A bi-objective optimization approach for wildfire detection

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Abstract. We consider the problem of buying and locating equipment for covering a given region. We propose two approaches, based on mathematical programming modelling and the epsilon-constraint method, that allow obtaining the efficient frontier of a bi objective optimization problem. In one of the approaches, lexicographic optimization is used to incorporate additional objectives – besides maximizing coverage and minimizing cost, we also consider maximizing double coverage and minimizing the maximum fire rate of spread of uncovered points. The latter objective comes from the specific application that motivated this work: wildfire detection. We present results from a case study in a portuguese landscape, as an example of the potential of optimization models and methods to support decision making in such a relevant field.

Keywords: Location · Multi-objective optimization · Wildfire detection.

1 Location problems and wildfire

In this paper, we consider the problem of buying and locating equipment for covering a given region.

Location problems have their modern roots in the 1960s. Nowadays, the existing body of work on models, methods and location applications is quite extensive. For a comprehensive work on location problems, we refer the reader to the book by Laporte et al. [10].

The problem we address in this paper can be seen as a variation of the well known maximal coverage problem [3]. The basic version of this problem can be stated as to decide which sites should be chosen (from a discrete set of potential sites) to install facilities to maximize the number of clients covered (e.g. within a given ray of a facility). In [12] an overview of more recent work in the maximal covering problems is provided, including applications, solution methods and variants.

The covering problem addressed in this paper extends the maximal coverage problem in two ways: firstly, it includes a budget for the equipment cost that is not known *a priori*; secondly, the coverage objective is addressed as a set of hierarchically related objectives. These extensions result on a bi-objective problem (cost vs. coverage) with a coverage objective function resulting from lexicographic optimization.

The motivation for this problem comes from the desire to improve wildfire detection in a Portuguese municipality. Optimization has been used in wildfire detection. In particular, [4] use the maximal covering problem in wildfire detection. We extend that work by using a bi-objective approach and consider additional objectives, namely to maximize double coverage (as defined in, e.g., [8]) and to minimize the potential of fire spread. We show how these objectives can be integrated (through using lexicographic optimization in the epsilon-constraint method) to provide the efficient frontier to a decision maker. We refer the reader to [11] and [6] for surveys on optimization and fire.

We describe a practical application of the models and methods proposed which consists in the use of drones (unmanned aerial vehicles - UAV) in fire detection. Fire detection comprises find and gathering information about ignitions. When an ignition alarm appears, a drone may be sent to the (approximate) location of the ignition to collect information about if the ignition is real or just resulting from a false alarm, the current status of the fire (e.g. perimeter and intensity) and its potential for spreading. This information may improve the decisions to be made about the initial attack resources and fire suppression tactics to be employed.

The paper is organized as follows: In Section 2 we define the problem, introduce the base model and describe the method to solve it. Section 3 addresses the additional objectives and how they are incorporated through lexicographic optimization in the solution approach described previoulsy. Section 4 described the practical application of proposed methods. The conclusions of this works are drawn in Section 5.

2 Bi-objective optimization for buying and locating equipment

2.1 Problem definition

We consider the problem of deciding which resources to buy and where to locate them in order to cover a given region. The available locations and the areas to be covered are discrete, i.e. they are set points. We characterize each type of resource by a unit cost and by the set of points it covers in each potential location. In the base version of the approach, we consider two conflicting objectives: to maximize the coverage and to minimize the cost.

The motivation for addressing this problem comes from its applicability in wildfire detection, where demands are associated with potential fire ignitions. With small adjustments, the proposed model is suitable for supporting decisions about, for example, locating vigilance towers, activating water sources, buy and positioning drones, pre-positioning fire fighting resources, buy and locating cameras and sensors.

2.2 Model

The mathematical notation to be used in the following models is:

- J set of demand points;
- K set of types of resources;
- I^k set of potential locations for resources k, $k \in K$;
- J^{ki} set of demand points covered by a resource of type k, $k \in K$, located at i, $i \in I^k$; $J^{ki} \subseteq J$;
- KI^j set of pairs (k, i) of a resource of type k, $k \in K$, and a location i, $i \in I^k$, that cover demand j;
- c_k cost of one unit of the resource of type k.

We note that the coverage relation is represented explicitly by the set of covered points for each pair type of resource - location through the sets J^{ki} and KI^j. This allows flexibility in the type of resources to be included in the model. Although elementary coverage rules, as defining the demand points covered as the ones that are inside a circle centered in the location with the ray characterized by the type of equipment, can still be used, virtually any function can be used. For example, we may use non linear functions of the distance for sensors or explicitly enumerate points that are not covered by vigilance towers because they stand in a valley.

If the single objective is minimizing cost, the well known set covering model can be used directly. We define the decision variables:

$$y_{ki} = \begin{cases} 1, & \text{if an equipment of type } k \text{ is located at point } i \\ 0, & \text{otherwise} \end{cases} \qquad k \in K, i \in I^k.$$

The model is:

$$\text{Minimize} \sum_{k \in K, i \in I^{ki}} c_k y_{ki} \tag{1}$$

Subject to:

$$\sum_{(k,i) \in KI^j} y_{ki} \geq 1 \qquad j \in J \tag{2}$$

$$y_{ki} \in \{0, 1\} \qquad k \in K, i \in I^k \tag{3}$$

The objective function (1) minimizes the cost of the equipment to buy and locate. Constraints (2) assure each demand is satisfied and constraints (3) define the domain of the decision variables.

This model is infeasible if there are demands that cannot be covered, even if all resources are used. Therefore, to avoid that, but also to increase the flexibility to the more elaborated models to be proposed next, we consider the inclusion of decision variables x_j related to the demand being satisfied or not:

$$x_j = \begin{cases} 1, & \text{if demand point } j \text{ is covered} \\ 0, & \text{otherwise} \end{cases} \qquad j \in J$$

A model to maximize coverage is then:

$$\text{Maximize} \sum_{j \in J} x_j \tag{4}$$

Subject to:

$$x_j \le \sum_{(k,i) \in KI^j} y_{ki} \qquad j \in J \tag{5}$$

$$y_{ki} \in \{0,1\} \qquad k \in K, i \in I^k \tag{6}$$

$$x_j \in \{0,1\} \qquad j \in J \tag{7}$$

The objective function (4) maximizes the coverage, while constraints (5) state that for a demand to be counted as covered it must effectively be covered, at least, by a resource location pair. The domains of the variables are given by (6) and (7).

In practice, both objectives, to minimize cost and to maximize coverage, are relevant and therefore we model the problem as a bi-objective problem:

$$\text{Minimize} \sum_{k \in K, i \in I^{ki}} c_k y_{ki} \tag{8}$$

$$\text{Maximize} \sum_{j \in J} x_j \tag{9}$$

Subject to:

$$x_j \le \sum_{(k,i) \in KI^j} y_{ki} \qquad j \in J \tag{10}$$

$$y_{ki} \in \{0,1\} \qquad k \in K, i \in I^k \tag{11}$$

$$x_j \in \{0,1\} \qquad j \in J \tag{12}$$

2.3 Solution method

Given the power of current state of the art mixed integer programming solvers and the relatively small size of the model for typical real world instances, we use the epsilon-constraint method [2], to obtain efficient solutions. We note that, with this method, the full efficient frontier can be obtained (including non-supported solutions) in opposition with weight-based approaches. We keep the coverage as an objective and address the cost as a constraint resulting in the following mixed integer programming model, termed $MIP(b)$:

The $MIP(b)$ model is:

$$
\begin{aligned}
&\text{Maximize } \sum_{j \in J} x_j \\
&\text{Subject to:} \\
&x_j \leq \sum_{(k,i) \in KI^j} y_{ki} && j \in J \\
&\sum_{k \in K, i \in I^{ki}} c_k y_{ki} \leq b && (13) \\
&y_{ki} \in \{0,1\} && k \in K, i \in I^k \\
&x_j \in \{0,1\} && j \in J
\end{aligned}
$$

Constraint (13) comes from the first objective of (8-12). For different values of the budget b, a solution that maximizes the coverage is obtained. If the b values are chosen systematically and with sufficiently small variations, the efficient frontier is obtained.

Algorihtm 1 corresponds to this approach and uses the notation:

- b current budget;
- z proportion of covered demands;
- x optimal solution of the current MIP;
- S set of efficient solutions;
- Δ step of the epsilon-constraint method (budget increment from one iteration to the next);
- $MIP(b)$ MIP model with b as the right hand side of the budget constraint.

After the initialization, at each iteration, the epsilon-constraint method solves the mixed integer programming model, $MIP(b)$, with a fixed budget. If the optimal solution of $MIP(b)$ was not found before, the optimal solution is added to the set of the efficient solutions. The iteration ends with the updating of the coverage and the budget. The algorithm ends when a 100% coverage is obtained.

Algorithm 1 Epsilon-constraint method

```
S ← ∅
b ← 0
z ← 0
while z < 1 do
    Solve MIP(b)
    S ← S ∪ {x}
    z ← (∑_{j∈J} x_j) / |J|
    b ← b + Δ
end while
```

3 Addressing additional objectives

3.1 Modeling double coverage and min max uncovered demand

Besides the objectives previously introduced, we consider two other objectives, both related with coverage. The first one is motivated by the potential need to satisfy different demands in the same time interval - solutions with better double coverage (i.e. with more points covered by, at least, two resources) are preferable. This is a straightforward way of addressing dynamic (time evolving) aspects of the problem: if a resource is busy, a demand can be satisfied by another one.

The model below maximizes double coverage and relies on the definition of the decision variables

$$w_j = \begin{cases} 1, & \text{if demand point } j \text{ is covered at least twice} \\ 0, & \text{otherwise} \end{cases} \qquad j \in J$$

$$\text{Maximize} \sum_{j \in J} w_j \tag{14}$$

Subject to:

$$2w_j \leq \sum_{(k,i) \in KI^j} y_{ki} \qquad j \in J \tag{15}$$

$$\sum_{k \in K, i \in I^{ki}} c_k y_{ki} \leq b \tag{16}$$

$$y_{ki} \in \{0,1\} \qquad k \in K, i \in I^k \tag{17}$$

$$w_j \in \{0,1\} \qquad j \in J \tag{18}$$

The objective function 14 maximizes the double coverage. Constraints 15 relate the double coverage variables with the resource location variables, not allowing that a non covered twice demand to be counted as covered. The domain of the varibales are defined by 18.

In the second additional objective, weights are associated with demand points. The objective is to minimize the weight of the uncovered demand with higher weight. The additional parameters and decision variables, and the model are presented below.

- parameters r_j - weight of demand point j, $j \in J$;
- decision variable z - weight of the uncovered demand point with higher weight.

$$\text{Minimize } z \tag{19}$$

Subject to:

$$z \geq r_j(1 - x_j) \qquad j \in J \tag{20}$$

$$x_j \leq \sum_{(k,i) \in KI^j} y_{ki} \qquad j \in J \tag{21}$$

$$\sum_{k \in K, i \in I^{ki}} c_k y_{ki} \leq b \tag{22}$$

$$x_{ki} \in \{0, 1\} \qquad k \in K, i \in I^k \tag{23}$$

$$z \geq 0 \qquad j \in J \tag{24}$$

Objective (19) and constraints (20) translate the well known linearization of minmax functions. The domain of the decision variable z is defined by 24. If all demands are covered, then $z = 0$.

3.2 Lexicographic optimization

Taking into account the motivation of this work, we consider the three coverage objectives hierarchically. Given the high importance of early detection of ignitions in the success of initial attacks (e.g. [9]), the first objective is to maximize the (single) coverage. Among solutions with the same coverage, a solution with double coverage is preferred. Among solutions with the same double coverage, a solution with the maximum uncovered demand as small as possible is chosen.

In algorithmic terms, for a fixed budget (b), problem $MIP(b)$ is first solved. Let f_1 be the value of its optimal solution (i.e. the optimal single coverage value). This value is used in a constraint (constraint (25) in the model below) that does not allow the single coverage to be deteriorated when the double coverage is maximized. The second problem to be solved is then:

$$\text{Maximize } \sum_{j \in J} w_j$$

Subject to:

$$\sum_{j \in J} x_j = f_1 \tag{25}$$

$$x_j \leq \sum_{(k,i) \in KI^j} y_{ki} \qquad j \in J$$

$$2w_j \leq \sum_{(k,i) \in KI^j} y_{ki} \qquad j \in J$$

$$\sum_{k \in K, i \in I^{ki}} c_k y_{ki} \leq b$$

$$y_{ki} \in \{0, 1\} \qquad k \in K, i \in I^k$$

$$x_{ki} \in \{0, 1\} \qquad k \in K, i \in I^k$$

$$w_j \in \{0, 1\} \qquad j \in J$$

Let f_2 be the value of the optimal solution of this second problem (i.e. the optimal double coverage value with a constraint on the optimal coverage value). The last problem to be solved is:

$$\begin{aligned}
&\text{Minimize } z \\
&\text{Subject to:} \\
&z \geq r_j(1-x_j) && j \in J \\
&\sum_{j \in J} x_j = f_1 && (26) \\
&\sum_{j \in J} w_j = f_2 && (27) \\
&x_j \leq \sum_{(k,i) \in KI^j} y_{ki} && j \in J \\
&2w_j \leq \sum_{(k,i) \in KI^j} y_{ki} && j \in J \\
&\sum_{k \in K, i \in I^{ki}} c_k y_{ki} \leq b \\
&x_{ki} \in \{0,1\} && k \in K, i \in I^k \\
&y_{ki} \in \{0,1\} && k \in K, i \in I^k \\
&w_j \in \{0,1\} && j \in J \\
&z \geq 0 && j \in J
\end{aligned}$$

Constraints (26) and (27) forbid the deterioration of the single and double coverages.

An interesting variation of this approach is to assign weights to the demands in the coverage objective(s) to take into account fire danger and/or fire spread potential (through fire rate of spread estimates as used in the unconvered demand objective). Another interesting variation is to consider the uncovered demand objective as the second most important when single coverage is not total (keeping double coverage as the second objective when coverage is total).

4 Practical application

4.1 Description

We applied the proposed method in a landscape that includes Baião, a municipality in the north of Portugal, in the problem of buying drones and establishing their base for wildfire detection. The local authorities provided six potential location for drones (e.g. firefighters headquarters) and defined a landscape of $217.7km^2$ centered in the municipality.

We characterized the demand through land use data, publicly available [5]. From the nine existing categories of land use (in the first categorization level), we selected forest and bushes (the flammable ones) as demand points (around 75% of the total number of nodes).

Land use was also used to determine the fuel category associated with each demand point (through a correspondence between the land use level 4 and the portuguese fuel models [7]). Based on the fuel models, slopes and (upslope) wind, a fire rate of spread was obtained with BehavePlus6 [1] corresponding to a worst-case value for each demand point.

We considered three types of drones characterized by a unit cost (1.5 k€, 2 k€, 15 k€) and a range (4 km, 6 km and 11 km, in the same order as the cost).

4.2 Cost vs. single coverage

By applying the method described in Section 2.3, we obtained the values of (all) efficient solutions displayed in Table 4.2.

Table 1. Values of the efficient solutions.

Budget (k€)	Coverage (%)
0	0.0
1.5	25.0
2	53.2
3.5	71.7
4	87.9
5.5	91.6
6	93.2
7.5	94.4
8	94.5
16.5	94.7
17	99.2
18.5	99.9
30	100.0

For example, with a budget of 4 k€ (two drones of the second type with the bases displayed in Figure 1) is enough to cover almost 90% of the landscape. A budget of 17 k€ allows covering 99 % of he landscape (Figure 2).

4.3 Cost vs. coverage with lexicographic optimization

The major results of the application of the methods described in this paper to the case study are displayed in Figure 3. The data series 'coverage' is the single coverage for each budget (the values of Table 1). Data series 'lex: double coverage' and 'lex: min max ros' refer to the values of the objective functions described in Section 3, while 'max coverage: double coverage' refers to the values of double

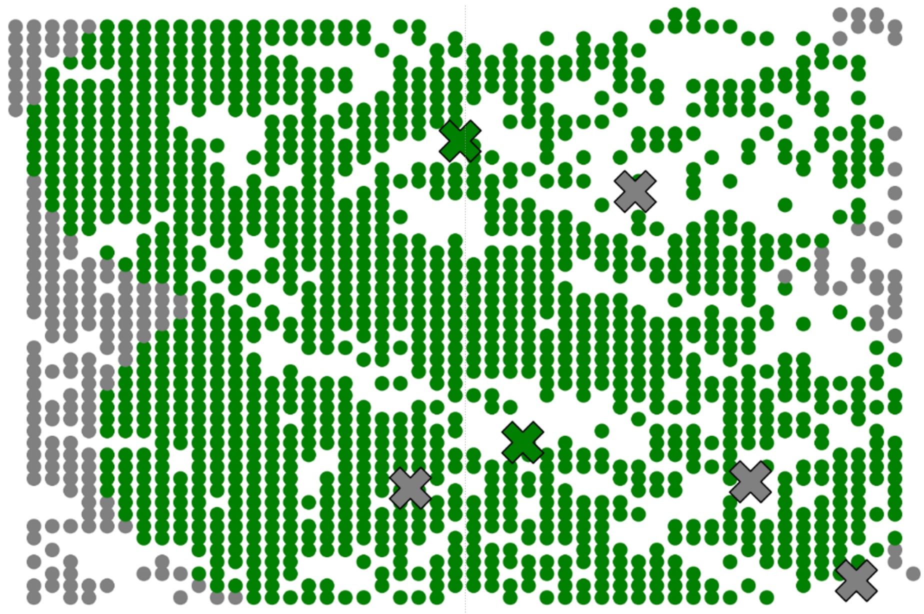

Fig. 1. Optimal solution for a budget of 4 k€.

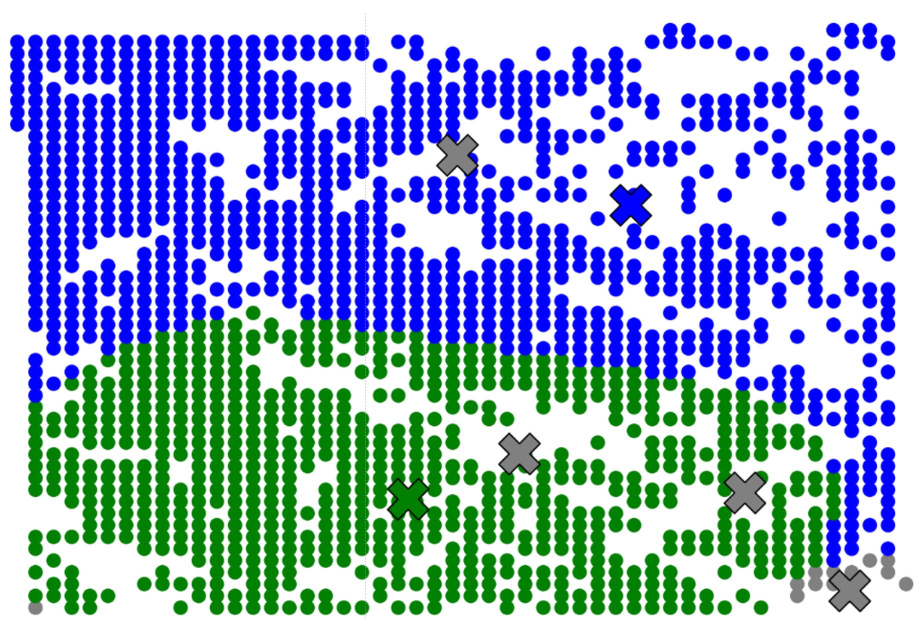

Fig. 2. Optimal solution for a budget of 17 k€.

coverage when only single coverage is optimized. The arrows signal particularly significant increases of the double coverage when the lexicographic optimization approach is used.

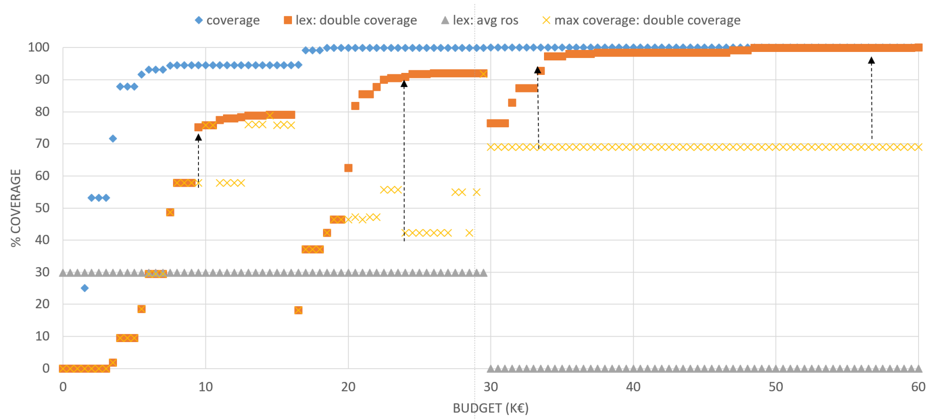

Fig. 3. Results for different approaches for buying and locating drones

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5 Conclusions

In this paper we addressed the problem of buying and locating equipment for covering a given spatial region. Our motivation came from the potential of optimization in practical wildfire detection. Using a combination of well-known models and methods (mixed integer programming, covering models, epsilon-constraint, lexicographic optimization) we proposed a general way of approaching this type of problems. We presented results on a case study where it was intended to buy drones and locate them to cover a landscape around a municipality. The efficient frontier of the bi-objective problem (cost vs. coverage) with lexicographic coverage optimization (double coverage and min max uncovered fire rate of spread are also optimized) was obtained, allowing the municipality to make an informed choice between the different alternatives.

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