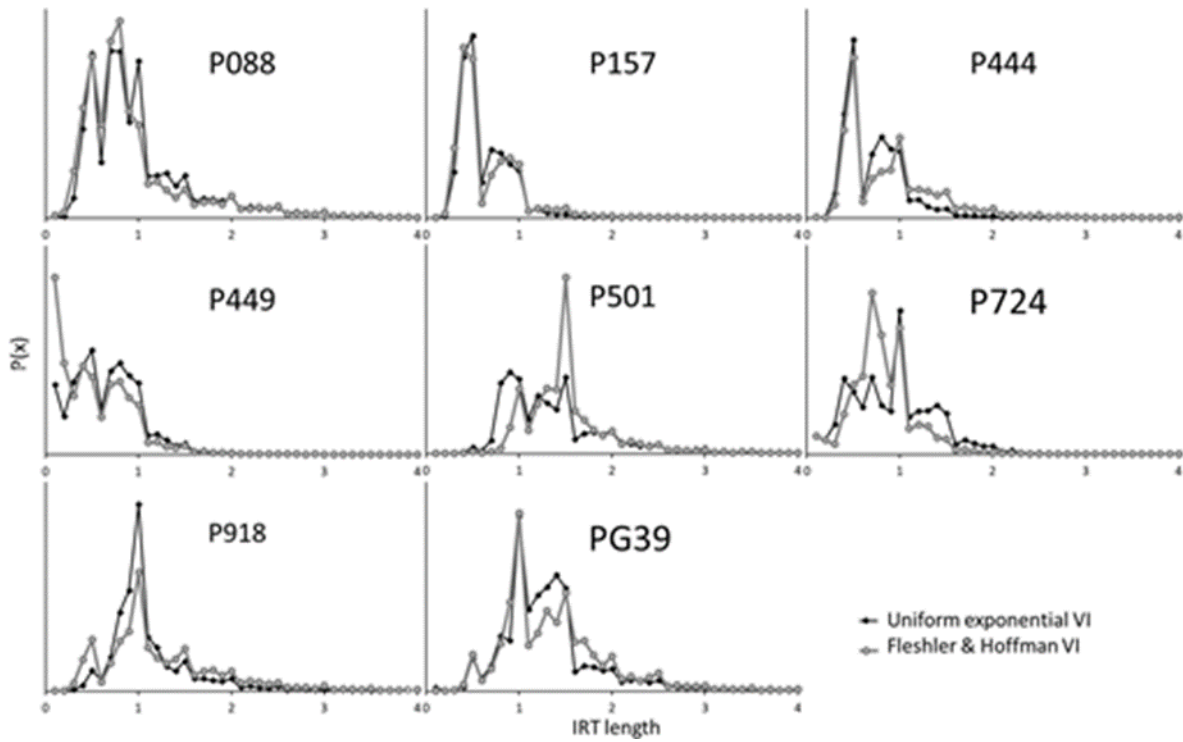


Figure 23: Distributions of the IRTs emitted during the last five sessions of uniform exponential VI (black line) and Fleshler & Hoffman VI (gray line), for each bird of Study 2 Experiment 1. IRTs are gathered in 0.1s bins.

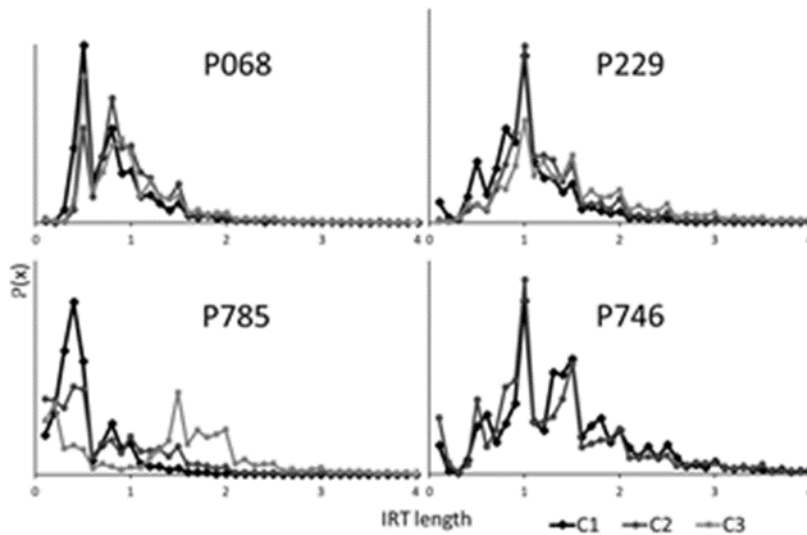


Figure 24: Distributions of the IRTs in Study 2 Experiment 2 at the end of each condition (C1: $2T=30s$; C2: $2T=60s$; C3: $2T=120s$) for the four bird of the experiment. IRTs were collected over 5 sessions and gathered in 0.1s bins.

APPENDIX 1: Novel representation of the number e derived from the uniform exponential VI distribution

The uniform exponential VI of Bugallo, Machado & Vasconcelos (2018) creates a constant hazard rate from the combination of a one distribution of reinforced intervals and of a distribution of unreinforced intervals. Interestingly, whereas these reinforced and unreinforced intervals form together a uniform distribution, the same intervals, when laid end to end until reinforcement, form an exponential distribution (Bugallo et al, 2018), as presented in Figure 25. In what follows, we extract a representation of the number e from this relationship.

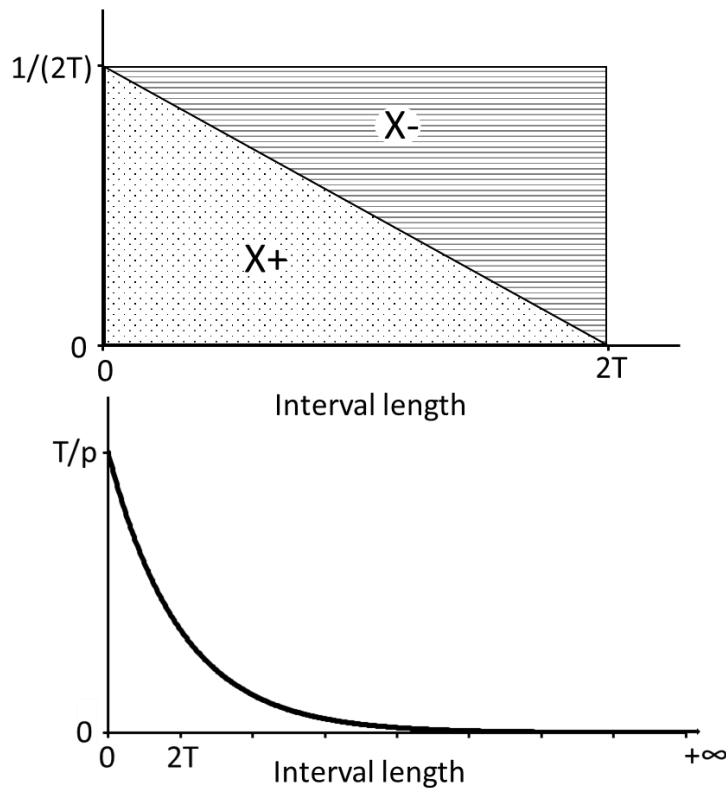


Figure 25: Distribution of the intervals of the uniform exponential VI. The top graph represents the combination of the distributions of reinforced intervals (area with dots) and unreinforced intervals (area with horizontal lines), which together form a uniform distribution from 0 to 2T, with mean interval T. When laid end to end until reinforcement occurs, the intervals drawn from this combination of distributions yield an exponential distribution (see Bugallo et al, 2018). The bottom graph represents this resultant exponential distribution. (In our example the, p , the probability of a reinforced trial was set at 0,5.)

Let us consider the exponential distribution defined by $i(x) = \lambda \cdot e^{-(\lambda \cdot x)}$, wherein x is interval length and λ the rate of reinforcement, in the context of its use for a VI. By integrating the function $i(x)$, from x to $+\infty$, we obtain a function, whose value at any x , corresponds to the proportion of intervals

from the initial distribution that are longer than x , defined by $j(x) = e^{-\lambda x}$ ($j(0) = 1$; at $x = 0$, all intervals from the initial distribution are longer than x).

We know from Bugallo, Machado & Vasconcelos (2018), that the intervals of the uniform exponential VI, when intervals are laid end to end until reinforcement follow an exponential distribution with $\lambda = T/\rho$ (ρ is the probability of a reinforced trial, it can only be comprised between 0 and 0,5 as shown in Bugallo et al, 2018); we call Y the variable corresponding to the length of the interval obtained when intervals from the uniform exponential VI are laid end to end until reinforcement. We consider the distribution of a uniform exponential VI with parameters $2T = 1$ and $\rho = 0.5$ (Figure 26 presents separately the density functions of the reinforced and unreinforced intervals obtained under these parameters); it corresponds to an exponential distribution with $\lambda = 1$.

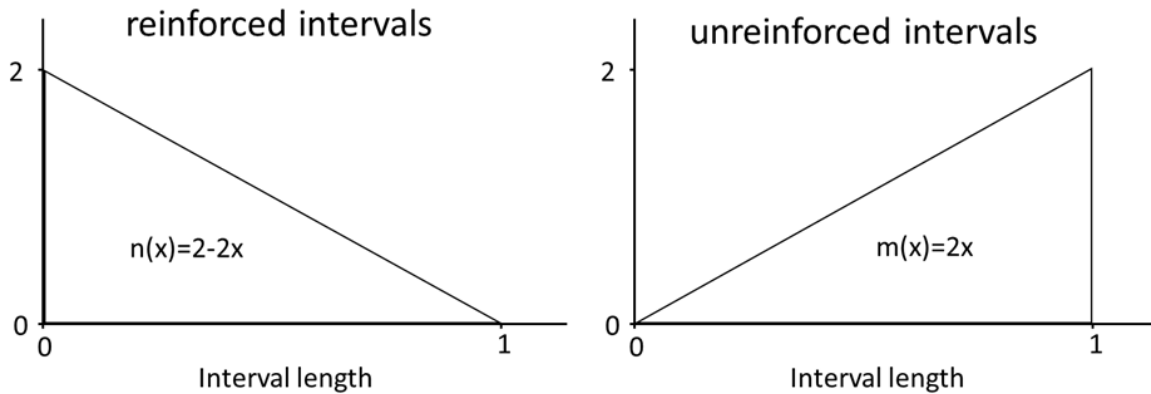


Figure 26: Separated distributions of the reinforced intervals (graph on the left) and of the unreinforced intervals (graph on the right), with parameters of maximal interval set at $2T=1$ and of reinforced trial probability set at $p=0,5$.

Let us then consider the outcome of $x = 1$ in $j(x)$ when $\lambda = 1$: ($e^{-1 \cdot x}$), it equals e^{-1} . That is, we have a height which is equal to $1/e$ and that corresponds to the proportion of the Y intervals longer than 1. And, we can calculate this height by summing, from $n=1$ to n is infinity, the products of, $P(Y=nX)$, the probability that an inter-reinforcement interval is made of n intervals, with $P(Y>1 | nX)$, the probability that an inter-reinforcement interval is longer than 1 given that it is made of n intervals. Of this sum we will take the inverse to obtain the number e .

Intervals are drawn randomly, one after the other, from the combined distribution of reinforced and unreinforced intervals, and the final interval, the reinforced interval, has always one chance in two to be drawn at each round. Therefore, the probability associated to the number of X intervals in a Y interval is defined by a geometric distribution, and,

$$P(nX) = \frac{1}{2^n}$$

The probability that an inter-reinforcement interval is longer than 1 given that it is made of n intervals, needs to be decomposed. $P(Y > 1 | nX)$ corresponds to the sum of, the probability that the sum of the unreinforced intervals is greater than 1, and of the probability that, Y is greater than 1 and the sum of the unreinforced intervals is smaller than 1. We have

$$P(Y > 1 | nX) = P\left(\sum_{n=1}^{n-1} X_{-n} > 1\right) + P\left(Y > 1 \cap \sum_{n=1}^{n-1} X_{-n} < 1\right)$$

(where X_{-n} is the interval length of unreinforced interval number n), which gives,

$$P(Y > 1 | nX) = P\left(\sum_{n=1}^{n-1} X_{-n} > 1\right) + P\left(\sum_{n=1}^{n-1} X_{-n} < 1\right) \times P\left(Y > 1 | \sum_{n=1}^{n-1} X_{-n} < 1\right)$$

We first consider the probability that the sum of the X -s is smaller than 1. The probability that the first X - is smaller than 1 is 1. To find the probability that the sum of the two first X -s is smaller than 1, we calculate the probability that this sum is greater than 1, which we will subtract to 1. For the sum of the two first X -s to be greater than 1, the length x obtained from the distribution of the first X - (the same for any X -: $m(x)=2x$; see Figure 26), must be associated to a second interval of a length greater than $1-x$. Within the distribution of the second X - on 0 to 1, we have to take at any x the proportion of intervals that would be longer than 1 if associated to a first X - of this certain x length. That is,

$$P(X_{-1} + X_{-2} > 1) = \int_0^1 \left(2x \int_{1-x}^1 2x dx \right) dx$$

Thus, we have,

$$P(X_{-1} + X_{-2} < 1) = \left[1 - \int_0^1 \left(2x \int_{1-x}^1 2x dx \right) dx \right]$$

To obtain then the probability that the sum of three X -s is smaller than 1, we multiply the probability that the sum of the first two X -s is smaller than 1 by the probability that the adding of the third X - would still not make 1 when the sum of the previous two X -s was smaller than 1. On the 0 to 1 segment, the distribution of the summed intervals corresponds to the shape of a power of x ; at any point of the area of the previous distribution of the summed X -s, on the 0 to 1 segment, each new X - comes to add its own distribution of intervals. Thus, we find the density function of the intervals formed by the sum of n X -s ($n > 1$), on the 0 to 1 segment,

$$z(x) = \frac{x^{2n-3}}{\int_0^1 x^{2n-3} dx}$$

And with it, we obtain the probability that the sum of n X-s is smaller than 1,

$$P\left(\sum_{n=1}^n X_{-n} < 1\right) = P\left(\sum_{n=1}^{n-1} X_{-n} < 1\right) \times \left(1 - \int_0^1 \left(\frac{x^{2n-3}}{\int_0^1 x^{2n-3} dx} \int_{1-x}^1 2x dx\right) dx\right)$$

with,

$$P(X_{-1} < 1) = 1$$

we resolve,

$$P\left(\sum_{n=1}^n X_{-n} < 1\right) = \frac{1}{\prod_{n=1}^n n(2n-1)}$$

We now consider the probability that the sum of all intervals, including the reinforced one, is greater than 1, given that the sum of the unreinforced intervals is smaller than 1. Pictured within the distribution of the reinforced intervals defined by $n(x)=2-2x$ (see Figure 26), it corresponds at any x to the proportion of reinforced intervals that will be greater than 1 when summed with an interval from the distribution of the sum of the X-s.

$$\begin{aligned} P\left(Y > 1 \mid \sum_{n=1}^{n-1} X_{-n} < 1\right) &= \int_0^1 \left((2-2x) \int_{1-x}^1 \frac{x^{2n-1}}{\int_0^1 x^{2n-1} dx} dx \right) dx \\ &= \frac{n}{n+1} \end{aligned}$$

Thus, we have all elements to obtain the probability that an inter-reinforcement interval is longer than 1. Being careful to stop the pi products from the probabilities concerning only the X-s at $n-1$ in this equation where n is the total number of intervals, we have:

$$P(Y > 1 | nX) = \left(1 - \frac{1}{\prod_{n=1}^{n-1} n(2n-1)}\right) + \frac{1}{\prod_{n=1}^{n-1} n(2n-1)} \times \frac{n}{n+1}$$

which simplifies in,

$$P(Y > 1 | nX) = 1 - \frac{1}{n \prod_{n=1}^{n-1} n(2n-1)}$$

and simplifies further in,

$$P(Y > 1 | nX) = 1 - \frac{1}{n(2n^2 - 5n + 3)!}$$

Finally, we assemble

$$e = \frac{1}{\sum_{n=1}^{\infty} \frac{1}{2^n} \left[1 - \frac{1}{n(2n^2 - 5n + 3)!} \right]}$$

APPENDIX 2: Methods of calculation for the probability of reinforcement of IRTs

In this appendix we explain how the probabilities of reinforcement as a function of IRTs length were calculated for each type of schedule used in this thesis. Our method is inspired by the Appendix 1 of Catania & Reynolds (1968), which we extend to deal with empty trials and continuous distribution of intervals.

An IRT is defined as the duration between two consecutive responses (a latency, the duration between trial onset and the first response is not an IRT); a reinforced IRT is an IRT which terminal response gives access to reinforcement. We call the first response of an IRT a start time, which can be used to grasp IRT occurrence. In an interval schedule, an IRT is reinforced when its start time occurs at a time into the trial interval which distance to reinforcement criterion is smaller than the length of the IRT considered. Hereof, the probability of reinforcement of an IRT can be defined as the ratio of, all start times belonging to a reinforced IRT, over all possible start times.

Yet, it is not possible to directly quantify all possible start times. Take the simplest example of an FI20 under which we want to know the probability of reinforcement of a 5s IRT. There are in fact an infinity of moments at which start times belonging to a reinforced IRT, or start times in general, can be emitted. To circumvent this, we consider the different segments of intervals along which the two categories of start time belong. In our example, the start times that will lead to a reinforced IRT are those happening from 15s to 20s into the interval, that is along a 5s segment, while start times in general can happen between 0s and 20s into the trial interval (responses emitted after the reinforcement criterion cannot be start times because they are reinforced responses that end the trials within which IRTs are considered). That is, the probability of reinforcement of a 5s IRT in a FI20 is equal to the ratio $5/20$ or $1/4$; probabilities with different IRT values are found by replacing their length value in the numerator of the previous ratio (e.g., 10s IRTs have a probability of reinforcement of $10/20=0.5$).

This principle can be easily extended to VIs made of discrete distributions of intervals, by summing across all possible intervals, on one hand, the segments of intervals in which happen start times belonging to reinforced IRTs, and on the other hand, the segments of intervals in which start times in general can happen.

In this paragraph we apply the method to a Fleshler & Hoffman VI60 as that used in Study 2 Experiment 1 but with five intervals instead of the fifty intervals for more clarity. These intervals are: 6.4s, 21.6s, 42.0s, 73.4s and 156.6s. The probability of reinforcement of an IRT is given by taking the ratio of, the sum of the segments corresponding to the IRT length within each interval, over the sum of the possible intervals. Each interval will so contribute to the sum in the numerator, or by its own length (when the IRT is longer than the interval) or by the IRT length (when the IRT is smaller than the interval). That is, for example, the probability of reinforcement of a 10s IRT is equal to $(6.4+10+10+10+10)/(6.4+21.6+42.0+73.4+156.6)$. From there, an excel table can easily be made to obtain the probability of a wide range of IRTs.

Let us now consider the case of schedules containing empty trials but still made of discrete distributions of intervals as the peak-FI/VIs conditions in Study 1. When there are as much reinforced trials as empty trials, as it is the case with the schedules that we used, we can get the picture in the simplest situation of a fixed reinforcement criterion that start times associated to reinforced IRT would still correspond to the IRT length (or interval length if the IRT is longer than the interval) while all possible start times would correspond to the criterion interval plus the duration of an empty trial. That is, in our peak-FI20 where empty trials lasted 140s on average, the probability of reinforcement of an IRT would be $IRT/(20+140)$ for any IRT shorter than 20s, and, it would be $20/(20+140)$ for any IRT longer than 20s. The probability of reinforcement of IRTs reaches an asymptote after the interval value; this asymptote, equals to $20/160=1/8$ corresponds to the proportion of time spent under reinforced trials and will be same for all peak-VI conditions sharing the mean interval value of 20s with mean empty trial of 160s, and be reached from the first IRT longer than the longest interval of the schedule.

For the peak-VIs, the probability of reinforcement of an IRT is then simply the ratio of the sum of the segments corresponding to the IRT length within each interval, over the sum of the possible intervals to which we add the mean empty trial duration multiplied by the number of possible intervals. For example, the probability of a 2s IRT in the peak-VI0/40 made of 41 intervals with 1s spacing is: $(0+1+2*39)/[(0+1+2+3+... +40)+140*41]$.

Let us consider now the case of VIs made of intervals from continuous distributions. It is not possible any longer to sum segments of intervals and intervals which are now uncountable; in place we will use areas. In a VI made of intervals from a continuous distribution, the probability of reinforcement of an IRT is the mass of the start times which belong to reinforced IRTs within the

distribution of the start times. The crucial step in the process to obtain this probability is to determinate the distribution of the start times from the distribution of the intervals. Any instant spent during an interval can be the occasion of a start time, so that the distribution of the start times can be conceived as the distribution of the instants. The height in this distribution of the instants must be, at any t value on the abscissa, proportional to the mass of the intervals longer than t in the original distribution of intervals. Hence, the distribution of the instants (or distribution of the start times) is obtained by, integrating the distribution of the intervals, at any t into the interval axes, from t to the maximum interval of the VI considered, and then by dividing the resulting function by its own integral from 0 to maximum interval value (this last step is to obtain a proper pdf of mass 1).

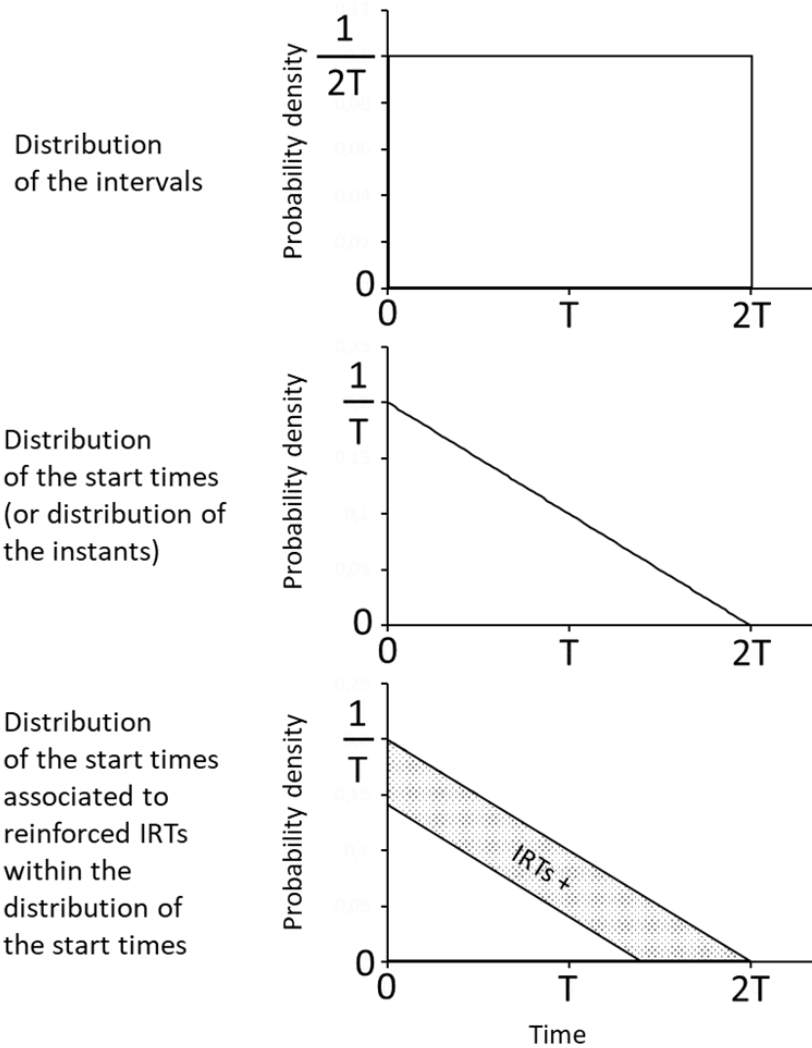


Figure 27: Intervals and start times distributions associated to the uniform VI. The dotted slice, the distribution of the start times belonging to reinforced IRTs, has an area corresponding to the probability of reinforcement of the IRT considered. The duration of the IRT considered determines the width of the slice.

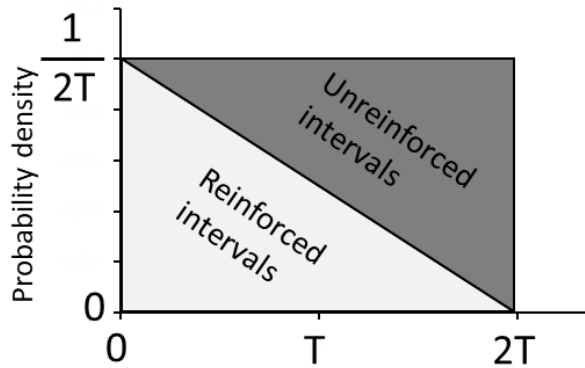
Before considering the uniform exponential VI which not only is made of a continuous distribution but also contains empty trials, let us treat the simpler case of the uniform VI. Figure 27 illustrates the method. We consider a VI with mean T_s , maximum interval $2T_s$ and with pdf $f(t)=1/2T$ (this distribution of interval is presented in the top graph of Figure 27). To obtain the distribution of the instants under the uniform VI we integrate $f(t)$ from t to $2T$, which gives $2*(2T-t)/(2T)^2$ and which we will call $g(t)$ (this distribution of the start times is presented in the middle graph of Figure 27). Next, we need the distribution of the start times belonging to non-reinforced IRTs within the distribution of the IRTs, and, take the complement of its mass to find the mass of the start times associated to reinforced IRTs (see bottom graph of Figure 27) . To obtain this distribution we need to shift the distribution of the start times by one IRT length to the left, so that would only be left start times belonging to non-reinforced IRTs; it is done by replacing t per $t + IRT$ in $g(t)$, this gives $2*(2T-t-IRT)/(2T)^2$, a function that we restrict to $0 \leq t \leq 2T-IRT$ and call $i(t)$. The mass of the start times of interest is the complement of the integral of $i(t)$ from 0 to $2T-IRT$. That is, the probability of reinforcement of an IRT in the uniform (continuous) VI is given by the formula:

$$P(+|IRT) = 1 - \left(\frac{2T - IRT}{2T} \right)^2$$

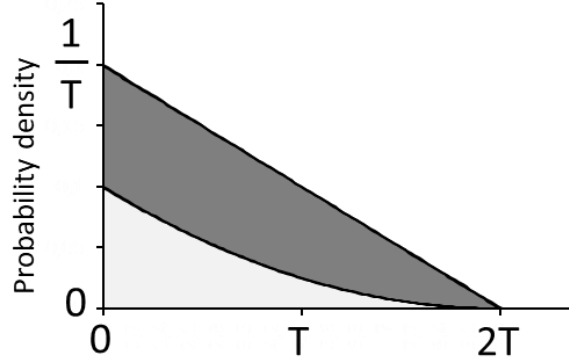
This formula is only valid from 0 to $2T$ (a corollary to the fact that the formula was constituted in part by a function defined on $0 \leq t \leq 2T-IRT$) where it reaches the asymptote of 1. For any IRT greater than $2T$, reinforcement is certain.

Finally, let us consider the case of the uniform exponential VI. Figure 28 illustrates the method. Since the overall distribution of intervals in this VI is uniform, its pdf is $f(t)=1/2T$ (see top graph of Figure 28) and the related distribution of start times is defined by the function $g(t)= 2*(2T-t)/(2T)^2$ (see middle graph of Figure 28). But this VI is made of a combination of a distribution of reinforced trials with a distribution of unreinforced trials. Thus, to obtain the probability of reinforcement of IRTs, we will need, to search for the distribution of the start times conditional to being in a reinforced trials, then, to obtain the mass of the start times belonging to non-reinforced IRTs within the latter distribution, from which mass we will take the complement that we will weight according to the mass of the start times happening during reinforced trials.

Distribution of the intervals



Distribution of the start times (or distribution of the instants)



Distribution of the start times associated to reinforced IRTs within the start times happening during reinforced trials

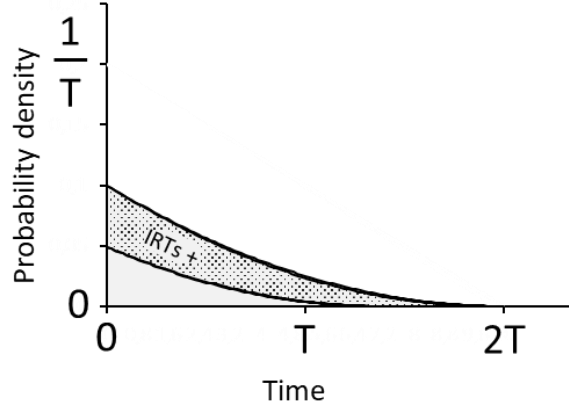


Figure 28: Intervals and start times distributions associated to the uniform exponential VI. The dotted slice, the distribution of the start times belonging to reinforced IRT, has an area corresponding to the probability of reinforcement of the IRT considered. The duration of the IRT considered determines the width of the slice. In this example the VI parameter p , is 0.5.

The distribution of the reinforced intervals is given by the function $w(t) = 2p^*(2T-t)/(2T)^2$ (the sub-distribution in light gray in the top graph of Figure 28), where p is the probability that a trial will be reinforced; it is only a portion of the complete distribution and has a mass of 0.5. To obtain the distribution of the start times happening during reinforced trials within the distribution of the start times we integrate this latter partial distribution from t to $2T$, which gives $2p^*(2T-t)^2/(2T)^3$ and which we call $v(t)$ (the sub-distribution in light gray in the middle graph of Figure 28). Now, we divide $v(t)$ by its

own integral from 0 to 2T in order to obtain the distribution of the start times given that we are in a reinforced trial, we call it $u(t) = 3 \cdot (2T - t)^2 / (2T)^3$; in the process we obtained the mass of $v(t)$ from 0 to 2T, it is equal to $2/3$ of p . The next step is to shift of one IRT to the left the distribution defined by $u(t)$ in order to obtain the distribution of the start times belonging to non-reinforced IRTs given that we are in a reinforced trial, which we do by replacing t per $t + IRT$ in $u(t)$; we obtain the function $j(t) = 3 \cdot [(2T - IRT)^2 - 2t(2T - IRT) + t^2] / (2T)^3$ which we restrict on $0 \leq t \leq 2T - IRT$ (in the bottom graph of Figure 28, the area of the start times belonging to reinforced IRTs is represented directly within the sub-division of the start times happening during reinforced intervals). By taking the complement of the integral of $j(t)$ from 0 to 2T we obtain the mass of the start times belonging to reinforced IRTs given that we are in a reinforced trial, which we multiply by the mass of start times happening during reinforced trial within the distribution of the start times, the mass of $v(t)$ from 0 to 2T; we obtain the formula of the probability of reinforcement of an IRT:

$$P(+|IRT) = \frac{2}{3}p \left[1 - \left(\frac{2T - IRT}{2T} \right)^3 \right]$$

This probability reaches the asymptote $2/3$ of p at 2T (the asymptote is thus $1/3$ in all uniform exponential VI that we used in Study 2). As with the previous case with the formula for the uniform VI, this formula is only valid for IRTs comprised between 0s and 2Ts, a corollary to the fact that the formula was constituted in part by a function defined on $0 \leq t \leq 2T - IRT$. For greater values of IRTs, the part between parenthesis in the formula, which correspond to the probability of reinforcement of an IRT given that we are in a reinforced trial, should be replaced by 1 (we justify this by the simple fact that an IRT which is longer than 2T, when the current trial is a reinforced one, will be reinforced for certain).

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