

Asymmetric Impedance Model for Grid-Forming Converters With Droop Control

Diego Pérez-Estévez, *Member, IEEE*, Diego Ríos-Castro, *Member, IEEE*, Pablo Marino Fernández-Abraldes, *Student Member, IEEE*, Vitor Monteiro, *Senior Member, IEEE*, J. G. Pinto, *Member, IEEE*, João L. Afonso, *Senior Member, IEEE*, and Jesús Doval-Gandoy, *Member, IEEE*

Abstract—This article introduces an impedance model for a grid-forming (GFM) converter with droop control (Droop-GFM converter) connected to a weak power grid. The proposed model offers a more accurate and simpler alternative to the conventional minor-loop criterion and the Generalized Nyquist Stability Criterion for studying the stability of a Droop-GFM converter, as it accounts for loading effects and uses only two transfer functions. Formulated in the $\alpha\beta$ frame, the model consists of only two balanced impedances, offering a clearer physical interpretation of the impedance asymmetry introduced by droop control, in contrast to dq -frame impedance matrix models. The model describes frequency coupling effects (the mirror frequency problem) across the entire frequency range, providing both a geometric and physical understanding of this phenomenon. It allows determination of the bandwidth of the frequency coupling region, and how this region changes based on the droop coefficients and the loading condition of the Droop-GFM converter. The theoretical findings are validated experimentally using a Droop-GFM converter in three distinct scenarios, including: weak power grid operation, a change in the droop coefficients, and a change in the power grid impedance.

Index Terms—Asymmetric Impedance Model, Droop Control, Output Impedance, Grid-Forming Converters, Stability.

NOMENCLATURE

General conventions

$\mathbf{G}(z), \mathbf{K}(z), \mathbf{Z}(z)$ Discrete-time transfer matrices.

$\mathbf{v}$ Column vector representation of the complex signal v in the dq frame.

$\mathbf{v}^T$ Transpose of $\mathbf{v}$.

$\tilde{v}$ Small-signal perturbation of v .

$G(z), K(z), Z(z)$ Discrete-time transfer functions.

Manuscript received xxxxx; revised xxxxx; accepted xxxxx. Date of current version xxxxx. This work was supported in part by the Government of Galicia under the grants ED481B-2023-019 and GPC-ED431B 2023/12, in part by the Ministry of Education and Universities under the grant FPU22/02102, and in part by the Spanish State Research Agency (AEI) under project PID2022-136908OB100/MCIN/AEI/10.13039/501100011033/FEDER-UE. (Corresponding author: Jesus Doval-Gandoy.)

D. Pérez-Estévez, Diego Ríos-Castro, Pablo Marino Fernández-Abraldes and J. Doval-Gandoy are with the Applied Power Electronics Technology Group (APET), CINTECX, University of Vigo, Vigo 36310, Spain (e-mail: dieperez@uvigo.es; diego.rios@uvigo.gal; pablmario.fernandez@uvigo.gal; jdoval@uvigo.es).

Vitor Monteiro, J. G. Pinto, and João L. Afonso are with the Centro Algoritmi, University of Minho, Guimarães, Portugal, (e-mail: vmonteiro@dei.uminho.pt; gpinto@dei.uminho.pt; jla@dei.uminho.pt).

Digital Object Identifier xxxxx

r^* Reference signal r .
 V, I Scalar signals.
 v, i Complex signals in the $\alpha\beta$ reference frame.
 $v^\dagger$ Conjugate of v .

Large-signal complex vectors

i_o Grid-forming (GFM) converter output current.
 v_C^* Undisturbed droop control reference voltage.
 v_C GFM converter output voltage.
 $v_C^{\text{droop}*}$ Droop control reference voltage perturbed by $\tilde{i}_{\text{dist}}$.
 $v_C^{\text{lin}*} = v_C^* + \tilde{v}_C^*$ Proposed linear model of $v_C^{\text{droop}*}$.
 v_g Power grid voltage.

Large-signal scalar variables

$\omega_g = 2\pi f_g$ Rated angular frequency of v_g .
 θ_g Phase of v_g .
 P Active power delivered by the GFM converter.
 Q Reactive power delivered by the GFM converter.
 V_0 Rated magnitude of v_g .

Small-signal complex vectors

$\tilde{i}_{\text{dist}}$ Current disturbance (small-signal perturbation) injected at the point of common coupling (PCC).
 $\tilde{i}_o$ Perturbation of i_o caused by $\tilde{i}_{\text{dist}}$.
 $\tilde{i}_o^{\parallel}$ Component of $\tilde{i}_o$ parallel to v_C .
 $\tilde{i}_o^{\perp}$ Component of $\tilde{i}_o$ perpendicular to v_C .
 $\tilde{i}_o^{\text{droopless}}$ Component of $\tilde{i}_o$ caused by $\tilde{i}_{\text{dist}}$ when droop control is inoperative.
 $\tilde{v}_C$ Perturbation of v_C caused by $\tilde{i}_{\text{dist}}$.
 $\tilde{v}_C^*$ Proposed linear model of droop control reference voltage perturbation caused by $\tilde{i}_{\text{dist}}$.
 $\tilde{v}_C^{*\parallel}$ Component of $\tilde{v}_C^*$ parallel to v_C .
 $\tilde{v}_C^{*\perp}$ Component of $\tilde{v}_C^*$ perpendicular to v_C .
 $\tilde{v}_C^{\text{droopless}}$ Component of $\tilde{v}_C$ caused by $\tilde{i}_{\text{dist}}$ when droop control is inoperative.
 $\tilde{v}_C^{\text{droop}}$ Component of $\tilde{v}_C$ caused by the response of the droop control to $\tilde{i}_{\text{dist}}$.

Small-signal scalar variables

$\tilde{\omega}$	Angular frequency perturbation of v_C^* caused by $\tilde{i}_{\text{dist}}$.
$\tilde{\theta}$	Phase perturbation of v_C^* caused by $\tilde{i}_{\text{dist}}$.
$\tilde{P}$	Active power perturbation caused by $\tilde{i}_{\text{dist}}$.
$\tilde{Q}$	Reactive power perturbation caused by $\tilde{i}_{\text{dist}}$.
$\tilde{V}_{\text{mag}}$	Magnitude perturbation of v_C^* caused by $\tilde{i}_{\text{dist}}$.

Impedances and other transfer functions

$G_{\text{cd}}(z)$	Current division ratio of $\tilde{i}_{\text{dist}}$ towards the converter.
$G_{\text{rt}}(z)$	Voltage controller reference tracking transfer function.
$K_{\text{droop}}(z)$ and $\mathbf{K}_{\text{droop}}(z)$	Droop equivalent gain (symmetrical and asymmetrical model, respectively).
$Z_{\text{PCC}}^{\text{droopless}}(z)$	Impedance of the PCC bus when the droop control is disabled.
$Z_{\text{PCC}}^{\text{droop}}(z)$	Impedance component of the PCC bus added by the droop control.
$Z_{\text{out}}^{\text{vc}}(z)$	Output impedance of a GFM converter with a voltage controller (without droop control).
$Z_{\text{critical}}(z)$	Critical impedance according to the impedance ratio criterion.
$Z_{\text{PCC}}(z)$	Impedance of the PCC bus.
$Z_{\text{PCC}}^+(z)$, $Z_{\text{PCC}}^-(z)$	Symmetrical components of Z_{PCC} .
$Z_{\text{PCC}}^{\text{droop}+}(z)$, $Z_{\text{PCC}}^{\text{droop}-}(z)$	Symmetrical components of $Z_{\text{PCC}}^{\text{droop}}$.
$Z_{\text{th}}(z) = 1/Y_{\text{th}}$	Thevenin equivalent impedance seen by the converter from its output terminals towards the load/grid.

I. INTRODUCTION

DISTRIBUTED energy resources are increasing their role in today's electrical power grid. This motivates the creation and update of requirements for the interconnection of these resources to the electric power grid [1]. It has been recognized that the characterization of converter-based resources in terms of their harmonic distortion is a generally inaccurate concept with today's technology [2]. Distributed energy resources (DERs) that meet current distortion limits when connected to an undistorted external system, such as during type testing, may exceed current distortion limits when measured in a field environment having voltage distortion. Many times, such current distortion flows in the presence of background voltage distortion at a level within the capabilities of the DER resource. However, it is possible for the DER to be in resonance with a system harmonic impedance, in which case the measured current distortion would be significantly beyond the acceptable limit [2] and in need of mitigation. Due to the previous reason, it has become increasingly important the characterization of converter-based resources in terms of their output impedance, their contribution to the impedance of the power grid, or the point of common coupling (PCC) bus.

Impedance modeling of power converters is a complex topic because it involves the physical parameters of the power converter, as well as the algorithms that control the converter. The

complexity of a power converter impedance model increases as the number of sensed variables (voltages and currents) fed back to the converter control algorithms increases. The first works used impedance models to study the stability of dc systems, as presented in [3]. DC-DC converters that operate as a constant power load often behave as a negative impedance (if the voltage rises at the DC-DC converter input, the input current decreases). Such negative converter equivalent impedance can resonate with the power grid impedance and cause stability and waveform quality problems. To study these stability problems, the impedance ratio criterion, also known as minor loop gain, was originally proposed in [3].

In addition to studying the stability of dc power converters [4], the impedance ratio criterion has also been applied to study the stability of three-phase DC-AC converters. First, it was applied to constant power loads [5]. However, the large number of control strategies and modes of operation developed for ac power converters throughout the last three decades has been continuously generating a need to develop new impedance models that account for the influence of these control algorithms, including grid-forming (GFM) converters, both in stand-alone and grid-connected modes [6, 7]. To study resonance and stability problems that arise due to interaction between power converters and other grid elements, such as capacitor banks and constant power loads, or in less critical scenarios waveform quality issues, an accurate impedance model of the ac power converters has to be defined. It has been shown that the load a converter sees at its output changes its output impedance [8] and its stability [9, 10]. This is particularly relevant in the case of grid-connected converters, where changes in the power grid impedance or the parallel aggregation of converter-based resources [11, 12], can significantly modify the impedance seen by the converter and cause performance or even stability problems.

In grid-connected scenarios, synchronization algorithms such as a phase-locked loop (PLL), in the case of grid following converters, or a droop control, in the case of GFM converters, are often not included in the impedance models developed for the converter due to their nonlinear characteristic [8, 13]. However, the influence of these synchronization algorithms on the impedance model increases under weak power grid conditions, specially in the low-frequency range [14]. Their nonlinear operation, among other effects, introduces a frequency coupling (also named mirror frequency problem) [15, 16] that cannot be modeled by a single balanced impedance.

To model this frequency coupling, different asymmetrical impedance models have been proposed in the literature. Two model structures are often used: transfer function matrices or complex transfer functions [17]. Transfer function matrices are applied in sequence domain models [18–26], harmonic state-space models [27], and multi-harmonic linearization models [28–30]. Transfer matrix models require more parameters compared to complex transfer function models (also named space vector models) [17, 31], which only use two transfer functions to describe an asymmetric impedance. Additionally, transfer matrix models often rely on the Generalized Nyquist Stability Criterion (GNSC) to assess stability, making them

more complex and computationally demanding than simply testing the stability of two transfer functions. Asymmetrical impedance models can also be classified based on whether they are defined using setup and control algorithm parameters, i.e., parametric models [19–21, 24–29, 31], or by current and voltage relations, i.e., non parametric models [17, 18, 22, 30].

Compared to existing impedance models for GFM converters with droop control [24–26], the presented asymmetric impedance model (AIM) is applied to a scenario where the GFM converter is connected to the grid. Contrary to previous solutions, the proposed AIM is applied and experimentally tested when the GFM converter sees a low impedance from its output. Such low impedance results in a large loading effect experienced by the GFM converter. As a result, the impedance ratio criterion (minor loop), which does not account for the loading effect, does not provide an accurate stability assessment. For this reason, the solutions presented in [24–26], which also neglect this effect, do not yield accurate results under certain loading conditions. The solution in [24] addresses the frequency stability of a single GFM converter in an islanded ac microgrid. The impedance seen by the GFM converter in the experimental results is 8.6 p.u. and the frequency response measurement of the converter output impedance is carried out without any loads connected at the output. The solutions in [25, 26] provide an impedance-based stability analysis of a microgrid supported by two grid-forming converters. The GFM converters use an LCL filter, where the grid-side inductors are considered as part of the line impedance between the GFM converter and the load. In order to derive the impedance model, the loading effects are not considered. The models in [25, 26] are used for stability analysis, but they are not experimentally verified using frequency response measurements. It should be noted that the droop control in a GFM converter performs several nonlinear operations; therefore, no existing method allows for the development of a single impedance model (i.e. a linear model) that can accurately describe the behavior of a GFM converter in stand-alone and grid-connected modes, simultaneously. A summary of the models discussed is listed in Table I.

This article presents an AIM for a grid-forming (GFM) converter with droop control (Droop-GFM converter) connected to a weak power grid, where the source of asymmetry is the droop control. The model characterizes the impedance of an ac bus supported by a Droop-GFM converter and connected to other elements, such as loads or a weak power grid. The model is presented in the $\alpha\beta$ frame and is composed of two balanced impedances. It has the following advantages:

- 1) It permits assessing the stability of a GFM converter with droop control for different loading conditions. Moreover, the proposed impedance model can be used to analyze the stability attending to passivity criteria. There are many stability studies in the literature that are based on analyzing passivity characteristics of power converters, without including the droop control [32]. The proposed model can aid to extend passivity studies to GFM converters with droop control.
- 2) It permits calculating how the impedance of the PCC

changes depending on the parametrization of the droop control (value of droop coefficients selected).

- 3) It permits determining the bandwidth of the frequency coupling region created by the droop control, the frequency region where mirror frequency disturbances can appear, and how this region changes depending on the droop coefficients and the loading condition of the GFM converter. The proposed model also permits directly assessing the frequency coupling region by evaluating a single transfer function, namely, the negative sequence impedance.

The developed model serves as a more accurate alternative to study the stability of a Droop-GFM converter, compared to the conventional minor-loop criterion, because it is able to model the asymmetry introduced by the droop control and the loading effect of the impedances connected at the converter output. Furthermore, compared to the GNSSC employed by impedance models that use transfer matrix models (cf. Table I), where the trajectory of the eigenvalues of a transfer matrix needs to be determined, the proposed AIM provides a simpler stability assessment that only requires to determine the stability of two transfer functions. It should be noted that, the presented AIM is a small-signal model that relies on a linear model for the droop control and the voltage controller commanding the GFM converter. Transient stability analysis where there is a large disturbance, such as a voltage sag, that may cause the GFM converter to enter into a current limiting mode or to switch to grid-following (GFL) mode are beyond the scope of this article.

The two transfer functions that compose the proposed AIM have a straightforward physical interpretation and they are experimentally measured for validation. The proposed AIM is validated and compared to experimental measurements of the impedance of an ac bus supported by a Droop-GFM converter and connected to a weak power grid. The experimental measurements are performed by another grid-connected converter operating in parallel as a current source. Three different operating conditions are tested, including: weak power grid operation, a change in the droop coefficients, and a change in the power grid impedance. After this Introduction, Section II presents the theoretical derivation of the proposed AIM. This proposal builds upon the impedance model developed for a GFM converter with a voltage control [13], by developing a linear model of a droop control. First, the rotationally invariant case is considered. Then, the droop model is extended to the rotationally variant (dq asymmetric) case, discussing some of the limitations of the model. Next, the presented AIM is used for stability analysis, discussing the minor loop criterion. In Section III, a brief explanation is provided to clarify the relationship between the proposed droop control model and virtual impedances. Section IV introduces experimental verifications and Section V is devoted to conclusions.

II. DROOP-CONTROLLED GFM CONVERTER OUTPUT IMPEDANCE

This section calculates the impedance of an ac bus denoted as PCC, Z_{PCC} , that is supported by a droop-controlled GFM

TABLE I
ASYMMETRIC IMPEDANCE MODELS FOR POWER CONVERTERS IN THE LITERATURE

Model	Ref.	Type	Struc.	SoA	Application	Characteristics	ET	FRD
Sequence domain	[18]	NPM	2TM	Faults	Fault calculations and base values for p.u. calculations.	Establishes relations between phasor, sequence, and dq domains.	×	×
	[19]	PM	2TM	PLL	Grid connected inverter with L filter and PLL.	Studies effect of PLL on impedance model and stability.	✓	×
	[20, 21]	PM	3TM	PLL & DC bus	Grid connected inverter with PLL & Vdc dynamics.	Provides connection with dq and phasor domains.	×	×
	[22]	NPM	2TM	PLL	Wind turbines connected to the power grid.	Provides an impedance measurement technique.	×	×
	[24]	PM	2TM	Droop	DGFM converter in an islanded microgrid.	Mirror freq. and loading effect not included in the model.	✓	✓
	[25]	PM	2TM	Droop	DGFM converter connected to a weak power grid.	Uses GNC to study stability, and loading effect not included in the model.	✓	×
	[26]	PM	2TM	Droop	DGFM converter connected to a weak power grid.	Uses GNC to study stability, and loading effect not included in the model.	✓	×
Modified seq. domain	[23]	NPM	2TM	—	Describes measurement algorithms.	Addresses limitations of prior seq. domain models.	×	×
Harm. state-space	[27]	PM	10TM	PLL & DC bus	Grid connected rectifier with LCL filter and PLL.	Based on a linear time-varying periodically theory.	✓	×
Multi-harm. lineariz.	[28, 29]	PM	5TM	PLL	Grid connected inverter with L filter.	Addresses asymmetry caused by PLL or DC bus volt. control.	×	×
	[30]	NPM	∞ TM	PLL	Grid connected inverter with L filter and PLL.	Black box model.	×	×
Complex space vectors	[17]	NPM	CTF	—	Seminal work that describes models for asymmetric systems.	Provides connection with transfer matrix models.	×	×
	[31]	PM	CTF	PLL	Grid connected inverter with L filter and PLL.	Studies effect of PLL on impedance model and stability.	✓	×
	Proposal	PM	CTF	Droop	DGFM converter connected to a weak power grid.	Accounts for loading effect and links AIM to traditional definition of balanced impedances.	✓	✓

¹ Model type: Parametric model (PM) is derived from physical parameters. Non-parametric model (NPM) is derived from voltage-current relations.

² Model structure: $n \times n$ transfer function matrix (n TM). Complex transfer function (CTF). ³ Source of asymmetry (SoA).

⁴ Experimentally tested (ET) ⁵ Experimentally tested comparing impedance model to measured frequency response data (FRD).

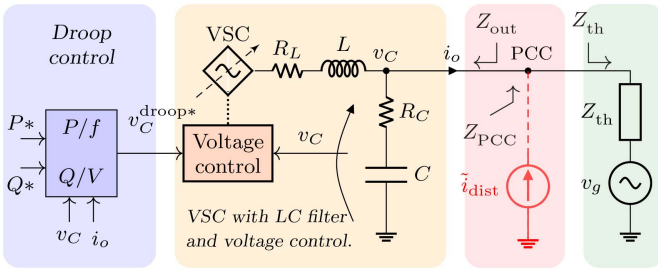


Fig. 1. Model of a Droop-GFM converter connected to a weak power grid with an equivalent impedance Z_{th} . A current source injects a current disturbance $\tilde{i}_{dist}$ at the PCC.

converter and connected to a power grid, as illustrated in Fig. 1. The GFM converter is composed of a voltage source converter (VSC) with an output LC filter, an ac-voltage controller, and a P - f Q - V droop control. The impedance seen by the GFM converter from its output affects its output impedance. In order to account for this loading effect in the model developed, the GFM converter sees at its output an equivalent impedance Z_{th} , which lumps together the impedance of any loads and external elements connected to the converter, including the weak power grid impedance. This parameter permits characterizing different loading conditions, as a result of changes in the power grid impedance or the loads connected to the PCC.

To measure the PCC bus impedance Z_{PCC} , a perturbation

current, $\tilde{i}_{dist}$, is introduced. This perturbation signal is injected at the PCC as shown in Fig. 1, or in [13, Fig. 3] for cases without droop control. The injected current disturbance $\tilde{i}_{dist}$ enables the measurement of the PCC bus impedance. This current disturbance can be created by connecting a high impedance load or another converter with a current control to the PCC bus [33–35].

The following summarizes the modeling steps carried out by the presented method to obtain the PCC impedance in Fig. 1:

- 1) First, the impedance of the PCC bus when the converter does not have a droop control, $Z_{PCC}^{droopless}$, is calculated.
- 2) Then, a linear model for the droop control is elaborated.
- 3) Last, the impedance change, Z_{PCC}^{droop} , that the PCC bus experiences when the droop control is added to the GFM converter is calculated. The impedance of the PCC bus under droop-control operation is modeled, as explained in the following, by an additive model:

$$Z_{PCC} = Z_{PCC}^{droopless} + Z_{PCC}^{droop}. \quad (1)$$

An additive model is chosen because the perturbation in the PCC voltage, caused by injecting a current perturbation at the PCC, is determined by the sum of two components: the perturbation that occurs without droop control and the response from the droop control.

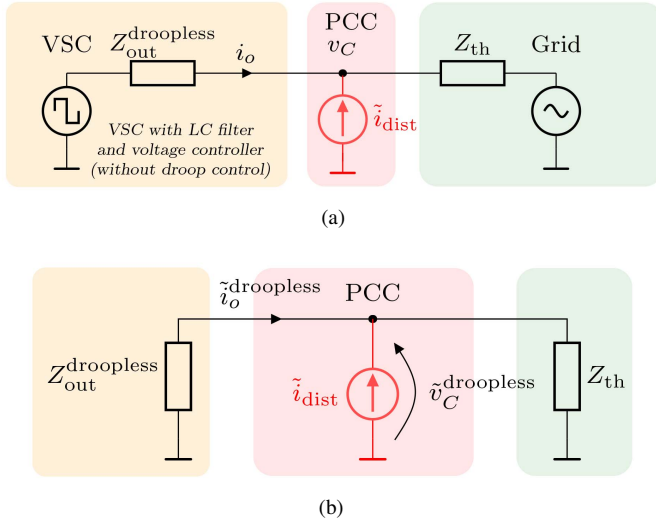


Fig. 2. Simplified model of a voltage controlled GFM converter connected to a power grid with an equivalent impedance Z_{th} . A current disturbance $\tilde{i}_{dist}$ is injected at the PCC. (a) Simplified model with three independent sources: the converter voltage, the power grid voltage, and the PCC disturbance current. (b) Simplified model with only one independent source, the disturbance current.

A. Impedance of an AC Bus Supported by a VSC With an AC-voltage Controller (Without Droop Control)

An impedance model for a voltage-controlled GFM converter was developed in [13]. When the droop control is not present and the GFM converter is commanded by a voltage control, the impedance of the PCC bus shown in Fig. 1 is:

$$Z_{PCC}^{droopless} = \left(\frac{1}{Z_{out}^{droopless}} + \frac{1}{Z_{th}} \right)^{-1}, \quad (2)$$

where $Z_{out}^{droopless}$ is the output impedance of a VSC with an LC filter and an ac-voltage controller [13].

When a current disturbance $\tilde{i}_{dist}$ is injected into the PCC, all state variables in the plant model are perturbed. In particular, the perturbation of the PCC bus voltage v_C and the converter output current i_o are of special importance when studying how the droop control changes the impedance of the PCC bus. The reason is that the droop control feeds back these two variables; hence, it responds to such current and voltage perturbations and, as a result, alters the impedance of the PCC bus compared to a scenario where there is not a droop control loop. In a scenario where the droop controller is removed, the simplified model shown in Fig. 2(a) becomes useful to calculate these two disturbances. Since only one input $\tilde{i}_{dist}$ is relevant to the impedance model, Fig. 2(a) can be further simplified by removing the independent voltage sources, as illustrated in Fig. 2(b). From the previous model, the disturbance current $\tilde{i}_{dist}$ generates the following voltage disturbance in the PCC bus:

$$\tilde{v}_C^{droopless} = \tilde{i}_{dist} Z_{PCC}^{droopless} = \tilde{i}_{dist} \left(\frac{1}{Z_{out}^{droopless}} + \frac{1}{Z_{th}} \right)^{-1}. \quad (3)$$

Similarly, a fraction of the current disturbance $\tilde{i}_{dist}$ flows towards the converter and causes a power disturbance at the converter output, as illustrated in Fig. 2. This converter output

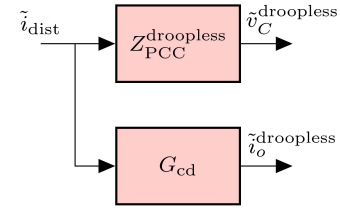


Fig. 3. Perturbation models for the output voltage and output current in a voltage-controlled GFM converter (without a droop control) perturbed by a current disturbance $\tilde{i}_{dist}$ injected at the PCC.

current disturbance is:

$$\tilde{i}_o^{droopless} = \tilde{i}_{dist} G_{cd} = \tilde{i}_{dist} \frac{-Z_{th}}{Z_{out}^{droopless} + Z_{th}}. \quad (4)$$

A negative sign appears in (4) because $\tilde{i}_o^{droopless}$ is positive when the current flows out of the converter, but a positive $\tilde{i}_{dist}$ causes a current that flows into the converter. Fig. 3 displays a block diagram representation of the linear relations between $\tilde{i}_{dist}$ and $\tilde{v}_C^{droopless}$ (3) and between $\tilde{i}_{dist}$ and $\tilde{i}_o^{droopless}$ (4).

B. A Linear Model for a P-f Q-V Droop Control

The droop control performs several nonlinear operations to the previously calculated disturbances ($\tilde{v}_C^{droopless}$ and $\tilde{i}_o^{droopless}$) in order to obtain the power flow disturbance that they cause at the output of the Droop-GFM converter. Then, the droop control changes the reference of the voltage controller according to the established droop characteristic. As a result of this voltage reference change commanded by the droop control, the impedance of the PCC bus also changes. All the nonlinear operations carried out by the droop control can be approximated by a linear model in order to study their effect on the impedance of the PCC bus. Small-signal analysis permits approximating the behavior of a nonlinear power electronics system, such as the droop control of a GFM converter, with a linear time-invariant model that is valid around an operating point of interest. This permits applying linear control theory to study and develop an impedance model for this nonlinear power system. In this section a small-signal analysis is developed to obtain a linear model of a P-f Q-V droop control. Next, the model of the droop control is combined with the linear model of the voltage-controlled converter and with the disturbance propagation model presented in Section II-A. The symbol “ $\sim$ ” is used to denote variables associated to the linear small-signal analysis and distinguish them from large signal variables.

The droop controller shown in Fig. 1 responds to a power disturbance at the output of the GFM converter and changes the voltage reference of the voltage controller v_C^{droop*} . This response of the droop controller is composed of two nonlinear operations: (i) a power calculation operation, represented in Fig. 4(a), and (ii) a reference correction operation, represented in Fig. 4(b). The power calculation operation obtains the active and reactive power according to the instantaneous power theory. When a current disturbance $\tilde{i}_o$ is considered, the power

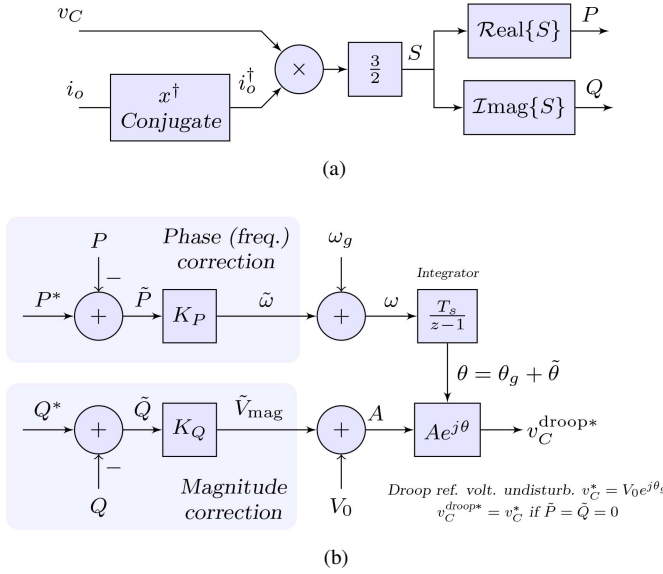


Fig. 4. Nonlinear operations carried out by a droop control. (a) Calculation of active and reactive power. (b) Calculation of the GFM converter voltage reference v_C^{droop*} .

disturbance is

$$\tilde{S} = \tilde{P} + j\tilde{Q} = \frac{3}{2}v_C \tilde{i}_o^\dagger, \quad (5)$$

where $\tilde{i}_o^\dagger$ denotes complex conjugate of the current $\tilde{i}_o$. The amplitude invariant $\alpha\beta$ reference frame is considered for all signals unless otherwise stated.

Second, the reference correction operation of the droop controller changes the voltage controller reference v_C^{droop*} with respect to the open loop (undisturbed) reference $v_C^* = V_0 e^{j\theta_g}$. To study this second operation, it is convenient to decompose the current disturbance $\tilde{i}_o$ into a parallel and a perpendicular component to the output voltage:

$$\tilde{i}_o = \tilde{i}_o^\parallel + \tilde{i}_o^\perp. \quad (6)$$

On the one hand, the component of the current disturbance that is parallel to the output voltage, $\tilde{i}_o^\parallel$, causes an active power disturbance $\tilde{P}$:

$$\tilde{P} = -\frac{3}{2}|v_C| \cdot |\tilde{i}_o^\parallel|. \quad (7)$$

Such active power disturbance generates a frequency disturbance $\tilde{\omega}$:

$$\tilde{\omega} = K_P \tilde{P} = -\frac{3}{2}K_P |v_C| \cdot |\tilde{i}_o^\parallel|. \quad (8)$$

The previous frequency disturbance is integrated over one sample period T_s and results in a phase disturbance $\tilde{\theta}$:

$$\tilde{\theta} = -\frac{3}{2} \frac{T_s}{z-1} K_P |v_C| \cdot |\tilde{i}_o^\parallel|. \quad (9)$$

These operations are illustrated in Fig. 4(b).

On the other hand, a component of the current disturbance that is perpendicular to the output voltage $\tilde{i}_o^\perp$ causes a reactive power disturbance $\tilde{Q}$:

$$\tilde{Q} = -\frac{3}{2}|v_C| \cdot |\tilde{i}_o^\perp|. \quad (10)$$

Such reactive power disturbance causes a disturbance in the magnitude of the droop voltage reference:

$$\tilde{V}_{mag} = K_Q \tilde{Q} = -\frac{3}{2}K_Q |v_C| \cdot |\tilde{i}_o^\perp|. \quad (11)$$

Combining both the phase and magnitude correction operations (9) and (11), the corrected reference voltage calculated by the droop control is obtained:

$$\underbrace{v_C^{droop*}}_{\text{Droop ref. volt. disturbed by } \tilde{i}_{dist}} = A e^{j\theta} = \underbrace{(\tilde{V}_{mag} + V_0)}_{\text{Mag. correct.}} \cdot \underbrace{e^{j\theta_g}}_{\text{Droop ref. volt. undisturbed } v_C^*} \cdot \underbrace{e^{j\tilde{\theta}}}_{\text{Phase correct.}}. \quad (12)$$

This correction operation carried out by the droop control is illustrated in Fig. 4(b).

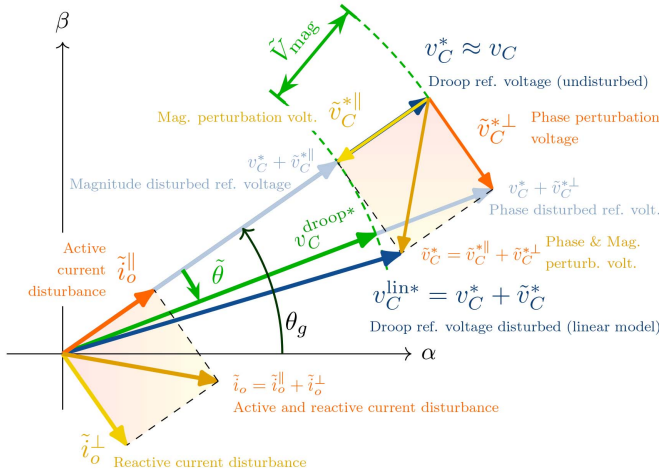
In order to develop an impedance model for the Droop-GFM converter, the nonlinear correction operations performed by the droop control (12) are modeled by a linear approximation. As a result, the disturbed droop reference voltage v_C^{droop*} is linearly approximated by the reference voltage v_C^{lin*} :

$$\underbrace{v_C^{droop*}}_{\text{Droop ref. volt. disturbed by } \tilde{i}_{dist}} \approx \underbrace{v_C^{lin*}}_{\text{Droop ref. volt. disturb. (lin. model)}} = \underbrace{v_C^*}_{\text{Droop ref. volt. undisturbed}} + \underbrace{\tilde{v}_C^*}_{\text{Mag. \& phase perturbation volt.}}, \quad (13)$$

where v_C^* is the large-signal steady-state reference voltage that serves as the operating point for the small-signal analysis ($v_C^* = V_0 \cdot e^{j\theta_g}$), and $\tilde{v}_C^*$ is a voltage that causes a similar magnitude and phase correction as the droop control.

The correction voltage $\tilde{v}_C^*$ is composed of two components: $\tilde{v}_C^* = \tilde{v}_C^{\parallel*} + \tilde{v}_C^{\perp*}$. The parallel component mainly changes the magnitude and the perpendicular component mainly changes the phase of the reference voltage v_C^* , as illustrated in Fig. 5. For illustration purposes, Fig. 5 highly magnifies the droop response, by showing a large correction vector $\tilde{v}_C^*$ compared to the output voltage v_C . For such large vector $\tilde{v}_C^*$, the magnitude correction component $\tilde{v}_C^{\parallel*}$ introduces also a small phase change, and the phase correction component $\tilde{v}_C^{\perp*}$ introduces also a small magnitude change (cosine error). For this reason, the magnitude of v_C^{lin*} is slightly different than the magnitude corrected voltage vector ($v_C^* + \tilde{v}_C^{\parallel*}$), and the phase of v_C^{lin*} is also slightly different than the phase corrected voltage vector ($v_C^* + \tilde{v}_C^{\perp*}$), cf. Fig. 5.

To obtain a mathematical expression for the parallel and perpendicular components of $\tilde{v}_C^*$, it is convenient to refer to Fig. 5. This figure shows that a current disturbance $\tilde{i}_o$ must result in a voltage reference disturbance $\tilde{v}_C^*$ that causes a magnitude and phase correction of the reference voltage equivalent to the droop control. The droop control magnitude correction is $\tilde{V}_{mag}$ and the phase correction is $\tilde{\theta}$. To achieve a similar magnitude and phase correction of the reference voltage, the proposed linear model calculates some suitable magnitude and phase correction vectors $\tilde{v}_C^{\parallel*}$ and $\tilde{v}_C^{\perp*}$, respectively. Their expressions are derived from the droop control equations (9) and (11) and the space vector relations depicted in Fig. 5. The phase



Linear model

$$\tilde{i}_o^{\parallel} \Rightarrow \tilde{v}_C^{\perp} \Rightarrow v_C^*, \text{ decelerates } (\omega \text{ decreases})$$

$$\tilde{i}_o^{\perp} \Rightarrow \tilde{v}_C^{\parallel} \Rightarrow |v_C^*| \text{ decreases}$$

Droop equations

$$\tilde{i}_o^{\parallel} > 0 \Rightarrow P > P^* \Rightarrow \tilde{P} < 0 \Rightarrow \tilde{\theta} < 0 \quad (\omega \text{ decreases})$$

$$\tilde{i}_o^{\perp} > 0 \Rightarrow Q > Q^* \Rightarrow \tilde{Q} < 0 \Rightarrow \tilde{V}_{mag} < 0 \quad (|v_C^*| \text{ decreases})$$

Fig. 5. Undisturbed droop reference, v_C^* , disturbed droop reference voltage, v_C^{droop*} , and proposed linear model of the disturbed droop reference voltage v_C^{lin*} for an output current disturbance $\tilde{i}_o$.

correction vector is:

$$\begin{aligned} \tilde{v}_C^{\perp} &= -j v_C^* \sin(\tilde{\theta}) \approx -j v_C^* \tilde{\theta} \\ &\approx -j \underbrace{\frac{3}{2} \frac{T_s}{z-1} K_P |v_C^*|^2}_{K_{droop}^P} \tilde{i}_o^{\parallel}, = -j K_{droop}^P \tilde{i}_o^{\parallel}, \end{aligned} \quad (14)$$

and the magnitude correction vector is:

$$\tilde{v}_C^{\parallel} = -j \underbrace{\frac{3}{2} K_Q |v_C^*|}_{K_{droop}^Q} \tilde{i}_o^{\perp} = -j K_{droop}^Q \tilde{i}_o^{\perp}, \quad (15)$$

where multiplication by the complex value $-j$ provides the 90° phase lag. The resultant expression for the correction vector is

$$\tilde{v}_C^* = \tilde{v}_C^{\perp} + \tilde{v}_C^{\parallel} = -j K_{droop}^P \tilde{i}_o^{\parallel} - j K_{droop}^Q \tilde{i}_o^{\perp}. \quad (16)$$

If the following relation $-j K_{droop}^P = -j K_{droop}^Q = -j K_{droop}$ is met, then the droop is rotationally invariant and it can be described by the following linear operation, which resembles the expression of a virtual impedance of value $j K_{droop}$:

$$\tilde{v}_C^* = -j K_{droop} \tilde{i}_o. \quad (17)$$

A rotationally invariant, or time invariant, droop controller responds in the same manner to a current disturbance irrespective of the phase of the current disturbance with respect to the output voltage. On the other hand, if the droop controller is rotationally variant ($K_{droop}^P \neq K_{droop}^Q$), which is often the case, then the output impedance of the GFM converter depends on

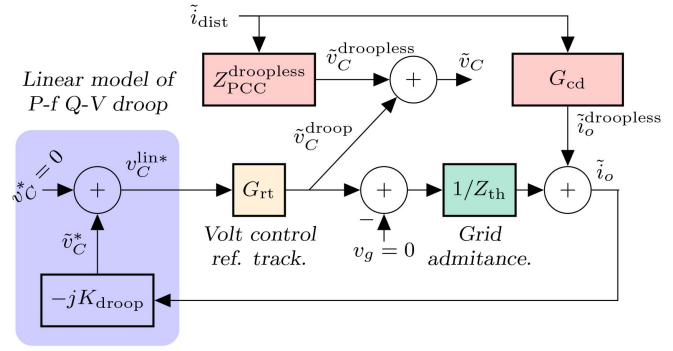


Fig. 6. Small-signal model ($v_g = 0$ and $v_C^* = 0$) of a rotationally invariant ($K_{droop}^P = K_{droop}^Q = K_{droop}$) droop-controlled GFM converter that sees an impedance Z_{th} at its output.

the phase of the current disturbance applied.

The previous equations (14)-(17) show how the droop responds to a current disturbance. However, the droop control operates in a closed-loop manner. The droop response to a current disturbance generates a voltage reference disturbance $\tilde{v}_C^*$ that is propagated to the GFM converter output following the voltage controller reference tracking response, which is often a low-pass response [36, Fig. 2]. The AIM presented in this article can be applied to any VSC with a linear voltage controller; therefore, in order to not limit the presentation to a specific voltage controller, the reference-tracking response of the voltage controller is denoted as $G_{rt}(z)$. A complete model of this feedback relationship in a Droop-GFM converter is constructed by combining the model that describes the response of a voltage-controlled GFM converter to a current disturbance $\tilde{i}_{dist}$ (Fig. 3) with the droop model (13), as shown in Fig. 6.

C. Impedance of the PCC when the GFM Converter is Commanded by a Rotationally Invariant Droop Control

A rotationally invariant droop control denotes a droop control that can be modeled by a transfer function, i.e., a linear time-invariant system. In a droop-controlled GFM converter, rotational invariance is analogous to the absence of saliency in a synchronous generator. As explained in the previous section, physically, this means that the output impedance experienced by a current disturbance remains the same, regardless of the disturbance's phase relative to the output or terminal voltage.

To calculate the impedance of the PCC bus Z_{PCC} when the droop controller is operating, the linear model shown in Fig. 6 becomes useful. The PCC bus impedance is the transfer function that relates the current disturbance $\tilde{i}_{dist}$ to the capacitor voltage disturbance $\tilde{v}_C$, in Fig. 6, assuming that the other two model inputs, namely, the voltage reference v_C^* and the power grid voltage v_g are zero:

$$Z_{PCC}(z) = \left. \frac{\tilde{v}_C}{\tilde{i}_{dist}} \right|_{v_C^*=v_g=0} = \underbrace{\frac{\tilde{v}_C^{droopless}}{\tilde{i}_{dist}}}_{Z_{PCC}^{droopless}} + \underbrace{\frac{\tilde{v}_C^{droop}}{\tilde{i}_{dist}}}_{Z_{PCC}^{droop}}. \quad (18)$$

Equation (18) shows that the impedance at the PCC bus is the vector addition of the impedance without droop $Z_{PCC}^{droopless}$

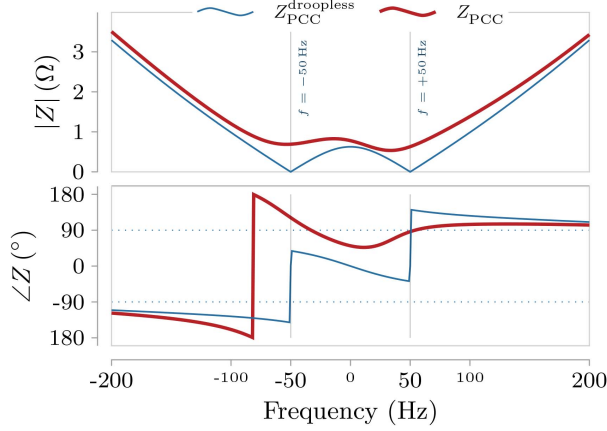


Fig. 7. Impedance of a PCC bus supported by GFM converter without a droop controller, $Z_{PCC}^{\text{droopless}}$, and with a rotationally invariant droop controller, Z_{PCC} . The droop gain, K_{droop} , is set to a value of 0.65 for illustration.

plus an impedance term Z_{PCC}^{droop} . This coincides with the model introduced in (1).

Fig. 7 compares the impedance of a PCC bus supported by a GFM converter with and without droop control for the LC filter parameters listed in Table III. A rotationally invariant droop gain, K_{droop} , with a value of 0.65 is selected. In both cases, a voltage controller with resonant action at the fundamental frequency (50 Hz) is used. When operating with only the voltage controller, the PCC impedance, $Z_{PCC}^{\text{droopless}}$, is zero for both the positive and negative sequences of the fundamental frequency due to the controller's resonant action. This occurs because, with only the voltage controller (without droop control), the steady-state output voltage at 50 Hz is unaffected by the output current. However, when droop control is enabled, the PCC impedance, Z_{PCC} , increases at the fundamental frequency. The reason is that the droop control causes the converter output voltage to depend on the output current, which is related to the output power flow, resulting in a nonzero impedance.

As shown by the previous equation, the droop control changes the impedance of the PCC bus by adding an additional impedance term Z_{PCC}^{droop} . This additional impedance term can be expressed in terms of the droop and setup parameters as:

$$Z_{PCC}^{\text{droop}}(z) = \frac{\tilde{v}_C^{\text{droop}}}{\tilde{i}_{\text{dist}}} = \frac{K_{\text{droop}} Z_{\text{th}} G_{\text{cd}} G_{\text{rt}}}{Z_{\text{th}} - K_{\text{droop}} G_{\text{rt}}} = \frac{K_{\text{droop}} G_{\text{cd}} G_{\text{rt}}}{1 - K_{\text{droop}} G_{\text{rt}} Y_{\text{th}}}. \quad (19)$$

The impedance term Z_{PCC}^{droop} depends on the droop gain, K_{droop} , the GFM voltage controller reference-tracking transfer function, G_{rt} , and the impedance G_{cd} and admittance Y_{th} seen from the PCC when the GFM converter operates only with a voltage controller. A block diagram representation of only the impedance term associated to the droop Z_{PCC}^{droop} is illustrated in Fig. 8(a).

When the voltage controller of the GFM converter includes a resonant action at the fundamental output frequency f_g [37], the expression for (18) and (19) at such frequency is greatly simplified. The resonant action of the voltage controller ensures that the impedance $Z_{PCC}^{\text{droopless}}$ is zero [13] and the transfer functions G_{rt} and G_{cd} are one and minus one, respectively, when evaluated at the frequency f_g . Therefore, (18) and (19)

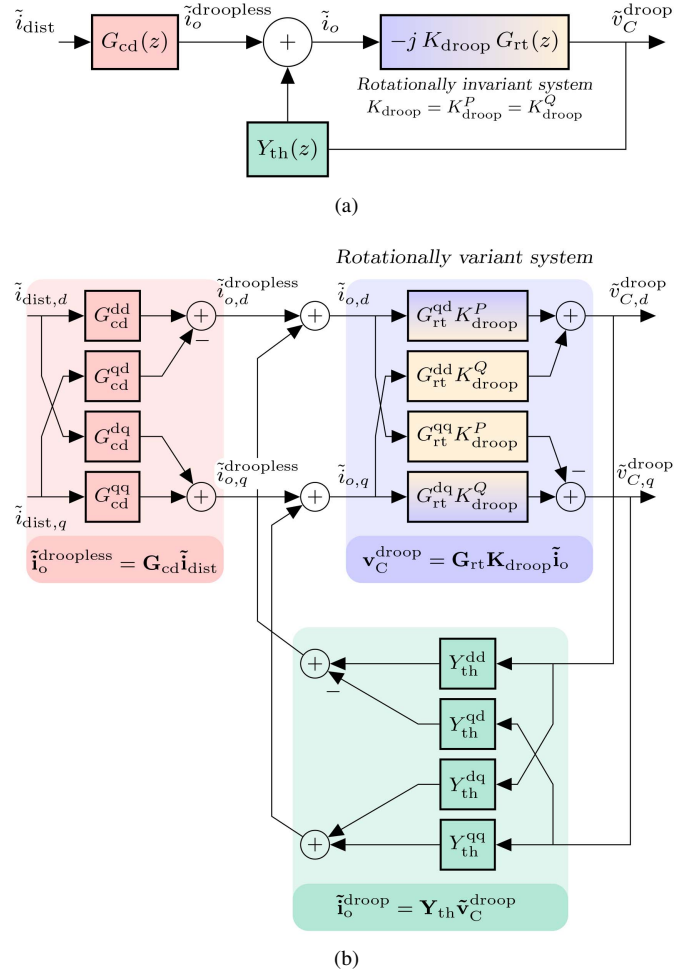


Fig. 8. Effect of droop control on the capacitor voltage, $\tilde{v}_C^{\text{droop}}$, in response to an output current disturbance, $\tilde{i}_{\text{dist}}$. (a) A rotationally invariant droop control. (b) A rotationally variant droop control.

become:

$$Z_{PCC}(f_g) = \frac{\tilde{v}_C}{\tilde{i}_{\text{dist}}} = Z_{PCC}^{\text{droop}}(f_g) = \frac{-K_{\text{droop}}}{1 - K_{\text{droop}} Y_{\text{th}}}. \quad (20)$$

As a result, using a voltage controller with zero steady-state error at the fundamental frequency (with a resonant action), provides a more predictable and simple expression for the PCC bus impedance under different loading conditions.

D. Impedance of the PCC when the GFM Converter is Commanded by a Rotationally Variant Droop Control

If the droop control is rotationally variant ($K_{\text{droop}}^P \neq K_{\text{droop}}^Q$), which is often the case, then the droop controller response cannot be accurately modeled using a single-input single-output (SISO) transfer function representation. The droop response is influenced by the phase difference between the output voltage and the output current disturbance. To effectively model this asymmetry, the most suitable reference frame is a rotating (dq) frame synchronized with the output voltage frequency. In this reference frame, the phase asymmetry introduced by the droop control can be represented as a static difference between the d and q axes, since there is a consistent (constant) phase

difference between the output voltage and the dq axes. To construct such two-input two-output (TITO) dq system, first matrix notation is adopted. Boldface denotes a matrix or a vector element, and the operator T denotes transpose.

Equation (17), when converted to a transfer function matrix model in the dq reference frame, becomes:

$$\tilde{\mathbf{v}}_{\mathbf{C}}^* = \mathbf{K}_{\text{droop}} \tilde{\mathbf{i}}_{\mathbf{o}}, \quad (21)$$

where $\tilde{\mathbf{v}}_{\mathbf{C}}^* = [\tilde{v}_{C,d}^* \ \tilde{v}_{C,q}^*]^T$, $\tilde{\mathbf{i}}_{\mathbf{o}} = [\tilde{i}_{o,d} \ \tilde{i}_{o,q}]^T$. The d and q components in (21) are approximately equivalent to the parallel and perpendicular components in (14) and (15). It should be noted that while the dq reference frame rotates at a constant frequency, the output voltage space vector can experience frequency fluctuations due to the droop control; therefore, components parallel (or perpendicular) to output voltage space vector are not necessarily perfectly aligned with the d axis (or q axis). The droop gain $\mathbf{K}_{\text{droop}}$ is a non-symmetric 90° rotation matrix, which is the non-symmetric version of the multiplication by $-j$ in (14) and (15):

$$\mathbf{K}_{\text{droop}} = \begin{bmatrix} 0 & K_{\text{droop}}^Q \\ -K_{\text{droop}}^P & 0 \end{bmatrix}, \quad (22)$$

where K_{droop}^P and K_{droop}^Q are defined according to (14) and (15), respectively.

The $\alpha\beta$ -frame transfer function models $G_{\text{rt}}(z)$, $G_{\text{cd}}(z)$, and $Y_{\text{th}}(z)$ become the dq -frame transfer function matrices $\mathbf{G}_{\text{rt}}(z)$, $\mathbf{G}_{\text{cd}}(z)$, and $\mathbf{Y}_{\text{th}}(z)$. This transformation is carried out in two steps. First, the $\alpha\beta$ -frame transfer function models are converted to the dq frame using the frequency shifting property of the z -transform [17]:

$$G_{\text{dq}}(z) = G_{\alpha\beta}(z \cdot e^{j\omega_g T_s}). \quad (23)$$

Second, the dq transfer functions are converted to transfer matrix notation [17]. The elements of the transfer function matrices are obtained from the dq -frame transfer functions using the relations given in the Appendix.

As a result of this model conversion, the block diagram shown in Fig. 8(a) becomes the block diagram shown in Fig. 8(b). From the model shown in Fig. 8(b), the impedance contribution of the droop control is:

$$\tilde{\mathbf{v}}_{\mathbf{C}}^{\text{droop}} = \underbrace{[(\mathbf{I} - \mathbf{G}_{\text{rt}} \mathbf{K}_{\text{droop}} \mathbf{Y}_{\text{th}})^{-1} \mathbf{G}_{\text{rt}} \mathbf{K}_{\text{droop}} \mathbf{G}_{\text{cd}}]}_{\mathbf{Z}_{\text{PCC}}^{\text{droop}}} \tilde{\mathbf{i}}_{\text{dist}}. \quad (24)$$

The previous equation is the rotationally variant counterpart to (19). Although the droop gain $\mathbf{K}_{\text{droop}}$ is the only rotationally variant element, the complete impedance model $\mathbf{Z}_{\text{PCC}}^{\text{droop}}$, which relates $\tilde{\mathbf{i}}_{\text{dist}}$ to $\tilde{\mathbf{v}}_{\mathbf{C}}^{\text{droop}}$, becomes also rotationally variant due to the connection of a rotationally variant droop block to the remaining elements.

E. Physical Interpretation of a Rotationally Variant Output Impedance

A rotationally variant system in the dq reference frame (dq asymmetric) has a different impedance in the d axis than in the q axis. Fig. 9 illustrates the voltages $\tilde{\mathbf{v}}_{C,k}^{\text{droop}}$ ($k = 1, 2, 3, 4$) generated when four unit-magnitude currents $\tilde{\mathbf{i}}_{\text{dist},k}$ ($k = 1, 2, 3, 4$),

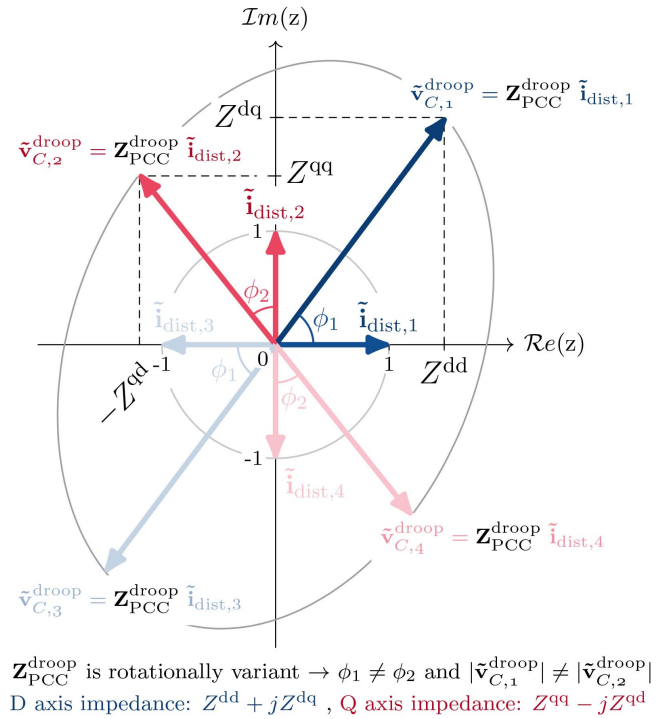


Fig. 9. Four voltages $\tilde{\mathbf{v}}_{C,k}^{\text{droop}}$ ($k = 1, 2, 3, 4$) generated when four unit-magnitude current disturbances $\tilde{\mathbf{i}}_{\text{dist},k}$ ($k = 1, 2, 3, 4$) flow through a dq asymmetric impedance $\mathbf{Z}_{\text{PCC}}^{\text{droop}}$. The four current disturbances represented have phase values 0, $\pi/2$, π , and $3\pi/2$. Each voltage current pair is displayed using a different color. Figure is not represented to scale for illustration purposes, actual impedance is often more asymmetric.

respectively, flow through a dq -asymmetric impedance, such as $\mathbf{Z}_{\text{PCC}}^{\text{droop}}$. For illustration purpose, the impedance represented has a smaller value than the impedance values provided by (24) for the setup parameters used in Section IV, which would result in a more elliptical trajectory and a greater asymmetry. Due to the asymmetry, this impedance cannot be defined in terms of a single value of magnitude and phase. Instead, four different impedance components Z^{dd} , Z^{dq} , Z^{qd} , and Z^{qq} become necessary. These impedance components relate the d and q components of a current that flows through an asymmetric impedance with the d and q components of the voltage generated:

$$\underbrace{\begin{bmatrix} \tilde{v}_{C,k,d}^{\text{droop}} \\ \tilde{v}_{C,k,q}^{\text{droop}} \end{bmatrix}}_{\tilde{\mathbf{v}}_{C,k}^{\text{droop}}} = \underbrace{\begin{bmatrix} Z^{\text{dd}} & -Z^{\text{qd}} \\ Z^{\text{dq}} & Z^{\text{qq}} \end{bmatrix}}_{\mathbf{Z}_{\text{PCC}}^{\text{droop}}} \underbrace{\begin{bmatrix} \tilde{i}_{\text{dist},k,d} \\ \tilde{i}_{\text{dist},k,q} \end{bmatrix}}_{\tilde{\mathbf{i}}_{\text{dist},k}} \quad (25)$$

The impedance seen by a current disturbance aligned with the d axis $\tilde{\mathbf{i}}_{\text{dist},1}$ (so called d-axis impedance) is $Z^{\text{dd}} + jZ^{\text{dq}}$ and it has a phase ϕ_1 , which is the phase difference between the current space vector and the voltage space vector that produces when it flows through the asymmetric impedance, cf. (24), whereas the impedance seen by a current disturbance aligned with the q axis $\tilde{\mathbf{i}}_{\text{dist},2}$ (q-axis impedance) is $Z^{\text{qq}} - jZ^{\text{qd}}$ and it has a phase ϕ_2 . Fig. 9 illustrates the value of a dq asymmetric impedance for a single frequency. To represent the dq asymmetric impedance for a wide frequency range,

different representations are possible. One possibility is to display the four impedance components that comprise the impedance matrix model using four bode plots. Fig. 10 shows the four bode plots of $\mathbf{Z}_{\text{PCC}}^{\text{droop}}$ in (24) for the setup parameters indicated in Table III. However, this representation does not provide a clear characterization of the impedance asymmetry. For example, it does not directly indicate the magnitude of the mirror frequency component nor how it compares to the non mirror component. The amplitude of the non mirror components is a weighted sum of the four impedances that compose the impedance matrix Z^{dd} , Z^{qq} , Z^{dq} , and Z^{qd} . Similarly, the amplitude of the mirror frequency component is a different weighted sum of these four impedance components. For this reason, this representation does not provide a simple connection with the traditional concept of impedance, which usually defines the magnitude and phase relation between a current component and a voltage component of the same frequency (non-mirror) or the opposite frequency (mirror frequency). In the following section, an alternative representation that leverages the theory developed for asymmetrical impedances in [17] is presented. This new model consists of only two transfer functions (instead of four). One impedance component is associated only to the non mirror component and the other only to the mirror component, hence it addresses the previous limitations of the transfer matrix model.

The impedance created by the droop control should not be confused with an asymmetric impedance in the stationary reference frame, such as the one that arises when three-phase branch impedances are not equal. Two main differences appear between a dq -asymmetric system and an $\alpha\beta$ - or abc -asymmetric system.

- 1) In a dq -asymmetric system, fundamental frequency current disturbances (dc current components in the dq frame) see different impedances depending on their phase, i.e., the GFM converter behaves as a time variant system. In contrast, in a stationary reference frame asymmetric system, a fundamental frequency current disturbance would generate both positive- and negative-sequence fundamental frequency voltage disturbances.
- 2) Harmonic and interharmonic current disturbances (frequencies other than dc in the dq frame), which follow circular trajectories in the dq plane, result in elliptical voltage trajectories (as a result of the unbalanced impedance). Such elliptical trajectories can be described as the superposition of two circular trajectories rotating in opposite directions, using a transformation analogous to the Fortescue transformation for asymmetric systems in the stationary reference frame [17]. This is often referred in the literature as the frequency coupling problem or mirror frequency problem, because a disturbance at a given frequency causes perturbations at said frequency and at an opposite (mirror) frequency in the dq reference frame. For example, a negative-sequence fifth-harmonic current disturbance in the $\alpha\beta$ becomes a negative-sequence sixth-harmonic current disturbance in the dq , which is the reference frame where the asymmetry exists. As a result, the flow of this disturbance

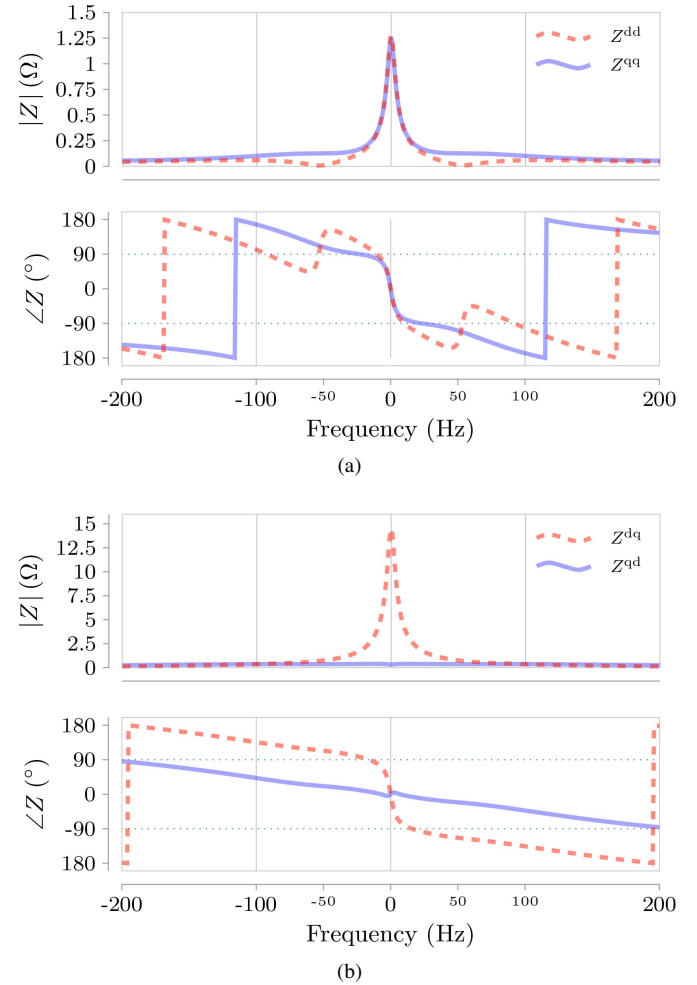


Fig. 10. Components of a dq -asymmetric impedance matrix $\mathbf{Z}_{\text{PCC}}^{\text{droop}}$. (a) Z^{dd} and Z^{qq} . (b) Z^{dq} and Z^{qd} .

current through the asymmetric impedance generates both a negative- and a positive-sequence (dq mirror frequency) sixth-harmonic voltage disturbances in the dq frame. When converted to the $\alpha\beta$ frame this two voltage disturbances become a negative-sequence fifth-harmonic voltage disturbance and a positive-sequence seventh-harmonic (dq mirror frequency).

F. DQ-Reference Frame Alignment for Impedance Model Validation

In order to experimentally validate the presented impedance model, the PCC impedance is measured by injecting a current disturbance into the PCC and measuring the resultant change in the PCC voltage. The implementation details are given in Section IV. Both magnitude and phase changes are measured. In order to measure phase changes in the PCC voltage, a rotating reference frame with a stable (constant) frequency is established. Such reference frame is synchronized to a voltage reference that does not accelerate or decelerate when the current disturbance is injected, i.e., an infinite ac bus. In the experimental results presented in this article, the dq frame is aligned with the power grid voltage, which is considered

TABLE II
FUNDAMENTAL FREQUENCY PCC IMPEDANCES

<i>Impedance seen by a d-axis current disturbance (d-axis impedance)</i>				
Model ^a	Load ^b	$ \tilde{i}_{\text{dist}} $	$\text{Re}\{Z\} + j\text{Im}\{Z\}$ (%)	$ Z (\%) \angle Z(^{\circ})$
Theory			$4.7 + j59.2$	$59.2 \angle 85$
Simul.	CC	0.1 %	$4.7 + j59.6$	$59.9 \angle 85$
Simul.	CC	1 %	$4.4 + j59.2$	$59.6 \angle 86$
Simul.	CC	10 %	$3.2 + j56.7$	$56.8 \angle 87$
Simul.	CC	20 %	$1.6 + j54.2$	$54.3 \angle 83$
Simul.	R	0.1 %	$4.7 + j59.6$	$59.9 \angle 85$
Simul.	R	1 %	$4.1 + j61.1$	$61.2 \angle 86$
Simul.	R	10 %	$3.8 + j62.4$	$62.5 \angle 86$
Simul.	R	20 %	$1.9 + j66.8$	$66.8 \angle 83$
<i>Impedance seen by a q-axis current disturbance (q-axis impedance)</i>				
Model ^a	Load	$ \tilde{i}_{\text{dist}} $	$\text{Re}\{Z\} + j\text{Im}\{Z\}$ (%)	$ Z (\%) \angle Z(^{\circ})$
Theory			$4.9 + j2.0$	$5.3 \angle 23$
Simul.	CC	0.1 %	$5.3 + j2.0$	$5.6 \angle 21$
Simul.	CC	1 %	$5.3 + j2.0$	$5.7 \angle 20$
Simul.	CC	10 %	$5.5 + j2.0$	$5.8 \angle 20$
Simul.	CC	20 %	$5.7 + j1.9$	$6.0 \angle 19$
Simul.	L	10 %	$4.7 + j2.0$	$5.1 \angle 24$
Simul.	L	20 %	$4.6 + j2.0$	$5.0 \angle 24$

^a Model used to obtain the PCC bus impedance: “Theory” denotes the proposed model (Linearized droop control), “Simul.” denotes a simulation model with a P - f Q - V droop control, as illustrated in Fig. 1.

^b Type of load used to generate the disturbance current $\tilde{i}_{\text{dist}}$: R (resistive), L (inductive), CC (constant current source).

to be stable against the current disturbances generated to measure the impedance of the PCC. It should be noted that access to a stable frequency reference is only required to experimentally validate the impedance model, and it is not required to parametrize or use the presented impedance model. In many scenarios, the power grid voltage cannot be measured because it is a remote voltage, the voltage behind the Thevenin equivalent impedance. If an experimental measurement needs to be done in such scenario, then the PCC voltage can be filtered using a PLL to obtain a stable frequency reference.

G. Limitations due to Linearization of the P - f Q - V Droop control

The linear droop control model presented in Section II-B assumes that the output voltage magnitude $|v_C|$ is constant in (14) and (15). This approximation is valid provided that the capacitor voltage perturbation $\tilde{v}_C$ associated to the current perturbation $\tilde{i}_{\text{dist}}$ is much smaller than the large signal value of the capacitor voltage: $|\tilde{v}_C| \ll |v_C| \approx V_0$. This is often the case, even for large current perturbations. Table II shows the impedance value provided by the impedance model for the fundamental frequency and the setup parameters listed in Table III. The impedance at the fundamental frequency is an asymmetric impedance, for this reason the impedance in the d axis is different from the impedance in the q axis. The theoretical value (denoted as “Theory”) is obtained from the proposed AIM evaluated at the fundamental frequency. Since the AIM is a linear model (linear time variant in the

case of a rotationally variant droop), the impedance values obtained do not depend on the amplitude of the perturbation current. Table II also shows the measured impedance value using a simulation in Matlab/Simulink of the Droop-GFM converter and different perturbation currents $\tilde{i}_{\text{dist}}$. The values obtained from the simulation model are denoted as “Simul.”. The perturbation currents in the simulation are generated by connecting to the PCC three types of loads: a resistive load, an inductive load, and an ideal current source with controllable power factor. On the one hand, the resistive load generates a current disturbance that is aligned with the PCC voltage; therefore, it is used to measure the impedance in the d axis. On the other hand, the inductive load generates a current disturbance that is in quadrature with the PCC voltage; therefore, it is used to measure the impedance in the q axis. Due to the asymmetry, the measured impedance value depends on the phase of the perturbation current with respect to the PCC voltage, for this reason the impedance value measured when using the resistive load is different from the value obtained with the inductive load. The impedance in the d axis has a large reactive value, while the impedance in the q axis has a smaller value and it is mainly resistive. Other current disturbances that have a different phase would perceive an impedance that is a combination of the previous d and q impedance values. The ideal current source can generate a current disturbance in the d or q axis by selecting a suitable power factor value. As expected from the nonlinear characteristics of the droop control, the measured impedance slightly changes when the amplitude of the perturbation current varies, and they approach the theoretical value as the perturbation current diminishes towards the ideal small-signal perturbation.

As previously indicated, the proposed AIM assumes that the d and q components of the current disturbance $\tilde{i}_o$ are parallel and perpendicular to the output voltage, respectively. However, the parallel and perpendicular components are aligned with the voltage at the PCC, which is a voltage that can experience frequency changes, while the dq axis components are aligned with a stable frequency reference, as explained in Section II-F. The error introduced by this approximation becomes larger as the power grid impedance increases and the PCC voltage experiences greater frequency fluctuations. However, compared to stand-alone operation, in grid-connected operation the PCC bus experiences small frequency fluctuations. The values shown in Table II correspond to a scenario where the Droop-GFM converter is connected to a weak power grid [$R_{\text{th}} = 10 \Omega$ (31 %), $L_{\text{th}} = 10 \text{ mH}$ (10 %)], where this effect is more significant. To simplify comparison across different voltage levels and component ratings the values are provided in per unit (p.u.) or per cent (%).

The previous characteristic explains why the q axis impedance measured by a current source that injects a current with an amplitude of 10 % in the q axis is slightly different from the impedance measured by a 10-p.u. inductive load that also causes a current flow of 10 %. Both loads cause the same initial load disturbance, but the droop control changes the voltage at the PCC in response to the initial current disturbance, and contrarily to the constant current source, the inductive load also changes the current disturbance generated

$$\underbrace{\tilde{\mathbf{v}}_C^{\text{droop}}}_{\text{Elliptical trajectory}} = \underbrace{\begin{bmatrix} Z_{dd} & -Z_{dq} \\ Z_{dq} & Z_{qq} \end{bmatrix}}_{\mathbf{Z}_{\text{PCC}}^{\text{droop}} \text{ (dq asymmetric)}} \underbrace{\begin{bmatrix} \tilde{i}_{\text{dist},d} \\ \tilde{i}_{\text{dist},q} \end{bmatrix}}_{\tilde{\mathbf{i}}_{\text{dist}} \text{ (Clockwise circular trajectory)}} = \underbrace{\begin{bmatrix} \frac{Z_{dd}+Z_{qq}}{2} & \frac{Z_{dq}+Z_{qd}}{2} \\ \frac{Z_{dq}+Z_{qd}}{2} & \frac{Z_{dd}+Z_{qq}}{2} \end{bmatrix}}_{\mathbf{Z}_{\text{PCC}}^{\text{droop},+} \text{ (dq symmetric (+))}} \underbrace{\begin{bmatrix} \tilde{i}_{\text{dist},d} \\ \tilde{i}_{\text{dist},q} \end{bmatrix}}_{\tilde{\mathbf{i}}_{\text{dist}} \text{ (Clockwise circular trajectory)}} + \underbrace{\begin{bmatrix} \frac{Z_{dd}-Z_{qq}}{2} & \frac{Z_{dq}-Z_{qd}}{2} \\ \frac{Z_{dq}-Z_{qd}}{2} & \frac{Z_{dd}-Z_{qq}}{2} \end{bmatrix}}_{\mathbf{Z}_{\text{PCC}}^{\text{droop},-} \text{ (dq symmetric (-))}} \underbrace{\begin{bmatrix} \tilde{i}_{\text{dist},d} \\ -\tilde{i}_{\text{dist},q} \end{bmatrix}}_{\tilde{\mathbf{i}}_{\text{dist}}^{\dagger} \text{ (Counter-clockwise circular trajectory)}} \quad (27)$$

as the PCC voltage changes.

H. Positive- and Negative-Sequence Impedances

To obtain a better understanding of how a Droop-GFM converter responds to output current disturbances, the asymmetric impedance $\mathbf{Z}_{\text{PCC}}^{\text{droop}}$ in (24) is decomposed into two symmetric impedances $\mathbf{Z}_{\text{PCC}}^{\text{droop},+}$ and $\mathbf{Z}_{\text{PCC}}^{\text{droop},-}$ using a symmetrical component decompositions [17].

$$\tilde{\mathbf{v}}_C^{\text{droop}} = \mathbf{Z}_{\text{PCC}}^{\text{droop}} \cdot \tilde{\mathbf{i}}_{\text{dist}} = \mathbf{Z}_{\text{PCC}}^{\text{droop},+} \cdot \tilde{\mathbf{i}}_{\text{dist}} + \mathbf{Z}_{\text{PCC}}^{\text{droop},-} \cdot \tilde{\mathbf{i}}_{\text{dist}}^{\dagger} \quad (26)$$

The components of the symmetric matrix impedance as a function of the asymmetric impedance matrix components are shown in (27). Once the decomposition into two symmetric impedance matrices is performed, complex notation can be used instead of matrix notation, which brings back the familiar concepts of magnitude and phase at each frequency. Moreover, complex notation is a more compact representation of the impedance model because it only uses one transfer function to model each symmetric impedance matrix, which otherwise requires four transfer functions per impedance matrix. The positive- and negative-sequence impedances are [17]:

$$Z_{\text{PCC}}^{\text{droop},+} = \frac{Z_{dd} + Z_{qq}}{2} + j \frac{Z_{dq} + Z_{qd}}{2}, \quad (28)$$

$$Z_{\text{PCC}}^{\text{droop},-} = \frac{Z_{dd} - Z_{qq}}{2} + j \frac{Z_{dq} - Z_{qd}}{2}. \quad (29)$$

The physical interpretation of these two impedances is illustrated in Fig. 11: a current disturbance, $\tilde{i}_{\text{dist}}$, of a given frequency generates a voltage disturbance, $\tilde{v}_C^{\text{droop},+}$, of the same frequency (the voltage and current pair rotate in the same direction) that has a magnitude and phase determined by $Z_{\text{PCC}}^{\text{droop},+}$, plus a voltage disturbance, $\tilde{v}_C^{\text{droop},-}$, with the opposite frequency in the dq reference frame (inverse or mirror sequence) that has a magnitude and phase determined by $Z_{\text{PCC}}^{\text{droop},-}$. It should be noted that $\tilde{i}_{\text{dist}}^{\dagger}$ is a current component that does not exist in the physical system. It is a virtual current component that facilitates the calculation of $\tilde{v}_C^{\text{droop},-}$. If the two circular trajectories followed by $\tilde{v}_C^{\text{droop},+}$ and $\tilde{v}_C^{\text{droop},-}$ are added, then an elliptical trajectory as shown in Fig. 9 is obtained. This two-impedance model is used to describe the unbalance that exists in the plant itself. The previous positive and negative sequence impedance components are different from the positive and negative sequence components of voltages and currents, which as explained by the geometrical interpretation provided in [38], are frequency components that follow a clockwise or counterclockwise circular trajectory, respectively, in the plane of the transformed reference frame (locus diagram).

Fig. 12(a) shows the frequency response of $Z_{\text{PCC}}^{\text{droop},+}$ and $Z_{\text{PCC}}^{\text{droop},-}$ for the setup parameters detailed in Table III. This figure contains the same information as Fig. 10, but represented in a more compact way. As shown, these two impedances, which are virtual impedances created by the droop control, have a large magnitude only in a narrow frequency range centered around 50 Hz (the fundamental frequency in the stationary reference frame), as expected from the low bandwidth of the droop control. The frequency region where the impedance $Z_{\text{PCC}}^{\text{droop},-}$ is larger than 1Ω causes noticeable frequency coupling. Frequencies outside this region do not create significant mirror frequency components.

Fig. 12(b) provides a comprehensive representation of how the PCC bus impedance progressively changes due to the control algorithms in a Droop-GFM converter. The stationary reference frame is selected for representation, hence the fundamental frequency is located at 50 Hz. An equivalent representation in the dq reference frame could be obtained by subtracting 50 Hz from the frequency axis. Fig. 12(c) is a zoomed representation of the low frequency range shown in Fig. 12(b), which is the frequency region where the droop control has a noticeable effect. The open-loop impedance of the PCC $Z_{\text{PCC}}^{\text{ol}}$ has a high value in the frequency region centered around the resonant frequency created by the LC filter of the GFM converter. Additionally, at low frequencies, the impedance of the PCC is low, but not zero, due to the equivalent series resistance of the converter and the power grid. The impedance of the PCC when the GFM converter is controlled by a voltage controller [37] $Z_{\text{PCC}}^{\text{droopless}}$ is significantly different to the impedance under open-loop control at low and high frequencies. The voltage controller damps the LC filter resonance, hence it reduces the impedance around the resonant frequency. The impedance of the PCC bus at the fundamental frequency becomes zero, due to the resonant action of the voltage controller. At other intermediate frequencies, the impedance slightly increases compared to the open-loop impedance. This is an unavoidable tradeoff that results from Bode's sensitivity integral theorem [37]. Finally, when a droop control is enabled, the PCC impedance increases in a narrow frequency range around the fundamental frequency as indicated by Z_{PCC}^+ , and it also becomes asymmetric. To model the asymmetry, an additional transfer function is needed, Z_{PCC}^- , which is a negative sequence impedance. The negative sequence impedance is non zero only in a narrow frequency range around the fundamental frequency; hence, the asymmetry introduced by the droop control is limited to this narrow frequency range.

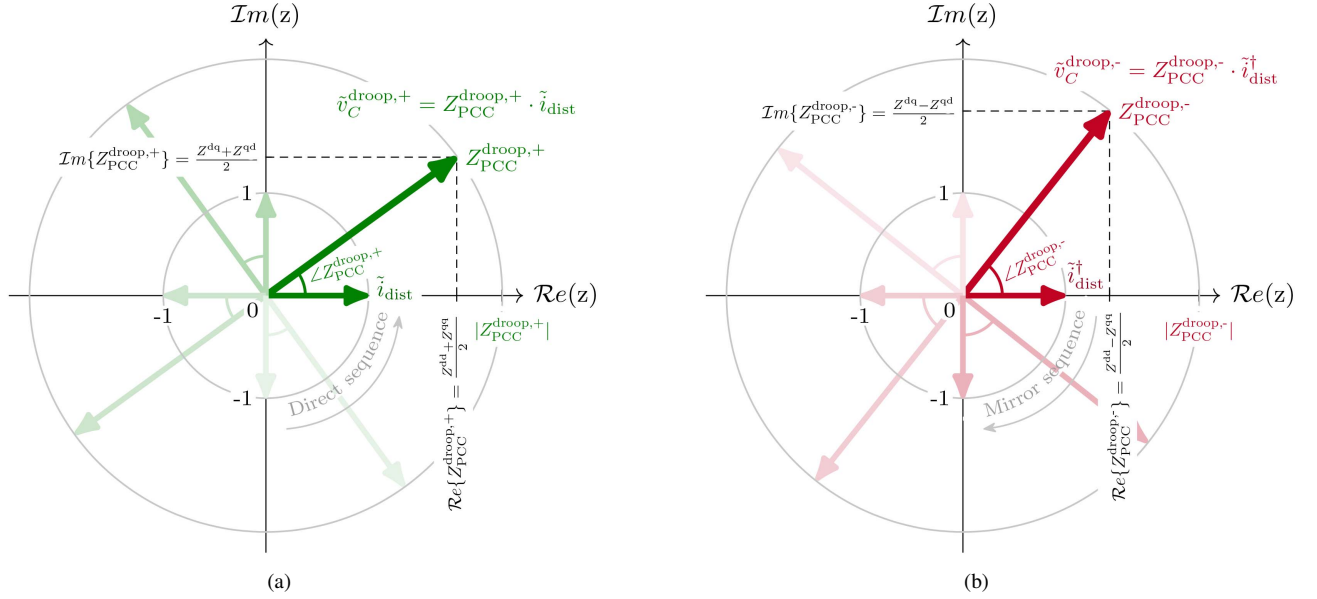


Fig. 11. Decomposition of a voltage disturbance, $\tilde{\mathbf{v}}_C^{\text{droop}}$, and a dq asymmetric impedance, $\mathbf{Z}_{\text{PCC}}^{\text{droop}}$, into their symmetrical (balanced) components, $\tilde{\mathbf{v}}_C^{\text{droop},+}$, $\tilde{\mathbf{v}}_C^{\text{droop},-}$, and, $\mathbf{Z}_{\text{PCC}}^{\text{droop},+}$, $\mathbf{Z}_{\text{PCC}}^{\text{droop},-}$, respectively. (a) Positive-sequence components. (b) Negative-sequence components.

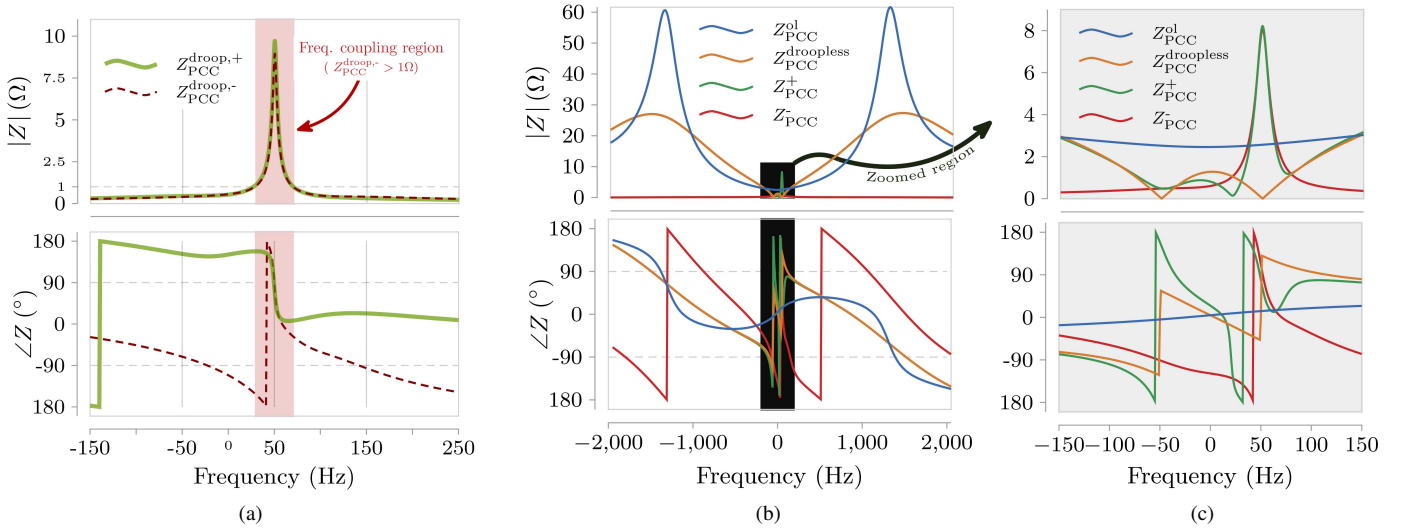


Fig. 12. Frequency response representations in the stationary reference frame (fundamental frequency is 50 Hz). (a) Impedance components created by the droop control, $\mathbf{Z}_{\text{PCC}}^{\text{droop},+}$, and, $\mathbf{Z}_{\text{PCC}}^{\text{droop},-}$, and frequency region where noticeable frequency coupling occurs ($\mathbf{Z}_{\text{PCC}}^{\text{droop},-} > 1 \Omega$). (b) PCC impedance when GFM converter does not incorporate feedback (open-loop operation), $\mathbf{Z}_{\text{PCC}}^{\text{ol}}$, when the GFM converter is controlled by a voltage controller with a resonant action at the fundamental frequency [37], $\mathbf{Z}_{\text{PCC}}^{\text{droopless}}$, and when the GFM converter is controlled by a voltage controller and a droop control, $\mathbf{Z}_{\text{PCC}}^+$, and, $\mathbf{Z}_{\text{PCC}}^-$. (c) Zoomed representation of the low frequency region in (b).

I. Effect of the Droop Controller Gains on the Stability

The impedance model developed (18) and its extension to dq -asymmetric systems (26), (28), and (29) can be used to assess the stability of a GFM converter with droop control for different droop gain configurations or power grid impedance values, in an analogous way to the small-signal stability analysis of synchronous generators. This type of study should not be confused with a large-signal (or transient) stability analysis. As explained in [13], the impedance of the PCC is a transfer function that can be used to determine the stability of a feedback controlled converter, because the stability is a

global property of the system, which does not only depends on the GFM converter, but on all impedances connected to the PCC bus. In the case of a Droop-GFM converter, the PCC bus exhibits an asymmetric impedance, which can be described by two transfer functions. For this asymmetric impedance to be stable, then both transfer functions have to be stable.

Fig. 13 shows the stability regions of a Droop-GFM converter as a function of the droop coefficients $\Delta\omega$ and ΔV and the equivalent impedance seen by the converter at its output $Z_{\text{th}} = R_{\text{th}} + j\omega L_{\text{th}}$, for the setup parameters detailed in Table III. The droop gains K_P and K_Q are calculated

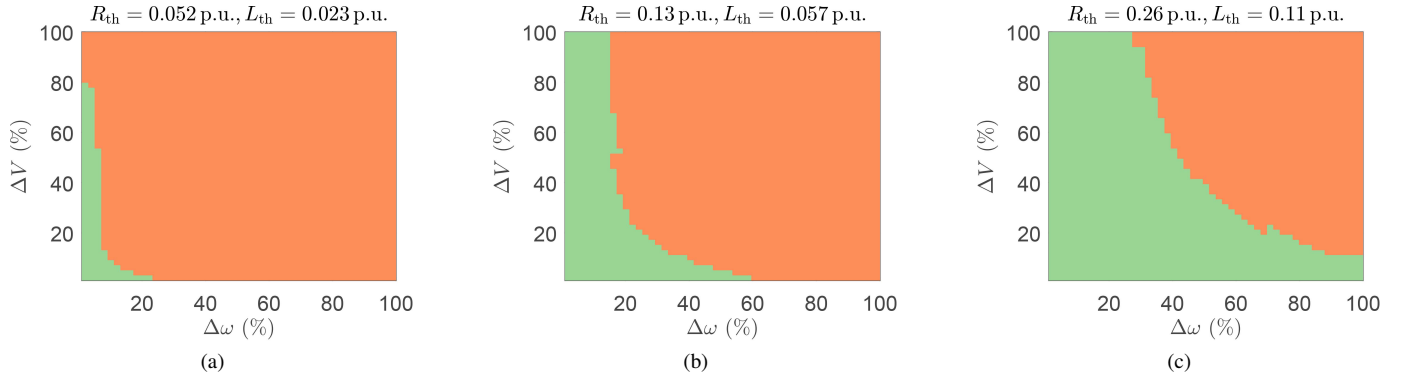


Fig. 13. Stability regions of a Droop-GFM converter as a function of droop coefficients, $\Delta\omega$, and, ΔV , and the equivalent impedance seen by the converter at its output $Z_{th} = R_{th} + j\omega L_{th}$ (green denotes a stable region, red denotes an unstable region).

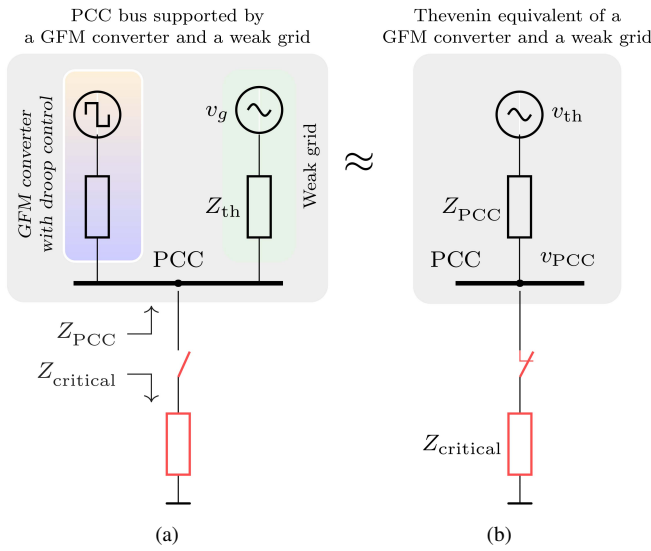


Fig. 14. Setup configuration where the impedance ratio criterion $Z_{PCC}/Z_{critical}$ is applied. (a) A critical load is fed from a Droop-GFM converter that operates in parallel with a power grid. (b) Simplified equivalent model.

according to the following expressions [39]:

$$K_P = \Delta\omega \cdot \omega_g / P_{base}, \quad (30)$$

$$K_Q = \Delta V \cdot V_0 / (2Q_{base}). \quad (31)$$

As shown in Fig. 13, the stability region diminishes when the impedance seen by the converter at its output Z_{th} becomes smaller. To calculate these stability regions, the proposed impedance model is evaluated once for each pair of values $\Delta\omega$ and ΔV represented. For each model evaluation, stability is determined by checking the poles of the resultant impedance model. If both impedances Z_{PCC}^+ and Z_{PCC}^- are stable transfer functions, then that pixel of the image is represented in green color. Conversely, if any of the transfer functions is unstable, then the pixel is represented in red color. The stability maps show that the droop control is more sensitive to an increase in the frequency droop coefficient $\Delta\omega$ compared to the magnitude droop coefficient ΔV .

J. Application of the Impedance Ratio Criterion

The impedance ratio criterion, also known as minor loop gain, is often used to predict the value of critical loads that cause a resonance, if connected to the output of a GFM converter (the PCC bus). Any element connected to the ac bus that is not part of a resonance perceives it as a frequency region where the ac bus impedance is high. In fact, at the resonant frequency, the impedance of the ac bus tends to infinite. The practical implication of such high impedance is that any small current distortion or noise close or at the resonant frequency results in a large voltage distortion and noise, that causes the ac bus to not meet voltage waveform quality requirements, such as total harmonic distortion or individual harmonic limits.

Mathematically, the impedance ratio criterion studies the stability of interconnecting two elements that are modeled by their Thevenin equivalent impedances. In the presented analysis, one of these two impedances is the Thevenin equivalent of a Droop-GFM converter connected to a weak power grid, which is denoted as Z_{PCC} in Fig. 14. The other impedance is a load, $Z_{critical}$. Unfortunately, the output impedance of the Droop-GFM converter is influenced by the value of $Z_{critical}$. As a result, the impedance ratio criterion does not provide an exact calculation method, as will be demonstrated later, because it overlooks this loading effect.

The scenario shown in Fig. 14(a) can be simplified to the scenario shown in Fig. 14(b), where the following relation holds:

$$v_{PCC} = \frac{1}{1 + \frac{Z_{PCC}}{Z_{critical}}} v_{th}. \quad (32)$$

From this relation the impedance ratio criterion, also known as minor loop gain, emerges. This stability criterion states that if there is a frequency $f_{critical}$ where

$$\frac{Z_{PCC}(f_{critical})}{Z_{critical}(f_{critical})} = -1, \quad (33)$$

then, the system becomes unstable. It is important to notice that (33) only needs to be met at a single frequency for the system to be unstable. Therefore, for every frequency value assigned to $f_{critical}$, a value of $Z_{critical}$ exists. Fig. 15 shows the values of $Z_{critical}$ that cause a resonance in the

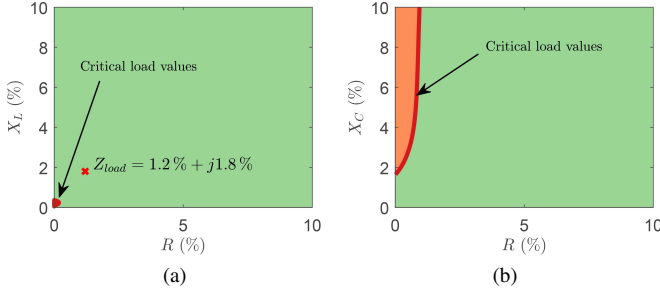


Fig. 15. Critical load values that cause a resonance in the system when connected to the PCC (red line) and stability regions (green denotes a stable region, red denotes an unstable region), as predicted by the impedance ratio criterion. The droop coefficients are $\Delta\omega = 2.5\%$ and $\Delta V = 5\%$ and the power grid impedance seen by the converter at its output is $Z_{th} = R_{th} + j\omega L_{th}$ where $R_{th} = 1.7\ \Omega$ (5.2%) and $L_{th} = 2.3\text{ mH}$ (2.3%). (a) Inductive load map. (b) Capacitive load map.

system according to the impedance ratio criterion, when the impedance $Z_{critical}$ is a capacitive or an inductive load.

It should be noticed that in the previous analysis, the small signal stability of the PCC bus voltage is studied using a single transfer function approximation of the PCC impedance. If the impedances are modeled using TITO dq systems (denoted in boldface), the impedance ratio becomes $\mathbf{Z}_{PCC}/\mathbf{Z}_{load}$. To study the stability of such TITO dq system, the GNSC is often used. The GNSC calculates the eigenvalues $\lambda_1(s)$ and $\lambda_2(s)$ of $\mathbf{Z}_{PCC}/\mathbf{Z}_{load}$ for the discrete frequency range from $-f_s/2$ to $f_s/2$. The resulting characteristic loci are plotted on the complex plane, which is shown in Fig. 16(a) for the case where the load is disconnected, and in Fig. 16(b) for the case where a load is connected. For the case where the load is connected, the Nyquist plot do not show any encirclements of the critical point in spite of this loading condition resulting in an unstable operation, as predicted by the proposed AIM [Fig. 17(a)] and experimentally verified in the following section. The reason is that the GNSC calculates the stability, assuming that the output impedance of the GFM converter does not change when a load is connected. However, it is known in the literature that the loads connected to a GFM converter affect its output impedance [8]. This is particularly relevant in the case of grid-connected operation, where the impedance seen by the GFM converter is very small and can cause a significant loading condition.

A limitation of the impedance ratio criterion, which is not always clearly stated, is that it assumes that the impedance Z_{PCC} is not affected by the impedance $Z_{critical}$. However, the value of the Thevenin equivalent that models the GFM converter and weak power grid shown in Fig. 14(a) changes when the switch is closed and the load $Z_{critical}$ is connected [13]. This effect is most significant for small values of $Z_{critical}$.

An alternative approach to overcome the previous limitation of the minor loop criterion is to include the loading effect of $Z_{critical}$ in the proposed AIM of the PCC bus. To this end, Z_{th} is replaced by the parallel equivalent of Z_{th} and $Z_{critical}$. When this change is performed, stability can be directly assessed by determining if the AIM for the PCC bus is a stable impedance. The new stability regions are shown

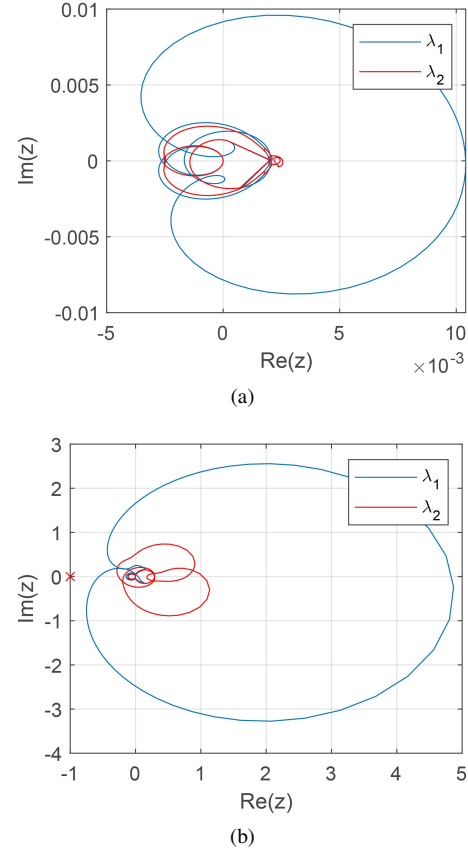


Fig. 16. Nyquist plot of the eigenvalues of $\mathbf{Z}_{PCC}/\mathbf{Z}_{load}$. Grid impedance is $R_g = 8\%$ and $L_g = 5\%$. (a) Load is disconnected ($R_{load} = 1000\%$ and $L_{load} = 1000\%$) (b) Load is connected ($R_{load} = 1.2\%$ and $L_{load} = 1.8\%$).

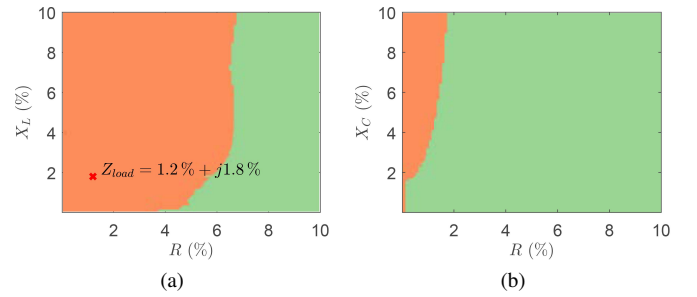


Fig. 17. Stability regions for different loading conditions, as predicted by the proposed impedance model (green denotes a stable region, red denotes an unstable region). The droop coefficients are $\Delta\omega = 2.5\%$ and $\Delta V = 5\%$ and the power grid impedance seen by the converter at its output is $Z_{th} = R_{th} + j\omega L_{th}$, where $R_{th} = 1.7\ \Omega$ (5.2%) and $L_{th} = 2.3\text{ mH}$ (2.3%). (a) Inductive load map. (b) Capacitive load map.

in Fig. 17. The actual stable region for inductive loads with a small resistive component is smaller than predicted by the impedance ratio criterion (Fig. 15). For capacitive loads, the stable region is comparable to the stable region delimited by the minor loop criterion, cf. Fig. 15(b).

III. RELATION BETWEEN VIRTUAL IMPEDANCE LOOPS AND DROOP CONTROL IN GFM CONVERTERS

The linear model developed for the droop control shown in Fig. 6 resembles the structure of a virtual impedance (VI) loop. In this model, the reference of the voltage controller v_C^{lin*}

TABLE III
SETUP PARAMETERS OF THE GFM CONVERTER

VSC (base reference)	Base power	S_{base}	5 kVA
	Base voltage	V_{base}	400 V
	Base frequency	f_{base}	50 Hz
	Base impedance	Z_{base}	32 Ω
	Switching frequency	f_{sw}	5.12 kHz
	Sampling frequency	f_s	5.12 kHz
	DC bus voltage	v_{dc}	700 V _{DC}
LC filter	Filter inductance	L	0.02 p.u.
	Filter capacitance	C	0.1 p.u.
Droop	Frequency droop	$\Delta\omega$	0.025 p.u.
	Amplitude droop	ΔV	0.05 p.u.

is modified by multiplying an output current disturbance $\tilde{i}_o$ by a gain $(-jK_{\text{droop}})$, similarly to a VI loop. For this reason, the impedance term associated to the droop control $Z_{\text{PCC}}^{\text{droop}}$ can be understood as a virtual impedance term that the droop control creates to improve the power sharing characteristics of the GFM converter.

Contrarily to a traditional VI, this impedance term has two characteristics which makes it special. First, the impedance is asymmetric in the dq reference frame, which permits independently tuning the response of the GFM converter to active and reactive power disturbances. Second, the impedance is mainly concentrated around the fundamental frequency. By providing a virtual impedance that is zero for all frequencies other than the fundamental frequency, the voltage source characteristics of the GFM converter are not degraded. This means that the capability of the GFM converter to supply distorted currents to nonlinear loads is not hindered, in spite of being able to have a relatively high output impedance at the fundamental frequency.

To obtain the combined effect of a virtual impedance loop and a droop control, the term $-jK_{\text{droop}}$ in Fig. 6 can be replaced by $-jK_{\text{droop}} + Z_{\text{VI}}$, where Z_{VI} is the desired virtual impedance term. However, it is not the objective of this article to study how virtual impedances affect the impedance model of GFM converters.

IV. EXPERIMENTAL RESULTS

This section presents the experimental tests. The experimental results are carried out in a 5-kW VSC with a 700-V dc bus v_{dc} that forms a 400-V line-to-line 50-Hz three-phase power grid. The GFM converter incorporates a P - f Q - V droop control and a voltage controller with a reference-tracking bandwidth of 300 Hz and a disturbance rejection bandwidth located at both sequences of the fundamental output frequency [37]. Both the VSC switching frequency and the sampling frequency of the control algorithms are 5.12 kHz. The experimental impedance measurements are obtained using an additional VSC with the same topology that operates as a current source that injects a current disturbance at the PCC. This auxiliary converter also operates at a switching frequency of 5.12 kHz and it uses a coupling impedance with a high value (300 Ω) so as to not cause a significant additional loading at

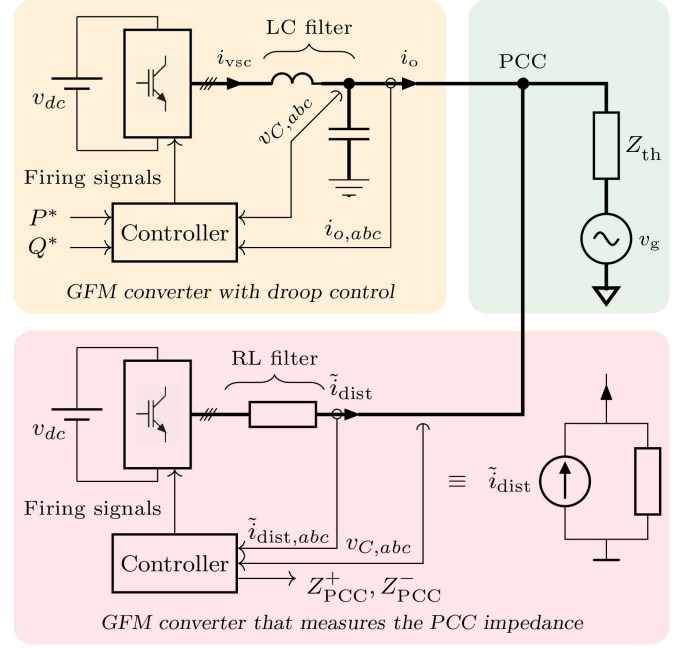


Fig. 18. Diagram of the experimental setup.

the PCC bus. The injected current $\tilde{i}_{\text{dist}}$ has a spread spectrum (similar to noise) and a root mean square (RMS) value of 1 A. To measure the PCC impedance in the low frequency range, the auxiliary converter senses the current $\tilde{i}_{\text{dist}}$ and the PCC voltage using an antialiasing filter and a sampling frequency of 512 Hz. Fig. 18 shows a simplified block diagram of the setup. Table III details the experimental setup parameters associated to the GFM converter.

Five tests are carried out to assess the agreement between the developed AIM [(18), (19), and (24)] and the measured impedance in an ac bus supported by a Droop-GFM converter connected to a weak power grid.

The first test changes the impedance seen by the GFM converter, Z_{th} , so as to alter its loading condition. To achieve such a loading condition change, the impedance Z_{th} is reduced from $R_{\text{th}} = 8\%$ and $L_{\text{th}} = 5\%$ to $R_{\text{th}} = 1\%$, $L_{\text{th}} = 1\%$. This reduction in Z_{th} is equivalent to connecting a load to the PCC bus of value $R_{\text{critical}} = 1.2\%$ and $L_{\text{critical}} = 1.8\%$. This load value is indicated by a red cross in the stability regions shown in Fig. 15(a) and Fig. 17(a). The oscilloscope capture shown in Fig. 19(a) displays the active power P and reactive power Q delivered by the Droop-GFM converter when the value of Z_{th} is reduced, which is marked by the arrows on top of the oscillogram. Prior to the loading condition change, the Droop-GFM converter delivers 2 kW of power to the power grid. When the impedance seen by the Droop-GFM converter decreases, $t = 45$ ms, the converter becomes unstable, and eventually the converter protections are tripped in $t = 65$ ms. Steady state P and Q fluctuations are mainly caused by grid voltage distortion and converter non linearities. When the converter trips, the output of the VSC is in high impedance because all the VSC power transistors are open. However, the LC filter remains connected to the grid, therefore, a small

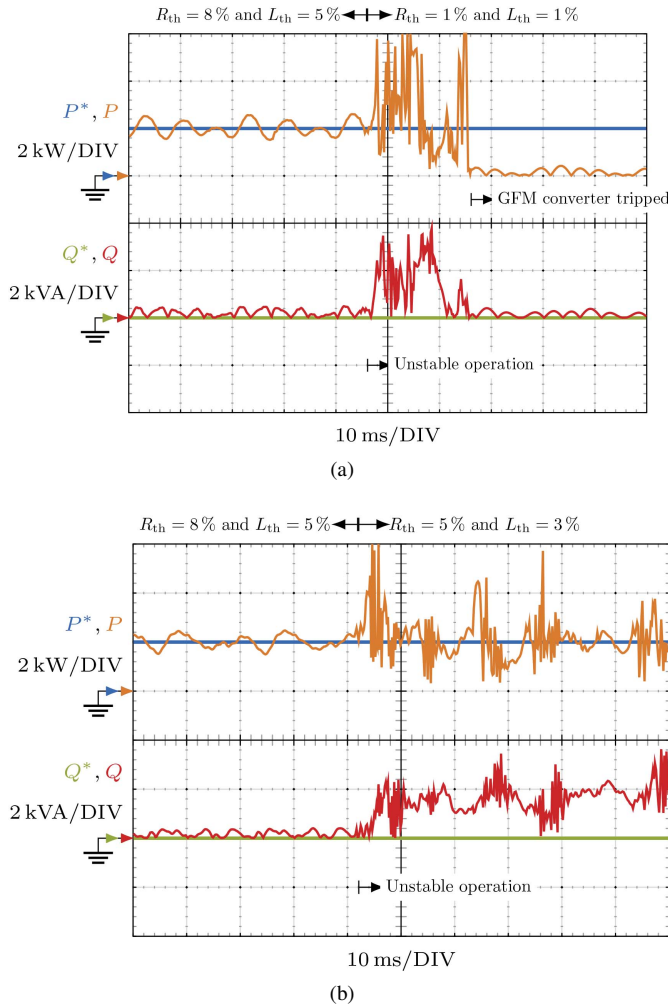


Fig. 19. Active power reference P^* , reactive power reference Q^* , active power P , and reactive power Q delivered by a GFM converter when a loading condition occurs. (a) The value of Z_{th} is reduced from a value of $R_{th} = 2.5 \Omega$ (8%) and $L_{th} = 5 \text{ mH}$ (5%) to a value of $R_{th} = 0.3 \Omega$ (1%), $L_{th} = 1 \text{ mH}$ (1%). (b) The value of Z_{th} is reduced from a value of $R_{th} = 2.5 \Omega$ (8%) and $L_{th} = 5 \text{ mH}$ (5%) to a value of $R_{th} = 1.6 \Omega$ (5%), $L_{th} = 3 \text{ mH}$ (3%).

power flow continues between the grid and the capacitor in the LC filter. This result agrees with the stability regions shown in Fig. 17(a), which indicate that the Droop-GFM converter becomes unstable when it sees at its output such a low impedance value. This test highlights the operation of the Droop-GFM converter under a loading condition where the impedance ratio criterion inaccurately predicts stable operation, as shown in Fig. 15(a). This discrepancy arises from the inherent approximations within the impedance ratio criterion. The next test, shown in Fig. 19(b), changes the grid impedance, Z_{th} , from a value of $R_{th} = 8\%$ and $L_{th} = 5\%$ to a value of $R_{th} = 5\%$ and $L_{th} = 3\%$. As expected from the stability analysis presented in Fig. 17(a), an unstable response is obtained. In Fig. 19(b), after the grid impedance value is changed, the VSC continues operating for a longer time, compared to Fig. 19(a), before experiencing an overcurrent condition and tripping. This response is in accordance with a greater damping provided by the grid impedance after the

loading condition change ($R_{th} = 5\%$), compared to Fig. 19(a) ($R_{th} = 1\%$).

The third test compares the experimentally measured PCC bus impedance to the theoretical impedance values provided by the proposed AIM. Fig. 20 illustrates the measured symmetrical components of the PCC bus impedance, $Z_{PCC,measured}^+$ and $Z_{PCC,measured}^-$, across the low-frequency range under three different operating conditions. The theoretical impedance values, $Z_{PCC,theoretical}^+$ and $Z_{PCC,theoretical}^-$, predicted by the proposed AIM for the same operating conditions, are overlaid using thick traces for comparison. The first measurement [Fig. 20(a)] corresponds to a scenario where the power grid impedance value is $R_{th} = 10 \Omega$ (31%) and $L_{th} = 10 \text{ mH}$ (10%), and the droop coefficients are $\Delta\omega = 2.5\%$, $\Delta V = 5\%$. As shown, the influence of the droop controller is limited to a narrow bandwidth centered around the fundamental frequency. As a result, the impedance of the PCC at the fundamental frequency (+50 Hz) significantly increases, compared to the impedance value at the corresponding negative sequence frequency component (-50 Hz). The phase of the measured negative sequence impedance $Z_{PCC,measured}^-$ significantly deviated from the theoretical trace for values outside the narrow bandwidth where the droop control has effect. The reason is that for values where $Z_{PCC} \approx 0$, the phase cannot be easily measured, for the same reason that the phase of the complex value zero is not determined.

Fig. 20(b) corresponds to a scenario where the droop coefficients are increased by a factor of two compared to Fig. 20(a). The new droop coefficients are $\Delta\omega = 5\%$, $\Delta V = 10\%$. The rest of the setup parameters remain the same. Increasing the droop coefficients results in a broader bandwidth for the positive- and negative-sequence impedances around the fundamental frequency (+50 Hz). Additionally, the impedance magnitude at this frequency also shows a moderate increase.

Fig. 20(c) corresponds to a scenario where the power grid impedance significantly decreases compared to Fig. 20(a) and Fig. 20(b). The new power grid impedance values are $R_{th} = 2.5 \Omega$ (8%) and $L_{th} = 5 \text{ mH}$ (5%). The rest of the setup parameters remain the same as in Fig. 20(a). Contrarily to a change in the droop coefficients, which only affect the impedance in a narrow frequency region, this change reduces the PCC impedance at all frequencies.

Finally, in order to experimentally test the frequency coupling effects introduced by the droop control, the following two experiments have been carried out: The first experiment uses the auxiliary converter to inject a current disturbance $\tilde{i}_{dist}$ with a frequency component of 49 Hz into the PCC and measures the voltage at the PCC, instead of calculating the PCC impedance. The 49-Hz current disturbance component is located within the narrow bandwidth of the droop control, therefore, it produces a voltage disturbance at 49 Hz with an amplitude determined by the value of the positive sequence impedance of the PCC at such frequency, $Z_{PCC}^+(49 \text{ Hz})$, and a voltage disturbance at its mirror frequency (51 Hz) with an amplitude determined by the value of the negative sequence impedance of the PCC at this other frequency, $Z_{PCC}^-(51 \text{ Hz})$. For the setup parameters used, the magnitude of both the positive and negative sequence impedances, is approximately

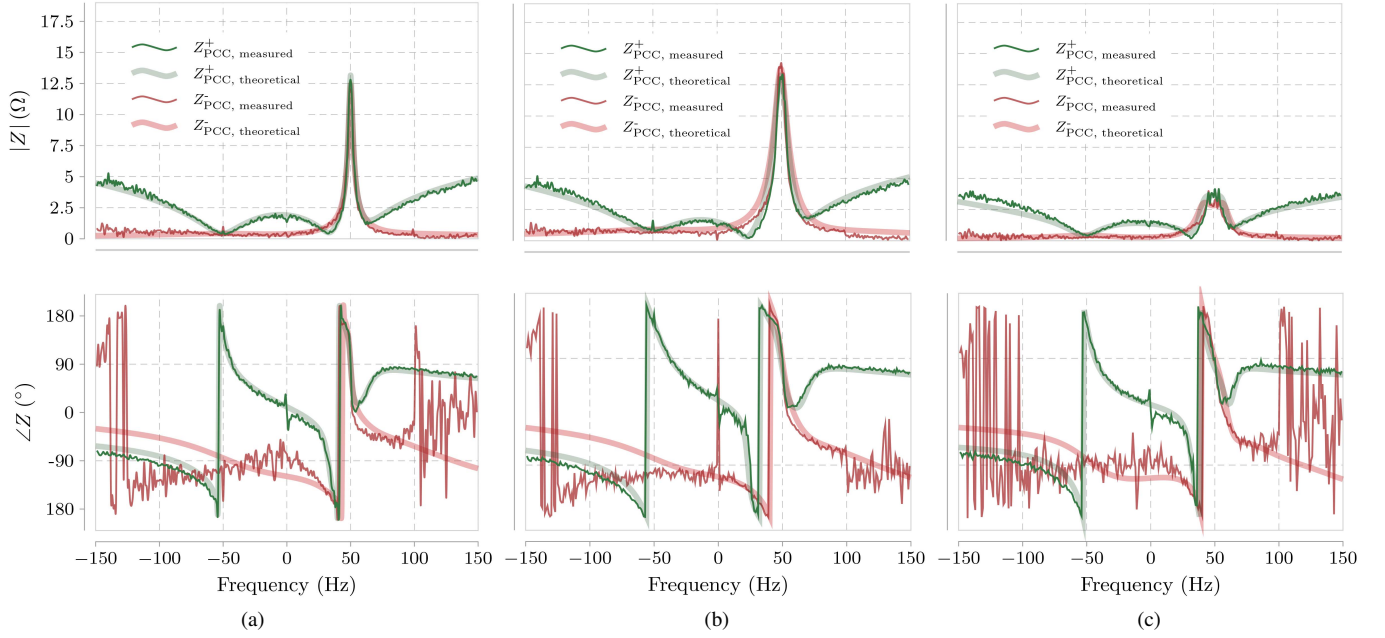


Fig. 20. Impedance of an ac bus supported by a Droop-GFM converter and a weak power grid. Experimentally measured symmetrical components, $Z_{PCC,measured}^+$ and $Z_{PCC,measured}^-$, and symmetrical components predicted by the AIM, $Z_{PCC,theoretical}^+$ and $Z_{PCC,theoretical}^-$, for different values of droop coefficients $\Delta\omega$ and ΔV and power grid impedance $Z_{th} = R_{th} + j\omega L_{th}$ (a) Nominal droop coefficients ($\Delta\omega = 2.5\%$, $\Delta V = 5\%$), and nominal power grid impedance [$R_{th} = 10\ \Omega$ (31%), $L_{th} = 10\text{ mH}$ (10%)]. (b) Increased droop coefficients ($\Delta\omega = 5\%$, $\Delta V = 10\%$) and nominal power grid impedance [$R_{th} = 10\ \Omega$ (31%) and $L_{th} = 10\text{ mH}$ (10%)]. (c) Nominal droop coefficients $\Delta\omega = 2.5\%$, $\Delta V = 5\%$ and decreased power grid impedance [$R_{th} = 2.5\ \Omega$ (8%) and $L_{th} = 5\text{ mH}$ (5%)].

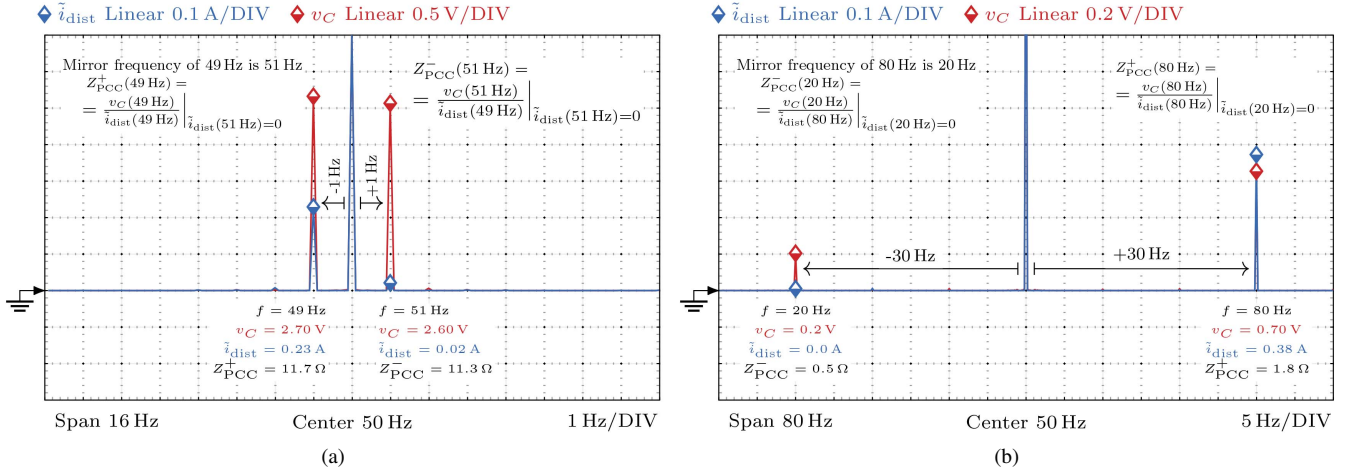


Fig. 21. Spectrum of the measured output voltage v_c and of the current disturbance $\tilde{i}_{dist}$. (a) The current disturbance $\tilde{i}_{dist}$ has a frequency component of 49 Hz. (b) The current disturbance $\tilde{i}_{dist}$ has a frequency component of 80 Hz.

12 Ω , as illustrated in Fig. 20(a). The spectrum of the measured output voltage and of the current disturbance are shown in Fig. 21(a). In addition to the frequency components previously described, $\tilde{i}_{dist}$ also contains a large frequency component at 50 Hz and a very small component at 51 Hz. These two frequency components of $\tilde{i}_{dist}$ appear because the auxiliary converter used to inject the 49 Hz current disturbance is not an ideal current source; therefore, the presence of frequency components in the PCC voltage result in additional unwanted current components in $\tilde{i}_{dist}$.

The second experiment repeats the previous measurement,

but instead of injecting a current component at 49 Hz (withing the bandwidth of the droop control), it injects a current components at 80 Hz (outside the bandwidth of the droop control). Now, the frequency coupling effect is small, as expected from the small value of the negative sequence impedance at the mirror frequency of 80 Hz (20 Hz). For the setup parameters used, the magnitude of the positive sequence impedance at 80 Hz, is approximately 2 Ω and the magnitude of the negative sequence impedance at 20 Hz is approximately 0 Ω , as illustrated in Fig. 20(a). Fig. 21(b) shows the spectrum of the measured output voltage and of the current disturbance.

V. CONCLUSION

This article presented an asymmetric impedance model (AIM) for a grid-forming (GFM) converter with droop control (Droop-GFM converter). The model is presented in the $\alpha\beta$ frame and comprises two balanced impedances. This two-impedance model offers a more clear physical interpretation, compared to models that use an impedance matrix in the dq frame. The presented impedance model accounts for the loading effect caused by changes in the power grid impedance or by changes in the number of loads in operation. The presented model also accurately describes the frequency coupling effects (often denoted as mirror frequency problem in the literature) that arise due to the operation of the droop control. Finally, the developed model serves as a more accurate alternative, compared to the minor-loop criterion, and a more simple alternative, compared to the Generalized Nyquist Stability Criterion, to study the stability of Droop-GFM converters for different operating conditions and droop coefficients. The proposed AIM has been experimentally tested using a Droop-GFM converter, to validate the theory in three different operating conditions, including: weak power grid operation, a change in the droop coefficients, and a change in the power grid impedance. Experimental impedance measurements in this operating conditions have been compared to the theoretical model. Experimental results are also presented to validate the frequency coupling effects predicted by the proposed impedance model.

APPENDIX TRANSFER MATRIX MODELS

This appendix details the steps carried out to convert the transfer function $G_{rt,dq}(z)$, $G_{cd,dq}(z)$, and $Y_{th,dq}(z)$ into transfer matrix models.

The transformation used to convert from complex notation to vector notation [17] is summarized below:

$$\text{Complex notation : } \underbrace{y_{dq}}_{y_{dq}=y_d+jy_q} = G_{dq}(z) \underbrace{x_{dq}}_{x_{dq}=x_d+jx_q} \quad (34)$$

$$\text{Vector notation : } \underbrace{\begin{bmatrix} y_d \\ y_q \end{bmatrix}}_y = \underbrace{\begin{bmatrix} G_{dd}(z) & -G_{dq}(z) \\ G_{dq}(z) & G_{qq}(z) \end{bmatrix}}_G \underbrace{\begin{bmatrix} x_d \\ x_q \end{bmatrix}}_x, \quad (35)$$

where

$$G^{dd}(z) = \text{Real}\{G_{dq}(z)\} = \frac{G_{dq}(z) + G_{dq}^\dagger(z^\dagger)}{2}, \quad (36)$$

$$G^{dq}(z) = \text{Imag}\{G_{dq}(z)\} = \frac{G_{dq}(z) - G_{dq}^\dagger(z^\dagger)}{2j}, \quad (37)$$

$$G^{qq}(z) = G^{dd}(z) \text{ and } G^{qd}(z) = G^{dq}(z). \quad (38)$$

As a result, the transfer function matrix $\mathbf{G}_{rt}(z)$ is constructed as follows:

$$\mathbf{G}_{rt}(z) = \begin{bmatrix} G_{rt}^{dd}(z) & -G_{rt}^{dq}(z) \\ G_{rt}^{dq}(z) & G_{rt}^{qq}(z) \end{bmatrix}, \quad (39)$$

where

$$G_{rt}^{dd}(z) = \frac{G_{rt,dq}(z) + G_{rt,dq}^\dagger(z^\dagger)}{2}, \quad (40)$$

$$G_{rt}^{dq}(z) = \frac{G_{rt,dq}(z) - G_{rt,dq}^\dagger(z^\dagger)}{2j}, \quad (41)$$

$$G_{rt}^{qq}(z) = G_{rt}^{dd}(z) \text{ and } G_{rt}^{qd}(z) = G_{rt}^{dq}(z). \quad (42)$$

The other transfer matrices $\mathbf{G}_{cd}(z)$ and $\mathbf{Y}_{th}(z)$ are obtained in an identical manner.

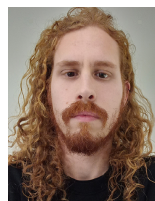
REFERENCES

- [1] "IEEE Guide to Using IEEE Std 1547 (TM) for Interconnection of Energy Storage Distributed Energy Resources with Electric Power Systems". In: *IEEE P1547.9, February 2022* (2022), pp. 1–84. doi: [10.1109/IEEEESTD.2022.9849493](https://doi.org/10.1109/IEEEESTD.2022.9849493).
- [2] "IEEE Standard for Interconnection and Interoperability of Distributed Energy Resources with Associated Electric Power Systems Interfaces". In: *IEEE Std 1547-2018 (Revision of IEEE Std 1547-2003)* (2018), pp. 1–138. doi: [10.1109/IEEEESTD.2018.8332112](https://doi.org/10.1109/IEEEESTD.2018.8332112).
- [3] R. Middlebrook. "Input Filter Considerations in Design and Application of Switching Regulators". In: *IEEE Industry Applications Society Annual Meeting*. 1976, pp. 366–382. doi: [10.1109/PESC.1993.472019](https://doi.org/10.1109/PESC.1993.472019).
- [4] A. Riccobono and E. Santi. "Comprehensive Review of Stability Criteria for DC Power Distribution Systems". In: *IEEE Trans. Ind. Appl.* 50.5 (2014), pp. 3525–3535. doi: [10.1109/TIA.2014.2309800](https://doi.org/10.1109/TIA.2014.2309800).
- [5] M. Belkhaty. "Stability Criteria for AC Power Systems With Regulated Loads". Purdue University, Oct. 1997.
- [6] J. Sun. "Impedance-Based Stability Criterion for Grid-Connected Inverters". In: *IEEE Trans. Power Electron.* 26.11 (2011), pp. 3075–3078. doi: [10.1109/TPEL.2011.2136439](https://doi.org/10.1109/TPEL.2011.2136439).
- [7] Z. Zou, X. Tang, X. Wang, Z. Wang, W. Chen, G. Buticchi, and M. Liserre. "Modeling and Control of a Two-Bus System With Grid-Forming and Grid-Following Converters". In: *IEEE Journal of Emerging and Selected Topics in Power Electronics* 10.6 (2022), pp. 7133–7149. doi: [10.1109/JESTPE.2022.3182366](https://doi.org/10.1109/JESTPE.2022.3182366).
- [8] L. Corradini, P. Mattavelli, M. Corradin, and F. Polo. "Analysis of Parallel Operation of Uninterruptible Power Supplies Loaded Through Long Wiring Cables". In: *IEEE Trans. on Power Electron.* 25.4 (2010), pp. 1046–1054. doi: [10.1109/TPEL.2009.2031178](https://doi.org/10.1109/TPEL.2009.2031178).
- [9] S. Mortazavian and Y. A.-R. I. Mohamed. "Investigation and Enhancement of Stability in Grid-Connected Converter-Based Distributed Generation Units With Dynamic Loads". In: *IEEE Access* 8 (2020), pp. 93426–93443. doi: [10.1109/ACCESS.2020.2991415](https://doi.org/10.1109/ACCESS.2020.2991415).
- [10] A. Kahrobaei and Y. A.-R. I. Mohamed. "Analysis and Mitigation of Low-Frequency Instabilities in Autonomous Medium-Voltage Converter-Based Microgrids With Dynamic Loads". In: *IEEE Trans. Ind. Electron.* 61.4 (2014), pp. 1643–1658. doi: [10.1109/TIE.2013.2264790](https://doi.org/10.1109/TIE.2013.2264790).
- [11] C. Gómez-Aleixandre, Á. Navarro-Rodríguez, C. Blanco, G. Villa, A. Suárez, and P. García. "Analysis of a Complex-Valued Droop Method for Complete Steady-State Frequency Compensation Using dq-Decomposition". In: *IEEE Trans. Ind. Appl.* 59.6 (2023), pp. 7657–7668. doi: [10.1109/TIA.2023.3300259](https://doi.org/10.1109/TIA.2023.3300259).
- [12] C. Blanco, D. Reigosa, J. C. Vasquez, J. M. Guerrero, and F. Briz. "Virtual Admittance Loop for Voltage Harmonic Compensation in Microgrids". In: *IEEE Trans. Ind. Appl.* 52.4 (2016), pp. 3348–3356. doi: [10.1109/TIA.2016.2547362](https://doi.org/10.1109/TIA.2016.2547362).
- [13] D. Pérez-Estévez, D. Ríos-Castro, and J. Doval-Gandoy. "Sensitivity-Based Impedance Modeling of AC Voltage-Controlled Converters". In: *IEEE Transactions on Industry Applications* (2024), pp. 1–13. doi: [10.1109/TIA.2024.3419668](https://doi.org/10.1109/TIA.2024.3419668).
- [14] S. Shah, P. Koralewicz, V. Gevorgian, H. Liu, and J. Fu. "Impedance Methods for Analyzing Stability Impacts of Inverter-Based Resources: Stability Analysis Tools for Modern Power Systems". In: *IEEE Electrification Magazine* 9.1 (2021), pp. 53–65. doi: [10.1109/MELE.2020.3047166](https://doi.org/10.1109/MELE.2020.3047166).
- [15] J. Sun. "Frequency-Domain Stability Criteria for Converter-Based Power Systems". In: *IEEE Open Journal of Power Electronics* 3 (2022), pp. 222–254. doi: [10.1109/OJPEL.2022.3155568](https://doi.org/10.1109/OJPEL.2022.3155568).

- [16] D. Wang and X. Yuan. "Interaction analysis between induction motor loads and STATCOM in weak grid using induction machine model". In: *Journal of Modern Power Systems and Clean Energy* 6.1 (2018), pp. 158–167. doi: [10.1007/s40565-017-0316-x](https://doi.org/10.1007/s40565-017-0316-x).
- [17] L. Harnefors. "Modeling of Three-Phase Dynamic Systems Using Complex Transfer Functions and Transfer Matrices". In: *IEEE Transactions on Industrial Electronics* 54.4 (2007), pp. 2239–2248. doi: [10.1109/TIE.2007.894769](https://doi.org/10.1109/TIE.2007.894769).
- [18] G. Paap. "Symmetrical components in the time domain and their application to power network calculations". In: *IEEE Transactions on Power Systems* 15.2 (2000), pp. 522–528. doi: [10.1109/59.867135](https://doi.org/10.1109/59.867135).
- [19] M. Kazem Bakhshizadeh, X. Wang, F. Blaabjerg, J. Hjerrild, L. Kocewiak, C. L. Bak, and B. Hesselbaek. "Couplings in Phase Domain Impedance Modeling of Grid-Connected Converters". In: *IEEE Trans. Power Electron.* 31.10 (2016), pp. 6792–6796. doi: [10.1109/TPEL.2016.2542244](https://doi.org/10.1109/TPEL.2016.2542244).
- [20] S. Shah and L. Parsa. "Sequence domain transfer matrix model of three-phase voltage source converters". In: *2016 IEEE Power and Energy Society General Meeting (PESGM)*. 2016, pp. 1–5. doi: [10.1109/PESGM.2016.7742009](https://doi.org/10.1109/PESGM.2016.7742009).
- [21] S. Shah and L. Parsa. "Impedance Modeling of Three-Phase Voltage Source Converters in DQ, Sequence, and Phasor Domains". In: *IEEE Transactions on Energy Conversion* 32.3 (2017), pp. 1139–1150. doi: [10.1109/TEC.2017.2698202](https://doi.org/10.1109/TEC.2017.2698202).
- [22] W. Ren and E. Larsen. "A Refined Frequency Scan Approach to Sub-Synchronous Control Interaction (SSCI) Study of Wind Farms". In: *IEEE Transactions on Power Systems* 31.5 (2016), pp. 3904–3912. doi: [10.1109/TPWRS.2015.2501543](https://doi.org/10.1109/TPWRS.2015.2501543).
- [23] A. Rygg, M. Molinas, C. Zhang, and X. Cai. "A Modified Sequence-Domain Impedance Definition and Its Equivalence to the dq-Domain Impedance Definition for the Stability Analysis of AC Power Electronic Systems". In: *IEEE Journal of Emerging and Selected Topics in Power Electronics* 4.4 (2016), pp. 1383–1396. doi: [10.1109/JESTPE.2016.2588733](https://doi.org/10.1109/JESTPE.2016.2588733).
- [24] F. Cavazzana, A. Khodamoradi, H. Abedini, and P. Mattavelli. "Analysis of an Impedance Modeling Approach for Droop-Controlled Inverters in System DQ Frame". In: *IEEE Energy Conversion Congress and Exposition (ECCE)*. 2019, pp. 5576–5583. doi: [10.1109/ECCE.2019.8913275](https://doi.org/10.1109/ECCE.2019.8913275).
- [25] R. Strunk and A. Mertens. "Impedance-Based Stability Analysis of Droop-Controlled AC Microgrids with the Generalized Nyquist Criterion". In: *European Conference on Power Electronics and Applications (EPE'23 ECCE Europe)*. 2023, pp. 1–10. doi: [10.23919/EPE23ECCEEurope58414.2023.10264611](https://doi.org/10.23919/EPE23ECCEEurope58414.2023.10264611).
- [26] R. Strunk, P. Sourkounis, and A. Mertens. "Impedance-Based Stability Analysis of Grid-Forming Inverters with Virtual Impedance or Angle Droop for Improved Robustness". In: *International Symposium on Power Electronics for Distributed Generation Systems (PEDG)*. 2024, pp. 1–6. doi: [10.1109/PEDG61800.2024.10667448](https://doi.org/10.1109/PEDG61800.2024.10667448).
- [27] J. Kwon, X. Wang, F. Blaabjerg, C. L. Bak, V.-S. Sulara, and C. Busca. "Harmonic Interaction Analysis in a Grid-Connected Converter Using Harmonic State-Space (HSS) Modeling". In: *IEEE Trans. Power Electron.* 32.9 (2017), pp. 6823–6835. doi: [10.1109/TPEL.2016.2625802](https://doi.org/10.1109/TPEL.2016.2625802).
- [28] I. Vieto, X. Du, H. Nian, and J. Sun. "Frequency-domain coupling in two-level VSC small-signal dynamics". In: *2017 IEEE 18th Workshop on Control and Modeling for Power Electronics (COMPEL)*. 2017, pp. 1–8. doi: [10.1109/COMPEL.2017.8013379](https://doi.org/10.1109/COMPEL.2017.8013379).
- [29] X. Wang and F. Blaabjerg. "Harmonic Stability in Power Electronic-Based Power Systems: Concept, Modeling, and Analysis". In: *IEEE Transactions on Smart Grid* 10.3 (2019), pp. 2858–2870. doi: [10.1109/TSG.2018.2812712](https://doi.org/10.1109/TSG.2018.2812712).
- [30] M. K. Bakhshizadeh, F. Blaabjerg, J. Hjerrild, X. Wang, L. Kocewiak, and C. L. Bak. "A Numerical Matrix-Based Method for Stability and Power Quality Studies Based on Harmonic Transfer Functions". In: *IEEE Journal of Emerging and Selected Topics in Power Electronics* 5.4 (2017), pp. 1542–1552. doi: [10.1109/JESTPE.2017.2742241](https://doi.org/10.1109/JESTPE.2017.2742241).
- [31] X. Wang, L. Harnefors, and F. Blaabjerg. "Unified Impedance Model of Grid-Connected Voltage-Source Converters". In: *IEEE Trans. Power Electron.* 33.2 (2018), pp. 1775–1787. doi: [10.1109/TPEL.2017.2684906](https://doi.org/10.1109/TPEL.2017.2684906).
- [32] Y. Liao, X. Wang, and F. Blaabjerg. "Passivity-Based Analysis and Design of Linear Voltage Controllers For Voltage-Source Converters". In: *IEEE Open Journal of the Industrial Electronics Society* 1 (2020), pp. 114–126. doi: [10.1109/OJIES.2020.3001406](https://doi.org/10.1109/OJIES.2020.3001406).
- [33] Q. Rong, P. Hu, L. Wang, Y. Li, Y. Yu, D. Wang, and Y. Cao. "Asymmetric Sampling Disturbance-Based Universal Impedance Measurement Method for Converters". In: *IEEE Trans. Power Electron.* 39.12 (2024), pp. 15457–15461. doi: [10.1109/TPEL.2024.3451403](https://doi.org/10.1109/TPEL.2024.3451403).
- [34] Z. Xie, W. Wu, Y. Chen, S. Cao, and Y. Xu. "Sequence-Admittance Measurement Method of Grid-Connected Inverter With Its Control System Disturbance". In: *IEEE Trans. Ind. Electron.* 70.8 (2023), pp. 8598–8602. doi: [10.1109/TIE.2022.3231305](https://doi.org/10.1109/TIE.2022.3231305).
- [35] D. Pérez-Estévez and J. Doval-Gandoy. "Method for estimating electrical parameters of a grid". EP3637574B1. Filed at the European Patent Office. 2024. doi: <https://register.epo.org/application?number=EP18382728&tab=main>.
- [36] D. Ríos-Castro, D. Pérez-Estévez, and J. Doval-Gandoy. "AC-Voltage Controller for Grid-Forming Converters". In: *IEEE Trans. Power Electron.* 38.4 (2023), pp. 4529–4543. doi: [10.1109/TPEL.2022.3233137](https://doi.org/10.1109/TPEL.2022.3233137).
- [37] D. Pérez-Estévez, J. Doval-Gandoy, and J. M. Guerrero. "AC-Voltage Harmonic Control for Stand-Alone and Weak-Grid-Tied Converter". In: *IEEE Trans. Ind. Appl.* 56.1 (2020), pp. 403–421. doi: [10.1109/TIA.2019.2942265](https://doi.org/10.1109/TIA.2019.2942265).
- [38] C. J. O'Rourke, M. M. Qasim, M. R. Overlin, and J. L. Kirtley. "A Geometric Interpretation of Reference Frames and Transformations: dq0, Clarke, and Park". In: *IEEE Transactions on Energy Conversion* 34.4 (2019), pp. 2070–2083. doi: [10.1109/TEC.2019.2941175](https://doi.org/10.1109/TEC.2019.2941175).
- [39] J. M. Guerrero, L. Hang, and J. Uceda. "Control of Distributed Uninterruptible Power Supply Systems". In: *IEEE Trans. Ind. Electron.* 55.8 (2008), pp. 2845–2859. doi: [10.1109/TIE.2008.924173](https://doi.org/10.1109/TIE.2008.924173).



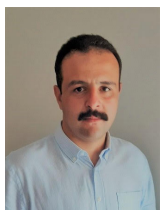
Diego Pérez-Estévez (Member, IEEE) received the M.Sc. and the Ph.D. degrees from the University of Vigo, Vigo, Spain, in 2014 and 2019, respectively. Since 2014, he has been with the Applied Power Electronics Technology Research Group. His research interests include control of grid-connected converters and distributed power generation systems.



Diego Ríos-Castro (Member, IEEE) received the B.Sc degree in electronic engineering and the M.Sc degree in telecommunication engineering from the University of Vigo, Vigo, Spain, in 2014 and 2020, respectively. He is a Ph.D student of the Applied Power Electronics Technology Research Group (APET), Universidade de Vigo. His research interests are in the areas of control of ac power converters.



Pablo Marino Fernández-Abraldes (Student Member, IEEE) received the B.Sc. degree in electronic engineering and the M.Sc. degree in industrial engineering from the University of Vigo, Vigo, Spain, in 2021 and 2023, respectively. He is currently working toward the Ph.D. degree in electrical engineering with Applied Power Electronics Technology Research Group, University of Vigo. His research interests include the areas of control of ac power converters.



Vitor Monteiro (Senior Member, IEEE) received the Ph.D. degree in Power Electronics and Energy Systems in 2016, and the M.Sc. in Industrial Electronics and Computer Engineering in 2010, both from the University of Minho, Portugal. Currently, he is an Assistant Professor at the Department of Industrial Electronics, School of Engineering, University of Minho, Portugal. His research activity has been developed at the Group of Energy and Power Electronics (GEPE), of the Centro ALGORITMI. He is chair of the IEEE PES Portugal Section Chapter,

and Member of IEEE IES, IEEE PELS, and IEEE PES. Vitor Monteiro has as main research interests the application of power electronics technologies to smart grids.



J. G. Pinto (Member, IEEE) was born in Guimarães, Portugal, in 1977. He received the degree in Industrial Electronics Engineering (2001), the MSc degree in Industrial Electronics Engineering (2004) and the PhD degree in Electronics and Computer Engineering (2012), all from the University of Minho, Portugal. He is, currently, an Assistant Professor in the Industrial Electronics Department, at the School of Engineering of the University of Minho. He is a Researcher at the Group of Energy and Power Electronics (GEPE) of the Centro AGORITMI of

the University of Minho and Integrated Researcher at the Intelligent Systems Associate Laboratory (LASI). His research interests relate with Power Electronics, Power Quality and Renewable Energy.



João L. Afonso (Senior Member, IEEE) is Full Professor of the Department of Industrial Electronics, University of Minho, Portugal. He received the B.Sc. and M.Sc. degrees in Electrical Engineering from the Federal University of Rio de Janeiro, Brazil, in 1986 and 1991, and the Ph.D. degree in Industrial Electronics from the University of Minho, in 2000. He has been with the Department of Industrial Electronics of the University of Minho since 1993. He is coordinator and researcher with the Group of Energy and Power Electronics (GEPE) of Centro

ALGORITMI. His research interests lie in the application of Power Electronics technologies to Power Quality, Renewable Energies, Electric Mobility, Smart Grids and Aerospace Engineering.



Jesús Doval-Gandoy (Member, IEEE) received the M.S. and Ph.D. degrees in electrical engineering from the Polytechnic University of Madrid, Madrid, Spain, and from the University of Vigo, Vigo, Spain, in 1991 and 1999 respectively. He is a Professor and the head of the Applied Power Electronics Technology Research Group (APET), University of Vigo. His research interests are in the areas of ac power conversion.