MULTI-OBJECTIVE OPTIMIZATION OF UNBIASED MODEL FOR EXTERNALLY BONDED CFRP SYSTEM CONTRIBUTION TO SHEAR RESISTANCE IN RC BEAMS

Amirhossein Mohammadi[1*], Joaquim A.O. Barros[2], José Sena-Cruz[3]

ABSTRACT:

Predicting the shear resistance of reinforced concrete (RC) beams strengthened with externally bonded reinforcement (EBR) using carbon fiber-reinforced polymer (CFRP) materials poses a significant challenge due to the complex nonlinear relationship with beam features and the inherent randomness in shear failure events, driven by their brittle nature. This paper proposes a model employing a multi-objective optimization technique, enhancing both the accuracy, and ensuring unbiased predictions. To highlight the achieved improvement, a comparison is made between the performance of the proposed model and a series of well-established models in the literature. The proposed model presents superior predictive performance and a smaller coefficient of variation in the model error distribution than of those models. In addition, it exhibits unbiased predictions, highlighting the advantages of incorporating multi-objective optimization. Furthermore, a modified demerit point classification is introduced and applied to all models, highlighting its superiority in producing more reliable and cost-effective predictions.

AUTHOR KEYWORDS: Shear strengthening, CFRP reinforcement, Multi-objective optimization, Model error, Guideline models.

[1] Ph.D. Candidate, ISISE, Dept. of Civil Engineering, Univ. of Minho, Azurém 4800-058, Guimarães, Portugal (corresponding author), *amir.xdbx@gmail.com

[2] Full Professor, ISISE, IBS, Dept. of Civil Engineering, Univ. of Minho, Azurém 4800-058, Guimarães, Portugal, barros@civil.uminho.pt

[3] Associate Professor, ISISE, IBS, Dept. of Civil Engineering, Univ. of Minho, Azurém 4800-058, Guimarães, Portugal, jsena@civil.uminho.pt

INTRODUCTION:

Over the past three decades, CFRP materials have been extensively employed to enhance structural behavior, such as improving the flexural and shear resistance of RC beams (Barros et al. 2007; Abbasi et al. 2022). The goal of predicting the contribution of CFRP strengthening to shear resistance in RC beams is challenging due to the inherent complexity of shear behavior in RC beams. Additionally, different types of potential failure modes, non-uniform strain profile along the CFRP composites and significant uncertainties associated with the strengthening systems further contribute to this complexity (Chen et al. 2022; Mohammadi et al. 2023b). Design guidelines often resort to employing a conventional truss model to simplify the representation of the shear contribution from CFRP reinforcement. Unlike steel reinforcement, where the stress in the stirrups bridging the critical diagonal crack (CDC) can be assumed to reach the yield strength, CFRP reinforcements do not follow this straightforward behavior. As shown by advanced numerical simulations and complex analytical models, the strains in CFRP reinforcements at the point of shear failure in beams vary significantly among these reinforcements crossing the CDC (Chen et al. 2013; Abbasi et al. 2022). Furthermore, the maximum strain level allowed to be mobilized by the CFRP shear reinforcements should allow the mobilization of other shear resisting mechanisms, like aggregate interlock. To attend these aspects in a relatively simple approach, the concept of effective strain (ε_{fe}) for these reinforcements has been used (Barros et al. 2007; Mohammadi et al. 2024). Consequently, the contribution of CFRP to the shear resistance of the beam can be obtained by the following equation:

$$V_f = A_{fpl} h_f E_f \varepsilon_{fe} \left(\cot\theta + \cot\alpha_f\right)\sin\alpha_f \tag{1}$$

where A_{fpl} is the ratio of CFRP reinforcement cross-sectional area provided per unit length, determined as $2w_f t_f \sin\alpha_f / s_f$ for strips and $2t_f \sin\alpha_f$ for continuous CFRP sheets, being w_f (in mm), t_f (in mm) and s_f (in mm) the width, the thickness and the spacing of CFRP reinforcement. In Eq. (1), h_f (in mm), α_f, and E_f (in MPa) are the height, fiber orientation angle, and modulus of elasticity of the CFRP reinforcement. θ is the inclination angle of CDC, for which a provisional value of 45º or more accurate estimates may be used (*f*ib 2023).

In Fig. 1, the symbols of the considered geometric properties of the RC beams strengthened with EBR-CFRP systems are illustrated, as utilized in this study.

[Fig. 1 above here.]

Different prediction models exist in the literature to obtain ε_{fe}, and consequently V_f. However, several studies have demonstrated that these models often fail to deliver reliable and accurate predictions. Additionally, these models tend to be highly sensitive to variations in input features such as the properties of the CFRP material, the geometric dimensions of the beams, etc. This sensitivity results in inconsistent prediction performance, where the accuracy of the model can vary significantly depending on the specific values of the input parameters. This inconsistency highlights the models' inability to be applicable across different scenarios, making them less reliable for practical applications. (Barros et al. 2007; D’Antino et al. 2020; Abbasi et al. 2022; Mohammadi et al. 2023a, 2024). The issue arises from the fact that these models ignore physically impactful features, such as the interaction between CFRP and existing transverse steel reinforcement, resulting in inconsistent and biased predictions along these parameters. This study introduces the main influential features and discusses their impact on the predictions.

To address the limitations of the existing models in terms of their predictive performance, various design methods have been explored, including the utilization of machine learning algorithms such as XGBoost and Artificial Neural Networks, with promising results in various applications (Karimzadeh et al. 2023a; b; Mohammadi et al. 2023c). While these methods have the potential of providing more accurate predictions, they are blackbox tools, not suitable for constituting design guidelines; therefore, in this study, a new empirical model utilizing multi-objective optimization is developed, considering influential features for providing unbiased predictions regarding these features.

To develop the model and compare it with other models, a sizable dataset consisting of 323 experimentally tested RC beams shear strengthened with EBR-CFRP reinforcement was employed (Mohammadi et al. 2023b). Some statistics of the dataset are given in Fig. 2. Most beams in the dataset are U-wrapped (73.1%), including all beams with T-shaped cross-sections, while the remaining beams are fully wrapped (26.9%). Nearly half of the beams (47.7%) utilize continuous CFRP sheets, and the rest of them (52.3%) are reinforced with CFRP strips. Among the beams, 69% exhibit a T-shape cross-section, while 31% have a rectangular cross-section. In 88.2% of the beams CFRP reinforcements are aligned vertically, and in the remaining 11.8% CFRP reinforcements are applied with a certain inclination to the beam axis. Additionally, 65.9% of the beams are reinforced with steel stirrups, while the remaining beams lack this type of reinforcement.

[Fig. 2 above here.]

Fig. 3 also presents boxplots for some primary features in the dataset: The concrete compressive strength (f_{cm}), the longitudinal reinforcement ratio (ρ_l), the transverse reinforcement ratio (ρ_{sw}), the FRP reinforcement ratio (ρ_f), the height of the beam (h), E_f, and shear span to depth ratio (a/d_s). f_{cm} ranges from approximately 10 to 53 MPa, with several cases beyond this range. ρ_l shows a range mostly between 0.01 and 0.058. ρ_{sw} predominantly falls between 0 and 0.002, but cases are up to 0.0085. ρ_f ranges mostly between 0 and 0.005, with several cases reaching up to 0.025. h varies widely from about 150 mm to 600 mm, indicating a diverse set of beam sizes in the dataset. E_f ranges from approximately 70000 MPa to 400000 MPa, with a significant number of beams with E_f around 230000 MPa. a/d_s ranges from 1.0 to 5, with an approximate median of 3.0.

[Fig. 3 above here.]

The predictive performance of the proposed model, along with several models commonly used by academics and designers, are evaluated using this dataset, namely: *f*ib Bulletin 90 (*f*ib 2019), CNR-DT 200 R1 (CNR 2013), ACI PRC-440.2-23 (ACI 2023), TR 55 (TR 55 2012), and CIDAR (CIDAR 2006). The formulation of these models is resumed in Appendix A. In addition to several statistical tests and comparisons to recognize the most accurate model, a modified demerit classification is introduced and utilized to facilitate the identification of the most reliable and cost-efficient model.

DETERMINATION OF EFFECTIVE STRAIN OF CFRP SHEAR REINFORCEMENT IN EBR SHEAR-STRENGTHENED RC BEAMS

This section outlines the development of the regression-based prediction model to estimate the effective strain of CFRP shear reinforcement in EBR shear-strengthened RC beams, denoted as ε_{fe}. The experimental values of ε_{fe} can be obtained from the collected dataset. This involves calculating the experimental contribution of CFRP to shear resistance (V_f^{exp} in N) by subtracting the shear resistance of a reference beam from that of a strengthened beam, along with considering other features of the beam:

$$\varepsilon_{fe} = \frac{V_f^{exp}}{A_{fpl} h_f E_f (\cot\theta + \cot\alpha_f)\sin\alpha_f} \tag{2}$$

Assuming that FRP strengthening has a marginal effect on CDC angle, θ can be determined based on the longitudinal strain at the mid-depth of the section (ε_x) as recommended in *f*ib Model Code 2020 (*f*ib 2023):

$$\theta = 29 + 7000\varepsilon_x \geq \theta_{\min} = 20 + 4000\varepsilon_x \tag{3}$$

ε_x is defined according to the following equation:

$$\varepsilon_x = \frac{1}{2E_s A_{sl}}\left(\frac{M_E}{z_v} + V_E\right) \geq 0 \tag{4}$$

where E_s is the elastic modulus of longitudinal steel reinforcement which is taken equal to 200×10^3 MPa for all beams and A_{sl} is its cross-sectional area. M_E and V_E are the bending moment and shear forces caused by external loads. z_v is the effective shear depth of the beam which is defined as the maximum of $0.9d_s$ and $0.72h$.

To develop a regression-based model, the main step is to recognize the features that influence the target value. Data analysis could be employed to determine the important features that influence ε_{fe}. However, the literature review shows that an increase in ρ_f and E_f promotes premature debonding, which means a decrease in ε_{fe} (Khalifa et al. 1998; Aghabagloo et al. 2024). Conversely, an increase in f_{cm}, or using full wrapping configuration rather than U-wrapping configuration, retard debonding failure, which increases ε_{fe} (Triantafillou and Antonopoulos 2000; Chen and Teng 2003). It is shown that when ρ_{sw} increases, the CFRP shear reinforcement becomes less effective. This is because, at the CFRP failure, strain in the steel stirrups crossing the CDC may fall below the steel yielding strain (ε_{swy}), thereby the reinforcement potential of these stirrups is not fully exploited (Chen et al. 2017; Li et al. 2018; Mohammadi et al. 2023b, 2024). Another important feature is ρ_l, which can significantly alter the CDC angle and crack end opening width, thereby affecting ε_{fe} (Mohammadi et al. 2024). It is reported that the ε_{fe} relies on the internal arm of steel reinforcement (d_s in mm); known as size effect in the shear behavior of RC beams strengthened in shear with externally bonded FRP (Abbasi et al. 2022). As far as the authors are aware, this can be mainly attributed to the fact that at a beam with larger d_s, the maximum crack width that leads to the failure of the CFRP occurs at a smaller crack tip opening angle. This results in a less uniform stress distribution profile, consequently resulting in a relatively smaller ε_{fe}. Moreover, it is demonstrated that rounding the corners with a larger corner radius (R_c in mm) helps reducing stress concentration and local rupture in the CFRP system (Li and Leung 2016; *f*ib Bulletin No. 90. 2019). Fig. 4 presents a scatterplot that displays the relationship between ε_{fe} and the introduced features. Fitted lines along with the Pearson coefficient of correlation (r) values are displayed on the plot to assess the level of correlation between ε_{fe} and each parameter. The observed data supports the aforementioned presumed dependence of ε_{fe} from indicated features.

[Fig. 4 above here.]

Different approaches have been utilized by existing design guidelines for the estimation of ε_{fe}. This study introduces a methodology for developing a regression model. Initially, a basis model is fitted between the most influential input features and the target value. Subsequently, modification factors are implemented in the model to ensure consistent and unbiased performance across the remaining influential features. As shown in Fig. 4, a robust correlation between ρ_f and ε_{fe} is evident, establishing it as the most influential individual feature among others. However, previous studies (Triantafillou and Antonopoulos 2000; Mohammadi et al. 2023b) show that combining ρ_f, E_f, and f_{cm} forms a composite feature (i.e., $K=\rho_f E_f/f_{cm}^{2/3}$) with more robust correlation with ε_{fe}. This observation has led to the decision to develop the basis model based on this feature. Trying to fit different functions to 323 experimental data points, the best-fitting function was determined to be a power function (i.e., $\varepsilon_{fe}=aK^b$). This choice was made not only based on its accuracy but also because it ensures prediction values remain above zero for even large values of K, which is more realistic. To ensure that the fitted function can produce equally accurate predictions for future unseen data, a 10-fold cross-validation was performed. This method involves dividing the dataset into ten subsets, training the model on nine subsets, and validating it on the remaining one, with the process repeated so each subset serves as the validation set once. The results of the 10-fold cross-validation showed that the fitted parameters (a and b) exhibited low coefficient of variation (CoV) (6.64% and 3.51%, respectively), indicating consistent performance across different folds. The mean value of r was 0.66 with a CoV of 10%, demonstrating a good linear relationship between predicted and actual values. Although the R^2 values displayed more variability with a CoV of 29.7%, the mean R^2 value of 0.4 indicates a reasonable fit. Therefore, it is concluded that the regression model, fitted using the average values from the cross-validation, is reliable and is expected to perform well on new unseen data as well. Fig. 5a shows the power function with the average values of fitted parameters along with the scatterplot of experimental data points. The power function with the average values of fitted parameters is shown as follows:

$$\varepsilon_{fe} = 0.0126\left(\frac{E_f \rho_f}{f_{cm}^{2/3}}\right)^{-0.52} \tag{5}$$

Fig. 5b shows the scatterplot comparing the predictions of the base model with the experimental values, resulting in an r of 0.64 and an R^2 of 0.40 .

[Fig. 5 above here.]

As discussed, the effective strain is influenced by various features beyond K, which renders the proposed model susceptible to bias concerning these additional features, namely R_c, ρ_{sw}, ρ_l, d_s, and C_f (strengthening configuration factor being 1.0 and 0 for fully wrapped and U-wrapped beams, respectively). Furthermore, as shown in Fig. 5b, the basic model leads to biased results, attributed to its inability to capture the entire nonlinear relationship between ε_{fe} and the associated features in K (i.e., ρ_f, E_f, f_{cm}). Therefore, it appears necessary to adjust the basis model to ensure consistent results across these parameters. Fig. 6 shows the distribution of model error $\chi_\varepsilon = \varepsilon_{fe}^{\exp} / \varepsilon_{fe}^{pred}$ across different values of these features, allowing an evaluation of their impact on model performance. A line is fitted between model error and the corresponding feature, with the r value indicating the level of correlation, to facilitate the recognition of bias in model error for each feature. As evident, the predictions generated by Eq. (5) exhibit bias, with model error showing a clear correlation to the different values of these features. To address this issue, it is common practice to adjust the basis model using modification factors (Mohammadi et al. 2023b; Shayanfar et al. 2023). These modifications aim to ensure consistent predictive performance across all feature ranges. Nevertheless, modifying the model for individual features one by one in a sequential manner may reintroduce significant bias within the features for which the model was initially modified. Therefore, this study presents a novel multi-objective optimization method aiming to maximize the overall predictive performance of the final model while simultaneously minimizing introduced bias across all the input features.

[Fig. 6 above here.]

BIAS HANDLING WITH MULTI-OBJECTIVE OPTIMIZATION

A predefined parametric model is introduced for predicting ε_{fe}, which includes Eq. (5) as the basis and a modification factor κ_m:

$$\varepsilon_{fe} = \kappa_m \left(0.0126 \left(\frac{E_f \rho_f}{f_{cm}^{2/3}} \right)^{-0.52} \right) \tag{6}$$

The global model modification factor (κ_m) calculated as the product of a series of linear equations:

$$\kappa_m = \prod_{j=1}^{p} \left(a_j x_j + b_j \right); \text{ where } \mathbf{x} = \left[d_s, C_f, \rho_l, \rho_f, f_{cm}, \rho_{sw}, R_c, E_f \right] \text{and } \; p = 8 \tag{7}$$

where $\mathbf{x}$ is the vector of selected features, p is the number of selected features (in this case p=8). The parameter vectors are defined as $\mathbf{a}$=[a_1,…,a_j,…,a_p] and b_j=[b_1,…,b_j,…,b_p], where a_j and b_j are the parameters associated with each feature x_j, and j is the index of selected features.

Optimization is used to obtain the unknown parameters of the previous equation. To enhance the optimization process and capture multiple facets of the problem, a multi-objective function (Ω) is used in this study:

$$\Omega = \frac{\sum_{i=1}^{N}\sqrt{\left(y_i-\gamma_i\right)^2}}{N} + \sum_{j=1}^{p}\left|\rho_{x_j,X}\right| + \left|\frac{\sum_{i=1}^{N} y_i/\gamma_i}{N} - 1\right| \tag{8}$$

where $\mathbf{y}$ and $\boldsymbol{\gamma}$ are the vector of known and predicted values of ε_{fe}, respectively, i is the index of beam sample in the dataset, and N is the number of samples. Traditionally, optimization is focused solely on minimizing the sum of squared errors between predicted and known values (first term of Eq.(8)). While this approach promises the highest accuracy in predictions, it may overlook the bias in model errors regarding the features influencing the variable in analysis (in the present case, the ε_{fe}). A second objective is introduced to address this limitation by minimizing the correlation coefficients between the model error and the considered features (second term of Eq. (8)). Finally, to ensure that the model error has a distribution with a mean value of unity, a third objective is adopted (third term of Eq. (8)). By combining these three objectives, the optimization algorithm seeks to balance achieving unbiased predictions and improved predictive accuracy while assuring that the mean model error is equal to 1.0.

A genetic algorithm developed in Python was employed to optimize the parameters of the model. A maximum number of iterations equal to 100, a population size of 100, a mutation probability of 0.1, an elitism ratio of 0.01, a crossover probability of 0.5, and a parent's portion of 0.3 was used. These parameters were determined based on experimentation and empirical knowledge to ensure effective optimization performance, resulting in the parameters shown in Table 1.

[Table 1 below here.]

After deriving the parameter vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ through optimization analysis, Eq. (7) can be transformed to:

$$\kappa_m = \prod_{j=1}^{p}\left(\frac{a_j x_j + b_j}{b_j}\right)\prod_{j=1}^{p} b_j = B\prod_{j=1}^{p}\left(a_j^{'} x_j + 1\right); \text{ where } B = \prod_{j=1}^{p} b_j \tag{9}$$

Therefore, using Eq. (9) and the value of B, Eq. (6) can be given as:

$$\varepsilon_{fe} = 0.016\prod_{j=1}^{p}\left(a_j^{'} x_j + 1\right)\left(\frac{E_f \rho_f}{f_{cm}^{2/3}}\right)^{-0.52} \text{ where } \mathbf{x}=\left[d_s, C_f, \rho_{sl}, \rho_f, f_{cm}, \rho_{sw}, R_c, E_f\right] \tag{10}$$

where $a'_j = a_j/b_j$. The sign of a'_j carries critical information. The positive sign is attributed to the inherent tendency of the basis model to underestimate the effective strain ($\varepsilon_{fe}^{pred} < \varepsilon_{fe}^{\exp}$) with an increase in the corresponding feature. Consequently, the modification process entails multiplying the value obtained from the basis model by a factor greater than 1.0, which increases with the corresponding feature. Conversely, the negative sign indicates that the model tends to overestimate the effective strain ($\varepsilon_{fe}^{pred} > \varepsilon_{fe}^{\exp}$) as the corresponding feature increases. In such cases, a modification factor less than 1.0 which decreases with the increase of the corresponding feature, is utilized.

Fig. 7 shows the scatterplot of model error obtained from Eq. (10) across different values of adopted features of the strengthened beams. The result demonstrates minimal bias concerning all the features, as evidenced by the *r* value between each feature and the model error. Fig. 8 shows the scatterplot of model error against *K* and predicted against the actual value of ε_{fe} before and after taking the modification factor into account (Eq. (5) and (10)). As can be seen, the modification factor has decreased the *CoV* in the model from 0.57 to 0.48. Additionally, Eq. (10) provided consistent predictive performance along the *K*. The modified model is demonstrated to yield an improved coefficient of determination (R^2) and *r*, with values of 0.57 and 0.76, respectively, compared to 0.40 and 0.64 in the basis model.

[Fig. 7 above here.]

[Fig. 8 above here.]

COMPARISON OF THE PREDICTIVE PERFORMANCE OF PROPOSED AND EXISTING MODELS

To showcase the predictive performance of the proposed model, a comparison is made with the one provided by the following existing models: *f*ib Bulletin 90, CNR-DT 200 R1, ACI PRC-440.2-23, TR 55, and CIDAR. The performance of the proposed model and considered existing models is evaluated by comparing their predictions in terms of the average CFRP shear contribution ($V_{R,f}^{\text{model}}$) to the experimental values ($V_f^{\exp}$). A summary of these models' formulation for obtaining $V_{R,f}^{\text{model}}$ is given in Appendix A. It must be noted that average values of material properties are used, and partial factors are set equal to 1.0. Throughout this study, the inclination of the shear crack is obtained from Eq. (3). Where not provided, R_c of 20 mm is assumed. In Fig. 9, a series of scatterplots demonstrates the predictive performance of these models. It is evident that the dispersion of predictions by the proposed model is comparatively lower than by other models.

[Fig. 9 above here.]

A set of performance metrics is employed to quantify the predictive performance of each model, including r, R^2, root mean squared error (*RMSE*), and mean absolute percentage error (*MAPE*), as defined in Eq. (11) to (14), respectively. Each metric provides distinctive insights into various aspects of model performance, collectively contributing to a comprehensive assessment.

$$r=\sum_{i=1}^{n}\left[\left(\boldsymbol{\gamma}_i-\bar{\gamma}\right)\left(\mathbf{y}_i-\bar{y}\right)\Big/\sqrt{\sum_{i=1}^{n}\left(\boldsymbol{\gamma}_i-\bar{\gamma}\right)^2\sum_{i=1}^{n}\left(\mathbf{y}_i-\bar{y}\right)^2}\right] \tag{11}$$

$$R^2=1-\left(\sum_{i=1}^{n}\left(\mathbf{y}_i-\boldsymbol{\gamma}_i\right)^2\Big/\sum_{i=1}^{n}\left(\mathbf{y}_i-\bar{y}\right)^2\right) \tag{12}$$

$$RMSE=\sqrt{\sum_{i=1}^{n}\left(\mathbf{y}_i-\boldsymbol{\gamma}_i\right)^2\Big/n} \tag{13}$$

$$MAPE=\frac{100\%}{n}\sum_{i=1}^{n}\left(\left|\left(\mathbf{y}_i-\boldsymbol{\gamma}_i\right)/\mathbf{y}_i\right|\right) \tag{14}$$

In these equations, $\mathbf{y}_i$ is the known value for the i^{th} record, $\bar{y}$ is the mean of known values, $\boldsymbol{\gamma}_i$ is the predicted value for the i^{th} record, and $\bar{\gamma}$ is the mean of predicted values; n is the total number of records. Fig. 10 shows that the proposed model has better predictive performance than of the other models, exhibiting the highest values of r and R^2 along with the lowest *RMSE* and *MAPE* (perfect predictions correspond to r=R=1, *RMSE*=*MAPE*=0). For detailed metric values of all models, refer to Table 2.

[Fig. 10 above here.]

[Table 2 below here.]

Another crucial aspect of model evaluation involves determining whether the models make consistent predictions along beam features. To assess the degree of bias in each model, the model error is computed as the ratio of the experimental value to the average CFRP shear contribution, i.e. $\chi_V=V_f^{\exp}\big/V_{R,f}^{\text{model}}$. Before assessing the consistency in the models' performance, the general distribution of model errors should be examined. The model with the best predictive performance has normal model error distribution, with mean, median, minimum, and maximum the closest to the unitary value and the minimum standard deviation (σ) and CoV. According to Fig. 11 and Table 3 the proposed model has better predictive performance, having a mean, median, minimum and maximum of 1.0, 0.92, 0.15, 2.67, respectively, with a σ and CoV of 0.48.

[Fig. 11 above here.]

[Table 3 below here.]

As demonstrated in the preceding section (Fig. 6), neglecting the impact of beam features may result into inconsistent predictive performance regarding these features. Similar to the analysis performed for effective strain, it is beneficial to evaluate whether the proposed model and the considered existing models are consistent along critical beam features on V_f prediction, namely ρ_{sw}, ρ_f, ρ_l and d_s. Fig. 12 shows the scatterplots of χ_V against ρ_l and d_s, while Fig. 13 presents the scatterplots of χ_V against ρ_{sw} and ρ_f for considered existing models and the proposed model. Each plot shows the relationship between the errors and different parameters on the x-axis for different models. For clarity, a regression line is fitted to the data in each plot. The assessment of bias in models involves examining whether there is a significant correlation between input features and χ_V. For this reason, a Pearson correlation test is conducted in which the null hypothesis is that no correlation between input feature and χ_V (H_0: $r = 0$) exists and the alternative hypothesis (H_1: $r \neq 0$) is that the model error is dependent to the input feature. The p-values are computed and displayed, using a significance level of 0.05, helping to decide whether to reject or accept the null hypothesis. If the p-value is less than 0.05, then the null hypothesis is rejected, indicating bias in predictions. On the other hand, if the p-value is greater than or equal to 0.05, then the null hypothesis is accepted, indicating that there is no significant bias in the predictions of the model for that specific feature. The figures and the reported p-values for each model show that the proposed model produces no bias in the predictions corresponding to all the features. All considered existing models are deficient in making consistent predictions regarding ρ_{sw} and ρ_f. CNR-DT 200 R1 and CIDAR also exhibited a significant bias regarding ρ_l and d_s.

[Fig. 12 above here.]

[Fig. 13 above here.]

MODIFIED DEMERIT POINTS CLASSIFICATION

The demerit points classification analysis was performed to provide an additional fair and comprehensive evaluation of the model's predictive accuracy from a closer perspective of a designer. The traditional demerit points classification is typically applied to model errors derived from design values rather than average values (Mohammadi et al. 2024). Given that the design values for the proposed model have not yet been determined due to the necessity for a thorough reliability analysis, a modified classification approach is introduced in this study. The modification ensures that the analysis remains valid and meaningful even in the absence of finalized design values. To ensure fairness in comparing model errors from average values, it is crucial that all distributions have a mean of 1.0. This is due to the fact that in reliability analysis, the bias in model error is taken into account, so normalizing the distributions by their means ensures an unbiased comparison. Thus, each distribution is divided by its mean to achieve this normalization. Similar to the traditional demerit points classification, the modified demerit points classification categorizes predictions for beam specimens into distinct levels, including extremely conservative, conservative, appropriately safe, relatively safe, dangerous, or extremely dangerous. The penalty points assigned to each category reflect the varying consequences of the model errors. For instance, extremely conservative predictions (which can lead to unnecessary costs) are penalized less severely than dangerous predictions (which can lead to failures). The total demerit points for each model are computed by summing the products of the percentage of specimens with χ_V falling within each defined range, multiplied by the corresponding penalty points for that range, and then divided by 100. The resulting score falls within the range of 0 to 10. A lower total demerit points indicates that the model produces more accurate and reliable predictions, highlighting the model's superiority. This classification provides a nuanced evaluation of model performance, highlighting not only the average error but also the distribution and severity of errors. Table 4 provides the breakdown of specimens with χ_V values falling into each classification, along with the total demerit points assigned to each model. The proposed model exhibits a notably lower total demerit score. Among the considered existing models, CIDAR and *fib* Bulletin 90 record the lowest and highest total demerit points, respectively.

[Table 4 below here.]

CONCLUSIONS

In this study, a dataset comprising 323 RC beams shear-strengthened with CFRP-EBR (fully or U-wrapped) was employed for the development of a regression-based model. A multi-objective optimization method was introduced to account for the diverse effects of various beam features and to avoid producing biased predictions regarding these features. The application of multi-objective optimization proved essential for successfully resolving biases in the regression model and markedly enhancing predictive performance. To underscore the improvement achieved by the proposed model, a rigorous comparison in terms of the predictive performance of the shear strengthening contribution of CFRP-BR was conducted with well-established models in *f*ib Bulletin 90, CNR-DT 200 R1, ACI PRC-440.2-23, TR 55, and CIDAR. The proposed model has shown the best predictive performance. In fact, it exhibited a notable linear correlation between predictions and experimental measurements, as evidenced by r of 0.79. Additionally, the model achieved the most well-balanced predictions, with an R^2 of 0.62, the lowest occurrence of erratic predictions, reflected in a *MAPE* of 54.4%, and the minimum significant prediction error with an *RMSE* of 35.8 kN.

The evaluation of models was extended by examining a dimensionless parameter known as model error. Statistical metrics related to model errors revealed that the proposed model generates model errors with a mean value of 1.0 and a *CoV* of 0.48. These characteristics position the proposed model as a superior choice compared to the other considered models. Significant variability still remains, which is attributed to inherent experimental uncertainties and the simplicity of the current model. Efforts are underway to develop more advanced models, such as ANN and XGBoost, to further enhance predictive accuracy. Furthermore, The trend of model errors across various predictive features of the beam was analyzed using a Pearson correlation test, and the findings indicate that only the proposed model can generate consistent predictions.

A modified demerit point classification was introduced as a further step in the evaluation of the model's predictive performance. This classification facilitated a robust comparison of model reliability and cost-effectiveness. Notably, the proposed model presented the lowest total demerit points and the smallest percentage of predictions falling into extremely dangerous (12%) and extremely conservative (3%) categories.

In summary, this study showcases the effectiveness of the proposed multi-objective optimization method in mitigating biases and enhancing predictive accuracy. The performance of the model, as evidenced by various metrics, positions it as a better tool for predicting the contribution of CFRP-EBR to the shear strengthening of RC beams. Despite its merits, the proposed model still demonstrates a notable margin of error in predictions, prompting consideration for the application of machine learning algorithms to enhance its performance. Additionally, a calibration of partial safety factors is deemed necessary to facilitate the utilization of the proposed model in design applications.

DATA AVAILABILITY STATEMENT

Some or all data, models, or code that support the findings of this study are available from the corresponding author upon reasonable request.

ACKNOWLEDGEMENTS

This study is a part of the project "Sticker –Innovative technique for the structural strengthening based on using CFRP laminates with multifunctional attributes and applied with advanced cement adhesives", with the reference POCI-01-0247-FEDER-039755. The first author also acknowledges the support provided by FCT PhD individual fellowship with the reference of 10.54499/2020.08876.BD (https://doi.org/10.54499/2020.08876.BD). This work was partly financed by FCT / MCTES through national funds (PIDDAC) under the R&D Unit Institute for Sustainability and Innovation in Structural Engineering (ISISE), under reference UIDB / 04029/2020 (doi.org/10.54499/UIDB/04029/2020), and under the Associate Laboratory Advanced Production and Intelligent Systems ARISE under reference LA/P/0112/2020.

NOTATION

a: shear span (mm);
α_f: fiber orientation angle;
A_{fpl}: ratio of CFRP reinforcement cross-sectional area provided per unit length (mm);
A_{fw}: area of FRP for shear strengthening measured perpendicular to the direction of the fibers (mm^2);
A_{sl}: cross-sectional area of longitudinal steel reinforcement (mm^2);
$\mathbf{a}_j$: regression coefficient;
$\mathbf{a}'_j$: regression coefficient;
B: regression coefficient;
$\mathbf{b}_j$: regression coefficient;
b_w: breadth of beam web (mm);
β_L: coefficient to compensate for insufficient FRP anchorage length;
β_w: FRP width-to-spacing ratio;
C_f: strengthening configuration factor;
CoV: coefficient of variation;
D_f: stress distribution factor;
d_{fv}: effective depth of FRP shear reinforcement (mm);
d_s: internal arm of steel reinforcement (mm);
E_f: modulus of elasticity of the CFRP reinforcement (MPa);
E_s: modulus of elasticity of longitudinal steel (MPa);
ε_f: strain in the FRP reinforcement;
ε_{fe}: effective strain in CFRP reinforcement;
ε_{fse}: effective strain in the FRP for shear strengthening according to TR 55;
ε_{fu}: ultimate strain of FRP at rupture (MPa);
$\varepsilon^{exp}{}_{fe}$: value of ε_{fe} obtained from experiments;
$\varepsilon^{pred}{}_{fe}$: predicted value of ε_{fe};
ε_{swy}: steel yielding strain;
ε_x: longitudinal strain at the mid-depth of the section;
f_{fbwm}: mean value bond strength of FRP reinforcement (MPa);
f_{ck}: characteristic compressive strength of concrete (MPa);
f_{ctk}: characteristic value of tensile strength of concrete (MPa);
f_{cm}: compressive strength of concrete (MPa);
f_{ctm}: mean value of tensile strength of concrete (MPa);
f_{fe}: effective stress in the FRP reinforcement as introduced in CNR-DT 200 R1 (MPa);
f_{fem}: mean value for the FRP effective stress as introduced in CIDAR (MPa);
f_{fmm}: FRP debonding stress according to CNR-DT 200 R1 (MPa);
f_{fm}: mean value of ultimate strength of FRP (MPa);
$f_{fwm,c}$: mean value of FRP strength attributed to a failure caused by FRP rupture (MPa);
$f_{fm,max}$: mean value of maximum stress in FRP as introduced in CIDAR (MPa);
f_{fu}: ultimate strength of FRP (MPa);

f_{fwm}: mean value of the average stress in the FRP intersected by the shear crack in the ultimate limit state (MPa);
Γ_{fm}: bonded joint specific fracture energy;
H_0: null hypothesis;
H_1: alternative hypothesis;
h: height of beam (mm);
h_f: height of CFRP reinforcement (mm);
h_{fe}: effective height of FRP reinforcement as introduced in *f*ib Bulletin 90 (mm);
h_w: height of beam web (mm);
K: a composite feature of the CFRP reinforcement that is obtained from $\rho_f E_f / f_{cm}^{2/3}$;
L_e(ACI): active bond length as introduced in ACI PRC-440.2-23 (mm);
L_e(CIDAR): effective bond length of FRP as introduced in CIDAR (mm);
L_e(*f*ib): effective bond length of FRP as introduced in *f*ib Bulletin 90 (mm);
l_{em}: effective bond length as introduced in CNR-DT 200 R1 (mm);
$l_{t,max}$: anchorage length required to develop full anchorage capacity (mm);
λ: ratio of the maximum available bond length to the effective bond length according to CIDAR;
MAPE: mean absolute percentile error;
M_E: bending moment caused by external loads (N·m);
N: number of specimens;
n_f: number of FRP layers;
n_{ss}: parameters of the TR 55 model representing type of strengthening scheme;
χ_ε: model error obtained for effective strain;
χ_V: model error obtained for V_f;
ρ_f: ratio of CFRP reinforcement;
ρ_l: longitudinal tensile steel reinforcement ratio;
ρ_{sw}: ratio of steel stirrups;
$\rho_{xj,\chi}$: coefficient of correlation between j^{th} parameter and model error;
R_c: corner radius of beam cross-section (mm);
r: Pearson coefficient of correlation;
R^2: coefficient of determination;
RMSE: root mean squared error (kN);
s_f: spacing of CFRP reinforcement (mm);
s_u: interface slip corresponding to full debonding (mm);
s_{0m}: mean value of ultimate slip (mm);
t_f: thickness of CFRP reinforcement (mm);
t'_f: effective total thickness of FRP reinforcement as introduced in *f*ib Bulletin 90 (mm);
t_0: thickness of each FRP layer (mm);
θ: inclination angle of CDC;
θ_{min}: minimum inclination angle of CDC;
τ_{b1m}: mean value of ultimate bond strength (MPa);
V_f: contribution of CFRP reinforcement to shear resistance of RC beams (kN);
$V^{exp}{}_f$: value of V_f obtained from experiments (kN);
$V^{model}{}_{R,f}$: predicted value of V_f obtained from each specific model (kN);
V_E: shear load caused by external loads (N.m);
x_j: input variable;
x', y', m', n': parameters of the *f*ib Bulletin 90 model;
$\mathbf{y}_i$: known value for specimen i;
$\boldsymbol{\gamma}_i$: predicted value for specimen i;
$\bar{y}$: mean of known values;
$\bar{y}$: mean of predicted values;

σ: standard deviation;
w_f: width of CFRP reinforcement (mm);
k_1, k_2, k_v: parameters of the ACI PRC-440.2-23 model representing concrete strength, type of strengthening scheme, and bond reduction coefficient, respectively;
k_R: reduction factor to consider stress concentration at corners;
k_b,k_g: geometrical corrective factor and additional corrective factor, respectively, as introduced in CNR-DT 200 R1;
κ_m: global model modification factor;
Ω: objective function;

REFERENCES

Abbasi, A., Z. El, A. Benzeguir, O. Chaallal, and G. El-saikaly. 2022. "FE Modelling and Simulation of the Size Effect of RC T-Beams Strengthened in Shear with Externally Bonded FRP Fabrics."

ACI. 2023. *PRC 440.2- Design and Construction of Externally Bonded Fiber-Reinforced Polymer (FRP) Systems for Strengthening Concrete Structures—Guide*. *Am. Concr. Inst.*

Aghabagloo, M., L. Carreras, M. Barahona, C. Barris, and M. Baena. 2024. "Experimental study and numerical prediction of the bond-slip law for concrete elements strengthened with FRP using metaheuristic algorithm." *Constr. Build. Mater.*, 411 (December 2023): 134712. Elsevier Ltd. https://doi.org/10.1016/j.conbuildmat.2023.134712.

Barros, J. A. O., S. J. E. Dias, and J. L. T. Lima. 2007. "Efficacy of CFRP-based techniques for the flexural and shear strengthening of concrete beams." *Cem. Concr. Compos.*, 29 (3): 203–217. https://doi.org/10.1016/j.cemconcomp.2006.09.001.

Chen, C., X. Xiao, Y. Zhou, Y. Yang, and L. Cheng. 2022. "FRP Shear-Strengthened RC Beams: Re-examining the Shear-Crack Effect." *J. Compos. Constr.*, 26 (5): 4022065. American Society of Civil Engineers.

Chen, G. M., S. W. Li, D. Fernando, P. C. Liu, and J. F. Chen. 2017. "Full-range FRP failure behaviour in RC beams shear-strengthened with FRP wraps." *Int. J. Solids Struct.*, 125: 1–21. https://doi.org/10.1016/j.ijsolstr.2017.07.019.

Chen, G. M., J. G. Teng, and J. F. Chen. 2013. "Shear Strength Model for FRP-Strengthened RC Beams with Adverse FRP-Steel Interaction." *J. Compos. Constr.*, 17 (1): 50–66. American Society of Civil Engineers (ASCE). https://doi.org/10.1061/(asce)cc.1943-5614.0000313.

Chen, J. F., and J. G. Teng. 2003. "Shear Capacity of Fiber-Reinforced Polymer-Strengthened Reinforced Concrete Beams: Fiber Reinforced Polymer Rupture." *J. Struct. Eng.*, 129 (5): 615–625. https://doi.org/10.1061/(asce)0733-9445(2003)129:5(615).

CIDAR. 2006. "CIDAR, Design guideline for RC structures retrofitted with FRP and metal plates: beams and slabs. Draft 3." Adelaide, Australia: The Univ. of Adelaide submitted to Standards Australia.

CNR. 2013. "DT 200 R1- Guide for the design and construction of externally bonded FRP systems for strengthening existing structures." *CNR*. Roma: CNR – Advisory Committee on Technical Recommendations for Construction.

D'Antino, T., F. Focacci, L. H. Sneed, and C. Pellegrino. 2020. "Shear Strength Model for RC Beams with U-Wrapped FRCM Composites." *J. Compos. Constr.*, 24 (1). https://doi.org/10.1061/(ASCE)CC.1943-5614.0000986.

*f*ib. 2019. "Bulletin 90. Externally Applied FRP Reinforcement for Concrete Structures. Technical Report." *f*ib Bulletins, (S. Matthys and T. Triantafillou, eds.), (May). *f*ib. The International Federation for Structural Concrete. https://doi.org/10.35789/*f*ib.BULL.0090.

*f*ib. 2023. *Model Code 2020: Towards a general code for both new and existing concrete structures*.

Karimzadeh, S., A. Mohammadi, S. M. S. Hussaini, D. Caicedo, A. Askan, and P. B. Lourenço. 2023a. "ANN-Based Ground Motion Model for Turkey using Stochastic Simulation of Earthquakes." *Geophys. J. Int.* https://doi.org/10.1093/gji/ggad432.

Karimzadeh, S., A. Mohammadi, U. Salahuddin, A. Carvalho, and P. B. Lourenço. 2023b. “Backbone ground motion model through simulated records and XGBoost machine learning algorithm: An application for the Azores plateau (Portugal).” *Earthq. Eng. Struct. Dyn.*, (833123): 1–26. https://doi.org/10.1002/eqe.4040.

Khalifa, A., W. J. Gold, A. Nanni, and A. A. M.I. 1998. “Contribution of Externally Bonded FRP to Shear Capacity of RC Flexural Members.” *J. Compos. Constr.*, 2 (4): 195–202. https://doi.org/10.1061/(asce)1090-0268(1998)2:4(195).

Li, W., C. Hu, Z. Pan, W. Peng, and Y. Yang. 2018. “A proposed strengthening model considering interaction of concrete-stirrup-FRP system for RC beams shear-strengthened with EB-FRP sheets.” https://doi.org/10.1177/0731684418760204.

Li, W., and C. K. Y. Leung. 2016. “Shear Span–Depth Ratio Effect on Behavior of RC Beam Shear Strengthened with Full-Wrapping FRP Strip.” *J. Compos. Constr.*, 20 (3): 1–14. https://doi.org/10.1061/(asce)cc.1943-5614.0000627.

Mohammadi, A., J. A. O. Barros, and J. Sena-cruz. 2024. “A Novel Design Model for Predicting the Shear Resistance of Reinforced Concrete Beams Strengthened with EBR-CFRP Systems.” *Compos. Struct.*, 117901. The Author(s). https://doi.org/10.1016/j.compstruct.2024.117901.

Mohammadi, A., J. A. O. Barros, and J. Sena-Cruz. 2023a. “Performance of models for predicting the shear capacity of FRP-strengthened RC beams.” https://doi.org/10.5281/ZENODO.8066241.

Mohammadi, A., J. A. O. Barros, and J. Sena-Cruz. 2023b. “A new model for predicting the shear strength of RC beams strengthened with externally bonded FRP sheets.” *Compos. Struct.*, 319 (March): 117081. Elsevier Ltd. https://doi.org/10.1016/j.compstruct.2023.117081.

Mohammadi, A., S. Karimzadeh, S. Amir, V. Ozsarac, and P. B. Lourenço. 2023c. “The potential of region-specific machine-learning-based ground motion models : Application to Turkey.” *Soil Dyn. Earthq. Eng.*, 172 (January): 108008. Elsevier Ltd. https://doi.org/10.1016/j.soildyn.2023.108008.

Shayanfar, J., J. A. O. Barros, M. Abedi, and M. Rezazadeh. 2023. “Unified Compressive Strength and Strain Ductility Models for Fully and Partially FRP-Confined Circular, Square, and Rectangular Concrete Columns.” *J. Compos. Constr.*, 27 (6). https://doi.org/10.1061/jccof2.cceng-4336.

TR 55. 2012. “Design guidance for strengthening concrete structures using fibre composite materials, Third Edition.” Camberley: Concrete Society.

Triantafillou, T. C., and C. P. Antonopoulos. 2000. “Design of Concrete Flexural Members Strengthened in Shear with FRP.” *J. Compos. Constr.*, 4 (4): 198–205. https://doi.org/10.1061/(ASCE)1090-0268(2000)4:4(198).

Models	Formulation
fib Bulletin 90	$V_{R,f} = A_{fpl} h_{fe} f_{fwm} (\cot\theta + \cot\alpha_f)\sin\alpha_f$ $A_{fpl} = \begin{cases} \frac{2t'_f w_f}{s_f} & \text{strips} \\ 2t'_f \sin\alpha_f & \text{Continuous sheet}\end{cases}$; $t'_f = \begin{cases} t_0 n_f & \text{if } n_f < 4 \\ t_0 (n_f)^{0.85} & \text{if } n_f \ge 4\end{cases}$ $h_{fe} = \min(h_f, h-0.1d_s)$ Fully wrapped: $f_{fwm} = f_{fwm,c} = k_R . f_{fm}$ U-wrapped: $f_{fwm} = \min(f_{fbwm}, f_{fwm,c})$ For continuous CFRP sheets: $f_{fbwm} = \begin{cases} \left[1-\frac{1}{3}\frac{L_e(fib)}{(h_{fe}/\sin\alpha_f)}\right] f_{fbm} & \text{for } x' \ge L_e(fib) \\ \frac{2}{3}\frac{h_{fe}/\sin\alpha_f}{L_e(fib)} f_{fbm} & \text{for } x' \le L_e(fib)\end{cases}$ For discrete CFRP strips: $f_{fbwd} = \begin{cases} f_{fbm} & \text{for } x' \ge L_e(fib);\ L_e(fib) \le y' \le x' \\ \left[1-\left(1-\frac{2}{3}\frac{m' s_f}{L_e(fib)}\right)\frac{m'}{n'}\right] f_{fbm} & \text{for } x' \ge L_e(fib); y' \le L_e(fib) \\ \frac{2}{3}\frac{(n' s_f)/[(\cot\theta+\cot\alpha_f)\sin\alpha_f]}{L_e(fib)} f_{fbm} & \text{for } x' \le L_e(fib); y' \le x'\end{cases}$ $x' = h_{fe}/\sin\alpha_f$ $y' = s_f/(\cot\theta+\cot\alpha_f)\sin\alpha_f$ $n' = \text{int}\lfloor h_{fe}(\cot\theta+\cot\alpha_f)/s_f \rfloor$ $m' = \text{int}\lfloor L_e(f\text{ib})(\cot\theta+\cot\alpha_f)\sin\alpha_f/s_f \rfloor$ $L_e(fib) = \frac{\pi}{2}\sqrt{\frac{E_f t'_f s_{0m}}{\tau_{b1m}}}$ $\tau_{b1m} = 0.72\sqrt{f_{cm} f_{ctm}}$ $s_{0m} = 0.24$ mm $f_{fbm} = \sqrt{\frac{E_f s_{0m}\tau_{b1m}}{t'_f}}$
CNR-DT 200 R1	$V_{R,f} = 0.9 d_s f_{fe} . 2t_f (\cot\theta+\cot\alpha_f)\frac{w_f}{s_f \sin\alpha_f}$ U-Wrapped: $f_{fe} = f_{fmm}\left[1-\frac{1}{3}\frac{l_{em}\sin\alpha_f}{\min\{0.9d_s, h_w\}}\right]$ Fully wrapped: $f_{fe} = f_{fmm}\left[1-\frac{1}{6}\frac{l_{em}\sin\alpha_f}{\min\{0.9d_s, h_w\}}\right] + \max\left(0, \left\langle \frac{1}{2}(\kappa_R . f_{fu} - f_{fmm})\left[1-\frac{l_{em}\sin\alpha_f}{\min\{0.9d_s, h_w\}}\right]\right\rangle\right)$ $\kappa_R = 0.2+1.6\frac{R_c}{b_w}$, *with* $0 \le \frac{R_c}{b_w} \le 0.5$; $f_{fmm} = \sqrt{\frac{2E_f \Gamma_{fm}}{t_f}}, \Gamma_{fm} = k_b k_G \sqrt{f_{cm} f_{ctm}}$; $t_f = n_f t_0$ $k_b = \sqrt{\frac{2 - w_f/s_f \sin\alpha_f}{1 + w_f/s_f \sin\alpha_f}} \ge 1$ (if $w_f/b_w < 0.25$, k_b is equal to 1.18); $k_G = \begin{cases} 0.037 & \text{Wet lay-up} \\ 0.023 & \text{pre-cured}\end{cases}$ For continuous sheets:

	$w_f = \min\{0.9d_s, h_w\}\dfrac{\sin(\theta+\alpha_f)}{\sin\theta}$, $s_f = w_f/\sin\alpha_f$ $l_{em} = \max(\dfrac{1}{f_{bm}}\sqrt{\dfrac{\pi^2 E_f t_f \Gamma_{Fm}}{2}}, 200mm)$, $f_{bm} = \dfrac{2\Gamma_{Fm}}{s_u}$, $s_u = 0.25\,\text{mm}$
ACI PRC-440.2-23	$V_{R,f} = \dfrac{A_{fw} f_{fe} d_{fv}(\sin\alpha_f + \cos\alpha_f)}{s_f}$ $A_{fw} = 2t_f w_f$; $f_{fe} = E_f \varepsilon_{fe}$ Fully wrapped: $\varepsilon_{fe} = 0.004 \le 0.75\varepsilon_{fu}$ U-wrapped: $\varepsilon_{fe} = \kappa_v \varepsilon_{fu} \le 0.004$ $\kappa_v = \left(\kappa_1\kappa_2 L_e\left(ACI\right)\right)/\left(11900.\varepsilon_{fu}\right) \le 0.75$; $L_e\left(ACI\right) = 23300/(t_f E_f)^{0.58}$ $\kappa_1 = \left(f_{ck}/27\right)^{2/3}$; $\kappa_2 = \left(d_{fv} - L_e\left(ACI\right)\right)/d_{fv}$
TR 55	$V_{Rd,f} = \dfrac{A_{fw}}{s_f}\left(d_{fv} - \dfrac{n_{ss}}{3} l_{t,\max}\cos\alpha_f\right) E_f \varepsilon_{fse}\left(\sin\alpha_f + \cos\alpha_f\right)$ $n_{ss} = \begin{cases} 0 & \text{,fully-wrapped} \\ 1 & \text{,U-wrapped} \end{cases}$, $l_{t,\max} = 0.7\sqrt{\dfrac{E_f t_f}{f_{ctk}}}$ $\varepsilon_{fse} = \min\left\{\varepsilon_{fu}/2, 0.5\sqrt{\dfrac{f_{ctk}}{E_f t_f}}, 0.004\right\}$ For continuous sheets: $w_f = \sin\alpha_f$,and $s_f = 1$ Note: ε_{fse} should be greater than 0.002. if not, this formulation is not applicable.
CIDAR 2006	$V_f = 2 f_{fem} t_f \dfrac{w_f}{s_f} h_{fe}(\cot\theta + \cot\alpha_f).\sin\alpha_f$ $f_{fem} = D_f . f_{fm,\max}$ Fully wrapped: $f_{fm,\max} = \begin{cases} f_{fu} \text{ ; } \varepsilon_f \le 1.5\% \\ E_f.\varepsilon_f ; \varepsilon_f > 1.5\% \end{cases}$; $D_f = 0.5\left(1 + \dfrac{z_t}{z_b}\right)$, $L_e\left(CIDAR\right) = \sqrt{\dfrac{E_f.t_f}{\sqrt{f_{ck}}}}$ For U- Wrapped: $D_f = \begin{cases} \dfrac{2}{\pi.\lambda}.\dfrac{1-\cos(\frac{\pi}{2}.\lambda)}{\sin(\frac{\pi}{2}.\lambda)} ; & \lambda = \dfrac{L_{\max}}{L_e\left(CIDAR\right)} \le 1 \\ 1 - \dfrac{\pi-2}{\pi\lambda} ; & \lambda = \dfrac{L_{\max}}{L_e\left(CIDAR\right)} > 1 \end{cases}$ **(** $L_{\max} = \dfrac{h_{fe}}{\sin\alpha_f}$ **)** $f_{fm,\max} = \min\begin{cases} f_{fu} \\ 0.35 \times \beta_L \times \beta_W . \sqrt{\dfrac{E_f.\sqrt{f_{ck}}}{t_f}} \end{cases}$ $\beta_L = \begin{cases} \lambda, & \lambda \le 1 \\ 1, & \lambda > 1 \end{cases}$ $\beta_w = \sqrt{\dfrac{2 - w_f/(s_f.\sin\alpha_f)}{1 + w_f/(s_f.\sin\alpha_f)}}$ $h_{fe} = z_b - z_t$, $z_b = 0.9d_s$, $z_t = d_{ft}$

APPENDIX B. SUPPLEMENTARY DATA

The following supplementary file contains the dataset with input features of the specimens accompanied by the predictions for each model: [Excel file provided]