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David António Vieira dos Santos Moura Neto

Optimization under uncertainty for forest fire containment

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David António Vieira dos Santos Moura Neto

Optimization under uncertainty for forest fire containment

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Filipe Pereira Pinto Cunha Alvelos

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Name: David António Vieira dos Santos Moura Neto

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ABSTRACT

Forest fires are a major problem that affects the entire world, causing tragic loss of life and serious injuries, which have been worsening due to global warming, making it essential to minimize the serious consequences of these phenomena. In this sense, this project addresses the problem of positioning resources to combat forest fires. As uncertainty is an important aspect in fire propagation modeling, stochastic approaches are used, such as the Equivalent Deterministic Model and the Sample Average Approximation. The purpose of these approaches is to determine the best locations to deploy a limited number of combat assets, for example fire crews. Another important point is to study how fire spreads in a forest given the region's topography, wind and other factors to incorporate fire propagation modeling with the management and planning of fire prevention and firefighting resources (optimization). Although there are several fire propagation simulation software, their integration with optimization problems is still very limited. In this work, this integration is achieved through the minimum travel time (MTT) principle that, when representing the forest by a network in which the transmission times between adjacent homogeneous forest zones are known, states the fire takes the quickest paths. This principle is used in mixed integer programming models to optimize the positioning of the available resources, both in a deterministic and in a stochastic setting. Computational experiments are conducted to validate the approach.

Keywords: Forest Fires, Fire Propagation Modeling, Planning of Fire Prevention and Firefighting Resources, Optimization

RESUMO

Os incêndios florestais são um problema grave que afeta todo o mundo, causando trágicas perdas de vidas e ferimentos graves, que se têm vindo a agravar devido ao aquecimento global, tornando-se essencial minimizar as graves consequências destes fenómenos. Neste sentido, este projeto aborda o problema do posicionamento de meios de combate a incêndios florestais. Como a incerteza é um aspeto importante na modelação da propagação do fogo, são utilizadas abordagens estocásticas, tais como Deterministic Equivalent Model e Sample Average Aproximation. O objetivo destas abordagens é determinar os melhores locais para colocar um número limitado de meios de combate, por exemplo, equipas de bombeiros. Outro ponto importante é estudar a forma como o fogo se propaga numa floresta, tendo em conta a topografia da região, o vento e outros fatores, para incorporar a modelação da propagação do fogo na gestão e planeamento dos recursos de prevenção e combate a incêndios (otimização). Embora existam vários softwares de simulação de propagação de incêndios, a sua integração com problemas de otimização é ainda muito limitada. Neste trabalho, esta integração é conseguida através do princípio do tempo mínimo de viagem (MTT) que, ao representar a floresta por uma rede em que se conhecem os tempos de transmissão entre zonas florestais homogéneas adjacentes, estabelece que o fogo toma os caminhos mais rápidos. Este princípio é utilizado em modelos de programação inteira mista para otimizar o posicionamento dos recursos disponíveis, tanto num contexto determinístico como num contexto estocástico. São efetuados testes computacionais para validar a abordagem.

Palavras-Chave: Incêndios Florestais, Modelação da Propagação do Fogo, Gestão e Planeamento dos Recursos de Prevenção e Combate a Incêndios, Otimização

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1 Introduction

1.1 FIRE IMPACT

Fires have a profound impact on the human lives, health, environment, economy, and society. In this introduction, we will provide a comprehensive overview on the effects of fires and the various ways in which they impact our world. Moreover, we will present some statistics related to wildfires in Portugal, with a focus on the foreseen intensification of these incidents in the future.

1.1.1 HUMAN HEALTH IMPACT

Forest fires have severe implications for human health resulting in the loss of numerous lives, for instance in 2017 alone, the forest fires in Portugal caused the dead of 116 civilians and a firefighter.

But the devastating impact of fires on communities cannot be overstated, as it extends far beyond the physical harm of individuals.

In the aftermath of a wildfire, those who were forced to evacuate or escape can experience a range of traumatic consequences, including direct physical trauma such as burns, lacerations, and other injuries, and those who cannot escape inevitably die. The reverberations of such events ripple through the fabric of families and communities, affecting the health and well-being of all those impacted [1]. Moreover, the well-being of those residing in close proximity might be alarmingly impacted. It has been brought to light that the emission of deleterious particles and gases during wildfires can exacerbate conditions such as asthma and heart disease, as well as induce irritations in the eyes, nose, and throat. Furthermore, the inhalation of smoke and ash can result in respiratory afflictions, including coughing, wheezing, and difficulty breathing, particularly for those with respiratory issues such as the elderly [1]–[3].

The devastating impact of fires on communities extends far beyond the physical health detriment. Regrettably, the psychological toll it imposes on affected populations is often overlooked. The World Health Organization acknowledges that individuals who have suffered the loss of their homes, belongings, or loved ones in a fire are vulnerable to emotional trauma, shock, and grief, which can manifest in the form of anxiety, depression, and insomnia. Additionally, the mere threat of a fire or the fear of losing one's belongings can result in chronic stress and anxiety, exacerbated by the added stress of evacuation, displacement, and uncertainty regarding one's future [4]. Tragically, those who have experienced or witnessed a traumatic event such as a fire are at risk of developing PTSD, characterized by persistent symptoms such as intrusive memories, avoidance behavior, and hypervigilance [5] [6].

Ultimately, it is not uncommon for individuals who have suffered the devastating consequences of a fire, to experience a dual burden of financial strain and emotional distress. This can stem from the loss of material possessions and the subsequent financial burden of rebuilding and recouping losses. Additionally, these individuals may face a daunting reality of displacement, housing instability and even homelessness, which can have adverse effects on both their physical and mental well-being [5] [6].

1.1.2 ENVIRONMENT IMPACT

Fires have the potential to ravage and significantly impair the habitats of wildlife, leading to a decrease in biodiversity and the transformation of ecosystems. A multitude of extensive examinations into the ramifications of wildfires on the environment and human health highlight the manner in which these blazes can affect the air through the emission of noxious pollutants and toxins into the atmosphere. These include detrimental substances such as carbon monoxide, nitrogen oxides, and particulate matter, which can have deleterious effects on the health of both wildlife and nearby human populations, as previously mentioned in section 1.1.1. [1] [3].

Furthermore, the dissemination of ashes and other substances into adjacent aquatic habitats can modify the chemical composition of the water, potentially hindering its suitability for drinking and irrigation. For instance, the presence of nitrogen and phosphorus in the ashes may induce alterations in the water's purity, resulting in augmented nutrient levels that can foster the proliferation of algae and other aquatic vegetation, ultimately leading to eutrophication. Additionally, wildfires can exacerbate soil erosion by removing the protective layer of vegetation that acts as a barrier for soil retention, resulting in a heightened occurrence of sedimentation. This, in turn, can contribute to a decline in the quality of water in nearby waterways, negatively impact aquatic ecosystems, clog irrigation systems and impede the storage and supply of water [1] [3].

The convergence of these adverse consequences can bring about the depletion of rich soil, hindering its potential to foster future growth and undermining the viability of the ecosystem to sustain animal life, thus resulting in a decrease in biodiversity. As a result, the recovery process of forests that have been ravaged by wildfires can be prolonged for decades or more, hindering the accessibility of sustenance and refuge for wildlife. Additionally, wildfires are a significant contributor of CO2 emissions, exacerbating the already detrimental impact of climate change [1] [3] [6].

1.1.3 ECONOMIC AND SOCIAL IMPACT

From a societal viewpoint, the ravages of fire have resulted in substantial destruction of property, and forced relocation of entire populations. Furthermore, the attractiveness of the region as a tourist destination may also be diminished by the despoiled vista and the perception of danger that lingers. Furthermore, the fires have placed a significant burden on the local economies, as well as emergency services and available resources. Moreover, the cultural ramifications of these blazes have been devastating, with historical and cultural landmarks being reduced to ashes, and traditional practices and ways of life being disrupted [7]–[9].

It has been elucidated that the raging wildfires can have a profound and lasting effect on the region's aptitude for agriculture and forestry, thus disturbing the local economies that depend on agribusinesses and industries. Furthermore, these wildfires may even diminish the worth of nearby properties [8]–[10]. Additionally, damage to homes, business and infrastructures, can result in significant economic losses for local property owners and the communities in which they are located, as well as increase insurance costs, rebuilding and recovery, and the cost of firefighting effort [8] [9].

1.2 WILDFIRES IN PORTUGAL

Portugal, being a country with a Mediterranean climate, experiences hot and dry summers, making it susceptible to wildfires. According to the Portuguese National Institute of Statistics (INE), between 2001 and 2017 Portugal experienced an average of over 5,000 forest fires per year, with an average annual area burned of around 120,000 hectares. The statistics in question illustrate the persistent difficulties that Portugal encounters in its endeavours to regulate and thwart forest fires, thereby emphasizing the requirement for sustained exertions aimed at diminishing their regularity and magnitude. To alleviate the ramifications of such blazes, Portugal has instituted a multitude of strategies, such as refined land utilization mapping, advanced fire control techniques, and heightened public consciousness. Nevertheless, the nation remains challenged in its efforts to deter and alleviate the consequences of forest fires, thereby necessitating continued endeavours to reduce their frequency and intensity. The expenditure incurred in extinguishing these blazes annually may escalate to the tune of tens of millions of euros, and the monetary repercussions stemming from the destruction of crops, the decline in tourism, and various other industries can be far more devastating. Furthermore, the detrimental effect on the well-being of the local population, their families, and the environment, which is arduous to quantify, exacerbates the situation [11] [12].

Recently, the scourge of fires in Europe has intensified, with an increase in both frequency and magnitude. Alarmingly, there are several ominous indicators that the likelihood of wildfires will escalate in the near future. Firstly, the ramifications of climate change cannot be ignored, as the soaring temperatures and altered precipitation regimes can give rise to conditions that are favourable for wildfires. This includes longer and more intense droughts, which in turn can escalate the availability of dry fuels, thereby exacerbating the risk of fires; 2) extreme human activities, such as land use changes, urbanization, and increased travel and recreation can escalate the risk of fires by creating more opportunities for fires to start and spreading, and by increasing the exposure of communities to fires and 3) Ineffective fire management practices, such as poor land use planning, limited funding for fire management, and a lack of firefighting resources, can increase the risk of fires and limit the ability of fire management agencies to effectively respond to fires [13]–[15].

In line with the phenomes described above, recently, a lamentable surge in the incidence of devastating wildfires has been observed in various European nations, such as Portugal, Spain, Greece, Italy, and the like. A poignant example of this was seen in 2020, when Portugal and Spain witnessed a historical high in the frequency of wildfires, resulting in thousands of fires scorching vast expanses of land. This phenomenon accentuates the urgency for the collective efforts of governments, communities, and individuals to undertake proactive measures to mitigate the risk of fires and curb their damaging effects, including the reduction of dry fuels, the promotion of fire-resistant landscapes, and the enhancement of effective fire management strategies [14]–[16].

The suppression of fires is a complex and demanding task that requires the use of resources in an effective manner to control the flames, limit damage, and protect life and property. In recent times, mathematical optimization models have been proposed to aid in this process, here referred as fire suppression models. The most relevant suppression models works will be further discussed in the next chapter, Chapter 2 – Literature Review.

1.3 CONTEXT AND MOTIVATION

As previously discussed, forest fires are a serious problem, which have been aggravated due to the climate changes. This causes environmental impacts, destroying wild areas in addition to being a danger to wildlife. Significantly affecting the different components of natural ecosystems. They also cause economic problems associated with the loss of forest resources. With the worsening of global warming, there have been even more situations of this terrible phenomenon all over the world. This problem has been a concern of Governments, with legislation on prevention and combat, namely, Regulatory Decree 55/81 or Decree-Law 156/2004 and 124/2006 [17].

Despite these initiatives, with important contributions to better prevention and organization of the combat operational device, no solution to the problem has yet been found.

Fire spread models to strategically allocate the resources to contain fire, are very useful and have increasingly gaining traction. One such type of models is based on calculate quickest paths the fire will take to reach each node, which can be obtained by, for example, Dijkstra algorithm [18]. In these type of models is mandatory to estimate the fire transmission time between adjacent nodes. . For this is used a very well-known formula to calculate the way the fire spreads given parameters of a homogeneous terrain and the direction and velocity of the wind. That formula is the Rothermel formula, which has been tested in laboratories and perfected over the years, given good predictions on the ways and shapes the fire expands in a given time [19] [20].

This dissertation motivation is then to add more models capable of decreasing the burned area of a forest fire not by applying the classical algorithm, but by incorporating stochastic components to the model, and therefore when applied on real life events reduce the impact caused by the fire.

1.4 OBJECTIVES

The main objective of this work is to contribute with an optimization approach to the fire suppression problem. It is considered the problem of positioning fire fighting resources with the objective of minimizing the burned area in a forest through the use of a model that takes into account uncertainty in the direction of the wind. Firstly, the forest is modeled as a grid, giving the nodes and edges information about the properties of the fuel and slope. Then, knowing the direction and velocity of the wind is possible, fire transmission times on the arcs are obtained through the Rothermel model for fire behaviour. Different wind intensities and directions define scenarios for the fire transmission times. Different approaches are used to decide the positions of the available resources to attack the wildfire taking into account simultaneously the scenarios, in a reasonable time to be acceptable.

2 Literature Review

Fire suppression models aim to anticipate the progression of fires and support decision making during fire suppression operations by strategically positioning resources. Mixed integer programming (MIP) models, in particular, have been suggested to tackle fire suppression issues, typically taking into account factors such as wind direction and speed, fuel type, and terrain to mimic the spread of fire.

In [21] is proposed a model that describes a spatial optimization approach for fire management in large land cells. The goal is to maximize the time delay for fire ignition in protected areas. The management effort variables are defined as the proportion of each cell receiving fire management or the magnitude of delay effort applied. Fire spread and burn time are predicted using fire prediction models such as BEHAVE or FARSITE, based on the management effort and conditions such as wind, moisture, fuel, and topography. The model has variables to track the fire front ignition time and exit time and includes equations to set ignition time, relate ignition times to exit times, relate fire front duration to available fuel, and relate fuel to treatment. The optimization problem is created by the limited resources for treatment.

The authors recognize several limitations to their work: 1) the model assumes the burn time of a cell is determined by its own fuel and not by the fuel or fire intensity of adjacent cells, ignoring cell-to-cell spread rate interaction and spot fires; 2) the model assumes that the spread of a fire and the timing of protection areas are predictable and that management efforts can slow the fire, which isn’t necessarily always true; 3) logistic delivery problems. Additionally, this model incorrectly estimated the time of arrival of the fire in cells that were not located on the binding path of the fire.

In [22] the fire suppression optimization is reshaped into a MIP feasibility problem by incorporating all necessary constraints to correctly determine fire arrival times in any node for several objective functions. As the previous one, this approach is based on the minimum travel time (MTT) principle: the fire shortest path problem was solved using the Dijkstra algorithm while taking into account the impact of fire suppression actions on the fire spread. The author conceived MIP for different objective functions such as safeguarding particular areas, minimizing the overall extent of the burned area, and two definitions of fire containment. The two containment problems differ in terms of their definition: one considers preventing new ignitions within a given time interval, while the other requires resources to be positioned along the entire perimeter of the fire.

In [23] the problem with the objective of minimizing the number of burned nodes and the weighted number of resources at a target instant, was addressed with an iterated local search (ILS) algorithm to allow solving approximately larger instances. ILS is a single-solution meta-heuristic that searches for a solution by exploring the neighbourhood of the current solution and moving to a better solution if possible. ILS is well-known for its fast computational running time and high-quality solutions, making it a compromise between speed and accuracy. This method, as expected, showed improved solution times compared to the exact model (CPLEX) used in [22] , especially for larger grid sizes.

In [24] is proposed a methodology for fire suppression that aims to determine the fire arrival time for each cell in a grid with the goal of minimizing the value of the area burned and the costs associated with the process. This methodology is similar to other integrated models, but has a unique feature in that it calculates fire intensity dynamically, based on the binding paths of the fire spread, rather than relying on fire simulators. The fire spread rate information used to determine fire arrival times and intensities is taken from FlamMap and is deterministic. The authors introduce the concept of beneficial fires and explore different fire management objectives. In one of the case studies, they examine how fire behavior can be altered rather than just suppressed using a multi-objective approach. However, the model has some limitations, as it assumes unlimited resources and does not correctly determine the timing of controls, nor does it consider completion times before fire arrival, and the fire control decisions supported by the model are highly simplified and cannot account for many important suppression activities or firefighting techniques.

In [25] was introduced a multistage model to limit resources based on their previous deterministic work [24] with the goal of minimizing cells affected by burning and controls (no weights associated). The weather tree represents uncertainty about when and how the weather may change. The model determines fire suppression decisions at various stages, which are referred to as "decision points". The decisions are conditioned on the weather and the outcomes of previous stages. The decision points can be at fixed or variable times. The weather tree becomes a "multistage decision tree" when decision stages are imposed on it. The text shows three examples of the weather tree and explains the implications of using different structures for decision timing. However, the stochasticity of the fire spread rate presents a challenge, and including the stochastic weather trees can significantly increase the computational time, or even make the problem unsolvable if beneficial fires are considered. The model restricts the number of controls within each period, but it does not address the issue of simultaneous controls. Nevertheless, the model is still in the development phase and needs to incorporate more realistic suppression decisions and additional enhancements, such as accounting for resource travel time and fire spread from spot fires and re-burns. The objective function can be customized to balance the benefits and losses from wildfires, and it influences the solution times.

The grid-based models stated above do include certain restrictions limiting the resources and their availability times, but they do so in a very basic manner. In [24] the assumption that resources are unlimited and only limit the controls in cells that cannot be reached in the reaction time by prohibiting the insertion of controls in certain precalculated cells. In order to account for limited resources, The article [25] restricts the number of controls to be used in each stage, but it does not include specific time constraints related to the arrival time of the fire or prevent simultaneity of controls within each period.

In [26] this problem is addressed with a model to determine the cells where suppression (controls) are needed aiming to minimize the expected area burned with least distance travelled. The fire brigade is responsible for placing the controls, traveling between adjacent nodes, spending time in traveling and also some additional time if a control in a cell is needed. The fire intensity is modelled dynamically, affecting the time required to control a cell similarly to [24]. However, three of the original fire spread equations from [24] were altered to allow both fire occurrence and suppression actions.

All the previous discussed works rely on the MTT principle, as well as the proposed in dissertation. However, different approaches could be taken. For instance, In [27] was developed a simulation-optimization scheme for simulating fire spread. The model uses the Huygens principle and simulates the spread stochastically in heterogeneous terrain with a wind-slope correction. The optimization module chooses the starting places and parameters of the curves that the brigades follow to close the perimeter based on specified curves [27]. It takes into account the consistent fireline output rate and the fact that one crew's beginning position is also its ending place. The research uses a combination of a Genetic Algorithm (GA) and the Traveling Salesman Problem (TSP) to determine the proper order of points for constructing firelines to control forest fires. Then, the fire suppression is simulated using the Monte Carlo method to determine the total area burned in the worst-case scenario. The simulation discards solutions deemed unacceptable or infeasible. Fire propagation and suppression influence each other in an iterative manner. The objective is to minimize the area enclosed by the curve if the fire is completely surrounded. If the fire escapes the enclosed area, the value is considered infinity. The GA-based simulation-optimization technique is computationally intensive but can handle a large number of resources, making it suitable for varying resource scenarios. The authors noted that this approach is completely data-driven and does not take into account expert knowledge, which plays a significant role in firefighting operations. A more convenient method would be one that combines both data-driven and expert knowledge-based approaches.

The reviewed publications provide an overview of some relevant approaches for optimizing wildfire suppression. The aim of firefighting is to manage and put out fires with minimal harm to life and property, and mathematical optimization models can provide significant support in accomplishing this objective.

3 Fire Spread Models

3.1 FIRE MODEL

3.1.1 RATE OF MAXIMUM SPREAD

Wildfire modeling aims to predict the fire behavior taking into account, mainly, the forest terrain, its topography, and the wind.

A major indicator of the fire behaviour is the fire rate of spread (ROS). Using homogenous conditions, the most common model for fire behaviour is Rothermel's [19] This model estimates the ROS taking into account the highest slope steepness and the influence of wind.

The wind direction and velocity and slope steepness, that directly determine the direction of maximum spread, are also involved in the calculation of the wind factor (ϕ_w)and slope factor (ϕ_s). Rothermel's equation for determining the maximum ROS is given by (1):

$$R = \frac{I_R \xi (1 + \phi_w + \phi_s)}{\rho_b \varepsilon Q_{ig}} \tag{1}$$

The numerator component of (1) represents the heat source, and the denominator is the heat sink, a lot of variables are used to reach this relationship, presented in a simplified way in Figure 1.

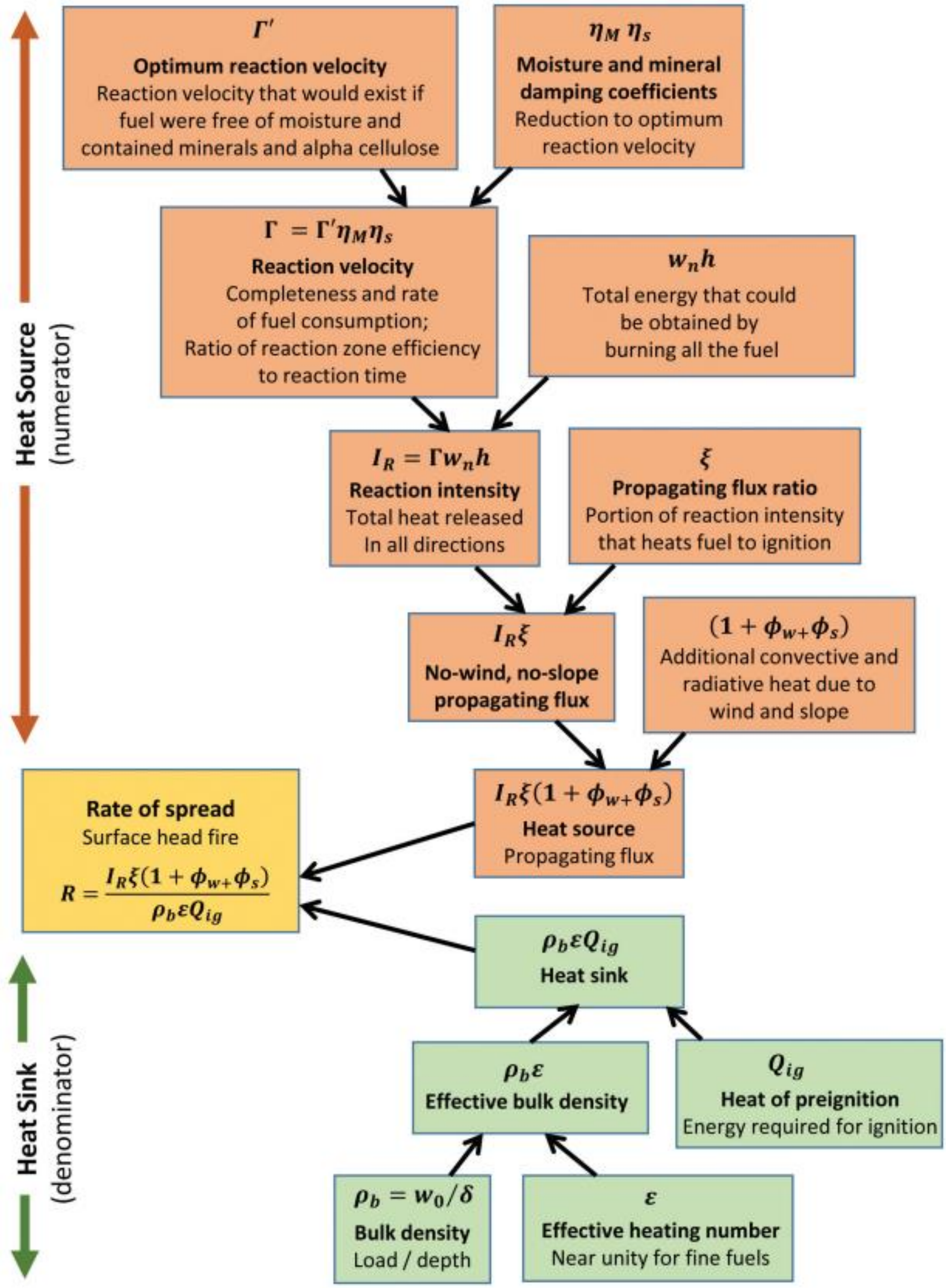

Figure 1 – Flow of variables and calculations in the basic fire spread model. Adapted from [20].

3.1.2 SHAPE OF THE FIRE FRONT FROM THE IGNITION POINT

Fire tends to propagate elliptically, with the direction of maximum spread being dictated by the conjunction of the wind vector and slope vector.

These factors will affect the eccentricity of the ellipse (e), making it flatter the greater the intensity of the wind and higher the upslope.
The shape of a generic fire front, as well as some relative distances can be seen in Figure 2.

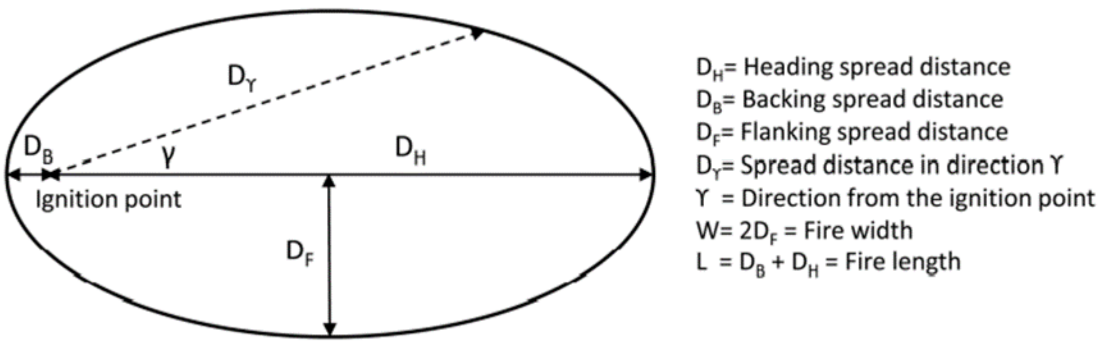

Figure 2 – Fire front shape and relative distances. Adapted from [20].

The basic Rothermel formula (1) is applied when the wind direction is upslope, but it is also possible to calculate the speed in all directions from the ignition point, for instance in the direction Dγ , knowing the angle γ [20].

The ROS in direction γ from the ignition point is given by (2).

$$R_{\gamma} = R_H(1 - e)/(1 - e \cos \gamma) \quad \textbf{(2)}$$

Back fire spreads in the direction opposite of the head fire, which is a simplification of equation (2). ROS of the back fire is then simplified to (3).

$$R_B = R_H(1 - e)/(1 + e) \quad \textbf{(3)}$$

3.1.3 IMPLEMENTED PRACTICAL EXAMPLE

In order to obtain the ROS in any direction, a python program was implemented. The following list describes the inputs required for the program:

1. R_0: velocity of the fire without any slope or wind affecting it;
2. Wind velocity;
3. Wind Direction;
4. Coordinates of the origin of the fire and final point: location where the fire started and location where the fire arrival time will be measured;
5. Height of the origin of the fire and Height of the final point: elevation of the location where the fire started and elevation of where the fire arrival time will be measured. This is important to calculate the slope factor between this points and also in conjunction with the coordinates to determine the distance between them;
6. Other inputs: β and σ, properties of the soil. β (Packing ratio) is given by the Bulk density (lb/ft3) divided by the Oven-dry particle density (lb/ft3), while σ (Surface-area-to-volume ratio) is the reason between the surface area and the volume (ft2/ft3).

It is noteworthy that R_0relies on the fuel type. In the given example, according to Rossa and Fernandes (2018), $R_0$was set as 3.937007874 ft/min as it is relative to the most common fuel type in Portugal (*Eucalyptus globulus*, Figure 3 adapted from [28] with Canva) [29].

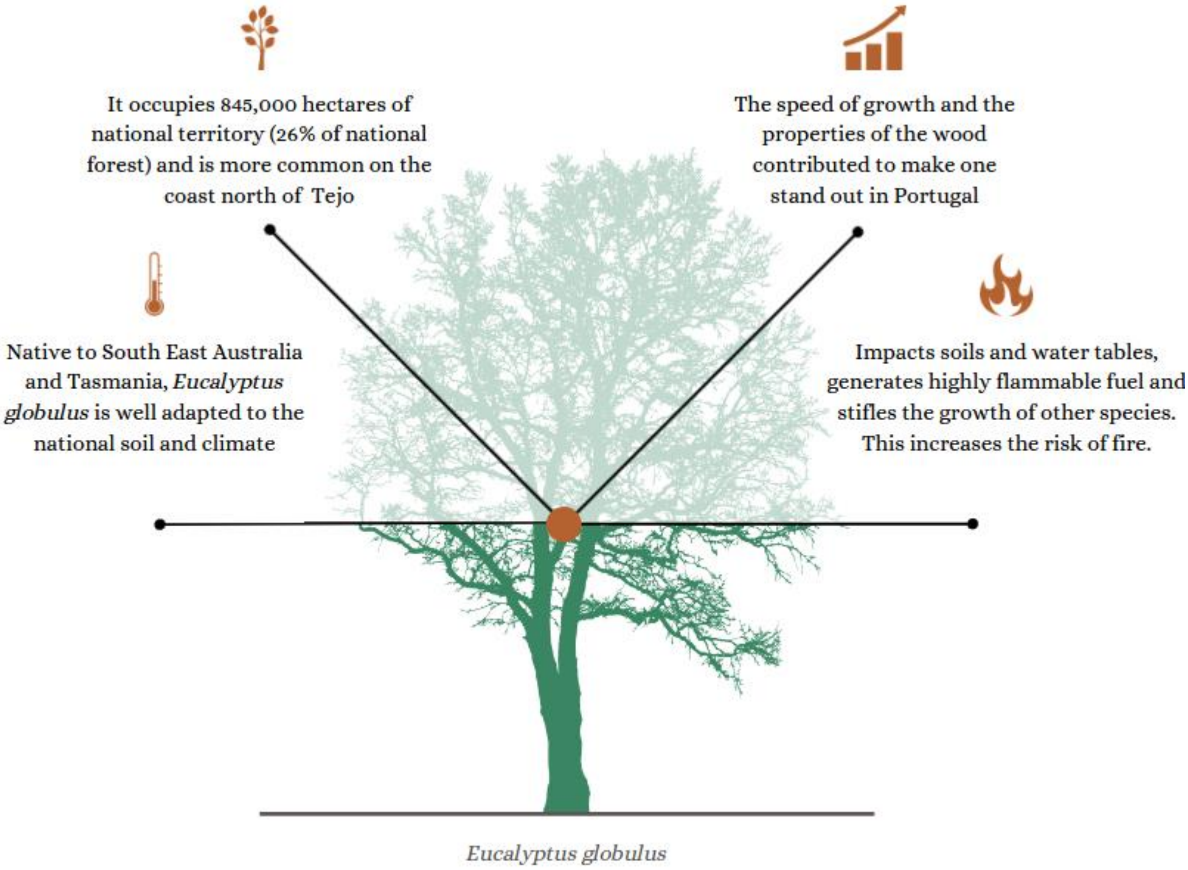

Figure 3 – *Eucalyptus globulus'* origin, reasons behind its predominance in Portugal and fire risk. Adapted from [28] with Canva

By using these inputs, the program can calculate the velocity of the fire in a given direction, taking into account the effects of wind, slope, and other environmental factors. The inputs used in the given example are described in Table 1.

Table 1 - Inputs used for the implemented practical example for prediction of the velocity of fire in a given direction. In the first column is the input type, followed by the given values used for the example, as well as units used, in the last column

Input Type	**Value**	**Units**
R_0	3.937007874	ft/min
Wind Velocity	25	Km/h
Wind Direction (From North clockwise)	79	Degrees

Coordinates of the origin point	(0,0)	(ft,ft)
Coordinates of the final point	(2105.2631578947367,200)	(ft,ft)
Height of the origin point	0	ft
Height of the final point	1500	ft
β	0.006	
σ	1700	(ft2/ft3)

Figure 4 represents the visual representation of the example's output.

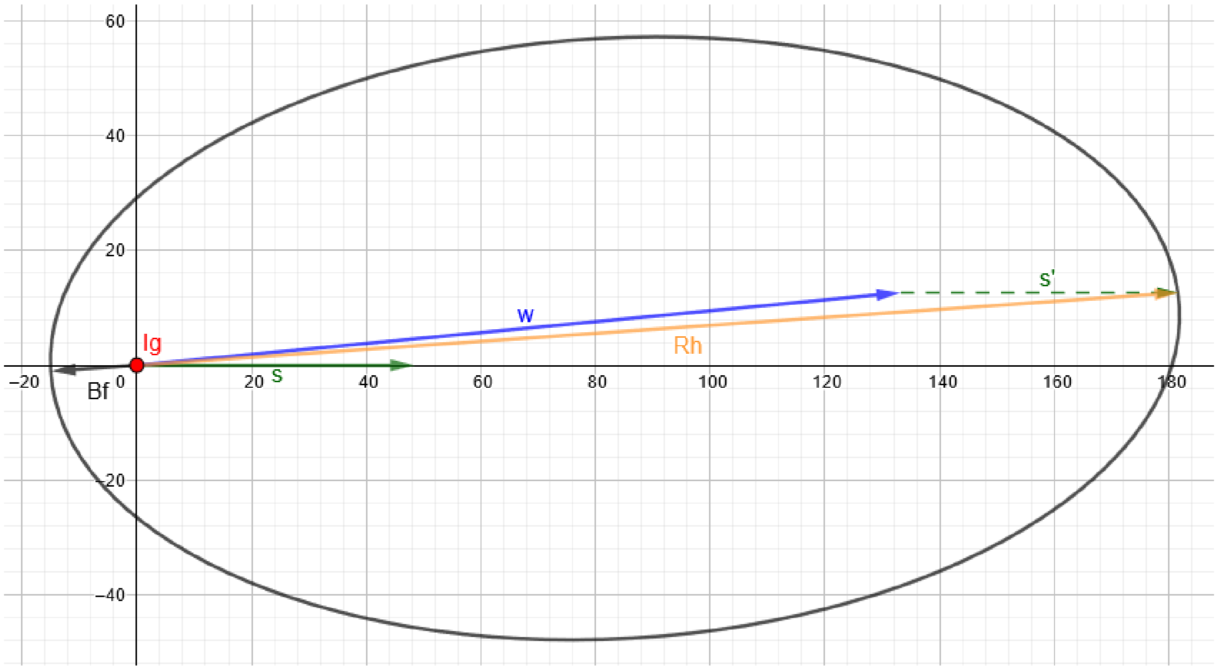

Figure 4 – Visual representation of the practice example output. In red is defined the ignition point (Ig) while the direction of maximum spread, Rh, is displayed as an yellow vector that results from the addition of the slope vector (S, in green) and the wind vector (w, in blue), which are represented as separate vectors in the same space. S portrays the effect of the slope on the spread of the wildfire, while the w represents the impact of wind on the spread. Additionally, in black is the backfire vector (Bf) that represents the minimum fire velocity from the ignition point (opposite direction of Rh). Created with Geogebra.

In **Figure 4**, we examine the relationship between the ignition point (Ig), rate of spread headfire (Rh), slope vector (S), wind vector (W), and backfire (Bf) in the context of wildfire behaviour. Specifically, we visualize these variables in a two-dimensional space, where the ignition point is represented at the origin of the x and y axis in red, forming an ellipse that shows the range of all the velocities resulting from all possible directions. In a simplistic way, the ellipse represents the velocity of the fire in all directions.

It is noteworthy that the vectors size and the angles formed are represented in scale, according to the values of the given example's output.

The output value of most interest for this analysis is the rate of spread, commonly referred to as Ros. The calculation of ROS involves the vector magnitude and the angle γ, as depicted in Figure 5, which is formed between the x-axis and the vector Rh, capturing the direction of maximum spread. This tri-dimensional representation further elucidates the application of the vectors depicted only two-dimensionally in the previous picture. Additionally it enables the understanding that the magnitude of the slope vector increases with the increase of α, whereby a larger value of α yields a steeper slope.

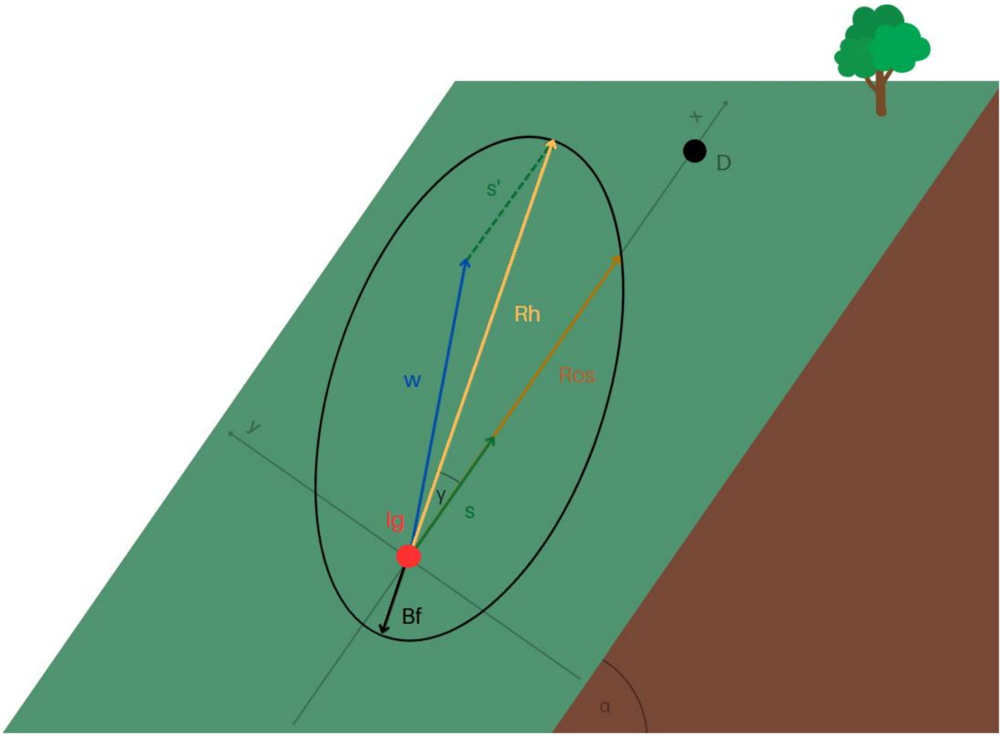

Figure 5 –Tri-dimensional representation of the practice example output (not in scale). In red is defined the ignition point (Ig) while the direction of maximum spread, Rh, is displayed as an yellow vector that results from the addition of the slope vector (S, in green) and the wind vector (w, in blue), which are represented as separate vectors in the same space. S portrays the effect of the slope on the spread of the wildfire (originated from α, which represents the slope stepness), while the w represents the impact of wind on the spread. The vector Ros represents the fire velocity in direction of the segment Ig to D, where D is the final point, and results from Rh being projected in respect to the ellipse with the angle γ. Additionally, in black is the backfire vector (Bf) that represents the minimum fire velocity from the ignition point (opposite direction of Rh). Created with Canva.

Upon calculation, the value of ROS was derived to be 183.56 ft/min, which corresponds to an equivalent conversion of 3.36 km/h.

3.2 APPLYING ROTHERMEL IN MTT EXTENSION

3.2.1 REPRESENTING THE LANDSCAPE

Graph theory is a branch of mathematics that studies relationships between objects in a given set. For this, structures called graphs are used. G (V, E), where V represents the set of objects called vertices and E is the set of edges. The edges allow the nodes to be related through adjacency relationships and may or may not be linked with an associated direction. If the graph has directed edges, it is a digraph.

Furthermore, both edges and vertices can have properties associated with them. Nodes can have different data in different fields characterizing and differentiating them from others, while edges usually have weights associated with them, which can represent distance or other factors.

Graphs are highly advantageous in portraying a real-life scenario with varying degrees of complexity, depending on the specific circumstances at hand.

We assume the landscape is represented by a square grid digraph, where each node corresponds to a location and each arc represents the potential direct transmission of fire between the nodes it links.

3.2.2 MINIMUM TRAVEL TIME PRINCIPLE

The minimum travel time principle states that the fire always follow the quickest path, which in the context of this problem as the landscape is represented by a graph, later explained, the edges that the fire takes will be known and the fire path can be calculated from an origin node.

3.2.3 FIRE TRAVEL TIMES AND FIRE PATHS

If we consider each node from a graph as being a possible ignition node where all nodes possesses properties of the terrain of the landscape and a global wind vector we can apply the Rothermel Model to determine the fire travel time from each node to all his neighbours by using the fire projection equation in the direction in question, associating a weight to each edge.

In the small digraph shown in Figure 6 will be illustrated an example of the application of the method described. The parameters used are shown in Table 2 below the figure.

Due to the large quantity of edges in a limited space available, the fire travel times be instead displayed in Table 3, below Table 2.

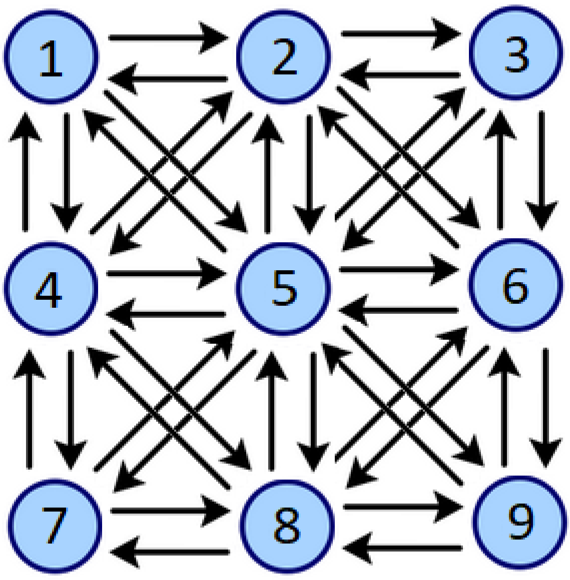

Figure 6 – Small digraph representing a small landscape showing all possible connections.

Table 2- Data used for the implemented practical example of the Ros Model to determine the fire arrival times at each node. In the first column is the input type, followed by the given values used for the example, as well as units used, in the last column

Parameters	**Value**	**Units**
R_0	1.2	meters/min
Wind Velocity	80	Km/h
Wind Direction (From North clockwise)	155	degrees
Horizontal distance between nodes	1039	meters
Vertical distance between nodes	707	meters
Heights of nodes (1,2,3,4,5,6,7,8,9)	(654,548,538,764,644,520,846,779,667)	meters

β	0.02	
σ	3000	(ft2/ft3)

Table 3 - Travel time between the nodes. In the first column, the origin node is depicted, while in the second column is represented the destination node. In the last column is stated the time that the fire takes to travel from the origin node to the destiny node, in minutes.

Origin node	**Destination node**	**Travel time (minutes)**
1	2	6.3
1	5	1.8
1	4	1.1
4	7	1.5
4	8	1.9
4	5	1.3
4	2	14.8
4	1	14.2
7	4	11.6
7	5	13.5
7	8	7.0
2	1	21.4
2	4	17.8
2	5	5.2
2	6	1.7
2	3	4.9
5	1	26.2
5	4	21.8
5	7	20.1
5	2	15.1
5	8	1.4
5	3	15.7
5	6	5.8
5	9	2.1
8	7	18.4
8	4	25.3
8	5	13.8
8	6	11.0
8	9	8.1
3	2	24.0
3	5	23.5
3	6	7.2
6	3	14.0
6	2	28.1
6	5	22.5
6	8	21.2
6	9	5.2
9	6	13.7
9	5	24.3
9	8	17.2

If we consider for instance the node 1 as the ignition node of the fire we will obtain the fire arrival times at each node as well as its path applying a shortest path tree algorithm, according to the MTT principle, as shown in Figure 7.

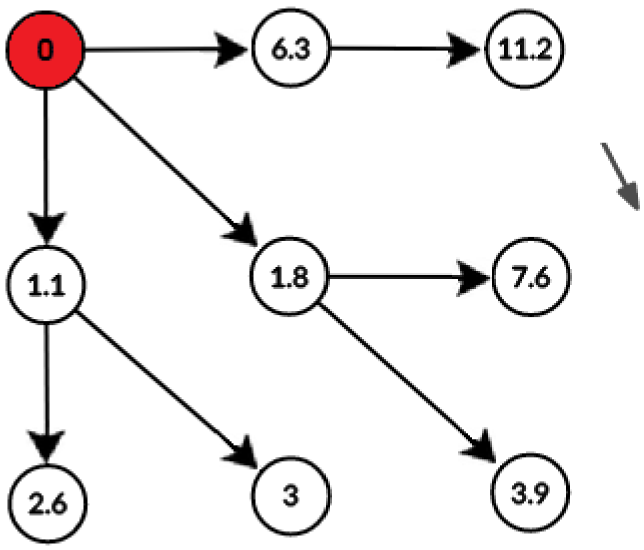

Figure 7 – Fire arrival time at each node and the fire paths

4 Deterministic Model

A deterministic model is a type of mathematical or computational model that produces the same output or result for a given set of inputs or initial conditions every time it is run. Put differently, it completely determines the output based on the input without any randomness or uncertainty.

In this chapter, the problem of positioning fire suppression resources to attack fire spread is addressed. It is assumed that the fire travels times are deterministic and therefore a deterministic mixed integer programming model is presented.

4.1 PROBLEM DEFINITION

As in the previous chapter, a network, where fire spreads, with travel times associated with edges is considered. Additionally, we consider a set of resources that are at a given location (node) when an ignition is detected in a node of the network. The problem is to select a position (node) to each resource such that the burned area (or, equivalently, the number of burned nodes) within a given time frame is minimized. Resources are first positioned in a given node and take time to reach their destination. When a resource is positioned in a node it will delay fire spread through that node implying that, in general, the fire paths will be different.

It is worth mentioning this is an integrated model (as termed in the categorization of [Mendes and Alvelos 2023]) in the sense that both fire spread (i.e. fire arrival times at each node) and resources positioning are optimized simultaneously.

With this information it is possible to set the goal to minimize the burned area of the landscape in an elapsing amount of time, using a limited amount of resources available to strategically position in order to delay and stop the fire from spreading, having each resource to be placed in a dry spot, requiring to reach before the fire, based in a factor of the Euclidian distance from the base to the respective node.

4.2 MIP MODEL

The notation and parameters used in the MIP model are the following:

N – set of cells (nodes);

A – set of ordered pairs of cells representing a potential fire transmission between adjacent cells (directed arcs);

v – time instant when the burned area is evaluated;

e_i –resource earlier arrival time at node $i, \forall i \in N$;

g – ignition node;

c_{ij} – travel time from cell i to cell j, $\forall ij \in A$;

D – delay in fire transmission if a resource is located at cell $i, \forall i \in N$;

B – number of resources.

The decision variables are.

Decision variables:

t_i – fire arrival time at cell i, $\forall i \in N$

$$x_i = \begin{cases} 1, if\ a\ resource\ is\ located\ at\ node\ i \\ 0, otherwise \end{cases}, \forall i \in N$$

$$y_i = \begin{cases} 1, if\ node\ i\ burns\ before\ instante\ v \\ 0, otherwise \end{cases}, \forall i \in N$$

The model is:

$$z = Min \sum_i y_i \tag{1}$$

subject to:

$$t_g = 0 \tag{2}$$

$$t_j \leq t_i + c_{ij} + Dx_i, \forall ij \in A \tag{3}$$

$$y_i \geq 1 - t_i/v, \forall i \in N \tag{4}$$

$$\sum_i x_i \leq B \tag{5}$$

$$e_i x_i \leq t_i, \forall i \in N \tag{6}$$

$$t_i \geq 0, \forall i \in N \tag{7}$$

$$x_i \in \{0,1\}, \forall i \in N \tag{8}$$

$$y_i \in \{0,1\}, \forall i \in N \tag{9}$$

The objective function (1) minimizes the number of burned nodes.

Constraints (2) forces ignition node to have fire arrival time 0.

Constraint (3) states that for all adjacent nodes the fire arrival time from node i to j is always less or equal to the sum of the fire arrival time to j with the fire transmission time from node i to j and potentially with a delay, if an allocation of a resource is decided to be placed on node i, its less or equal constrain since the fire can take quicker paths. Which, by others words means, that the fire will always follow the quickest path to reach each node, and the resources placed will cause a delay making the fire follow other paths to reach some specific zones, because that delay changed what was the previously established quickest path.

Constraints (4) state that a node is burned if the fire arrives earlier than the evaluation instant.

Constraints (5) states that the number of resources that can be used are limited by number of available ones.

Constraints (6) dictates that a resource cannot be placed on a burned node, since the release time associated with that node has always to be less or equal to the fire arrival time at the node, in order to arrive earlier than the fire and be possibly positioned there, otherwise its forced to be excluded from placement at that particular node.

Constraints (7-9) state the domain of the decision variables.

4.3 EXAMPLE

Let’s assume a represention of the landscape of Baião (north of Portugal) in an 11x11 grid where the ignition starts at the left center node and the wind is blowing at 80 km per hour in the direction represented by the black arrow as seen in the image underneath. A base containing all the resources is located at the bottom right node of the graph. The e_i was calculated based on a factor of the Euclidian distance between the base and the respective node.

Assuming we have available a budget of 10 resources, the result obtained can be seen in the same image (Figure 8 as shown below).

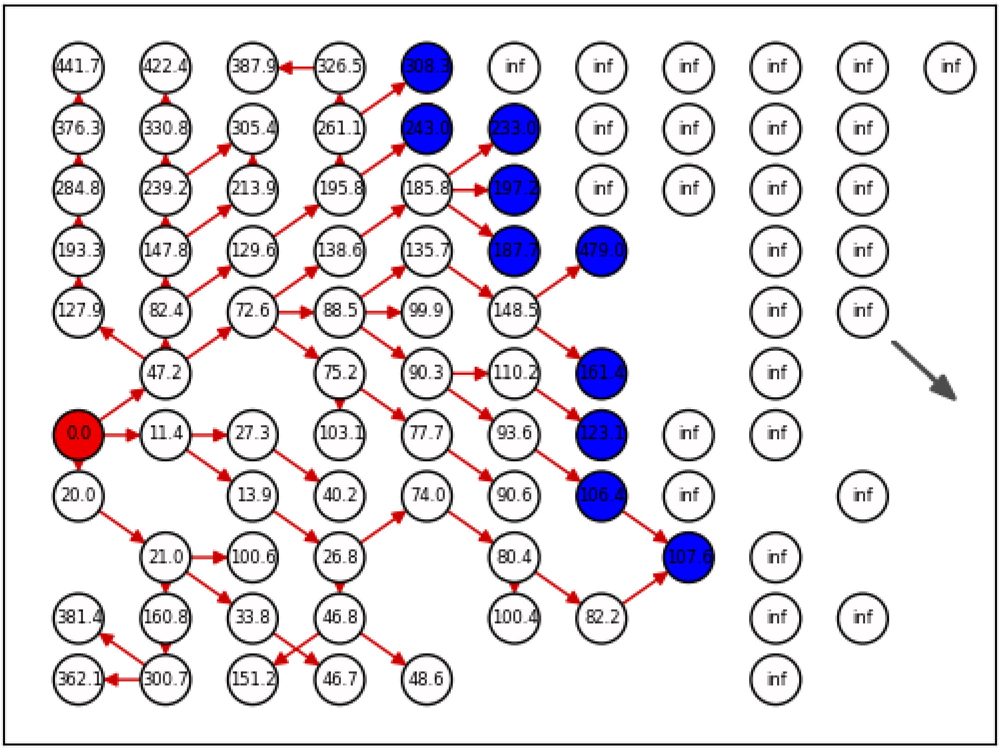

Figure 8 - Placement of the resources and fire paths in an 11x11 grid. The values in the nodes are the fire arrival times (in minutes), the red one is the ignition node, the red edges represent the fire paths from node to node, the black arrow shows the wind direction and the blue nodes represent the places where the resources are placed in order to minimize the burned area. Nodes where the fire does not reach are labeled ‘inf’.

5 Stochastic Optimization Models

In the first section of this chapter we assume that there is uncertainty in the wind direction and that this uncertainty can be represented by a small set of potential scenarios each associated with a probability of occurrence and a wind speed and direction. In this case, the stochastic optimization problem can be converted into a deterministic one.
In the second section, we consider that the wind uncertainty is characterized by a probability distribution.

5.1 DETERMINISTIC EQUIVALENT MODEL

A deterministic equivalent model is a simplified version of a stochastic optimization problem that is converted into a deterministic problem. In a stochastic optimization problem, there is uncertainty related to the data or parameters involved, which can stem from factors like random variations, incomplete information, or future events. The goal is to find an optimal decision or solution that considers this uncertainty and maximizes or minimizes a specific performance measure.

Suppose we have a set of scenarios, each one with a probability of occurrence associated, the scenarios share all of the parameters except the wind direction in the landscape.

Knowing this we set the goal to minimize the expected burned area of the landscape in a given time frame, taking into account all possible scenarios, using a limited amount of resources available to strategically position in order to delay and stop the fire from spreading. Having each resource to be placed in a dry spot for all the scenarios requiring to reach before the fire for all cases, as we do not know which scenario will occur yet. The resources earlier arrival time is again calculated based in a factor of the Euclidian distance from the base to the respective node.

When isolated, each scenario can be seen as a deterministic problem. However, given that the decisions must be taken before the scenario that will occur is known, the optimization must take into account all scenarios at the same time. That is accomplished by the following mixed integer programming model.

Parameters

S – set of scenarios;

p_s – probability of scenario $s, \forall s \in S$;

N – set of cells (nodes);

A – set of ordered pairs of cells representing a potential fire transmission between adjacent cells (directed arcs);

g – index of the ignition node

v – time instant when the burned area is evaluated

e_i –resource earlier arrival time at node $i, \forall i \in N$

c_{sij}– fire transmission time from cell i to cell j on scenario s, $\forall ij \in A$, $\forall s \in S$;

D – delay in fire transmission if a resource is located at cell $i, \forall i \in N$;

B – number of resources.

Decision variables:

t_{si} – instant of the fire arrival time at cell i in scenario s , $\forall i \in N$, $\forall s \in S$

$$x_i = \begin{cases} 1, if\ a\ resource\ is\ located\ at\ cell\ i \\ 0, otherwise \end{cases}, \forall i \in N$$

$$a_{si} = \begin{cases} 1, if\ resource\ positioned\ at\ cell\ i\ is\ not\ effective\ in\ scenario\ s \\ 0, otherwise \end{cases}, \forall i \in N, \forall s \in S$$

$$y_{si} = \begin{cases} 1, if\ cell\ i\ burns\ before\ instante\ v\ in\ scenario\ s \\ 0, otherwise \end{cases}, \forall i \in N, \forall s \in S$$

Model:

$$z = Min \sum_s \sum_i p_s y_{si} \tag{1}$$

subject to:

$$t_{sg} = 0, \forall s \in S \tag{2}$$

$$t_{sj} \leq t_{si} + c_{sij} + D(x_i - a_{si}), \forall ij \in A, \forall s \in S \tag{3}$$

$$\sum_i x_i \leq B \tag{4}$$

$$a_{si} \leq x_i, \forall i \in N, \forall s \in S \tag{5}$$

$$x_i \leq t_{si}/e_i + a_{si}, \forall i \in N, \forall s \in S \quad (6)$$

$$y_{si} \geq 1 - t_{si}/v, \forall i \in N, \forall s \in S \quad (7)$$

$$x_i \in \{0,1\}, \forall i \in N \quad (8)$$

$$a_{si} \in \{0,1\}, \forall i \in N, \forall s \in S \quad (9)$$

$$y_{si} \in \{0,1\}, \forall i \in N, \forall s \in S \quad (10)$$

$$t_{si} \geq 0, \forall i \in N, \forall s \in S \quad (11)$$

The objective function (1) minimizes the expected number of burned nodes.

Constraints (2) forces ignition nodes to have fire arrival time 0 in the respective scenario.

A major difference with respect to the deterministic model is that we may decide to position a resource in a given node but, for some scenario(s) the fire may arrive earlier. Variables a_{si} allow to model that situation by removing the effect of a resource being placed in a position that is infeasible for a given scenario. Let us suppose that a resource is positioned at node i, $x_i = 1$. Let us suppose that for a scenario s, the fire arrives earlier than the resource, $\frac{t_{si}}{e_i} < 1$. Without the a_{si} variable, the model would be infeasible because of (6) (what makes sense as the resource would burn in that scenario). With variable a_{si}, $a_{si} = 1$ and the delay of D in (3) will not take effect. Constraints (3) states that for all adjacent nodes the fire arrival time from node i to j is always less or equal to the sum of the fire arrival time to j with the fire transmission time from node i to j and potentially with a delay, if an allocation of a resource is to be located on node i, unless it is forced to be removed due to not being effective in that scenario, meaning the fire reached that node earlier in that specific scenario(if $a_{si} = 1$) Put simply means, that the fire will always follow the quickest path to reach each node, and the resources placed will cause a delay making the fire follow other paths to reach some specific zones, but the resources are only allowed to stay on a specific scenario if they reach the node earlier than the fire for that scenarios, otherwise they are not placed in the first place.

Constraints (4) states that the number of resources that can be used are the number of available ones.

Constraints (5) dictates that for each node in each scenario a resource can only be removed in case it is ineffective, only if it was originally first placed there.

Constraints (6) states that a resource at node i, can only be originally placed if its arrival time is less or equal than the fire arrival at that node for all the scenarios taken into account, or if the resource would be ineffective in some scenario in the first place. Meaning that the resource can only be placed in a node if it arrives earlier than the fire for all scenarios.

Constraints (7) states that for each scenario a node is only burned if the time passed (variable v) is greater than the fire arrival time at that node.

Constraints (8-11) state the domain of the decision variables.

This model can be used directly if the number of scenarios is small or as a component in the sample average approximation (SAA), which considers distribution probabilities.

5.2 SAMPLE AVERAGE APPROXIMATION

Sample Average Approximation (SAA) is a method employed to estimate the expected value of an objective function when dealing with uncertain variables in optimization and decision-making problems. It relies on Monte Carlo Simulation to produce random samples or scenarios for these uncertain variables. By calculating the objective function for each sample and averaging the results, SAA provides an approximation of the objective function's average behavior under uncertain conditions. SAA is particularly useful in scenarios where the underlying model is complex, lacks closed-form solutions, or demands computationally expensive exact solutions. It serves as a practical and efficient approach for approximating the expected value and facilitating decision-making in the presence of uncertainty [30].

5.2.1 MONTE CARLO SIMULATION

Monte Carlo Simulations are simulations involving randomness. The idea is to obtain a considerable amount of random unbiased samples to obtain the expected value of the average or other statistical measures of the global population using only random samples. The more samples generated the greater will be our confidence about the result (law of large numbers) [31].

This method is very useful when dealing with a large or even infinite population.

5.2.2 PROBLEM DEFINITION AND A TOY EXAMPLE

Suppose we have a wildfire occurring, all the parameters except the wind direction in the landscape are known. Since we do not know the wind direction, and will never know as it is the parameter where the uncertainty relies, we need to rely in simulations to obtain the solution which minimizes the average burned area for all the simulated scenarios, and therefore the usage of the SAA Model.

In the first phase of the model M independent set of random scenarios, each one with m random scenarios is be generated (where only the wind direction changes). Every set of scenarios M is fed to a deterministic (DE) model given as output an equal number of candidate solutions.

In the second phase a large number of n random scenarios (where only the wind changes) are generated, to be then applied each candidate solution. Evaluating the results, the candidate solution with the lowest average number of burned nodes for all the n scenarios is selected as the best solution.

A base containing all resources is located at the bottom right node for each scenario.

Example

First Phase

In this case we generated 4 sets of scenarios each one with 3 random scenarios, and applied the DE model for each one, obtained the 4 solutions shown in Figure 9.

M = 4

m = 3

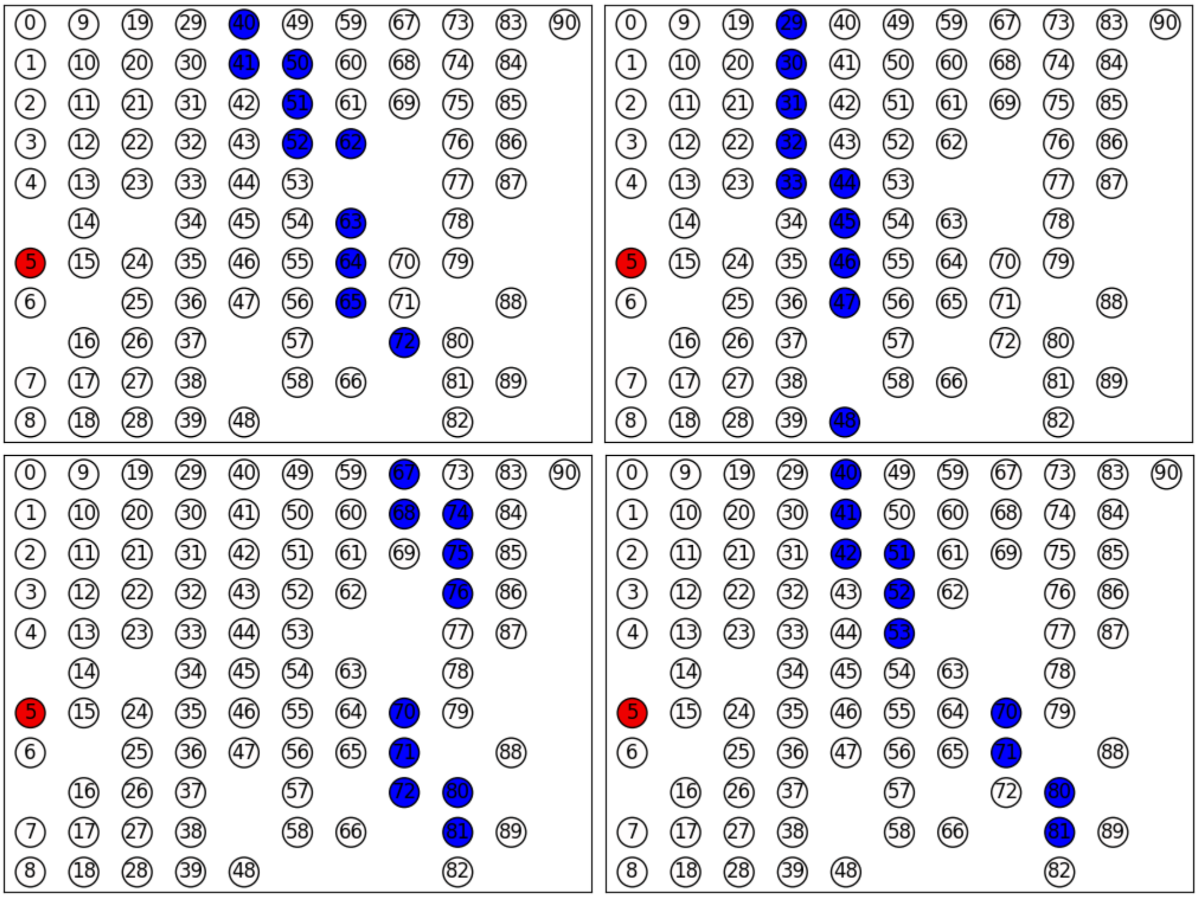

Figure 9 – Solutions obtained using four DE models each one with different 3 random scenarios. Red nodes represent the ignition node, the solution for each scenario is the set of nodes in blue, where the resources are placed.

Second Phase

3 different scenarios were randomly generated to evaluate the solutions.

n = 3

Random Scenario 1:

Four results of the first random scenario are represented below in Figure 10, Figure 11, Figure 12 and Figure 13, resorting to the first, second, third and fourth solution, respectively.

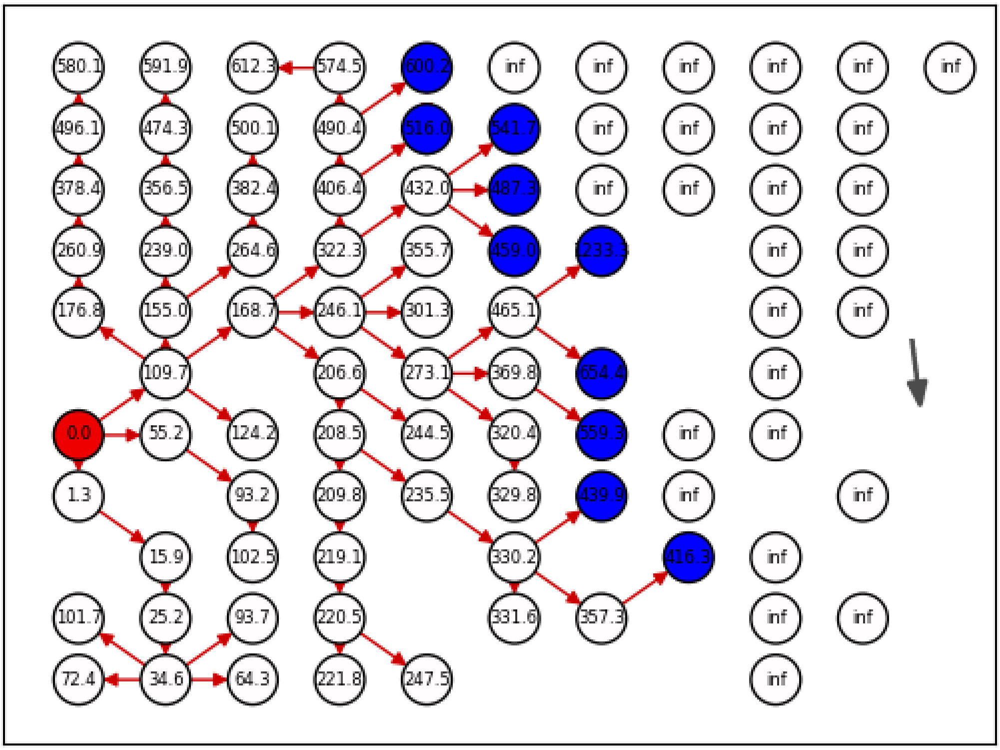

Figure 10- Solution 1 applied to the first Scenario with a number of burned nodes of 54. The red one is the ignition node, the red edges represent the fire paths from node to node, the black arrow indicates the wind direction and the blue nodes represent the places where the resources are placed.

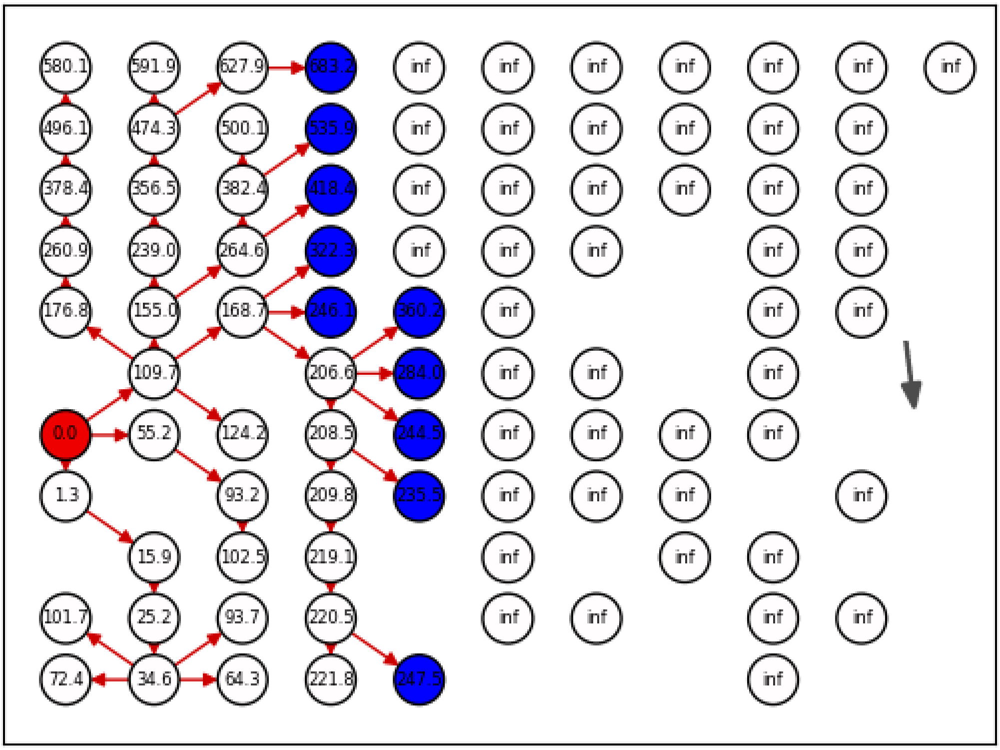

Figure 11 - Solution 2 applied to the first Scenario with a number of burned nodes of 35. The red one is the ignition node, the red edges represent the fire paths from node to node, the black arrow indicates the wind direction and the blue nodes represent the places where the resources are placed.

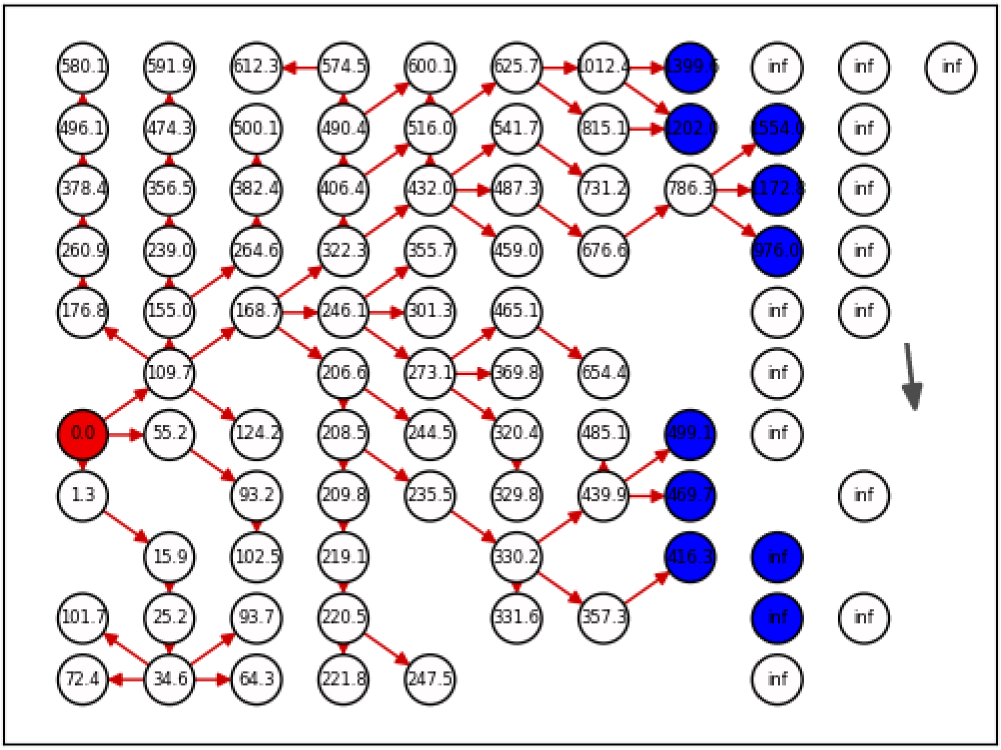

Figure 12 - Solution 3 applied to the first Scenario with a number of burned nodes of 67. The red one is the ignition node, the red edges represent the fire paths from node to node, the black arrow indicates the wind direction and the blue nodes represent the places where the resources are placed.

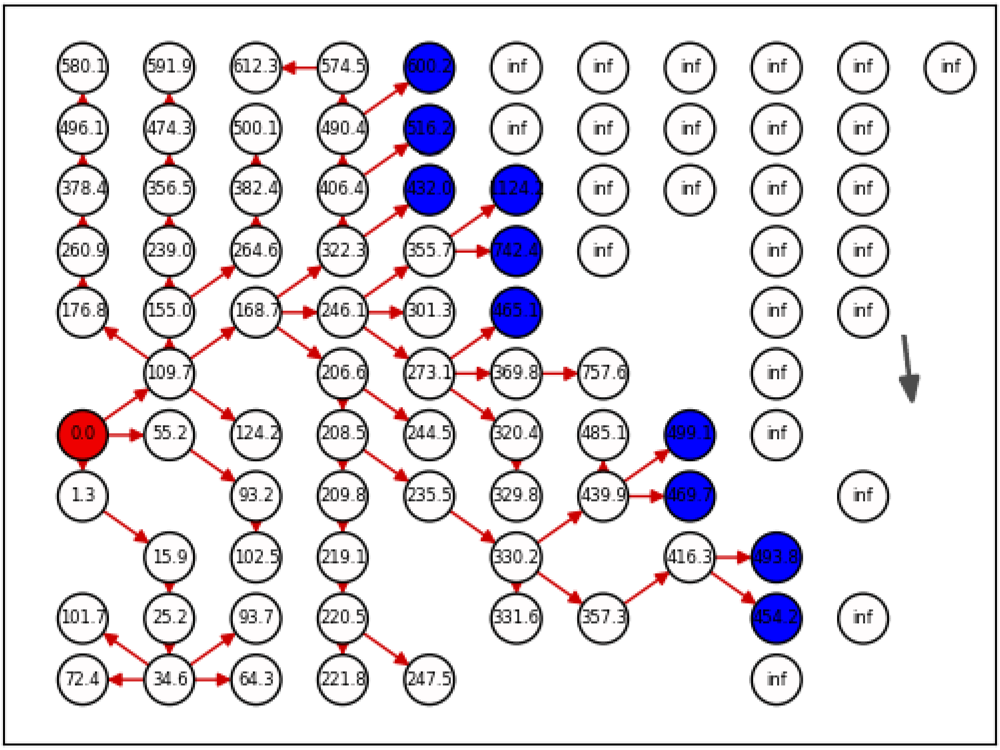

Figure 13- Solution 4 applied to the first Scenario with a number of burned nodes of 56. The red one is the ignition node, the red edges represent the fire paths from node to node, the black arrow indicates the wind direction and the blue nodes represent the places where the resources are placed.

Random Scenario 2:

Equally, four results of the second random scenario are represented below in Figure 14, Figure 15, Figure 16 and Figure 17, resorting to the first, second, third and fourth solution, respectively.

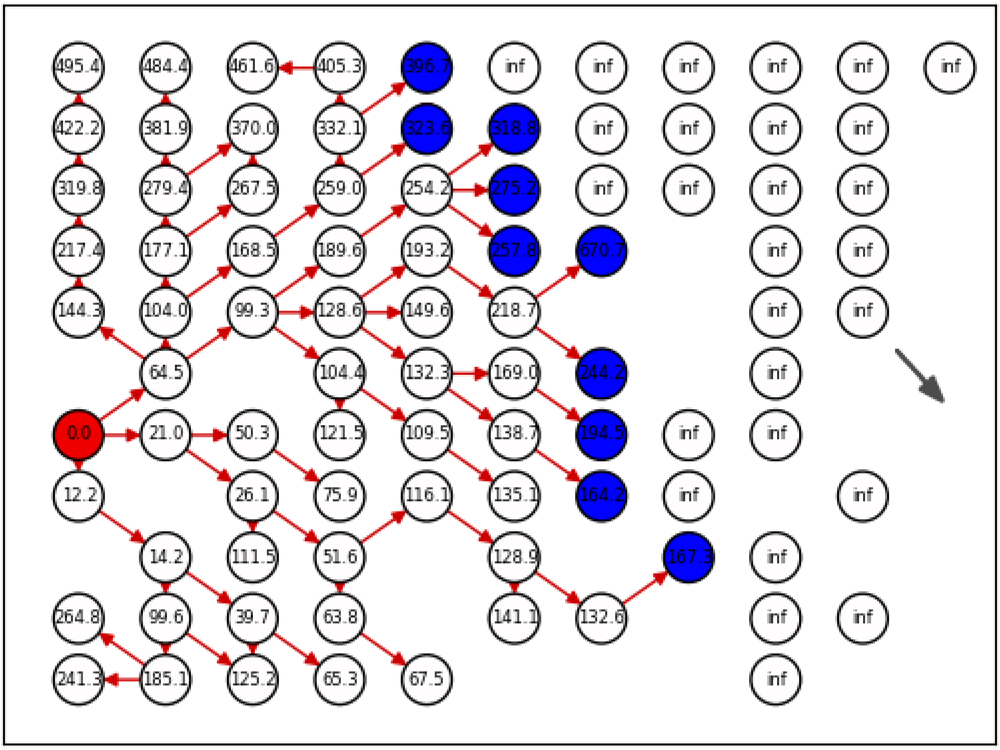

Figure 14 - Solution 1 applied to the second Scenario with a number of burned nodes of 54. The red one is the ignition node, the red edges represent the fire paths from node to node, the black arrow indicates the wind direction and the blue nodes represent the places where the resources are placed.

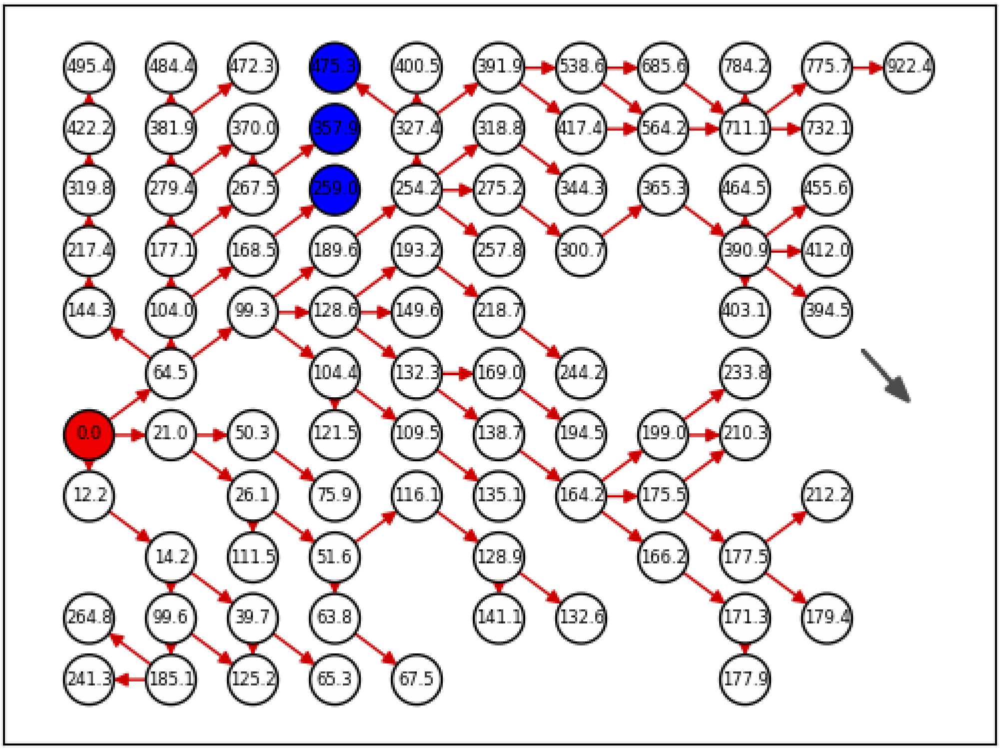

Figure 15 - Solution 2 applied to the second Scenario with a number of burned nodes of 88. The red one is the ignition node, the red edges represent the fire paths from node to node, the black arrow indicates the wind direction and the blue nodes represent the places where the resources are placed.

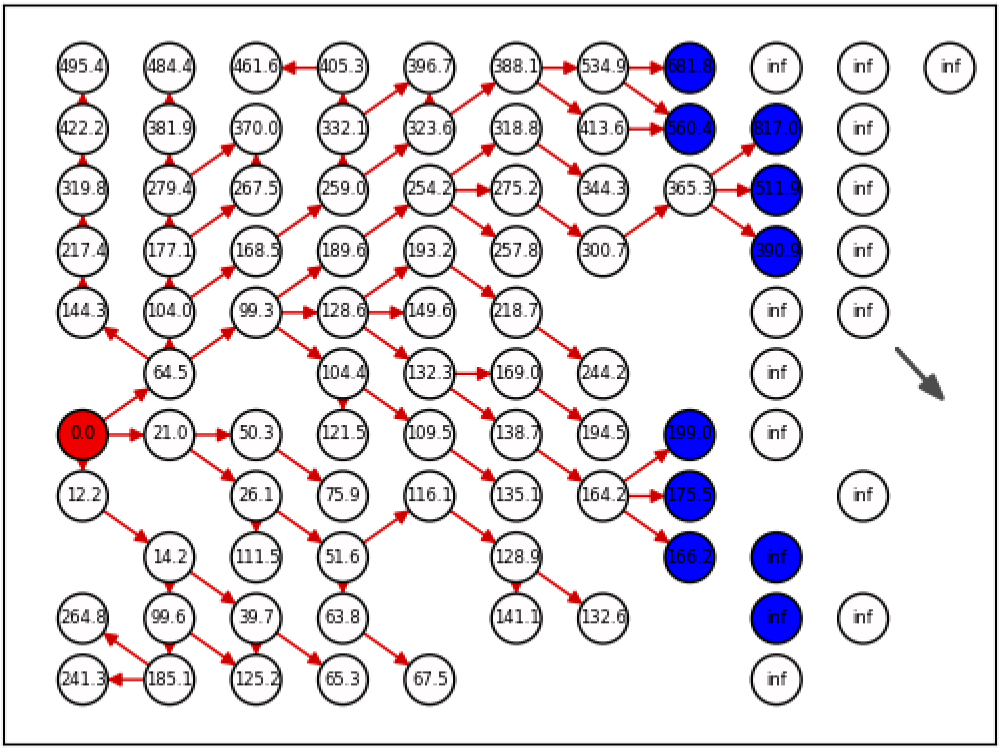

Figure 16 - Solution 3 applied to the second Scenario with a number of burned nodes of 68. The red one is the ignition node, the red edges represent the fire paths from node to node, the black arrow indicates the wind direction and the blue nodes represent the places where the resources are placed.

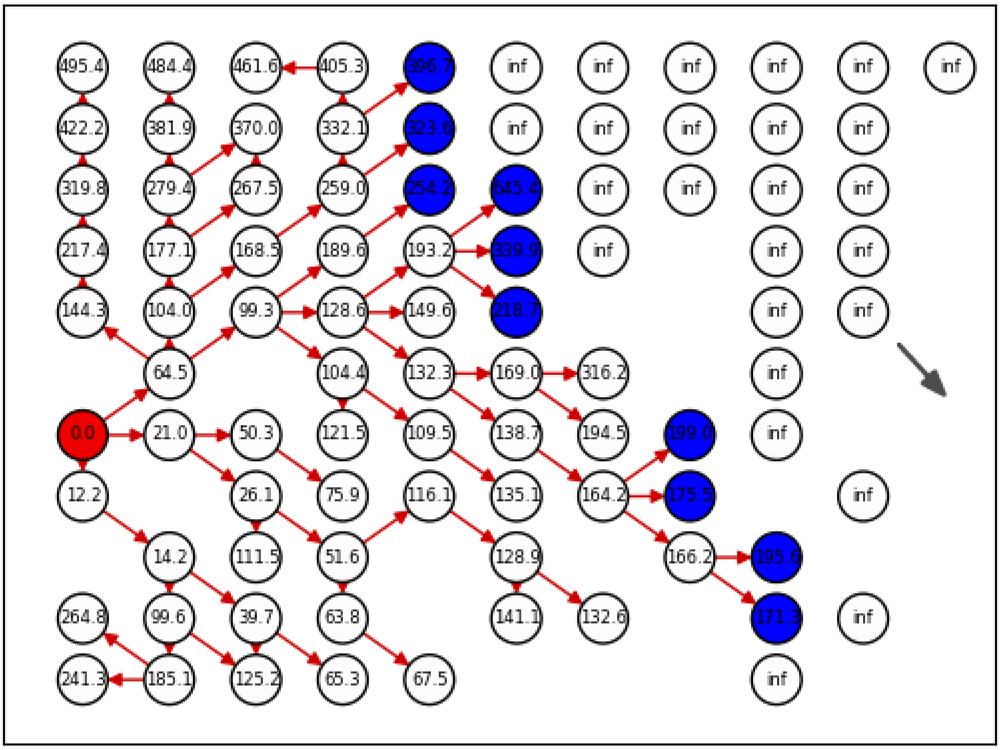

Figure 17 - Solution 4 applied to the second Scenario with a number of burned nodes of 56. The red one is the ignition node, the red edges represent the fire paths from node to node, the black arrow indicates the wind direction and the blue nodes represent the places where the resources are placed.

Random Scenario 3:

Once again, four results of the third random scenario are represented below in Figure 18, Figure 19, Figure 20 and Figure 21, resorting to the first, second, third and fourth solution, respectively.

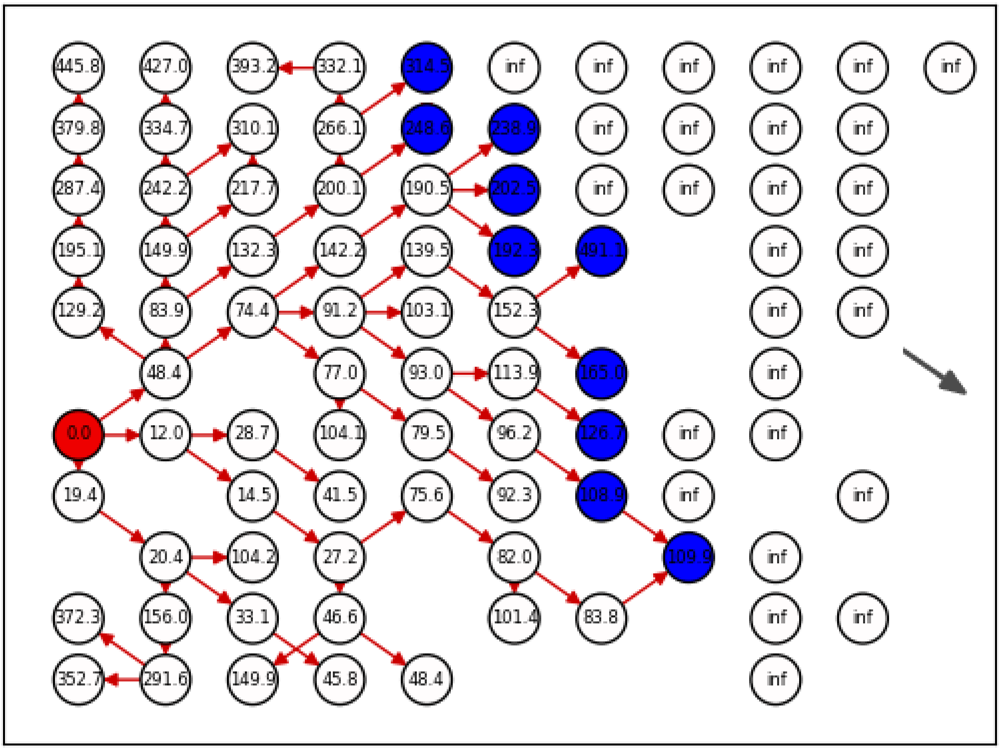

Figure 18- Solution 1 applied to the third Scenario with a number of burned nodes of 54. The red one is the ignition node, the red edges represent the fire paths from node to node, the black arrow indicates the wind direction and the blue nodes represent the places where the resources are placed.

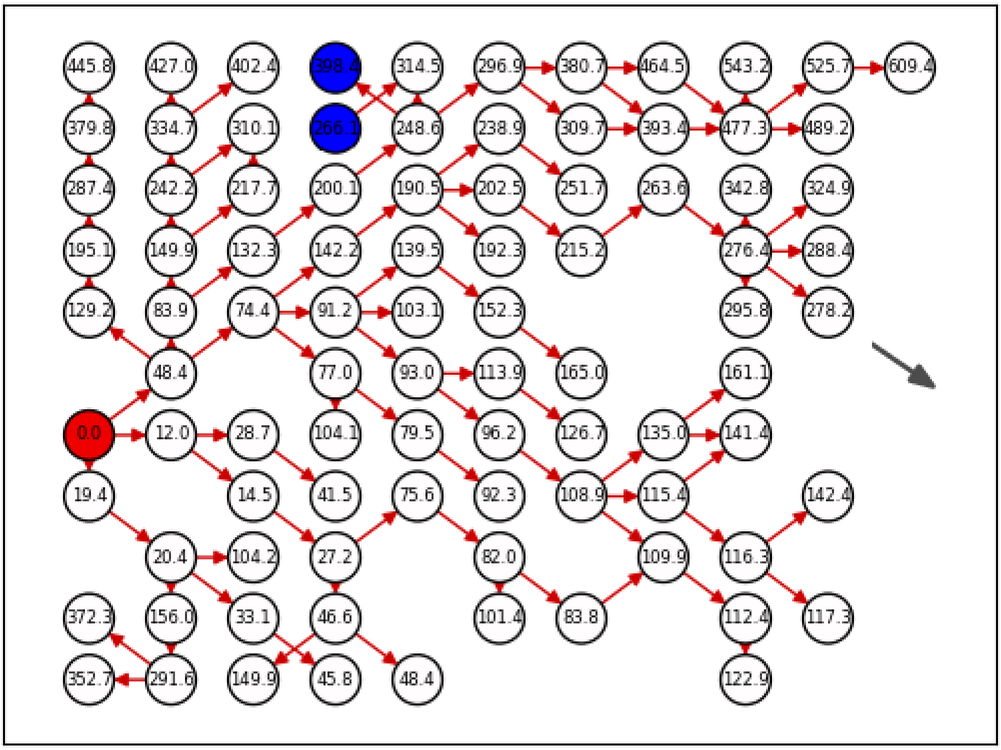

Figure 19- Solution 2 applied to the third Scenario with a number of burned nodes of 89. The red one is the ignition node, the red edges represent the fire paths from node to node, the black arrow indicates the wind direction and the blue nodes represent the places where the resources are placed.

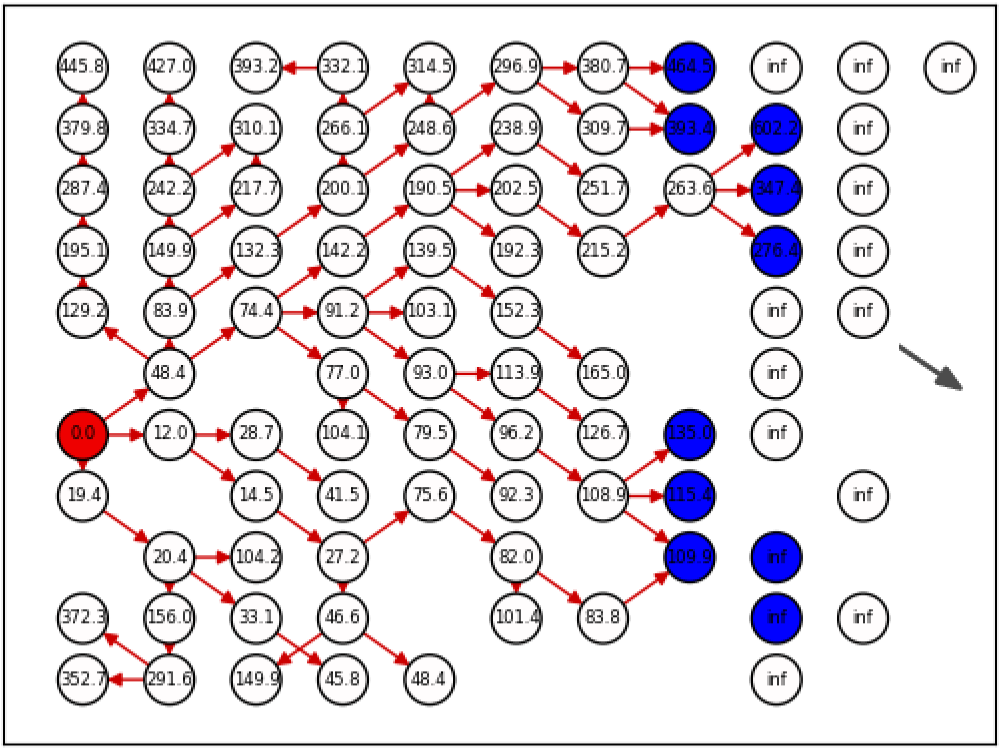

Figure 20- Solution 3 applied to the third Scenario with a number of burned nodes of 68. The red one is the ignition node, the red edges represent the fire paths from node to node, the black arrow indicates the wind direction and the blue nodes represent the places where the resources are placed.

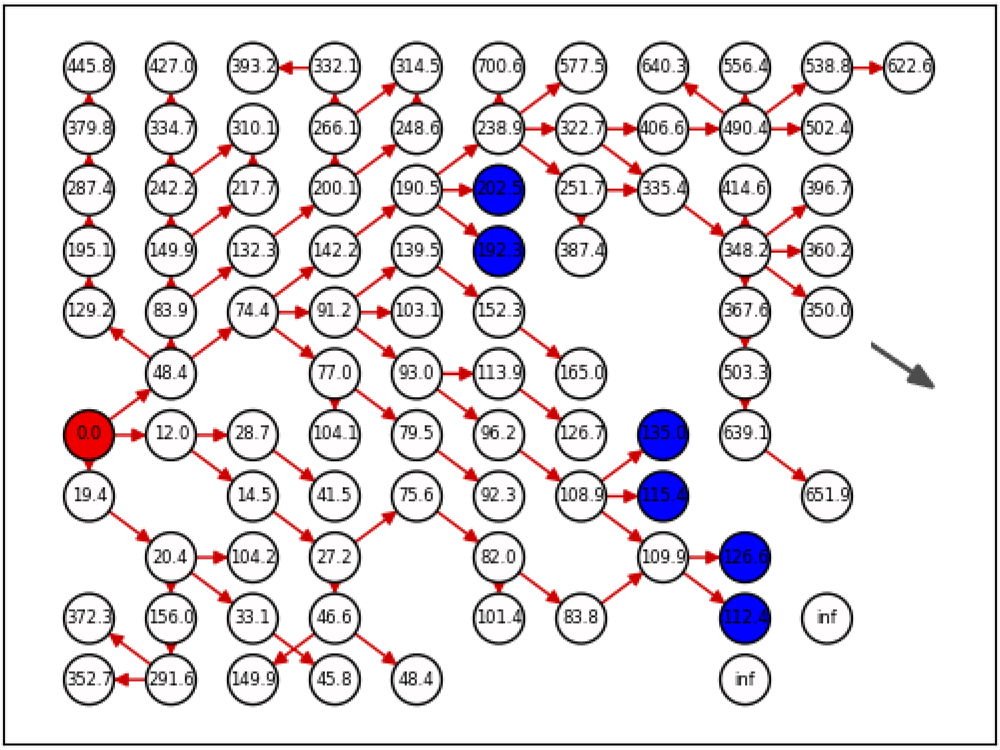

Figure 21- Solution 4 applied to the third Scenario with a number of burned nodes of 83. The red one is the ignition node, the red edges represent the fire paths from node to node, the black arrow indicates the wind direction and the blue nodes represent the places where the resources are placed.

Table 4 compares the results obtained when using different solutions.

Table 4 - Results obtained when applying different solutions to the Scenarios as well as evaluations of each solution. In bold is highlighted the best solution, solution 1.

	Solution 1	**Solution 2**	**Solution 3**	**Solution 4**
Scenario 1 # burned nodes	**54**	35	67	56
Scenario 2 # burned nodes	**54**	88	68	56
Scenario 3 # burned nodes	**54**	89	68	83
Average # burned nodes	**54**	70.66	67.66	65

As it is possible to observe from table 3, the solution 1 is the solution that presents the lowest average number of burned nudes, and therefore would be selected as the best solution. We can analyse the other solutions despite already knowing the solution 1 is the chosen one based on the criteria of the lowest average number of burned nodes. Solution 2 despite having only 35 burned nodes on scenario 1, on the other scenarios the number of burned nodes is extremely high, so it is not very good. Both Solution 3 and 4 have a number of burned nodes higher than solution 1 in every scenario, therefore showing right away the bad quality of both. What we are aiming is the lower expected number of burned nodes and therefore solution 1 was chosen as explained before.

Obviously, this is a small example just for the purpose of demonstrating the process. To really obtain good results a larger number of random solutions and scenarios would be necessary.

6 Computational Experiments

6.1 EXPERIMENTAL SETTING

This section aims to evaluate the performance of the SAA model using four different cases, where the ignition node will vary between the center and center left nodes, and the grid resolution will vary from 11x11 to 21x21 grid. The base with the resources is allocated at the bottom right of the grid, with 10 available resources for the 11x11 grid and 40 for the 21x21 grid, it is assumed that the time it takes from a resource to arrive at a node is directly proportional to the Euclidian distance between it and the base of the resources.

All the tests performed were programmed in python (version 3.9), in the PyCharm ide, with the package networkx (version 2.6.3). The optimization core is gurobi (version 9.5.0).

The topography data used on the tests were from the region of Baião, sadly, a well-known region for forest fires in Portugal. Uncertainty will be present in the wind that will follow a kernel density estimation (KDE) probability density function obtained from real historical data presented in [32], illustrated in Figure 22, in this tests we will only be considering the wind in range between blowing from west to east and from north to south, as it is not reasonable to consider the other directions because of the assumed ignition location and the rare frequency of occurrence in other directions. The wind speed considered will be 80 km/h. The instance of evaluation for this tests is considered to be at the end of the fire.

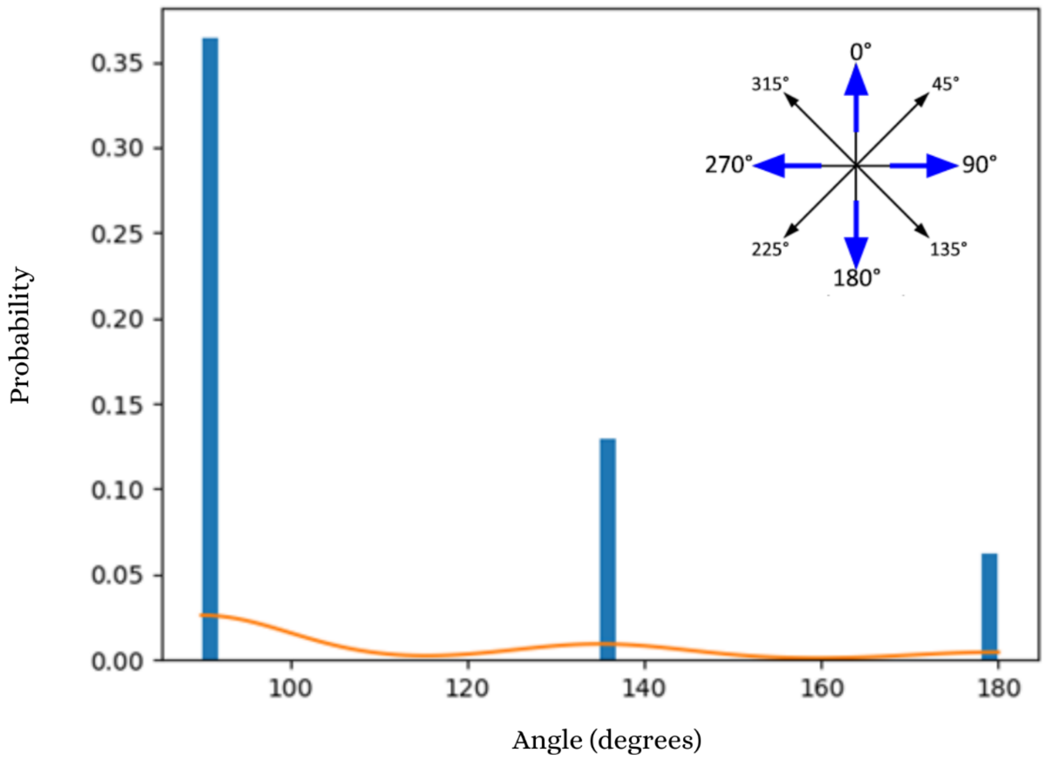

Figure 22 – KDE plot representing the wind blowing direction occurrence probabilities. Obtained from data present in [32], represented by the histogram in blue. In the x axis are the wind blowing directions (0º being North increasing clockwise) in the y axis are the probabilities of occurrence.

In each of the four cases in the first phase of the SAA model 10 DE models were generated having each 5 different random scenarios and in the second phase of the model 1000 other random scenarios were generated.

In order to understand the quality of the solution's result obtained and therefore the importance of using a stochastic approach for this problem, it is imperative to have some other results to compare and for that, it is necessary to obtain the expected value solution and to introduce the concept of perfect information, both explained next.

6.2 EXPECTED VALUE SOLUTION

The expected value solution is the optimal solution of the deterministic model where the uncertain values are replaced by their average. This approach will simply use a deterministic model considering a scenario with average wind direction of the KDE density probability function, and therefore removing the stochastic part of the problem, with that, a solution is obtained for the "average" scenario. The next step of this method is to simply try to apply the same solution to all the random scenarios generated in the second phase of the SAA. If we call "D" the average number of burned nodes using the same solution for all the random scenarios, we are expected to obtain a positive result for the Value of the stochastic solution (VSS), which can be described as VSS = D - $Z_n\ (x^*)$.

6.3 PERFECT INFORMATION

Perfect information is a theoretical concept in decision making that assumes that a decision-maker has complete and accurate information about all the available options. In other words, it assumes that the decision-maker has access to all the information needed to make an optimal decision [33].

In the context of this problem each random scenario generated in the second phase of the SAA, if solved individually using the deterministic approach can be classified as a perfect information problem, as all the information is known and the best solution for that specific scenario will be obtained. If we call PI the average number of burned nodes using the best solution for all the random scenarios individually, we are expected to obtain a positive result for the Expected Value of Perfect Information (EVPI), which can be formulated as such: EVPI = $Z_n\ (x^*)$ - PI.

6.4 COMPARATIVE ANALYSES BETWEEN PI AND DE RESULTS

We now exemplify the calculation of perfect information solutions and their relation with the optimal solution of the deterministic equivalent.

Five scenarios with the ignition on the center node were randomly generated with the purpose of comparing the results obtained taking into account the possibility of occurrence of all 5 (DE), or simply applying the best solution to each one individually (PI). Figure 23 presents the results obtained when resorting to the DE model.

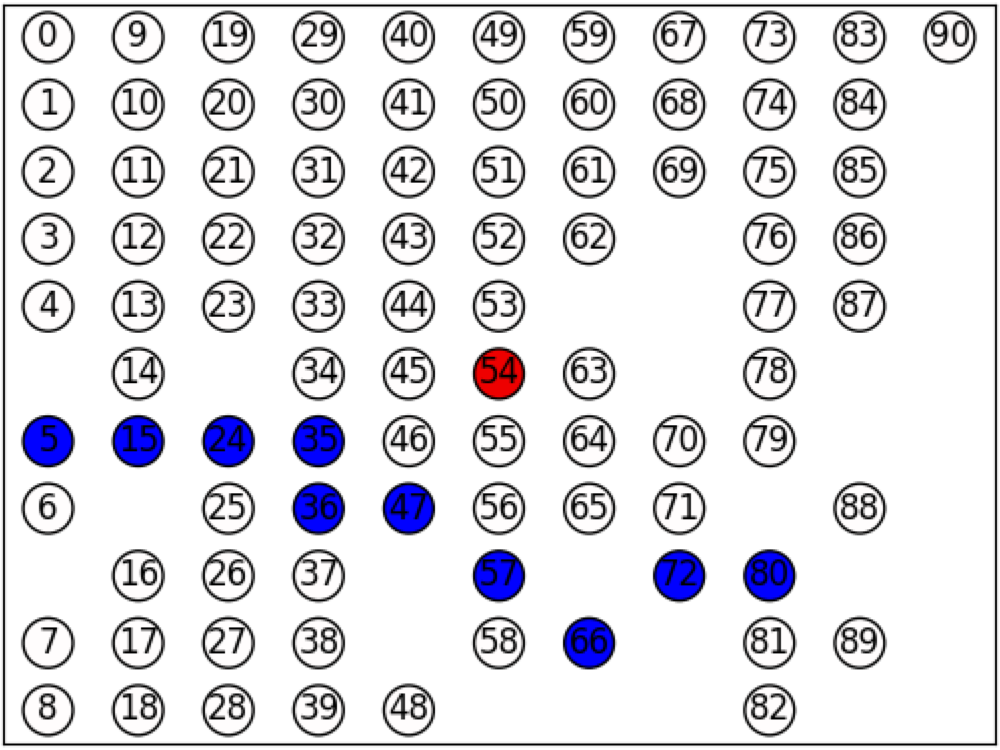

Figure 23 - Solution using the DE model with 5 random scenarios, with the grid representing the region of Baião. Ignition node is marked in red and in blue are represent the locations where the resources are placed.

The solutions obtained with perfect information on the five different scenarios, scenario 1, scenario 2, scenario 3, scenario 4 and scenario 5 are shown in Figure 24, Figure 25, Figure 26, Figure 27 and Figure 28, respectively.

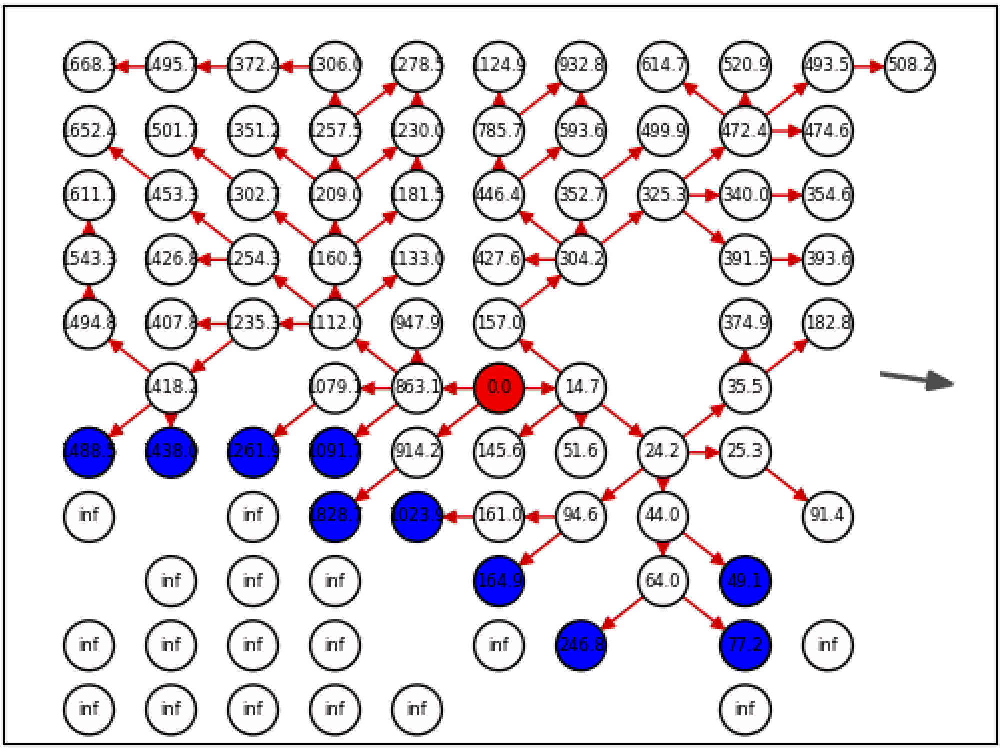

Figure 24 - Placement of the resources and fire paths in scenario 1, the first random Scenario. The numbers on the nodes are the fire arrival times in minutes, the red one is the ignition node, the red edges represent the fire paths from node to node, the black arrow indicates the wind direction and the blue nodes represent the places where the resources are placed using PI.

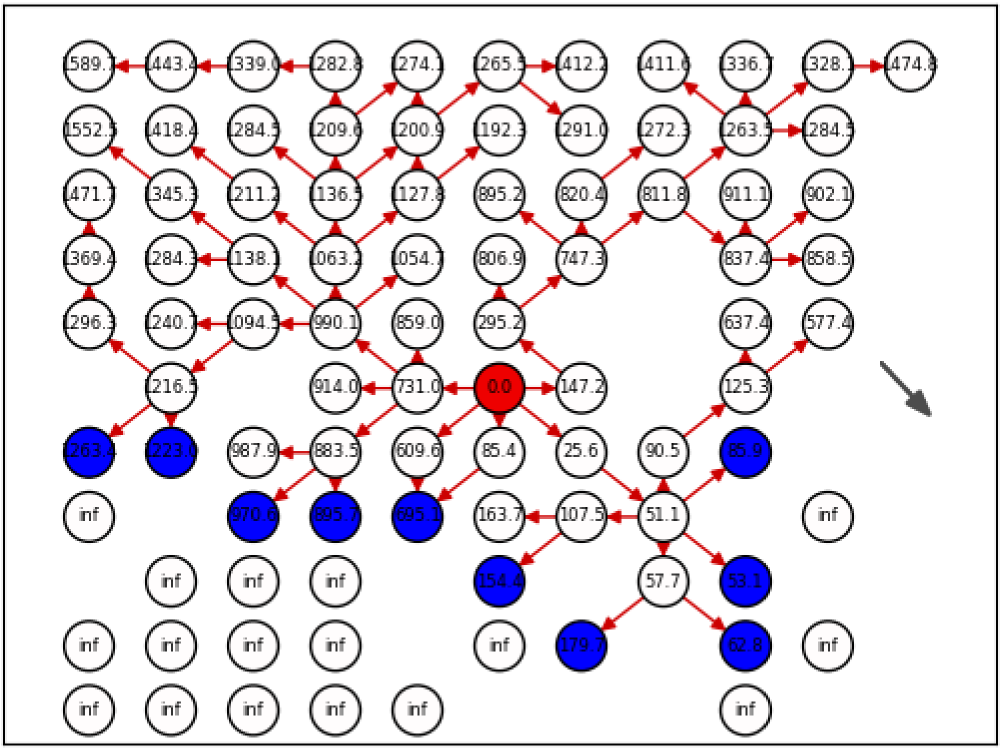

Figure 25 - Placement of the resources and fire paths in scenario 2, the second random Scenario. The numbers on the nodes are the fire arrival times in minutes, the red one is the ignition node, the red edges represent the fire paths from node to node, the black arrow indicates the wind direction and the blue nodes represent the places where the resources are placed using PI.

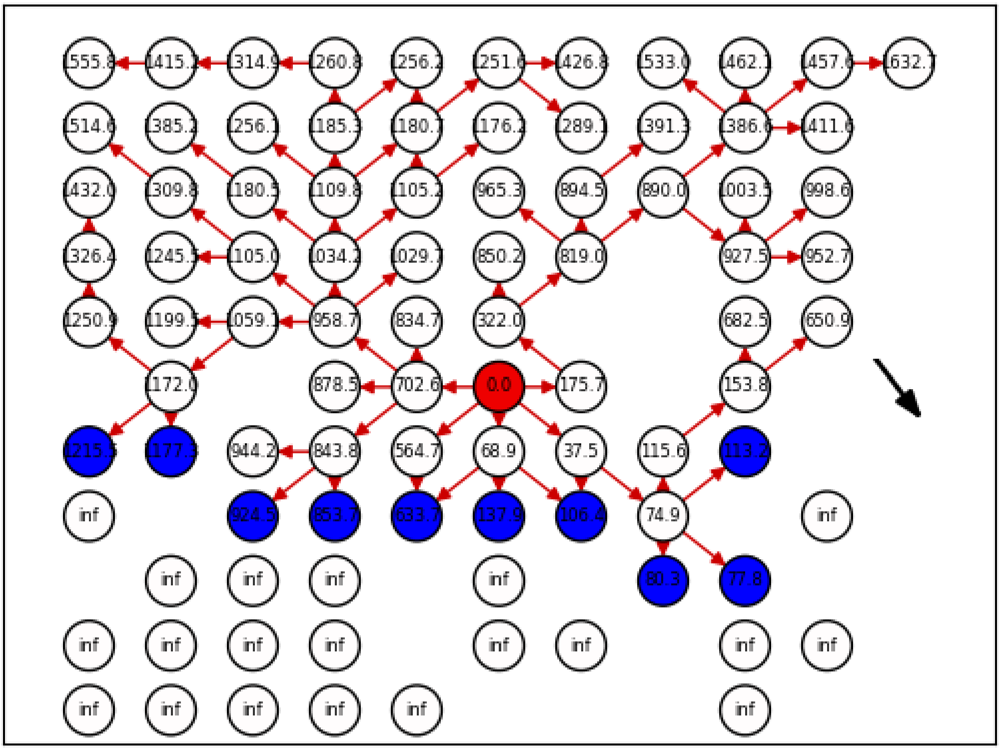

Figure 26 - Placement of the resources and fire paths in scenario 3, the third random Scenario. The numbers on the nodes are the fire arrival times in minutes, the red one is the ignition node, the red edges represent the fire paths from node to node, the black arrow indicates the wind direction and the blue nodes represent the places where the resources are placed using PI.

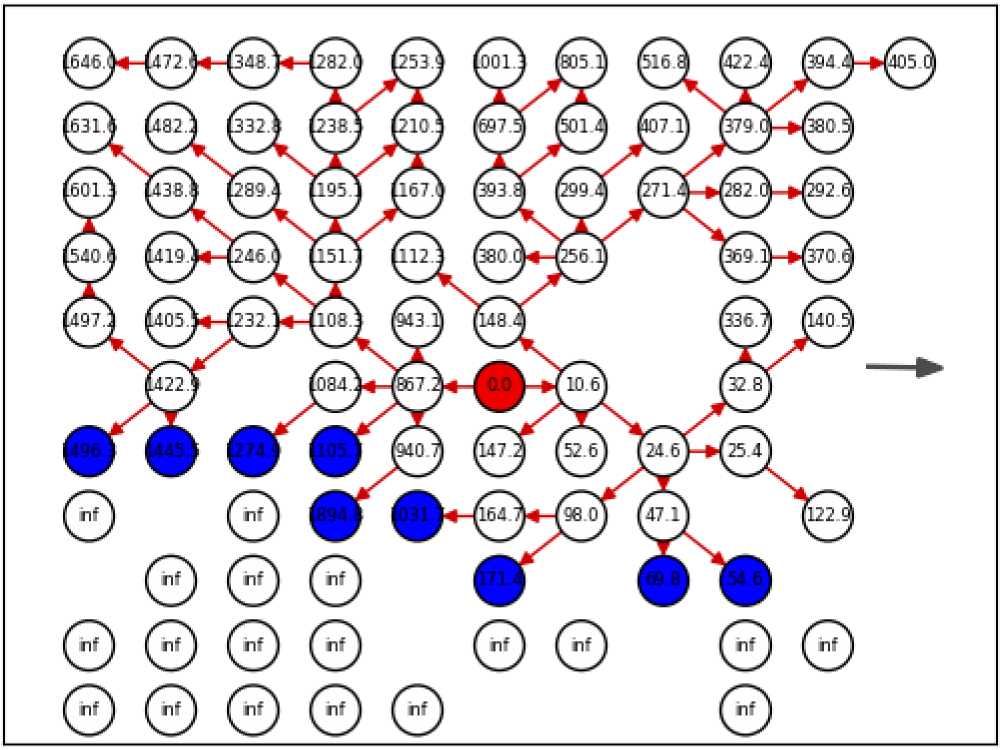

Figure 27 - Placement of the resources and fire paths in scenario 4, the fourth random Scenario. The numbers on the nodes are the fire arrival times in minutes, the red one is the ignition node, the red edges represent the fire paths from node to node, the black arrow indicates the wind direction and the blue nodes represent the places where the resources are placed using PI.

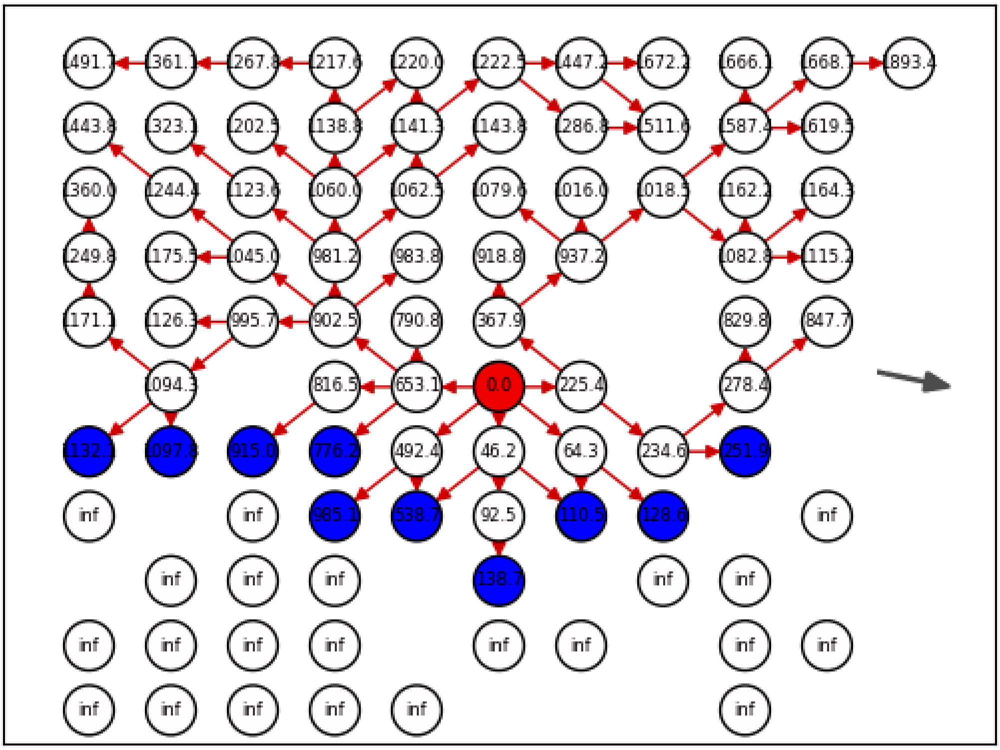

Figure 28 - Placement of the resources and fire paths in scenario 5, the fifth random Scenario. The numbers on the nodes are the fire arrival times in minutes, the red one is the ignition node, the red edges represent the fire paths from node to node, the black arrow indicates the wind direction and the blue nodes represent the places where the resources are placed using PI.

Average number of burned nodes using DE: 68.2

Average number of burned nodes using PI: 62.2

The DE model will always take into account all the scenarios possible to occur with the associated probabilities and therefore give a worst solution than the PI, that in the other hand looks at each scenario individually, giving the best solution for each one and naturally having a better average. But with problems with uncertainty associated it is not possibly to rely on the PI approach.

6.5 RESULTS

In order to test the model, four different tests were performed: Test 1, detailed described in section 6.5.1 and the others: Test 2, Test 3 and Test 4 are described in the attachments (Appendix I, Appendix II and Appendix III, respectively). Amongst the four tests, there are two factors changes, therefore resulting in the four different combinations, as depicted in Table 5.

Table 5 - Different combinations of parameters that originated the four different test of the model

		Grid size	
		11x11	21x21
Ignition	Center of the grid	Test 1	Test 3
	Center left of the grid	Test 2	Test 4

The parameters ignition and grid size were previously explained and used in the example of section 3.1.3.

As the grid size changes, the number of available resources necessarily changes as well. Therefore, the number of available resources for an 11x11 grid is assigned as 10, and for a 21x21 grid, it is 40.

In Table 6 are shown a summary of the results obtained from the Tests performed.

Table 6 - Summary of the results obtained

	$\boldsymbol{Z_n\ (x^*)}$	**D**	**PI**	**VSS = D -** $\boldsymbol{Z_n\ (x^*)}$	**EVPI =** $\boldsymbol{Z_n\ (x^*)}$ **- PI**	**runtime**
Test 1	66.19	80.97	63.46	14.78	2.73	28 min
Test 2	75.73	79.32	65.46	3.59	10.27	25 min
Test 3	232.33	275.97	226.30	43.64	6.03	105 min
Test 4	220.17	273.92	177.24	53.75	42.93	111 min

6.5.1 DETAILED DISCUSSION OF TEST1

Ignition: Center of the Grid

Grid Size: 11x11

Number of Resources: 10

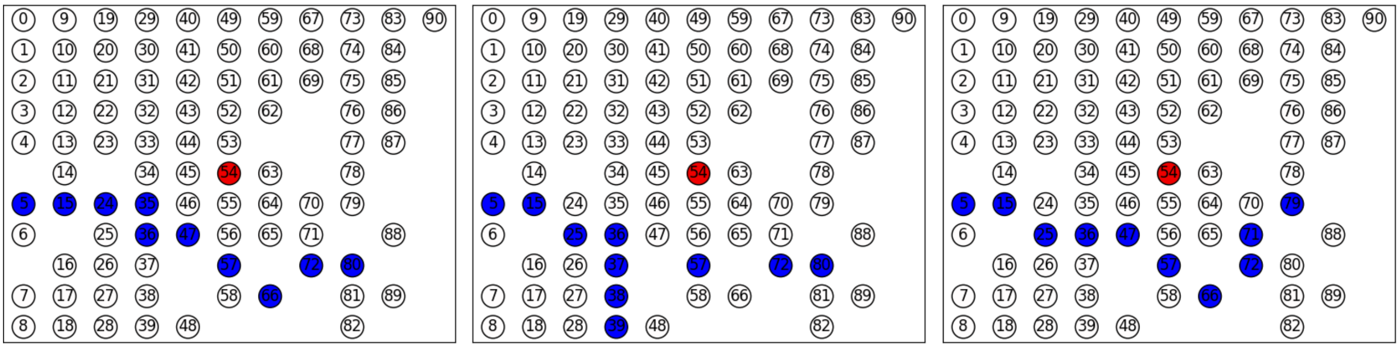

Figure 29 – Some of the Solutions obtained using DE models in the first phase each one with different 5 random scenarios. Red nodes represent the ignition node, the solution for each scenario is the set of nodes in blue, where the resources are placed.

To better illustrate the computation experiments four different Scenarios for Test 1 are presented where the solutions above are applied. A total of 1000 scenarios were generated, which makes it implausible to present them all in this work.

Random Scenario 1:

Three results of the first random scenario are represented below in Figure 30, Figure 31 and Figure 32, resorting to the first, second, third solution, respectively.

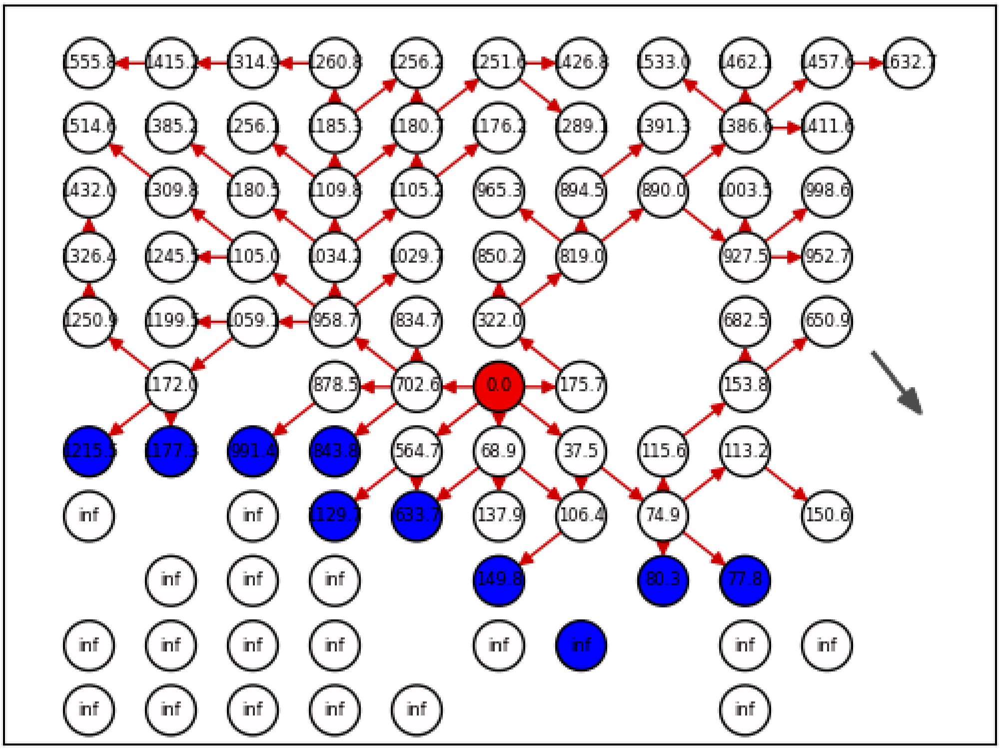

Figure 30- Solution 1 applied to the first Scenario with a number of burned nodes of 63. The red one is the ignition node, the red edges represent the fire paths from node to node, the numbers on the nodes are the fire arrival times in minutes, the black arrow indicates the wind direction and the blue nodes represent the places where the resources are placed.

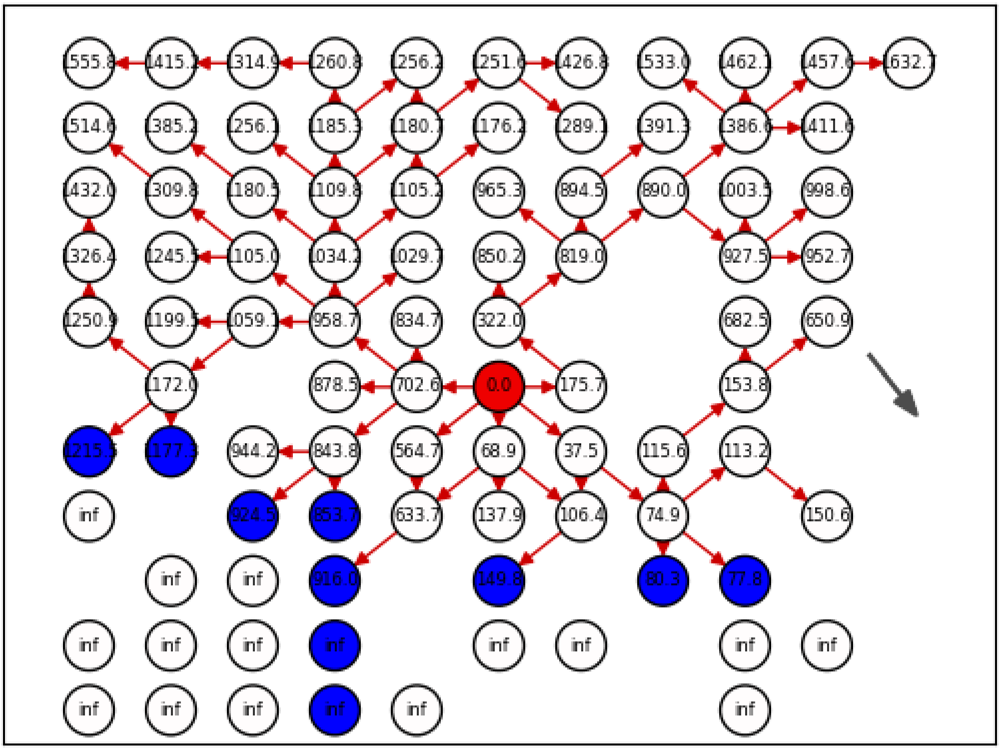

Figure 31 - Solution 2 applied to the first Scenario with a number of burned nodes of 66. The red one is the ignition node, the red edges represent the fire paths from node to node, the numbers on the nodes are the fire arrival times in minutes, the black arrow indicates the wind direction and the blue nodes represent the places where the resources are placed.

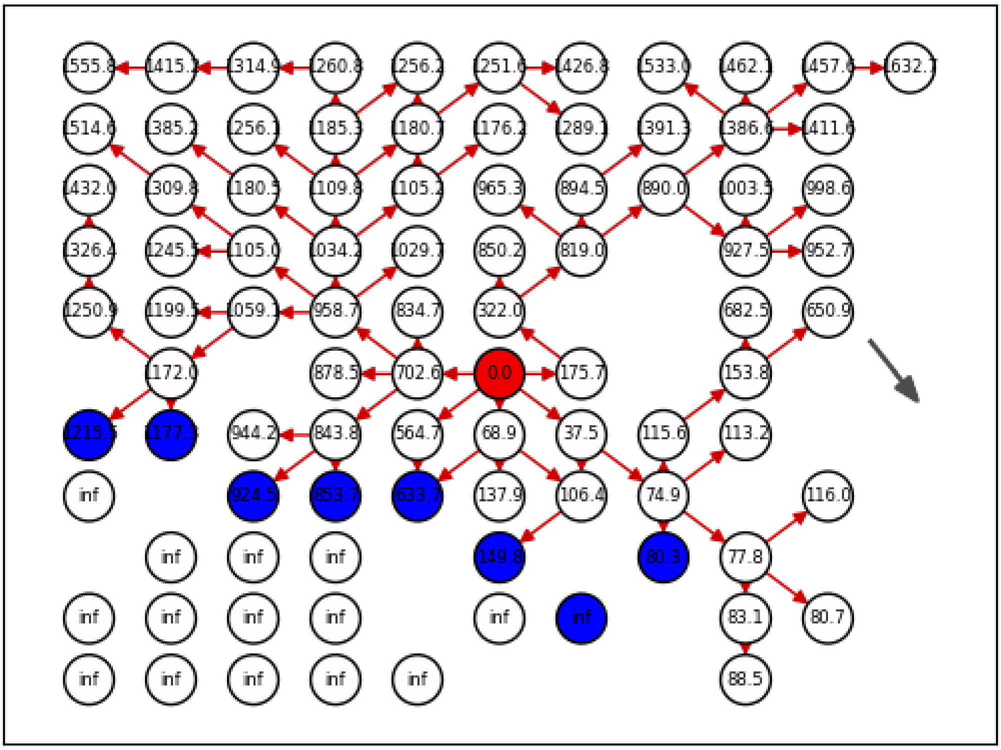

Figure 32 - Solution 3 applied to the first Scenario with a number of burned nodes of 69. The red one is the ignition node, the red edges represent the fire paths from node to node, the numbers on the nodes are the fire arrival times in minutes, the black arrow indicates the wind direction and the blue nodes represent the places where the resources are placed.

Random Scenario 2:

Again, three results of the second random scenario are represented below in Figure 33, Figure 34 and Figure 35, resorting to the first, second and third solution, respectively.

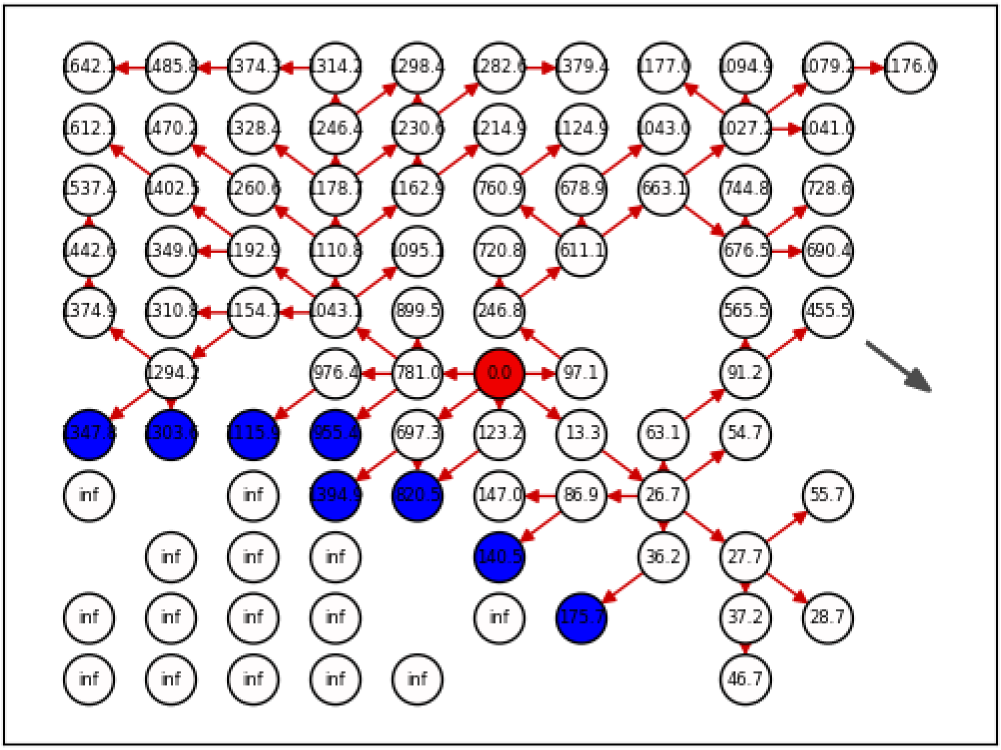

Figure 33 - Solution 1 applied to the second Scenario with a number of burned nodes of 68. The red one is the ignition node, the red edges represent the fire paths from node to node, the numbers on the nodes are the fire arrival times in minutes, the black arrow indicates the wind direction and the blue nodes represent the places where the resources are placed.

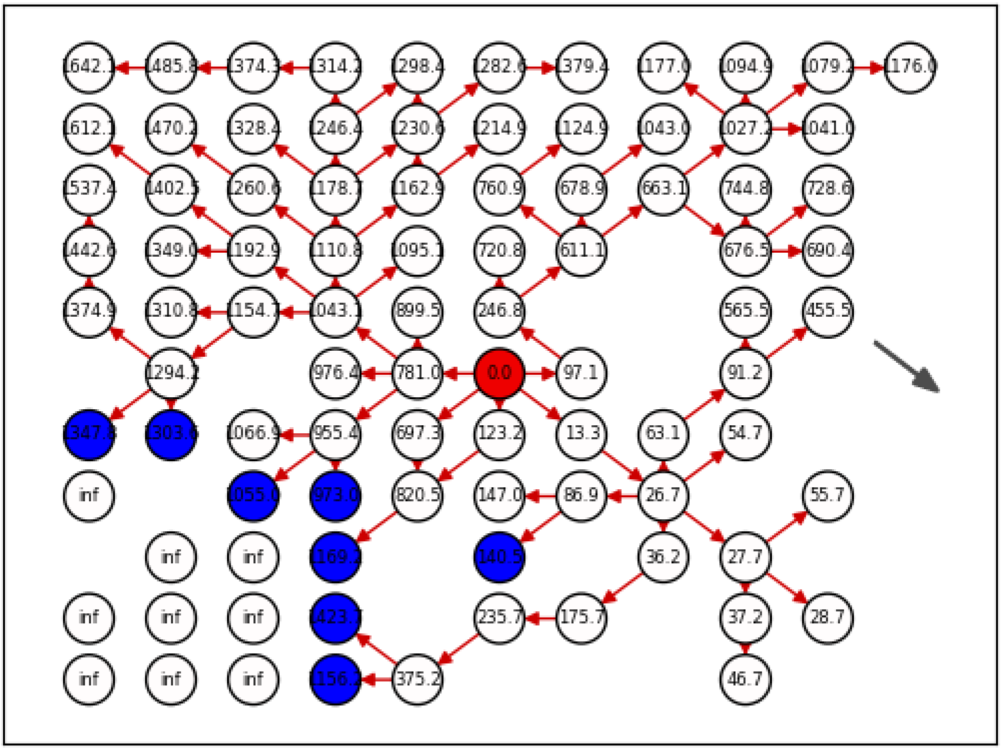

Figure 34 - Solution 2 applied to the second Scenario with a number of burned nodes of 74. The red one is the ignition node, the red edges represent the fire paths from node to node, the numbers on the nodes are the fire arrival times in minutes, the black arrow indicates the wind direction and the blue nodes represent the places where the resources are placed.

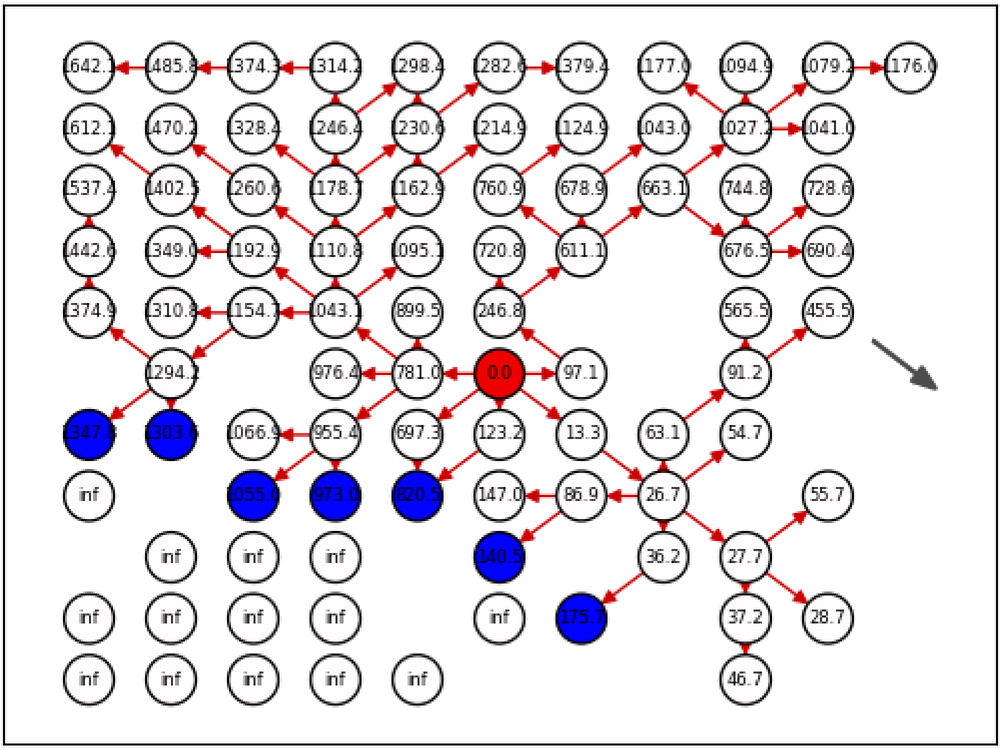

Figure 35 - Solution 3 applied to the second Scenario with a number of burned nodes of 70. The red one is the ignition node, the red edges represent the fire paths from node to node, the numbers on the nodes are the fire arrival times in minutes, the black arrow indicates the wind direction and the blue nodes represent the places where the resources are placed.

Random Scenario 3:

Once again, three results of the third random scenario are represented below in Figure 36, Figure 37 and Figure 38, resorting to the first, second, third and fourth solution, respectively.

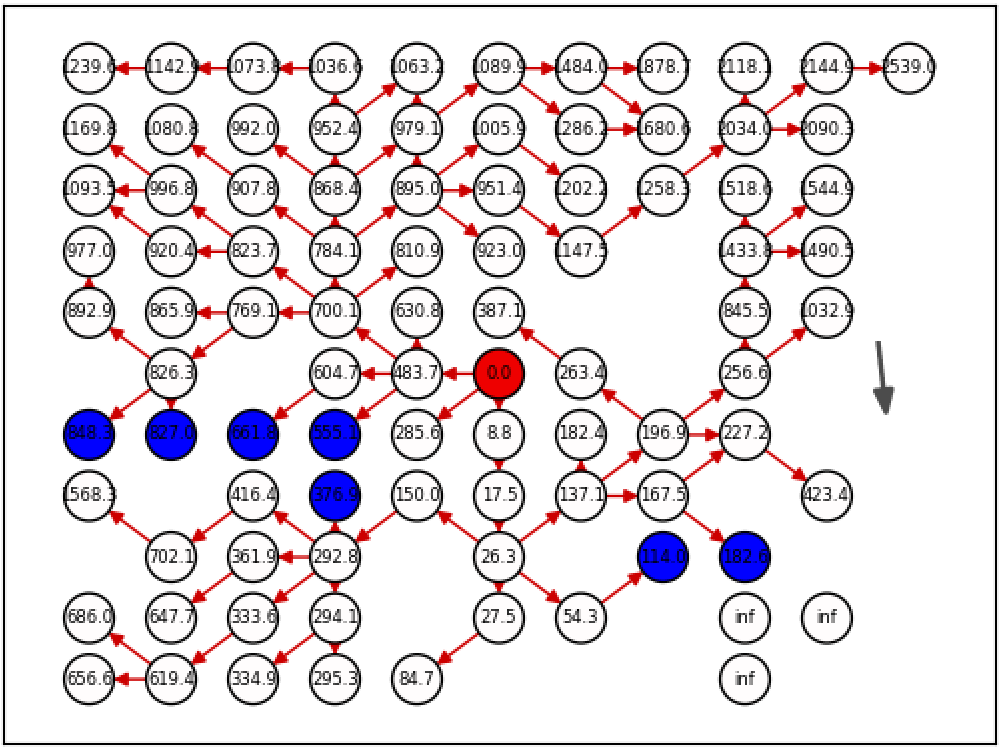

Figure 36 - Solution 1 applied to the third Scenario with a number of burned nodes of 81. The red one is the ignition node, the red edges represent the fire paths from node to node, the numbers on the nodes are the fire arrival times in minutes, the black arrow indicates the wind direction and the blue nodes represent the places where the resources are placed.

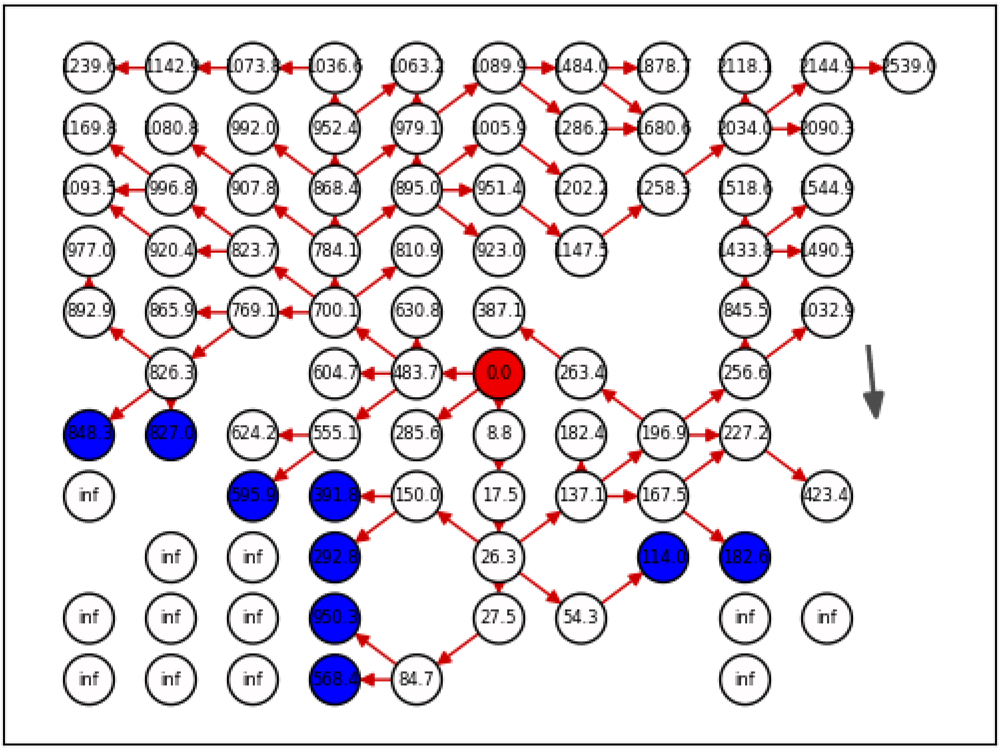

Figure 37- Solution 2 applied to the third Scenario with a number of burned nodes of 70. The red one is the ignition node, the red edges represent the fire paths from node to node, the numbers on the nodes are the fire arrival times in minutes, the black arrow indicates the wind direction and the blue nodes represent the places where the resources are placed.

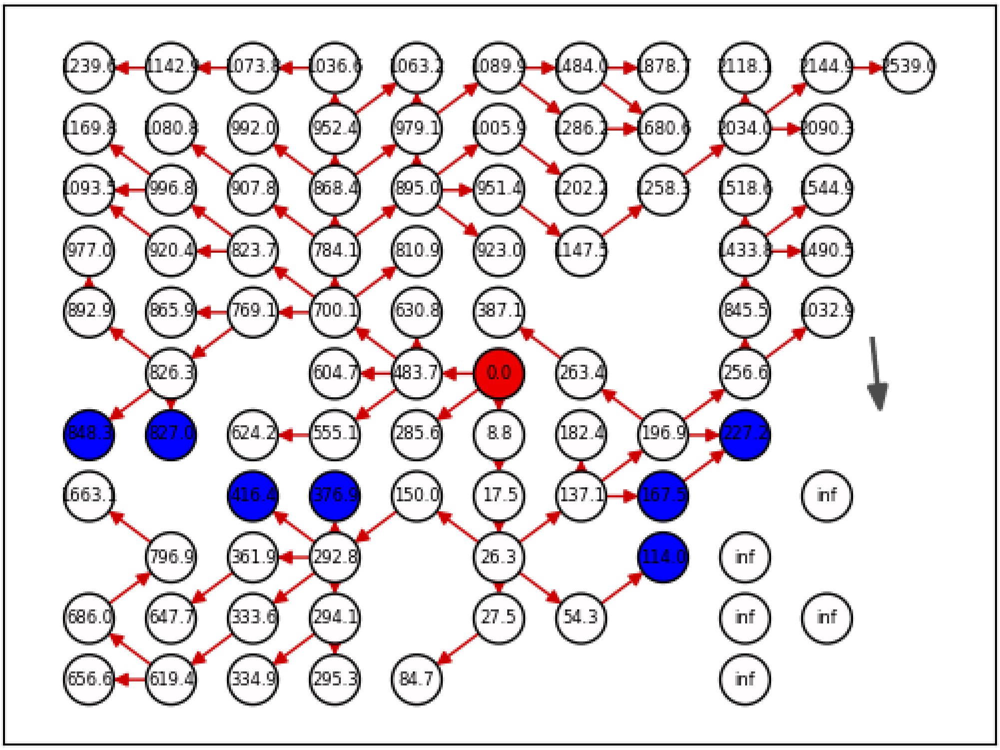

Figure 38 - Solution 3 applied to the third Scenario with a number of burned nodes of 79. The red one is the ignition node, the red edges represent the fire paths from node to node, the numbers on the nodes are the fire arrival times in minutes, the black arrow indicates the wind direction and the blue nodes represent the places where the resources are placed.

Random Scenario 4:

One last time, three results of the fourth random scenario are represented below in Figure 39, Figure 40 and Figure 41, resorting to the first, second and third solution, respectively.

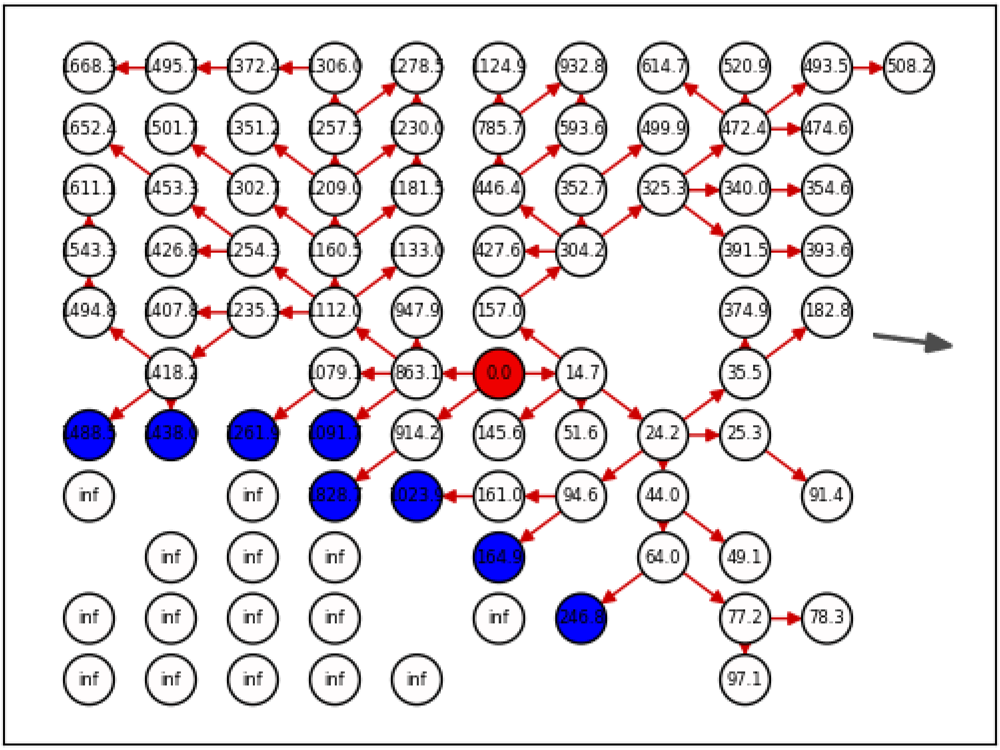

Figure 39 - Solution 1 applied to the fourth Scenario with a number of burned nodes of 68. The red one is the ignition node, the red edges represent the fire paths from node to node, the numbers on the nodes are the fire arrival times in minutes, the black arrow indicates the wind direction and the blue nodes represent the places where the resources are placed.

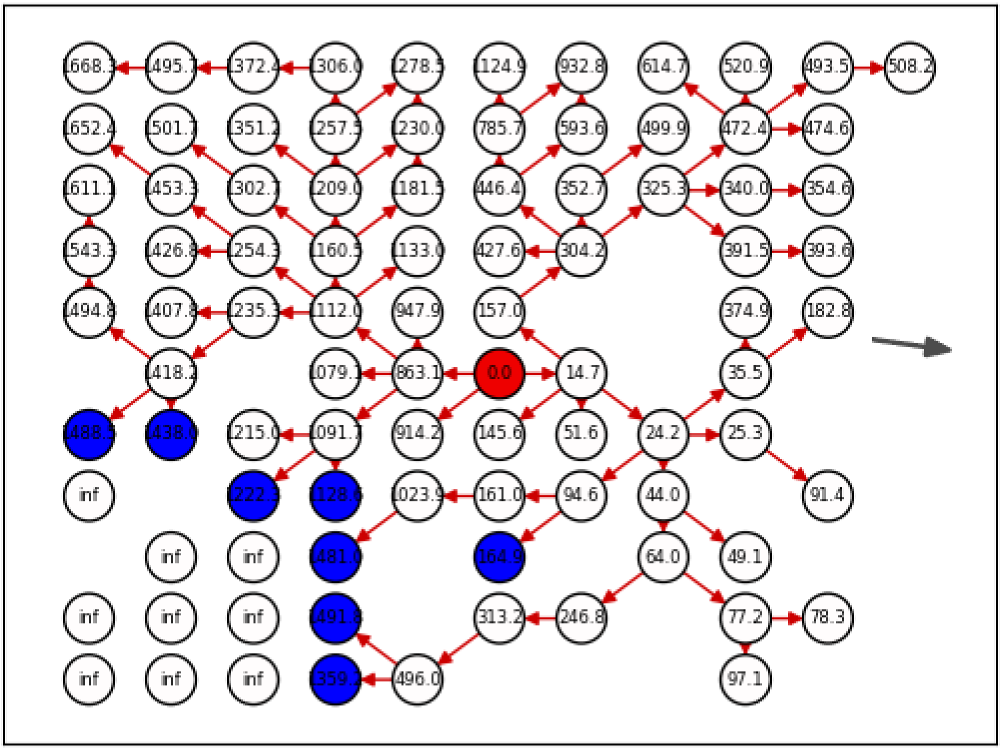

Figure 40 - Solution 2 applied to the fourth Scenario with a number of burned nodes of 74. The red one is the ignition node, the red edges represent the fire paths from node to node, the numbers on the nodes are the fire arrival times in minutes, the black arrow indicates the wind direction and the blue nodes represent the places where the resources are placed.

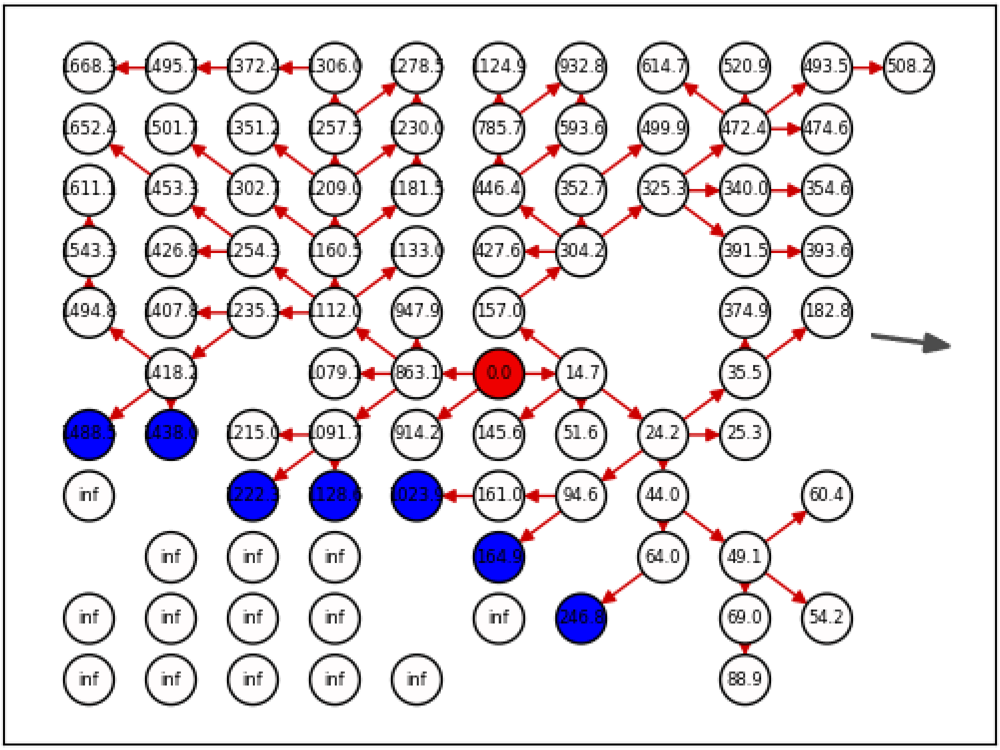

Figure 41 - Solution 3 applied to the fourth Scenario with a number of burned nodes of 70. The red one is the ignition node, the red edges represent the fire paths from node to node, the numbers on the nodes are the fire arrival times in minutes, the black arrow indicates the wind direction and the blue nodes represent the places where the resources are placed.

Results:

Best Solution: [5, 15, 24, 35, 36, 47, 57, 66, 80, 81]

Average Number of Burned Nodes = $Z_n\ (x^*)$: 66.19

D = 80.97

PI = 63.46

Validation:

VSS = D - $Z_n\ (x^*)$ = 14.78

EVPI = $Z_n\ (x^*)$ - PI = 2.73

runtime: 28 min

In this test was obtained an average number of burned nodes of 66.19, that when compared to the result of the PI, which was 63.46, shows how close the result attained was to the perfect solution when considering each scenario individually (EVPI = 2.23), meanwhile the solution D (obtained for the “average” scenario exhibits a significantly poorer outcome of 80.97 (VSS = 14.78) in comparison to the aforementioned results. The model's runtime of 28 minutes demonstrates its practical usability in real-life situations.

Figure 42 represents the acquired solution for test 1.

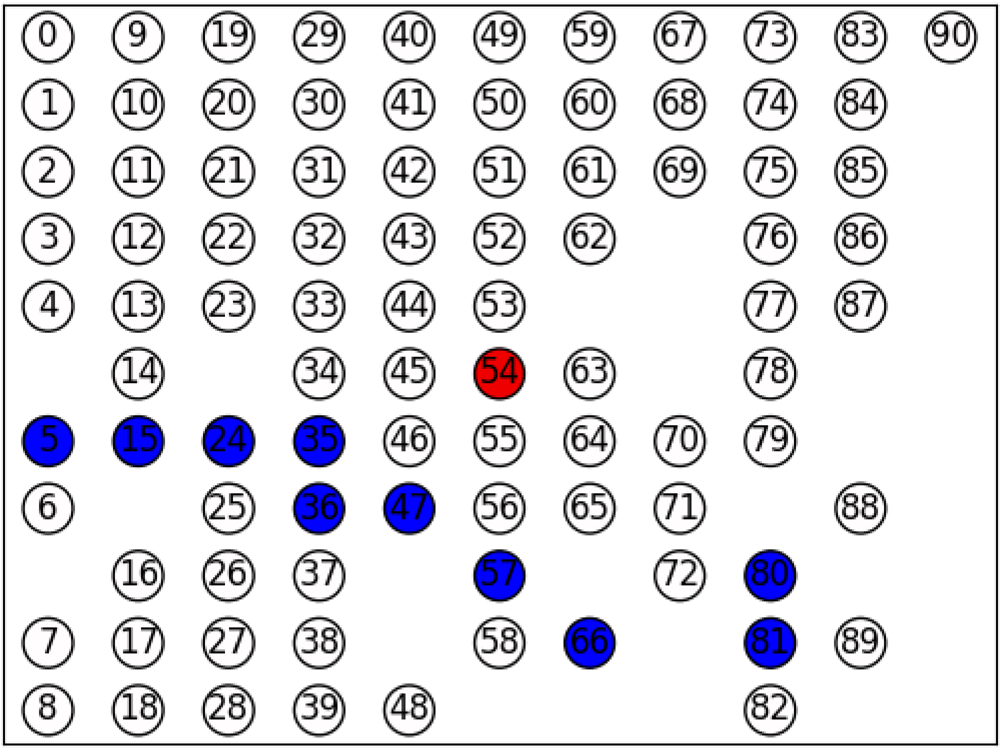

Figure 42 - Solution for Test 1 (ignition point at the center of the 11x11 grid) representing the region of Baião. Ignition node is marked in red, blue nodes are the best solution and represent the locations where the resources are placed.

The solution represented above for most of the scenarios will lead to a good result, as depicted bellow on Figure 43. However, the presented model does not guarantee always a good result, as is shown in Figure 44.

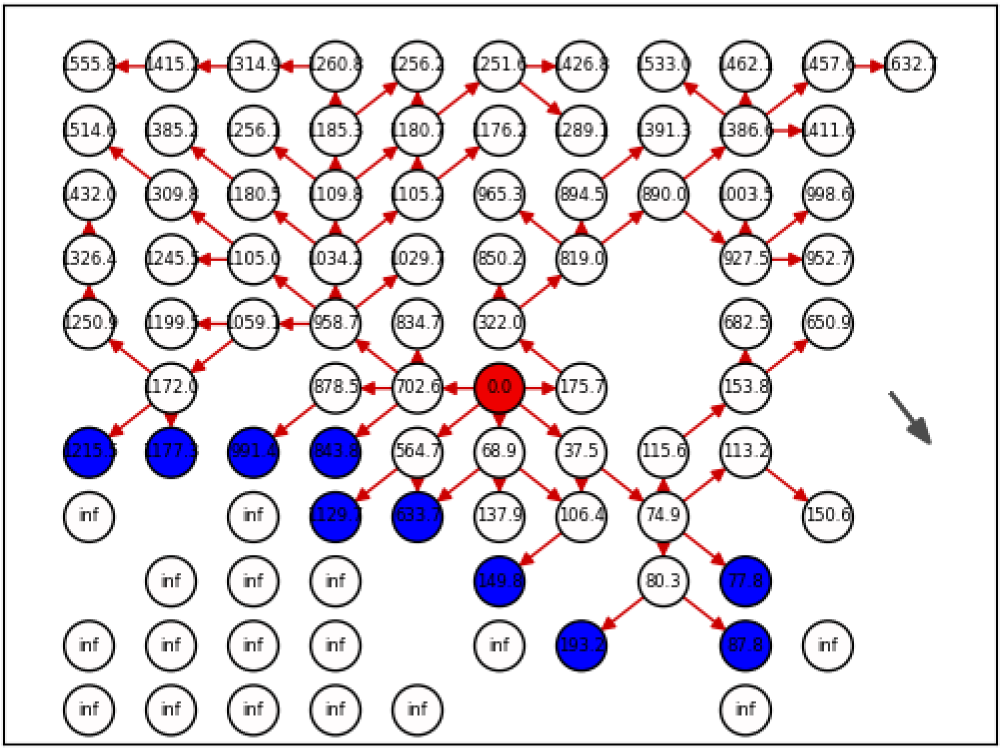

Figure 43 – Example of a Scenario where the solution obtained presented a good result for test 1. The Placement of the resources and fire paths in a specific Scenario are shown. The numbers on the nodes are the fire arrival times in minutes, the red one is the ignition node, the red edges represent the fire paths from node to node, the black arrow indicates the wind direction and the blue nodes represent the places where the resources are placed in order to minimize the burned area.

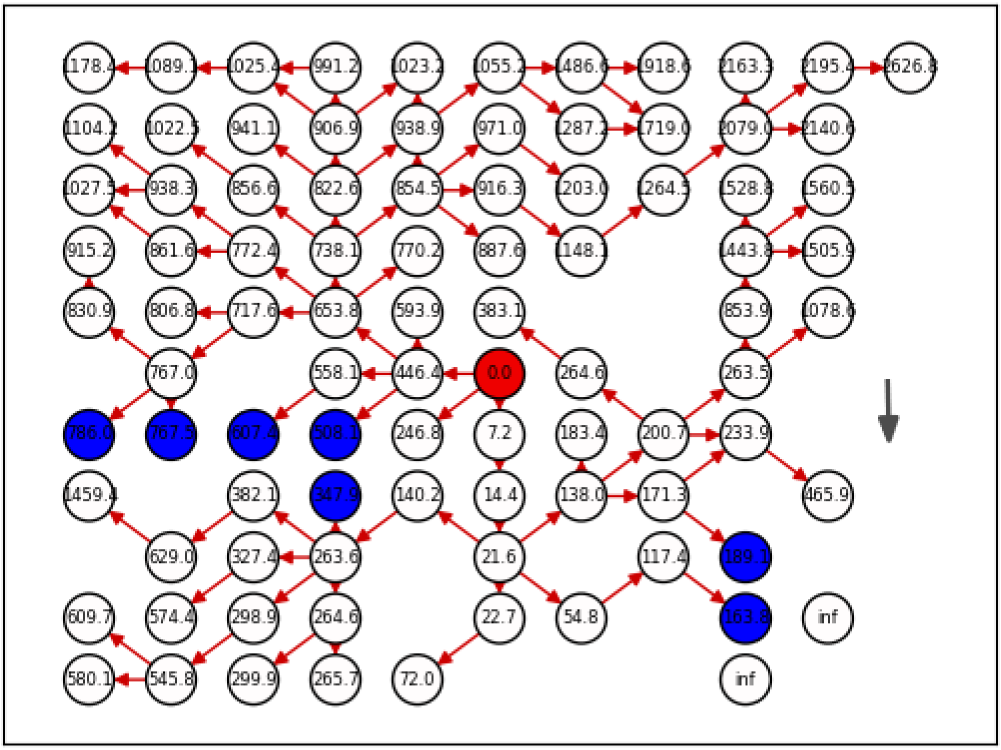

Figure 44 – Example of a Scenario where the solution obtained presented a bad result for test 1. The Placement of the resources and fire paths in a specific Scenario are shown. The numbers on the nodes are the fire arrival times in minutes, the red one is the ignition node, the red edges represent the fire paths from node to node, the black arrow indicates the wind direction and the blue nodes represent the places where the resources are placed in order to minimize the burned area.

7 Conclusions

7.1 CONCLUSION

The SAA model was tested several times and presented a good performance under a reasonable amount of time, and by visualizing the graphs it was evident that the wind and slope factors wielded paramount influence in the fire paths, as expected by the Rothermel formula. For future work more factors

Forest fires pose a significant global challenge, causing widespread destruction and taking human lives. The exacerbation of this problem by the effects of global warming underscores the urgent need to mitigate the severe repercussions of such phenomena. In light of this, this project was develop to assess the possibility of using optimization models in supporting forest fires fighting.

This dissertation had the purpose to help to reduce the impact forest fire, using models that deal with uncertainty that is naturally connected with this problem. The model was applied to networks with different resolutions representing the region of Baião.

Using Rothermel's fire behaviour model it is possible to estimate the fire travel times between adjacent nodes and with mixed integer programming it is possible to optimal position resources for fixed conditions. We extended the MIP model to account for uncertainty in the wind direction.

Considering different possible scenarios where the wind direction is different based on a probability distribution, the SAA model was applied in order to minimize the expected number of burned nodes.

Computational tests showed the feasibility of this approach for positioning resources in fire suppression under wind uncertainty.

7.2 FUTURE PERSPECTIVES

This work exhibited good results by the usage of the SAA model as already explained. However, it in the future, in the first stage of the model, more combination of the number of sets of Scenarios and the number of Scenarios within them can be tested to compare the scalability of the approach. And instead of evaluating the state of the fire only at the end, the evaluation and decision making of resource allocation can be performed at certain time windows.

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APPENDIX

APPENDIX I - RESULTS OF TEST 2

Ignition: Center Left of the Grid

Grid Size: 11x11

Number of Resources: 10

Results:

Best Solution: [49, 50, 51, 52, 62, 70, 71, 72, 80, 81]

Average Number of Burned Nodes = $Z_n\ (x^*)$: 75.73

D = 79.32

PI = 65.46

Validation:

VSS = D - $Z_n\ (x^*)$ = 3.59

EVPI = $Z_n\ (x^*)$ - PI = 10.27

runtime: 25 min

For this test the average number of burned nodes obtained was 75.73. The outcome for the PI was 65.46 and the result for the “average” scenario was 79.32. If we compare the value of VSS (3.59) with the value of EVPI (10.27), we can conclude that the performance of the SAA on this test was worst than the test 1, due to being closer to be result obtained from D, which does not mean it was a bad result, it means it could have been better. The fact that the value of VSS is positive already shows some value of the application of the model and in a 25 minute runtime is reasonable. One possible explanation for this could be that in the test 1 the fact that the ignition node was on the center of the grid, and therefore in closer proximity to the base of resources, created a smaller range of options on where to allocate the recourses, and thus less nodes possible for placement, resulting in similar solutions that will have similar outcomes. In this case the ignition is way far from the base.

Figure A1 represents the acquired solution for Test 2, while Figure A2 and Figure A3 depict examples of good and bad solutions, respectively.

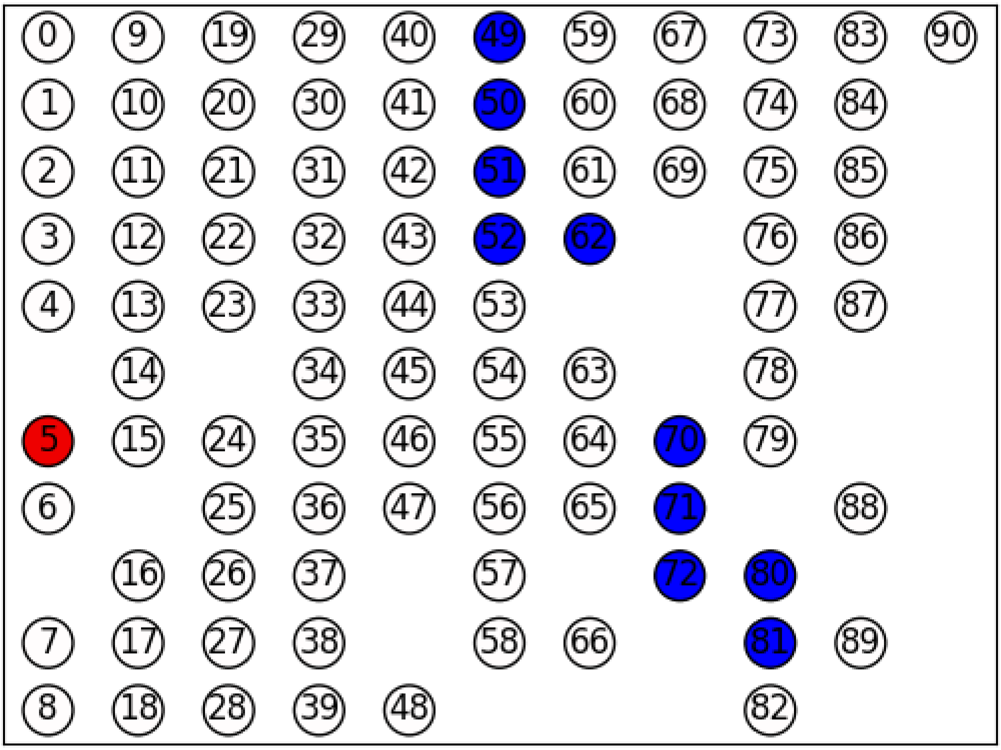

Figure A1 - Solution for Test 2 (ignition point at the left center of the 11x11 grid) representing the region of Baião. Ignition node is marked in red, blue nodes are the best solution and represent the locations where the resources are placed.

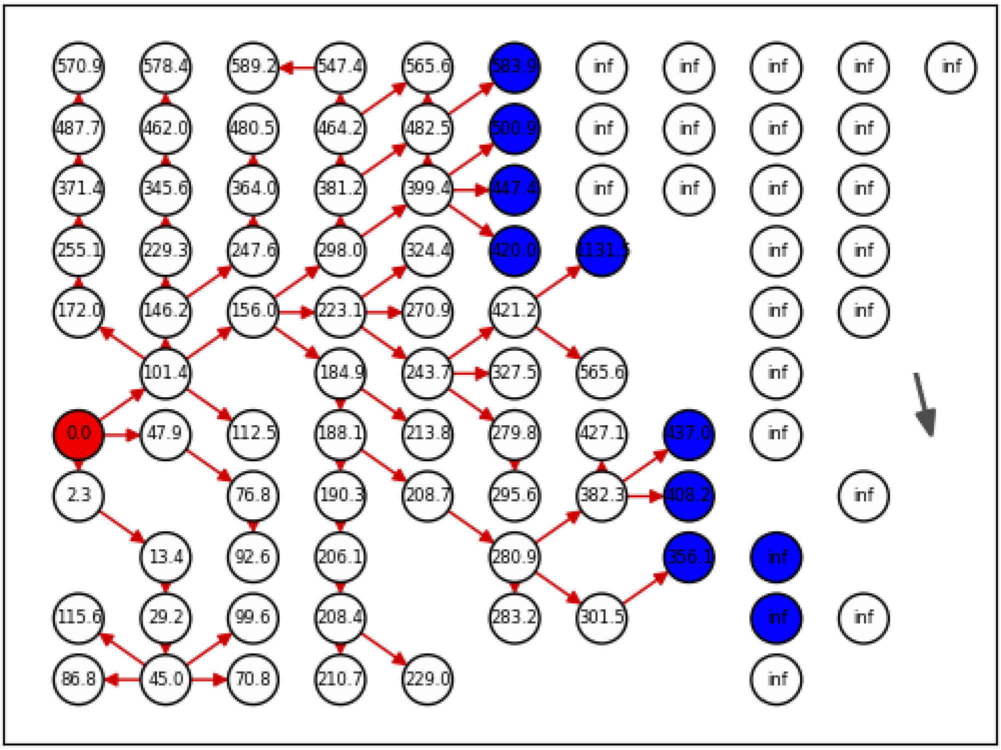

Figure A2 – Example of a Scenario where the solution obtained presented a good result for test 2. The Placement of the resources and fire paths in a specific Scenario are shown. The numbers on the nodes are the fire arrival times in minutes, the red one is the ignition node, the red edges represent the fire paths from node to node, the black arrow indicates the wind direction and the blue nodes represent the places where the resources are placed in order to minimize the burned area.

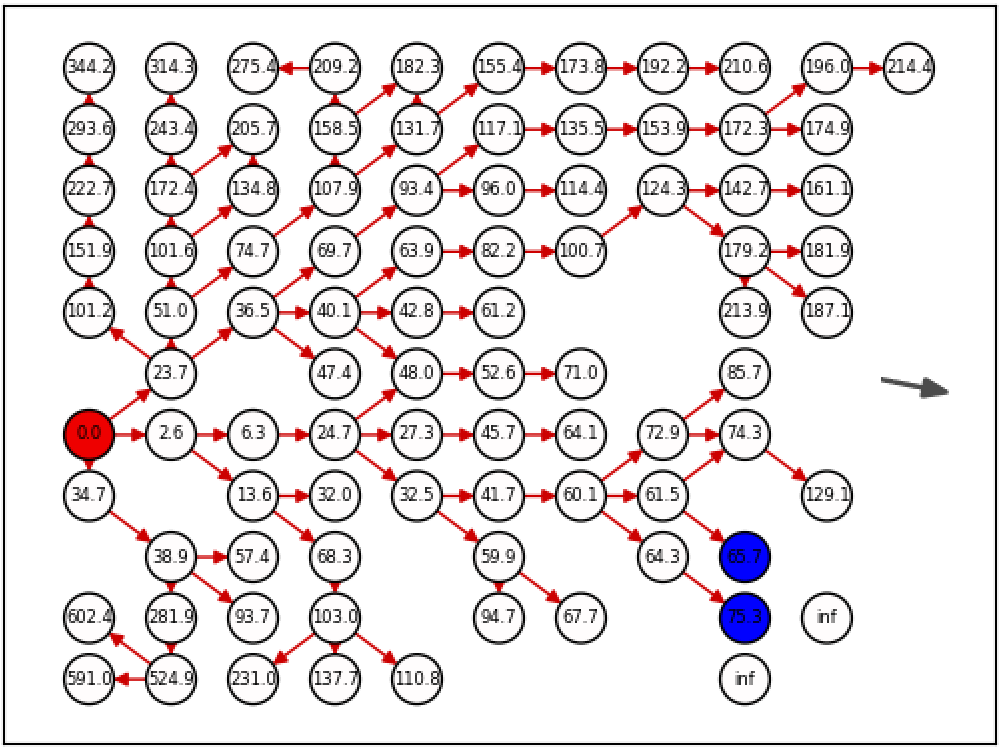

Figure A3 – Example of a Scenario where the solution obtained presented a bad result for test 2. The Placement of the resources and fire paths in a specific Scenario are shown. The numbers on the nodes are the fire arrival times in minutes, the red one is the ignition node, the red edges represent the fire paths from node to node, the black arrow indicates the wind direction and the blue nodes represent the places where the resources are placed in order to minimize the burned area.

APPENDIX II - RESULTS OF TEST 3

Ignition: Center of the Grid

Grid Size: 21x21

Number of Resources: 40

Results:

Best Solution: [25, 28, 44, 62, 66, 79, 97, 101, 117, 121, 133, 134, 151, 167, 168, 186, 188, 190, 220, 234, 237, 245, 249, 250, 264, 269, 270, 271, 272, 285, 289, 290, 304, 305, 306, 307, 308, 321, 322, 334]

Average Number of Burned Nodes = $Z_n\ (x^*)$: 232.33

D = 275.97

PI = 226.30

Validation:

VSS = D - Z_n (x^*) = 43.64

EVPI = Z_n (x^*) - PI = 6.03

runtime: 105 min

Similar to test 1 the solution obtained from the SAA model has an average number of burned nodes that closely approaches the result obtained using perfect solution for each individual scenario while being way better when compared to the outcome of the "average" scenario .It is also valid for real-life application with a reasonable 105 runtime. The higher runtime is explained by the bigger size grid, providing a higher resolution of the topography of the region.

Once again, Figure A4 represents the acquired solution for Test 3, while Figure A5 and Figure A6 depict examples of good and bad solutions, respectively.

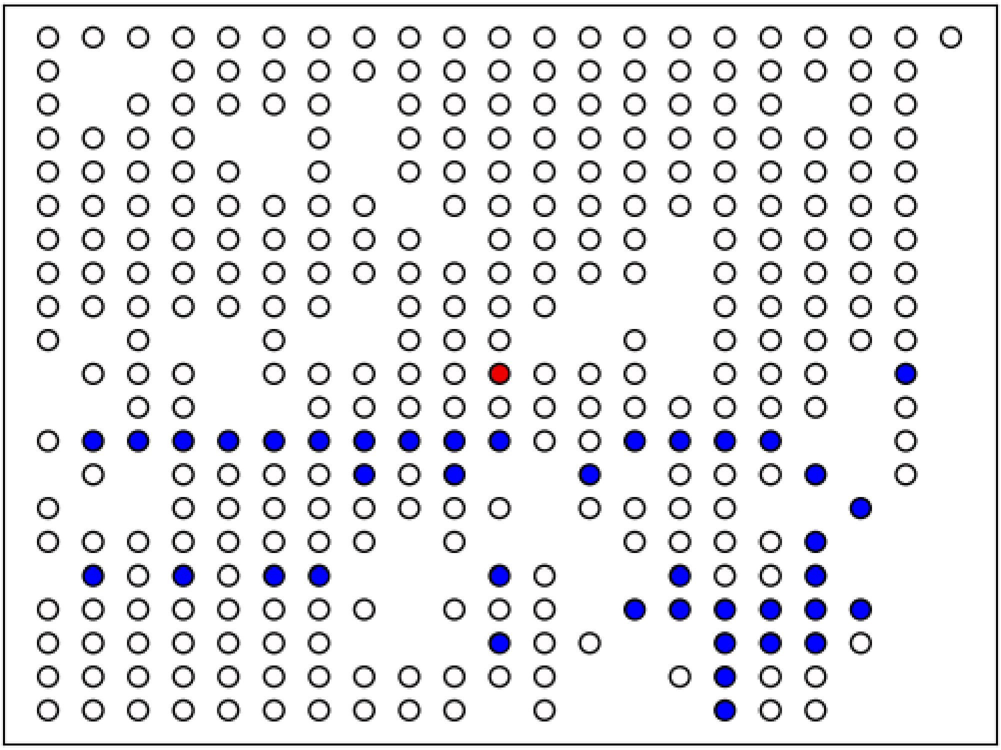

Figure A4 - Solution for Test 3 (ignition point at the center of the 21x21 grid) representing the region of Baião. Ignition node is marked in red, blue nodes are the best solution and represent the locations where the resources are placed.

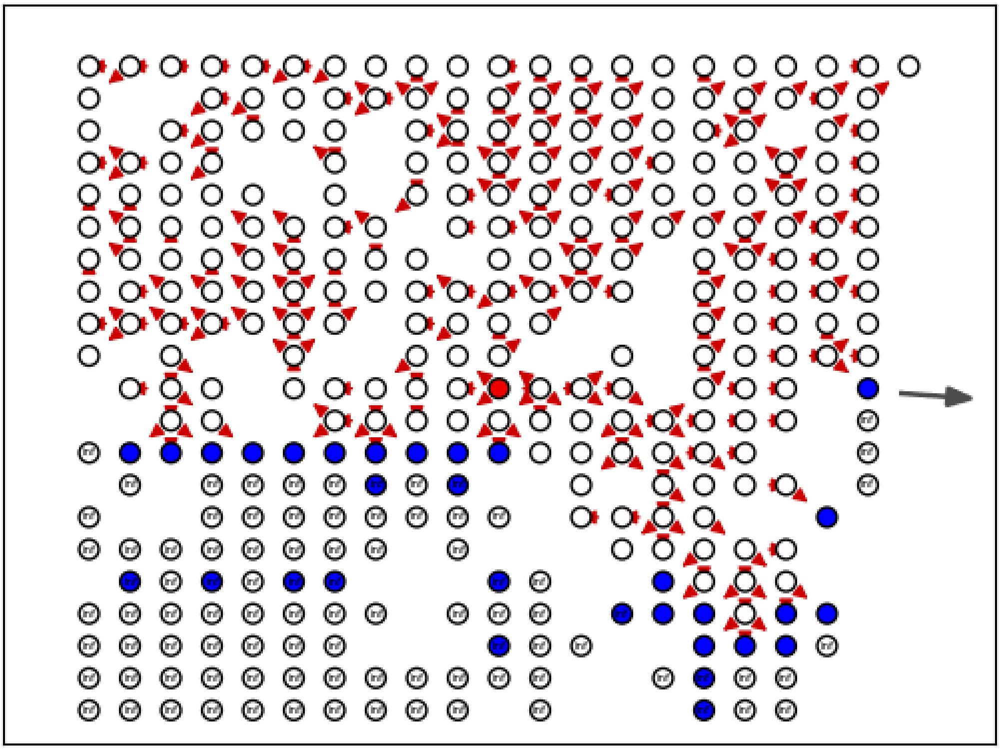

Figure A5 – Example of a Scenario where the solution obtained presented a good result for test 3. The Placement of the resources and fire paths in a specific Scenario are shown. The red node is the ignition point, the red edges represent the fire paths from node to node, the black arrow indicates the wind direction and the blue nodes represent the places where the resources are placed in order to minimize the burned area.

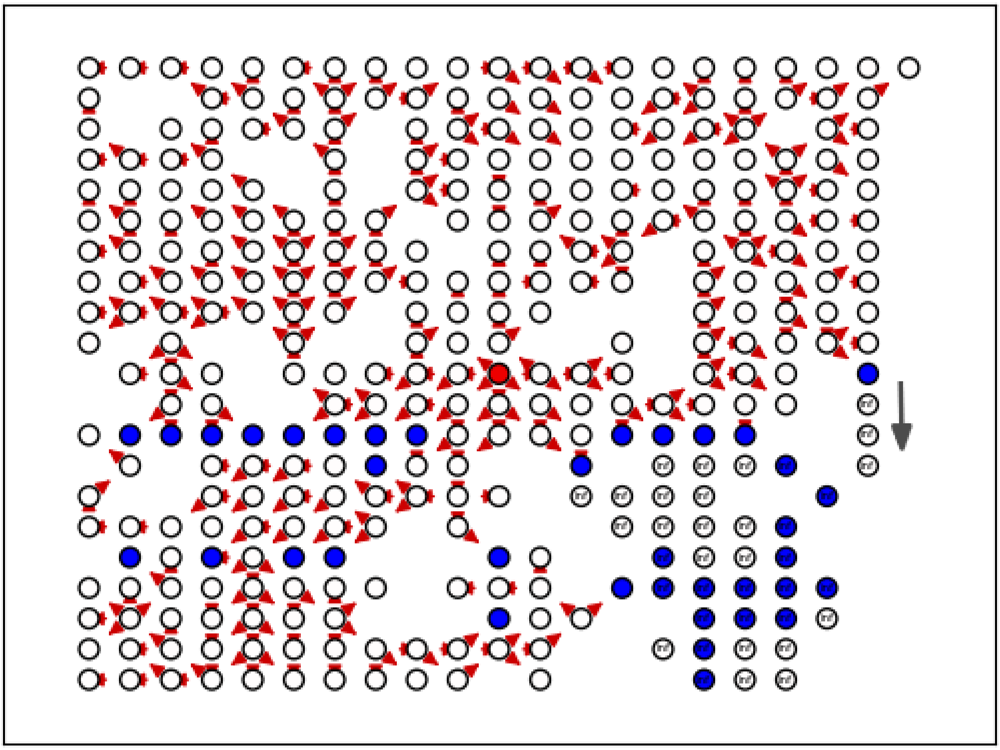

Figure A6 – Example of a Scenario where the solution obtained presented a bad result for test 3. The Placement of the resources and fire paths in a specific Scenario are shown. The red one is the ignition node, the red edges represent the fire paths from node to node, the black arrow indicates the wind direction and the blue nodes represent the places where the resources are placed in order to minimize the burned area.

APPENDIX III - RESULTS OF TEST 4

Ignition: Center Left of the Grid

Grid Size: 21x21

Number of Resources: 40

Results:

Best Solution: [2, 18, 33, 35, 54, 109, 144, 161, 162, 163, 164, 183, 184, 185, 186, 220, 232, 233, 234, 238, 239, 240, 241, 242, 243, 248, 249, 250, 256, 258, 261, 267, 277, 278, 279, 282, 283, 287, 302, 321]

Average Number of Burned Nodes = $Z_n(x^*)$: 220.17

Validation:

D = 273.92

PI = 177.24

VSS = D - $Z_n\,(x^*)$ = 53.75

EVPI = $Z_n\,(x^*)$ - PI = 42.93

runtime: 111 min

In this case the average number of burned nodes obtained when using the SAA model solution was 220.17 the PI result was 177.24 and the "average" scenario provided a solution with result 273.92.

The fact that the VSS value was higher than the previous case indicates us a better performance of the model. When also looking at the runtime of 111 minutes, it can be considerable to be applied in real-life situations.

Once again, Figure A7 represents the acquired solution for Test 4, while Figure A8 and Figure A9 depict examples of good and bad solutions, respectively.

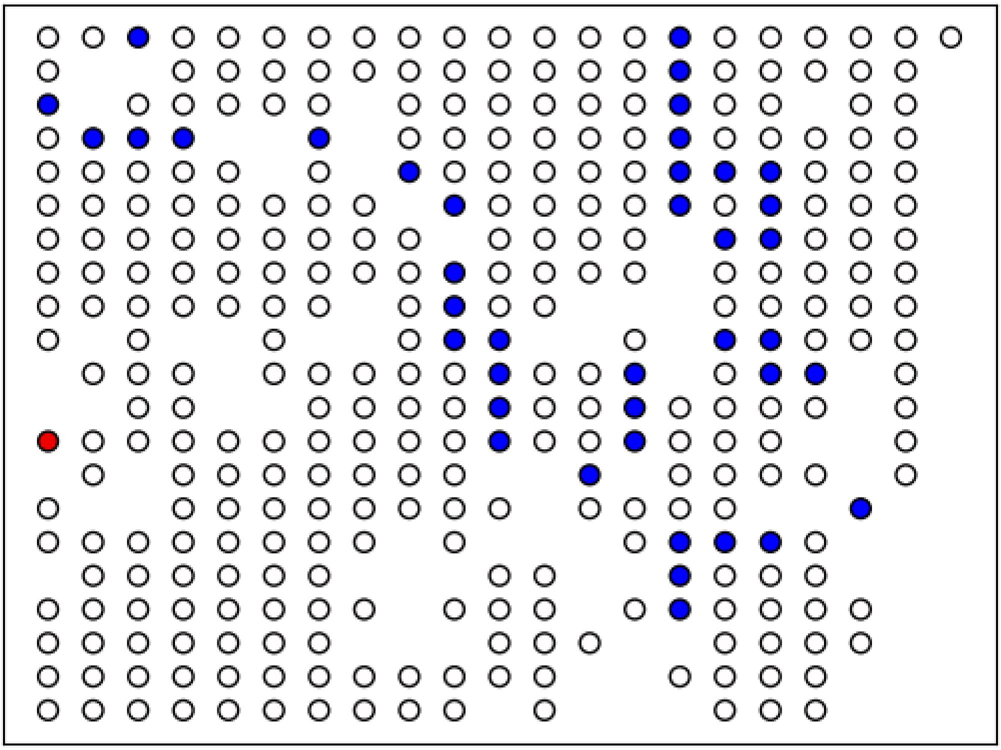

Figure A7 - Solution for Test 4 (ignition point at the left center of the 21x21 grid) representing the region of Baião. Ignition node is marked in red, blue nodes are the best solution and represent the locations where the resources are placed.

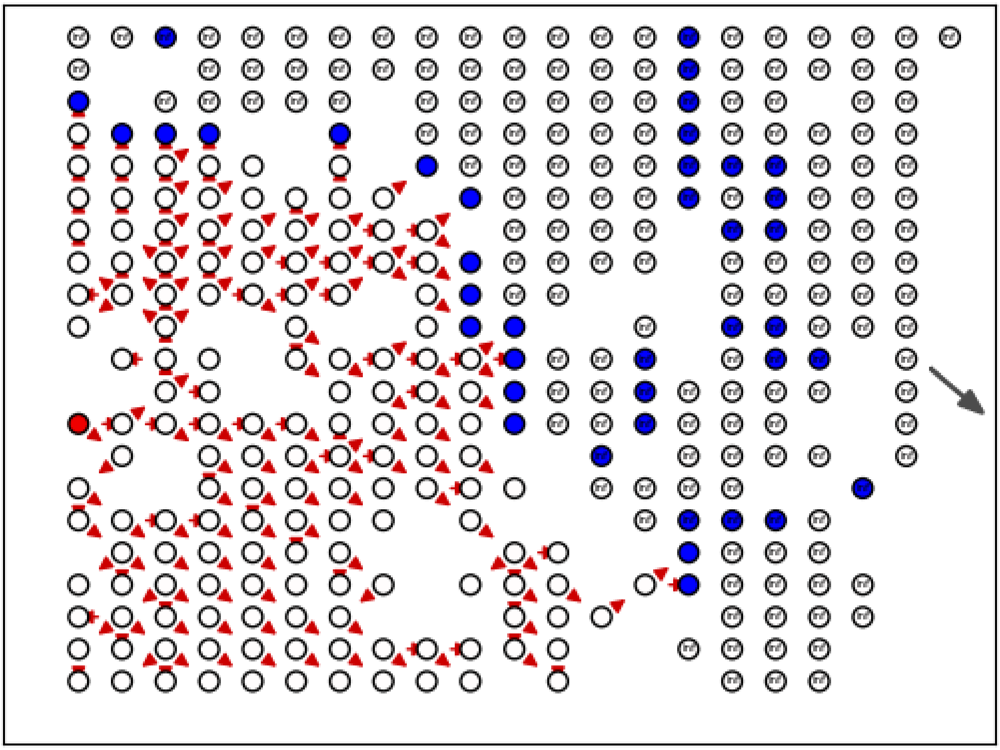

Figure A8 – Example of a Scenario where the solution obtained presented a good result for test 4. The Placement of the resources and fire paths in a specific Scenario are shown. The red node is the ignition point, the red edges represent the fire paths from node to node, the black arrow indicates the wind direction and the blue nodes represent the places where the resources are placed in order to minimize the burned area.

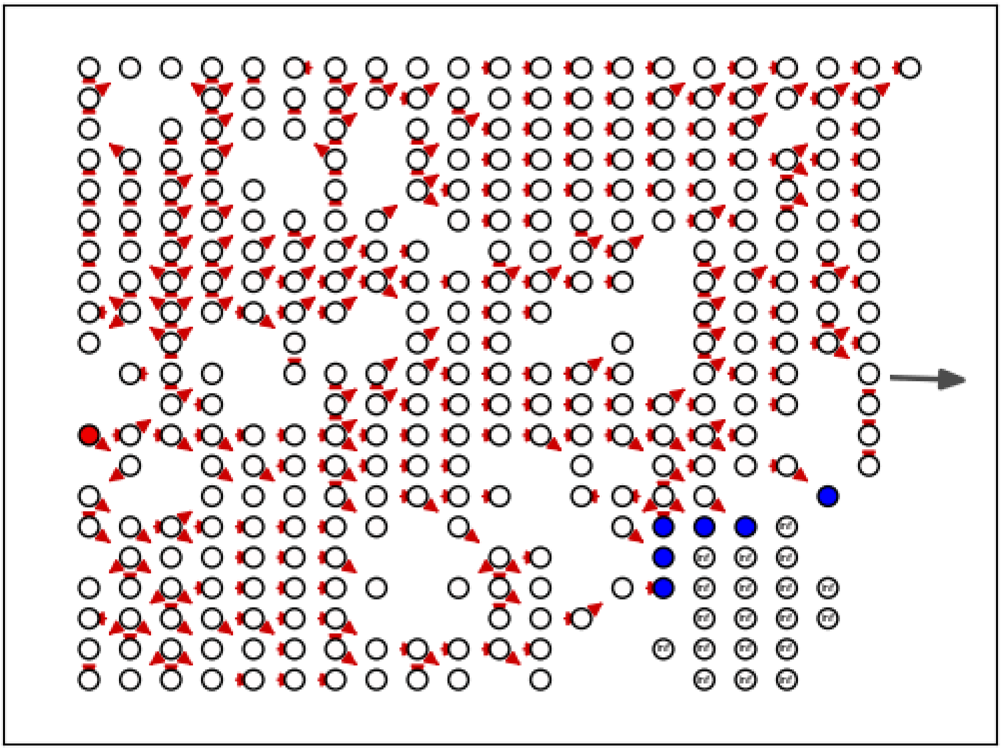

Figure A9 – Example of a Scenario where the solution obtained presented a bad result for test 4. The Placement of the resources and fire paths in a specific Scenario are shown. The numbers on the nodes are the fire arrival times in minutes, the red one is the ignition node, the red edges represent the fire paths from node to node, the black arrow indicates the wind direction and the blue nodes represent the places where the resources are placed in order to minimize the burned area.