Risk analysis and risk measures applied to the furniture industry

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Abstract. In this work we show how concepts and methods from actuarial risk theory can be applied to risk analysis in industry. Risk analysis consists in identifying, quantifying and classifying or ordering risks. In the proposed methodology, the risks identified in industrial setting, are modeled by loss random variables. The loss random variables are used to calculate the expected loss, the loss variance and exceedance probabilities permitting risk quantification. In order to classify and order risks, besides measures of uncertainty, risk measures, such as Value-at Risk and Tail-Value-at-Risk (or Expected Shortfall) are determined and, together with risk quantification, the risk levels are analysed. To exemplify this methodology, a case study for risk analysis and classification of occupational accidents in the furniture industry was carried out.

Keywords: risk analysis, risk measures, value-at-risk, expected shortfall, industry.

1 Introduction

Risk analysis is important for many problems in industrial settings. In industry, risks are random losses which can for example represent monetary values or injuries as consequences of accidents. Risks can be modeled by loss random variables, which allow the description of exposure to risk, in particular the quantification and classification or ordering of risks. The level of exposure to risk can be described by risk measures, which inform the risk manager about the degree that the industry is liable to be confronted with specific details of risk. Risk measures have been developed in the context of actuarial risk theory and of risk management, however they are useful for the evaluation of any random variable. For example, risk managers often pay attention to the probability of an adverse outcome, which can be represented by the Value-at-Risk (VaR) at a specific level of probability. VaR refers to a loss level and it can represent an upper bound for the loss for a given confidence level and in a fixed time interval. This risk measure can be useful and relevant for the study of the risk of occupational accidents. So far, VaR was applied to the description of accident risks in various different sectors and activities, for example of accidents in the energy sector [1], in highway hazmat shipments [2] or in [3], where VaR permitted the identification of relevant accident scenarios in furniture industries. In [3] risks corresponding to occupational accidents were analyzed and classified taking into account the different injury categories. The data used in that study corresponds to accidents in the furniture industry in Portugal in the year 2010, this industry being one of the most relevant activity sectors in Portugal [4], which consists predominantly of small and medium-sized enterprises [5].

In this work, we present a more general analysis than in [3], showing how to apply the methodology to the analysis of risks in industry, and then extend the analysis of occupational accidents by generalizing certain concepts, such as the general prediction of lost work days due to accidents in industry, including a new risk measure, namely the Tail-Value-at-Risk (TVaR), also known as Expected Shortfall.

2 Risk Theory

In many problems a risk can be modeled by a loss random variable X defined as follows (see e.g. [6], [7]):

$$X = IB, \tag{1}$$

where B corresponds to a random amount of loss and I is an indicator random variable, with values $I = 1$ or $I = 0$; $I = 1$ indicates that some event, in particular a loss, has occurred.

The indicator random variable I can be described by a Bernoulli(q) distribution, $0 \leq q \leq 1$, where the probability of the event is $q = P(I = 1)$ and $1 - q = P(I = 0)$ corresponds to the probability of no risk occurrence. If $I = 1$ the loss X is drawn from the distribution of the random variable B and if $I = 0$, then $X = 0$, meaning that there is a zero loss.

The moments of X can be calculated with the iterative formula of conditional expectations. The expected loss is calculated in the following way

$$\mathrm{E}[X] = \mathrm{E}[\mathrm{E}[X|I]] = P(I = 1)\mathrm{E}[B] = q\mathrm{E}[B]. \tag{2}$$

Note that if $B = b$ is a fixed amount of loss, then the expected loss is simply given by

$$\mathrm{E}[X] = \mathrm{E}[Ib] = \mathrm{E}[I]b = qb. \tag{3}$$

The variance of $X = IB$ can also be determined using the conditional distribution of B, given I, from the following variance decomposition rule

$$\mathrm{Var}[X] = \mathrm{Var}[\mathrm{E}[X|I]] + \mathrm{E}[\mathrm{Var}[X|I]] = \mathrm{E}[B]^2 q(1-q) + \mathrm{Var}[B]q. \tag{4}$$

For a fixed amount of loss $B = b$ the variance of the loss is simply given by

$$\mathrm{Var}[X] = \mathrm{Var}[Ib] = b^2 q(1-q). \tag{5}$$

If X is the sum of a large number of independent random variables, i.e. when the general loss corresponds to the sum of independent random individual losses, then the probability for the random loss to exceed a certain value α can be calculated using the Central Limit Theorem:

$$P(X > \alpha) \approx 1 - \Phi\left(\frac{\alpha - \mathrm{E}[X]}{\sqrt{\mathrm{Var}[X]}}\right), \tag{6}$$

where $\Phi(\cdot)$ is the cumulative distribution function of a standard normal random variable; and $\mathrm{E}[X]$ and $\mathrm{Var}[X]$ are determined from (2) or (3) and (4) or (5), respectively.

A risk measure is a statistical tool used to assess risks by associating a real value to a random variable representing the loss. The real value is intended to quantify risk exposure. Examples of risk measures are the Value-at-Risk (VaR) or the Tail-Value-at-Risk (TVaR), also known as Expected Shortfall. The Value-at-Risk (VaR) is a standard risk measure, which is used in actuarial risk theory to assess the exposure to risk (see e.g. [6], [7]). This measure is also called a quantile risk measure. The VaR of a loss random variable X at the 100p% level, represented by $\mathrm{VaR}_p(X)$, is the 100p percentile (or quantile) of the distribution of X. We use the notation π_p for the value $\mathrm{VaR}_p(X)$ satisfying

$$P(X \leq \pi_p) = p. \tag{7}$$

The VaR at the confidence level p, $0 < p < 1$, is the quantity π_p that will maximally be lost with probability p, so that there is a $1-p$ chance of exceeding π_p: $P(X > \pi_p) = 1 - p$.

The VaR has the disadvantage that it only informs about the probability of the shortfall of X over π_p being positive. However, the size of the shortfall should also be taken into account. Risk measures which consider the size of the shortfall $(X - \pi_p > 0)$ when the amount (e.g. capital) π_p is available, include the Tail-Value-at-Risk (TVaR). The TVaR, also known as Expected Shortfall or Mean Excess Loss (see e.g. [6], [7]), of a random variable X at the 100p% level $\mathrm{TVaR}_p(X)$ is the expected loss, given that the loss exceeds the 100p percentile (or quantile) of the distribution of X, and is defined by

$$\mathrm{TVaR}_p(X) = \mathrm{E}[X|X > \pi_p] = \frac{\int_{\pi_p}^{\infty} x f(x)\mathrm{d}x}{1 - F(\pi_p)}, \tag{8}$$

where f and F denote, respectively, the density function and the cumulative distribution function of X. TVar gives therefore more information about the tail of the distribution than VaR does.

A simple way to judge risk is to consider the effect of the expected value of the risky outcome $\mathrm{E}[X]$ and also its variation or uncertainty given by the variance $\mathrm{Var}[X]$ or standard deviation $\sqrt{\mathrm{Var}[X]}$. In risk analysis it is therefore useful, first, to analyse and classify risks according to this criterion based on the mean and variance. Then one can take into account certain probabilities for the random loss to exceed certain values α using (6). And the risks can then be better classified and ordered using the risk measures $\mathrm{VaR}_p(X)$ and $\mathrm{TVaR}_p(X)$. Given two risks X and Y, we say that X is riskier than Y if

$$\mathrm{VaR}_p(X) > \mathrm{VaR}_p(Y) \tag{9}$$

and

$$\mathrm{TVaR}_p(X) > \mathrm{TVaR}_p(Y). \tag{10}$$

3 Application to risk analysis of occupational accidents in industry

In this section we show how to generally apply the risk theory presented in the previous section to the risk analysis of occupational accidents in the furniture industry, generalizing the analysis done in [3], and extending that analysis with further results, including the risk measure TVaR, which will be a useful tool to decide the risk ordering of the injury categories with intermediate risk level. Workers from Portuguese furniture industries face several risks that can jeopardize their safety. These risks are related to common hazards in the sector, such as unsafe machinery, worker's unsafe behaviors and manual tasks (to saw, drill, cut, plane, polish or manual material handling) [4]. As a consequence, the accident frequency rate in furniture sector remains high [4]. Information about the most important accident mechanisms in the furniture sector, including the probability of the expected consequences, is critical to overall risk management process. This information will help enterprises to decide about the need of control measures and will contribute to authorities develop effective intervention programs. Official accident reports data provided by the Portuguese Office of Strategy and Planning (GEP) from the year 2010, which are aligned with European Statistics on Accidents at Work (ESAW III), described six categories of contact-modes of injuries, denoted by $i = 1, \dots, 6$, which occurred in the furniture industry in Portugal in 2010 (see Table 1.).

Table 1. Contact mode of injury categories.

i	injury category
1	Contact with electrical voltage, temperatures, hazardous substances
2	Horizontal or vertical impact with or against a stationary object (victim in motion)
3	Struck by object in motion, collision with
4	Contact with sharp, pointed, rough, coarse Material Agent
5	Trapped, crushed, etc.
6	Physical or mental stress

Different injuries will lead to different numbers of lost work days; it is also possible to have zero lost working days. Therefore, the risk of accident is characterized by the probability of its occurrence and its severity, measured as lost work days.

The number of lost work days will be modeled using the loss random variable

$$X = IB, \tag{11}$$

which depends on the accidents' occurrence probability and on the estimated number of lost work days. The random variable B represents the number of lost work days. The random indicator variable I indicates if an accident leads to more than one lost work days, in which case $I = 1$, or if an accident has associated zero lost work days, in which case $I = 0$. In general, in the furniture industry one can identify a total number of $n = 4313$ accidents, from which $n0 = 1023$ had associated no lost work days. Therefore, the probability that an accident in the furniture industry will lead to at least one lost work day, $P(I = 1)$, is

$$q = \frac{n - n0}{n} = \frac{3290}{4313} = 0.76$$

and the probability that an accident in the furniture industry will lead to no lost work days, $P(I = 0)$, is

$$1 - q = \frac{n0}{n} = \frac{1023}{4313} = 0.24.$$

Table 2 contains the information about the number of accidents for each injury category, represented by n_i, $i = 1, \ldots, 6$, and about the number of accidents which had associated zero lost work days, denoted by $n0_i$, $i = 1, \ldots, 6$. Note that $n = \sum_{i=1}^{6} n_i = 4313$ and $n0 = \sum_{i=1}^{6} n0_i = 1023$. The probabilities $P(I = 1)$ for each injury category will be estimated by

$$q_i = \frac{n_i - n0_i}{n_i} \tag{12}$$

and $P(I = 0)$ by

$$1 - q_i = \frac{n0_i}{n_i}, \tag{13}$$

which represents the probability for injury category i that an accident will lead to zero lost work days.

Table 2 also contains the estimated number of lost days associated with an accident of category i given by

$$b_i = \frac{b_{Ti}}{n_i}, \tag{14}$$

where b_{T_i} stands for the total number of lost days associated with accidents of category i, and the occurrence probabilities of an accident of category i represented by

$$p_i = \frac{n_i}{n}. \tag{15}$$

Table 2. Results for the contact modes of injury categories.

i	n_i	$n0_i$	q_i	$1-q_i$	b_{Ti}	b_i	p_i
1	97	20	0.79	0.21	1135	11.70	0.02
2	523	162	0.69	0.31	17457	33.38	0.12
3	958	373	0.61	0.39	18082	18.87	0.22
4	1406	218	0.84	0.16	53661	38.17	0.33
5	331	61	0.82	0.18	13594	41.07	0.08
6	998	189	0.81	0.19	27062	27.12	0.23

In general, one can describe the risk X of lost days due to accidents in the furniture industry in the following way using the statistics and risk measures presented in Section 2.

In order to calculate the mean and variance of X one needs the mean and variance of B:

$$\mathrm{E}[B] = \sum_{i=1}^{6} p_i b_i = 30.51, \tag{16}$$

$$\mathrm{Var}[B] = \mathrm{E}[B^2] - \mathrm{E}[B]^2 = 68.82. \tag{17}$$

The expected number of lost work days due to accidents in the furniture industry is then given by (2)

$$\mathrm{E}[X] = q\mathrm{E}[B] = 23.19, \tag{18}$$

and the variance (4) by

$$\mathrm{Var}[X] = \mathrm{E}[B]^2 q(1-q) + \mathrm{Var}[B]q = 222.09. \tag{19}$$

The probabilities that the number of lost work days exceed one week and half a month are, respectively,

$$P(X > 7) = 0.86 \text{ and } P(X > 15) = 0.71. \tag{20}$$

The application of the Central Limit Theorem is justified by the fact that X corresponds in fact to a sum of a large number of independent random variables, since, assuming that the $n = 4313$ accidents are independent, one could write $X = X_1 + \cdots + X_{4313}$ or, taking into account the six injury categories with $n_1 = 97, \cdots, n_6 = 998$ (see Table 2), $X = X_{1,1} + \cdots + X_{1,97} + \cdots + X_{6,1} + \cdots + X_{6,998}$, where in (16) we used for each i the sum of n_i estimated number of lost days b_{T_i}, which is divided by n in order to have the expected number of lost work days due to an accident (cf. (14), (15)), and we used $P(I = 1) = q$ for each accident in the furniture industry.

The risk measures $\mathrm{VaR}_p(X)$ and $\mathrm{TVaR}_p(X)$ at the 95% level can be determined from (7) and (8) and one thus obtains

$$\mathrm{VaR}_{0.95}(X) = 47.7 \text{ and } \mathrm{TVaR}_{0.95}(X) = 53.95, \tag{21}$$

where we considered that X has approximately a Normal distribution, due to the reasons explained before.

These results indicate that given the occurrence of an accident in the furniture industry, there is a high probability, as indicated in (20), that the number of lost work days exceed one week or half a month. In fact the expected number of lost work days is 23.19 (approximately three weeks), with a high variance of 222.09, meaning that there is a considerable risk of occurring more extreme values of lost work days. Concerning the risk measures, the VaR indicates that the probability of lost work days being less than $\pi_{0.95} = 47.7$ days is 0.95, so that there is a 5% chance that the number of lost work days will exceed that number. The TVaR indicates that the expected number of lost work days being higher than $\pi_{0.95} = 47.7$ is 53.95, or, given that the number of lost work days exceed the threshold $\pi_{0.95} = 47.7$, the mean excess loss will be 53.95.

A detailed analysis about the risk of accident occurrence and number of lost work days due to each injury category can be found in [3]. Here we complement those results by calculating the risk measure TVaR for each injury category. For that purpose we use the particular loss random variable associated to each injury category $i = 1, \ldots, 6$,

$$X_i = I_i b_i, \tag{22}$$

representing the lost work days due to an accident of category i, where I_i has a Bernoulli(p_i) distribution, with p_i and b_i given in Table 2. Note that in this case b_i is modeled as a fixed number of lost work days and therefore formulas (3) and (5) can be used to determine the mean and variance of X_i for each injury category. Calculating the risk measure TVaR at a 95% level for each injury category, one obtains the results listed in Table 3. This Table also contains the values of the risk measure VaR.

Table 3. Risk measures VaR and TVaR for each injury category.

i	1	2	3	4	5	6
$\text{VaR}_{0.95}$	3.12	21.97	17.10	41.87	21.14	25.09
$\text{TVaR}_{0.95}$	3.82	26.53	20.35	49.32	25.67	29.81

In [3], the injury categories were ordered in the following way according to their risk (see Table 1 for the identification of the injuries 1 to 6). Injury 4 has the highest risk level and injury 6 is the second problematic one for the industry, whereas injury 1 has the lowest risk level. Considering the injury categories with intermediate risk level, namely injuries 2, 3 and 5, their ordering was not straightforward taking into account the expected number of lost work days, the variance, the exceedance probabilities of 7 and 15 lost work days and the risk measure VaR. Additionally taking into account the new information about the risk measure TVaR for each injury category, the results reveal that for injuries

2, 3 and 5 with intermediate risk level the order would be: injury category 2 is riskier than injury category 5 and both are riskier than 3. Also with TVaR there is a strong evidence for classifying injury category 4 as the most problematic one, followed by category 6, and injury category 1 as the less risky one.

4 Conclusions

In this work we presented a methodology for risk analysis which can be adapted to the analysis of risks in the context of industry. The methodology was applied here to analyze in general the occurrence of accidents in the furniture industry, focussing on the risk of lost work days due to accidents. Based on the two risk measures, VaR and TVaR, both give similar evidence for the most problematic and for the less risky category in the furniture industry. These risk measures help to determine the ranking of the categories considered to be of intermediate risk, where the identification based on, for instance, the uncertainty measure or on the expected loss was not straightforward. The results indicate that there is a high probability of accidents leading to lost work days in this industry sector and the expected number of lost work days in the case of accident occurrence is around three weeks.

The general results presented in this work for the furniture industrial sector are relevant and useful if one wants to analyze in general how risk in the furniture industrial sector has evolved since 2010 and if one wants to compare the risk of accidents in the furniture industrial sector with other industrial sectors.

In the future we want to apply this methodology in order to compare accident risks, and in particular the risk of lost work days due to accidents, between different industry sectors and also continue the study in the furniture industry context using more actual data.

Acknowledgments

The authors acknowledge support from FCT, through the projects UIDB/00013/2020, UIDP/00013/2020 and UID/CEC/00319/2019.

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