



Original article

Generative modelling of Monopteros and Tholos temples using existing data: The case study of Vesta temple in Tivoli

Annalaura Vuoto^a, Marco F. Funari^{b,*}, Shaghayegh Karimzadeh^a, Paulo B. Lourenço^a^a Department of Civil Engineering, ISE, ARISE, University of Minho, Guimarães, Portugal^b School of Engineering, University of Surrey, Guildford, UK

ARTICLE INFO

Article history:

Received 8 February 2024

Accepted 9 December 2024

Keywords:

Generative programming

Visual programming

3D modelling

Seismic assessment

ABSTRACT

This paper presents a novel method for generating geometric models of architectural heritage in the absence of a digital survey. The method employs a Generative Programming (GP) algorithm for geometric model generation, with the Temple of Vesta in Tivoli chosen as a case study. Francesco Piranesi's 18th-century etchings are utilised as references to identify the architectural layout and modularity. The effectiveness of the proposed generative workflow is highlighted through its time efficiency and the reusability of the algorithm. The workflow includes the capability to generate an export file suitable for structural simulation software packages. The generated geometric model is then used to conduct nonlinear dynamic analysis using a concurrent continuous/block-based approach within a Finite Element environment. The simulations are performed with the structure in its current state and do not account for retrofitting interventions, i.e. anchorages and tie rods are not taken into account. The numerical model reveals how local failure mechanisms of columns and entablature affect the structural safety of the Vesta temple.

© 2024 The Author(s). Published by Elsevier Masson SAS.

This is an open access article under the CC BY license (<http://creativecommons.org/licenses/by/4.0/>)

Introduction

Today, technologies for digital geometric modelling of existing buildings enable the creation of accurate representations, often referred to as Digital Twins (DTs) [1–4]. Even if often used within recent literature, this definition can be misleading [5,6]; hence, the term “Geometric Digital Twin” is thus more fitting for emphasising the geometric features without encompassing the broader functional and data-driven dimensions of a complete DT [7].

Even though state-of-the-art processes enable high levels of geometric fidelity [8], it remains crucial to leverage available tools to streamline and optimise tasks that would otherwise be complex and time-consuming. Reality-based modelling, based on laser scanning or digital photogrammetry, is recommended for architectural heritage where structural elements cannot be accurately represented using geometric primitives [9–14]. The most advanced computational tools now enable increased automation in both data acquisition and processing phases. These models can be applied to capturing features crack patterns [15–17], or even the full structural geometries [18,19]. However, reality-based approaches are still not fully automated, are challenging to implement for large buildings, and are time-consuming. These limitations hinder

their widespread application in large-scale heritage preservation projects.

Therefore, simplified methods that still incorporate advanced geometric modelling techniques can be used [20,21]. For instance, masonry microstructure can be generated by algorithms and then used as input for numerical simulations [22–24]. At the building scale, object-based parametric modelling is particularly effective for regular structures characterised by modules, repetitions, and symmetries [25], significantly accelerating the modelling process. Input data can be sourced from available documentation, such as historical records and drawings [26,27], which is especially advantageous in situations where the site is not accessible [28–30]. When the geometrical layout permits, visual and textual programming languages (V/T PL) [31,32] can be employed to generate faster and more efficient models. While TPL enables the writing of executable code for generating geometric model outputs and necessitates a high level of expertise, VPL utilises a flowchart system created through nodes and connections, making them more accessible to practising engineers and architects [31]. When combined with the concept of Generative Programming (GP), VPL parametric modelling has demonstrated efficiency across various engineering fields, though its applications to historical masonry buildings remain limited in the literature [25,31,33,34].

In this context, architectures built during the classical era often comprise elements arranged in the space according to specific

* Corresponding author.

E-mail address: m.funari@surrey.ac.uk (M.F. Funari).

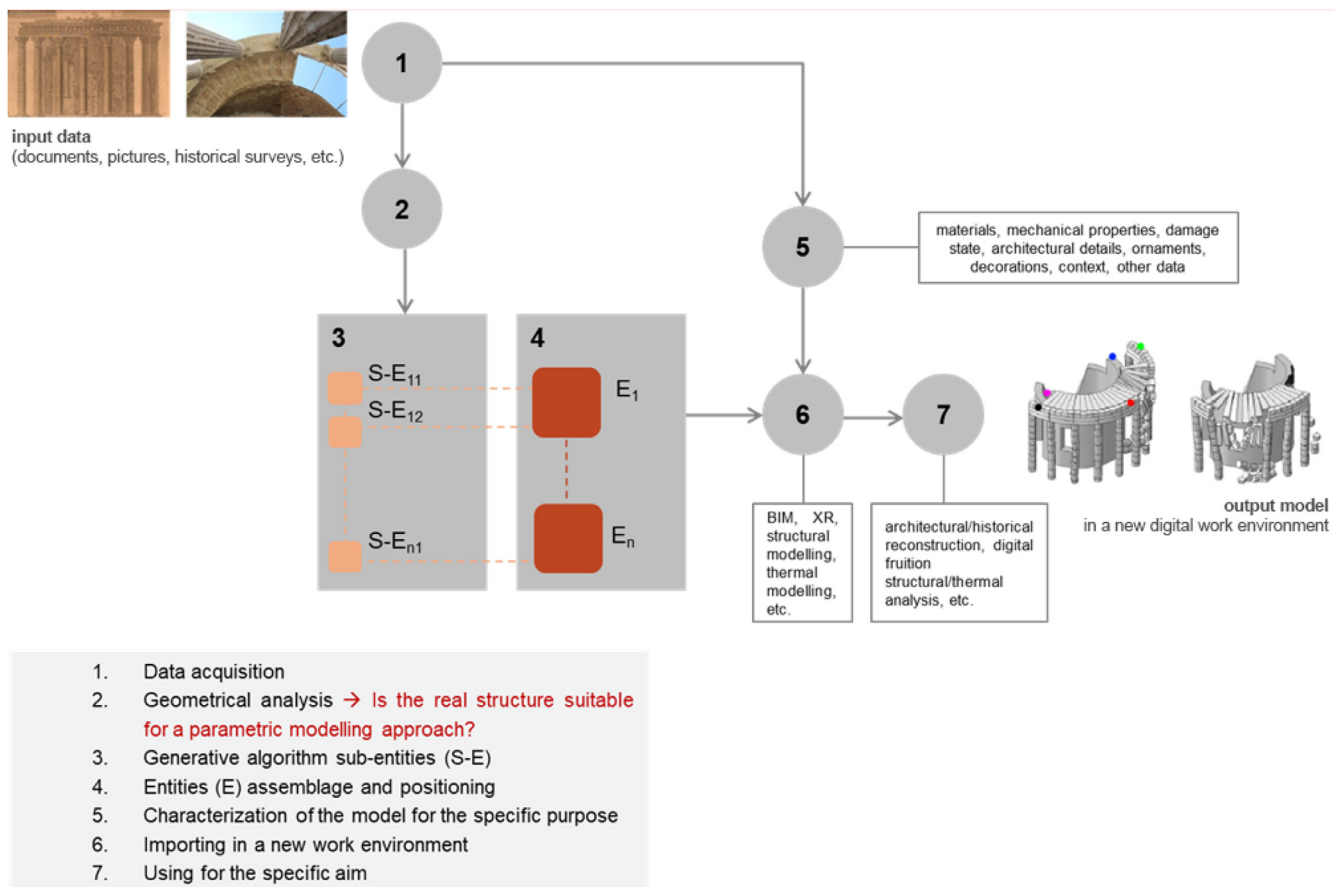


Fig. 1. Workflow of the proposed approach (the images are related to the case study presented in this work and are intended for illustrative purposes).

modular layouts, facilitating their interpretation through a metrological approach. By comparing the recurrence of elements with the support of known ancient units of measurement [35], it becomes feasible to analyse the geometric layout and uncover the architectural composition rules employed during the construction. Such an approach has proven effective in the study of a Roman bridge dating to the 2nd century BC, a late Roman settlement in Egypt [36–38], and a Roman residence in Cyprus [39], among others.

This work explores the potential of 3D parametric modelling of architectural heritage using GP informed by written documents and existing surveying. A modelling workflow is developed and validated using the Vesta Temple in Tivoli, Italy, as a proof of concept. The 3D model obtained is subsequently tested for applicability for structural simulations. Specifically, nonlinear dynamic analyses are performed to assess the structure's seismic response [40].

The paper is organised as follows: after addressing the problem of geometrical modelling for historical masonry buildings and introducing the framework in Section *Introduction*, the developed general methodology is described in detail in Section *Methodology overview*. The proposed framework is tested in Section *Application of the methodology to a tholos temple*. Final remarks and suggestions for extending the application of the proposed framework are provided in Section *Conclusion*.

Methodology overview

This work aims to develop a methodology for the 3D parametric modelling of architectural heritage using existing data. Due to the architectural features, state of preservation, and accessibility, the direct acquisition of geometrical information is not always

feasible. However, obtaining geometrical models that perform efficiently according to their intended use has become an imperative step for all conservation-related applications. Therefore, this study aims to optimise the geometrical modelling phase of architectural heritage, incorporating morphological characteristics into a parametric model informed by existing data, such as historical photographs, reports, previous surveys, and drawings. A workflow based on generative algorithms for modelling geometric components is implemented in a VPL, enabling the development of a procedure that minimises errors and offers high reusability for modelling similar layouts.

To achieve this vision, this study leverages the potential of GP. In parametric design, the shapes of elements are defined by specific parameters, with geometric relationships algorithmically expressed to articulate the architectural composition [41]. Specifically, generative code is developed using rational rules that define the distribution layout of these elements [25,42].

The input consists of geometrical information obtained from existing documents and drawings, whereas the output is a 3D geometrical model of the investigated asset. The pipeline steps are outlined below and schematised in Fig. 1:

1. Data acquisition from existing sources.
2. Geometrical analysis of the structure. It allows for the assessment of its suitability for a parametric modelling approach by identifying (i) entities, (ii) sub-entities, (iii) modules and repetitions, and (iv) symmetries for the parametric discretisation of the structural components.
3. Implement instance-based parametric components (sub-entity, S-E) using a generative modelling paradigm implemented in a visual programming environment. Once the ra-

tional rules adopted by the ancient masons are identified in step 2, they are transposed into a set of coding rules using Python programming language to get the full geometry of the structure.

4. Geometrical modelling of the structure. The sub-entities are moved, copied, and assembled using the same script to obtain the entities (E) that make up the temple.
5. Integration of the geometrical features with other data and information needed for the final use of the model (i.e. materials, physical/mechanical/thermal properties, damage state, architectural details and ornaments, environmental context, boundary conditions, etc.).
6. Import the geometrical model in a simulation software package for the final use, i.e. Building Information Modelling (BIM), Extended Reality (XR), clouds, structural modelling, thermal modelling, etc.
7. Usage of the model for specific purposes, including architectural or historical reconstruction, digital presentation through extended reality, the development of an informed model linked to databases, and structural analyses, etc.

Application of the methodology to a tholos temple

The workflow described in Section *Methodology overview* has been applied to the Vesta Temple in Tivoli, Italy. In this instance, the geometrical model has subsequently been used to evaluate the structural behaviour of the asset under seismic loading conditions. After completing steps 1–4 of the workflow, the geometrical model has been imported into a structural finite element software package, where the mechanical properties of materials, as well as boundary and loading conditions, were defined. Nonlinear dynamic analyses have been performed using ground motions selected and scaled in accordance with the target response spectrum for Tivoli as seismic input. The analysis results produced collapse scenarios that have been interpreted to assess the structural safety of the temple and to gain insights into its structural behaviour.

The Vesta Temple in Tivoli

The Vesta Temple in Tivoli is classified as *tholos*, which is a type of circular temple featuring a colonnade and an inner cella [43]. Dating to about 80 BC, it consists of a cella and a peristyle, with eighteen columns, of which only ten still stand (Fig. 2(a), (b)).

The temple stands on a stylobate, or circular podium (Fig. 2(c)), directly supporting the columns with an Attic base (Fig. 2(d)) [44]. The Hellenistic capital (Fig. 2(e), (j)) is the most striking and valuable element of the temple, notable for its double row of curly-leaved acanthus leaves with the addition of long fern-like leaves [45]. The entablature (Fig. 2(f)) has a plain architrave, a cornice with simple mouldings and a frieze decorated with complete ox-heads connected by garlands (see the reconstruction in Fig. 2 (i)), topped by a ceiling with a double row of coffers, having a rose in their centre (Fig. 2(g)). Travertine, a local limestone, is the most used material across the temple, according to the use of the golden age of Augustus. Only the cella is in *opus reticulatum*, made by irregular polygonal tuff stones and cement (Fig. 2(j)). The cella originally had three openings, namely the monumental travertine door and two windows on its left and right sides.

The Vesta Temple was extensively studied and surveyed during the Renaissance. However, these representations often exhibit inconsistencies and inaccuracies regarding the temple's architectural details and dimensions. According to Richardson [46], this variability is to be expected when different individuals measure and interpret an ancient, weathered ruin using various instruments, especially during a time of limited technological resources. Some of the historical documents are presented in Section *Data acquisition*.

A cross-comparison of these documents has been conducted to validate the information used for the geometrical modelling. Although some data may vary slightly, the different analyses share significant features in common.

Data acquisition

The workflow starts with analysing geometrical data from historical documentation (step 1 in the workflow of Section *Methodology overview*). While using existing documents and drawings offers significant benefits, it also has limitations, particularly concerning buildings that have been subjected to modifications. To mitigate these issues, it is advisable to cross-compare multiple sources from different periods, prioritising recurrent and consistent information. The reliability of documents also depends on the intended use of the geometrical model; in this case, accuracy in the dimensions of structural elements and their proportional relationships is essential for creating numerical models to perform simulations efficiently.

Several sources have been collected and compared. The reference source for the knowledge of the temple has been the analysis proposed by Palladio [45] until the 17th century, when Antoine Desgodetz's study partly superseded it in 1682 [47]. From the 18th century onwards, the temple became one of the most studied ancient archaeological sites; the surveys by the English architects George Dance the Younger and Sir John Soane are worth mentioning. In Italy, a full archaeological survey was carried out by Francesco Piranesi in the 1760s and published in 1780 in the first volume of '*Raccolta de' tempi antichi*' [48]. According to Sebastiani [49], Piranesi was the first to restore a rigorous reconstruction of the temple, although including minor amendments later removed by Uggeri [50] and Valadier [51]. In the 1980s, the architect Mark Wilson Jones provided an updated survey, whose main dimensions closely align with both Dance and Piranesi's drawings [46]. After cross-checking the geometrical information contained in the sources listed above, Piranesi's etchings have been chosen as input data for this study. They contain drawings of the temple's plan, front and side elevations, and the reconstruction of numerous architectural and decorative details. All drawings were dimensioned using the Roman *palmus maior*, i.e. greater or major palm, equal to 22.34 cm, as a unit of measurement.

Geometrical analysis

In this subsection, the creation of the geometrical model (steps 2–4 of the workflow in Section *Methodology overview*) is described. In step 2, the suitability of a parametric approach for the geometrical modelling of the Vesta Temple has been tackled by conducting a geometrical analysis aimed at identifying i) entities, ii) sub-entities, iii) recurring modular elements, and iv) symmetries for the parametric discretisation of the architecture to be modelled. Four main entities (E_n) have been identified (see Fig. 3(a)): i) E1 – colonnade, ii) E2 – cella, iii) E3 – coffered ceiling, and iv) E4 – entablature. These entities are composed of the primary architectural elements, i.e. sub-entities ($S-E_{ni}$), whose structural behaviour influences the overall seismic response of the temple, as opposed to the ornamental elements, which have been neglected in this paper.

Secondly, a search has been conducted to find a module or metrological unit to define the key dimensions of structural elements and parameterise the temple's overall geometry. Considering that at the time of construction, elements' dimensions were established based on modularity, each dimension has been defined as a multiple of the module. One should note that a certain tolerance due to i) type of material, ii) precision of stonework, iii) state of conservation, and iv) methods and tools of measurement [52],



Fig. 2. The Vesta Temple in Tivoli. Upper: (a) view of the temple and the cliff, (b) global view. Middle: architectural details (c) podium, (d) column without plinth, (e) capital, (f) entablature, (g) coffered ceiling, (h) cella in irregular *opus reticulatum*. Bottom: reconstruction of architectural details (i) Royal Academy lecture's drawing of the entablature by Sir John Soane at Sir John Soane Museum with the reconstruction of the frieze's decoration, (j) full-size replica of the capital at Sir John Soane's Museum [43].

must be accounted for. As far as temples are concerned, this basic module is often the diameter of the column. Indeed, in most of the historical descriptions of the Vesta Temple [46,53], the dimensions of the main elements were expressed as multiples of the column diameter and through ratios between the elements. For example, the height of the capital is the same as the diameter of the column [46,53]. The intercolumn is about two diameters [53], and the height of the column, including the base and capital, is 9.5 diameters [46,53], etc. (Fig. 3). Some descriptions (Dance, Soane, Wilson Jones [46]) provided the measurement of the diameter of the column, varying between 74 and 77 cm, corresponding to ten Roman *palmus minor*, i.e. lesser or minor palm (Fig. 3(b)). One *palmus minor* measures 7.35–7.42 cm, roughly one-quarter of

a *pes*, i.e. foot, which is the basic module of the Roman system of measurement, equal to 29.4–29.7 cm [52]. While the *palmus maior* seen in Piranesi's etchings has been regularly used, the *palmus minor* has been used in contexts requiring smaller units to describe the geometry modularity. Therefore, as depicted in Fig. 3(a),(c), the metrological unit of one *palmus minor* has been chosen as a linear module (*m*) to parameterise the entire geometry. Consequently, all the dimensions given in major palms in Piranesi's drawings have been converted into minor palms, with a minor palm being about one-third of a major palm (22.34 cm). An additional angular module (i) corresponding to one intercolumn (20°) has been identified and used for the radial discretisation of entablature and coffered ceiling, visible in Fig. 3(d).

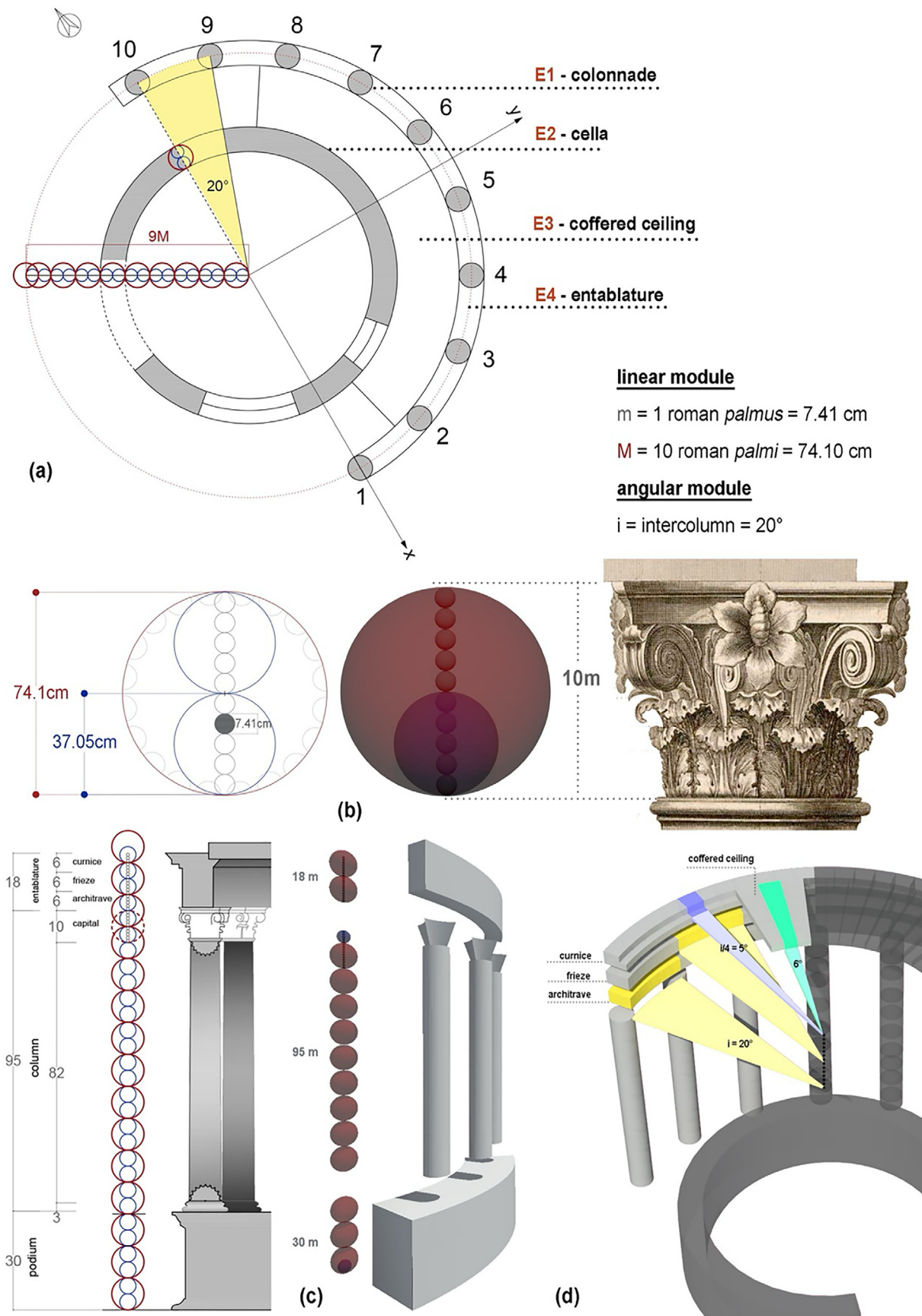
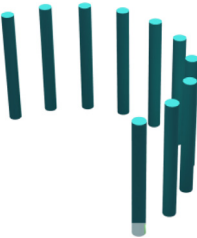
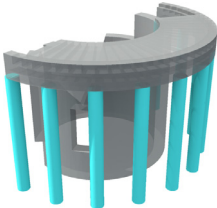
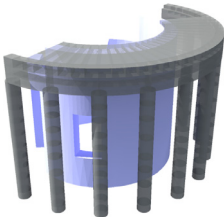
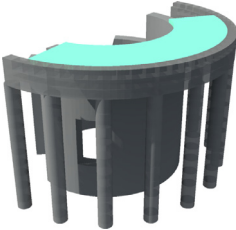
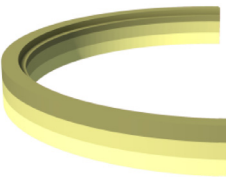
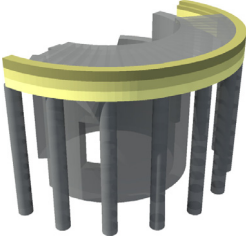


Fig. 3. Geometrical analysis of the Vesta Temple: (a) identification of the entities (En) and plan parametrisation; (b) definition of the linear module (m) equal to 1 Roman palmus (7.41 cm); (c) elevation parametrisation; (d) discretisation of entablature and coffered ceiling by using the angular module (i) equal to the intercolumn (20°).

Table 1
Abacus of entities (E), sub-entities (S-E), attributes' parametrisation and spatial position.

Entity [E]	Parametric model [E]	Sub-entity [S-E]	Attribute		Parametric model [S-E]	Position in the full model
Colonnade E1		Column (x10)	radius height	5 m 95 m		
Cella E2			radius (int) height thickness	50 m 110 m 5 m		
Coffered ceiling E3			base height radius (block)	10 m 6 m i/3.33		
Entablature E4		Architrave	base height radius (block)	10 m 6 m i		
		Frieze	base height radius (block)	10 m 6 m i		
		Cornice	base height radius (block)	10 m 6 m i/4		

Downstream of the geometrical analysis, a generative algorithm has been implemented in the Visual Programming environment provided by Rhinoceros [54] and Grasshopper [55] software (step 3 of the workflow in Section *Methodology overview*). The modelling script is available at the following link: Generative Modelling of Monopteros and Tholos Temples: Vesta Temple in Tivoli (zenodo.org) [56]. Firstly, the 3D modelling of the sub-entities has been carried out by extrapolating the primitive geometries that matched the sub-entities. Various attributes have been selected as input variables to model such geometries, i.e. radius, height, etc., and a generative algorithm has been implemented for each sub-entity through the GHPython component available in Grasshopper [55]. As reported in Table 1, the geometrical attributes have been modelled by using both linear and angular modules as a unit of measurement. Their dimensions are expressed in the table in

terms of these modules (see ‘Attribute’ column). The obtained sub-entities have been collected and used to generate the four main entities in the next step of the workflow.

The assemblage of the identified entities has been carried out by implementing the geometric relationships between the architectural elements that define the structure’s layout (step 4 of Fig. 1). This has also been performed by using a GHPython script, which positioned the sub-entities by moving, spacing, and rotating them to get the entities placed in a spatial arrangement to fit the original layout of the temple according to the scheme reported in Fig. 3.

In Fig. 4, the modelling of the entity E1 – colonnade is depicted to provide a clearer understanding of the procedure. All the columns comprising the colonnade have been modelled by stacking their component blocks. The actual heights of the blocks,

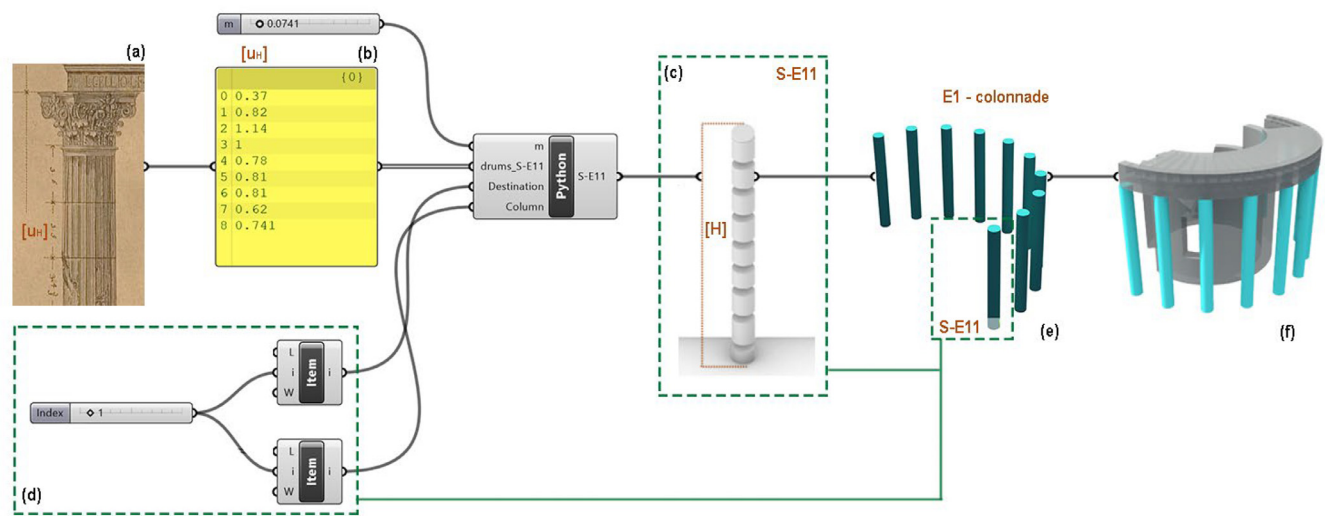


Fig. 4. Modelling workflow for column 1 (S-E11) implemented in GHPython for Grasshopper.

Table 2

Material properties of the inner cella.

E_0 [MPa]	ν	ρ [kg/m ³]	Dilatation angle	Eccentricity	f_{b0}/f_{c0}	K_c	Viscosity parameter
1000	0.2	1450	10°	0.1	1.16	2/3	1e-5

Compressive behavior			Tensile behavior		
Stress [MPa]	Inelastic strain	d_c	Stress [MPa]	Inelastic strain	d_t
2.00	0	0	0.15	0	0
2.20	0.004	0	0.001	0.002	0.9
0.2	0.010	0.9			

as reported in Piranesi's etchings (Fig. 4(a) and (b)), have been used. The columns have been finally indexed and positioned (Fig. 4(d),(e)) according to the actual layout of the temple to obtain the entire colonnade (Fig. 4(f)).

The 3D model of the Vesta Temple is provided in Fig. 5(a–d). One can note that the existing *lacuna* has been introduced in the cella through Boolean subtraction, which allowed for the reproduction of its current shape.

One should note that sub-entities can be stored in libraries and utilised for other objects, and entire scripts can be reused to model similar structures, such as other *monopteros/tholos* temples, with minor adjustments in nearly no time. The code's reusability has been tested to generate a geometrical model of the temple's original configuration based on Francesco Piranesi's hypothesis. The model in Fig. 5(e) and (f) has been created by adjusting the script, increasing sub-entities to complete the circular plan, and adding a dome as a new entity. Additional capabilities have been coded for modelling primitive geometries required for the dome.

FE modelling

The structural modelling of the Vesta Temple has been performed using a concurrent continuous/block-based approach [40]. In particular, the cella has been modelled via the macro-modelling approach [57]. Masonry non-linearities have been taken into account via the so-called Concrete Damage Plasticity (CDP), which couples plasticity with a scalar-based damage model [58]. The quasi-brittle nature of masonry is represented by a linear type of softening in tension. In compression, a plateau exists after the compressive strength, followed by a linear type of softening. Damage variables are adopted when softening is active and aim at re-

ducing the initial (undamaged) elastic modulus through the following expressions:

$$\sigma_c = (1 - d_c)E_0(\varepsilon_c - \varepsilon_c^{pl}) \tag{1}$$
$$\sigma_t = (1 - d_t)E_0(\varepsilon_t - \varepsilon_t^{pl})$$

where E_0 is the elastic modulus of the undamaged masonry, σ_i is the effective stress value; ε_i is the total strain value, and ε_i^{pl} is the inelastic (plastic) strain value. The subscript i reads as c or t if associated with the compressive or tensile regime, respectively. A scalar-based damage model describes the damage in tension d_t (cracking) and compression d_c (crushing), which can assume a value between zero (no damage) and one (fully damaged). When cyclic loading is applied, loss of stiffness in the unloading phase due to cracking and crushing is likely to happen. CDP assumes a non-associative flow rule given as a Drucker–Prager hyperbolic function and requires the definition of physically based parameters. Material properties adopted in the numerical analysis are reported in Table 2.

Travertine blocks, forming the columns, ceiling, and entablature ($\rho = 2500$ [kg/m³]), have been assumed to be deformable discrete blocks following an isotropic and linear elastic constitutive law ($E = 63$ GPa, $\nu = 0.2$). The dry-assemblage of blocks has been represented by zero-thickness interfaces, which include a non-associative plastic flow rule and a classical Mohr–Coulomb failure surface criterion. Normal and tangential contact behaviours assume an infinitesimal interpenetration between blocks. A linear relationship between the over-closure displacements and the applied stress has been defined by the normal and tangential stiffness values of $k_n = 5 \times 10^9$ Pa/m and $k_s = 2 \times 10^9$ Pa/m, respec-

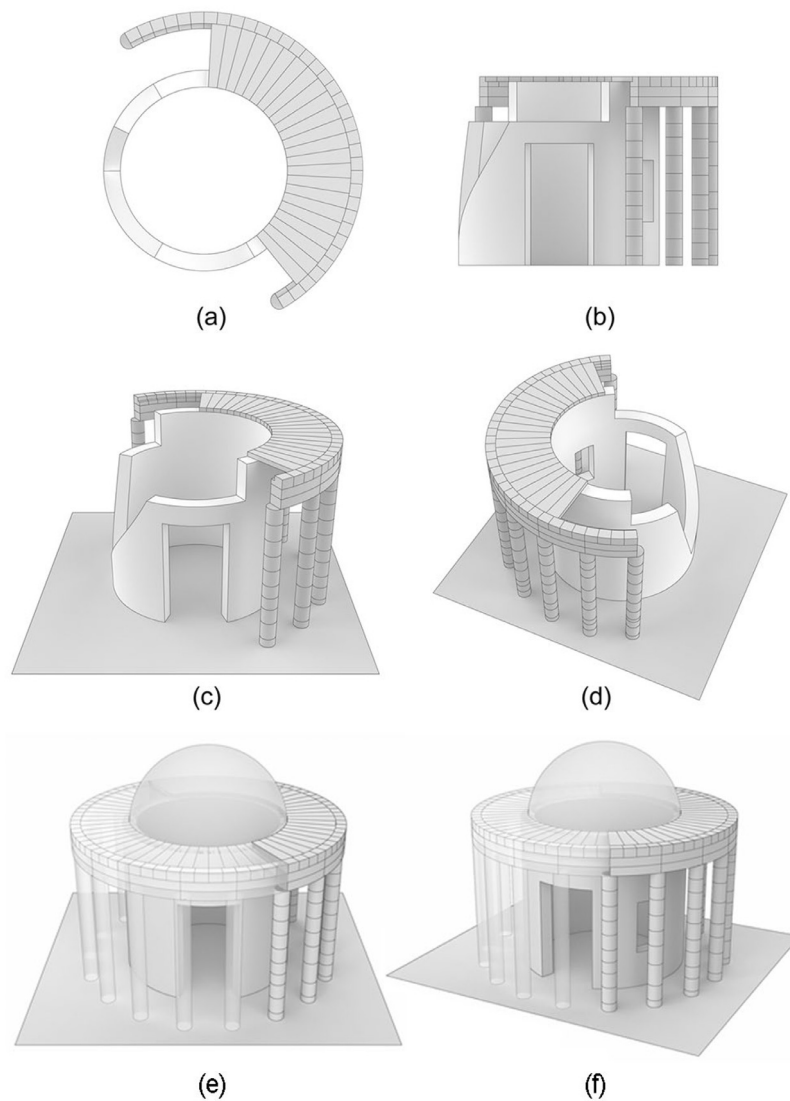


Fig. 5. Final 3D models of the Vesta Temple: actual configuration (a)–(d); (a) plan, (b) front elevation, (c) and (d) 3D views; reconstructive hypothesis according to F. Piranesi (e) and (f).

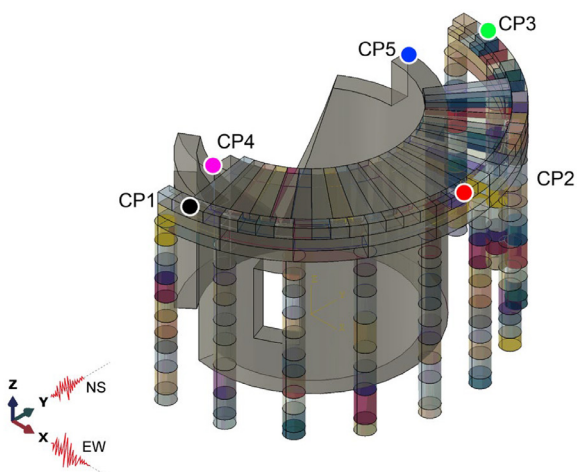


Fig. 6. FE model: geometry, boundary conditions and control points.

tively. A friction coefficient ($\mu = 0.70$) completing the shear contact behaviour has been adopted. No viscous damping has been set to the contact interfaces [59].

The three-dimensional FE model (Fig. 6) enforces the use of three-dimensional (solid) solid elements; therefore, the mesh discretisation has been achieved using tetrahedron FEs for the cella (TETC3D4) and hexahedral FEs for the remaining structural components (C3D8R) having a characteristic dimension of 200 mm. Appropriate boundary and loading conditions have been implemented to run the simulations. Numerical analyses have been performed by applying two phases that idealised the load process: at first, the gradual application of gravity loads and, subsequently, ground motion records simultaneously in North-South (NS) (Y) and East-West (EW) (X) directions. An explicit time integration scheme has been adopted to integrate the equation of motion, while nonlinear geometries have been taken into consideration [60].

Ground motion selection and scaling

The code-based selection of ground motions is a crucial step in achieving the seismic assessment of a historic building, as it ensures that structural design and assessment are performed using representative and realistic seismic inputs. Engineers utilise specific building codes and seismic design standards to guide the selection of ground motion records that reflect the seismic hazard at a particular location. By adhering to established seismic design

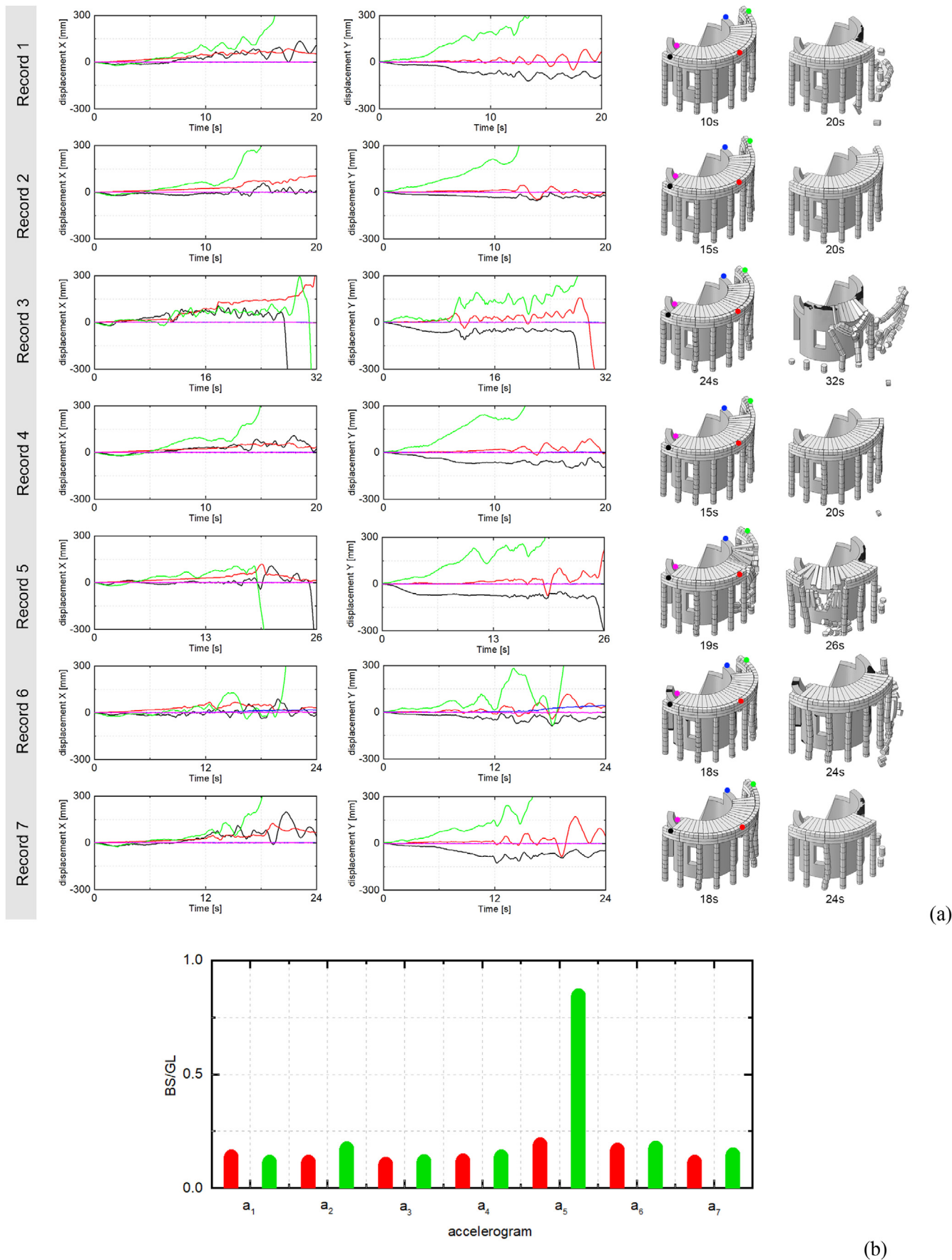


Fig. 7. (a) Time history displacement of selected control points (different colours) and failure mechanisms (b) Maximum base shear / gravity loads (BS/GL) in -X (red) and -Y (green) directions.

Table 3
Summary of selected unscaled ground motion records with PGA and PGV for the selected scaled records, along with their corresponding scaling factors.

Record No	Event Name	Year	Station Name	M _w	Fault Mechanism	Vs30 (m/s)	R _{JB} (km)	Scale factor	PGA (g)		PGV (cm/s)	
									EW	NS	EW	NS
1	Irpinia_ Italy-01	1980	Arienzo	6.9	Normal	612.78	52.93	3.97	0.11	0.14	11.07	12.01
2	Chi-Chi_ Taiwan-05	1999	CHY042	6.2	Reverse	665.20	62.84	3.12	0.15	0.12	11.92	13.95
3	Chi-Chi_ Taiwan-05	1999	TTN001	6.2	Reverse	484.60	58.74	3.58	0.10	0.12	13.85	14.84
4	Chi-Chi_ Taiwan-06	1999	TCU029	6.3	Reverse	406.53	64.17	4.75	0.15	0.15	10.03	13.72
5	Chuetsu-oki_ Japan	2007	ISK001	6.8	Reverse	487.67	106.86	4.40	0.13	0.16	9.85	12.26
6	Iwate_ Japan	2008	AKT018	6.9	Reverse	485.20	54.09	2.95	0.32	0.28	18.5	18.61
7	Christchurch_ New Zealand	2011	LSRC	6.2	Reverse	561.04	74.72	3.67	0.14	0.12	12.14	14.53

codes, engineers can systematically choose ground motion records encompassing a range of intensities, frequencies, and spectral shapes, helping to assess the potential earthquake effects on structures accurately. The code-based selection process involves considering factors such as the site’s seismic hazard, the building’s structural characteristics, and the desired performance objectives to identify suitable ground motion inputs for structural analysis [61].

In this study, seven records with two horizontal components (NS and EW) have been selected and scaled in the time domain according to the target spectrum of the Italian seismic code [62] calculated for the limit state *Safeguard of human life*. This limit state corresponds to the return period of 475 years, consistent with the 10 % probability of exceedance in a nominal life of 50 years. The selection process criterion is the mean of the square root of the sum of squares (SRSS) of the Pseudo-Spectral Acceleration (pSa) for two horizontal components of the selected records within the period range of 0.5 s to a maximum of 2.0 s and $1.5 \times T_1$, where T_1 is the fundamental period of the structure, should be between 90 % and 130 % of the target spectrum. Additionally, the mean spectra for the selected motions at $T = 0$ s must not be less than the target spectral amplitude at the site of interest.

The seismic selection criteria are determined based on the characteristics of the target structure. Depending on the region, earthquakes with magnitudes ranging from moment magnitude (M_w) of 5.5 to 7.5, featuring normal and reverse faulting mechanisms, are considered, specifically at stations located within an epicentral distance of 50 to 150 km. The soil class for the temple is assumed to be type B, thus restricting the use of ground motions recorded on soil Type B [63,64], with the average shear wave velocity within the upper 30 m of the soil profile (V_{s30}) values ranging from 360 to 800 m/s, in the selection process. The scaling procedure is performed using the PEER ground motion selection platform, which minimises the mean square error, with a maximum scaling factor capped at 5.0.

Table 3 presents comprehensive data about unscaled ground motion parameters, encompassing event name, the year of occurrence, the station name, M_w , fault mechanism, V_{s30} , and Joyner and Boore distance (R_{JB}). Table 3 also lists statistical insights into ground motion parameters, specifically focusing on peak ground acceleration (PGA) and peak ground velocity (PGV) derived from the scaled records with their respective scale factors. Notably, PGA exhibits a range of 0.10 to 0.32 g, while PGV spans from 9.85 to 18.61 cm/s. Record number 6 stands out as the strongest when compared to the other records, boasting a maximum PGA of 0.32 g and a PGV of 18.61 cm/s. The chosen records exhibit varying durations and frequency content when compared to one another.

Nonlinear dynamic analysis

This section discusses the results of the nonlinear dynamic analyses. Seven analyses have been performed, and the results are shown in terms of response measures. Specifically, the displacement of control points and the maximum base shear have been

analysed. Fig. 7(a) represents the displacement time histories obtained for the considered ground motion records. Displacements of control points located on top of the entablature and cella have been monitored. All the simulations end up with a local failure mechanism involving columns and entablature, whereas the cella is unlikely to be damaged, and the control point presents a negligible displacement compared to those located on the top of the entablature. The maximum base shear for each direction is reported in Fig. 7(b). The results are normalised for the gravity loads. One should note how the results are consistent except for Record 5, which presents a base shear magnitude (Y direction) four times greater than the overall trend. This can be attributed to Record 5 higher spectral amplitudes around the fundamental period of the structure.

Conclusion

This work presents a workflow implemented in a Generative Programming (GP) environment that drives the Visual Programming Language (VPL) parametric modelling of architectural heritage using existing data (i.e. historical documentation, drawings, surveys, etc.). In order to be applied, this methodology implies that symmetries, repetition of modules and architectural orders characterise the construction. Such features enable the creation of parametric models using minimal metric information, effectively overcoming limitations related to cost, speed, and accessibility while ensuring reliability even in the absence of digital surveying.

The capabilities of the implemented framework have been tested and validated by applying it to model the Vesta Temple in Tivoli, a Roman temple in a state of ruin. Subsequently, the effectiveness of the obtained 3D model has been assessed in a different work environment by performing a seismic analysis of the existing structure using finite element (FE) structural software. Seven records, each with two horizontal components, have been selected and scaled according to the target spectrum outlined in the Italian seismic code for the limit state safeguarding human life. The reusability of the workflow has been further demonstrated by modelling the same structure in its original configuration, based on hypotheses by Francesco Piranesi, highlighting significant advantages in terms of code reuse capability and execution speed.

The following conclusions are derived from the implementation and testing of the presented framework:

- The use of historical data is proven effective for the acquisition of dimensional and geometrical information aimed at the 3D modelling of specific typologies of architectural heritage buildings.
- The GP paradigm is an efficient method for creating geometrical models of structures featuring symmetries, repeated modules, and similar characteristics.
- The suggested tool demonstrates flexibility and the ability to swiftly replicate the geometrical definition of multiple structures of the same typology in nearly no time.

- The proposed framework enables the efficient creation of 3D models, effectively addressing the well-known challenges associated with meshing heritage buildings obtained through reality-based techniques. It ensures straightforward and rapid meshing operations in other work environments, as demonstrated by the application of the 3D model in structural software.
- The non-linear dynamic analyses conducted highlight that the primary source of failure for the investigated structure is the lack of connection between the cella and columns, particularly under varying ground motion conditions.

Future developments will test the implemented algorithm's reusability by modelling other monopteros/tholos temples and adapting it to different building typologies.

Acknowledgments

This work was partly financed by FCT / MCTES through national funds (PIDDAC) under the R&D Unit Institute for Sustainability and Innovation in Structural Engineering (ISISE), under reference UIDB / 04029/2020 (doi.org/10.54499/UIDB/04029/2020), and under the Associate Laboratory Advanced Production and Intelligent Systems ARISE under reference LA/P/0112/2020. This study was partly funded by the STAND4HERITAGE project that received funding from the European Research Council (ERC) under the European Union's Horizon 2020 research and innovation program (Grant agreement No. 833123), as an Advanced Grant. This work was also partly financed by Fundação para a Ciência e a Tecnologia PhD Grants, under reference PRT/BD/152822/2021.

References

- [1] M. Mohammadi, M. Rashidi, V. Mousavi, A. Karami, Y. Yu, B. Samali, Case study on accuracy comparison of digital twins developed for a heritage bridge via UAV photogrammetry and terrestrial laser scanning, In 10th International Conference on Structural Health Monitoring of Intelligent Infrastructure, SHMII, 10, 2021, pp. 1713–1720.
- [2] M. Mohammadi, M. Rashidi, V. Mousavi, A. Karami, Y. Yu, B. Samali, Quality evaluation of digital twins generated based on uav photogrammetry and tls: bridge case study, Remote Sens. (Basel) 13 (17) (2021), doi:10.3390/RS13173499.
- [3] J. Moyano, I. Gil-Arizón, J.E. Nieto-Julián, D. Marín-García, Analysis and management of structural deformations through parametric models and HBIM workflow in architectural heritage, J. Build. Eng. 45 (2022), doi:10.1016/j.jobe.2021.103274.
- [4] A. Scalas, M. Mortara, M. Spagnuolo, A pipeline for the preparation of artefacts that provides annotations persistence, J. Cult. Herit. 41 (2020) 113–124, doi:10.1016/j.culher.2019.06.006.
- [5] M. Grieves, J. Vickers, Digital Twin: Mitigating Unpredictable, Undesirable Emergent Behavior in Complex Systems, Springer, Cham, 2016 no. August 2017, doi:10.1007/978-3-319-38756-7.
- [6] A. Vuoto, M.F. Funari, P.B. Lourenço, Shaping digital twin concept for built cultural heritage conservation: a systematic literature review, Int. J. Archit. Heritage 18 (11) (2024) 1762–1795, doi:10.1080/15583058.2023.2258084.
- [7] B.G. Pantoja-Rosero, S. Saloustros, R. Achanta, K. Beyer, Image-based geometric digital twinning for stone masonry elements, Autom. Constr. 145 (2023) 104632 November 2022, doi:10.1016/j.autcon.2022.104632.
- [8] X. Yang, P. Grussenmeyer, M. Koehl, H. Macher, A. Murtiyo, T. Landes, Review of built heritage modelling: integration of HBIM and other information techniques, J. Cult. Herit. 46 (2020) 350–360, doi:10.1016/j.culher.2020.05.008.
- [9] G. Castellazzi, A. Maria D'altri, G. Bitelli, I. Selvaggi, A. Lambertini, From laser scanning to finite element analysis of complex buildings by using a semi-automatic procedure, Sensors 15 (2015) 18360–18380, doi:10.3390/s150818360.
- [10] G. Bitelli, G. Castellazzi, A.M. D'altri, S. De Miranda, A. Lambertini, I. Selvaggi, Automated voxel model from point clouds for structural analysis of cultural heritage, Int. Arch. Photogramm., Remote Sens. Spatial Inf. Sci. - ISPRS Arch. 41 (July) (2016) 191–197, doi:10.5194/isprsarchives-XLI-B5-191-2016.
- [11] A. Marra, S. Gerbino, A. Greco, G. Fabbrocino, Combining integrated informative system and historical digital twin for maintenance and preservation of artistic assets, Sensors 21 (17) (2021), doi:10.3390/S21175956.
- [12] G. Patrucco, et al., 3D models and non-destructive investigations: towards a meeting in digital twins, Int. Arch. Photogramm., Remote Sens. Spatial Inf. Sci. - ISPRS Arch. 43 (B2–2022) (2022) 845–852, doi:10.5194/isprs-archives-XLI-B2-2022-845-2022.
- [13] G. Sammartano, G. Patrucco, S. Perri, R. Ceravolo, E. Lenticchia, A. Spanò, Documenting complexity for the 20th century heritage: the enriched 3D models of the turin exposition Nervi's halls digitization, ISPRS Ann. Photogramm., Remote Sens. Spatial Inf. Sci. 8 (M–1–2021) (2021) 141–148, doi:10.5194/isprs-annals-VIII-M-1-2021-141-2021.
- [14] C. Chevrier, J.P. Laser, and R. Data, "Laser range data, photographs and architectural components Christine Chevrier, Jean-Pierre Perrin To cite this version : HAL Id : halshs-00280953," 2008.
- [15] B.G. Pantoja-Rosero, R. Achanta, K. Beyer, Damage-augmented digital twins towards the automated inspection of buildings, Autom. Constr. 150 (2023) 104842, doi:10.1016/j.autcon.2023.104842.
- [16] B.G. Pantoja-Rosero, K.R.M. dos Santos, R. Achanta, A. Rezaie, K. Beyer, Determining crack kinematics from imaged crack patterns, Constr. Build. Mater. 343 (2022), doi:10.1016/j.conbuildmat.2022.128054.
- [17] B.G. Pantoja-Rosero, et al., TOPO-Loss for continuity-preserving crack detection using deep learning, Constr. Build. Mater. 344 (2022) 128264, doi:10.1016/j.conbuildmat.2022.128264.
- [18] D. Galanakis, et al., SVD-based point cloud 3D stone by stone segmentation for cultural heritage structural analysis – The case of the Apollo Temple at Delphi, J. Cult. Herit. 61 (2023) 177–187, doi:10.1016/j.culher.2023.04.005.
- [19] B.G. Pantoja-Rosero, R. Achanta, K. Beyer, Automated image-based generation of finite element models for masonry buildings, Bull. Earthq. Eng. (0123456789) (2023), doi:10.1007/s10518-023-01726-7.
- [20] F. Diara, F. Rinaudo, From reality to parametric models of cultural heritage assets for HBIM, Int. Arch. Photogramm., Remote Sens. Spatial Inf. Sci. - ISPRS Arch. 42 (2/W15) (2019) 413–419, doi:10.5194/isprs-archives-XLII-2-W15-413-2019.
- [21] S.M. Haakonsen, A. Rønquist, N. Labonnote, Fifty years of shape grammars: a systematic mapping of its application in engineering and architecture, Int. J. Archit. Comput. 21 (1) (2023) 5–22, doi:10.1177/14780771221089882.
- [22] S. Zhang, M. Hofmann, K. Beyer, A 2D typology generator for historical masonry elements, Constr. Build. Mater. 184 (2018) 440–453, doi:10.1016/j.conbuildmat.2018.06.085.
- [23] M. Shaqfa, K. Beyer, A virtual microstructure generator for 3D stone masonry walls, Eur. J. Mech., A/Solids 96 (2022) 104656, doi:10.1016/j.euromechsol.2022.104656.
- [24] S. Szabó, M.F. Funari, P.B. Lourenço, A mason-inspired pattern generator for historic masonry structures using quality indexes, Eng. Struct. 304 (2024) 117604, doi:10.1016/j.engstruct.2024.117604.
- [25] M.F. Funari, A.E. Hajjat, M.G. Masciotta, D.V. Oliveira, P.B. Lourenço, A parametric scan-to-FEM framework for the digital twin generation of historic masonry structures, Sustainability (Switzerland) 13 (19) (2021), doi:10.3390/su131911088.
- [26] G. Angeliu, D. Coronelli, and G. Cardani, "Development of the simulation model for Digital Twin applications in historical masonry buildings: the integration between numerical and experimental reality," 2020, doi: 10.1016/j.compstruc.2020.106282.
- [27] M.G. Bevilacqua, M. Russo, A. Giordano, R. Spallone, 3D reconstruction, digital twinning, and virtual reality: architectural heritage applications, in: Proceedings - 2022 IEEE Conference on Virtual Reality and 3D User Interfaces Abstracts and Workshops, VRW 2022, 2022, pp. 92–96, doi:10.1109/VRW55335.2022.00031.
- [28] F. Guzzetti, K.L.N. Anyabolu, F. Biolo, R. Dell'orto, BIM and castello sforzesco in milan. a particular approach to digitisation of the architectural and information heritage, Int. Arch. Photogramm., Remote Sens. Spatial Inf. Sci. - ISPRS Arch. 46 (5/W1–2022) (2022) 115–122, doi:10.5194/isprs-archives-XLVI-5-W1-2022-115-2022.
- [29] A. Marra, I. Trizio, and G. Fabbrocino, "Digital tools for the knowledge and safeguard of historical heritage," Lect. Notes Civil Eng., vol. 156, pp. 645–662, 2021, doi: 10.1007/978-3-030-74258-4_41.
- [30] A. Barontini, C. Alarcon, H.S. Sousa, D.V. Oliveira, M.G. Masciotta, M. Azenha, Development and demonstration of an HBIM framework for the preventive conservation of cultural heritage, Int. J. Archit. Heritage 16 (10) (2022) 1451–1473, doi:10.1080/15583058.2021.1894502.
- [31] M. Calvano, L. Martinelli, F. Calcerano, E. Gigliarelli, Parametric processes for the implementation of HBIM-visual programming language for the digitisation of the index of masonry quality, Geo-Information (2022), doi:10.3390/ijgi11020093.
- [32] R. Spallone, M. Calvano, Parametric experiments on palladio's 5 by 3 villas, Nexus Netw. J. 24 (2) (2022) 287–313, doi:10.1007/s00004-022-00592-1.
- [33] M.F. Funari, S. Spadea, P. Lonetti, F. Fabbrocino, R. Luciano, Visual programming for structural assessment of out-of-plane mechanisms in historic masonry structures, J. Build. Eng. 31 (2020) 101425, doi:10.1016/j.jobe.2020.
- [34] S.C. Chase, Generative design tools for novice designers: issues for selection, Autom. Constr. 14 (6) (2005) 689–698, doi:10.1016/j.autcon.2004.12.004.
- [35] G. Germano, Ancient metrology in architecture: a new approach in the study of the Roman bridge of Canosa di Puglia (Italy), Acta IMEKO 11 (1) (2022) 1–7, doi:10.21014/acta_imeko.v11i1.1092.
- [36] C. Rossi, F. Fiorillo, A metrological study of the late Roman fort of umm al-Dabadib, Kharga Oasis (Egypt), Nexus. Netw. J. 20 (2) (2018) 373–391, doi:10.1007/s00004-018-0388-6.
- [37] F. Fiorillo, C. Rossi, 3D survey and metric analysis of the Late Roman Fort of Umm al-Dabadib (Egypt), Acta IMEKO 7 (3) (2018) 57–63, doi:10.21014/acta_imeko.v7i3.590.

- [38] F. Fiorillo, C. Rossi, 3D parametric modelling based on point cloud for the interpretation of the archaeological remains, *Disegnarecon* 14 (26) (2021) 21–211, doi:[10.20365/disegnarecon.26.2021.2](https://doi.org/10.20365/disegnarecon.26.2021.2).
- [39] A. Brzozowska-Jawornicka, A. Kubicka-Sowińska, In search of the module in the architectural design of the 'Hellenistic' House in Nea Paphos, Cyprus, *Etudes et Travaux* (34) (2021) 167–181, doi:[10.12775/ETUDTRAV.34.006](https://doi.org/10.12775/ETUDTRAV.34.006).
- [40] M.F. Funari, L.C. Silva, N. Savalle, P.B. Lourenço, A concurrent micro/macro FE-model optimised with a limit analysis tool for the assessment of dry-joint masonry structures, *Int. J. Multiscale Comput. Eng.* 20 (5) (2022) 65–85, doi:[10.1615/IntJMultCompEng.2021040212](https://doi.org/10.1615/IntJMultCompEng.2021040212).
- [41] S. Milena, M. Ognen, Application of generative algorithms in architectural design, in: *International Conference on Mathematical and Computational Methods in Science and Engineering - Proceedings, 2010*, pp. 175–180.
- [42] S.C. Chase, Generative design tools for novice designers: issues for selection, *Autom. Constr.* 14 (6) (2005) 689–698, doi:[10.1016/j.AUTCON.2004.12.004](https://doi.org/10.1016/j.AUTCON.2004.12.004).
- [43] M. Richardson, "John Soane and the Temple of Vesta at Tivoli," vol. 46, no. 2003, pp. 127–146, 2014.
- [44] S. Serlio, *I sette libri dell'Architettura*. 1537.
- [45] A. Palladio, *I quattro libri dell'Architettura*. 1570.
- [46] M. Richardson, "John Soane and the Temple of Vesta at Tivoli," vol. 46, no. 2003, pp. 127–146, 2014.
- [47] A. Desgodetz, *Les edifices antiques de Rome*. 1682.
- [48] F. Piranesi, *Tempi Antichi*, Vol. I. 1780.
- [49] F.A. Sebastiani, *Viaggio a Tivoli. Antichissima città latino-sabina*. 1825.
- [50] A. Uggeri, *Giornate pittoriche degli edifici antichi di Roma e de' suoi contorni*. 1800.
- [51] G. Valadier, V. Feoli, and F.A. Visconti, *Raccolta delle piu insigni fabbriche di Roma antica ... /dichiarate dall'Architetto GiuseppeGiuseppe Valadier ; ill. da Filippo Aurelio Visconti ; incise da Vincenzo Feoli*. Roma.
- [52] A. Brzozowska-Jawornicka and A. Kubicka-Sowińska, "In search of the module in the architectural design of the 'Hellenistic' House in Nea Paphos, Cyprus," *Etudes et Travaux*, no. 34, pp. 167–181, 2021, doi: [10.12775/ETUDTRAV.34.006](https://doi.org/10.12775/ETUDTRAV.34.006).
- [53] F.A. Sebastiani, *Viaggio a Tivoli. Antichissima città latino-sabina*. 1825.
- [54] "Rhino—Rhinoceros 3D. 2020." Accessed: jun. 11, 2022. [Online]. Available: <https://www.rhino3d.com>.
- [55] "Grasshopper—Algorithmic Modeling for Rhino. 2020." Accessed: jun. 11, 2022. [Online]. Available: <https://www.grasshopper3d.com>.
- [56] A. Vuoto, M.F. Funari, P.B. Lourenço, Generative Modelling of Monopteros and Tholos Temples: Vesta Temple in Tivoli, 2023, doi:[10.5281/zenodo.13923776](https://doi.org/10.5281/zenodo.13923776).
- [57] G. Fortunato, M.F. Funari, P. Lonetti, Survey and seismic vulnerability assessment of the Baptistery of San Giovanni in Tumba (Italy), *J. Cult. Herit.* 26 (2017), doi:[10.1016/j.culher.2017.01.010](https://doi.org/10.1016/j.culher.2017.01.010).
- [58] J. Lubliner, J. Oliver, S. Oller, E. Oñate, A plastic-damage model for concrete, *Int. J. Solids Struct.* 25 (3) (1989) 299–326, doi:[10.1016/0020-7683\(89\)90050-4](https://doi.org/10.1016/0020-7683(89)90050-4).
- [59] E. Erdogmus, B. Pulatsu, A. Gaggioli, M. Hoff, Reverse engineering a fully collapsed ancient roman temple through geoarchaeology and DEM, *Int. J. Archit. Heritage* 15 (12) (2021) 1795–1815.
- [60] V. Abaqus, 6.14 Documentation, Dassault Syst. Simul. Corp. 651 (6.2) (2014).
- [61] I. Iervolino, G. Manfredi, A review of ground motion record selection strategies for dynamic structural analysis, *Mod. Test. Tech. Struct. Syst.* (2008) 131–163.
- [62] N. Mordà, A. Mancini, Norme tecniche per le costruzioni (NTC 2018) D. Min. Infrastrutture e Trasporti 17 gennaio 2018, *Legislazione Tecnica* (2018).
- [63] G. Forte, E. Chioccarelli, M. De Falco, P. Cito, A. Santo, I. Iervolino, Seismic soil classification of Italy based on surface geology and shear-wave velocity measurements, *Soil Dyn. Earthq. Eng.* 122 (2019) 79–93.
- [64] F. Bozzoni, A. Furiosi, C.G. Lai, Probabilistic assessment of the earthquake-induced soil liquefaction hazard at national scale: macrozonation of the Italian territory, *Nat. Hazards* 115 (3) (2023) 2237–2255.